

Thermal properties of the scalar glueballs from holography

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Based on a machine learning holographic QCD model, we construct a systematical framework to investigate the properties of the scalar glueballs continuously from zero temperature to finite temperature. By using both the quasinormal frequencies and the spectral functions, we extract the pole masses, thermal widths, screening masses and dispersion relation of the scalar glueballs in hot medium. It is shown that the pole masses almost remain the vacuum values at temperatures far below the critical temperature T_c , and then decrease with the increasing of temperature until a temperature lower than T_c . This result qualitatively agrees with earlier lattice simulations. While the pole masses increase monotonically above the critical temperature T_c , which agrees with recent lattice calculation. Meanwhile, it is shown that the thermal widths increase monotonically with temperature, which also agrees with the near T_c lattice simulations. The screening mass exhibits a similar temperature-dependent behavior to the pole mass, while the dispersion relation increasingly deviates from the relativistic one as the temperature rises. It is interesting to note that we obtain the imaginary corrections in the thermal correlators, which contains both the temporal and spatial information and might be helpful for the four-dimensional calculations. Furthermore, by comparing the quasinormal modes and the spectral functions, we note that it requires more careful analysis when applying the spectral functions in studying thermal hadrons from holography, since there could be other types of quasinormal modes which are not related with bound states while they may contribute to the peaks of the spectral functions.

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I. INTRODUCTION

A glueball is the most direct prediction from the gluon self-interactions feature of QCD, which attracts much attention from both theory and experiment. The glueball spectra has been investigated by using nonperturbative methods, e.g., lattice QCD calculations [1–3], QCD sum rules [4–7], and recently by using the holographic QCD method [8–12] as well as in the Sakai-Sugimoto model [13–15]. From the experimental side, searching for glueballs has also made much progress [16], and recently the evidence of C-odd odderons were announced by the D0 and TOTEM Collaborations [17], and the glueball-like particle X(2370) was observed at BESIII [18,19].

The investigation on glueballs, which are the only hadronic excitations at zero temperature in pure SU(3) Yang-Mills theory, is of essential meaning both in understanding the nonperturbative perspective of the strong interaction and in revealing the dynamics of the deconfinement phase transition. For the scalar glueballs, the lattice simulations [1,2,20,21] predict a ground-state mass of around 1.475–1.730 GeV, and indicate a linear dependence of the mass square on the radial excitation quantum number n , i.e., $m_n^2 \sim n$, which could be due to the linear confinement. At finite temperature, the breaking of Lorentz symmetry may lead to a splitting between the pole masses and the screening masses, which characterize temporal correlations and spatial correlations respectively. For the temporal aspect, the early lattice simulations find decreasing pole masses of the lowest-lying scalar glueball with the increase of temperature before the deconfinement phase transition [22–24]. However, when including a constant contribution in the meson correlations, the recent lattice calculation [25] observes a different behavior, with the pole masses staying unchanged below T_c and increasing with temperature above T_c . Thus, it is still useful to conduct further research to examine whether the constant term

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originating from the deconfinement transition can have a significant impact in the low-temperature region. Moreover, compared with the temporal correlations, the spatial correlations of the scalar glueballs seems to be less studied. So, it is still interesting to investigate the full thermal properties of glueballs using other nonperturbative methods to obtain more detailed and valuable information.

As an important approach in dealing with the strong coupling problems in quantum field theories, the holographic method based on the anti-de Sitter/conformal field theory correspondence (AdS/CFT) [26–28] turns out to be an effective and powerful method. A famous example is the prediction of the small value of shear viscosity to entropy density $\eta/s = 1/(4\pi)$ [29–31], which matches the order used in fitting the data from the Relativistic Heavy Ion Collider (RHIC) [32–35]. A variety of models have been developed within this framework, including bottom-up models such as the hard-wall model [36], the soft-wall model [37], V-QCD model [38], light-front holographic QCD [39], deformed AdS metric models, and improved holographic QCD models [40–42], which give good descriptions of hadronic physics, thermodynamics, and transport properties of QCD matter.

The scalar glueball spectra of the vacuum from lattice simulations can be well-described by modifying the metric or the dilaton field in dynamical holographic QCD which incorporates a gravity-dilaton system as the background, allowing for a more self-consistent description of the coupling between the dilaton field and the metric [9,12]. It is interesting to see that by dynamically solving the dilaton system, the linear confinement in the glueball spectra can be well-characterized. Besides the scalar glueball spectra, the higher-spin pomeron or odderon spectra can also be described by introducing the tensor fields [10,43–45]. At finite temperature, Ref. [46] introduces the AdS-Schwarzschild metric in the quadratic dilaton model and read the thermal properties from the spectral functions. It gives a decreasing pole masses at very low temperatures. Meanwhile, Ref. [47] employed the same model but obtained the thermal properties through the corresponding quasinormal frequencies, thereby capturing the behavior at higher temperatures. Due to the simplicity of the AdS-Schwarzschild metric and its lack of information about the phase transition, the behavior at finite temperature and its relation with the critical temperature still require further investigation. On the other hand, in order to get more understanding on the thermal properties, it is still quite necessary to do the investigation in a holographic model which has been well-established and tested in many other aspects, at least the thermodynamics.

In this work, to compare with the lattice simulations, we will focus on the scalar glueballs in the pure gluon system. So its dual description would be scalar perturbative above the dual-quenched gravity model. When extending this model to finite temperature and finite density systems,

additional degrees of freedom are introduced to form the Einstein-Maxwell-Dilaton system as the background [48–63]. This approach can quantitatively describe the strongly coupled quark-gluon plasma (QGP) under extreme conditions. When using this model to solve problems, it is necessary to provide an ansatz by specifying the form of one of the following; the dilaton field, the warp factor in the metric, or the potential function of the dilaton field. Thus, there are two predominant methods to obtain solutions of the Einstein-Maxwell-Dilaton model. The first involves inserting the form of the dilaton potential and numerically constructing a family of five-dimensional black holes [49,50,59,60,64]. The second method, known as the potential reconstruction method [51–54,56,57,62,63,65], involves inputting the dilaton or a metric function to determine the dilaton potential. Although this approach results in a temperature-dependent dilaton potential, the model can nevertheless capture many QCD properties through analytical solutions. However, an ansatz that performs well for solving one type of problem may prove inadequate when applied to another type of problem. For example, a quadratic dilaton field can yield very accurate results when solving for the hadronic spectra at zero temperature, but when extending the system to finite temperature by introducing a black hole, this choice imposes a relatively high minimum temperature limit below which it is hard to obtain the temperature-dependent properties of hadrons.

Furthermore, with the advancement of deep learning, related applications have emerged in the field of holographic QCD recently [66,67]. This technique allows for more exploration of the forms of the introduced fields. In Ref. [67], the authors constructed a neural network to learn the equation of state results and baryon number susceptibility at finite temperature and density, thereby obtaining a specific form of the warp factor. Since this model has been carefully examined in describing thermodynamic quantities, we will extend it to study the thermal properties of the scalar glueballs.

In this paper, we will use a modified dynamical holographic models with the aforementioned warp factor to calculate the pole mass, thermal width, screening mass, and dispersion relation of the scalar glueball at finite temperature. In Sec. II, we will introduce the setup of the holographic model and present the black hole solutions derived from the equations of motion after inputting the specific form of the warp factor into the background action. In Sec. III, we will construct a holographic model describing the scalar glueball and use it to study the glueball's properties at both zero and finite temperatures. At zero temperature, we will calculate the glueball spectrum and fix the model. At finite temperature, we employ two complementary methods—calculating the spectral function and determining the complex frequencies of quasinormal modes (QNMs)—to extract the mass and width of glueballs. Additionally, by analyzing the poles of the two-point

correlation function, we obtain the screening mass and dispersion relations at various temperatures. Finally, in Sec. IV, we will provide a summary and discuss the results.

II. HOLOGRAPHY MODEL

To compare with the lattice simulations, we will focus on the pure gluon system and try to consider the corresponding excitations in a dual thermal background. The total action of the five-dimensional holographic QCD model including glueball excitations can generally be taken as

$$S_{\text{total}}^s = S_b^s + S_g^s, \quad (1)$$

where S_b^s is the action for the background in the string frame and S_g^s is the action describing glueballs in the string frame. Since the glueballs are excitations on the thermodynamic background and will be considered as a perturbation, here, S_g^s will not affect the thermodynamics, i.e., taking the probe limit. Therefore, in this section, we will provide a detailed description of the background action and its solutions, while the action describing glueballs will be left to the next section.

To describe QCD matter at finite temperature, a simple choice is to consider the gravity-dilaton system. The gravity-dilaton action S_b^s for the background includes a gravity field g_{MN}^s and a neutral dilatonic scalar field Φ_s , which in string frame is given by

$$S_b^s = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g^s} e^{-2\Phi_s} \times [R_s + 4\partial_M \Phi_s \partial^M \Phi_s - V_s(\Phi_s)], \quad (2)$$

where $V(\Phi)$ is the potential of the dilaton field and G_5 is the Newton constant in five dimensions. Generally, one can choose a specific form of the dilaton potential $V(\Phi)$ and solve the system. In this work, we will take the potential reconstruction approach, in which the explicit forms of the dilaton potential can be solved consistently from the equations of motion (EoM) after assuming the profiles of the dilaton field or the warp factor of the metric.

To facilitate the derivation of the equations of motion later on, we transform the action from the string frame to the Einstein frame using the following transformations:

$$\Phi_s = \sqrt{\frac{3}{8}}\Phi, \quad (3a)$$

$$g_{MN}^s = g_{MN} e^{\sqrt{\frac{2}{3}}\Phi}, \quad (3b)$$

$$V_s(\Phi_s) = e^{-\sqrt{\frac{2}{3}}\Phi} V(\Phi). \quad (3c)$$

The background actions in Einstein frame become

$$S_b = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left[R - \frac{1}{2} \partial_M \Phi \partial^M \Phi - V(\Phi) \right]. \quad (4)$$

The EoMs derived from the action are, respectively,

$$R_{MN} - \frac{1}{2} g_{MN} R - T_{MN} = 0, \quad (5a)$$

$$\partial_M [\sqrt{-g} \partial^M \Phi] - \sqrt{-g} \frac{\partial V}{\partial \Phi} = 0, \quad (5b)$$

with

$$T_{MN} = \frac{1}{2} \left[\partial_M \Phi \partial_N \Phi - \frac{1}{2} g_{MN} (\partial \Phi)^2 - g_{MN} V(\Phi) \right]. \quad (6)$$

We take the following ansatz of metric:

$$ds^2 = \frac{L^2 e^{2A(z)}}{z^2} \left[-f(z) dt^2 + \frac{dz^2}{f(z)} + d\vec{x}^2 \right], \quad (7)$$

where z is the fifth-dimensional holographic coordinate and the AdS radius L of AdS_5 space is set to be 1 (in fact, when one considers a flat boundary, L could be absorbed in other dimensional parameters everywhere). The bulk geometry of this form is a black hole solution with an event horizon z_h where the blackening factor $f(z)$ vanishes. Using the above ansatz of the metric, the equations of motion and constraints for the background fields can be obtained as

$$\Phi'' + \Phi' \left(-\frac{3}{z} + \frac{f'}{f} + 3A' \right) - \frac{L^2 e^{2A}}{z^2 f} \frac{\partial V}{\partial \Phi} = 0, \quad (8a)$$

$$f'' + f' \left(-\frac{3}{z} + 3A' \right) = 0, \quad (8b)$$

$$A'' + \frac{f''}{6f} + A' \left(-\frac{6}{z} + \frac{3f'}{2f} \right) - \frac{1}{z} \left(-\frac{4}{z} + \frac{3f'}{2f} \right) + 3A'^2 + \frac{L^2 e^{2A} V}{3z^2 f} = 0, \quad (8c)$$

$$A'' - A' \left(-\frac{2}{z} + A' \right) + \frac{\Phi'^2}{6} = 0, \quad (8d)$$

where the first equation comes from the equation of motion for the dilaton field, and the next three equations correspond to the z -direction part, the temporal part, and the spatial part of Einstein's equations, respectively. Only three of the above four equations are independent.

To solve those equations, we need to specify the boundary conditions first. For the blackening factor near the horizon, one should impose

$$f(z_h) = 0. \quad (9)$$

Near the boundary $z \rightarrow 0$, we require the metric in string frame to be asymptotic to AdS_5 . The boundary conditions are

$$A(0) = -\sqrt{\frac{1}{6}}\Phi(0) = 0, \quad f(0) = 1. \quad (10)$$

Since there are four unknown functions in the three independent equations, we need to specify the explicit form of one of them in order to solve for the others. When we choose $A(z)$ as the input function, the EoMs can be analytically solved as

$$\Phi'(z) = \sqrt{-6\left(A'' - A'^2 + \frac{2}{z}A'\right)}, \quad (11a)$$

$$f(z) = 1 - \frac{\int_0^z y^3 e^{-3A} dy}{\int_0^{z_h} y^3 e^{-3A} dy}, \quad (11b)$$

$$V(z) = -3z^2 f e^{-2A} \left[A'' + 3A'^2 + \left(\frac{3f'}{2f} - \frac{6}{z} \right) A' - \frac{1}{z} \left(\frac{3f'}{2f} - \frac{4}{z} \right) + \frac{f''}{6f} \right]. \quad (11c)$$

To obtain a solution of the system, we will use the following ansatz:

$$A(z) = d \ln(az^2 + 1) + d \ln(bz^4 + 1), \quad (12)$$

which is taken from Ref. [67], with 4 undetermined parameters included G_5 which will be fixed by comparing the result of the equation of state with the lattice results using the help of machine learning. The result is $a = 0$, $b = 0.072$, $d = -0.584$, $G_5 = 1.326$, and critical temperature $T_c = 0.265$ GeV for pure gluon case. Here, though the first term in Eq. (12) is irrelevant when $a = 0$, we still keep it since it could be turned on to describe systems with flavors.

With the above warp factor, the expression for $\Phi(z)$ can be obtained as

$$\begin{aligned} \Phi(z) = & \frac{\sqrt{438}}{125} \left(4\sqrt{13} \text{ArcTan} \left[\frac{125\sqrt{417} + 9\sqrt{417}z^4 + 3z^2\sqrt{78125 + 3753z^4}}{500\sqrt{13}} \right] \right. \\ & \left. + \sqrt{417} \text{ArcTanh} \left[\frac{3z^2}{\sqrt{\frac{78125}{417} + 9z^4}} \right] \right) - \frac{4}{125} \sqrt{5694} \text{ArcTan} \left[\frac{\sqrt{\frac{417}{13}}}{4} \right]. \end{aligned} \quad (13)$$

It should be noted that the coefficients in the expression depend on the specific values of b and d . It exhibits the following asymptotic behavior in the infrared and ultra-violet regions:

$$\Phi(z \rightarrow 0) = \frac{3}{25} \sqrt{\frac{438}{5}} z^2 - \frac{7497\sqrt{\frac{219}{10}}}{1953125} z^6 + \mathcal{O}(z^7), \quad (14)$$

$$\Phi(z \rightarrow \infty) = \text{constant} + \frac{6\sqrt{20294}}{125} \ln(z). \quad (15)$$

After solving the solution, the temperature can be obtained from

$$T = \frac{|f'(z_h)|}{4\pi}, \quad (16)$$

and the corresponding thermodynamic quantities and properties could be studied.

Using Eq. (11), one obtains the results of the relationship between temperature and the horizon which are plotted with a red line in Fig. 1. It can be seen that, except for a

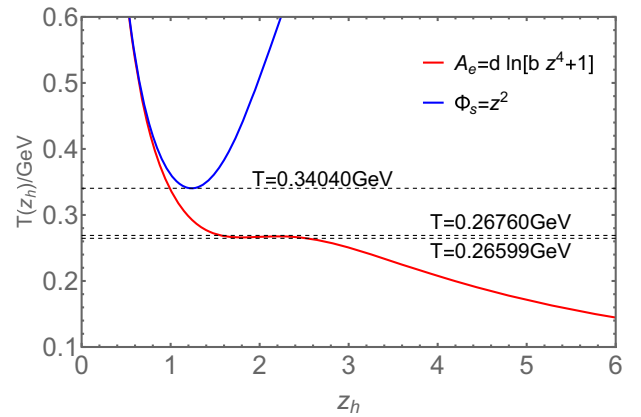


FIG. 1. The temperature T as a function of the black hole horizon z_h . The red line represents the results obtained using $A(z) = d \ln(az^2 + 1) + d \ln(bz^4 + 1)$. In the range of $[1.78, 2.20]$, the function exhibits a slight monotonic increase. The blue line represents the results obtained using $\Phi = z^2$ while $A(z)$ is obtained by solving the equation of motion. There exists a minimum temperature $T_{\min} \approx 0.34$ GeV.

small region $1.78 \text{ GeV}^{-1} \leq z \leq 2.20 \text{ GeV}^{-1}$ where the function increases with temperature, it decreases monotonically in other areas. It can also be proven that when z_h is sufficiently large, the behavior of T is similar to z_h^{-1} and it approaches 0 at infinity. These properties can be used to realize a holographic description of the deconfinement phase transition [54]; the intermediate branch of black hole solutions, where the temperature increases with z_h , is unstable and must be discarded. As a result, only the two disconnected branches are physically relevant—the right branch exists at low temperatures, while the left branch exists at high temperatures. In the narrow temperature range where both solutions coexist, the dominant one is determined by which has the smaller free-energy density. The temperature at which the free energy densities of the two branches are equal corresponds to the critical temperature T_c ; below T_c , the right branch dominates, while above T_c , the left branch does.

For comparison, we also show in the figure the result obtained by introducing $\Phi(z) = z^2$ into the equation of motion, which is another widely used input. In this case, the system's temperature exhibits a minimum value. When $T < T_{\min}$, only the thermal gas solution exists [68], where $A(z)$ and $\Phi(z)$ remain consistent with the black hole solution presented earlier but $f(z) = 1$. This configuration still satisfies the equations of motion. Within this temperature range, the system exhibits the same properties as at zero temperature and does not vary with temperature, which is clearly unphysical. Therefore, within the framework of the present model, the input we adopted (the red line) is more reasonable.

Generally, because we input A_e whose form is independent of temperature, we can derive $V(\Phi)$ with temperature dependent $V(\Phi, T)$. However, as shown in Fig. 2, the difference in V at different temperatures are very small, with relative deviations less than 10^{-2} . It is also noteworthy that Ref. [67] states that the dilaton potential in this model closely resembles the potential in the DeWolfe-Gubser-Rosen model [50].

Finally, we present in Fig. 3, a comparison between the equation of state obtained from our model and the lattice

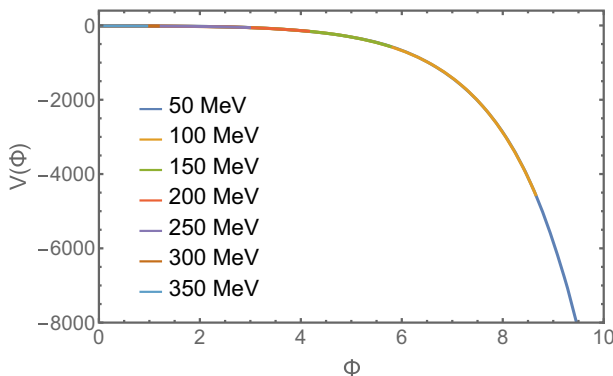


FIG. 2. The dilaton potential $V(\Phi)$ at different temperature.

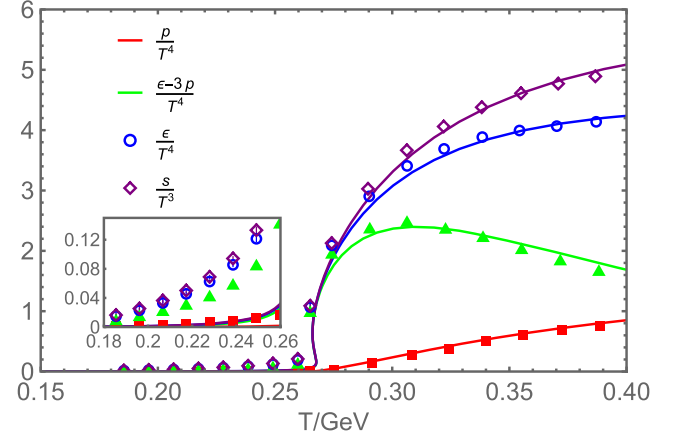


FIG. 3. The results (lines) of the entropy density, pressure, energy density, and trace anomaly as a function of the temperature for a pure gluon system, compared with lattice results [69] in different symbols. Redrawn based on Fig. 2 in Ref. [67].

results [69], as a reproduction of Fig. 2 in Ref. [67]. It can be seen that, through the use of deep learning, the holographic model reproduces the lattice results quite well above T_c , while yielding smaller values below T_c . This discrepancy may arise from the fact that the deep learning fitting process did not specifically emphasize the confined phase, or it may be due to the limitations of the model itself, which might require higher-order corrections to the gravitational action [70].

In addition, in the confined phase, since glueballs themselves can contribute to the thermodynamics, incorporating backreaction—namely, treating the glueball action S_g and the background action S_b self-consistently—may also have a positive effect on the results. However, this would not only significantly increase the complexity of solving the equations of motion but also render the input we adopted invalid. Furthermore, in a more rigorous approach, the scalar glueball action S_g^s should correspond to the four-dimensional gluon-condensate operator $G_{\mu\nu}^2$, whose effect is important for the thermodynamics and should be included in the background part S_b^s . Thus, S_g^s should be derived from the perturbation of S_b^s by imposing the perturbation of $G_{\mu\nu}^2$. However, as is widely done in most holographic QCD models, the effects of operators are not included explicitly. Instead, they are mimicked by several parameters of the models, for example, the parameters in the dilaton potential of the Einstein-Maxwell-Dilaton model. In this way, it is not easy to identify the corresponding operator of the glueballs explicitly. Nevertheless, for the scalar glueball, the form of S_g^s should be a simple action for a 5D massless scalar field, and we will simply model it rather than try to derive it from S_b^s . In this sense, it is reasonable to treat it as a probe and do not consider its backreaction.

Therefore, although our focus in this work is on glueballs rather than on the thermodynamics of the confined phase,

the modifications mentioned above would also affect the glueball properties we aim to study. Meanwhile, the modeling approach we adopt has been widely used in various holographic studies and has yielded many qualitatively or even quantitatively valuable results. Hence, it is necessary to reasonably assess the results obtained from this model for temperatures below T_c . Overall, we believe that our results remain highly valuable at least in a qualitative sense.

III. SPECTRA OF GLUEBALLS

In this section, we discuss the spectra of scalar glueballs at zero temperature and finite temperature. The action of the related glueball fields will be introduced and the glueballs will be treated as excitations above the background. At zero temperature, we obtain the glueball spectrum by solving the eigenvalue problem of the equations of motion. At finite temperature, we study the properties of the glueball spectrum by combining the calculation of the spectral function and the QNM frequencies.

In the string frame, the action describing scalar glueballs can be written as

$$S_g^s = -c_g \int d^5x \sqrt{-g_s} h(\Phi_s) \left(\frac{1}{2} g_s^{MN} \partial_M S \partial_N S + \frac{1}{2} M_5^2 S^2 \right), \quad (17)$$

where s denotes the string frame and c_g describes the coupling strength of the glueball's part in the whole theory [12]. The scalar fields S (we will always use S_{5D} to represent the five-dimensional action in the remainder of this work), are five-dimensional fields that are dual to scalar glueball operator. The five-dimensional mass M_5^2 is given by the AdS₅/CFT₄ correspondence dictionary. Based on the dictionary, the conformal dimension Δ of a q -form operator at the UV boundary is related to the five-dimensional mass square M_5^2 of its dual field in the bulk as follows:

$$M_5^2 = (\Delta - q)(\Delta + q - 4). \quad (18)$$

For the scalar glueball, the dual operator is the scalar gluon operator $\text{Tr}(G^2)$ with $\Delta = 4$, $q = 0$. So M_5^2 can be easily obtained as 0 and only the kinetic term is kept in Eq. (17).

In the original soft-wall model, the coupling function $h(\Phi_s)$ is chosen as $h(\Phi_s) = e^{-\Phi_s}$. It is not difficult to accept such a coupling from the point of view of open string coupling. However, since the scale of the string is far from the scale of phenomenological interests, it is also acceptable to apply other forms of this factor by fitting the experimental observables. As we will see later, with the former case $h = e^{-\Phi_s}$, the model fails to produce bound states for glueballs. Therefore we will take the latter point of view and fix the factor $h(\Phi_s)$ by the vacuum spectra of

the scalar glueball from lattice simulations. It is noticed that a recent machine learning work [71] showed a modified metric can combine both glueball spectra at zero temperature and equation of state. However, this modified model with $h = e^{-\Phi_s}$ still cannot investigate the system continuously from zero to high temperature.

Furthermore, to reduce the dependence of the final conclusion on the form of this factor, we will take three kinds of the form of $h(\Phi_s)$. Since $\Phi(z)$ is monotonical, it is possible to take h as a function of z and obtain the numerical form of it as a function of Φ ,

$$h(z) = e^{-kz^2} \text{ (Choice I)}, \quad (19a)$$

$$h(z) = e^{-\Phi_s - lz^4} \text{ (Choice II)}, \quad (19b)$$

$$-\ln[h(z)] = 3A_s(z) - jz^2 + 3 \ln \left[\frac{2^{\frac{3}{8}} \times 3^{\frac{1}{8}} \Gamma(\frac{5}{4}) I_{\frac{1}{4}} \left(\frac{\sqrt{\frac{8}{3}} j z^2}{\sqrt{6}} \right)}{\sqrt{\frac{8}{3}} j^{\frac{1}{4}} \sqrt{z}} \right] \text{ (Choice III)}, \quad (19c)$$

where j , k , and l are parameters, $\Gamma(z)$ is the Euler gamma function, and $I_n(z)$ is the modified Bessel function of the first kind. For later convenience, we will call Choice I/II/III in this model as MDHM (modified dynamical HQCD model) I/II/III, respectively.

We now provide some explanation for the complicated form of Choice III. In Ref. [9], by substituting $\Phi_s(z) = jz^2$ into the equations of motion, one obtains

$$A(z) = \ln \left[\frac{2^{\frac{3}{8}} \times 3^{\frac{1}{8}} \Gamma(\frac{5}{4}) I_{\frac{1}{4}} \left(\frac{\sqrt{\frac{8}{3}} j z^2}{\sqrt{6}} \right)}{\sqrt{\frac{8}{3}} j^{\frac{1}{4}} \sqrt{z}} \right] \quad (20)$$

and can derive a glueball spectrum that fits well with the lattice results by setting $j = 1 \text{ GeV}^2$. If we denote the two equations above as $\Phi_{s,\text{quadratic}}$ and $A_{\text{quadratic}}$, respectively (note that $A_{\text{quadratic}}$ is in the Einstein frame), and then set $-\ln[h(z)] = 3A_s(z) - 3A_{s,\text{quadratic}} + \Phi_{s,\text{quadratic}}$, according to Eq. (25), we can see that this leads to an effective potential for the glueball equation of motion in our model that is exactly the same as that obtained in the model used in Ref. [9], which implies that it can produce an identical glueball spectrum. The purpose of doing this is to demonstrate the diversity of possible forms of $h(\Phi)$ that can lead to glueball spectra in good agreement with lattice results. Moreover, if the three chosen forms yield similar results when studying the properties of glueballs at finite temperature (as will be shown in the following sections), this would strengthen the model independence of the related conclusions.

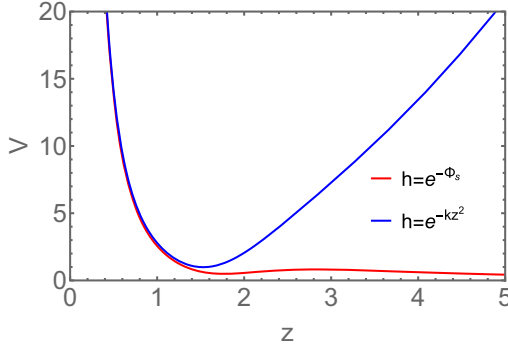


FIG. 4. The effective potentials of scalar glueballs.

In the calculation process, this is equivalent to keeping the $e^{-\phi_s(z)}\mathcal{L}_g^s$ coupling form while setting $\phi_s = -\ln[h(z)]$. Therefore, to keep the formulas in this paper as consistent as possible with those in other works using holographic methods, the ϕ_s that appears later is defined in this way, rather than simply referring to the dilaton field in the string frame Φ_s .

After the above clarification, we can derive the EoM from the action S_g^s ,

$$\partial_z(\sqrt{-g_s}e^{-\phi_s}g_s^{zz}\partial_z S) + \sqrt{-g_s}e^{-\phi_s}g_s^{\mu\nu}\partial_\mu\partial_\nu S = 0, \quad (21)$$

and in the momentum space, it satisfies

$$\partial_z\left(\frac{e^{3A_s-\phi_s}}{z^3}f(z)\partial_z S(p, z)\right) + \frac{e^{3A_s-\phi_s}}{z^3}\left(\frac{\omega^2}{f(z)} - \mathbf{p}^2\right)S(p, z) = 0. \quad (22)$$

A. Glueball at zero temperature

When considering the spectra at zero temperature, we have $f(z) = 1$ and $z \in (0, +\infty)$, and via the substitution

$$S_n \rightarrow z^{\frac{3}{2}}e^{-\frac{1}{2}(3A_s-\phi_s)}S_n, \quad (23)$$

the EoM can be brought into Schrödinger-like equation,

$$-\partial_z^2 S_n + V_s S_n = m_n^2 S_n, \quad (24)$$

with the five-dimensional effective Schrödinger potential

$$V_s = \frac{3A_s'' + \frac{3}{z} - \phi_s''}{2} + \frac{(3A_s' - \frac{3}{z} - \phi_s')^2}{4}. \quad (25)$$

As introduced in Ref. [37], $m_n^2 = \omega^2 - \mathbf{p}^2$ in this equation is the mass of the glueball, which can be obtained by solving for the eigenvalues of this Schrödinger equation. We show the shapes of the effective potentials for the cases $h = e^{-\Phi_s}$ and $h = e^{-kz^2}$ with $k = (0.95 \text{ GeV})^2$, respectively in Fig. 4. As shown in the figure, for the conventional choice ($h = e^{-\Phi_s}$), although the effective potential has a very shallow well, it approaches zero as $z \rightarrow \infty$, indicating that bound states cannot be formed. Therefore, we attempt to overcome this issue by modifying the model. After adjusting the coupling form between the dilaton field and the glueball action, the resulting change in the effective potential enables the formation of glueball bound states.

Taking the lattice results in [1,2,20,21] (shown in Table I), we fit the parameters left in the three choices as $k = 0.95^2 \text{ GeV}^2$ (I), $l = 0.0354 \text{ GeV}^4$ (II), and $j = 1 \text{ GeV}^2$ (III) and work out the glueball masses, as shown in the Table I and Fig. 5. It can be seen that a wide range of choices can yield results that agree well with the lattice data. However, to ensure the Regge trajectory that the squared glueball mass M_n^2 exhibits a linear relationship with the excitation number n over a wide range, the behavior of h needs to be similar to that of e^{-kz^2} at large z . Therefore, although MDHM II can still reproduce a glueball spectrum very close to the lattice results through parameter fitting, its overall behavior lacks a Regge pattern and thus carries limited phenomenological reliability. However, as mentioned earlier, we made these different choices to enhance the independence of the finite-temperature conclusions from the specific form of $h(\Phi)$.

It is also worth noting that the glueball action part of this model is quite similar to that of the soft-wall model, although the latter is set in an AdS_5 spacetime. The soft-wall model, however, struggles to simultaneously describe both the ground-state mass and the Regge slope accurately.

TABLE I. The scalar glueball masses obtained using three different choices of $h(z)$ are presented (MDHM I-III). The parameters are set as $k = 0.95^2 \text{ GeV}^2$ (I), $l = 0.0354 \text{ GeV}^4$ (II), and $j = 1 \text{ GeV}^2$ (III), respectively. The lattice QCD results are taken from Refs. [1,2,20,21], respectively.

	M_1/GeV	M_2/GeV	M_3/GeV	M_4/GeV
MDHM I	1.826	2.777	3.420	3.940
MDHM II	1.593	2.581	3.372	4.086
MDHM III	1.593	2.618	3.311	3.877
LQCD1	1.710(50)(80)	...	...	...
LQCD2	1.730(50)(80)	2.670(180)(130)	...	...
LQCD3	1.653(26)	2.842(40)	...	...
LQCD4	1.475(30)(65)	2.755(70)(120)	3.370(100)(150)	3.990(210)(180)

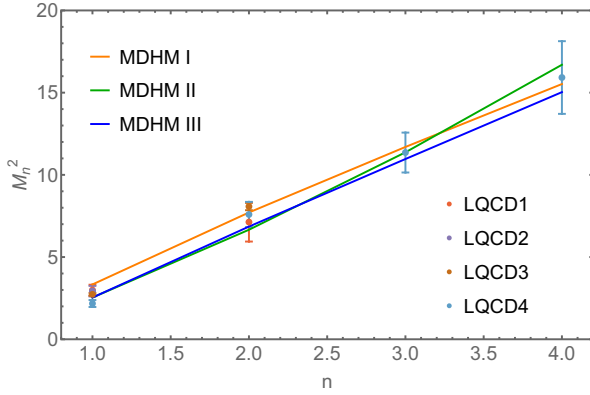


FIG. 5. Comparison between the squared masses of scalar glueball excitations obtained from different choices and their dependence on the excitation number with the lattice results. The lattice results are also taken from [1,2,20,21].

In contrast, by using the improved metric in our model, the results are significantly improved.

B. Glueball at finite temperature

At finite temperature, due to the breaking of Lorentz symmetry, the original definition of mass is no longer appropriate. Instead, two distinct concepts are used; the pole mass and the screening mass, which describe the correlations of the two-point Green function in the temporal and spatial directions, respectively. The pole mass is considered the physically observable mass of a thermal hadron, while the screening mass characterizes the length scale of strong interactions within the thermal medium, i.e., $G(x) \sim e^{-m_{\text{scr}}|x|}/|x|$ when $|x| \gg m_{\text{scr}}^{-1}$. Moreover, due to interactions with the surrounding thermal medium, even originally stable hadrons may become unstable and acquire a thermal width.

On the other hand, these quantities can be related to the pole of the two-point correlation function in momentum space; the pole mass and thermal width correspond to the real and imaginary parts, respectively, of the pole in the retarded two-point correlation function, while the screening mass is given by the pole in the spatial correlation function.

In this section, we use the holographic method to calculate the correlation functions, and thereby obtain the pole mass, screening mass, and thermal width of the scalar glueball at finite temperature, as well as their temperature-dependent behavior.

In detail, the partial function can be obtained from the equivalence of the two sides,

$$S(\omega, \mathbf{p}, z) \rightarrow (z_h - z)^{\mp i\omega \frac{g(z_h)}{g'(z_h)}} \left[1 - \frac{g(z_h)}{2z_h g'(z_h)^2 (2\omega g(z_h) \pm i g'(z_h))} \left((2g'(z_h)^2 (\mp i p^2 z_h + 3\omega - 3z_h \omega A'_s(z_h) + z_h \omega \phi'_s(z_h)) \right. \right. \\ \left. \left. \pm 2iz_h \omega^2 g(z_h) g''(z_h)) - z_h \omega g'(z_h) g''(z_h) \right) (z_h - z) + O[(z_h - z)^2] \right], \quad (30)$$

$$\left\langle e^{i \int d^4 x X_0(x) O_G(x)} \right\rangle_{\text{CFT}} = e^{i S_{5D}[X(x, z)]}, \quad (26)$$

where $S_{5D}[X(x, z)]$ is the on shell action of the bulk field $X(x, z)$ dual to $O_G(x)$, with $X_0(x)$ as a source term. Therefore the two-point correlation function can be derived by differentiating the right-hand side of Eq. (26) with respect to the source twice.

By combining Eqs. (17) and (21), we obtain the on shell five-dimension action,

$$S_{5D}^{\text{on-shell}} = \int d^4 x \frac{c_g}{2} \sqrt{g_s} e^{-\phi_s(z)} g_s^{zz} S(x, z) \partial_z S(x, z) \big|_{z=0}. \quad (27)$$

Then, the two-point correlation function is

$$G(\omega, \mathbf{p}) = -i \int d^4 x \frac{1}{i} \frac{\delta}{\delta S(x, 0)} \frac{\delta S_{5D}^{\text{on-shell}}}{\delta S(0, 0)} e^{-ipx} \\ = -\frac{c_g}{2} \sqrt{g_s} e^{-\phi_s(z)} g_s^{zz} [K(-\omega, -\mathbf{p}, z) \partial_z K(\omega, \mathbf{p}, z) \\ + K(\omega, \mathbf{p}, z) \partial_z K(-\omega, -\mathbf{p}, z)] \big|_{z=0} \\ = -\frac{c_g e^{3A_s - \phi_s}}{z^3} f(z) \\ \times \text{Re}[K^*(\omega, \mathbf{p}, z) \partial_z K(\omega, \mathbf{p}, z)] \big|_{z=0}, \quad (28)$$

where we extracted the value of the Fourier transform $S(\omega, \mathbf{p}, z)$ of the scalar field $S(x, z)$ at the boundary $S_0(\omega, \mathbf{p}) = S(\omega, \mathbf{p}, 0)$ so that $S(\omega, \mathbf{p}, z)$ can be written as the product of its value at the boundary $S_0(\omega, \mathbf{p})$ and the propagator in the z -direction $K(\omega, \mathbf{p}, z)$. Thus, $S(\omega, \mathbf{p}, z) = S_0(\omega, \mathbf{p}) K(\omega, \mathbf{p}, z)$ and $K(\omega, \mathbf{p}, z)$ satisfies Eq. (22) with boundary conditions Eqs. (29) ($k_0 = 1$) and (30). In addition, the third equality of Eq. (28) holds because $K^*(\omega, \mathbf{p}, z) = K(-\omega, -\mathbf{p}, z)$.

To solve for $K(\omega, \mathbf{p}, z)$, we need to specify the boundary conditions. Near the boundary $z = 0$, the expansion of the solution can be obtained from the equation of motion as

$$S(\omega, \mathbf{p}, z) \rightarrow k_0 + \frac{k_0(\omega^2 - \mathbf{p}^2)}{4} z^2 + k_1 z^4 - \frac{k_0}{16} (\omega^2 - \mathbf{p}^2) \\ \times [\omega^2 - \mathbf{p}^2 + 6A_s''(0) - 2\phi_s''(0)] z^4 \log(z) \\ + \dots, \quad (29)$$

with k_0 the value of source term. Similarly, near the horizon $z = z_h$, it is not difficult to obtain the expansion as

with $g(z) = \int_0^z x^3 e^{-3A(x)} dx$. It can be seen that there are two solutions for $S(\omega, \mathbf{p}, z)$ in the near horizon expansions, known as the incoming wave condition (negative sign in the exponent) and the outgoing wave condition (positive sign

in the exponent), respectively. Choosing the former (latter) corresponds to obtaining the retarded (advanced) correlation function [72].

When we substitute Eq. (29) into Eq. (28), we find that

$$G_R(\omega, \mathbf{p}) = \lim_{z \rightarrow 0} c_g e^{3A_s - \phi_s} f(z) \left(\frac{1}{16} (\mathbf{p}^2 - \omega^2)^2 + \frac{-\mathbf{p}^2 + \omega^2}{2z^2} + \frac{4k_1}{k_0} + \frac{3}{8} (\mathbf{p}^2 - \omega^2) A_s''(0) - \frac{1}{8} (\mathbf{p}^2 - \omega^2) \phi_s''(0) - \frac{1}{4} (\mathbf{p}^2 - \omega^2) \ln(z) (\mathbf{p}^2 - \omega^2 - 6A_s''(0) + 2\phi_s''(0)) \right) + O(z^2). \quad (31)$$

The terms involving logarithms and z^{-2} can be removed through a regularization procedure; therefore, they are irrelevant for the mass poles and spectral functions. When ω and $\mathbf{p}$ takes specific values such that $k_0 = 0$, i.e., $S(\omega, \mathbf{p}, 0) = 0$, the Green's function diverges, indicating that these values correspond to singularities. Under such boundary conditions, the solution $S(\omega, \mathbf{p}, z)$ is referred to as a quasinormal mode, a term that describes the solutions to perturbations in a black hole background, which is consistent with the scenario we are studying, and ω is its complex frequency.

By setting $\mathbf{p} = 0$ and $\omega = 0$ respectively, we obtain the correlation functions in the temporal and spatial directions. By applying the shooting method to determine ω and $|\mathbf{p}|$ under the corresponding boundary conditions, we can extract the pole mass, thermal width, and screening mass. If we keep both ω and $\mathbf{p}$ and solve for the complex frequency corresponding to a given $|\mathbf{p}|$, we can obtain the dispersion relation $\omega(|\mathbf{p}|)$.

Moreover, the retarded Green's function can be used to compute the spectral function,

$$\rho(\omega) = -2 \text{Im} G_R(\omega). \quad (32)$$

If the spectral function exhibits peak structures, the mass and width of the glueball at finite temperature can be extracted. Furthermore, the changes in the spectral function at different temperatures reveal the variations in the properties of glueballs.

In the following, we will present the temperature dependence of the glueball pole mass and thermal width obtained by calculating the QNM frequencies, and simultaneously compute the spectral function for cross-validation. We will then also show the screening mass and dispersion relations at different temperatures.

1. Pole mass and thermal width

As mentioned at the beginning of this section, under the condition that boundary condition Eq. (30) with $\mathbf{p} = 0$ is satisfied, the values of ω that make $S(\omega, 0) = 0$ correspond to the complex frequencies of the QNMs. The real part of ω represents the glueball pole mass, while the imaginary part

corresponds to the thermal width. Using the shooting method to solve the equation, we can obtain the temperature dependence of several QNM complex frequencies in the case of choice I [$h(z) = e^{-kz^2}$], as shown in Fig. 6.

At the same temperature, we can obtain many complex frequencies. Among them, a subset has real parts that approach the masses of the glueball bound states at zero temperature, as discussed in the previous subsection, while their imaginary parts approach zero as the temperature tends to zero. This allows us to associate these modes with the mass and thermal width of glueballs at finite temperature. Figure 6 shows the complex frequencies corresponding to the first four glueball bound states at different temperatures.

These complex frequencies exhibit a common trend as the temperature changes; at low temperatures, the real parts decrease very slowly, making the change almost unnoticeable in the plot; hence, they can be considered approximately constant. Once the temperature increases to a certain value T_a (which we define as the temperature at which the reduction reaches 1% of the zero-temperature mass), the rate of decrease begins to accelerate, continuing until a point T_w , where the real part starts increasing with temperature. The total decrease by this point is about 0.07 GeV. After that, as the temperature reaches T_c , the real part rises sharply due to the phase transition and then increases linearly with temperature. In contrast, the imaginary parts continuously increase with temperature. However, their growth rate is also very small at low temperatures when the scattering effect is not large enough to produce a large thermal width. Around T_r (which we also define as the temperature at which the width increases by 1 MeV), the rate of increase becomes more significant, followed by a sharp rise near T_c , and then a linear increase beyond that point; the case of $n = 1$ is somewhat different, featuring a relatively broad temperature region where it transitions toward linear growth. This indicates the scattering effect from in-medium particles is driving the glueballs to be unstable. We summarize some of the obtained numerical results in Table II.

By comparing the behavior of these frequencies, we can observe the following. Before T_c , both the real and

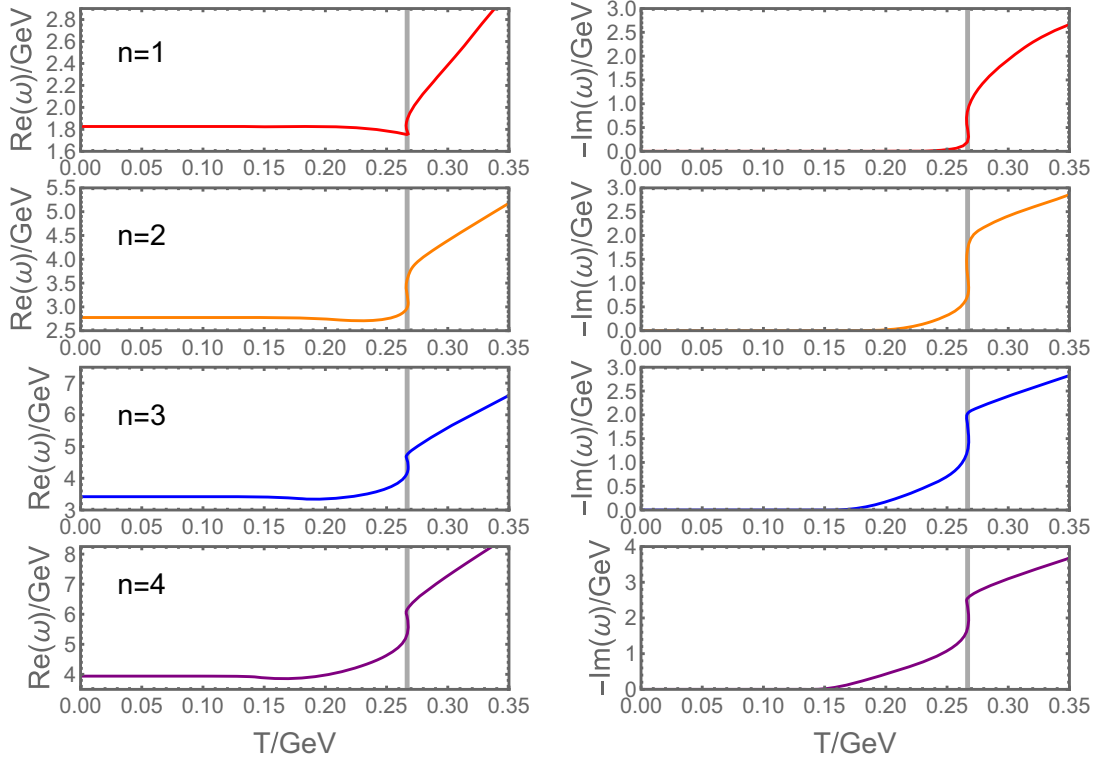


FIG. 6. The temperature dependence of the singularities corresponding to the first four glueball bound states at zero temperature are arranged from top to bottom. The left column shows the real parts of the singularities, while the right column shows the imaginary parts. The gray region in the figure indicates where the phase transition occurs.

imaginary parts of the frequencies corresponding to higher excited states are consistently larger than those of the lower excited states. Additionally, both T_a and T_w decrease as the excitation number increases. After the phase transition around T_c , the ordering of the real parts by magnitude remains unchanged. However, for the imaginary parts, aside from the ground-state frequency—which still has the smallest value—the ordering of their widths for the other excited states become uncertain; in our results, the width of the second excited state eventually exceeds that of the third excited state.

It is worth noting that although we previously mentioned that there are various possible forms of $h(z)$ that can produce similar glueball spectra at zero temperature. We present in Fig. 7 the temperature dependence of the glueball ground-state mass and width obtained under the three different choices, which shows that these different choices still yield similar results at finite temperature. Therefore, the results presented above can be considered, to some extent, model-independent. And for the sake of simplicity and its ability to produce a linear spectrum, we will focus primarily on Choice I.

TABLE II. The complex frequencies $\omega_R - i\omega_I$ corresponding to the lowest four scalar glueball bound states at different temperatures. Here, T_r denotes the temperature at which the thermal width reaches 1 MeV, T_a denotes the temperature at which the real part of the complex frequency has decreased by 1% compared to the corresponding zero-temperature bound-state mass, T_w denotes the temperature at which the real part reaches its minimum before T_c which denotes the phase-transition temperature.

$n = 1$ (m = 1.826 GeV)			$n = 2$ (m = 2.777 GeV)			$n = 3$ (m = 3.420 GeV)			$n = 4$ (m = 3.941 GeV)		
T/GeV	ω_R /GeV	ω_I /GeV	T/GeV	ω_R /GeV	ω_I /GeV	T/GeV	ω_R /GeV	ω_I /GeV	T/GeV	ω_R /GeV	ω_I /GeV
$T_r = 0.218$	1.819	0.001	$T_r = 0.180$	2.767	0.001	$T_r = 0.146$	3.414	0.001	$T_r = 0.138$	3.924	0.001
$T_a = 0.234$	1.808	0.007	$T_a = 0.195$	2.751	0.009	$T_a = 0.167$	3.387	0.010	$T_a = 0.147$	3.902	0.009
$T_c = 0.265$	1.755	0.147	$T_w = 0.230$	2.708	0.137	$T_w = 0.193$	3.344	0.123	$T_w = 0.169$	3.854	0.131
0.268	1.925	0.964	$T_c = 0.265$	2.913	0.627	$T_c = 0.265$	4.040	1.101	$T_c = 0.265$	5.159	1.553
0.350	2.993	2.746	0.268	3.645	1.845	0.268	4.796	2.058	0.268	6.234	2.603
			0.350	5.178	2.857	0.350	6.616	2.826	0.350	8.609	3.686

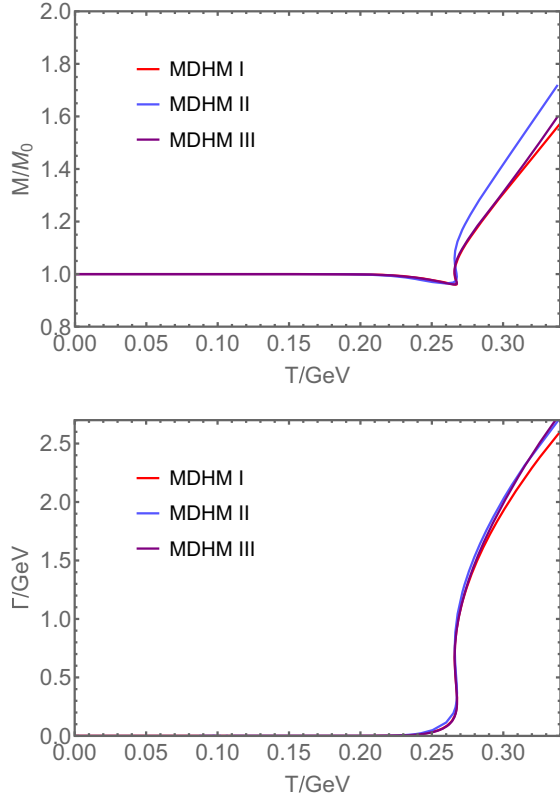


FIG. 7. Comparison of the temperature dependence of the glueball ground-state mass and width under the three different choices of $h(\phi)$.

Therefore, we can qualitatively summarize the behavior of the glueball mass and thermal width at finite temperature. The glueball mass remains approximately constant as the temperature initially increases, then begins to decrease slightly, followed by an upward trend. At the critical temperature T_c , it undergoes a sharp increase due to the phase transition, and beyond that point, it grows linearly with temperature. Meanwhile, the glueball width continuously increases with rising temperature. However, within

the temperature range where the mass remains nearly constant, the increase in width is also minimal. Beyond that, aside from a sharp rise near T_c and the nonlinear increase in the adjacent region, the growth rate becomes significantly larger and follows a linear trend.

In addition, some existing studies have obtained data on the mass and width of the scalar glueball ground state at finite temperature. References [23,24] used lattice QCD combined with Breit-Wigner fitting and concluded that the glueball mass starts to decrease around T_c and continues to decrease at temperatures above T_c , while the width increases near T_c . In contrast, Ref. [25] additionally considered the effect of the constant term in the correlation function that reflects the deconfinement phase transition, and concluded that the glueball mass remains approximately constant before the phase transition and increases afterward. In addition, [47] using holographic methods have investigated the glueball mass and thermal width at finite temperature within the soft-wall model under the AdS-BH (black hole) background. These studies found that the mass first decreases and then increases with temperature (with a turning point at around several tens of MeV), while the width increases rapidly throughout. Although different choices of critical temperature T_c and uncertainties in the ground-state mass at zero temperature prevent a fully quantitative comparison, we can still compile the relevant results for qualitative and even semiquantitative comparison. We summarize the above results in Fig. 8. For ease of comparison, we use rescaled coordinates $\frac{M}{M_0}$ and $\frac{T}{T_c}$ in the figure, where M_0 represents the mass at the lowest temperature for each dataset. This allows the results at lower temperatures to nearly overlap, making the differences in their temperature dependence more visually apparent.

It can be seen that before T_c , the mass obtained from our model agrees reasonably well with the older lattice results, but shows some deviations with the newest lattice results in the region near T_c . In contrast, the results from the soft-wall model in the AdS-black hole (AdS-BH) background show

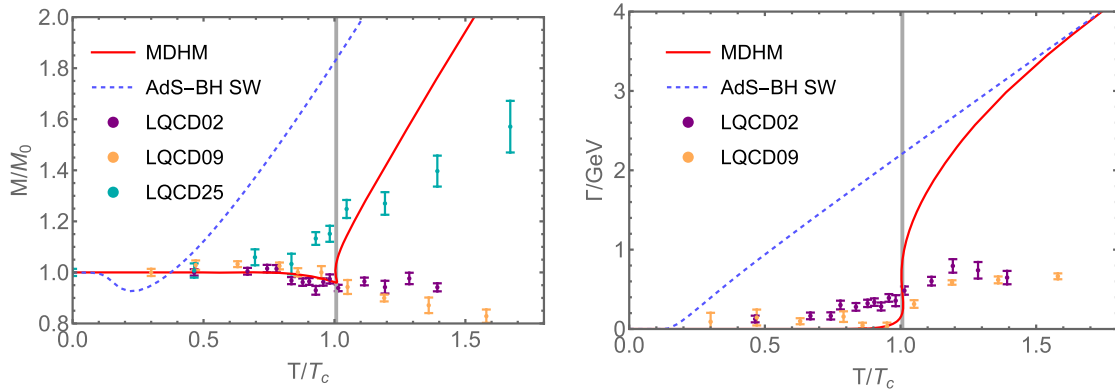


FIG. 8. Comparison of the temperature dependence of the scalar glueball ground state mass and width. The red line represents the result obtained in this work using the MDHM model. The blue line shows the result from [47], where $c = 0.317$ GeV. The lattice results shown in purple, orange, and cyan are taken from [23,24,25], respectively.

a clear trend of mass first decreasing and then increasing linearly with temperature, with the turning points occurring at much lower temperatures leading to significant deviations from the lattice QCD results. After the phase transition at T_c , our results show a linear increase in mass with temperature. Although the rate of increase is higher than that in the lattice data [25], it still supports the conclusion that glueball mass begins to rise after T_c . As for the width, our results exhibit a similar overall trend to the lattice data, but remain smaller before T_c and then rise rapidly after the transition. Eventually, they reach values much larger than those from lattice QCD, and closely approach the results from the soft-wall model under the AdS-BH background. This indicates that glueballs are more stable before T_c , but become rapidly destabilized following the phase transition.

Overall, the temperature dependence of the pole mass obtained from our study can be seen as a reconciliation of previous lattice results; it agrees with the older results below T_c , while above T_c it qualitatively aligns with the latest results. However, quantitative discrepancies still remain. The discrepancy in the mass growth rate above T_c may originate from the probe approximation, which neglects the backreaction of glueballs on the background geometry and from the simplified choice of the coupling function. Another possible source lies in the model's inadequate description of the thermodynamics in the confining phase; the equation of state being smaller than the lattice results may explain the relatively small thermal width observed below T_c . Nevertheless, our results still provide valuable qualitative insights into the behavior of the pole mass after the phase transition and represent a significant improvement over previous holographic studies.

2. Spectral function

In this section, we will use the procedure outlined above to calculate the spectral function of the glueball in order to cross-validate the temperature-dependent behavior of the glueball mass and width obtained in the previous section, but we will follow the procedure of the membrane paradigm as explained in Ref. [73] which is a more straightforward approach.

We first introduce the bulk response function,

$$\zeta(z, \omega) = \frac{\pi(z, \omega)}{\omega S(z, \omega)}, \quad (33)$$

where $\pi(z, \omega)$ is the radial canonical momentum conjugate to the scalar fluctuation $S(z, \omega)$,

$$\pi(z, \omega) = \frac{\delta S_{5D}}{\delta(\partial_z S(z, \omega))} = c_g \frac{e^{3A_s - \phi_s}}{z^3} f(z) \partial_z S(z, \omega). \quad (34)$$

So the second-order differential equation Eq. (22) reduced to a first-order nonlinear equation,

$$\partial_z \zeta + \frac{z^3 \omega}{e^{3A_s(z) - \phi_s(z)} f(z) c_g} \zeta^2 + \frac{e^{3A_s(z) - \phi_s(z)} c_g \omega}{z^3 f(z)} = 0. \quad (35)$$

Requiring regularity at the horizon, one obtains the following boundary condition:

$$\zeta = i \frac{e^{3A_s(z) - \phi_s(z)} c_g}{z^3} \Big|_{z=z_h}, \quad (36)$$

which is equivalent to the incoming-wave boundary condition.

For the retarded Green function we have the following relation:

$$\begin{aligned} G_R(\omega) &= -\frac{c_g e^{3A_s - \phi_s}}{z^3} f(z) K(\omega, z) \partial_z K(\omega, z) \Big|_{z=0} \\ &= \omega \lim_{z \rightarrow 0} \zeta(\omega, z). \end{aligned} \quad (37)$$

It is worth noting that, compared to Eq. (28), we have made use of the fact that $K(\omega, z)$ is real at $z = 0$. Then from Eq. (32), we can obtain the spectral function.

For the scalar glueball, we have obtained the spectral function shown in Fig. 9. Because for large ω the spectral

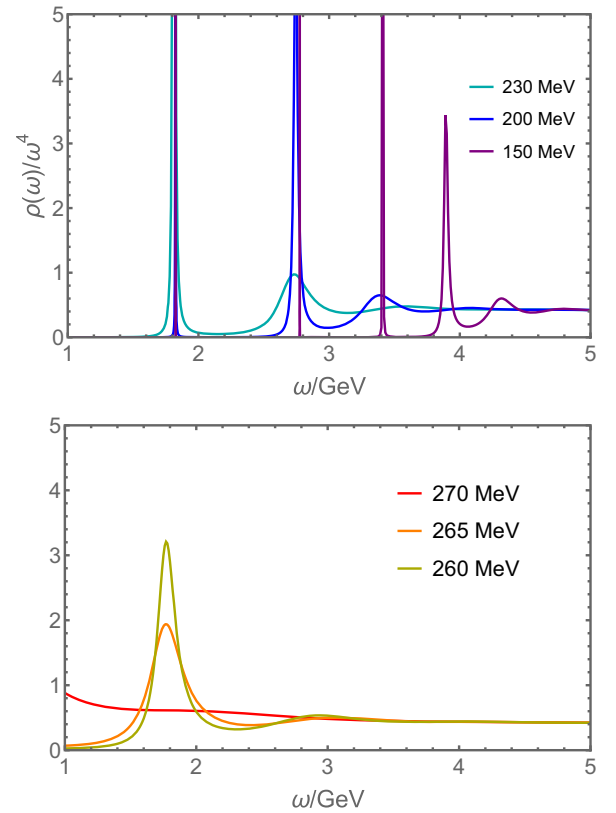


FIG. 9. The scaled spectral functions of the scalar glueball at different temperatures is shown. Upper panel: From 150 MeV to 230 MeV. Lower panel: From 260 MeV to 270 MeV.

function scales as ω^4 , we have used the scaled spectral function

$$\tilde{\rho}(\omega) = \frac{\rho(\omega)}{\omega^4}. \quad (38)$$

From the graph, we can observe how the spectral function changes with temperature. The spectral function exhibits multiple peak structures, whose central values and widths can be approximately associated with the glueball pole masses and thermal widths. The temperature-dependent behavior of these peaks shows the same patterns as those obtained from the QNM frequency calculations in the previous section; the widths of the peaks broaden as the temperature increases, and at the same temperature, peaks corresponding to lower energies are narrower. Additionally, the positions of the peaks shift leftward at first and then rightward as the temperature rises; this behavior is more obvious for peaks at higher energies.

Moreover, the spectral function provides a more intuitive way to observe the variation in the strength of the glueball two-point correlation function through changes in peak height. Although the QNM frequencies allow us to extract glueball masses and widths at relatively high temperatures, the spectral plots reveal that the peak height decreases as the temperature increases. Once the temperature reaches a certain value, the peak becomes indistinguishable due to the combined effect of its broadening and height reduction—a phenomenon often referred to as “melting” in the literature. This indicates that, at such temperatures, the correlation between gluons becomes so weak that it is hardly discernible from the thermal medium. It can be seen that the higher the excitation level, the lower its melting temperature. After undergoing the phase transition at T_c , all states melt due to the drastic increase in their widths.

After obtaining the spectral function, to quantitatively analyze the temperature dependence of the glueball mass and width, it is necessary to extract both by fitting the spectral function using the Breit-Wigner form,

$$\rho(\omega) = \frac{rm\Gamma\omega^s}{(\omega^2 - m^2)^2 + m^2\Gamma^2} + t\omega^4, \quad (39)$$

with parameters m , Γ , r , s , and t , where m and Γ represent the extracted mass and width, respectively. It is worth noting that although it is not the standard Breit-Wigner formula $\rho(\omega) \sim 1/[(\omega - m)^2 + \Gamma^2]$, when the thermal width Γ is not too large and the frequency ω is close to the peak center, it is equivalent to the standard form and gives a better fitting in our case. In fact, this form has been applied in many other references, e.g., [46,74]. The last term in the above equation accounts for the influence of the thermal medium (as the spectral function exhibits a ω^4 behavior at large ω). By removing this contribution, the spectral function can be more accurately fitted using the Breit-Wigner formula.

Since we have already obtained the quantitative behavior of the glueball mass and width at finite temperature through the calculation of QNM frequencies, we will not extract these two quantities by fitting the spectral function. However, it should be noted that the results obtained from these two methods may differ, with the discrepancy increasing as the width becomes larger. Typically, the values obtained from the spectral function fitting are greater than those from the QNM method. A detailed discussion has already been provided in Refs. [23,75].

In summary, the conclusions we obtain from the spectral function are consistent with those derived from the QNM frequency calculations. However, the latter method provides more detailed information, some of which may not be reflected in the spectral function. We discuss this point further in the Appendix. We also want to emphasize that the dissociation of a certain composite particle does not mean the disappearance of the corresponding QNM, but the appearance of a large imaginary part (width). Even when the particles dissociate, the corresponding correlations might still be meaningful and can be characterized by the masses and widths.

3. Screening mass and dispersion relations

If we set $\omega = 0$ in Eq. (30) and solve for the value of $|\mathbf{p}|$ that satisfies this boundary conditions along with $S(\mathbf{p} = i m_{\text{Screen}}, 0) = 0$, we can then compute the pole of the spatial correlation function and obtain the screening mass m_{Screen} . The results we obtained are presented in Fig. 10. For ease of comparison, the pole mass results are plotted together. We also have included part of the screening mass data in Table III.

From the figure, it can be observed that the screening mass exhibits a temperature dependence similar to that of the pole mass; both masses initially decrease very slightly—so slightly that the change can be considered negligible—then the rate of decrease increases, followed by a reversal where the mass increases with temperature. At the critical temperature T_c , a sharp increase occurs, and beyond this point, the mass grows linearly with temperature. Moreover, the higher the excitation level, the greater the extent of both the decrease and the subsequent increase. Therefore, we can adopt the same temperature markers T_a and T_w , defined as in Sec. III B 1 on the pole-mass computation, to categorize the temperature ranges where these different behaviors occur. The corresponding values are also included in Table III.

In terms of their relative magnitudes, the screening mass is generally larger than the pole mass throughout most of the temperature range, specifically, in the region where the mass remains approximately constant and beyond T_c . However, for all excited states except the ground state, there exists a temperature interval before T_c in which the pole mass exceeds the screening mass.

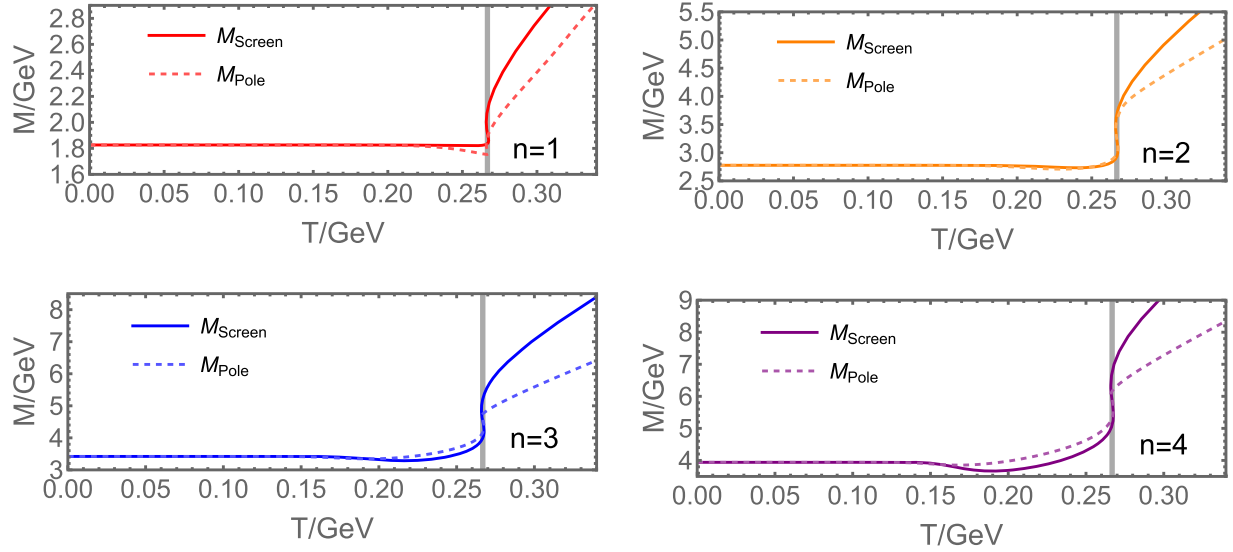


FIG. 10. The temperature dependence of the screening masses for each excited state of the scalar glueball, with the corresponding pole mass results plotted as dashed lines for comparison. The gray region in the figure indicates where the phase transition occurs.

Meanwhile, if we retain both ω and $\mathbf{p}$ in Eq. (30) and solve the equation of motion to obtain a set of $(\omega, \mathbf{p})$ data pairs that satisfy $S(\omega, \mathbf{p}, 0) = 0$, we can then extract the relation between energy and momentum of the glueball at finite temperature, that is, the dispersion relation. When we plot the dispersion relation curve on the $\mathbf{p}^2$ - $\text{Re}(\omega)$ plane, the screening mass and the pole mass correspond to the intersections of the curve with the horizontal and vertical axes, respectively. Although the square of the spatial momentum $\vec{p}^2$ is usually taken as real in the dispersion relation, we extend it to the negative half-axes in order to connect with the screening pole.

At zero temperature, hadrons follow the relativistic dispersion relation $\omega(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m^2}$. At low temperatures, however, the dispersion relation is typically modified to the form

$$\text{Re}(\omega(\mathbf{p})) = \sqrt{u^2 \mathbf{p}^2 + m^2}, \quad (40)$$

where m is the pole mass and u is interpreted as a velocity, which is therefore constrained by $u < 1$ [76]. In fact, such a modification is obtained when we neglected the thermal width and the diffusion effect of the corresponding modes. In general, at high temperatures, as we can see below, due to those effects the dispersion relation will be significantly modified.

We take the ground state as an example and extract the dispersion relations at three different temperatures ($T = 250, 300, 400$ MeV) in Fig. 11. To show the behavior of the dispersion relations more explicitly, we plot the square of the real part of the poles and the imaginary part as functions of the momentum square $\mathbf{p}^2$. It is found that $\text{Re}(\omega)^2$ at $T = 250$ MeV (the red solid line) and $T = 300$ MeV (the orange solid line) almost lie in straight lines with $\mathbf{p}^2$. Also, in the area of positive $\mathbf{p}^2$ the imaginary parts are almost linear with $\mathbf{p}^2$ in the plot region selected. When moving towards the negative momentum square, the square of the real parts reaches zero at a point $\mathbf{p}_0^2 > -m_{\text{Screen}}^2$, and

TABLE III. The screen mass m_s corresponding to the lowest four scalar glueball bound states at different temperatures. Here, $\Delta_m = \frac{m_s - m_{\text{pole}}}{m_{\text{pole}}}$ represents the relative difference between the screening mass and the pole mass with respect to the pole mass, T_a denotes the temperature at which the screen mass has decreased by 1% compared to the corresponding zero-temperature bound-state mass, T_w denotes the temperature at which the screen mass reaches its minimum before T_c which denotes the phase-transition temperature.

$n = 1$ ($m = 1.826$ GeV)			$n = 2$ ($m = 2.777$ GeV)			$n = 3$ ($m = 3.420$ GeV)			$n = 4$ ($m = 3.941$ GeV)		
T/GeV	m_s /GeV	Δ_m	T/GeV	m_s /GeV	Δ_m	T/GeV	m_s /GeV	Δ_m	T/GeV	m_s /GeV	Δ_m
$T_w = 0.257$	1.820	2.715%	$T_a = 0.220$	2.749	1.299%	$T_a = 0.181$	3.386	0.886%	$T_a = 0.153$	3.902	0.504%
$T_c = 0.265$	1.827	4.256%	$T_w = 0.240$	2.732	0.525%	$T_w = 0.217$	3.286	-3.337%	$T_w = 0.190$	3.667	-6.324%
0.268	2.152	11.776%	$T_c = 0.265$	2.869	-1.667%	$T_c = 0.265$	3.864	-4.543%	$T_c = 0.265$	4.911	-5.009%
0.350	3.438	14.867%	0.268	3.804	3.598%	0.268	5.409	12.739%	0.268	7.002	12.292%
			0.350	6.114	18.060%	0.350	8.699	31.490%	0.350	11.258	30.767%

remains zero until $p^2 = -m_{\text{Screen}}^2$, where the imaginary part crosses the horizontal axes. Based on those observations, one can summarize the real and imaginary parts in a unique equation as

$$\omega(p) = \sqrt{u^2 p^2 + m^2} - i(\Gamma + Dp^2), \quad (41)$$

where m , u , Γ , D are four parameters. It is easy to understand that m and Γ stand for the pole mass and thermal width since they are the real and imaginary parts of ω when $p = 0$. At low temperatures, Γ , D is very small, since all the diffusion effects in this model comes from the temperature. In this case, the relation will be reduced to Eq. (40) and u represents the sound velocity of the corresponding mode. Moreover, since the imaginary part at the real momentum space has the form $\omega \sim -iDp^2$, D represents the diffusion constant of the in-medium mode. Of course, all these parameters are temperature-dependent quantities. To show this this behavior directly, we also impose a fitting to the numerical data using this formalism, it does give a good description to data, as shown in Fig. 11. There, the fitting parameters are given as $m = 1.786$ GeV, $\Gamma = 0.032$ GeV, $u = 0.984$, $D = 0.004$ for $T = 250$ MeV (the blue dashed line) and $m = 2.385$ GeV, $\Gamma = 1.927$ GeV, $u = 1.136$, $D = -0.015$ for $T = 300$ MeV (the brown dashed line). From this equation, it is also easy to understand the above numerical results analytically. When $p^2 > -m^2/u^2$, only contribution to the real part is $\sqrt{u^2 p^2 + m^2}$, and thus its square is linear with p^2 . Then, if $p^2 < -m^2/u^2$, $\sqrt{u^2 p^2 + m^2}$ becomes pure imaginary. Thus, the real part disappears and the imaginary part becomes the curve of a square-root type. Here, due to the effect of thermal broadening and diffusion, the screening mass is no longer $m_{\text{Screen}} = m/u$. Instead, it should be solved from the vanishing of the imaginary part, i.e., $\sqrt{-u^2 m_{\text{Screen}}^2 + m^2} - i(\Gamma - Dm_{\text{Screen}}^2) = 0$. From this equation one gets a more general relation between the two mass parameters with the corrections from thermal effects, and it reads

$$m_{\text{Screen}}^2 = \frac{2(m^2 + \Gamma^2)}{u^2 + 2D\Gamma + \sqrt{u^4 - 4D^2 m^2 + 4D\Gamma u^2}}. \quad (42)$$

At low temperatures when D and Γ are sufficiently small, it reduces to $m_{\text{Screen}} = m/u$.

However, for $T = 400$ MeV (the purple solid lines in Fig. 11), we find that both the real and imaginary part exhibit certain nonlinear behavior with p^2 , and Eq. (41) (with $m = 3.583$ GeV, $\Gamma = 3.357$ GeV, $u = 1.237$, $D = -0.056$, the pink dashed lines) can only partially describe the numerical data. Of course, by considering more powers of p^2 , it is possible to describe those high temperature poles. However, since at such high temperatures,

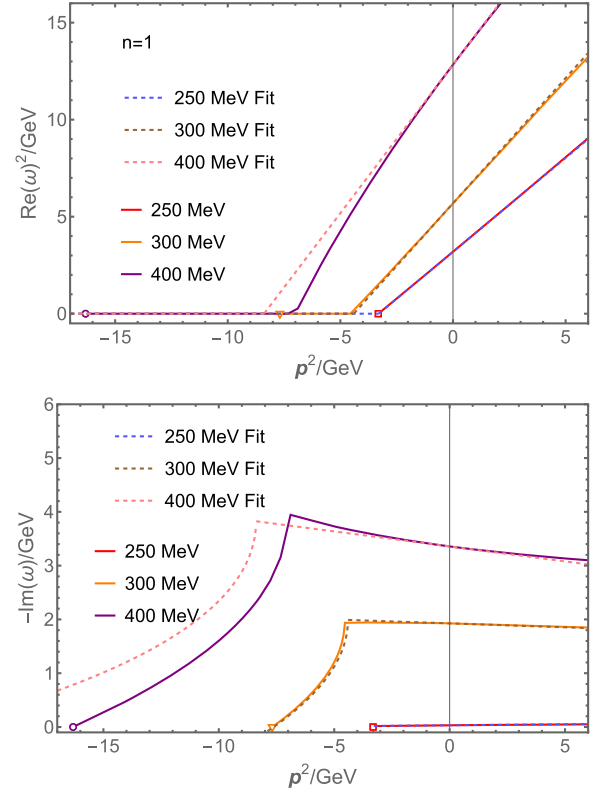


FIG. 11. The dispersion relations of the glueball ground state at three different temperatures. Upper panel: The relation between the square of the real part of the complex frequency and momentum squared. Lower panel: The relation between the imaginary part and momentum squared. We mark the pole positions $(-m_{\text{Screen}}^2, 0)$ at different temperatures in the figure using hollow points. The dashed lines represent the results obtained by fitting the dispersion relations using Eq. (41).

the particles might already dissociate, we will not go deeper in this direction.

Finally, we note that in our calculation, we find that when $p^2 < -m^2/u^2$, there is another branch of solution corresponding to choosing an opposite sign in the square root. This indicate that the full propagator at finite temperature is proportional to

$$G(\omega, p) \sim \frac{1}{[\omega + i(\Gamma + Dp^2)]^2 - u^2 p^2 - m^2}, \quad (43)$$

which includes the propagation and diffusion effect together (we do not try to get information of the decay constants in the numerator). Though from the thermal field theory, it is also possible to obtain such a correction from the imaginary part of the self-energy, it gives a more convenient way to get the thermal coefficients from the holographic framework. It is interesting to study the effects from those corrections on the thermodynamics. We will leave it to the future.

IV. DISCUSSION AND CONCLUSION

In this work, based on a machine learning holographic QCD model, a systematical framework has been constructed to investigate the properties of the scalar glueballs continuously from zero temperature to finite temperature. The thermal scalar glueballs are considered as excitations on gravitational backgrounds. These backgrounds are obtained by assuming a specific form of the warp factor $A(z)$ in the gravity-dilaton system and have been carefully tested in describing the thermodynamic quantities of the pure gluon system [67]. The coupling function $h(\Phi_s)$ between the glueball sector and the background is fixed by the vacuum spectrum of the glueballs, and it is found to provide a good description of the spectrum. Subsequently, the pole mass, screening mass, thermal widths, and dispersion relations—quantities that characterize the correlations of glueballs—are computed and subjected to a detailed analysis by using both the quasinormal frequencies and the spectral functions.

For the temporal components, we find that the pole mass remains approximately constant at first as the temperature increases, then decreases slightly, and eventually increases again with a sharp rise at the critical temperature T_c , after which it grows linearly. Meanwhile, the width consistently increases with temperature, except for a rapid growth during the phase transition. In particular, the temperature dependence of the ground state shows good agreement with lattice results below T_c , and supports the recently proposed linear growth behavior above T_c . By employing three distinct forms of the coupling function, we observe that this qualitative behavior is, to some extent, independent of the specific form of $h(\Phi_s)$. This finding may lend support to earlier lattice results [22–24] and indicates that the contribution of constant terms at low temperatures [25] warrants further investigation. For the spatial component, we found that the screening mass exhibits a similar trend to the pole mass. Moreover, except for a narrow temperature range before T_c for the excited glueball states, the screening mass is always larger than the pole mass. For the dispersion relation, we find that at relatively low temperatures, it can be well-described by the formula $\omega(\mathbf{p}^2) = \sqrt{u^2 \mathbf{p}^2 + m^2} - i(\Gamma + D\mathbf{p}^2)$, which indicates that the hot meson correlators might have the form of $G(\omega, \mathbf{p}) \sim \frac{1}{[\omega + i(\Gamma + D\mathbf{p}^2)]^2 - u^2 \mathbf{p}^2 - m^2}$. As the temperature increases, however, the nonlinear features of the dispersion relation become more prominent.

However, it should be noted that our model still has several limitations. First, its description of the thermodynamics in the confining phase remains inadequate, which limits its applicability in this temperature regime. In the deconfined phase, the glueball pole masses predicted by the model still show quantitative discrepancies compared with lattice results. Nevertheless, our results can still provide valuable qualitative insights, and the constructed model

serves as a useful reference for how to describe hadronic properties self-consistently across the entire temperature range within the holographic framework. Future improvements, including considering higher-order corrections to the gravitational action, the incorporation of backreaction, and a refined coupling function $h(\Phi)$ (since the results obtained from the three choices show only minor differences, there may be limited room for improvement in this aspect), may help achieve a more accurate holographic description of glueball thermal properties.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: NONBOUND-STATE POLES IN QNM FREQUENCY AND SPECTRAL FUNCTIONS METHOD

In the main text, we have presented the calculation of the glueball pole mass and thermal width at finite temperature using both the QNM frequency method and the spectral function method. We also highlighted the characteristics of each approach; the former allows for more accurate computations and is applicable over a wider temperature range, while the latter, though becoming increasingly difficult to apply as the peak width grows, offers a more intuitive representation since it is in the real frequency space.

However, during the calculation process, we discovered another feature of the QNM method. In the main text, we focused on the poles whose real parts approach the zero-temperature glueball masses and whose imaginary parts approach zero as temperature decreases. In fact, we also identified other types of poles whose real and imaginary parts both tend toward zero at zero temperature, and whose behavior with respect to temperature differs from those that correspond to glueball masses. We will refer to them as nonbound-state poles for now. Nevertheless, the effects of them may not necessarily be reflected in the spectral function.

Let us first give an example that clearly illustrates this feature. When using the model presented in the main text with $h(\Phi) = e^{-\Phi_s}$, i.e., the unmodified dynamical holographic model, we obtain the spectral function shown in Fig. 12.

From this figure, we can clearly observe a peak structure in the spectral function, which shows little variation at low

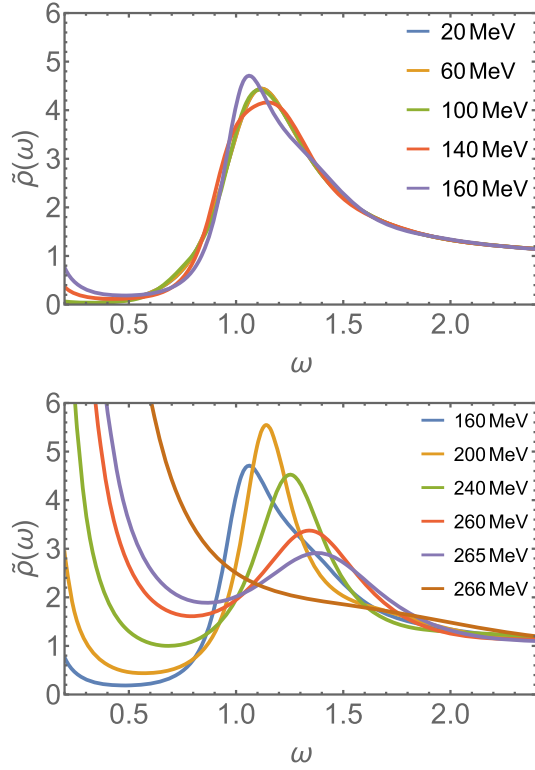


FIG. 12. Spectral functions at different temperatures obtained with $A(z) = d \ln(bz^4 + 1)$ in the dynamical holographic model.

temperatures but both the peak position and its width increase with rising temperature at higher temperatures. At low temperatures, the center of this peak is slightly above 1.0 GeV, which might suggest that it corresponds to the glueball ground state. However, as we showed in Sec. III A, this system cannot produce bound states at zero temperature, so such a correspondence lacks a reasonable foundation. If we instead determine the positions of the poles by calculating the QNM complex frequencies, we obtain the results in Fig. 13. It is noticed that there are other QNMs, but we only show the relevant ones.

In this figure, we show only a subset of the pole evolutions; in addition to these, there exist many other poles with different values but similar behavior. Using this method, we observe that all the poles approach zero as the temperature decreases, but there is no pole that tends toward a constant value. When we compare these results with those obtained from fitting the spectral function using the Breit-Wigner method (as described in detail in Sec. III B 2), we find that at higher temperatures, the fit closely matches the lowest QNM frequency, while at lower temperatures, a noticeable deviation occurs. We can conclude that the peak structure of the spectral function at low temperatures in this model results from the combined contributions of multiple poles, which causes it to remain nearly unchanged over a considerable temperature range. As the temperature increases, the spacing between these poles grows, and

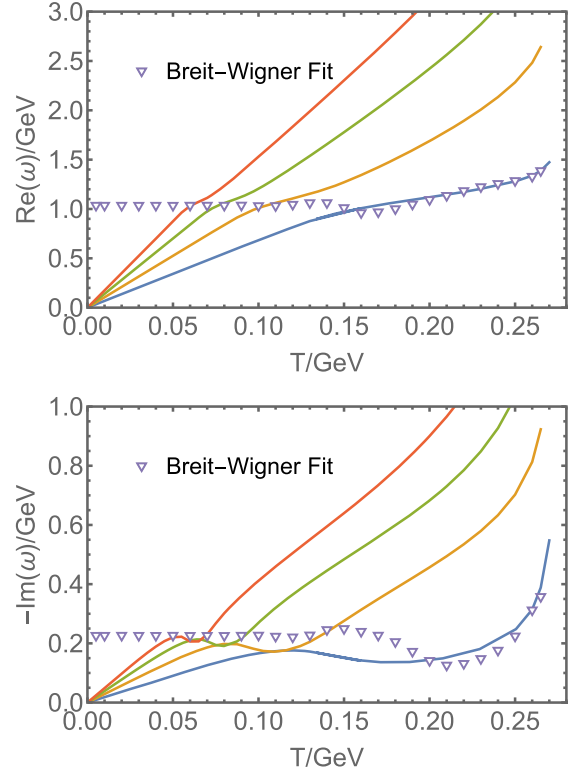


FIG. 13. Temperature dependence of four pole positions in the correlation function obtained via the QNM complex frequency method (colored solid lines), compared with the results extracted from the spectral function using Breit-Wigner fitting (purple inverted triangles). In addition, many other poles with similar behavior have been omitted.

the peak gradually becomes dominated by the lowest-lying pole. During the transition between these two regimes, a decrease in both the peak position and width appears; this is also evident in the unusual shape of the spectral function at 160 MeV shown in Fig. 12.

Returning to the holographic model used in the main text and the soft-wall model under the AdS-BH background, we can likewise observe the existence of such poles that do not correspond to zero-temperature bound states. There are likewise many such poles at a given temperature. We select one of them and compare its temperature-dependent behavior with that of the pole corresponding to the ground state. The results are shown in the Fig. 14. These non-bound state poles exhibit relatively consistent behavior; both their real and imaginary parts increase approximately linearly with temperature. However, the growth slopes differ among different poles. Additionally, since the metric in the MDHM model encodes information about the phase transition, the slopes also change before and after the transition.

However, unlike the previous example, these poles do not appear to have any noticeable effect on the spectral function in this case. This is because, in the two models

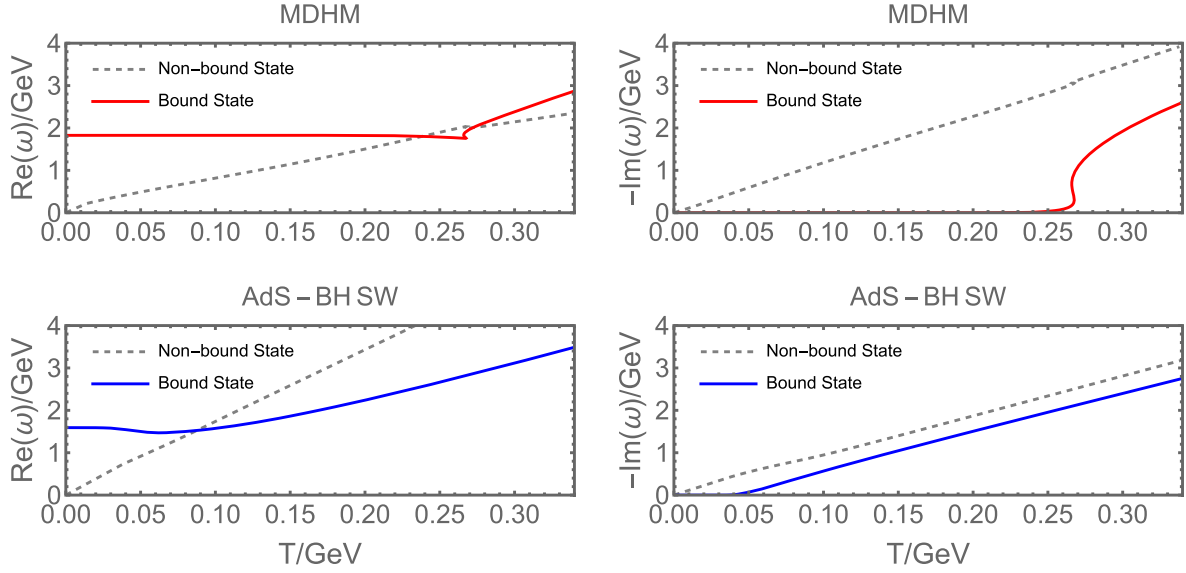


FIG. 14. Comparison of the temperature dependence between a nonbound-state pole and the pole corresponding to the ground state in the improved dynamical holographic model (MDHM) and the soft-wall model under the AdS-BH background (AdS-BH SW). In each model, the nonbound-state poles are indicated by gray dashed lines, while the bound-state poles are represented by solid lines.

capable of producing bound states, the imaginary parts of the nonbound state poles grow much faster with temperature—especially at low temperatures—than those of the bound-state poles. As a result, a “melting” effect occurs at very low temperatures, the width at which bound states melt is often already reached by the nonbound-state poles at around 10 MeV. Therefore, within the temperature range of interest, the influence of nonbound-state poles on the spectral function becomes negligible. In contrast, in the model from the first example, the growth rate of the imaginary parts of nonbound-state poles is significantly slower, allowing visible peak structures to appear even before T_c . It remains unclear whether this peculiar property is related to the model’s inability to produce bound states.

Physically, the states corresponding to these nonbound-state poles can be interpreted as certain excitations of the thermal medium. At finite temperature, these states typically possess relatively large thermal widths, leading to

rapid decay; as a result, they usually produce observable effects only at very low temperatures. However, at higher temperatures, as the widths of bound states increase rapidly, it is possible for some of these excitations—with widths smaller than those of the bound states—to become the dominant contributors to the spectral function (albeit very weakly at that stage).

Moreover, the appearance of these nonbound-state poles highlights another advantage of the QNM frequency method: its ability to comprehensively identify all the poles of the two-point correlation function. This is something that cannot be achieved through the spectral function method alone. Relying solely on the latter may even lead to misleading interpretations, as illustrated in the first example where nontarget states dominate the spectral signal. Therefore, in practical studies, a combination of multiple methods is essential to obtain more robust and reliable conclusions.

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