

One-loop exactness of gravity partition function

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In a previous work, we showed that the two- and three-loop contributions to the partition function of three-dimensional gravity in flat space vanish. This is in agreement with the expected one-loop exactness dictated by the underlying symmetry at the quantum level. To highlight the distinctive nature of the $D = 3$ case, we extend the three-loop computation to arbitrary spacetime dimensions D . In higher dimensions, the number of contributions increases substantially, reinforcing the view that one-loop exactness is a unique feature of three-dimensional gravity.

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I. INTRODUCTION

Einstein gravity in three dimensions has special features. One is the absence of local degrees of freedom. Another peculiarity appears at the quantum level: the partition function turns out to be one-loop exact.

In the case of gravity around AdS_3 , the one-loop exactness was argued by Maloney and Witten in a paper in which they explicitly computed the sum over metrics in the Euclidean setting [1]. The argument is based on the analysis of the classical asymptotic symmetries in AdS_3 carried out by Brown and Henneaux [2]. In summary, the idea is that the Hilbert space of a sector of quantum gravity organizes as Virasoro descendants; therefore, the partition function is a Virasoro character. From the full form of the character, one can read the effective action and conclude that the only correction to the classical action is of order zero in what plays the role of the Planck constant. This is precisely what one would obtain as the one-loop effective action in perturbation theory around AdS_3 . This means that if one performs the perturbative computation of the partition function, it will terminate at one loop, with all the higher loop contributions vanishing. Although the status of pure gravity around AdS_3 as a consistent quantum theory remains under debate (see, for example, [3] and [4,5]), the one-loop exactness rests on solid theoretical grounds.

For three-dimensional gravity in flat space, the idea that the partition function is one loop exact was first discussed by Witten in [6], where he performed a computation using the first-order formalism and argued that the perturbative series should terminate at one loop. More recently, in [7], a similar argument to the AdS_3 case was presented for flat space, now relying on the asymptotic symmetry at null infinity, namely the Bondi-Metzner-Sachs (BMS) group. This line of reasoning suggests that the partition function should coincide with the BMS character. In analogy to the AdS_3 case, one can read the effective action and conclude that the only correction to the classical action is again of order zero in what plays the role of the Planck constant. Then, it follows that the partition function of three-dimensional Einstein gravity in flat space is indeed one-loop exact. As they pointed out, the one-loop exactness does not follow from the absence of local degrees of freedom, as one can see in theories with topological degrees of freedom, such as Chern-Simons theory [7].

Despite the previous argument, the authors of [7] stated that it would be interesting to verify the one-loop exactness of three-dimensional gravity through a direct perturbative computation. We would like to understand how the phenomenon of one-loop exactness manifests in perturbation theory when one computes the vacuum diagrams. Is it the case that each individual diagram vanishes in $D = 3$ due to specific features revealed through dimensional regularization? Or does the cancellation occur more subtly—perhaps through precise compensations between contributions from graviton interactions and those from ghost fields?

We carried out precisely this type of computation in a previous work [8], where we carefully worked out the Feynman diagrams up to three loops in the perturbative expansion for flat space gravity. We found that at

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two-loops, each diagram vanishes trivially in a generic dimension due to dimensional regularization. This result agrees with the literature [9,10]. Therefore, up to two loops, nothing special occurs in $D = 3$. However, a more complex contribution appears at three loops, given that several diagrams are not trivially zero and the number of terms

associated with each diagram grows notably with the dimension. For simplicity, we started with the $D = 3$ case in [8]. We found that each nonvanishing diagram contributes with a given three-loop integral weighted with a coefficient leading to a beautiful zero:

$$\begin{array}{cccccccccc}
 \text{Diagram 1} & + & \text{Diagram 2} & + & \text{Diagram 3} & + & \text{Diagram 4} & + & \text{Diagram 5} & + & \text{Diagram 6} & + & \text{Diagram 7} & + & \text{Diagram 8} & + & \text{Diagram 9} & = 0 \\
 -61/16 & & 1 & & -1/2 & & -7/4 & & -3/8 & & 3/2 & & 11/4 & & -13/8 & & 45/16 &
 \end{array}$$

The numbers below each diagram encode the total coefficient of a single three-loop integral, which includes the symmetry factor. The zero is a result of an intricate cancellation that takes place among the different diagrams appearing at three loops.

One might wonder whether this feature is exclusive to $D = 3$ or not. It might seem that this result will also occur in generic dimensions, as it was the case at two loops, where the contribution was zero regardless of the dimension of the space-time. With that aim, we set out to compute the three-loop correction to the partition function around flat space in arbitrary dimension D .

The paper is organized in the following way: in Sec. II, we briefly review the main considerations regarding the partition function in the AdS_3/flat case. Then, in Sec. III, we write the action in a form suitable for our purposes. In Sec. IV, we summarize the previous results of [8] on the vanishing of the two- and three-loop computations in $D = 3$ for the flat case. Finally, in Sec. V, we present the case of generic dimension. We conclude with some remarks.

II. BRIEF REVIEW OF THE COMPUTATION OF THE PARTITION FUNCTION OF 3D GRAVITY

Before considering our specific result, we want to expand on the general argument given in the introduction concerning the computation and the group theoretical reasons leading to the conclusion that the partition function of 3D gravity around globally AdS space or flat space should be one-loop exact.

As in any quantum theory (in any spacetime dimension), in quantum gravity, the partition function

$$Z = \int \mathcal{D}g_{ab} e^{-S_E[g_{ab}]}, \quad (1)$$

is a quantity of significant interest, given that it captures essential information about its spectrum. As we mentioned before, the three-dimensional model offers an appealing scenario in which certain computations can be performed more easily. In particular, for the negative cosmological constant case, Maloney and Witten [1] were able to

compute the perturbative partition function in the AdS_3 sector by performing an explicit summation over classical configurations and their respective perturbations, whose boundary has the topology of a torus with τ as the modular parameter. If, for simplicity, we take $\tau = i\beta$, then the partition (1) will correspond to the integration of Euclidean geometries with period β , and from the Hilbert space perspective, (1) will coincide with the following trace:

$$Z = \text{Tr} e^{-\beta H}. \quad (2)$$

In [1], the computation explained above was done for generic complex τ . For the particular sector of geometries connected with global AdS_3 , a group theoretic argument can be made: the fact that the Hilbert space of this sector is comprised of Virasoro descendants of the vacuum (more technically, a coadjoint orbit of the Virasoro group) means that its partition function must be given by the vacuum Virasoro character. The result is

$$Z_{\text{AdS}}(\tau) = |q|^{-2k} \prod_{n=2}^{\infty} \frac{1}{|1 - q^n|^2}, \quad (3)$$

where $k \sim \frac{l}{G}$ ($l = \text{ratio of } \text{AdS}_3 \text{ and } G \text{ the Newton constant}$).¹

What is important to notice about (3) is that its expression is 1-loop exact. This can be understood as follows: the effective action is given by $I_{\text{eff}} = I_{\text{cl}} + \sum_{n=1}^{\infty} k^{-n} I_n$ where I_{cl} is the classical action and I_n are the n -loop corrections. Simply by comparison with (3) we realize that $I_{\text{cl}} = -4\pi k \text{Im}(\tau)$ is the on-shell Einstein-Hilbert action,² and the contribution of the vacuum descendants goes exactly like

¹The full partition function is actually given by an infinite regularized sum over $Z_{\text{AdS}}(\tau)$ and all the corresponding geometries related by the modular transformation group acting on τ . The final result was not physically sensible, and this was explored, for instance, in [4,5].

²Actually, the on-shell Einstein-Hilbert action diverges, and it is necessary to add the Gibbons-Hawking term and an intrinsic boundary term to regularize it [11,12].

the 1-loop correction with no higher orders. Therefore, the contribution to the Euclidean partition function coming from the AdS_3 orbit is one-loop exact (see also [13], where the authors derive the one loop exactness from localization arguments). The one-loop contribution (3) was computed explicitly as the appropriate functional determinant in [14] and the result agrees with the one expected from the Virasoro character. In connection with that, the survival of the Virasoro symmetry after the gauge fixing procedure was clarified in [15] and its accessibility in [16].

An analogous computation can be performed in the case of zero cosmological constant. In [7] the flat space one-loop partition function was computed, taking advantage of the fact that the states of the gravity theory organize into representations of the asymptotic symmetry group; in this case, is the BMS_3 group [17,18]. A different perspective on the matter can be found in [19]. From the BMS character, as in the case of AdS_3 , it can be inferred that the partition function is one-loop exact.

III. PERTURBATIVE EXPANSION IN FLAT GRAVITY

In this section, we review the perturbative expansion of the Einstein-Hilbert action in a manner suitable for our purposes.

The gravitational action is given by

$$S_{\text{EH}} = -\frac{1}{2\kappa^2} \int d^d x \sqrt{|g|} (R - 2\Lambda) + S_{\text{bdy}}, \quad (4)$$

where S_{bdy} is an appropriate boundary term and $\kappa^2 = 8\pi G$, with G the Newton constant.

It turns out that the following is a very convenient way to rewrite the Einstein-Hilbert action (4)

$$S_{\text{EH}} = -\frac{1}{2\kappa^2} \int_{M_d} d^d x \sqrt{|g|} g^{ab} g^{mn} g^{rs} (\partial_m g_{ab} \partial_n g_{rs} - \partial_m g_{ar} \partial_n g_{bs} + 2\partial_m g_{br} \partial_a g_{ns} - 2\partial_m g_{na} \partial_b g_{rs}), \quad (5)$$

up to boundary terms [20].

Next, we need to expand the action around a spacetime background that, in our case, is flat space

$$g_{ab} = \eta_{ab} + \kappa h_{ab}, \quad (6)$$

where η is the Minkowski metric. Replacing this into (5) we obtain

$$S_{\text{EH}} = -\frac{1}{2} \int_{M_d} d^d x \sqrt{|g|} g^{ab} g^{mn} g^{rs} (\partial_m h_{ab} \partial_n h_{rs} - \partial_m h_{ar} \partial_n h_{bs} + 2\partial_m h_{br} \partial_a h_{ns} - 2\partial_m h_{na} \partial_b h_{rs}). \quad (7)$$

What is particularly useful about the above expression is the following: to obtain any graviton vertex at any

perturbation order, we only need to expand the inverse metric g^{ab} and the measure $\sqrt{|g|}$ in terms of h_{ab} . In particular, the graviton propagator turns out to be

$$\Delta_{(\text{gr})}^{mnab}[k] = \frac{i}{2k^2} \left(\eta^{ma} \eta^{nb} + \eta^{mb} \eta^{na} - \frac{2}{d-2} \eta^{mn} \eta^{ab} \right) \quad (8)$$

with k being the momentum.

As usual, it is necessary to introduce a gauge-fixing term

$$S_{\text{gf}} = \int_{M_d} d^d x f^m \eta_{mn} f^n, \quad (9)$$

where the gauge-fixing function is

$$f^m = \left(\eta^{lm} \bar{\partial}^n - \frac{1}{2} \eta^{ln} \bar{\partial}^m \right) h_{nl}; \quad (10)$$

and the corresponding ghost field action is

$$S_{\text{gh}} = \int_{M_d} d^d x \bar{c}^m \frac{\delta f_m}{\delta h_{rs}} \mathcal{L}_c g_{rs}, \quad (11)$$

where c^s and $\bar{c}^l$ are the ghost and antighost fields, with $\mathcal{L}_c g_{rs}$ being the Lie derivative of the full metric g_{ab} with respect to the ghost field c^l

$$\mathcal{L}_c g_{rs} = 2\eta_{l(s} \partial_r) c^l + c^l \partial_l h_{rs} + 2h_{l(s} \partial_r) c^l. \quad (12)$$

More explicitly, up to a total derivative, the ghost action takes the form

$$S_{\text{gh}} = \int_{M_d} d^d x \left[\eta_{ls} \bar{c}^s \partial^2 c^l + \bar{c}^r \partial_l \partial_r c^l - \bar{c}^m \partial_m \partial_l c^l - \kappa \left(\partial^r \bar{c}^s \partial_l h_{sr} c^l + \partial^r \bar{c}^s h_{ls} \partial_r c^l + \partial^s \bar{c}^r h_{ls} \partial_r c^l - \frac{1}{2} \partial_m \bar{c}^m \partial_l h_r^l c^l - \partial_m \bar{c}^m h_{ls} \partial^s c^l \right) \right]. \quad (13)$$

The corresponding ghost propagator is

$$\Delta_{(\text{gh})}^{ab}[k] = -i \frac{\eta^{ab}}{k^2}. \quad (14)$$

Again, k is the momentum.

A. Remarks

What is important to notice is that:

- (i) all the vertices, both graviton and ghost, are quadratic in the momenta,
- (ii) there is a single ghost vertex which involves a ghost, a antighost and one graviton leg.

The expression of the graviton vertex increases in complexity with the number of legs. Just for illustration purposes,

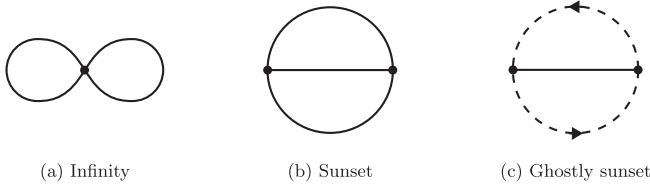


FIG. 3. Two-loop diagrams.

One can see that these contributions trivially vanish not only in $D = 3$ but in any dimension, by noticing that one integral always reduces to $\int dk_1$ or turns out to be odd in the momentum. For instance

$$\begin{aligned}
 & \oint \prod_{i=1}^2 d^D k_i \frac{(k_1 \cdot k_2)^2}{k_1^2 k_2^2 (k_1 + k_2)^2} \\
 &= \oint \prod_{i=1}^2 d^D k_i \frac{1}{4} \frac{((k_1 + k_2)^2 - k_1^2 - k_2^2)^2}{k_1^2 k_2^2 (k_1 + k_2)^2} \\
 &= \frac{1}{4} \oint d^D k_1 d^D k_2 \left(\frac{1}{k_1^2} + \frac{1}{k_2^2} + \dots \right) = 0, \quad (18)
 \end{aligned}$$

where the dots refer to similar terms containing the integral of 1 in k_1 or k_2 (vanishing by the Veltman lemma) or either $\frac{k_1 \cdot k_2}{k_1^2 k_2^2}$ (vanishing by symmetry).

Therefore:

$$S_{\text{eff}}^{(2)} = 0, \quad (19)$$

in arbitrary dimension.

B. Three-loop contribution in $D = 3$

The three-loop contribution to the partition function is significantly more complex. This can be seen from the variety of diagrams that appear at this perturbative order (refer to Figs. 4–7). Although some diagrams turn out to be zero under dimensional regularization [for example, the diagrams Figs. 4(a)–4(c), 5(a), and 5(b)], the remaining graphs contribute with several terms that are nonzero three-loop integrals. After lengthy work, discarding integrals that are zero by dimensional regularization, one can reduce all of them to a single master integral I

$$I = \oint \prod_{i=1}^3 d^3 k_i \frac{(k_1 \cdot k_2)(k_1 \cdot k_3)}{k_1^2 k_2^2 k_3^2 (k_1 + k_2 + k_3)^2}. \quad (20)$$

So all nonzero diagrams contribute to the partition function with a term proportional to (20). This is summarized in Table I. The three-loop effective action is

$$\begin{aligned}
 S_{\text{eff}}^{(3)} &= \text{[Diagrams: two circles joined at a point, a cylinder, a dashed cylinder, a dashed cylinder with arrows, a dashed cylinder with arrows and a central dot, a circle with three internal lines meeting at a central dot, a circle with three internal lines meeting at a central dot and arrows, a circle with three internal lines meeting at a central dot and arrows and a central dot, a circle with three internal lines meeting at a central dot and arrows and a central dot and a central dot]} \\
 &= \left(-\frac{61}{16} + 1 - \frac{1}{2} - \frac{7}{4} - \frac{3}{8} + \frac{3}{2} + \frac{11}{4} - \frac{13}{8} + \frac{45}{16} \right) I = 0.
 \end{aligned}$$

As we anticipated, the sum of the diagrams yields a vanishing result in $D = 3$.

V. THE PARTITION FUNCTION IN D DIMENSIONS

The argument given by [1,7] leading to the one-loop exactness of the partition function used specific features of the algebraic structure of the three dimensional case. We want to determine whether the vanishing of the three-loop contribution to the effective action is something specific to the $D = 3$ case.

The generic D dimensional case is much more complex. Several diagrams that vanished in the $D = 3$ case no longer

do so, with the notable exception of the trefoil diagram 4(a), which remains zero in any dimension. More importantly, the diagrams now contain many additional terms and do not simplify to a single master integral, as they did in three dimensions. Instead, all momentum integrals decompose into linear combinations of several master integrals: $I, J_1, K_1, K_2, A, B, C$ (C1). This result was obtained after a lengthy and tedious computation. The individual contributions of each diagram are summarized in Tables II and III.

Collecting all the terms, the three-loop effective action is given by

$$\begin{aligned}
 S_{\text{eff}}^{(3)} &= (D-3) \left[\frac{D^2(D+6)(2D-9)}{128(D-2)} I + \frac{D(18D^3 + 80D^2 - 518D - 107)}{576(D-2)} J_1 - \frac{D(18D^3 - 126D^2 + 71D + 362)}{576(D-2)} K_2 \right. \\
 &\quad \left. - \frac{(D-3)D^2(D^2 - 4D + 2)}{64(D-2)} K_1 + \frac{D^2(D^2 - 3D - 6)}{64(D-2)} A - \frac{(D-6)D^2}{256(D-2)} B - \frac{(D-3)D^2}{64} C \right]. \quad (21)
 \end{aligned}$$

One can see that each coefficient is a polynomial in D which has a natural root only in $D = 3$ (except for the one accompanying B). Of course, without knowing the precise relations among the integrals $I, J, \dots, C$ (depending on D) one cannot claim definitively that this contribution vanishes only in $D = 3$. A more detailed analysis of these three-loop integrals, which are generically divergent, is required. Nonetheless, this result indicates that the special nature of the partition function computation lies in the appearance of dimension-dependent polynomials that vanish specifically at $D = 3$. This suggests that there is an underlying algebraic mechanism unique to three dimensions that enables these remarkable cancellations.

VI. CONCLUDING REMARKS

One of the main challenges in the computation lies in the huge number of terms involved. Each graviton vertex contains multiple contributions, all of which are quadratic in the momenta. As previously mentioned, the three-graviton vertex—depicted in Fig. 1—is the simplest case. This gives an idea of how tedious the calculation becomes for more complex diagrams, such as the Mercedes-Benz diagram 7(a), which involves four such vertices. Even more intricate is the Glasses diagram Fig. 4(e), which includes a four-leg vertex that barely fits on the page.

A second major difficulty arises in reducing the resulting expressions to a set of master integrals, discarding those that vanish under dimensional regularization. This is where the number of dimensions becomes crucial: in $D = 3$, many terms are absent. This simplification was precisely the reason why our earlier work [8] began with the case $D = 3$.

Our ultimate goal is to understand the mechanism behind the cancellation of contributions in three dimensions. While the algebraic structure of the Hilbert space already explains why the partition function must be one-loop exact, it remains an open question how this exactness emerges diagrammatically. In the previous work, we took a small step toward explicitly verifying the vanishing of the three-loop contribution in $D = 3$. Here, we encountered significantly more involved contributions appearing in $D > 3$ (21). The calculation in $D = 3$ reveals a nontrivial and intricate cancellation among diagrams, offering no immediate intuition for how this occurs. Understanding the deeper diagrammatic structure responsible for these cancellations is an interesting direction that we leave for future work.

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: VACUUM DIAGRAMS AT THREE LOOPS

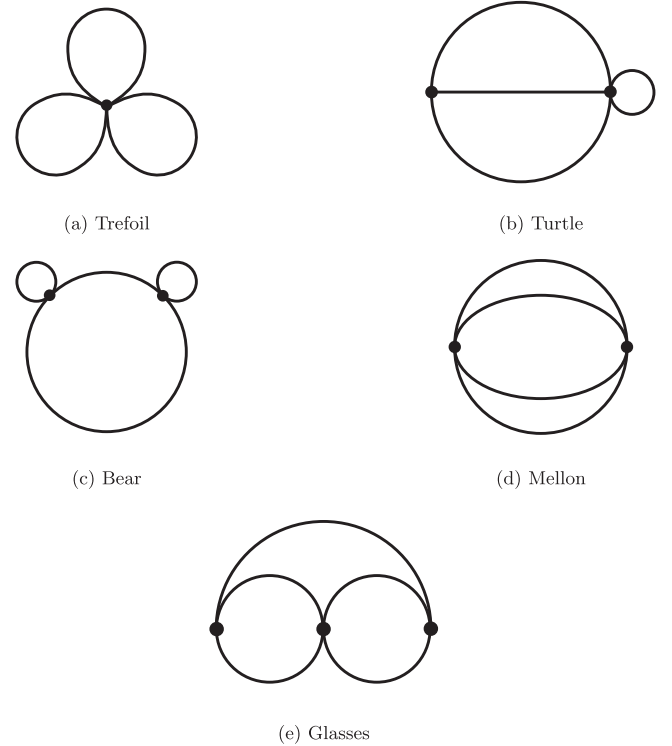


FIG. 4. Three-loop contributions.

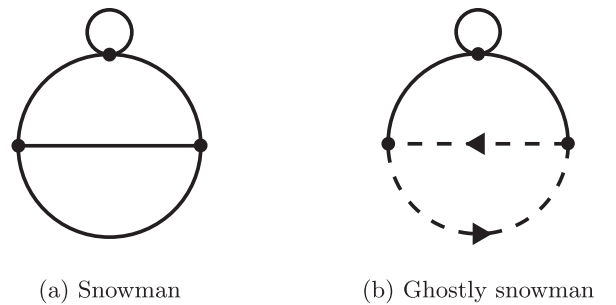


FIG. 5. Snowman-type three-loop diagrams.

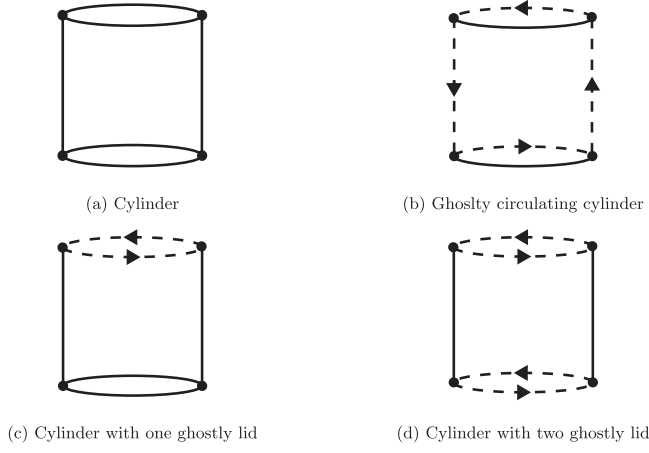


FIG. 6. Cylinder-type three-loop diagrams.

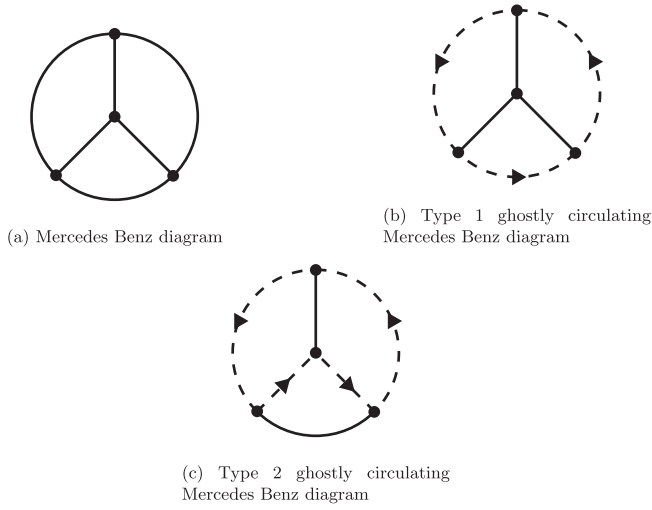


FIG. 7. Mercedes Benz-type three-loop diagram.

APPENDIX B: TABLES

The contribution of the three-loop diagram in arbitrary dimensions is split into two different tables. The first one contains diagrams proportional to the master integral I

TABLE I. Contribution of the nonvanishing 3-loops diagrams in $D = 3$. The symmetry factor of each diagram is stated in the second column. The result of the momentum integral is depicted in the third column (in $D = 3$ all the integrals are proportional to the master integral I).

Diagram	Symmetry factor	Integral
	$\frac{1}{8}$	$-\frac{61}{2} I$
	$\frac{1}{16}$	$16I$

(Table continued)

TABLE I. (Continued)

Diagram	Symmetry factor	Integral
	$-\frac{1}{2}$	I
	$-\frac{1}{4}$	$7I$
	$\frac{1}{4}$	$-\frac{3}{2} I$
	$\frac{1}{24}$	$36I$
	$-\frac{1}{3}$	$-\frac{33}{4} I$
	$-\frac{1}{4}$	$\frac{13}{2} I$
	$\frac{1}{48}$	$135I$

TABLE II. Nonvanishing diagrams whose momentum integral is proportional the master integral I .

Diagram	Symmetry factor	Integral
	$-\frac{1}{2}$	$-\frac{(D^5-2D^4-11D^3+24D^2+8D-16)}{16(D-2)^2} I$
	$-\frac{1}{4}$	$-\frac{(D^4-2D^3+3D^2+10D-56)}{16(D-2)} I$
	$\frac{1}{4}$	$\frac{D^3-D^2+12D-60}{16(D-2)} I$
	$\frac{1}{24}$	$-\frac{(D^6-69D^5+740D^4-2710D^3+3188D^2-64D+576)}{128(D-2)^2} I$
	$-\frac{1}{3}$	$\frac{D^5-D^4-10D^3-8D^2+80D-16}{16(D-2)^2} I$
	$-\frac{1}{4}$	$\frac{3D^4-35D^3-2D^2+472D-800}{64(D-2)^2} I$
	$\frac{1}{48}$	$\frac{D^5+22D^4-169D^3+184D^2+308D+48}{32(D-2)} I$

TABLE III. Nonvanishing diagrams whose contributions in D dimensions are not given solely by the master integral I . The coefficient of the first four diagrams is zero in $D = 3$.

Diagram	Symmetry factor	Integral
	$\frac{1}{12}$	$\frac{(D-3)D(9D^3-79D^2+138D+112)}{64(D-2)} J_1$
	$\frac{1}{16}$	$-\frac{(D-3)^2(D-5)D^3}{64(D-2)} K_1$
	$\frac{1}{8}$	$-\frac{(D-3)D(7D^4-76D^3+253D^2-246D-72)}{64(D-2)^2} J_1$
	$-\frac{1}{144}$	$-\frac{(D-3)D(D^2-4D-1)}{576(D-2)} J_1 + \frac{(D-3)D}{576} K_2$
	$\frac{1}{8}$	$-\frac{(3D^6+108D^5-1135D^4+3284D^3-2256D^2-1112D-1280)}{64(D-2)^2} I - \frac{(D-3)D(3D^3-97D^2+270D+232)}{64(D-2)} J_1$
	$\frac{1}{16}$	$\frac{5D^6+68D^5-789D^4+2200D^3-1228D^2-1472D-64}{128(D-2)^2} I + \frac{(D-3)^2D(3D^3-37D^2+78D+16)}{64(D-2)^2} J_1$ $+ \frac{(D-3)D(D^3-9D^2+18D)}{64(D-2)} K_1 + \frac{(D-3)D(2D^3-18D^2+28D+32)}{64(D-2)} K_2$ $+ \frac{(D-3)^2D^2}{64} A + \frac{-(D-3)^2D^2}{64} C$

while the second one depicts those diagrams whose contributions are not given solely by I .

APPENDIX C: MASTER INTEGRALS

In the following, $\int$ means $\int$:

$$I = \int \prod_{i=1}^3 d^D k_i \frac{(k_1 \cdot k_2)(k_1 \cdot k_3)}{k_1^2 k_2^2 k_3^2 (k_1 + k_2 + k_3)^2} \quad (\text{C1a})$$

$$J_1 = \int \prod_{i=1}^3 d^D k_i \frac{(k_1 \cdot k_3)^2}{k_1^2 k_2^2 (k_1 + k_2)^2 k_3^2} \quad (\text{C1b})$$

$$K_1 = \int \prod_{i=1}^3 d^D k_i \frac{(k_2 \cdot k_3)^2}{k_1^4 k_2^2 k_3^2} \quad (\text{C1c})$$

$$K_2 = \int \prod_{i=1}^3 d^D k_i \frac{(k_1 \cdot k_2)^2}{k_1^4 k_2^2 k_3^2} \quad (\text{C1d})$$

$$A = \int \prod_{i=1}^3 d^D k_i \frac{(k_2 \cdot k_3)^3}{k_1^2 (k_1 + k_2)^2 (k_1 + k_3)^2 k_2^2 k_3^2} \quad (\text{C1e})$$

$$B = \int \prod_{i=1}^3 d^D k_i \frac{k_1^6}{(k_1 + k_2)^2 (k_1 + k_3)^2 k_2^2 k_3^2 (k_2 - k_3)^2} \quad (\text{C1f})$$

$$C = \int \prod_{i=1}^3 d^D k_i \frac{(k_2 \cdot k_3)^4}{k_1^4 (k_1 + k_2)^2 (k_1 + k_3)^2 k_2^2 k_3^2} \quad (\text{C1g})$$

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