

Higher loop corrections to the current of created pairs in the lengthy electric pulse

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The Schwinger current, arising from vacuum breakdown in the presence of a strong constant electric pulse background, is well known and has been calculated in numerous studies. It originates from the amplification of zero-point fluctuations. However, we find that higher-loop corrections involve hidden redistributions of the level populations of charged particles and photons. These redistributions lead to corrections to the current, which become significant for sufficiently long pulses and ultimately modify Schwinger’s expression for the current.

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I. INTRODUCTION

In his seminal paper [1], Schwinger calculated the particle creation rate in QED under a constant electric field background, obtaining $w \sim E^2 e^2 \exp[-\pi m^2/eE]$. Subsequent works, such as those in [2–6], extended Schwinger’s calculations to other types of background electric fields. In the case of a spatially constant electric pulse background, it is straightforward to estimate the current arising from vacuum breakdown: the current along the field direction is given by $j \sim ewT$, where T represents the duration of the background pulse. This expression for the current has been derived in numerous studies [7–19] (see also [20–23]).

If the background field persists for a sufficiently long duration, one can expect that the created pairs, once accelerated, will radiate photons. These radiated photons can then decay into additional pairs of charged particles. Such processes are not forbidden in the presence of the background field. This raises a natural question: how can one estimate the effects of these processes on the current during vacuum breakdown? Will these new processes contribute significantly to the current, potentially altering Schwinger’s expression for the current? Alternatively, how long must the background field act for these effects to become significant?

While these questions are currently of academic interest given the achievable values of strong electric fields, they can still provide insights into other types of strong background fields, such as gravitational fields, which were likely present in the very early Universe (see, e.g., [20,24–30] for related discussions). In this context, our primary goal is to derive these effects from the first principles of quantum field theory, rather than merely providing estimates based on general physical arguments, which are not well established—especially for strong background fields, such as those where $m^2/e \leq E$.

The questions formulated above have been addressed in the context of a constant, eternal electric field in [21,22]. The results indicate that the phenomena under consideration are embedded within infrared loop effects, which arise in the presence of a strong background field. These effects indeed modify Schwinger’s expression for the current, which is derived from tree-level propagators. Specifically, the Schwinger current originates from the amplification of zero-point fluctuations. However, within loop corrections, one observes at least a redistribution of the population levels of higher energy states. This redistribution leads to a change in the current over sufficiently long observation times.

However, a constant eternal electric field background is not physically realizable in a laboratory and is, in fact, quite pathological. In particular, the tree-level current in such a background is strictly zero for a reasonable initial state (e.g., one that is Hadamard, time-translation invariant, and time-reversal invariant). However, in loop corrections, the current is nonzero and exhibits infrared divergences if the initial Cauchy surface is shifted to past infinity. The issue is that, even after introducing a UV cutoff, loop corrections remain infinite in the presence of such background fields when calculations are performed over the entire space-time.

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To regularize these divergences, one must introduce an initial Cauchy surface, which breaks the time-translation and time-reversal symmetries of both the constant eternal electric field and the initial state. This breaking of symmetry generates a nonzero current in the loops that grows with time. This observation suggests that the answer to the questions formulated above depends on the initial conditions, which motivates the consideration of a pulselike background rather than an eternal field.

In our previous paper [31], we performed similar calculations in the context of an electric pulse background. We demonstrated that, unlike the case of an eternal field, one-loop corrections to the current contribute nothing more than the UV renormalization of the tree-level expression. (In the present paper, we show that this occurs due to a subtle cancellation, which does not persist in higher loops.) However, corrections to the photon propagator exhibit growth with the pulse duration and may influence higher-loop corrections to the current. For this reason, in this paper, we investigate higher-loop corrections to the electric current and examine their effects. We observe that higher loops grow faster than expected and significantly alter the current, precisely due to the absence of the aforementioned cancellation.

Let us emphasize that large loop corrections are also frequently discussed in QED with other types of external fields (e.g., fields with different values of the Lorentz invariants for the field strength). For instance, the increase of loop contributions with field strength has been extensively studied in the context of the Ritus-Narozhny conjecture for constant crossed fields ($\mathbf{E} \perp \mathbf{H}$, $E = H$) [32–34], as well as in the case of a constant magnetic field [35]. For $E < H$, there is no vacuum breakdown, but nontrivial scattering processes—such as the decay of a photon into a pair of charged particles—are still allowed, and loop corrections to these processes grow with the intensity of the external field. However, in this work, we focus on large (time-growing) loop corrections that arise due to the change in the state of the theory caused by vacuum breakdown. To calculate loop corrections in the case of $E < H$, one can use the in-out Feynman diagrammatic technique, whereas for particle-creating backgrounds with $E > H$, the in-in Schwinger-Keldysh formalism must be applied.

Let us mention, however, certain qualitative similarities between the situation that we consider in this paper and the case of constant crossed fields. First, in the latter case, one-loop corrections do not share the same expansion parameter as higher-loop corrections [33,34]. Specifically, it might appear from the first two orders that the expansion parameter is $\alpha\chi^{1/3}$, where $\chi = \frac{e}{m^2} \sqrt{-(F_{\mu\nu}p^\nu)^2}$ and $\alpha = e^2/(4\pi)$. However, it was shown in [33] that higher-loop diagrams grow even faster, and the true expansion parameter should be $\alpha\chi^{2/3}$. Second, those higher-loop corrections are leading in χ , which include photon propagator dressed with scalar

bubbles (see [36] for a recent review). In this paper we will demonstrate that, in the electric pulse background, the two-loop correction to the current (where the photon line dressed with a scalar bubble) is also growing faster with the pulse duration T as compared to the first-loop correction.

II. SETUP OF THE PROBLEM

A. Action and modes

In this paper we consider scalar electrodynamics with an external source j_μ^{cl} ,

$$S[\phi, \phi^\dagger; A^\mu] = \int d^4x \left[|\partial_\mu \phi + ieA_\mu \phi|^2 - m^2 |\phi|^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j_\mu^{\text{cl}} A^\mu \right], \quad (2.1)$$

and divide the vector potential A^μ into the classical and quantum parts, $A^\mu = A_{\text{cl}}^\mu + a^\mu$, where A_{cl}^μ solves the classical equations of motion with the source. The source is such that the classical field is the lengthy, $eET^2 \gg 1$, homogeneous electric pulse background,

$$A_\mu^{\text{cl}} = (0; A_1(t); 0; 0), \quad A_1(t) = ET \tanh \frac{t}{T}. \quad (2.2)$$

For the gauge field a_μ we choose the Feynman gauge and quantize it in the standard way with $\hat{\alpha}_{\mathbf{q}\mu}^\dagger, \hat{\alpha}_{\mathbf{q}'\nu}$ being the creation and annihilation operators, correspondingly. The external electric field does not affect the free field quantization of photons. For the scalar field we have the following decomposition:

$$\hat{\phi}(t, \mathbf{x}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left[\hat{a}_{\mathbf{p}} e^{i\mathbf{p}\mathbf{x}} f_{\mathbf{p}}(t) + \hat{b}_{\mathbf{p}}^\dagger e^{-i\mathbf{p}\mathbf{x}} f_{-\mathbf{p}}^*(t) \right], \quad (2.3)$$

where the temporal part of the mode function $f_{\mathbf{p}}(t)$ solves the equation

$$\left(\partial_t^2 + [\mathbf{p} + e\mathbf{A}(t)]^2 + m^2 \right) f_{\mathbf{p}}(t) = 0. \quad (2.4)$$

We require the harmonic function $f_{\mathbf{p}}(t)$ to be the in-mode, i.e., the single wave at the past infinity:

$$f_{\mathbf{p}}^{\text{in}}(t/T \rightarrow -\infty) \simeq \frac{e^{-i\omega_- t}}{\sqrt{2\omega_-}}, \quad \omega_\pm(\mathbf{p}) = \sqrt{(\mathbf{p} + \mathbf{A}(\pm\infty))^2 + m^2}. \quad (2.5)$$

The exact form of the modes expressed via the hypergeometric function is not necessary for us below (see, e.g., [37]). Meanwhile, the asymptotic form of the modes $f_{\mathbf{p}}(t)$ in the future infinity, $t/T \rightarrow \infty$, follows from the behavior of

the hypergeometric function for the large argument and is given by

$$f_{\mathbf{p}}^{\text{in}}(t/T \rightarrow +\infty) = \mathcal{A}_+(\mathbf{p})e^{i\omega_+ t} + \mathcal{A}_-(\mathbf{p})e^{-i\omega_+ t}, \quad (2.6)$$

where

$$|\mathcal{A}_-(\mathbf{p})|^2 = \frac{1}{2\omega_+(\mathbf{p})} + |\mathcal{A}_+(\mathbf{p})|^2, \\ |\mathcal{A}_+(\mathbf{p})|^2 = \frac{1}{2\omega_+(\mathbf{p})} \cdot \exp \left[-\frac{\pi(\mathbf{p}_\perp^2 + m^2)}{eE} \right] \quad (2.7)$$

and $\mathbf{p}_\perp = (p_2, p_3)$.

For the future reference let us stress that, as is shown in [31], for $|t| < T$ the exact mode function introduced above, $f_{\mathbf{p}}(t)$, can be approximated up to the order of $\mathcal{O}(1/T^2)$ by the parabolic cylinder function,

$$f_{\mathbf{p}}(t) \simeq g_{\mathbf{p}_\perp}(p_1 + eEt) \\ = c(\mathbf{p}_\perp) D_{-\frac{1}{2} + i\frac{\mathbf{p}_\perp^2 + m^2}{2eE}} \left[-\frac{(1-i)(p_1 + eEt)}{\sqrt{eE}} \right], \quad (2.8)$$

where

$$c(\mathbf{p}_\perp) = \frac{2^{ieET^2}}{2^{\xi_0/2} [eE]^{1/4}} [eET^2]^{-i\frac{\mathbf{p}_\perp^2 + m^2}{4eE}} \cdot e^{-i\pi/8} \\ \cdot \exp \left[-\frac{\pi(\mathbf{p}_\perp^2 + m^2)}{8eE} \right]. \quad (2.9)$$

The initial state that we consider in the calculations below is the Fock space in-ground state: $\hat{a}_{\mathbf{p}}|\text{in}\rangle = 0$.

In the present paper we are interested in the calculation of the level populations, $\langle a^\dagger a \rangle$ (for the corresponding ladder operators), anomalous quantum expectation values, $\langle aa \rangle$ and $\langle a^\dagger a^\dagger \rangle$ (for both photons and charged complex scalars), and electric currents *during the electric pulse duration*. The point is that when the pulse is switched off we are dealing with the standard scalar QED, i.e., without any background field, but with an unusual initial state (right after the pulse is switched off) due to the presence of created scalars, photons, and the anomalous expectation values for the corresponding fields. Such a situation is similar to the one considered in [38], but in a bit different theory. In any case, on general grounds, one can expect that after the switching off of the background electric field there will be a thermalization process, at least for small enough anomalous expectation values. That is a different physical situation and, thus, a different problem, which of course can have its own specifics and academic interest. We are interested in the kinetics during the electric pulse duration. The fact that we consider the particle creation process during the pulse is the reason why in the expressions below we frequently use the mode functions in the approximate form $g_{\mathbf{p}_\perp}(p_1 + eEt)$ rather than $f_{\mathbf{p}}(t)$.

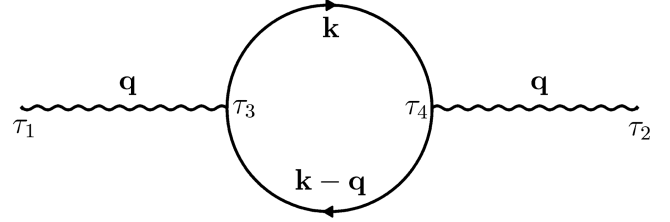


FIG. 1. One-loop correction to the photon's propagator. In the Schwinger-Keldysh technique in the one-loop order, one has to sum diagrams of such a form over plus and minus types of the internal (and external) vertices, depending on whether the vertex belongs to the upper or lower part of the Keldysh's time contour. For a review of the technique, one can consult, e.g., [39].

B. Tree-level current and one-loop corrections

The tree-level expression for the current in the long pulse, $eET^2 \gg 1$, in the future infinity, $t/T \gg 1$, has been calculated in many places both for the scalar and standard (spinor) QED [3,7,10,12,23] and has the following form:

$$j_1(t) \simeq \frac{E^2 e^3 T}{2\pi^3} \cdot \exp \left[-\frac{\pi m^2}{eE} \right]. \quad (2.10)$$

This expression has a clear physical explanation: during the pulse the pairs of charged particles are created with the Schwinger's probability rate per unit four-volume $w \sim E^2 e^2 \exp[-\pi m^2/eE]$. Hence, by the end of the pulse duration, one obtains the current density of the form $j_1 \sim ewT$. Our goal here is to check if loop corrections modify this expression substantially for long enough pulse.

In [31] it was shown that the one-loop correction to the tree-level current being suppressed by the extra power of the fine structure constant, e^2 , in comparison with the tree-level current, does not grow faster than (2.10) with T . Hence, even for a lengthy pulse the tree-level current is not drastically modified by the first quantum loop correction.

However, in [31] it was also shown that the one-loop correction to the photon's Keldysh propagator, shown in Fig. 1, does grow¹ with T . Namely, as is shown in [31], the correction to the photon's Keldysh propagator in the limit $|\tau_1 - \tau_2| \ll |\tau_1 + \tau_2|/2 \equiv |T| < T$ can be written in the following form:

$$\Delta G_{\mu\nu}^K(\mathbf{q}; \tau_1, \tau_2) \simeq n_{\mu\nu}(\mathbf{q}, T) \left[\frac{e^{-i|\mathbf{q}|(\tau_1 - \tau_2)}}{2|\mathbf{q}|} + \text{H.c.} \right], \quad (2.11)$$

where $n_{\mu\nu}(\mathbf{q})\delta(\mathbf{q} - \mathbf{q}') = \langle \hat{\alpha}_{\mathbf{q}\mu}^\dagger \hat{\alpha}_{\mathbf{q}'\nu} \rangle$ and $\hat{\alpha}_{\mathbf{q}\mu}^\dagger$, $\hat{\alpha}_{\mathbf{q}'\nu}$ are the photon's creation and annihilation operators, correspondingly. In the leading order in powers of large T and $|T| < T$, the result is [31]

¹For the constant background electric field, such a secular growth (actually secular divergence) was found in [21,22].

$$n_{\mu\nu}(\mathbf{q}, T) \approx -2e^2(\mathcal{T} + T) \int_{-\infty}^{+\infty} d\tau \frac{e^{-2i|\mathbf{q}|\tau}}{2|\mathbf{q}|} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \mathbb{D}_{\mu\nu}[g_{\mathbf{k}_\perp}(k_1 + eE\tau)g_{\mathbf{k}_\perp}^*(k_1 - eE\tau); g_{\mathbf{k}_\perp - \mathbf{q}_\perp}^*(k_1 - q_1 - eE\tau)g_{\mathbf{k}_\perp - \mathbf{q}_\perp}(k_1 - q_1 + eE\tau)], \quad (2.12)$$

where the function $g_{\mathbf{k}_\perp}(k_1 + eE\tau)$ is defined in Eq. (2.8) and the operator $\mathbb{D}_{\mu\nu}$ acts as

$$\begin{aligned} & \mathbb{D}_{\mu\nu}[g_{\mathbf{k}_\perp}(k_1 + eE\tau)g_{\mathbf{k}_\perp}^*(k_1 - eE\tau); g_{\mathbf{k}_\perp - \mathbf{q}_\perp}^*(k_1 - q_1 - eE\tau)g_{\mathbf{k}_\perp - \mathbf{q}_\perp}(k_1 - q_1 + eE\tau)] \\ &= \left[D_\mu g_{\mathbf{k}_\perp}(k_1 + eE\tau)g_{\mathbf{k}_\perp - \mathbf{q}_\perp}(k_1 - q_1 + eE\tau) - g_{\mathbf{k}_\perp}(k_1 + eE\tau)D_\mu^\dagger g_{\mathbf{k}_\perp - \mathbf{q}_\perp}(k_1 - q_1 + eE\tau) \right] \\ &\quad \times \left[g_{\mathbf{k}_\perp}^*(k_1 - eE\tau)D_\nu g_{\mathbf{k}_\perp - \mathbf{q}_\perp}^*(k_1 - q_1 - eE\tau) - D_\nu^\dagger g_{\mathbf{k}_\perp}^*(k_1 - eE\tau)g_{\mathbf{k}_\perp - \mathbf{q}_\perp}^*(k_1 - q_1 - eE\tau) \right]. \end{aligned} \quad (2.13)$$

The expression (2.12) explicitly grows with $\mathcal{T}$ and T . Such a growth of the photon level population is due to the fact that photons are created together with scalar particles from the initial state. The process in question is present in the interacting theory and is due to the presence of the vertex $a_\mu|\phi|^2$. Meanwhile, the anomalous quantum averages for the photon field, $\langle\alpha\alpha\rangle$ and $\langle\alpha^+\alpha^+\rangle$, do not grow with time, which is in agreement with the expectation that the initial Fock space ground state for the photon field remains to be the ground state of the Hamiltonian of the free photon theory.

In any case, due to this growth of the photon's two-point function, we expect that in the two-loop order the correction to the current of the created pairs (from the diagram shown in Fig. 2), although being suppressed by the extra power of the fine structure constant, will bring an additional power of growth with T (higher than linear growth, which is present in the tree-level contribution to the current). The

goal of this paper is to check if this expectation is correct, check the contributions of other two-loop diagrams, and discuss physical consequences of these observations.

III. TWO-LOOP CORRECTIONS TO THE CURRENT

In this section, we estimate two-loop corrections to the two-point correlation functions. We begin with the diagrams of the form shown in Fig. 2, since, as we mentioned above, such diagrams potentially can bring growing faster with T contributions to the current in comparison with the tree-level expression (2.10), which grows linearly.

A. Leading two-loop diagram

It is convenient to rewrite the contribution coming from the diagrams shown in Fig. 2 in terms of the advanced, retarded, and Keldysh propagators,

$$G^R = G^{--} - G^{-+}, \quad G^A = G^{--} - G^{+-}, \quad G^K = \frac{1}{2} [G^{-+} + G^{+-}]. \quad (3.1)$$

In this form, two-loop correction to the current has the following structure:

$$\begin{aligned} \Delta j_1(t) &= 2e \int \frac{d^3\mathbf{p}}{(2\pi)^3} (p_1 + eET) \Delta D^K(\mathbf{p}; t, t) \\ &= 2e \int \frac{d^3\mathbf{p}}{(2\pi)^3} (p_1 + eET) \int d\tau_1 d\tau_2 \int \frac{d^3\mathbf{q}}{(2\pi)^3} \mathbb{D}^{\mu\nu} \left[D^R(\mathbf{p}; t, \tau_1) \Sigma^K(\mathbf{p}, \mathbf{q}; \tau_1, \tau_2) D^A(\mathbf{p}; \tau_2, t) \right. \\ &\quad \left. + D^K(\mathbf{p}; t, \tau_1) \Sigma^A(\mathbf{p}, \mathbf{q}; \tau_1, \tau_2) D^A(\mathbf{p}; \tau_2, t) + D^R(\mathbf{p}; t, \tau_1) \Sigma^R(\mathbf{p}, \mathbf{q}; \tau_1, \tau_2) D^K(\mathbf{p}; \tau_2, t) \right], \end{aligned} \quad (3.2)$$

where $\mathbb{D}^{\mu\nu}$ acts on mode functions of the scalar field as in (2.13), and Σ is the self-energy, which in the Schwinger-Keldysh technique is the matrix of the form:

$$\begin{pmatrix} 2\Sigma^K & \Sigma^R \\ \Sigma^A & 0 \end{pmatrix} = - \begin{pmatrix} 2D^K \Delta G^K + \frac{1}{2} [D^A \Delta G^A + D^R \Delta G^R] & D^K \Delta G^R + D^R \Delta G^K \\ D^K \Delta G^A + D^A \Delta G^K & 0 \end{pmatrix}, \quad (3.3)$$

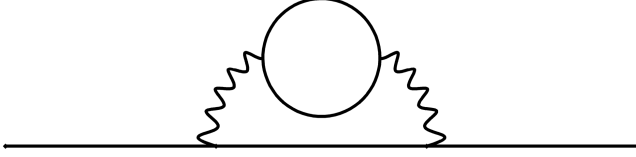


FIG. 2. Two-loop diagrams of such a form contain an additional power of growth with T .

where $D^{K,R,A} = D^{K,R,A}(\mathbf{p} - \mathbf{q}; \tau_1, \tau_2)$ is the tree-level Keldysh (retarded, advanced) propagators of the scalar field, while $\Delta G^{K(R,A)} = \Delta G_{\mu\nu}^{K(R,A)}(\mathbf{q}; \tau_1, \tau_2)$ is the one-loop

$$\Delta j_1 \approx -2e \int \frac{d^3 \mathbf{p}}{(2\pi)^3} (p_1 + eET) \int d\tau_1 d\tau_2 \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \Delta G_{\mu\nu}^K(\mathbf{q}; \tau_1, \tau_2) \mathbb{D}^{\mu\nu} \left[D^R(\mathbf{p}; t, \tau_1) D^K(\mathbf{p} - \mathbf{q}; \tau_1, \tau_2) D^A(\mathbf{p}; \tau_2, t) \right. \\ \left. + D^K(\mathbf{p}; t, \tau_1) D^A(\mathbf{p} - \mathbf{q}; \tau_1, \tau_2) D^A(\mathbf{p}; \tau_2, t) + D^R(\mathbf{p}; t, \tau_1) D^R(\mathbf{p} - \mathbf{q}; \tau_1, \tau_2) D^K(\mathbf{p}; \tau_2, t) \right]. \quad (3.4)$$

After the substitution of the expression for the tree-level propagators $D^{R,K,A}$ via the mode functions, we can rewrite the leading correction to the current as

$$\Delta j_1(t) = 2e \int \frac{d^3 \mathbf{p}}{(2\pi)^3} (p_1 + eET) \left[|f_{\mathbf{p}}(t)|^2 n_{\mathbf{p}}(t) + (f_{\mathbf{p}}^*(t))^2 \kappa_{\mathbf{p}}(t) + (f_{\mathbf{p}}(t))^2 \kappa_{\mathbf{p}}^*(t) \right], \quad (3.5)$$

where

$$n_{\mathbf{p}}(t) \simeq -2e^2 \int_{t_0}^t d\tau_1 \int_{t_0}^t d\tau_2 \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \Delta G_{\mu\nu}^K(\mathbf{q}; \tau_1, \tau_2) \mathbb{D}^{\mu\nu} \left[f_{\mathbf{p}}^*(\tau_1); f_{\mathbf{p}-\mathbf{q}}^*(\tau_1) f_{\mathbf{p}-\mathbf{q}}(\tau_2); f_{\mathbf{p}}(\tau_2) \right] \quad (3.6)$$

and

$$\kappa_{\mathbf{p}}(t) \simeq 2e^2 \int_{t_0}^t d\tau_1 \int_{t_0}^{\tau_1} d\tau_2 \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \Delta G_{\mu\nu}^K(\mathbf{q}; \tau_1, \tau_2) \mathbb{D}^{\mu\nu} \left[f_{\mathbf{p}}(\tau_1); f_{\mathbf{p}-\mathbf{q}}^*(\tau_1) f_{\mathbf{p}-\mathbf{q}}(\tau_2); f_{\mathbf{p}}(\tau_2) \right]. \quad (3.7)$$

These expressions can be used both during the pulse duration and after its switching off. That is the reason why in them we denote the mode functions as $f_{\mathbf{p}}(t)$ rather than $g_{\mathbf{p}\perp}(p_1 + eEt)$. The physical meaning of the obtained expression for the correction to the current is clear: while the tree-level current (2.10) appears due to the amplification of the zero-point fluctuations, the correction to the current (3.5) appears due to the excitation of the higher levels and due to the generation of the anomalous quantum expectation values.

To estimate the obtained expression in (3.6) and (3.7), we make the following change of integration variables:

$$\tau = \frac{\tau_1 - \tau_2}{2}, \quad \mathcal{T} = \frac{\tau_1 + \tau_2}{2}, \quad (3.8)$$

and recall that the propagator $\Delta G_{\mu\nu}^K(\mathbf{q}; \tau_1, \tau_2)$ contains growth with T only for $|\tau| \ll |\mathcal{T}|$, where it has the form (2.11). Because of the oscillating functions in (2.11), the

corrected photon's Keldysh (retarded, advanced) propagators. To obtain this correction to the current (3.2), we use the contribution to the Keldysh propagator coming from the diagram shown in Fig. 2 at the coincident space-time points, as is expressed in the first row of (3.2).

Only those terms that contain $\Delta G_{\mu\nu}^K(\mathbf{q}; \tau_1, \tau_2)$ contribute the leading corrections at large T , since it is only the Keldysh propagator that contains the extra power of T , as is shown in [31], and we have discussed above. Our goal is to single out the leading contribution for the large T .

In all, the leading correction to the current in the large T limit can be written as

integral over $\mathbf{q}$ for large τ is saturated within a short region. Hence, we can extend the integration over τ onto the entire real axis.

As we explained above, we are interested only in the time region inside the pulse duration. Then, we restrict the integration over $\mathcal{T}$ to the region $|\mathcal{T}| < T$. This allows us to use the parabolic cylinder function approximation (2.8) for the modes and apply the expression (2.12) for $n_{\mu\nu}(\mathbf{q}, T)$. After the change of variables,

$$\chi = \frac{p_1 + eET}{\sqrt{eE}}, \quad \chi_i = \frac{p_1 + eE\tau_i}{\sqrt{eE}}, \quad \mathbf{Q} = \frac{\mathbf{q}}{\sqrt{eE}}, \\ X = \frac{\chi_1 + \chi_2}{2}, \quad \tilde{\chi} = \chi_1 - \chi_2, \quad (3.9)$$

we obtain the following expressions:

$$n_{\mathbf{p}}(T) \simeq \mathcal{N}_{\mathbf{p}_{\perp}}(\chi) = -2e^2 \int_{\chi-2\sqrt{eE}T}^{\chi} dX \int_{-\infty}^{+\infty} d\tilde{\chi} \int \frac{d^3\mathbf{Q}}{(2\pi)^3} \Delta G_{\mu\nu}^K(\mathbf{Q}; X, \tilde{\chi}) \mathbb{D}^{\mu\nu} \left[g_{\mathbf{p}_{\perp}}^*(X + \tilde{\chi}/2) g_{\mathbf{p}_{\perp}-\mathbf{q}_{\perp}}^*(X + \tilde{\chi}/2 - Q_1) \right. \\ \left. \times g_{\mathbf{p}_{\perp}-\mathbf{q}_{\perp}}(X - \tilde{\chi}/2 - Q_1) g_{\mathbf{p}_{\perp}}(X - \tilde{\chi}/2) \right], \quad (3.10)$$

and

$$\kappa_{\mathbf{p}}(T) \simeq \mathcal{K}_{\mathbf{p}_{\perp}}(\chi) = 2e^2 \int_{\chi-2\sqrt{eE}T}^{\chi} dX \int_0^{+\infty} d\tilde{\chi} \int \frac{d^3\mathbf{Q}}{(2\pi)^3} \Delta G_{\mu\nu}^K(\mathbf{Q}; X, \tilde{\chi}) \mathbb{D}^{\mu\nu} \left[g_{\mathbf{p}_{\perp}}(X + \tilde{\chi}/2) g_{\mathbf{p}_{\perp}-\mathbf{q}_{\perp}}^*(X + \tilde{\chi}/2 - Q_1) \right. \\ \left. \times g_{\mathbf{p}_{\perp}-\mathbf{q}_{\perp}}(X - \tilde{\chi}/2 - Q_1) g_{\mathbf{p}_{\perp}}(X - \tilde{\chi}/2) \right]. \quad (3.11)$$

We want to extend the integration over X to the entire real axis using the asymptotic form of the parabolic cylinder function with the large argument,

$$g_{\mathbf{p}_{\perp}}(X) \simeq \alpha_{\pm}(\mathbf{p}_{\perp}) \frac{X^{i\frac{m^2+\mathbf{p}_{\perp}^2}{2eE}}}{X^{1/2}} e^{\frac{iX^2}{2}} + \beta_{\pm}(\mathbf{p}_{\perp}) \frac{X^{-i\frac{m^2+\mathbf{p}_{\perp}^2}{2eE}}}{X^{1/2}} e^{\frac{-iX^2}{2}}, \quad (3.12)$$

where the sign $\pm$ corresponds to the large positive and negative X values, correspondingly, and

$$|\alpha_{+}(\mathbf{p}_{\perp})|^2 = \frac{1}{2\sqrt{eE}} \exp \left[-\frac{\pi(\mathbf{p}_{\perp}^2 + m^2)}{eE} \right], \quad |\alpha_{-}(\mathbf{p}_{\perp})|^2 = \frac{1}{2\sqrt{eE}} \\ |\beta_{+}(\mathbf{p}_{\perp})|^2 = \frac{1}{2\sqrt{eE}} + |\alpha_{+}(\mathbf{p}_{\perp})|^2, \quad |\beta_{-}(\mathbf{p}_{\perp})|^2 = 0. \quad (3.13)$$

We can extend the lower limit of integration to minus infinity, since there is only one exponent in (3.12) for $X < 0$ and the integrands in (3.10) and (3.11) are fast oscillating functions in the asymptotic past.

However, there are several obstacles to extend the upper limit to plus infinity. First, the photon propagator contains additional growth with X . Second, it is not clear whether $\mathbb{D}^{\mu\nu}$ contains a growth with X after the convolution with $\Delta G_{\mu\nu}$ or not. Therefore, we examine in more detail the operator $\mathbb{D}^{\mu\nu}$ in the asymptotic region $1 \ll X < \chi$:

$$\mathbb{D}^{\mu\nu} = \left[[D^{\mu}]_1^{\dagger} - D_1^{\mu} \right] \cdot \left[[D^{\nu}]_2^{\dagger} - D_2^{\nu} \right], \quad (3.14)$$

where

$$[D^1]_1^{\dagger} - D_1^1 = i(2X - Q_1 + \tilde{\chi})\sqrt{eE}; \\ [D^1]_2^{\dagger} - D_2^1 = i(2X - Q_1 - \tilde{\chi})\sqrt{eE}, \quad (3.15)$$

and

$$[D^0]_1^{\dagger} - D_1^0 = \pm i(2X - Q_1 + \tilde{\chi})\sqrt{eE}; \\ [D^0]_2^{\dagger} - D_2^0 = \pm i(2X - Q_1 - \tilde{\chi})\sqrt{eE}. \quad (3.16)$$

From (3.15) and (3.16) it is clear that the trace $\mathbb{D} = \mathbb{D}_{\mu}^{\mu}$, which is contained in the one-loop correction to the

current, is independent² of X . In (3.10), the convolution is less trivial and does not reduce to the combination $\mathbb{D}_0^0 + \mathbb{D}_1^1$, i.e., to the trace. Because of components of $\mathbb{D}^{\mu\nu}$ with $\mu, \nu = 0, 1$, we have an additional growth of order X^2 in (3.10). The argument of the operator $\mathbb{D}^{\mu\nu}$ is of the order of $1/X^2$, as follows from (3.12). Thus, we find that the integral over X can be of the order of T^2 , which means that the correction to the electric current potentially contains even faster growth with T than we expected in the second-loop order due to the linear growth of the photon's self-energy.

To check the power of the growth of the two-loop correction to the current with T , one has to estimate the integrals in (3.10) and (3.11) for the more explicit form of the mode functions. In our forthcoming publications we will do that separately in two limits: for the weak and strong background electric fields. In particular, in our next paper we will show that for the weak field background the growth with T is actually faster than quadratic.

At this moment, we see that the two-loop correction being suppressed by the extra power of the fine structure constant is growing faster than T^2 , which is faster than the linear growth of the tree-level contribution to the current

²That actually was the reason why the first-loop correction to the current did not grow faster than the first power of T .

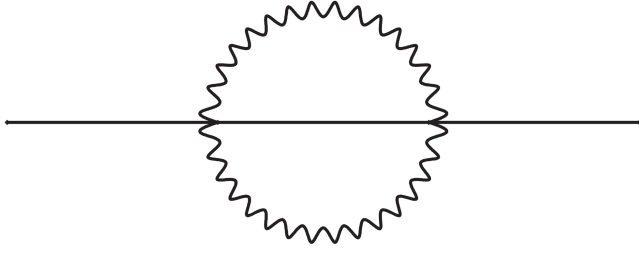


FIG. 3. Two-loop sunset diagrams potentially can contain additional power of T .

(2.10). This is the main conclusion of the present paper: for the lengthy enough background pulse, the Schwinger's expression for the current of the created particles is

drastically modified. Below we estimate other two-loop contributions to the current.

B. Other two-loop corrections to the current

We start with three two-loop diagrams, which also can potentially give an additional power of T to the current. They are shown in Figs. 3–5. Then we will continue with other types of two-loop diagrams which in principle can be present.

1. Sunset diagram

Let us start with the consideration of the sunset diagram from Fig. 3. This correction to the scalar propagator in the momentum space can be expressed as follows:

$$\Delta D^{ab}(\mathbf{p}; t_1, t_2) = 4 \sum_{c,d} \text{sgn}(c) \text{sgn}(d) e^4 \int d\tau_1 \int d\tau_2 \int \frac{d^3 \mathbf{q}_1}{(2\pi)^3} \int \frac{d^3 \mathbf{q}_2}{(2\pi)^3} G^{cd}(\mathbf{q}_1, \tau_1, \tau_2) G^{cd}(\mathbf{q}_2, \tau_1, \tau_2) \times D^{ac}(\mathbf{p}, t_1, \tau_1) D^{cd}(\mathbf{p} - \mathbf{q}_1 - \mathbf{q}_2, \tau_1, \tau_2) D^{db}(\mathbf{p}, \tau_2, t_2), \quad (3.17)$$

where $a, b = \pm$ are the standard indexes in the Schwinger-Keldysh technique. Since the product of the two tree-level photon propagators has a similar time dependence as the single one, the procedure of the calculation of this two-loop diagram does not differ much from the one-loop calculation performed in [31]. The correction to the current again can be expressed in the standard form (3.5), where

$$n_{\mathbf{p}}(t) \simeq -2e^4 \int_{t_0}^t d\tau_1 \int_{t_0}^{\tau_1} d\tau_2 \int \frac{d^3 \mathbf{q}_1}{(2\pi)^3} \int \frac{d^3 \mathbf{q}_2}{(2\pi)^3} \frac{e^{i(|\mathbf{q}_1| + |\mathbf{q}_2|)(\tau_1 - \tau_2)}}{|\mathbf{q}_1| \cdot |\mathbf{q}_2|} f_{\mathbf{p}}^*(\tau_1) f_{\mathbf{p}-\mathbf{q}_1-\mathbf{q}_2}^*(\tau_1) f_{\mathbf{p}-\mathbf{q}_1-\mathbf{q}_2}(\tau_2) f_{\mathbf{p}}(\tau_2), \quad (3.18)$$

and

$$\kappa_{\mathbf{p}}(t) \simeq 2e^4 \int_{t_0}^t d\tau_1 \int_{t_0}^{\tau_1} d\tau_2 \int \frac{d^3 \mathbf{q}_1}{(2\pi)^3} \int \frac{d^3 \mathbf{q}_2}{(2\pi)^3} \frac{e^{i(|\mathbf{q}_1| + |\mathbf{q}_2|)(\tau_1 - \tau_2)}}{|\mathbf{q}_1| \cdot |\mathbf{q}_2|} f_{\mathbf{p}}(\tau_1) f_{\mathbf{p}-\mathbf{q}_1-\mathbf{q}_2}^*(\tau_1) f_{\mathbf{p}-\mathbf{q}_1-\mathbf{q}_2}(\tau_2) f_{\mathbf{p}}(\tau_2). \quad (3.19)$$

After such a change of variables as (3.8), we can extend the integration over τ in (3.18) onto the entire real axis, as we did above:

$$n_{\mathbf{p}}(t) \simeq -4e^4 \int_{t_0}^t dT \int_{-\infty}^{+\infty} d\tau \int \frac{d^3 \mathbf{q}_1}{(2\pi)^3} \int \frac{d^3 \mathbf{q}_2}{(2\pi)^3} \frac{e^{2i(|\mathbf{q}_1| + |\mathbf{q}_2|)\tau}}{|\mathbf{q}_1| \cdot |\mathbf{q}_2|} f_{\mathbf{p}}^*(\tau_1) f_{\mathbf{p}-\mathbf{q}_1-\mathbf{q}_2}^*(\tau_1) f_{\mathbf{p}-\mathbf{q}_1-\mathbf{q}_2}(\tau_2) f_{\mathbf{p}}(\tau_2), \quad (3.20)$$

and similarly for the anomalous quantum average.

Let us stress at this moment that, in contrast to the one-loop correction to the current, which was considered in [31], in (3.20) there is a secular growth with t if $t \gg T$. Please do not confuse the latter one with the growth with T ; i.e., this time we discuss the growing with time change in the level population and anomalous average after the switch off of the background field. This secular growth apparently is due to the fact that delta functions establishing the energy conservation are no longer present. In fact, if the modes $f_{\mathbf{p}}(t)$ were single exponents, $e^{-i\omega(\mathbf{p})t}/\sqrt{2\omega(\mathbf{p})}$, then in (3.20) from the integral

$$\int_{-\infty}^{+\infty} d\tau e^{2i(|\mathbf{q}_1| + |\mathbf{q}_2|)\tau} f_{\mathbf{p}}^*(\tau_1) f_{\mathbf{p}-\mathbf{q}_1-\mathbf{q}_2}^*(\tau_1) f_{\mathbf{p}-\mathbf{q}_1-\mathbf{q}_2}(\tau_2) f_{\mathbf{p}}(\tau_2),$$

one would obtain $\delta(|\mathbf{q}_1| + |\mathbf{q}_2| + \omega(\mathbf{p}) + \omega(\mathbf{p} - \mathbf{q}_1 - \mathbf{q}_2))$. The argument of the latter delta function is never zero and, hence, $n_{\mathbf{p}}(t)$ is vanishing as it should be in the absence of the strong background field. The same is true for $\kappa_{\mathbf{p}}(t)$. Since in the presence of the strong background field the mode functions, $f_{\mathbf{p}}(t)$, are no longer pure exponents, one obtains a smearing of the delta function establishing the energy conservation: in fact, in the time dependent

background under consideration there is no energy conservation. For example, in the limit $t = (t_1 + t_2)/2 \gg |t_1 - t_2|$ from (3.20), we approximately obtain the following contributions to the level population:

$$n_{\mathbf{p}}(t) \simeq -2e^4 t \int \frac{d^3 \mathbf{q}_1 d^3 \mathbf{q}_2}{(2\pi)^5 |\mathbf{q}_1| |\mathbf{q}_2|} |\mathcal{A}_+(\mathbf{p}) \mathcal{A}_+(\mathbf{p} - \mathbf{q}_1 - \mathbf{q}_2)|^2 \delta(|\mathbf{q}_1| + |\mathbf{q}_2| - \omega_+(\mathbf{p}) - \omega_+(\mathbf{p} - \mathbf{q}_1 - \mathbf{q}_2)). \quad (3.21)$$

To fetch this expression, we used in (3.20) the asymptotic form (2.6) of the modes in the future infinity, where the coefficients $\mathcal{A}_{\pm}(\mathbf{p})$ are defined in (2.7). A similar expression one also obtains for the anomalous quantum expectation values.

There are two comments in order at this point. First, one can see that the secular (growing with time of observation, t , and/or with the duration of the pulse, T) contributions that we obtain in the background fields are essentially due to the presence of nontrivial collision integrals in strong background fields. Namely, in the strong background field under consideration, there is no energy conservation and, hence, various processes, such as creation of charged particles together with photons from the initial state of the theory, are allowed. Such processes are forbidden without background fields. In fact, the growing with time level population from (3.20) is just the collision integral integrated over the time of the evolution of the system. The same is true for (3.10) and (3.11).

Second, as we have already discussed, after the pulse is turned off the time change of the level population and anomalous average is a setup for a different problem. In

fact, during the duration of the background electric pulse, there can be generated in principle some level population and anomalous averages. The latter will set up an initial data for the thermalization process after the switching off of the background field. Such a process in a bit different theory was discussed in [38]. During this thermalization process, which is just a redistribution of the level populations and anomalous averages for different levels in a situation without a background field, there can appear an electric current, but in this paper we are not discussing this problem, as we have already explained above.

Thus, we concentrate on the situation within the duration of the background pulse, i.e., on the time period $|t| < T$. The integration over the region $T < -T$ in (3.20), i.e., before the start of the pulse, is suppressed, since all expectation values are taken with respect to the standard Poincaré invariant state (the in-state before the pulse). At the same time, as we have explained above, within the pulse duration we can approximate the exact modes with the parabolic cylinder functions. Then, for the level population we obtain the following expression:

$$n_{\mathbf{p}}(T) \simeq \mathcal{N}_{\mathbf{p}_{\perp}}(\chi) = -2e^5 E \int_{\chi - 2\sqrt{eET}}^{\chi} dX \int_{-\infty}^{+\infty} d\tilde{\chi} \int \frac{d^3 \mathbf{Q}_1 d^3 \mathbf{Q}_2}{(2\pi)^6} \frac{e^{i(|\mathbf{Q}_1| + |\mathbf{Q}_2|)\tilde{\chi}}}{|\mathbf{Q}_1| |\mathbf{Q}_2|} g_{\mathbf{p}_{\perp}}^*(X + \tilde{\chi}/2) g_{\mathbf{p}_{\perp} - \mathbf{q}_{1\perp} - \mathbf{q}_{2\perp}}^*(X + \tilde{\chi}/2 - Q_1 - Q_2) \times g_{\mathbf{p}_{\perp} - \mathbf{q}_{1\perp} - \mathbf{q}_{2\perp}}(X - \tilde{\chi}/2 - Q_1 - Q_2) g_{\mathbf{p}_{\perp}}(X - \tilde{\chi}/2), \quad (3.22)$$

where we have performed the change of variables (3.1).

As in the case of the one-loop correction to the current [31], we can extend the integration over X onto the entire

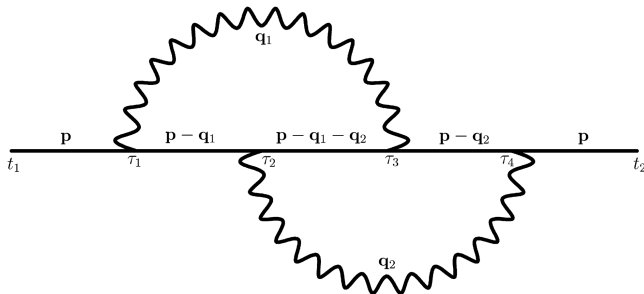


FIG. 4. Shifted sunset diagram.

real axis with accuracy $\mathcal{O}(1/T)$ and obtain that $n_{\mathbf{p}}(T)$ does not depend on T in the leading order. Thus, the corresponding correction to the current also does not give an additional power of growth with T . For the anomalous average $\kappa_{\mathbf{p}}(T)$, the situation is similar. Thus, the diagram shown in Fig. 3, being suppressed by the extra power of the fine structure constant, does not provide a correction which grows with T faster than the tree-level contribution to the current.

2. Shifted sunset diagram

Consider now the diagram from Fig. 4. The corresponding general expression for the correction to the propagator can be written as

$$\Delta D^{ab}(\mathbf{p}; t_1, t_2) = e^4 \sum_{cdef} \text{sgn}(c) \cdots \text{sgn}(f) \int d\tau_1 \cdots d\tau_4 \int \frac{d^3 \mathbf{q}_1 d^3 \mathbf{q}_2}{(2\pi)^6} G^{ce}(\mathbf{q}_1; \tau_1, \tau_3) G^{df}(\mathbf{q}_2; \tau_2, \tau_4) \\ \times \mathbb{D} \left[D^{ac}(\mathbf{p}; t_1, \tau_1) D^{cd}(\mathbf{p} - \mathbf{q}_1; \tau_1, \tau_2) D^{de}(\mathbf{p} - \mathbf{q}_1 - \mathbf{q}_2; \tau_2, \tau_3) D^{ef}(\mathbf{p} - \mathbf{q}_2; \tau_3, \tau_4) D^{fb}(\mathbf{p}; \tau_4, t_2) \right], \quad (3.23)$$

where the indexes, a, b , etc. in this expression are the same as in (3.17) and

$$\mathbb{D}^{\mu\nu\alpha\beta} = \left([D_1^\mu]^\dagger - D_1^\mu \right) \left([D_2^\nu]^\dagger - D_2^\nu \right) \left([D_3^\alpha]^\dagger - D_3^\alpha \right) \left([D_4^\beta]^\dagger - D_4^\beta \right), \quad (3.24)$$

while $\mathbb{D} = \mathbb{D}^{\mu\nu\alpha\beta} g_{\mu\alpha} g_{\nu\beta}$.

To represent this expression and the corresponding correction to the current in terms of modes, we rewrite the tree-level propagators in the following form:

$$D^{ab}(\mathbf{p}; t_1, t_2) = A^{ab}(\mathbf{p}; t_1, t_2) + B^{ab}(\mathbf{p}; t_1, t_2), \quad (3.25)$$

where

$$A^{ab}(\mathbf{p}; t_1, t_2) = \begin{pmatrix} \theta(t_1 - t_2) & 0 \\ 1 & \theta(t_2 - t_1) \end{pmatrix} f_{\mathbf{p}}(t_1) f_{\mathbf{p}}^*(t_2), \\ B^{ab}(\mathbf{p}; t_1, t_2) = \begin{pmatrix} \theta(t_2 - t_1) & 1 \\ 0 & \theta(t_1 - t_2) \end{pmatrix} f_{\mathbf{p}}^*(t_1) f_{\mathbf{p}}(t_2), \quad (3.26)$$

and similarly for the photon propagator:

$$G^{ab}(\mathbf{p}; t_1, t_2) = \mathcal{A}^{ab}(\mathbf{p}; t_1, t_2) + \mathcal{B}^{ab}(\mathbf{p}; t_1, t_2). \quad (3.27)$$

To simplify the expressions below, let us introduce the following notation $A^{ab}(\mathbf{p}; t_1, t_2) = A_{12}^{ab}$. Furthermore, we also use the following notations: $A^{ab}(\mathbf{p}; t, \tau_1) = A_{01}^{ab}$, etc. Then, one can restore the expanded expression (with all the arguments explicitly written) unambiguously. Hence, e.g.,

$$D^{+c}(\mathbf{p}; t, \tau_1) D^{f-}(\mathbf{p}; \tau_4, t) = A_{01}^{+c} A_{40}^{f-} + B_{01}^{+c} B_{40}^{f-} \\ + A_{01}^{+c} B_{40}^{f-} + B_{01}^{+c} A_{40}^{f-}. \quad (3.28)$$

The first two terms from the right side of (3.28) are proportional to $|f_{\mathbf{p}}(t)|^2$ and, hence, contribute to $n_{\mathbf{p}}(t)$. The third term is proportional to $f_{\mathbf{p}}^2(t)$ and, thus, contributes to $\kappa_{\mathbf{p}}^*(t)$, as can be seen from (3.5). The last term contributes to $\kappa_{\mathbf{p}}(t)$.

After the substitution of (3.25) into (3.23), we obtain a sum of integrals over $\tau_{1,2,3,4}$, whose integration domains are

specified by inequalities of the form $\tau_i < \tau_j$, for some i, j . However, from the causality of the Schwinger-Keldysh technique (see, e.g., [40]), we know for sure that all τ_i do not exceed t : otherwise the loop contribution is vanishing.

Since the product $G^{ce}(\mathbf{q}_1; \tau_1, \tau_3) G^{df}(\mathbf{q}_2; \tau_2, \tau_4)$ contains terms proportional to the product of exponents $e^{\pm i|\mathbf{q}_1|(\tau_1 - \tau_3)} e^{\pm i|\mathbf{q}_2|(\tau_2 - \tau_4)}$, after the change of variables,

$$\mathcal{T}_1 = \frac{\tau_1 + \tau_3}{2}, \quad \mathcal{T}_2 = \frac{\tau_2 + \tau_4}{2}; \\ \tilde{\tau}_1 = \tau_1 - \tau_3, \quad \tilde{\tau}_2 = \tau_2 - \tau_4, \quad (3.29)$$

the integral over $\mathbf{q}_{1,2}$ is saturated in the region $|\tilde{\tau}_{1,2}| \ll |\mathcal{T}_{1,2}|$ (for a large $\tilde{\tau}_i$, the exponent $e^{\pm 2i|\mathbf{q}_i|\tilde{\tau}_i}$ is fast oscillating, and the integral over $\mathbf{q}_i$ is suppressed) and we can extend the integration over $\tilde{\tau}_{1,2}$ onto the entire real axis or the real semiaxis [some terms can contain theta functions, such as $\theta(\tau_1 - \tau_3)$ etc.]. In any of these cases, after such an extension, the integrals over $\tilde{\tau}_i$ no longer contain t within the limits of integration.

As was explained above, we are interested only in the integration regions inside the pulse, $-T < \mathcal{T}_i < t < T$. In such a case, we can use the parabolic cylinder functions (2.8) as the approximate form of the modes. Below we will use that the coefficient in (2.8) obeys

$$|c(\mathbf{p}_\perp)|^2 = \frac{1}{\sqrt{2eE}} \exp \left[-\frac{\pi(\mathbf{p}_\perp^2 + m^2)}{4eE} \right]. \quad (3.30)$$

One can see from (3.25) and (3.26) that $D^{a_1 a_2}(\mathbf{k}, t_i, t_j) \propto |c(\mathbf{k}_\perp)|^2$ and does not depend on T in the leading order. Thus, all of the dependence on T in (3.23) is contained only in the limits of integration over $\mathcal{T}_i$.

For each integral over $\mathcal{T}_1$, one has the integrand of the form

$$\left([D_1^\mu]^\dagger - D_1^\mu \right) \left([D_{3,\mu}]^\dagger - D_{3,\mu} \right) \left[g_{\mathbf{p}_\perp}(p_1 + eE\tau_1) g_{\mathbf{p}_\perp - \mathbf{q}_{1,\perp}}(p_1 - q_1 + eE\tau_1) \right. \\ \left. \times g_{\mathbf{p}_\perp - \mathbf{q}_{1,\perp} - \mathbf{q}_{2,\perp}}(p_1 - q_1 - q_2 + eE\tau_3) g_{\mathbf{p}_\perp - \mathbf{q}_{2,\perp}}(p_1 - q_2 + eE\tau_3) \right], \quad (3.31)$$

where some modes g can contain the sign of the complex conjugation. To single out the leading contribution for large T from the integrals under consideration, we ought to use the asymptotic behavior of the parabolic cylinder functions with a large argument. Regardless of whether the mode is complex conjugate or not, it has the asymptotic behavior of the form (3.12). Consequently, different contributions will differ only in the coefficients, which are products of α 's and β 's and their arguments. Meanwhile, the operator

$$\left([D_1^\mu]^\dagger - D_1^\mu\right)\left([D_{3,\mu}]^\dagger - D_{3,\mu}\right)$$

acts on the asymptotic form (3.12) as follows:

$$\begin{aligned} & \left([D_1^\mu]^\dagger - D_1^\mu\right)\left([D_{3,\mu}]^\dagger - D_{3,\mu}\right) \\ & \mapsto (2\mathbf{p}_\perp - \mathbf{q}_{1,\perp})(2\mathbf{p}_\perp - 2\mathbf{q}_{2,\perp} - \mathbf{q}_{1,\perp}). \end{aligned} \quad (3.32)$$

In this case such an integrand as (3.31) is of the order $1/T_1^2$, which means that the integral over $\mathcal{T}_1$ converges at infinity and does not grow with T . Up to the order of $\mathcal{O}(1/T)$, one can extend the integration over $\mathcal{T}_1$ to the entire real axis. For the integral over $\mathcal{T}_2$, the situation is similar. Thus, in the case of the diagram under consideration, the quantum corrections again do not give an additional growth with

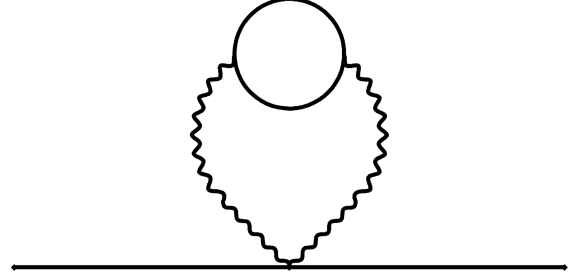


FIG. 5. Two-loop tadpole diagram.

T , which means that the final contribution to the current will be just suppressed in comparison with the tree-level current by the extra power of the fine structure constant (and will not contain any higher power of T). In all, we find that the diagram shown in Fig. 4 is suppressed for large values of T .

3. Two-loop tadpole diagram

In this subsection we consider the contribution of the two-loop tadpole type diagram shown in Fig. 5. The general expression for the correction to the matrix of propagator corresponding to this diagram is as follows:

$$\Delta D^{ab}(\mathbf{p}; t_1, t_2) = ie^2 \sum_c \text{sgn}(c) \int d\tau \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \Delta G^{cc}(\mathbf{q}; \tau, \tau) D^{ac}(\mathbf{p}; t_1, \tau) D^{cb}(\mathbf{p}; \tau, t_2), \quad (3.33)$$

where $\Delta G^{cc}(\mathbf{q}; \tau, \tau)$ is the one-loop corrected photon propagator taken at coinciding times. It was calculated in [31]. The expressions for $\Delta G^{A,R}(\mathbf{q}; \tau_1, \tau_2)$ have the same retarded (advanced) structure as the corresponding tree-level propagators. In particular, $\Delta G^{A,R}(\mathbf{q}; \tau, \tau) = 0$ for all τ . In this case, we obtain that

$$\Delta G^{cc}(\mathbf{q}; \tau, \tau) = \Delta G^K(\mathbf{q}; \tau, \tau) \quad (3.34)$$

for all $c = \pm 1$. Furthermore, the integral over $\mathbf{q}$ in (3.33) is factorizable, as in the case of the one-loop tadpole correction. However, in this case the factorized integral depends on τ . Namely, the leading contribution is as follows:

$$\int \frac{d^3 \mathbf{q}}{(2\pi)^3} \Delta G^K(\mathbf{q}; \tau, \tau) = \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \frac{n_\mu^\mu(\mathbf{q}, \tau)}{|\mathbf{q}|} = e^2(\tau + T)C', \quad (3.35)$$

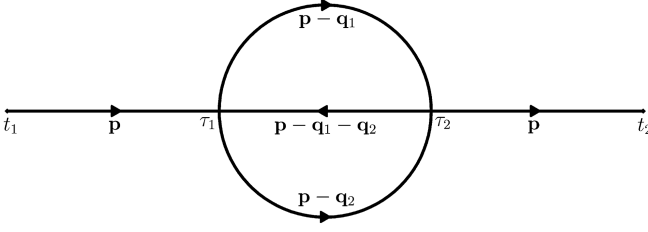
where C' is some constant similar C from the one-loop tadpole diagram which was calculated in [31]. Thus, in contrast to the one-loop tadpole diagram, the correction to the current contains an additional dependence on τ in its integrand:

$$\Delta j_1^{(\text{tad})}(t) = 2e \int \frac{d^3 \mathbf{p}}{(2\pi)^3} (p_1 + eET)[f_\mathbf{p}^2(t)\chi_\mathbf{p}(t) + \text{H.c.}], \quad (3.36)$$

where

$$\chi_\mathbf{p}(t) = ie^4 C' \int_{t_0}^t (\tau + T)[f_\mathbf{p}^*(\tau)]^2 d\tau. \quad (3.37)$$

However, similarly to the one-loop tadpole corrections, one can show that the integral over τ in $\Delta j_1^{(\text{tad})}$ can grow with T only for $\tau \in [-T, T]$, i.e., during the pulse. One also can show, using the asymptotics (3.12), that the sum of the integrals over τ in $\Delta j_1^{(\text{tad})}$ [the integral in (3.37) and its H.c.] converges and is saturated from the vicinity of such τ that $\sqrt{eE}\tau \sim 1$, for which $(T + \tau) \simeq T$. Taking this into account, one can see that all calculations in this case are the same as for the one-loop tadpole correction. Thus, the correction under consideration can be absorbed by just a shift of the mass by the quantity $C_{2\text{-loop}} = e^2 TC'$, following from (3.35), rather than attributed to a growth of the current.

FIG. 6. Sunset diagram for the $\lambda|\phi|^4$ theory.

4. Taking into account the $\lambda|\phi|^4$ self-interaction

In principle, the scalar electrodynamics can also contain an additional self-interaction term, such as, e.g., $\lambda|\phi|^4$. Then the action of the theory is

$$\Delta D^{ab}(\mathbf{p}; t_1, t_2) = \lambda^2 \sum_{cd} \text{sgn}(c) \text{sgn}(d) \int d\tau_1 d\tau_2 \int \frac{d^3 \mathbf{q}_1 d^3 \mathbf{q}_2}{(2\pi)^6} \times D^{ac}(\mathbf{p}; t_1, \tau_1) D^{cd}(\mathbf{p} - \mathbf{q}_1; \tau_1, \tau_2) D^{dc}(\mathbf{p} - \mathbf{q}_1 - \mathbf{q}_2; \tau_2, \tau_1) D^{cd}(\mathbf{p} - \mathbf{q}_2; \tau_1, \tau_2) D^{db}(\mathbf{p}; \tau_2, t_2). \quad (3.39)$$

The dependence on time in the expression (3.39) is the same as of the other sunset diagrams, which we have considered above. Namely, the limits of integration over $\tau_{1,2}$ for $n_{\mathbf{p}}(t)$ and $\kappa_{\mathbf{p}}(t)$ are the same as in the expressions (3.18) and (3.19) correspondingly. Then, when $\tau_{1,2}$ are in the region before the pulse is turned on the situation is trivial, i.e., the same as in the absence of any external field. After the pulse is turned off, via a Bogolyubov transformation, one can again reduce everything down to the situation without the external field, but with the initial quantum expectation values being not equal to zero. It is known that in such a situation there is a secular growth with t associated with the process of thermalization similar to the one considered in [38], as we have mentioned above.

Let us now concentrate on the situation within the duration of the external pulse, $|\tau_{1,2}| < T$. One can notice that the diagram from Fig. 4 can be reduced to the diagram from Fig. 6 if one shrinks photon propagators into points. As a result, to obtain (3.39) from (3.23), one should perform the following changes:

$$\begin{aligned} \text{sgn}(e) G^{ce}(\mathbf{q}_1; \tau_1, \tau_3) &\mapsto \delta_{ce} \delta(\tau_1 - \tau_3), \\ \text{sgn}(f) G^{df}(\mathbf{q}_2; \tau_2, \tau_4) &\mapsto \delta_{df} \delta(\tau_2 - \tau_4), \quad e^4 \mathbb{D} \mapsto \lambda^2. \end{aligned} \quad (3.40)$$

This means that the integrals over $\tau_{1,2}$ in (3.39) are very similar to the integrals over $\mathcal{T}_{1,2}$ considered in Sec. III B 2: using the asymptotic form (3.12), we find that the integrals over τ_i also have the integrands of the order $1/\tau_i^2$ and do not grow with T .

In all, we find that taking into account the self-interaction of the scalar field does not lead to an additional growth of

$$S[\phi, \phi^\dagger; a^\mu] = \int d^4x \left[|D_\mu \phi + ie a_\mu \phi|^2 - m^2 |\phi|^2 - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} + \frac{\lambda}{4!} |\phi|^4 \right]. \quad (3.38)$$

In such a case, there is another two-loop diagram which may lead to a secular contribution to the current. This diagram is shown in Fig. 6. (The one-loop tadpole correction to the Keldysh propagator for the $\lambda|\phi|^4$ theory can be shown also to give a shift in the mass in the theory.)

The general expression for the correction to the matrix of propagators following from this diagram is as follows:

the electric current with T , in comparison with the tree-level result.

IV. DISCUSSION AND CONCLUSION

We have shown that, at the second-loop order, the only contribution to the current that grows with the pulse duration T fast enough ($\sim T^2$ or faster) to substantially alter the tree-level current (which grows linearly with T) comes from the diagrams depicted in Fig. 2. Other diagrams from Figs. 3–6 either lead to mass renormalization or are suppressed in the limit of large T . (However, they may contain contributions that grow with the time of measurement t , which accumulate after the pulse is switched off.) Therefore, a significant modification of the tree-level Schwinger current does occur at the two-loop level and is due to the creation of photons, which subsequently decay into pairs.

Thus, the main conclusion of the present paper is that, for a sufficiently long background pulse, Schwinger's expression for the current of created particles is drastically modified. This, in particular, implies that, for a long enough pulse, one must perform a resummation of the leading contributions from all loop orders. The form of these leading contributions may depend on whether the background field is weak ($eE \ll m^2$) or strong ($eE \geq m^2$). We will carry out the corresponding calculations in our forthcoming papers. We expect that each charged particle loop is accompanied by an additional Schwinger factor $e^{-\frac{\pi m^2}{eE}}$, on top of the power e^2 of the electric charge. As a result, for a weak field, there is no need to perform a resummation, as the pulse would need to be impractically long to observe the effect: at present day achievable values of electric fields, the

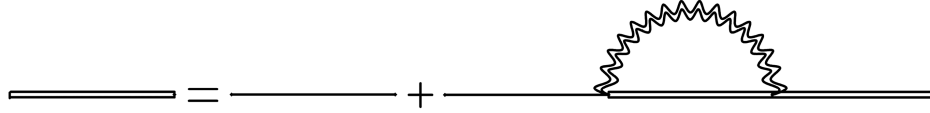


FIG. 7. Dyson-Schwinger equation for the scalar propagator. Here one has to work with the Schwinger-Keldysh matrix of propagators and consider an exact expression only for the Keldysh component of the matrix of propagators of photons and charged scalars (we denote exact propagators by double lines).

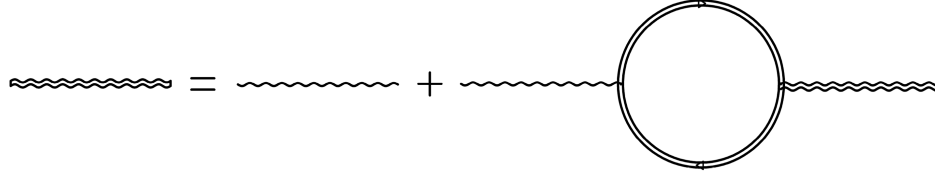


FIG. 8. Dyson-Schwinger equation for the photon propagator. Here one has to work with the Schwinger-Keldysh matrix of propagators and consider an exact expression only for the Keldysh component of the matrix of propagators of photons and charged scalars (we denote exact propagators by double lines).

Schwinger's factor is of the order of e^{-1000} , which makes the necessary duration of the pulse to be much longer than the life of the Universe. However, for a strong pulse (an overcritical electric field), the Schwinger factor is of the order of unity, and a resummation becomes necessary.

For a strong electric pulse, our observations show that, at higher-loop levels, the leading diagrams are those involving a dressed Keldysh photon propagator with scalar bubbles and a dressed Keldysh scalar propagator with corrected photon lines. Diagrams with corrected vertices do not contribute to the leading order. Therefore, for the exact propagators, we obtain Dyson-Schwinger equations of the type shown in Figs. 7 and 8.

The Dyson-Schwinger equations are equivalent to the leading diagrams resummation procedure and are a self-consistent system of equations that describes the behavior of the exact propagators.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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