

Light cone QCD sum rules study of the rare radiative $\Xi_{bb}^* \rightarrow \Xi_b \gamma$ decay

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The rare radiative decay $\Xi_{bb}^* \rightarrow \Xi_b \gamma$ is investigated within the light cone QCD sum rules approach. This decay proceeds through the flavor changing neutral current $b \rightarrow s$ transition in the Standard Model. The hadronic matrix element of the considered decay is parameterized in terms of four tensor form factors $T_1^V(q^2)$, $T_2^V(q^2)$, $T_1^A(q^2)$, and $T_2^A(q^2)$. The sum rules for these form factors describing the $\Xi_{bb}^* \rightarrow \Xi_b \gamma$ decay are derived at $q^2 = 0$ point using the Ξ_b distribution amplitudes. The results of the form factors are employed to calculate the corresponding decay width. Our finding indicates that this weak radiative decay could be within reach of future high statistics studies of doubly heavy baryons at LHCb and upcoming facilities.

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I. INTRODUCTION

The existence of doubly heavy baryons composed of two heavy quarks Q and one light quark q is predicted by the quark model. Doubly heavy baryons are divided into two families, Ξ_{QQ} and Ω_{QQ} . The Ξ_{QQ} family contains a light u or d quark, whereas the Ω_{QQ} family contains a strange quark s . Experimentally, the first evidence for a doubly heavy baryon was reported by the SELEX Collaboration [1]. In 2017, the LHCb Collaboration observed the doubly heavy baryon Ξ_{cc}^{++} in the decay channel $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$ [2] with mass $3621.55 \pm 0.23(\text{stat}) \pm 0.30(\text{syst}) \text{ MeV}/c^2$ [3]. Later, this state was confirmed in the analyses of $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ decay [4]. It should be noted that the spin-parity quantum numbers J^P of the Ξ_{cc}^{++} have not yet been determined. Despite intensive experimental studies, other members of the doubly heavy baryon multiplets have not yet been observed by LHCb and BELLE Collaborations. Experimental searches for these states are ongoing within these collaborations [5]. These observations have stimulated extensive theoretical studies of the properties of doubly heavy baryons.

The mass spectrum of doubly heavy baryons has been investigated within various theoretical approaches, such as the relativistic quark model [6–9], QCD sum rules [10–16],

potential models [17], and lattice QCD calculations [18–20], etc. Studies of the electromagnetic and strong decays also play an exceptional role in understanding the dynamics of doubly heavy baryons (see the review [21] and references therein, as well as [22–36]).

Flavor-changing neutral current processes governed by the transitions $b \rightarrow s$ constitute a promising area for precise tests of the Standard Model (SM). In the SM this transition is forbidden at tree level and occurs only at one-loop level. Hence, such decays allow us to scrutinize the gauge structure of the SM and provide a platform to search for new physics beyond the SM. The semileptonic decays of baryons induced at the quark level by $b \rightarrow s$ transitions represent an important opportunity to obtain more precise information on the properties of doubly heavy baryons. Moreover, weak radiative decays of doubly heavy baryons are highly informative for acquiring additional insight into the dynamics of these systems.

The present work is devoted to the study of weak radiative decay of spin- $\frac{3}{2}$ Ξ_{bb}^* baryon within the light-cone QCD sum rules (LCSR) method [37]. Note that in LCSR the operator product expansion (OPE) is organized in terms of the *twist* of operators, rather than their dimension as in traditional SVZ-type sum rules [38].

The paper is arranged as follows. In Sec. II we derive the light-cone QCD sum rules for the transition form factors, at $q^2 = 0$ point, describing the $\Xi_{bb}^* \rightarrow \Xi_b \gamma$ decay. In our calculations we employ the heavy Ξ_b baryon distribution amplitudes (DAs). In Sec. III we perform the numerical analysis for the form factors obtained in the previous section at $q^2 = 0$ point. We also provide a numerical prediction for the corresponding decay width. The last section contains a discussion of the obtained results and concluding remarks.

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II. LCSR FOR THE $\Xi_{bb}^* \rightarrow \Xi_b \gamma$ DECAY FORM FACTORS

The exclusive decay $\Xi_{bb}^* \rightarrow \Xi_b \gamma$ at the quark level is governed by the flavor-changing neutral current transition $b \rightarrow s \gamma$ in the SM. This transition takes place at loop level and is described by an effective Hamiltonian. Within this framework, the exclusive decays are represented as a convolution of short and long distance contributions. The short-distance dynamics are encoded in the Wilson coefficients, while the long-distance parts are described by the matrix elements between the initial and final baryon states, which are parametrized in terms of a set of transition form factors. The form factors belong to the nonperturbative domain of QCD; hence their calculation requires some nonperturbative methods. Among these nonperturbative approaches, the QCD sum rules method occupies a special place because it is based on the fundamental QCD Lagrangian. In the present work we employ the LCSR approach to compute the relevant form factors for the $\Xi_{bb}^* \rightarrow \Xi_b \gamma$ transition responsible for rare radiative decays.

The rare $b \rightarrow s \gamma$ transition is described by the effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[\sum_{i=1}^6 C_i(\mu) \mathcal{O}_i(\mu) + C_7(\mu) \mathcal{O}_7(\mu) + C_8(\mu) \mathcal{O}_8(\mu) \right], \quad (1)$$

where $C_i(\mu)$ are the Wilson coefficients and $\mathcal{O}_i(\mu)$ are the local operators. Explicit forms of the local operators are (see Ref. [39])

$$\mathcal{O}_1 = (\bar{s}_\alpha \gamma_\mu (1 - \gamma_5) c_\beta) (\bar{c}_\beta \gamma^\mu (1 - \gamma_5) b_\alpha), \quad (2)$$

$$\mathcal{O}_2 = (\bar{s}_\alpha \gamma_\mu (1 - \gamma_5) c_\alpha) (\bar{c}_\beta \gamma^\mu (1 - \gamma_5) b_\beta), \quad (3)$$

$$\mathcal{O}_3 = (\bar{s}_\alpha \gamma_\mu (1 - \gamma_5) b_\alpha) \sum_q (\bar{q}_\beta \gamma^\mu (1 - \gamma_5) q_\beta), \quad (4)$$

$$\mathcal{O}_4 = (\bar{s}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta) \sum_q (\bar{q}_\beta \gamma^\mu (1 - \gamma_5) q_\alpha), \quad (5)$$

$$\mathcal{O}_5 = (\bar{s}_\alpha \gamma_\mu (1 - \gamma_5) b_\alpha) \sum_q (\bar{q}_\beta \gamma^\mu (1 + \gamma_5) q_\beta), \quad (6)$$

$$\mathcal{O}_6 = (\bar{s}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta) \sum_q (\bar{q}_\beta \gamma^\mu (1 + \gamma_5) q_\alpha), \quad (7)$$

$$\mathcal{O}_7 = \frac{e}{4\pi^2} \bar{s}_\alpha \sigma_{\mu\nu} (m_b R + m_s L) b_\alpha F^{\mu\nu}, \quad (8)$$

$$\mathcal{O}_8 = \frac{g_s}{4\pi^2} \bar{s}_\alpha \sigma_{\mu\nu} (m_b R + m_s L) T_{a\beta}^a b_\beta G^{a\mu\nu}, \quad (9)$$

where

$$R = \frac{1 + \gamma_5}{2}, \quad L = \frac{1 - \gamma_5}{2},$$

α and β denote color indices, and e and g_s are the electromagnetic and strong coupling constants, respectively. The tensors $F^{\mu\nu}$ and $G^{a\mu\nu}$ represent the electromagnetic and strong interaction field strength tensors, respectively. In the next discussions we will neglect the terms containing m_s in $\mathcal{O}_7$ and $\mathcal{O}_8$ since $m_s \ll m_b$. The values of the Wilson coefficients C_i can be found in [39]. Since the main contribution comes from operator $\mathcal{O}_7$, the effective Hamiltonian relevant for $b \rightarrow s \gamma$ transition takes the following form

$$\mathcal{H}_{b \rightarrow s \gamma} = -\frac{G_F e}{4\pi^2 \sqrt{2}} V_{tb} V_{ts}^* C_7(m_b) \bar{s} \sigma_{\mu\nu} (m_b R + m_s L) b F^{\mu\nu}. \quad (10)$$

The matrix element of the exclusive $\Xi_{bb}^* \rightarrow \Xi_b \gamma$ decay is obtained by sandwiching this effective Hamiltonian between the initial and final states:

$$\mathcal{M} = \langle \Xi_{bb}^*(p', s') \gamma(q, \epsilon) | \mathcal{H}_{b \rightarrow s \gamma} | \Xi_b(p, s) \rangle. \quad (11)$$

Replacing $\sigma_{\mu\nu} F^{\mu\nu} = -2i \sigma_{\mu\nu} \epsilon^\mu q^\nu$ in the $\mathcal{O}_7$ operator, the matrix element of interest can be written as

$$\langle \Xi_{bb}^*(p') | \bar{s} i \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b | \Xi_b(p) \rangle. \quad (12)$$

In general this matrix element, in terms of form factors, is parametrized in the following way

$$\begin{aligned} \langle \Xi_{bb}^*(p') | \bar{s} i \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b | \Xi_b(p) \rangle \\ = \bar{u}_{\Xi_{bb}^*, \rho}(p') \gamma_5 [T_1^V \mathcal{P}_1^{\rho\mu} + T_2^V \mathcal{P}_2^{\rho\mu} + T_3^V \mathcal{P}_3^{\rho\mu}] u_{\Xi_b}(p) \\ + \bar{u}_{\Xi_{bb}^*, \rho}(p') [T_1^A \mathcal{P}_1^{\rho\mu} + T_2^A \mathcal{P}_2^{\rho\mu} + T_3^A \mathcal{P}_3^{\rho\mu}] u_{\Xi_b}(p). \end{aligned} \quad (13)$$

The Lorentz structures $\mathcal{P}_i^\mu$ are defined as

$$\begin{aligned} \mathcal{P}_1^{\rho\mu} &= \frac{1}{m_{\Xi_b}^2} [q^\rho p'^\mu - p' \cdot q g^{\rho\mu}], \\ \mathcal{P}_2^{\rho\mu} &= \frac{1}{m_{\Xi_b}} [q^\rho \gamma^\mu - \not{q} g^{\rho\mu}], \\ \mathcal{P}_3^{\rho\mu} &= \frac{1}{m_{\Xi_b}^2} [q^2 g^{\rho\mu} - q^\rho q^\mu]. \end{aligned} \quad (14)$$

For real photons, $\mathcal{P}_3^{\rho\mu}$ does not contribute and the following endpoint relations take place at the $q^2 = 0$ point (see also Ref. [40]),

$$T_1^V(0) = T_1^A(0), \quad T_2^{V(A)}(0) = T_2^{A(V)}(0) \mp \frac{m_{\Xi_{bb}^*}}{m_{\Xi_b}} T_1^{A(V)}(0), \quad (15)$$

where $m_{\Xi_{bb}^*}$ and m_{Ξ_b} denote the masses of Ξ_{bb}^* and Ξ_b baryons, respectively, and the $-(+)$ sign holds for $T_2^V(0)$ [$T_2^A(0)$] form factor. From the end-point relations it follows that, to describe the $\Xi_{bb}^* \rightarrow \Xi_b \gamma$ decay at $q^2 = 0$, it suffices to know only two form factors, e.g., $T_1^V(0)$ and $T_2^V(0)$.

For calculation of these form factors we apply light cone QCD sum rules method. We start by considering the following correlation function:

$$\Pi_{\mu\nu}(p, p') = i \int d^4x e^{iq \cdot x} \langle 0 | T \{ J_\mu(0) \bar{b}(x) \times i\sigma_{\nu\alpha} q^\alpha (1 + \gamma_5) s(x) \} | \Xi_b(p) \rangle, \quad (16)$$

where J_μ is the interpolating current of the doubly heavy spin- $\frac{3}{2}$ Ξ_{bb}^* baryon, and $J_\nu^{\text{tr}} = \bar{b} i\sigma_{\nu\alpha} q^\alpha (1 + \gamma_5) s$ is the weak transition current. The explicit current of the doubly heavy spin- $\frac{3}{2}$ baryon is

$$J_\mu = \frac{1}{\sqrt{3}} \epsilon_{abc} \{ 2(q^{a,T} C \gamma_\mu b^b) b^c + (b^{a,T} C \gamma_\mu b^b) q^c \}, \quad (17)$$

where C is the charge-conjugation matrix, ϵ_{abc} is the totally antisymmetric tensor in color space.

According to the standard procedure of the QCD sum rules method, the correlation function is represented in terms of the form factors of the hadron and calculated via the light cone OPE. Sum rules for the relevant physical quantities are obtained by matching the hadronic representation to the OPE result using the quark-hadron duality ansatz. The hadronic dispersion relation for correlation function is obtained by calculating the imaginary part with respect to variable p'^2 . Inserting a complete set of baryon states carrying the same quantum numbers as the interpolating current the following relation is obtained

$$\text{Im}_{p'^2} \Pi_{\mu\nu}(p, p') = \pi \delta(p'^2 - M_{\Xi_{bb}^*}^2) \sum_{s'} \langle 0 | J_\mu(0) | \Xi_{bb}^*(p', s') \rangle \langle \Xi_{bb}^*(p', s') | \bar{b} i\sigma_{\nu\alpha} q^\alpha (1 + \gamma_5) b | \Xi_b(p, s) \rangle + \dots \quad (18)$$

Here the sum $\sum_{s'}$ indicates the summation over the polarizations of the intermediate Ξ_{bb}^* baryon; the dots denote higher states and continuum. The matrix element $\langle 0 | J_\mu(0) | \Xi_{bb}^*(p') \rangle$ is defined as

$$\langle 0 | J_\mu(0) | \Xi_{bb}^*(p') \rangle = \lambda u_\mu(p'), \quad (19)$$

where λ is the residue and $u_\mu(p')$ is Rarita-Schwinger spinor for a spin- $\frac{3}{2}$ doubly heavy baryon, respectively.

The matrix element of the weak transition current between the initial and final baryon is determined via the form factors as given in Eq. (13). Using Eqs. (13) and (19) and performing the summation over the spin of the Ξ_{bb}^* baryon as,

$$\sum_{s'} u_\mu(p', s') u_\nu(p', s') = -(\not{p}' + m_{\Xi_{bb}^*}) \left[g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{2}{3} \frac{p'_\mu p'_\nu}{m_{\Xi_{bb}^*}^2} + \frac{1}{3} \frac{p'_\mu \gamma_\nu - p'_\nu \gamma_\mu}{m_{\Xi_{bb}^*}} \right], \quad (20)$$

for the hadronic part of the correlation function we get

$$\begin{aligned} \Pi_{\mu\nu}^{\text{(phys)}} &= \frac{\langle 0 | J_\mu | \Xi_{bb}^*(p') \rangle \langle \Xi_{bb}^*(p') | \bar{b} i\sigma_{\nu\alpha} q^\alpha (1 + \gamma_5) s | \Xi_b(p) \rangle}{m_{\Xi_{bb}^*}^2 - p'^2} + \dots \\ &= \frac{\lambda}{m_{\Xi_{bb}^*}^2 - p'^2} (\not{p}' + m_{\Xi_{bb}^*}) \left\{ \frac{1}{m_{\Xi_b}^2} (q_\mu p'_\nu - g_{\mu\nu} q \cdot p') (T_1^V \gamma_5 + T_1^A) \right. \\ &\quad \left. + \frac{1}{m_{\Xi_b}} (q_\mu \gamma_\nu - g_{\mu\nu} \not{q}) (-T_2^V \gamma_5 + T_2^A) \right\} u_{\Xi_b}(p) + \dots, \end{aligned} \quad (21)$$

where $\dots$ represents the terms taking contributions from higher states, and the states with spin- $\frac{1}{2}$. Here we would like to make the following remarks. The interpolating current J_μ couples not only to spin- $\frac{3}{2}$ state but also spin- $\frac{1}{2}$ state through the matrix element between vacuum and one particle state

$$\left\langle 0 | J_\mu | B \left(p', s = \frac{1}{2} \right) \right\rangle = (A p'_\mu + B \gamma_\mu) u(p', s). \quad (22)$$

This indicates that the terms proportional to the p'_μ or γ_μ on the left carry contribution from spin- $\frac{1}{2}$ states also. Therefore, to isolate the sole contribution of spin- $\frac{3}{2}$ state we choose the structures not having these factors. Besides, we consider certain order for Dirac structures, such as $\gamma_\mu \not{q}_\nu \not{p}$, to eliminate the present dependent structures. Finally, after the Borel transformation, the correlation function in physical side takes the form

$$\tilde{\Pi}_{\mu\nu}^{(\text{phys})} = \tilde{\Pi}_1 \not{q}_5 g_{\mu\nu} + \tilde{\Pi}_2 \not{q}_5 q_\mu v_\nu + \tilde{\Pi}_3 \not{q}_\mu v_\nu + \tilde{\Pi}_4 q_\mu v_\nu + \dots, \quad (23)$$

where $\tilde{\Pi}$ represents Borel transformed correlation function, $v_\nu = p_\nu / m_{\Xi_b}$ and

$$\begin{aligned} \tilde{\Pi}_1 &= e^{-\frac{m_{\Xi_b}^2}{M^2}} \frac{\lambda(m_{\Xi_b} + m_{\Xi_b^*})(m_{\Xi_b^*} T_1^V - m_{\Xi_b}(T_1^V + 2T_2^V))}{2m_{\Xi_b}^2}, \\ \tilde{\Pi}_2 &= e^{-\frac{m_{\Xi_b}^2}{M^2}} \frac{\lambda T_1^V}{m_{\Xi_b}}, \\ \tilde{\Pi}_3 &= e^{-\frac{m_{\Xi_b}^2}{M^2}} \frac{\lambda T_1^A}{m_{\Xi_b}}, \\ \tilde{\Pi}_4 &= -e^{-\frac{m_{\Xi_b}^2}{M^2}} \frac{\lambda(m_{\Xi_b^*} T_1^A + m_{\Xi_b}(T_1^A + 2T_2^A))}{m_{\Xi_b}}. \end{aligned} \quad (24)$$

The QCD side of the calculation is performed in deep Euclidean region where $p'^2 \ll 0$. The same correlation function, Eq. (16), is used with the interpolating current given in Eq. (17) and the weak transition current. After the application of the Wick theorem, the correlation function is expressed in terms of a heavy quark propagator and the DAs of the heavy Ξ_b baryon as

$$\begin{aligned} \Pi_{\mu\nu}^{\text{QCD}}(p', p) &= \frac{2i}{\sqrt{3}} \int d^4x e^{iq \cdot x} q'^\nu \sum_{i=1}^4 \Lambda_i \{ \text{Tr}[\Gamma_i(\sigma_{\nu\nu'}(1 + \gamma_5))^T S_b^T(-x) C \gamma_\mu] \\ &\quad + S_b(-x) \sigma_{\nu\nu'}(1 + \gamma_5) \Gamma_i^T C \gamma_\mu + \Gamma_i \sigma_{\nu\nu'}(1 + \gamma_5) S_b^T(-x) C \gamma_\mu] \} u_{\Xi_b}(p, s), \end{aligned} \quad (25)$$

where $S_b(-x)$ is the propagator of the b quark. Equation (25) is calculated using the DAs for the heavy Ξ_b baryon obtained using heavy quark effective field theory [41] and Λ_i and Γ_i are

$$\begin{aligned} \Lambda_1 &= \frac{1}{8} f^{(2)} \psi_2(t_1, t_2) & \Gamma_1 &= \not{n} \gamma_5 C^{-1}, \\ \Lambda_2 &= -\frac{1}{8} f^{(1)} \psi_{3\sigma}(t_1, t_2) & \Gamma_2 &= i \sigma_{\xi\varphi} \bar{n}^\xi n^\varphi \gamma_5 C^{-1}, \\ \Lambda_3 &= \frac{1}{4} f^{(1)} \psi_{3s}(t_1, t_2) & \Gamma_3 &= \gamma_5 C^{-1}, \\ \Lambda_4 &= \frac{1}{8} f^{(2)} \psi_4(t_1, t_2) & \Gamma_4 &= \not{n} \gamma_5 C^{-1}, \end{aligned} \quad (26)$$

where $\psi_2, \psi_{3\sigma}(\psi_{3s})$, and ψ_4 are the DAs with twist 2, 3, and 4, respectively, and the light cone vectors n and $\bar{n}$ are defined as,

$$\begin{aligned} n_\alpha &= \frac{1}{vx} x_\alpha, \\ \bar{n}_\alpha &= 2v_\alpha - \frac{1}{vx} x_\alpha. \end{aligned} \quad (27)$$

The DAs of heavy baryon Ξ_b are defined as

$$\psi(t_1, t_2) = \int_0^\infty dw w \int_0^1 du e^{-i w(t_1 u + t_2 \bar{u})} \psi(u, w), \quad (28)$$

with $\bar{u} = 1 - u$, $t_1 = 0$, $t_2 = vx$, and w is momentum of the light diquark. Considering the coefficients of the structures $\not{q}_5 g_{\mu\nu}$, $\not{q}_5 q_\mu v_\nu$, $\not{q}_\mu v_\nu$, and $q_\mu v_\nu$ we obtain the sum rules for the form factor by matching the results obtained from the QCD side to that of physical side.

$$\begin{aligned}
\Pi_{\mu\nu}^{\text{QCD}}(p', p) = \int du \int dw \Bigg\{ & \left[-\frac{2}{\sqrt{3}\Delta} (2f^{(1)}\hat{\psi}_{3\sigma}(u, w)\bar{u} - f^{(2)}\psi_2(u, w)w(2m_b + \bar{u}w) \right. \\
& - f^{(1)}\psi_{3s}(u, w)w(m_b + 2\bar{u}w)) - \frac{2\bar{u}}{\sqrt{3}\Delta^2} (f^{(2)}(\hat{\psi}_2(u, w) - \hat{\psi}_4(u, w))\bar{u}w(2m_b + \bar{u}w) \\
& + 2f^{(1)}\hat{\psi}_{3\sigma}(u, w)(m_b + 2\bar{u}w)q \cdot v) \Big] \not{q}\gamma_5 g_{\mu\nu} - \left[\frac{4}{\sqrt{3}\Delta m_{\Xi_b}} f^{(2)}\psi_2(u, w)\bar{u}w^2 \right. \\
& - \frac{4}{\sqrt{3}\Delta^2 m_{\Xi_b}} f^{(2)}(\hat{\psi}_2(u, w) - \hat{\psi}_4(u, w))\bar{u}^3 w^2 \Big] \not{q}\gamma_5 q_\mu v_\nu - \left[\frac{4}{\sqrt{3}\Delta m_{\Xi_b}} f^{(2)}\psi_2(u, w)\bar{u}w^2 \right. \\
& - \frac{4}{\sqrt{3}\Delta^2 m_{\Xi_b}} f^{(2)}(\hat{\psi}_2(u, w) - \hat{\psi}_4(u, w))\bar{u}^3 w^2 \Big] \not{q}q_\mu v_\nu + \left[\frac{4}{\sqrt{3}\Delta} (f^{(2)}\psi_2(u, w)w(m_b + \bar{u}w) \right. \\
& - f^{(1)}(\hat{\psi}_{3\sigma}(u, w)\bar{u} - \psi_{3s}(u, w)\bar{u}w^2) - \frac{4\bar{u}^2 w}{\sqrt{3}\Delta^2} (2q \cdot v f^{(1)}\hat{\psi}_{3\sigma}(u, w) \\
& + f^{(2)}(\hat{\psi}_2(u, w) - \hat{\psi}_4(u, w))(m_b + \bar{u}w)) \Big] q_\mu v_\nu + \dots \Bigg\} u_{\Xi_b}(v). \quad (29)
\end{aligned}$$

The dots in Eq. (29) represent the contributions coming from other structures and the functions $\hat{\psi}(u, w)$ and Δ are given as

$$\hat{\psi}(u, w) = \int_0^w d\tau \tau \psi(u, \tau), \quad (30)$$

and

$$\Delta = \frac{p'^2 \bar{u}w}{m_{\Xi_b}} - m_{\Xi_b} \bar{u}w + \bar{u}^2 w^2 - m_b^2. \quad (31)$$

Having completed the calculation of both sides of the correlation function, the next stage is equating the coefficients of the same structure in hadronic and QCD sides to obtain the sum rules for the form factors, T_1^V , T_2^V , T_1^A , and T_2^A . The final step for obtaining the sum rules for the form factors is performing a Borel transformation from both sides of the correlation function. Finally for the form factors we get the following sum rules:

$$\begin{aligned}
T_1^V &= e^{\frac{m_{\Xi_b}^2}{M^2}} \frac{m_{\Xi_b} \tilde{\Pi}_2^{\text{QCD}}}{\lambda}, \\
T_2^V &= e^{\frac{m_{\Xi_b}^2}{M^2}} \frac{(-2m_{\Xi_b} \tilde{\Pi}_1^{\text{QCD}} - m_{\Xi_b}^2 \tilde{\Pi}_2^{\text{QCD}} + m_{\Xi_b}^2 \tilde{\Pi}_2^{\text{QCD}})}{2\lambda(m_{\Xi_b} + m_{\Xi_b}^*)}, \\
T_1^A &= e^{\frac{m_{\Xi_b}^2}{M^2}} \frac{m_{\Xi_b} \tilde{\Pi}_3^{\text{QCD}}}{\lambda}, \\
T_2^A &= -e^{\frac{m_{\Xi_b}^2}{M^2}} \frac{(m_{\Xi_b} \tilde{\Pi}_3^{\text{QCD}} + m_{\Xi_b}^* \tilde{\Pi}_3^{\text{QCD}} + \tilde{\Pi}_4^{\text{QCD}})}{2\lambda}, \quad (32)
\end{aligned}$$

where $\tilde{\Pi}_i^{\text{QCD}}$ represents the Borel transformed results of the calculations obtained from the QCD side corresponding to the coefficients of the structures $\not{q}\gamma_5 g_{\mu\nu}$, $\not{q}\gamma_5 q_\mu v_\nu$, $\not{q}q_\mu v_\nu$, and $q_\mu v_\nu$ given in Eq. (29). The Borel transformation and the continuum subtraction are performed with the help of the following master formula:

$$\begin{aligned}
\int_0^1 du \int dw \frac{\rho(u, w)}{\Delta^k} &= (-1)^k \left\{ \int_0^1 du \int_0^{w_0} dw e^{-\frac{s}{M^2}} \frac{\rho(u, w)}{(k-1)! \left(\frac{\bar{u}w}{m_{\Xi_b}}\right) (M^2)^{k-1}} \right. \\
&+ \left. \int_0^1 du \left[\frac{1}{(k-1)!} e^{-\frac{s_0}{M^2}} \sum_{j=1}^{k-1} \frac{1}{(M^2)^{k-j-1}} \frac{1}{s'} \left(\frac{d}{dw} \frac{1}{s'}\right)^{j-1} \frac{\rho(u, w)}{\left(\frac{\bar{u}w}{m_{\Xi_b}}\right)^k} \right]_{w=w_0} \right\}, \quad (33)
\end{aligned}$$

where the integration region is the region in the uw plane where $s > 4m_b^2$. In Eq. (33) $s = m_{\Xi_b}(m_{\Xi_b} + \frac{m_b^2}{\bar{u}w} - \bar{u}w)$, and w_0 is obtained from the solution of the equation $s = s_0$, where s_0 is the continuum threshold.

The analytic results obtained in this section are used in the following section to obtain the results for the form factors numerically.

III. NUMERICAL ANALYSES

This section is devoted to numerical analyses of the analytic results for the form factors given in Sec. II at $q^2 = 0$. Additionally, using the obtained form factors we present the corresponding decay width.

In our numerical analysis the main input parameters are the DAs of the Ξ_b baryon, which are [41]

$$\begin{aligned}\psi_2(u, w) &= w^2 \bar{u} u \sum_{n=0}^2 \frac{a_n C_n^{3/2}(2u-1)}{\varepsilon_n^4 |C_n^{3/2}|^2} e^{-w/\varepsilon_n}, \\ \psi_4(u, w) &= \sum_{n=0}^2 \frac{a_n C_n^{1/2}(2u-1)}{\varepsilon_n^2 |C_n^{1/2}|^2} e^{-w/\varepsilon_n}, \\ \psi_3^{(\sigma,s)}(u, w) &= \frac{w}{2} \sum_{n=0}^2 \frac{a_n C_n^{1/2}(2u-1)}{\varepsilon_n^3 |C_n^{1/2}|^2} e^{-w/\varepsilon_n}.\end{aligned}\quad (34)$$

The parameters a_0 , a_1 , a_2 , and ε_0 , ε_1 , ε_2 in Eq. (34) are given in Ref. [42], $A = 1/2$, $C_n^\lambda(2u-1)$ is the Gegenbauer polynomial, and $|C_n^\lambda|^2$ is

$$|C_n^\lambda|^2 = \int_0^1 du [C_n^\lambda(2u-1)]^2. \quad (35)$$

Besides DAs, we also need other input parameters such as current coupling constant $\lambda = (0.22 \pm 0.03) \text{ GeV}^3$ [43], $f_1 = f_2 = (0.032 \pm 0.009) \text{ GeV}^3$ [44], $|V_{tb}| = 1.014 \pm 0.029$, $|V_{ts}| = (41.5 \pm 0.9) \times 10^{-3}$, $m_b = 4.78 \pm 0.06 \text{ GeV}$, $m_s = 93.4_{-3.4}^{+8.6} \text{ MeV}$, $m_{\Xi_b^-} = (5797.0 \pm 0.6) \text{ MeV}$ [45], $m_{\Xi_{bb}^{*-}} = 10.237 \text{ GeV}$ [46].

In addition to these input parameters, in the results there are auxiliary parameters, such as Borel parameter M^2 and threshold parameter s_0 . The threshold parameter is related to the excited state of the considered baryon, and considering these its value is fixed to the interval $119 \text{ GeV}^2 \leq s_0 \leq 123 \text{ GeV}^2$. Imposing that the contribution of the leading twist-2 term dominates over the power corrections and continuum contributions, the interval for the Borel parameter is determined as $9 \text{ GeV}^2 \leq M^2 \leq 11 \text{ GeV}^2$. Figure 1 presents the dependence of the form factors, T_1^V , T_2^V , T_1^A , and T_2^A , at $q^2 = 0$, on Borel parameter M^2 within the given interval of the Borel parameter, and at different values of threshold parameters. To determine the numerical values of the form factors and their corresponding uncertainties, we carry out the analysis in which the Borel parameter M^2 and the threshold parameter s_0 are randomly sampled within their prespecified intervals. With 5000 random sets of these parameters, we obtain the corresponding histograms presented in Fig. 2, and the mean values and standard deviations of the form factors. From these analyses we obtain the form factors as follows:

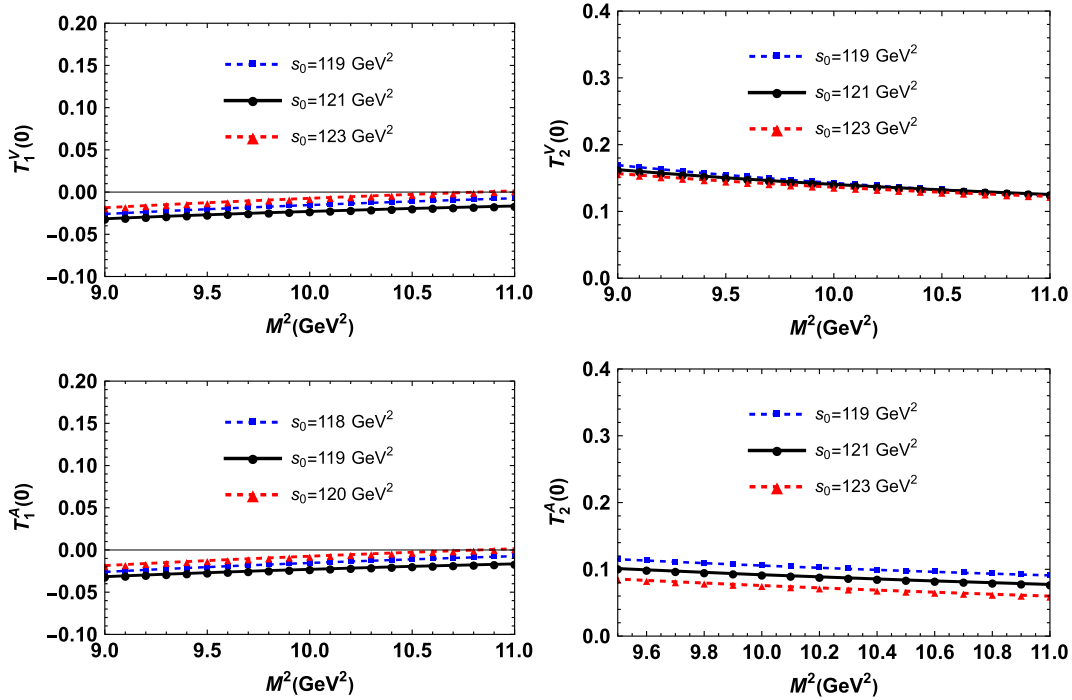


FIG. 1. Variation of the form factors of $\Xi_{bb}^* \rightarrow \Xi_b \gamma$ transition as a function of M^2 at different values of s_0 .

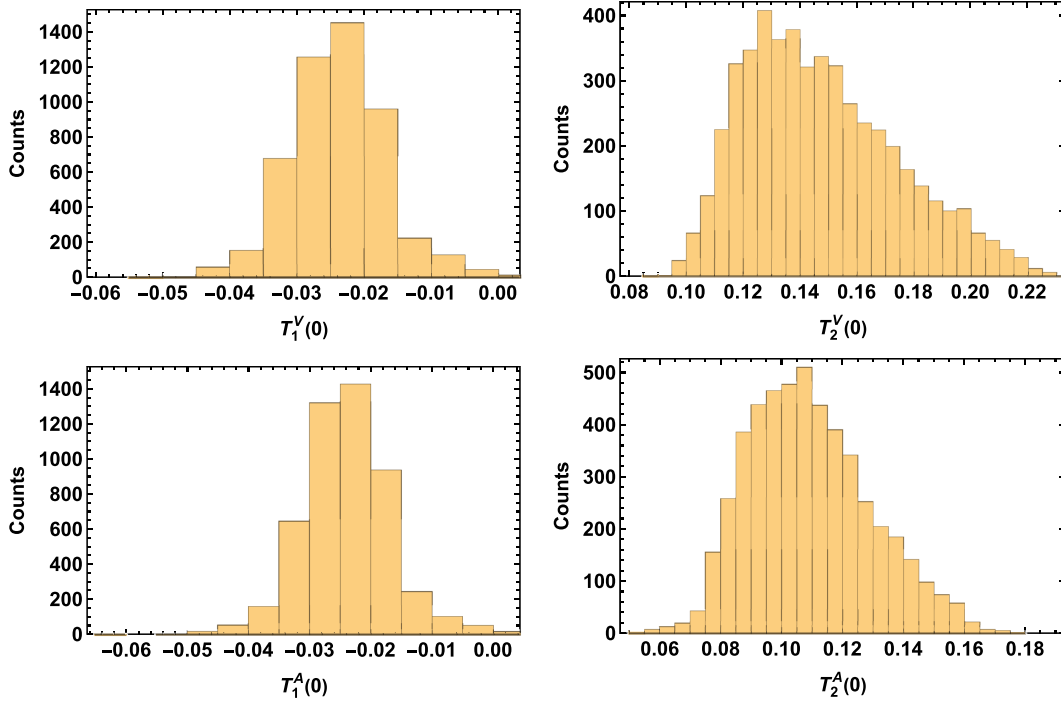


FIG. 2. Histograms of the form factors obtained using randomly selected values of the auxiliary parameters M^2 and s_0 .

$$\begin{aligned} T_1^V(q^2 = 0) &= -0.024 \pm 0.007 \text{ GeV}, & T_2^V(q^2 = 0) &= 0.15 \pm 0.03 \text{ GeV}, \\ T_1^A(q^2 = 0) &= -0.024 \pm 0.007 \text{ GeV}, & T_2^A(q^2 = 0) &= 0.11 \pm 0.02 \text{ GeV}. \end{aligned} \quad (36)$$

We see that the endpoint relations for T_1^V and T_1^A , namely $T_1^V(0) = T_1^A(0)$, are satisfied exactly, and the relations between $T_2^V(q^2)$ and $T_2^A(q^2)$ are satisfied with good accuracy.

With the obtained values of the form factors given in Eq. (36), we predict the width of considered decay. After some calculations for the decay width we get

$$\begin{aligned} \Gamma = & \frac{\alpha_{\text{em}} G_F^2 |C_{7\gamma}^{(0)\text{eff}}(m_b)|^2 |V_{tb} V_{ts}^*|^2 (m_{\Xi_b^*}^2 - m_{\Xi_b}^2)^3}{768\pi^4 m_{\Xi_b^*}^4 m_{\Xi_b}^5} \left[-2m_b m_s (m_{\Xi_b^*}^4 (|T_1^A|^2 - |T_1^V|^2) \right. \\ & + m_{\Xi_b} m_{\Xi_b^*}^3 (2|T_1^A|^2 + 5\text{Re}[T_1^A T_2^{A*}] + 2|T_1^V|^2 - 5\text{Re}[T_1^V T_2^{V*}]) + m_{\Xi_b}^2 m_{\Xi_b^*}^2 (|T_1^A|^2 - |T_1^V|^2 \\ & + 4\text{Re}[T_1^A T_2^{A*}] + 7|T_2^A|^2 + 4\text{Re}[T_1^V T_2^{V*}] - 7|T_2^V|^2) + m_{\Xi_b}^4 (|T_2^A|^2 - |T_2^V|^2) \\ & - m_{\Xi_b}^3 m_{\Xi_b^*} (\text{Re}[T_1^A T_2^{A*}] + 4|T_2^A|^2 - \text{Re}[T_1^V T_2^{V*}] + 4|T_2^V|^2) + m_b^2 (m_{\Xi_b^*}^4 (|T_1^A|^2 + |T_1^V|^2) \\ & - m_{\Xi_b}^3 m_{\Xi_b^*} (\text{Re}[T_1^A T_2^{A*}] + 4|T_2^A|^2 + \text{Re}[(T_1^V - 4T_2^V) T_2^{V*}]) + m_{\Xi_b}^4 (|T_2^A|^2 + |T_2^V|^2) \\ & + m_{\Xi_b} m_{\Xi_b^*}^3 (2|T_1^A|^2 + 5\text{Re}[T_1^A T_2^{A*}] - 2|T_1^V|^2 + 5\text{Re}[T_1^V T_2^{V*}]) + m_{\Xi_b}^2 m_{\Xi_b^*}^2 (|T_1^A|^2 \\ & + |T_1^V|^2 + 4\text{Re}[T_1^A T_2^{A*}] - 4\text{Re}[T_1^V T_2^{V*}] + 7(|T_2^A|^2 + |T_2^V|^2)) + m_s^2 (m_{\Xi_b^*}^4 (|T_1^A|^2 + |T_1^V|^2) \\ & - m_{\Xi_b}^3 m_{\Xi_b^*} (\text{Re}[T_1^A T_2^{A*}] + 4|T_2^A|^2 + \text{Re}[(T_1^V - 4T_2^V) T_2^{V*}]) + m_{\Xi_b}^4 (|T_2^A|^2 + |T_2^V|^2) \\ & + m_{\Xi_b} m_{\Xi_b^*}^3 (2|T_1^A|^2 + 5\text{Re}[T_1^A T_2^{A*}] - 2|T_1^V|^2 + 5\text{Re}[T_1^V T_2^{V*}]) + m_{\Xi_b}^2 m_{\Xi_b^*}^2 (|T_1^A|^2 + |T_1^V|^2 \\ & \left. + 4\text{Re}[T_1^A T_2^{A*}] - 4\text{Re}[T_1^V T_2^{V*}] + 7(|T_2^A|^2 + |T_2^V|^2)) \right]. \end{aligned} \quad (37)$$

Using the numerical values of the quantities entering in Eq. (37), for the decay width we obtain

$$\Gamma(\Xi_{bb}^* \rightarrow \Xi_b \gamma) = (1.23 \pm 0.23) \times 10^{-19} \text{ GeV}. \quad (38)$$

At the end of this section we would like to note that the calculations are directly applicable for estimating $\Xi_{bb}^* \rightarrow \Lambda_b \gamma$, $\Xi_{cc}^* \rightarrow \Xi_c \gamma$, $\Xi_{bc}^* \rightarrow \Xi_b(\Lambda_b) \gamma$, $\Xi_{cc}^* \rightarrow \Xi_c(\Lambda_c) \gamma$ transitions as well. After making appropriate replacements one can immediately estimate the transition form factors as well as the corresponding decay widths. For instance considering the ratio,

$$\frac{\Gamma(\Xi_{bb}^* \rightarrow \Xi_b \gamma)}{\Gamma(\Xi_{bb}^* \rightarrow \Lambda_b \gamma)} \simeq \frac{|V_{tb} V_{ts}^*|^2}{|V_{tb} V_{td}^*|^2} \simeq 20, \quad (39)$$

in the SU(3) symmetry limit,

$$\Gamma(\Xi_{bb}^* \rightarrow \Lambda_b \gamma) \simeq \frac{1}{20} \Gamma(\Xi_{bb}^* \rightarrow \Xi_b \gamma) \simeq 6 \times 10^{-21} \text{ GeV}. \quad (40)$$

IV. SUMMARY AND CONCLUSION

In the present work, the form factors of the rare radiative $\Xi_{bb}^* \rightarrow \Xi_b \gamma$ decay were calculated using the light cone QCD sum rules method and the heavy Ξ_b baryon distribution

amplitudes. At the quark level, the process is driven by the flavor changing neutral current transition $b \rightarrow s \gamma$, and the decay amplitude can be written in terms of four tensor form factors $T_1^V(q^2)$, $T_2^V(q^2)$, $T_1^A(q^2)$, $T_2^A(q^2)$. The numerical values of the form factors were obtained at $q^2 = 0$ as $T_1^V(0) = -0.024 \pm 0.007 \text{ GeV}$, $T_2^V(0) = 0.15 \pm 0.03 \text{ GeV}$, $T_1^A(0) = -0.024 \pm 0.007 \text{ GeV}$, and $T_2^A(0) = 0.11 \pm 0.02 \text{ GeV}$, which satisfy the endpoint relation within their uncertainties. Using the obtained form factors, we computed the decay width as $\Gamma(\Xi_{bb}^* \rightarrow \Xi_b \gamma) = (1.23 \pm 0.23) \times 10^{-19} \text{ GeV}$. The presented calculation scheme is applicable for estimating corresponding rare radiative decays, like $\Xi_{cc}^* \rightarrow \Xi_c(\Lambda_c) \gamma$, $\Xi_{bc}^* \rightarrow \Xi_b(\Lambda_b) \gamma$, $\Xi_{cc}^* \rightarrow \Xi_c(\Lambda_c) \gamma$ as well. Although the predicted width is small, it falls in a range that may become accessible with future high luminosity data on doubly heavy baryons at LHCb or similar experiments.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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- [1] M. Mattson *et al.* (SELEX Collaboration), *Phys. Rev. Lett.* **89**, 112001 (2002).
 - [2] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **119**, 112001 (2017).
 - [3] R. Aaij *et al.* (LHCb Collaboration), *J. High Energy Phys.* **02** (2020) 049.
 - [4] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **121**, 162002 (2018).
 - [5] R. Aaij *et al.* (LHCb Collaboration), *Phys. Rev. Lett.* **121**, 052002 (2018).
 - [6] D. Ebert, R. N. Faustov, V. O. Galkin, and A. P. Martynenko, *Phys. Rev. D* **66**, 014008 (2002).
 - [7] W. Roberts and M. Pervin, *Int. J. Mod. Phys. A* **23**, 2817 (2008).
 - [8] M. Karliner and J. L. Rosner, *Phys. Rev. D* **90**, 094007 (2014).
 - [9] D. H. He, K. Qian, Y. B. Ding, X. Q. Li, and P. N. Shen, *Phys. Rev. D* **70**, 094004 (2004).
 - [10] E. Bagan, M. Chabab, and S. Narison, *Phys. Lett. B* **306**, 350 (1993).
 - [11] Z. G. Wang, *Eur. Phys. J. A* **45**, 267 (2010).
 - [12] V. V. Kiselev and A. K. Likhoded, *Phys. Usp.* **45**, 455 (2002).
 - [13] J. R. Zhang and M. Q. Huang, *Phys. Rev. D* **78**, 094007 (2008).
 - [14] T. M. Aliev, K. Azizi, and M. Savci, *Nucl. Phys. A* **895**, 59 (2012).
 - [15] X.-H. Hu, Y.-L. Shen, W. Wang, and Z.-X. Zhao, *Chin. Phys. C* **42**, 123102 (2018).
 - [16] Y.-J. Shi, W. Wang, and Z.-X. Zhao, *Eur. Phys. J. C* **80**, 568 (2020).
 - [17] J. M. Richard and F. Stancu, *Bled Workshops Phys.* **6**, 25 (2005).
 - [18] R. Lewis, N. Mathur, and R. M. Woloshyn, *Phys. Rev. D* **64**, 094509 (2001).
 - [19] J. M. Flynn, F. Mescia, and A. S. B. Tariq (UKQCD Collaboration), *J. High Energy Phys.* **07** (2003) 066.
 - [20] L. Liu, H.-W. Lin, K. Orginos, and A. Walker-Loud, *Phys. Rev. D* **81**, 094505 (2010).
 - [21] H.-X. Chen, W. Chen, X. Liu, Y.-R. Liu, and S.-L. Zhu, *Rep. Prog. Phys.* **80**, 076201 (2017).
 - [22] H.-Y. Cheng, *Chin. J. Phys. (Taipei)* **78**, 324 (2022).
 - [23] D. Ebert, R. N. Faustov, V. O. Galkin, and A. P. Martynenko, *Phys. Rev. D* **70**, 014018 (2004).
 - [24] W. Roberts and M. Pervin, *Int. J. Mod. Phys. A* **24**, 2401 (2009).
 - [25] T. Branz, A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, and B. Oehl, *Phys. Rev. D* **82**, 074022 (2010).
 - [26] C. Albertus, E. Hernández, and J. Nieves, *Phys. Lett. B* **690**, 265 (2010).
 - [27] H. Bahtiyar, K. U. Can, G. Erkol, M. Oka, and T. T. Takahashi, *Phys. Rev. D* **98**, 114505 (2018).
 - [28] Q. Qin, Y.-J. Shi, W. Wang, G.-H. Yang, F.-S. Yu, and R. Zhu, *Phys. Rev. D* **105**, L031902 (2022).
 - [29] B. Eakins and W. Roberts, *Int. J. Mod. Phys. A* **27**, 1250153 (2012).

- [30] L. Y. Xiao, K. L. Wang, Q. F. Lü, X. H. Zhong, and S. L. Zhu, *Phys. Rev. D* **96**, 094005 (2017).
- [31] L. Y. Xiao, Q. F. Lü, and S. L. Zhu, *Phys. Rev. D* **97**, 074005 (2018).
- [32] H.-Z. He, W. Liang, and Q.-F. Lü, *Phys. Rev. D* **105**, 014010 (2022).
- [33] T. Mehen, *Phys. Rev. D* **96**, 094028 (2017).
- [34] Y.-L. Ma and M. Harada, *J. Phys. G* **45**, 075006 (2018).
- [35] M. J. Yan, X. H. Liu, S. Gonzalez-Solis, F. K. Guo, C. Hanhart, U.-G. Meißner, and B. S. Zou, *Phys. Rev. D* **98**, 091502 (2018).
- [36] B. Chen, S.-Q. Luo, K.-W. Wei, and X. Liu, *Phys. Rev. D* **105**, 074014 (2022).
- [37] V. M. Braun, in *4th International Workshop on Progress in Heavy Quark Physics* (1997), pp. 105–118, [arXiv:hep-ph/9801222](#).
- [38] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, *Nucl. Phys.* **B147**, 385 (1979).
- [39] A. J. Buras, [arXiv:hep-ph/9806471](#).
- [40] G. Hiller and R. Zwicky, *J. High Energy Phys.* **11** (2021) 073.
- [41] A. Ali, C. Hambrock, and A. Y. Parkhomenko, *Theor. Math. Phys.* **170**, 2 (2012).
- [42] A. Ali, C. Hambrock, A. Y. Parkhomenko, and W. Wang, *Eur. Phys. J. C* **73**, 2302 (2013).
- [43] T. M. Aliev, K. Azizi, and M. Savci, *J. Phys. G* **40**, 065003 (2013).
- [44] Z.-G. Wang, *Eur. Phys. J. C* **68**, 479 (2010).
- [45] S. Navas *et al.* (Particle Data Group), *Phys. Rev. D* **110**, 030001 (2024).
- [46] D. Ebert, R. N. Faustov, V. O. Galkin, and A. P. Martynenko, *Phys. Rev. D* **66**, 014008 (2002).