

Anti-de Sitter flag superspace

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This work aims to develop a global formulation for $\mathcal{N} = 2$ harmonic/projective anti-de Sitter (AdS) superspace $\text{AdS}^{4|8} \times S^2 \simeq \text{AdS}^{4|8} \times \mathbb{CP}^1$ that allows for a simple action of superconformal (and hence AdS isometry) transformations. First of all, we provide an alternative supertwistor description of the $\mathcal{N}$ -extended AdS superspace in four dimensions, $\text{AdS}^{4|4\mathcal{N}}$, which corresponds to a realization of the connected component $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})$ of the AdS isometry supergroup as $\text{SU}(2, 2|\mathcal{N}) \cap \text{OSp}(\mathcal{N}|4; \mathbb{C})$. The proposed realization yields the following properties: (i) $\text{AdS}^{4|4\mathcal{N}}$ is an open domain of the compactified $\mathcal{N}$ -extended Minkowski superspace, $\bar{\mathbb{M}}^{4|4\mathcal{N}}$; (ii) the infinitesimal $\mathcal{N}$ -extended superconformal transformations naturally act on $\text{AdS}^{4|4\mathcal{N}}$; and (iii) the isometry transformations of $\text{AdS}^{4|4\mathcal{N}}$ are described by those superconformal transformations which obey a certain constraint. The obtained results for $\text{AdS}^{4|4\mathcal{N}}$ are then applied to develop a supertwistor formulation for an AdS flag superspace $\text{AdS}^{4|8} \times \mathbb{F}_1(2)$ that we identify with the $\mathcal{N} = 2$ harmonic/projective AdS superspace. This construction makes it possible to read off the superconformal and AdS isometry transformations acting on the analytic subspace of the harmonic superspace.

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I. INTRODUCTION

As is well known, there exist two fully fledged superspace approaches to formulate off-shell $\mathcal{N} = 2$ rigid supersymmetric field theories in four dimensions: (i) harmonic superspace [1,2], and (ii) projective superspace [3–5]. They make use of the same superspace

$$\mathbb{M}^{4|8} \times \mathbb{CP}^1 \simeq \mathbb{M}^{4|8} \times \text{SU}(2)/\text{U}(1) \quad (1.1)$$

which was introduced for the first time by Rosly [6]. However, they differ in the following conceptual points: (i) the structure of off-shell supermultiplets used, and (ii) the supersymmetric action principle chosen.¹ In particular, they deal with different off-shell realizations for the so-called charged hypermultiplet: (i) the q -

hypermultiplet [1] in harmonic superspace, and (ii) the polar hypermultiplet [4] in projective superspace.²

In 2007, both the harmonic and projective superspace approaches were extended to the case of $\mathcal{N} = 1$ supersymmetric theories in AdS_5 [12,13]. The projective superspace construction of [12,13], in conjunction with the concept of superconformal projective multiplets [14,15], has proved to be powerful for nontrivial generalizations. It has been used for developing off-shell formulations for general supergravity-matter systems, first in five dimensions [16–18], and soon after in four [19–21], three [22], and six [23] dimensions. In a locally supersymmetric framework, the superspaces $\text{AdS}^{4|8}$ and $\text{AdS}^{5|8}$ originate as maximally supersymmetric solutions in the 4D $\mathcal{N} = 2$ [24,25] and 5D $\mathcal{N} = 1$ [26] AdS supergravity theories obtained by coupling the corresponding Weyl multiplet to two conformal compensators: (i) the vector multiplet, and (ii) the $\mathcal{O}(2)$ multiplet.

Extending the covariant harmonic-superspace approach developed for AdS_5 [12,13], or its four-dimensional analog introduced recently in [27], to local supersymmetry has turned out to be a nontrivial technical problem. To explain this issue, it suffices to restrict our attention to the 4D $\mathcal{N} = 2$ case and consider the $\text{SU}(2)$ superspace formulation for $\mathcal{N} = 2$ conformal supergravity [19]. Let

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¹The relationship between the harmonic and projective superspace formulations is spelled out in [7–10].

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²The terminology “polar hypermultiplet” was introduced in the influential paper [11].

$\mathcal{D}_A = (\mathcal{D}_a, \mathcal{D}_\alpha^i, \bar{\mathcal{D}}_{\dot{\alpha}}^i)$ be the corresponding covariant derivatives for curved superspace $\mathcal{M}^{4|8}$, and let v_i^+ be the homogeneous coordinates for $\mathbb{CP}^1$. The algebra of supergravity covariant derivatives [19] implies that the spinor operators $\mathcal{D}_\alpha^+ := v_i^+ \mathcal{D}_\alpha^i$ and $\bar{\mathcal{D}}_{\dot{\alpha}}^+ := v_i^+ \bar{\mathcal{D}}_{\dot{\alpha}}^i$ satisfy the anti-commutation relations:

$$\begin{aligned} \{\mathcal{D}_\alpha^+, \mathcal{D}_\beta^+\} &= 4Y_{\alpha\beta} \mathbb{J}^{++} + 4S^{++} M_{\alpha\beta}, \\ \{\mathcal{D}_\alpha^+, \bar{\mathcal{D}}_{\dot{\beta}}^+\} &= 8G_{\alpha\dot{\beta}} \mathbb{J}^{++}, \end{aligned} \quad (1.2)$$

with $\mathbb{J}^{++} := v_i^+ v_j^+ \mathbb{J}^{ij}$ and $S^{++} := v_i^+ v_j^+ S^{ij}$. Here $M_{\alpha\beta}$ and $\mathbb{J}^{ij}$ are the Lorentz and $\text{SU}(2)$ generators, while $Y_{\alpha\beta}$, $G_{\alpha\dot{\beta}}$, and S^{ij} are torsion tensors. With the notation $\mathcal{D}_\alpha^+ = (\mathcal{D}_\alpha^+, \bar{\mathcal{D}}_{\dot{\alpha}}^+)$, a Grassmann analytic superfield Q is a scalar superfield on curved superspace $\mathcal{M}^{4|8}$ which is v dependent and obeys the covariant Grassmann analyticity constraints

$$\mathcal{D}_\alpha^+ Q = 0. \quad (1.3)$$

These constraints have the integrability conditions $\{\mathcal{D}_\alpha^+, \mathcal{D}_{\dot{\beta}}^+\} Q = 0$. These conditions are automatically satisfied for the *projective multiplets*, which are characterized by the property [19]

$$\mathbb{J}^{++} Q = 0. \quad (1.4)$$

However, the integrability conditions do not hold for general *harmonic multiplets*.³

A covariant harmonic-superspace formulation for general $\mathcal{N} = 2$ supergravity-matter systems was developed 10 years ago by Butter [29], who also presented a plethora of nontrivial applications. In his approach, the conventional harmonic superspace $\mathcal{M}^{4|8} \times S^2$ is replaced with $\mathcal{M}^{4|8} \times TCP^1$, where the internal space is the tangent bundle of $\mathbb{CP}^1$.⁴ In the present paper we will advocate for a different internal space, namely a flag manifold $\mathbb{F}_1(2)$, which is often denoted $F(1, \mathbb{C}^2)$. Instead of considering a generic $\mathcal{N} = 2$ curved superspace $\mathcal{M}^{4|8}$, our attention will be restricted to the AdS case. Our analysis applies to the following AdS superspaces with auxiliary dimensions:

³This problem does not occur in the case of AdS superspace $\text{AdS}^{4|8}$ where $Y_{\alpha\beta} = 0$ and $G_{\alpha\dot{\beta}} = 0$ [28]. The conditions $Y_{\alpha\beta} = 0$ and $G_{\alpha\dot{\beta}} = 0$ also hold in on-shell AdS supergravity upon imposing an appropriate super-Weyl gauge [24,25]. The superspace geometry of on-shell supergravity is determined by the chiral super-Weyl tensor $W_{\alpha\beta}$ and the real isotriplet S^{ij} .

⁴This approach in [29] was inspired by the biharmonic construction for the quaternion Kähler manifolds developed by Galperin, Ivanov, and Ogievetsky [30].

(i) $\mathcal{N} = 2$ AdS₄ flag superspace⁵

$$\text{AdS}^{4|8} \times \mathbb{F}_1(2), \quad \mathbb{F}_1(2) = \text{GL}(2, \mathbb{C}) / \tilde{\mathbb{H}}_1(2). \quad (1.5a)$$

(ii) $\mathcal{N} = 1$ AdS₅ flag superspace

$$\text{AdS}^{5|8} \times \mathbb{F}_1(2), \quad \mathbb{F}_1(2) = \text{GL}(2, \mathbb{C}) / \tilde{\mathbb{H}}_1(2), \quad (1.5b)$$

where $\tilde{\mathbb{H}}_1(2)$ is the group of nonsingular lower triangular matrices,

$$\tilde{\mathbb{H}}_1(2) := \left\{ \tilde{r} = \begin{pmatrix} a & 0 \\ b & c \end{pmatrix} \in \text{GL}(2, \mathbb{C}) \right\}. \quad (1.6)$$

Here the flag manifold $\mathbb{F}_1(2)$ is the space of flags $V_1 \subset V_2 = \mathbb{C}^2$, with V_1 a one-dimensional subspace of $\mathbb{C}^2$. Of course, $\mathbb{F}_1(2)$ can be viewed as $\mathbb{CP}^1$ or as $S^2 \simeq \text{SU}(2)/\text{U}(1)$, which are precisely the realizations corresponding to the projective and harmonic superspaces, respectively. However, its description as $\text{GL}(2, \mathbb{C}) / \tilde{\mathbb{H}}_1(2)$ is most useful when dealing with $\mathcal{N} = 2$ superconformal transformations in the analytic subspace of harmonic superspace [36]. An important fact is that the three equivalent realizations of $\mathbb{F}_1(2)$ are naturally associated with different functional types of (super)fields. This point will be elaborated upon in Secs. II and III which are devoted to the discussion of $\mathcal{N} = 2$ supersymmetric field theories on Minkowski flag superspace $\mathbb{M}^{4|8} \times \mathbb{F}_1(2)$. In the remainder of this paper we will concentrate on a global description of the flag superspace (1.5a) and develop its supertwistor realization.

The supertwistor realizations for the $\mathcal{N}$ -extended AdS superspaces $\text{AdS}^{4|4\mathcal{N}}$ and $\text{AdS}^{5|8\mathcal{N}}$ were developed in Refs. [37–39] and [40], respectively. In the present paper (Secs. IV and V) we provide an alternative supertwistor description of $\text{AdS}^{4|4\mathcal{N}}$, which corresponds to a realization of the connected component $\text{OSP}_0(\mathcal{N}|4; \mathbb{R})$ of the AdS isometry supergroup as $\text{SU}(2, 2|\mathcal{N}) \cap \text{OSp}(\mathcal{N}|4; \mathbb{C})$.⁶ The

⁵The flag superspaces associated with (complexified) $\mathcal{N}$ -extended Minkowski superspace were considered in [6,31–34]. In the case of $\mathcal{N} = 3$ supersymmetry, the relevant flag manifold is $F(1, 2, \mathbb{C}^3)$ [6], with its points being all possible sequences $V_1 \subset V_2 \subset \mathbb{C}^3$, where V_1 and V_2 are one- and two-dimensional subspaces of $\mathbb{C}^3$. This flag manifold in the realization $\text{SU}(3)/[\text{U}(1) \times \text{U}(1)]$ was used in [32] to construct the complete $\mathcal{N} = 3$ harmonic formalism and to present, for the first time, an off-shell superfield action of $\mathcal{N} = 3$ SYM theory. Several important multiplets in $\mathcal{N} = 3$ conformal supergravity, including the super Bach tensor, are naturally defined on the manifold $\mathcal{M}^{4|12} \times F(1, 2, \mathbb{C}^3)$ [35].

⁶It is well known that the $\mathcal{N}$ -extended AdS superalgebra in four dimensions, $\mathfrak{osp}(\mathcal{N}|4; \mathbb{R})$, is a subalgebra of the $\mathcal{N}$ -extended superconformal algebra $\mathfrak{su}(2, 2|\mathcal{N})$; see [41,42] as well as [43] for a recent discussion.

advantage of doing so is that it allows us to read off the superconformal and isometry transformation rules for $\text{AdS}^{4|4\mathcal{N}}$ from those known for the compactified Minkowski superspace. Section VI is devoted to deriving a supertwistor realization of $\text{AdS}^{4|8} \times \mathbb{F}_1(2)$.

The main body of the paper is accompanied by three technical appendixes. Appendix A contains a brief review of $\mathcal{N} = 2$ conformal Killing supervector fields. Appendix B describes another similarity transformation for the AdS supergroup. Appendix C provides an alternative derivation of the (conformal) Killing supervector fields for $\text{AdS}^{4|4\mathcal{N}}$.

Our two-component spinor notation and conventions follow [44] and are similar to those used in [45]. In particular, two-component spinor indices are raised and lowered,

$$\begin{aligned}\psi^\alpha &:= \varepsilon^{\alpha\beta} \psi_\beta, & \psi_\alpha &= \varepsilon_{\alpha\beta} \psi^\beta; \\ \bar{\phi}^{\dot{\alpha}} &:= \varepsilon^{\dot{\alpha}\dot{\beta}} \bar{\phi}_{\dot{\beta}}, & \bar{\phi}_{\dot{\alpha}} &= \varepsilon_{\dot{\alpha}\dot{\beta}} \bar{\phi}^{\dot{\beta}},\end{aligned}\quad (1.7)$$

using the spinor metrics

$$\varepsilon^{\alpha\beta} = -\varepsilon^{\beta\alpha}, \quad \varepsilon_{\alpha\beta} = -\varepsilon_{\beta\alpha}, \quad \varepsilon^{12} = \varepsilon_{21} = 1; \quad (1.8a)$$

$$\varepsilon^{\dot{\alpha}\dot{\beta}} = -\varepsilon^{\dot{\beta}\dot{\alpha}}, \quad \varepsilon_{\dot{\alpha}\dot{\beta}} = -\varepsilon_{\dot{\beta}\dot{\alpha}}, \quad \varepsilon^{\dot{1}\dot{2}} = \varepsilon_{\dot{2}\dot{1}} = 1. \quad (1.8b)$$

One can convert between vector and spinor indices as follows:

$$\begin{aligned}x_{\alpha\dot{\alpha}} &= x^a (\sigma_a)_{\alpha\dot{\alpha}}, \\ x^{\dot{\alpha}\alpha} &= x^a (\tilde{\sigma}_a)^{\dot{\alpha}\alpha} \Leftrightarrow x_a = -\frac{1}{2} (\tilde{\sigma}_a)^{\dot{\alpha}\alpha} x_{\alpha\dot{\alpha}} = -\frac{1}{2} (\sigma_a)_{\alpha\dot{\alpha}} x^{\dot{\alpha}\alpha},\end{aligned}\quad (1.9)$$

where the matrices σ_a and $\tilde{\sigma}_a$ are given by

$$\begin{aligned}\sigma_a &= (\mathbb{1}_2, \vec{\sigma}) = ((\sigma_a)_{\alpha\dot{\alpha}}), & \tilde{\sigma}_a &= (\mathbb{1}_2, -\vec{\sigma}) = ((\tilde{\sigma}_a)^{\dot{\alpha}\alpha}), \\ (\tilde{\sigma}_a)^{\dot{\alpha}\alpha} &= \varepsilon^{\alpha\beta} \varepsilon^{\dot{\alpha}\dot{\beta}} (\sigma_a)_{\beta\dot{\beta}}.\end{aligned}\quad (1.10)$$

II. THREE REALIZATIONS OF $\mathbb{F}_1(2)$

The elements of $\mathbb{F}_1(2)$ are complete flags in $\mathbb{C}^2$. They may be identified with nonsingular 2×2 complex matrices⁷

$$\begin{aligned}\mathbf{w} &= (w_i, v_i) \in \text{GL}(2, \mathbb{C}) \Leftrightarrow \det \mathbf{w} = v^i w_i \\ &\equiv (v, w) \neq 0, & v^i &= \varepsilon^{ij} v_j\end{aligned}\quad (2.1a)$$

⁷We often make use of antisymmetric 2×2 matrices ε^{ij} and ε_{ij} normalized as $\varepsilon^{12} = \varepsilon_{21} = 1$.

defined modulo equivalence transformations of the form

$$\mathbf{w} \rightarrow \tilde{\mathbf{w}} \Leftrightarrow v_i \rightarrow c v_i, \quad w_i \rightarrow a w_i + b v_i, \quad ac \neq 0. \quad (2.1b)$$

These relations imply that v_i can be interpreted as the homogeneous coordinate for $\mathbb{CP}^1$, while w_i may be made arbitrary modulo the restriction $(v, w) := v^i w_i \neq 0$. In other words, w_i is a purely “gauge” degree of freedom.

The same flag manifold, $\mathbb{F}_1(2)$, can also be realised as

$$\begin{aligned}\mathbb{F}_1(2) &= \text{SL}(2, \mathbb{C}) / \mathbb{H}_1(2), \\ \mathbb{H}_1(2) &:= \left\{ r = \begin{pmatrix} c^{-1} & 0 \\ b & c \end{pmatrix} \in \text{SL}(2, \mathbb{C}) \right\}.\end{aligned}\quad (2.2)$$

In this realization, the elements of $\mathbb{F}_1(2)$ are unimodular 2×2 complex matrices

$$\mathbf{v} = (v_i^-, v_i^+) \in \text{SL}(2, \mathbb{C}) \Leftrightarrow \det \mathbf{v} = v^{+i} v_i^- = 1 \quad (2.3a)$$

defined modulo equivalence transformations of the form

$$\mathbf{v} \rightarrow \mathbf{v} \mathbf{r} \Leftrightarrow v_i^+ \rightarrow c v_i^+, \quad v_i^- \rightarrow c^{-1} v_i^- + b v_i^+. \quad (2.3b)$$

Here the superscript $\pm$ carried by $v^\pm$ indicates the degree of homogeneity, $v^\pm \rightarrow c^{\pm 1} v^\pm$, under the scale transformation with parameter c . The above freedom in the choice of $\mathbf{v}$ may be used to choose a representative

$$\mathbf{u} = (u_i^-, u_i^+) \in \text{SU}(2) \Leftrightarrow u^{+i} = \overline{u_i^-}, \quad u^{+i} u_i^- = 1. \quad (2.4a)$$

There still remain residual equivalence transformations (2.3b) of the form

$$u_i^\pm \rightarrow e^{\pm i\varphi} u_i^\pm, \quad \varphi \in \mathbb{R}. \quad (2.4b)$$

This leads to the third realization of the flag manifold $\mathbb{F}_1(2)$,

$$\mathbb{F}_1(2) = \text{SU}(2) / \text{U}(1) \simeq S^2. \quad (2.5)$$

Both realizations (2.1) and (2.3) are useful when a nonunitary group, say $\text{GL}(2, \mathbb{C})$, acts on $\mathbb{F}_1(2)$. It suffices to consider the action of $\text{SL}(2, \mathbb{C})$ on $\mathbb{F}_1(2)$, and in this case it is natural to use the realization (2.3) of $\mathbb{F}_1(2)$. Let $g \approx \mathbb{1} + \Lambda \in \text{SL}(2, \mathbb{C})$ be an infinitesimal group transformation, $\text{tr } \Lambda = 0$. Then

$$g\mathbf{v} \approx \mathbf{v} + \Lambda\mathbf{v}, \quad \Lambda\mathbf{v} = (\Lambda_i^j v_j^-, \Lambda_i^j v_j^+). \quad (2.6)$$

This transformation can be accompanied by an infinitesimal equivalence transformation (2.3b), $g\mathbf{v} \sim g\mathbf{v} \mathbf{r}$, such that v_i^- remains unchanged. We end up with

$$\delta v_i^- = 0, \quad \delta v_i^+ = -\Lambda^{++}(v) v_i^-, \quad \Lambda^{++}(v) = \Lambda^{jk} v_j^+ v_k^+. \quad (2.7)$$

This transformation law implies that $\text{SL}(2, \mathbb{C})$ acts on $\mathbb{F}_1(2)$ by holomorphic transformations.

Realization (2.1) is suitable to understand how the internal space $T\mathbb{CP}^1$ used in [29] originates. We introduce symmetric 2×2 matrices

$$\begin{aligned} (\Sigma^I)_{ij} &= (i\mathbb{1}, -\sigma_1, -\sigma_3) = (\Sigma^I)_{ji}, \\ (\Sigma^I)^{ij} &:= \varepsilon^{ik} \varepsilon^{jl} (\Sigma^I)_{kl} = (i\mathbb{1}, \sigma_1, \sigma_3), \end{aligned} \quad (2.8)$$

with $I = 1, 2, 3$ and $i, j = 1, 2$. Their properties are

$$\sum_{I=1}^3 (\Sigma^I)_{ij} (\Sigma^I)_{kl} = 2\varepsilon_{i(k} \varepsilon_{l)j}, \quad (\Sigma^I)_{ij} (\Sigma^I)^{ij} = -2\delta^{IJ}. \quad (2.9)$$

Next we introduce a complex 3-vector

$$\vec{Z} = (Z^I) \in \mathbb{C}^3, \quad Z^I = \frac{1}{(v, w)} v^i (\Sigma^I)_{ij} w^j, \quad (2.10a)$$

with the property

$$\vec{Z} \cdot \vec{Z} = 1. \quad (2.10b)$$

The complex hypersurface in $\mathbb{C}^3$ defined by Eq. (2.10b) provides a global realization for $T\mathbb{CP}^1$. Indeed, if we introduce the real and imaginary parts of $\vec{Z}$, $\vec{Z} = \vec{X} + i\vec{Y}$, then the constraint (2.10b) can be recast in the form:

$$\vec{R} \cdot \vec{R} = 1, \quad \vec{R} \cdot \vec{Y} = 0, \quad \vec{R} := \frac{\vec{X}}{\sqrt{1 + \vec{Y} \cdot \vec{Y}}}. \quad (2.11)$$

The expression for Z^I in terms of v^i and w^i , Eq. (2.10a), is invariant under the scale transformations (2.1b) described by the parameters a and c . However Z^I is not invariant under the b -transformation (2.1b).

Of course, the realization (2.3) is also suitable to describe the internal space $T\mathbb{CP}^1$. Here the expression (2.10a) for Z^I turns into

$$Z^I = v_i^+ (\Sigma^I)^{ij} v_j^-. \quad (2.12)$$

This complex three-vector is invariant under the scale c -transformation (2.3b). However, Z^I is not invariant under the b -transformation (2.3b).

Associated with the three realizations of $\mathbb{F}_1(2)$ considered above are three types of fields. In the case of (2.1), it is natural to deal with functions $\phi^{(p,q)}(v, w)$ that are homogeneous in v_i and, independently, in w_i ,

$$\phi^{(p,q)}(cv, aw) = c^p a^{-q} \phi^{(p,q)}(v, w). \quad (2.13)$$

By replacing $\phi^{(p,q)}(v, w) \rightarrow (v, w)^q \phi^{(p,q)}(v, w)$ we can always make $q = 0$, and thus it suffices to work with functions $\Phi^{(n)}(v, w) \equiv \phi^{(n,0)}(v, w)$,

$$\Phi^{(n)}(cv, aw) = c^n \Phi^{(n)}(v, w). \quad (2.14)$$

Invariance under the b -transformations in (2.1b) will be imposed on an action functional.

Realization (2.3) is obtained from (2.1) by introducing the variables

$$v_i^+ := v_i, \quad v_i^- := (v, w)^{-1} w_i. \quad (2.15)$$

Then the function $\Phi^{(n)}(v, w)$, Eq. (2.14), turns into $\Phi^{(n)}(v^+, v^-)$ such that

$$\Phi^{(n)}(cv^+, c^{-1}v^-) = c^n \Phi^{(n)}(v^+, v^-), \quad c \in \mathbb{C}^* \equiv \mathbb{C} - \{0\}. \quad (2.16)$$

Finally, in the case of the harmonic realization (2.4) one deals with functions over $\text{SU}(2)$, $\Psi^{(n)}(u^+, u^-)$, of $\text{U}(1)$ charge $n \in \mathbb{Z}$, with the defining property

$$\Psi^{(n)}(e^{\pm i\varphi} u^\pm) = e^{in\varphi} \Psi^{(n)}(u^\pm), \quad \varphi \in \mathbb{R}. \quad (2.17)$$

In accordance with Schur's lemma, an arbitrary function $\Psi(u^\pm)$ over $\text{SU}(2)$ is a linear combination of functions of definite charge,

$$\Psi(u^\pm) = \sum_{n \in \mathbb{Z}} \Psi^{(n)}(u^\pm). \quad (2.18)$$

Any function $\Psi^{(n)}(u^\pm)$ over $\text{SU}(2)$ proves to be represented by a convergent Fourier series of the form (see, e.g., [1,2,46])

$$\Psi^{(n)}(u^\pm) = \sum_{k=0}^{+\infty} \Psi^{(i_1 \dots i_{n+k} j_1 \dots j_k)} u_{i_1}^+ \dots u_{i_{n+k}}^+ u_{j_1}^- \dots u_{j_k}^-, \quad (2.19)$$

where the charge is assumed to be non-negative, $n \geq 0$. An analogous representation holds in the $n < 0$ case.

As a generalization of (2.19), we formally represent a holomorphic function $\Phi^{(n)}(v^\pm)$ satisfying the homogeneity condition (2.16) as

$$\Phi^{(n)}(v^\pm) = \sum_{k=0}^{+\infty} \Phi^{(i_1 \dots i_{n+k} j_1 \dots j_k)} v_{i_1}^+ \dots v_{i_{n+k}}^+ v_{j_1}^- \dots v_{j_k}^-, \quad n \geq 0, \quad (2.20a)$$

where the variables $v_i^\pm$ are related to $u_i^\pm$ as

$$v_i^+ = cu_i^+, \quad v_i^- = c^{-1}u_i^- + b^{--}u_i^+, \quad c \in \mathbb{C}^* \quad (2.20b)$$

for arbitrary $b^{--} \in \mathbb{C}$. One may think of $\Phi^{(n)}(v^\pm)$ to be an analytic continuation of (2.19), assuming that the series in (2.20a) is convergent when b^{--} in (2.20b) is equal to zero.

III. MINKOWSKI FLAG SUPERSPACE $\mathbb{M}^{4|8} \times \mathbb{F}_1(2)$

In this section we argue that $\mathbb{M}^{4|8} \times \mathbb{F}_1(2)$ is suitable to describe off-shell $\mathcal{N} = 2$ supersymmetric theories for all realizations of $\mathbb{F}_1(2)$ discussed earlier.

A. Harmonic superspace approach: The $u^\pm$ realization

Within the 4D $\mathcal{N} = 2$ harmonic superspace approach one works with analytic superfields $\mathcal{Q}^{(n)}(z, u^\pm)$ that are defined on $\mathbb{R}^{4|8} \times S^2$ and obey the Grassmann analyticity constraints

$$D_\alpha^+ \mathcal{Q}^{(n)} = 0, \quad \bar{D}_{\dot{\alpha}}^+ \mathcal{Q}^{(n)} = 0, \\ D_\alpha^\pm := u_i^\pm D_\alpha^i, \quad \bar{D}_{\dot{\alpha}}^\pm := u_{\dot{i}}^\pm \bar{D}_{\dot{\alpha}}^{\dot{i}}. \quad (3.1)$$

With respect to the harmonic variables $u_i^\pm$, $\mathcal{Q}^{(n)}(z, u^\pm)$ is a smooth function on $\text{SU}(2)$ of $\text{U}(1)$ charge n ,

$$\mathcal{Q}^{(n)}(z, e^{\pm i\varphi} u^\pm) = e^{in\varphi} \mathcal{Q}^{(n)}(z, u^\pm), \quad \varphi \in \mathbb{R}, \quad (3.2a)$$

$$\mathcal{Q}^{(n)}(z, u) = \sum_{k=0}^{+\infty} \mathcal{Q}^{(i_1 \dots i_{k+n} j_1 \dots j_k)}(z) u_{i_1}^+ \dots u_{i_{k+n}}^+ u_{j_1}^- \dots u_{j_k}^-, \quad (3.2b)$$

where the charge is assumed to be non-negative, $n \geq 0$. The harmonic superspace action is

$$S[\mathcal{L}^{(+4)}] = \int d^4x \int du (D^-)^4 \mathcal{L}^{(+4)}(x, \theta, \bar{\theta}, u^\pm)|_{\theta=\bar{\theta}=0}, \\ (D^-)^4 = \frac{1}{16} (D^-)^2 (\bar{D}^-)^2. \quad (3.3)$$

The integral over $\text{SU}(2)$ is defined in accordance with [1]

$$\int du \mathcal{Q}^{(n)}(u^\pm) = \delta_{n,0} \mathcal{Q}. \quad (3.4)$$

Here $\mathcal{Q}$ is the harmonic-independent coefficient in the Fourier series for $\mathcal{Q}^{(0)}(u^\pm)$,

$$\mathcal{Q}^{(0)}(u^\pm) = \mathcal{Q} + \sum_{k=1}^{+\infty} \mathcal{Q}^{(i_1 \dots i_k j_1 \dots j_k)} u_{i_1}^+ \dots u_{i_k}^+ u_{j_1}^- \dots u_{j_k}^-. \quad (3.5)$$

The action (3.3) is known to be $\mathcal{N} = 2$ supersymmetric; see the next subsection for the proof.

B. The $v^\pm$ realization

Now we analytically continue the superfield (3.2) to the v -variables (2.20b),

$$\mathcal{Q}^{(n)}(z, c v^+, c^{-1} v^-) = c^n \mathcal{Q}^{(n)}(z, v^\pm), \quad c \in \mathbb{C}^* \equiv \mathbb{C} - \{0\}, \quad (3.6a)$$

$$\mathcal{Q}^{(n)}(z, v) = \mathcal{Q}(z) + \sum_{k=1}^{+\infty} \mathcal{Q}^{(i_1 \dots i_{k+n} j_1 \dots j_k)}(z) v_{i_1}^+ \dots v_{i_{k+n}}^+ v_{j_1}^- \dots v_{j_k}^-, \quad (3.6b)$$

and the integer n is said to be the weight of $\mathcal{Q}^{(n)}$. This superfield is still Grassmann analytic,

$$D_\alpha^+ \mathcal{Q}^{(n)}(z, v) = 0, \quad \bar{D}_{\dot{\alpha}}^+ \mathcal{Q}^{(n)}(z, v) = 0, \\ D_\alpha^\pm := v_i^\pm D_\alpha^i, \quad \bar{D}_{\dot{\alpha}}^\pm := v_{\dot{i}}^\pm \bar{D}_{\dot{\alpha}}^{\dot{i}}. \quad (3.7)$$

We can formally extend the algebraic definition of the integral (3.4) to the variables v ,

$$\int dv \mathcal{Q}^{(n)}(v^\pm) := \delta_{n,0} \mathcal{Q}. \quad (3.8)$$

Finally, we define the flag-superspace action

$$S[\mathcal{L}^{(+4)}] = \int d^4x \int dv (D^-)^4 \mathcal{L}^{(+4)}(x, \theta, \bar{\theta}, v^\pm)|_{\theta=\bar{\theta}=0}, \quad (3.9)$$

where the spinor covariant derivatives D_α^- and $\bar{D}_{\dot{\alpha}}^-$ are defined in (3.7).

The integrand in (3.9) is obviously invariant under arbitrary rescalings $v_i^+ \rightarrow c v_i^+$ and $v_{\dot{i}}^- \rightarrow c^{-1} v_{\dot{i}}^-$. Let us give a small disturbance to the variables v^- ,

$$\delta v_{\dot{i}}^- = b^{--} v_{\dot{i}}^+, \quad (3.10)$$

while keeping v^+ fixed.⁸ The Lagrangian changes as

$$\delta \mathcal{L}^{(+4)} = b^{--} D^{++} \mathcal{L}^{(+4)} = D^{++} (b^{--} \mathcal{L}^{(+4)}), \\ D^{++} = v_i^+ \frac{\partial}{\partial v_{\dot{i}}^-}. \quad (3.11)$$

Of special importance is the fact that applying the operator D^{++} to any Grassmann analytic superfield $\mathcal{Q}^{(n)}$ results in a Grassmann analytic one,

$$[D^{++}, D_\alpha^+] = 0, \quad [D^{++}, \bar{D}_{\dot{\alpha}}^+] = 0. \quad (3.12)$$

It holds that

$$D_\alpha^+ \Xi^{++} = 0, \quad \bar{D}_{\dot{\alpha}}^+ \Xi^{++} = 0 \Rightarrow S[D^{++} \Xi^{++}] = 0. \quad (3.13)$$

⁸Notation b^{--} in (3.10) indicates that any rescaling $v_i^+ \rightarrow c v_i^+$ and $\delta v_{\dot{i}}^- \rightarrow c^{-1} \delta v_{\dot{i}}^-$ results in $b^{--} \rightarrow c^{-2} b^{--}$.

We conclude that the action (3.9) is invariant under the transformations (2.3b). Therefore the model is defined on the flag superspace $\mathbb{M}^{4|8} \times \mathbb{F}_1(2)$, although the integrand in (3.9) is a composite superfield on $\mathbb{M}^{4|8} \times T\mathbb{C}P^1$.

The action (3.9) is supersymmetric. It is actually superconformal provided the Lagrangian $\mathcal{L}^{(+4)}$ transforms as a primary dimension-2 superfield⁹

$$\delta_\xi \mathcal{L}^{(+4)} = (\xi - \Lambda^{++}[\xi] D^{--}) \mathcal{L}^{(+4)} + 2\Sigma[\xi] \mathcal{L}^{(+4)},$$

$$D^{--} = v_i^- \frac{\partial}{\partial v_i^+}. \quad (3.14)$$

Here $\xi = \xi^A(z) D_A$ is an arbitrary $\mathcal{N} = 2$ conformal Killing supervector field (see Appendix A), and $\Lambda^{++}[\xi]$ and $\Sigma[\xi]$ are its descendants,

$$\Lambda^{++}[\xi] := v_i^+ v_j^+ \Lambda^{ij}[\xi], \quad (3.15a)$$

$$\Sigma[\xi] := v_i^+ v_j^- \Lambda^{ij}[\xi] + \frac{1}{2}(\sigma[\xi] + \bar{\sigma}[\xi]). \quad (3.15b)$$

The descendants $\Lambda^{ij}[\xi]$ and $\sigma[\xi]$ of ξ are defined in Appendix A. The important property of the building blocks (3.15), which appear in (3.14), is their Grassmann analyticity

$$D_\alpha^+ \Lambda^{++}[\xi] = 0, \quad \bar{D}_{\dot{\alpha}}^+ \Lambda^{++}[\xi] = 0, \quad (3.16a)$$

$$D_\alpha^+ \Sigma[\xi] = 0, \quad \bar{D}_{\dot{\alpha}}^+ \Sigma[\xi] = 0. \quad (3.16b)$$

To massage the variation (3.14), we point out the identity

$$\xi = \bar{\xi} = \xi^a(z) \partial_a - (\xi^{+\alpha} D_\alpha^- + \bar{\xi}^{+\dot{\alpha}} \bar{D}_{\dot{\alpha}}^-) + (\xi^{-\alpha} D_\alpha^+ + \bar{\xi}^{-\dot{\alpha}} \bar{D}_{\dot{\alpha}}^+), \quad (3.17)$$

with $\xi^{\pm\alpha} = \xi^{ai} v_i^\pm$ and $\bar{\xi}^{\pm\dot{\alpha}} = \bar{\xi}^{ai} v_i^\pm$. Then, making use of the properties of ξ , (3.14) may be brought to the form:

$$\delta \mathcal{L}^{(+4)} = \partial_a (\xi^a \mathcal{L}^{(+4)}) + D_\alpha^- (\xi^{+\alpha} \mathcal{L}^{(+4)}) + \bar{D}_{\dot{\alpha}}^- (\bar{\xi}^{+\dot{\alpha}} \mathcal{L}^{(+4)}) - D^{--} (\Lambda^{++}[\xi] \mathcal{L}^{(+4)}); \quad (3.18)$$

see [14,15] for similar derivations. Here the first three terms on the right do not contribute to the variation of the action,

$$\delta_\xi S[\mathcal{L}^{(+4)}] = \int d^4x \int dv (D^-)^4 \delta_\xi \mathcal{L}^{(+4)}(x, \theta, \bar{\theta}, v^\pm) |_{\theta=\bar{\theta}=0}. \quad (3.19)$$

The last term in (3.18) also does not contribute to the variation of the action since

⁹See [14] for the five-dimensional counterpart of the transformation law (3.14).

$$\int dv D^{--} \mathcal{Q}^{++}(v^\pm) = 0. \quad (3.20)$$

Analyzing the transformation law (3.14), one observes that it includes a transformation of the complex harmonics $v_i^\pm$ of the form (2.7).

C. Projective superspace approach:

The (w, v) realization

In this approach, off-shell supermultiplets are described in terms of weight- n Grassmann analytic superfields $Q^{(n)}(z, v)$,

$$D_\alpha^{(1)} Q^{(n)} = \bar{D}_{\dot{\alpha}}^{(1)} Q^{(n)} = 0, \quad Q^{(n)}(z, cv) = c^n Q^{(n)}(z, v), \quad c \in \mathbb{C}^* \quad (3.21)$$

which are independent of w ,

$$\frac{\partial}{\partial w_i} Q^{(n)} = 0. \quad (3.22)$$

In other words, $Q^{(n)}(z, v)$ is a holomorphic function on an open domain of $\mathbb{C}P^1$, with v_i being the homogeneous coordinates for $\mathbb{C}P^1$.

The projective superspace action principle is¹⁰

$$S := -\frac{1}{2\pi} \oint_\gamma v_i dv^i \int d^4x \Delta^{(-4)} L^{(2)}(z, v) |_{\theta=\bar{\theta}=0}. \quad (3.23)$$

Here γ denotes a closed contour in $\mathbb{C}P^1$, $v^i(t)$, parametrized by an evolution parameter t . The action makes use of the following fourth-order differential operator:

$$\Delta^{(-4)} := \frac{1}{16} \nabla^\alpha \nabla_\alpha \bar{\nabla}_{\dot{\beta}} \bar{\nabla}^{\dot{\beta}}, \quad \nabla_\alpha := \frac{1}{(v, w)} w_i D_\alpha^i, \quad \bar{\nabla}_{\dot{\beta}} := \frac{1}{(v, w)} w_i \bar{D}_{\dot{\beta}}^i, \quad (3.24)$$

where $(v, w) := v^i w_i$. Here w_i is a fixed isotwistor chosen to be arbitrary modulo the condition $(v, w) \neq 0$ along the integration contour. The action proves to be independent of w_i ; see [15] for the proof. Thus the action is invariant under arbitrary transformations (2.3b).

IV. NEW REALIZATION OF THE ADS SUPERGROUP

In this section we introduce a new realization for the connected component $\text{OSP}_0(\mathcal{N}|4; \mathbb{R})$ of the AdS isometry

¹⁰In the super-Poincaré case, this action was introduced in [3]. It was reformulated in a manifestly projective-invariant form in [47]. The superconformal case was studied in [15].

supergroup as $\text{SU}(2, 2|\mathcal{N}) \cap \text{OSp}(\mathcal{N}|4; \mathbb{C})$. It will be used in Sec. V.

A. The superconformal group and supertwistors

The $\mathcal{N}$ -extended superconformal group in four dimensions is $\text{SU}(2, 2|\mathcal{N})$.¹¹ By definition, it consists of all supermatrices

$$\hat{g} = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right), \quad \hat{g} \in \text{SL}(4|\mathcal{N}; \mathbb{C}) \quad (4.1)$$

satisfying the master equation

$$\hat{g}^\dagger \Omega \hat{g} = \Omega, \quad \Omega = \left(\begin{array}{c|c|c} 0 & \mathbb{1}_2 & 0 \\ \hline \mathbb{1}_2 & 0 & 0 \\ \hline 0 & 0 & -\mathbb{1}_\mathcal{N} \end{array} \right). \quad (4.2)$$

In accordance with [48], a supertwistor T is a column vector

$$T = (T_A) = \left(\begin{array}{c} T_{\hat{\alpha}} \\ T_i \end{array} \right), \quad \hat{\alpha} = 1, 2, 3, 4, \quad i = 1, \dots, \mathcal{N}. \quad (4.3)$$

In the case of *even* supertwistors, $T_{\hat{\alpha}}$ is bosonic and T_i is fermionic. In the case of *odd* supertwistors, $T_{\hat{\alpha}}$ is fermionic while T_i is bosonic. The even and odd supertwistors are called pure. We introduce the parity function $\epsilon(T)$ defined as $\epsilon(T) = 0$ if T is even and $\epsilon(T) = 1$ if T is odd. If we define

$$\epsilon_A = \begin{cases} 0, & A = \hat{\alpha} \\ 1, & A = i \end{cases} \quad (4.4)$$

then the components T_A of a pure supertwistor have the following Grassmann parities:

$$\epsilon(T_A) = \epsilon(T) + \epsilon_A \pmod{2}. \quad (4.5)$$

The space of even supertwistors is naturally identified with $\mathbb{C}^{4|\mathcal{N}}$, while the space of odd supertwistors may be identified with $\mathbb{C}^{\mathcal{N}|4}$. The supergroup $\text{SU}(2, 2|\mathcal{N})$ acts on the space of even supertwistors and on the space of odd supertwistors,

$$T \rightarrow \hat{g}T \Rightarrow T^\dagger \Omega \rightarrow T^\dagger \Omega \hat{g}^{-1}. \quad (4.6)$$

It holds that $\epsilon(\hat{g}T) = \epsilon(T)$. The supertwistor space is equipped with the $\text{SU}(2, 2|\mathcal{N})$ -invariant inner product

$$\langle T|S \rangle = T^\dagger \Omega S. \quad (4.7)$$

¹¹The case $\mathcal{N} = 4$ is somewhat special, but the corresponding details will not be discussed here.

B. The AdS supergroup

In this paper, the connected component $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})$ of $\text{OSp}(\mathcal{N}|4; \mathbb{R})$ will be identified with the $\mathcal{N}$ -extended AdS supergroup in four dimensions. We recall that the supergroup $\text{OSp}(\mathcal{N}|4; \mathbb{C})$ consists of those supermatrices

$$f = (f_A{}^B) = \left(\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right) \in \text{GL}(4|\mathcal{N}; \mathbb{C}) \quad (4.8)$$

which satisfy the master equation

$$f^{\text{sT}} \mathbb{J} f = \mathbb{J}, \quad (4.9a)$$

where f^{sT} denotes the supertranspose of f ,

$$(f^{\text{sT}})_B^A := (-1)^{\epsilon_A \epsilon_B + \epsilon_B} f_B^A \Leftrightarrow f^{\text{sT}} = \left(\begin{array}{c|c} A^T & -C^T \\ \hline B^T & D^T \end{array} \right), \quad (4.9b)$$

and the symplectic supermatrix $\mathbb{J}$ is given by

$$\mathbb{J} = \left(\begin{array}{c|c|c} 0 & \mathbb{1}_2 & 0 \\ \hline -\mathbb{1}_2 & 0 & 0 \\ \hline 0 & 0 & i\mathbb{1}_\mathcal{N} \end{array} \right). \quad (4.9c)$$

The elements of $\text{OSp}(\mathcal{N}|4; \mathbb{R}) \subset \text{OSp}(\mathcal{N}|4; \mathbb{C})$ satisfy the reality condition

$$f^\dagger = f^{\text{sT}}. \quad (4.9d)$$

The supergroup $\text{OSp}(\mathcal{N}|4; \mathbb{C})$ naturally acts on the supertwistor space.¹² This action is characterized by the $\text{OSp}(\mathcal{N}|4; \mathbb{C})$ -invariant inner product

$$\langle T|S \rangle_\mathbb{J} := T^{\text{sT}} \mathbb{J} S, \quad (4.10)$$

where the supertranspose T^{sT} of T is defined as

$$T^{\text{sT}} := (T_{\hat{\alpha}}, -(-1)^{\epsilon(T)} T_i) = (T_A (-1)^{\epsilon(T) \epsilon_A + \epsilon_A}). \quad (4.11)$$

Now, let us restrict our attention to the action of $\text{OSp}(\mathcal{N}|4; \mathbb{R})$ on the supertwistor space. Then there exists the involution $*$ defined as

$$T \rightarrow *T, \quad (*T)_A = (-1)^{\epsilon(T) \epsilon_A + \epsilon_A} \overline{T_A}, \quad (4.12)$$

where $\overline{T_A}$ denotes the complex conjugate of T_A . Its key properties are

¹²The supertwistor space is defined as in the previous subsection. However, in this subsection our attention is restricted to the action of $\text{OSp}(\mathcal{N}|4; \mathbb{C})$ or of its subgroup $\text{OSp}(\mathcal{N}|4; \mathbb{R})$ on the supertwistor space.

$$*(*T) = T, \quad (4.13a)$$

$$f(*T) = *(fT), \quad \forall f \in \text{OSp}(\mathcal{N}|4; \mathbb{R}). \quad (4.13b)$$

A supertwistor is said to be real if it satisfies the reality condition

$$*T = T \Leftrightarrow T^\dagger = T^{sT}. \quad (4.14)$$

C. New realization of the AdS supergroup

For our purposes it is useful to work with an alternative realization of the AdS supergroup, as a subgroup of the superconformal group. Let $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})_{\mathfrak{U}}$ be the subgroup of $\text{SU}(2, 2|\mathcal{N})$ consisting of those supermatrices $g \in \text{SL}(4|\mathcal{N}; \mathbb{C})$ which are singled out by the conditions

$$g^\dagger \Omega g = \Omega, \quad (4.15a)$$

$$g^{sT} \mathfrak{F} g = \mathfrak{F}, \quad (4.15b)$$

where $\mathfrak{F}$ denotes follows symplectic supermatrix

$$\mathfrak{F} = \left(\begin{array}{c|c|c} \varepsilon & 0 & 0 \\ \hline 0 & -\varepsilon & 0 \\ \hline 0 & 0 & i\mathbb{1}_{\mathcal{N}} \end{array} \right), \quad \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (4.16)$$

The supergroup $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})_{\mathfrak{U}}$ proves to be isomorphic to $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})$. The proof is based on the following supermatrix correspondence:

$$f \rightarrow g = \mathfrak{U}^{-1} f \mathfrak{U}, \quad \forall f \in \text{OSp}_0(\mathcal{N}|4; \mathbb{R}). \quad (4.17)$$

Here the supermatrix $\mathfrak{U}$ is defined as

$$\begin{aligned} \mathfrak{U} &= \left(\begin{array}{c|c} \mathfrak{m} & 0 \\ \hline 0 & \mathbb{1}_{\mathcal{N}} \end{array} \right), \\ \mathfrak{m} &= \frac{1}{2} \left(\begin{array}{c|c} \alpha \mathbb{1}_2 + \bar{\alpha} \varepsilon & \bar{\alpha} \mathbb{1}_2 + \alpha \varepsilon \\ \hline -\alpha \mathbb{1}_2 + \bar{\alpha} \varepsilon & \bar{\alpha} \mathbb{1}_2 - \alpha \varepsilon \end{array} \right), \\ \alpha &= e^{i\pi/4} = \frac{1+i}{\sqrt{2}}. \end{aligned} \quad (4.18)$$

It obeys the useful relations

$$\mathfrak{U}^\dagger = \mathfrak{U}^{-1}, \quad (4.19a)$$

$$\mathfrak{U}^\dagger \mathbb{J} \mathfrak{U} = -i\Omega, \quad (4.19b)$$

$$(\mathfrak{U})^{sT} \mathbb{J} \mathfrak{U} = \mathfrak{F}. \quad (4.19c)$$

It can be constructed making use of the alternative realizations for $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})$ and $\text{SU}(2, 2|\mathcal{N})$ provided in [14, 49]. Specifically,

$$\mathfrak{U}^{-1} = M \Sigma \mathbb{U}, \quad (4.20)$$

with

$$M = \frac{1}{\sqrt{2}} \left(\begin{array}{c|c|c} \mathbb{1}_2 & -\varepsilon & 0 \\ \hline -\varepsilon & \mathbb{1}_2 & 0 \\ \hline 0 & 0 & \sqrt{2} \mathbb{1}_{\mathcal{N}} \end{array} \right), \quad (4.21a)$$

$$\Sigma = \frac{1}{\sqrt{2}} \left(\begin{array}{c|c|c} \mathbb{1}_2 & -\mathbb{1}_2 & 0 \\ \hline \mathbb{1}_2 & \mathbb{1}_2 & 0 \\ \hline 0 & 0 & \sqrt{2} \mathbb{1}_{\mathcal{N}} \end{array} \right), \quad (4.21b)$$

$$\mathbb{U} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|c|c} \mathbb{1}_2 & i\mathbb{1}_2 & 0 \\ \hline i\mathbb{1}_2 & \mathbb{1}_2 & 0 \\ \hline 0 & 0 & \sqrt{2} \mathbb{1}_{\mathcal{N}} \end{array} \right). \quad (4.21c)$$

The relations (4.19) can be proven with the aid of the following properties:

$$\mathbb{U} \mathbb{J} \mathbb{U}^\dagger = -i\mathbb{I}, \quad (\mathbb{U}^{-1})^{sT} \mathbb{J} \mathbb{U}^{-1} = \mathbb{J}, \quad (4.22a)$$

$$\Sigma \mathbb{I} \Sigma^\dagger = \Omega, \quad (\Sigma^{-1})^{sT} \mathbb{J} \Sigma^{-1} = \mathbb{J}, \quad (4.22b)$$

$$M \Omega M^\dagger = \Omega, \quad (M^{-1})^{sT} \mathbb{J} M^{-1} = \mathfrak{F}, \quad (4.22c)$$

where $\mathbb{I}$ is defined as

$$\mathbb{I} = \left(\begin{array}{c|c|c} \mathbb{1}_2 & 0 & 0 \\ \hline 0 & -\mathbb{1}_2 & 0 \\ \hline 0 & 0 & -\mathbb{1}_{\mathcal{N}} \end{array} \right). \quad (4.23)$$

Further, the matrices M , Σ , and $\mathbb{U}$ are unitary,

$$M^{-1} = M^\dagger, \quad \Sigma^{-1} = \Sigma^\dagger, \quad \mathbb{U}^{-1} = \mathbb{U}^\dagger. \quad (4.24)$$

These properties imply that the supermatrix g defined by Eq. (4.17) obeys the conditions (4.15), and hence $g \in \text{OSp}_0(\mathcal{N}|4; \mathbb{R})_{\mathfrak{U}}$. In Appendix B we introduce another supermatrix that relates the realizations $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})$ and $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})_{\mathfrak{U}}$.

Associated with $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})_{\mathfrak{U}}$ are two invariant inner products,

$$\langle T|S \rangle_\Omega := T^\dagger \Omega S, \quad (4.25a)$$

$$\langle T|S \rangle_{\mathfrak{F}} := T^{sT} \mathfrak{F} S, \quad (4.25b)$$

for arbitrary pure supertwistors T and S .

The supergroup elements g satisfy the reality condition

$$g^\dagger = \Upsilon^{-1} g^{\text{sT}} \Upsilon, \quad \Upsilon = \mathfrak{U}^{\text{sT}} \mathfrak{U} = \begin{pmatrix} 0 & i\varepsilon & 0 \\ -i\varepsilon & 0 & 0 \\ 0 & 0 & \mathbb{1}_{\mathcal{N}} \end{pmatrix}. \quad (4.26)$$

Then, making use of Eq. (4.26), one can introduce an involution operation $\star$ defined as

$$T \rightarrow \star T, \quad (\star T)_A = (-1)^{e(T)\epsilon_A + e_A} (\Upsilon^{-1} \tilde{T})_A. \quad (4.27)$$

Its key properties are

$$\star(\star T) = T, \quad (4.28a)$$

$$g(\star T) = \star(gT). \quad (4.28b)$$

In our new realization of the AdS supergroup, a supertwistor T is said to be real if it satisfies

$$\star T = T. \quad (4.29)$$

Further, we observe that

$$\star T^{\text{sT}} = T^\dagger \Upsilon^{-1}, \quad (4.30)$$

which, in conjunction with the relations (4.19), yields

$$\langle \star T | S \rangle_{\mathfrak{F}} = -i \langle T | S \rangle_{\Omega}. \quad (4.31)$$

V. THE SUPERTWISTOR REALIZATIONS OF $\text{AdS}^{4|\mathcal{N}}$

Before introducing the supertwistor realization of $\text{AdS}^{4|\mathcal{N}}$ in terms of the supergroup $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})_{\mathfrak{U}}$, we recall the original construction given in [37] and formulated in terms of the supergroup $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})$.

A. Original realization

Here we will make use of the supergroup $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})$. Let us consider the space of complex even supertwistors, which can be identified with $\mathbb{C}^{4|\mathcal{N}}$. In this space, we consider complex two-planes which are generated by two even supertwistors

$$\mathfrak{T} = (T_A^\mu), \quad \mu = 1, 2, \quad (5.1)$$

such that the bodies of T^1 and T^2 are linearly independent. By construction, the supertwistors T^μ are defined modulo the equivalence relation

$$T^\mu \sim \tilde{T}^\mu = T^\nu R_\nu^\mu, \quad R = (R_\nu^\mu) \in \text{GL}(2, \mathbb{C}), \quad (5.2)$$

as the bases $\{T^\mu\}$ and $\{\tilde{T}^\mu\}$ span the same two-plane. We restrict our attention to those two-planes which satisfy the constraints

$$\varepsilon_{\mu\nu} \langle T^\mu | T^\nu \rangle_{\mathbb{J}} \neq 0, \quad (5.3a)$$

$$\langle \star T^\mu | T^\nu \rangle_{\mathbb{J}} = 0. \quad (5.3b)$$

Here the supertwistor $\star T$ denotes the conjugate of T defined by (4.12). These conditions are preserved under the action of the supergroup $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})$. We say that any pair of supertwistors satisfying the constraints (5.3) constitutes a frame, and the space of frames is denoted $\mathfrak{F}_{\mathcal{N}}$.

The supergroup $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})$ acts on the space of frames as

$$T^\mu \rightarrow f T^\mu, \quad f \in \text{OSp}_0(\mathcal{N}|4; \mathbb{R}). \quad (5.4)$$

This action is naturally extended to the quotient space $\mathfrak{F}_{\mathcal{N}}/\sim$, where the equivalence relation is given by (5.2). As shown in [37], $\text{AdS}^{4|4\mathcal{N}}$ can be identified with this quotient space

$$\text{AdS}^{4|4\mathcal{N}} = \mathfrak{F}_{\mathcal{N}}/\sim. \quad (5.5)$$

B. New realization

In this subsection we will show how the AdS superspace $\text{AdS}^{4|4\mathcal{N}}$ arises as an open domain of compactified $\mathcal{N}$ -extended Minkowski superspace, $\bar{\mathbb{M}}^{4|4\mathcal{N}}$, the latter being studied in [14].

As discussed in [14], $\bar{\mathbb{M}}^{4|4\mathcal{N}}$ is the space of null two-planes in the space of complex even supertwistors. Given such a two-plane, it may be described by two even supertwistors

$$\mathfrak{T} = (T_A^\mu), \quad \mu = 1, 2, \quad (5.6)$$

such that the bodies of T^1 and T^2 are linearly independent. That the two-planes are null means they satisfy the constraint

$$\langle T^\mu | T^\nu \rangle_{\Omega} = 0. \quad (5.7)$$

By construction, the supertwistors T^μ are defined modulo the equivalence relation

$$T^\mu \sim \tilde{T}^\mu = T^\nu R_\nu^\mu, \quad R = (R_\nu^\mu) \in \text{GL}(2, \mathbb{C}), \quad (5.8)$$

as the bases $\{T^\mu\}$ and $\{\tilde{T}^\mu\}$ span the same two-plane. The condition (5.7) is preserved under the action of the superconformal group $\text{SU}(2, 2|\mathcal{N})$.

Let us restrict our attention to those two-planes which satisfy the additional condition

$$\langle T^\mu | T^\nu \rangle_{\mathfrak{F}} \neq 0. \quad (5.9)$$

Then, making use of the equivalence relation (5.8), we can normalize the two-planes such that

$$\langle T^\mu | T^\nu \rangle_{\Omega} = 0, \quad (5.10a)$$

$$\langle T^\mu | T^\nu \rangle_{\mathfrak{F}} = \ell \varepsilon^{\mu\nu}, \quad (5.10b)$$

for some constant parameter $\ell > 0$. The conditions (5.10) are preserved under the action of the AdS supergroup $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})_{\mathcal{U}}$, and under equivalence transformations of the form

$$T^\mu \sim \tilde{T}^\mu = T^\nu N_\nu{}^\mu, \quad N = (N_\nu{}^\mu) \in \text{SL}(2, \mathbb{C}). \quad (5.11)$$

We say that any pair of supertwistors satisfying the relations (5.10) constitutes a frame, and the space of frames is denoted $\mathfrak{F}_{\mathcal{N}}$. The supergroup $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})_{\mathcal{U}}$ acts on $\mathfrak{F}_{\mathcal{N}}$ by the rule

$$T^\mu \rightarrow g T^\mu, \quad g \in \text{OSp}_0(\mathcal{N}|4; \mathbb{R})_{\mathcal{U}}. \quad (5.12)$$

This action is naturally extended to the quotient space $\mathfrak{F}_{\mathcal{N}}/\sim$, which was identified with AdS superspace in [37],

$$\text{AdS}^{4|\mathcal{N}} = \mathfrak{F}_{\mathcal{N}}/\sim. \quad (5.13)$$

C. The north chart of $\text{AdS}^{4|\mathcal{N}}$

In what follows, we will set $\ell = 1$. It is instructive to write the two-plane explicitly as

$$\mathfrak{T} = (T^\mu) = \begin{pmatrix} F \\ \underline{G} \\ \varphi \end{pmatrix}. \quad (5.14)$$

Here, F and G are 2×2 matrices, and φ is an $\mathcal{N} \times 2$ matrix. Then, the conditions (5.10) imply the following:

$$F^\dagger G + G^\dagger F - \varphi^\dagger \varphi = 0, \quad (5.15a)$$

$$(\det F - \det G)\varepsilon - i\varphi^T \varphi = \varepsilon. \quad (5.15b)$$

Let us define the north chart to consist of those normalized two-planes for which $\det F \neq 0$. Then, making use of the equivalence relation (5.11), we can choose

$$F = \lambda \mathbb{1}_2, \quad (5.16)$$

for some parameter λ . We then find that the two-planes are of the form

$$\mathfrak{T} = \lambda \begin{pmatrix} \mathbb{1}_2 \\ -i\tilde{x}_+ \\ 2\theta \end{pmatrix}, \quad \tilde{x}_+ = (x_+^{\dot{\alpha}\alpha}), \quad \theta = (\theta_i^\alpha), \quad (5.17)$$

where the bosonic $(x_+^{\dot{\alpha}\alpha})$ and fermionic (θ_i^α) variables are *chiral* and satisfy the conditions

$$\tilde{x}_+ - \tilde{x}_- = 4i\theta^\dagger \theta, \quad \tilde{x}_- = (\tilde{x}_+)^{\dagger}. \quad (5.18)$$

The solution to the above condition is given by

$$\tilde{x}_{\pm} = \tilde{x} \pm 2i\theta^\dagger \theta, \quad \tilde{x}_{\pm} = x_{\pm}^a \tilde{\sigma}_a, \quad \tilde{\sigma}_a = (\mathbb{1}_2, -\vec{\sigma}), \quad (5.19)$$

where $\vec{\sigma}$ are the Pauli matrices. The parameter λ takes the form

$$\lambda = (1 - x_+^2 + 2i\theta^2)^{-\frac{1}{2}}, \quad \theta^2 = \text{tr}(\theta^T \theta \varepsilon). \quad (5.20)$$

It follows that the north chart is parametrized by the chiral coordinates x_+^a and θ_i^α .

To describe the action of the AdS supergroup on the north chart, it is instructive to begin with an element of the superconformal group, which can be represented as

$$\hat{g} = e^L, \quad L = \begin{pmatrix} -K_\alpha{}^\beta - \frac{1}{2}\Delta\delta_\alpha{}^\beta & ib_{a\dot{\beta}} & 2\eta_\alpha{}^j \\ -ia^{\dot{\alpha}\beta} & \bar{K}^{\dot{\alpha}}{}_\beta + \frac{1}{2}\bar{\Delta}\delta^{\dot{\alpha}}{}_\beta & 2\bar{\varepsilon}^{\dot{\alpha}j} \\ 2\varepsilon_i{}^\beta & 2\bar{\eta}_{i\dot{\beta}} & \frac{1}{\mathcal{N}}(\bar{\Delta} - \Delta)\delta_i{}^j + \Lambda_i{}^j \end{pmatrix}, \quad (5.21)$$

with

$$K = (K_\alpha{}^\beta), \quad \text{tr} K = 0, \quad \Lambda = (\Lambda_i{}^j), \\ \Lambda^\dagger = -\Lambda, \quad \text{tr} \Lambda = 0. \quad (5.22)$$

Here, the matrix elements correspond to a Lorentz transformation $(K_\alpha{}^\beta, \bar{K}^{\dot{\alpha}}{}_\beta)$, Poincaré translation $a^{\dot{\alpha}\beta}$,

special conformal transformation $b_{a\dot{\beta}}$, Q -supersymmetry $(\varepsilon_i^\alpha, \bar{\varepsilon}^{\dot{\alpha}i})$, S -supersymmetry $(\eta_\alpha^i, \bar{\eta}_{i\dot{\alpha}})$, combined chiral and scale transformation Δ , and $\text{SU}(\mathcal{N})$ transformation $\Lambda_i{}^j$.

It can be shown (see [14] for the derivation) that, under infinitesimal superconformal transformations, the coordinates of the north chart transform as

$$\begin{aligned}\delta\tilde{x}_+ &= \tilde{a} + \frac{1}{2}(\Delta + \bar{\Delta})\tilde{x}_+ + \bar{K}\tilde{x}_+ + \tilde{x}_+K \\ &\quad - \tilde{x}_+b\tilde{x}_+ + 4i\tilde{e}\theta - 4\tilde{x}_+\eta\theta,\end{aligned}\quad (5.23a)$$

$$\begin{aligned}\delta\theta &= \epsilon + \frac{1}{2\mathcal{N}}((\mathcal{N}-2)\Delta + 2\bar{\Delta})\theta + \theta K \\ &\quad + \Lambda\theta - \theta b\tilde{x}_+ - i\tilde{\eta}\tilde{x}_+ - 4\theta\eta\theta.\end{aligned}\quad (5.23b)$$

The AdS transformations can be singled out as those superconformal transformations which preserve the AdS condition (5.10b). This requirement proves to impose the following constraints on the parameters in (5.23)

$$b^a = -a^a, \quad (5.24a)$$

$$\eta_\alpha^i = i\delta^{ij}\epsilon_{\alpha j}, \quad (5.24b)$$

$$\Delta = 0. \quad (5.24c)$$

Further, only the antisymmetric component of Λ remains

$$\Lambda = -\Lambda^T. \quad (5.25)$$

D. Invariant supermetric on $\text{AdS}^{4|4\mathcal{N}}$

In this section we will elucidate some more details about the supertwistor construction above, and use it to introduce an $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})_{\text{U}}$ -invariant supermetric on $\text{AdS}^{4|4\mathcal{N}}$. Our analysis is similar to that given in [50] for the $2n$ -extended supersphere $S^{3|4n}$.

Given a superconformal transformation $\hat{g} \in \text{SU}(2, 2|\mathcal{N})$ that preserves the condition (5.9) on an open domain of $\text{AdS}^{4|4\mathcal{N}}$, a two-plane $\mathfrak{T}$ transforms as

$$\begin{aligned}\mathfrak{T} &\rightarrow \hat{g}\mathfrak{T} \sim \mathfrak{T}' = \hat{g}\mathfrak{T}R(\hat{g}, \mathfrak{T}), \\ R(\hat{g}, \mathfrak{T}) &\in \text{GL}(2, \mathbb{C}).\end{aligned}\quad (5.26)$$

Here, the matrix $R(\hat{g}, \mathfrak{T})$ serves two purposes: (i) it is used to preserve the parametrization of $\mathfrak{T}$, Eq. (5.17); and (ii) it is used to restore the normalization condition (5.10b). Indeed, for a generic superconformal transformation, the two-plane $\hat{g}\mathfrak{T}$ does not satisfy (5.10b). However, provided it still satisfies (5.9), that is

$$\langle \hat{g}T^\mu | \hat{g}T^\nu \rangle_{\mathfrak{F}} \neq 0, \quad (5.27)$$

one can always make use of the equivalence relation (5.8) to restore the normalization

$$\langle T'^\mu | T'^\nu \rangle_{\mathfrak{F}} = \ell \epsilon^{\mu\nu}. \quad (5.28)$$

The situation differs slightly for AdS transformations. Given an element of the AdS supergroup $g \in \text{OSp}_0(\mathcal{N}|4; \mathbb{R})_{\text{U}}$, a two-plane $\mathfrak{T}$ transforms as

$$\mathfrak{T} \rightarrow g\mathfrak{T} \sim \mathfrak{T}' = g\mathfrak{T}N(g, \mathfrak{T}), \quad N(g, \mathfrak{T}) \in \text{SL}(2, \mathbb{C}). \quad (5.29)$$

That the matrix $N(g, T)$ belongs to $\text{SL}(2, \mathbb{C})$ follows from the fact that the AdS transformations preserve the condition (5.10b). To prove this, let us consider a two-plane belonging to the north chart of $\text{AdS}^{4|4\mathcal{N}}$, Eq. (5.17). Then, for the AdS transformation g , we have

$$g\mathfrak{T} = \lambda(x_+, \theta) \begin{pmatrix} A(g, x_+, \theta) \\ B(g, x_+, \theta) \\ \chi(g, x_+, \theta) \end{pmatrix}. \quad (5.30)$$

Here we have explicitly indicated the dependence of λ on the coordinates x_+^a and θ_i^a . Further, the matrices A, B, χ are coordinate dependent as well as determined by the specific transformation g under consideration. For simplicity, let us assume that gT also belongs to the north chart of $\text{AdS}^{4|4\mathcal{N}}$, that is $\det A \neq 0$. We then introduce the unimodular matrix

$$N = A^{-1} \det A^{\frac{1}{2}} \Rightarrow \det N = 1. \quad (5.31)$$

Making use of the equivalence relation (5.11), we have

$$\begin{aligned}g\mathfrak{T} \sim g\mathfrak{T}N &= \lambda(x_+, \theta) \det A^{\frac{1}{2}} \begin{pmatrix} \mathbb{1}_2 \\ BA^{-1} \\ \chi A^{-1} \end{pmatrix} \\ &\equiv \gamma(x'_+, \theta') \begin{pmatrix} \mathbb{1}_2 \\ -i\tilde{x}'_+ \\ 2\theta' \end{pmatrix}.\end{aligned}\quad (5.32)$$

Finally, the symplectic condition (5.10b) implies

$$\gamma(x'_+, \theta') = (1 - x_+^2 + 2i\theta'^2)^{-\frac{1}{2}} = \lambda(x'_+, \theta'). \quad (5.33)$$

This completes the proof.

Now that we have determined the transformation properties of the two-planes $\mathfrak{T}$ under both finite superconformal and finite AdS transformations, let us introduce the matrix two-point function

$$\mathcal{E}(\mathfrak{T}_1, \mathfrak{T}_2) = \mathfrak{T}_1^\dagger \Omega \mathfrak{T}_2, \quad (5.34)$$

for two two-planes $\mathfrak{T}_1$ and $\mathfrak{T}_2$. Given the null condition (5.7), it follows that

$$\mathcal{E}(\mathfrak{T}_1, \mathfrak{T}_1) = 0. \quad (5.35)$$

The two-point function $\mathcal{E}(\mathfrak{T}_1, \mathfrak{T}_2)$ transforms homogeneously under superconformal transformations

$$\mathcal{E}(\mathfrak{T}'_1, \mathfrak{T}'_2) = R^\dagger(\hat{g}, \mathfrak{T}_1) \mathcal{E}(\mathfrak{T}_1, \mathfrak{T}_2) R(\hat{g}, \mathfrak{T}_2), \quad (5.36)$$

and under AdS transformations

$$\mathcal{E}(\mathfrak{Z}'_1, \mathfrak{Z}'_2) = N^\dagger(g, \mathfrak{Z}_1) \mathcal{E}(\mathfrak{Z}_1, \mathfrak{Z}_2) N(g, \mathfrak{Z}_2). \quad (5.37)$$

Associated with $\mathcal{E}(\mathfrak{Z}_1, \mathfrak{Z}_2)$ is the two-point function

$$\omega(\mathfrak{Z}_1, \mathfrak{Z}_2) = \det \mathcal{E}(\mathfrak{Z}_1, \mathfrak{Z}_2), \quad (5.38)$$

with the superconformal transformation law

$$\omega(\mathfrak{Z}'_1, \mathfrak{Z}'_2) = \det R^\dagger(\hat{g}, \mathfrak{Z}_1) \det R(\hat{g}, \mathfrak{Z}_2) \omega(\mathfrak{Z}_1, \mathfrak{Z}_2). \quad (5.39)$$

Given the relation (5.37) and the fact that $N(g, \mathfrak{Z}) \in \text{SL}(2, \mathbb{C})$, it follows that $\omega(\mathfrak{Z}_1, \mathfrak{Z}_2)$ is invariant under the AdS transformations.

If we restrict our attention to the AdS supergroup only, we can introduce chiral and antichiral two-point functions

$$\mathcal{E}_+(\mathfrak{Z}_1, \mathfrak{Z}_2) = \mathfrak{Z}_1^{\text{ST}} \mathfrak{Z}_2, \quad (5.40a)$$

$$\mathcal{E}_-(\mathfrak{Z}_1, \mathfrak{Z}_2) = (\star \mathfrak{Z}_1)^{\text{ST}} \mathfrak{Z}_2 \star, \quad (5.40b)$$

and

$$\omega_+(\mathfrak{Z}_1, \mathfrak{Z}_2) = \det \mathcal{E}_+(\mathfrak{Z}_1, \mathfrak{Z}_2), \quad (5.41a)$$

$$\omega_-(\mathfrak{Z}_1, \mathfrak{Z}_2) = \det \mathcal{E}_-(\mathfrak{Z}_1, \mathfrak{Z}_2). \quad (5.41b)$$

The two-point functions ω , ω_+ , and ω_- are invariant under the AdS transformations (5.29). However, under equivalence transformations (5.8), they scale as

$$\omega(\mathfrak{Z}_1, \mathfrak{Z}_2) \rightarrow \det R_1^\dagger \det R_2 \omega(\mathfrak{Z}_1, \mathfrak{Z}_2), \quad (5.42a)$$

$$\omega_+(\mathfrak{Z}_1, \mathfrak{Z}_2) \rightarrow \det R_1 \det R_2 \omega_+(\mathfrak{Z}_1, \mathfrak{Z}_2), \quad (5.42b)$$

$$\omega_-(\mathfrak{Z}_1, \mathfrak{Z}_2) \rightarrow \det R_1^\dagger \det R_2^\dagger \omega_-(\mathfrak{Z}_1, \mathfrak{Z}_2). \quad (5.42c)$$

Making use of the above analysis, we can construct a two-point function that is invariant under both the AdS transformations (5.29) and arbitrary equivalence transformations of the form (5.8), as follows:

$$\tilde{\omega}(\mathfrak{Z}_1, \mathfrak{Z}_2) = \frac{\omega(\mathfrak{Z}_1, \mathfrak{Z}_2)}{\sqrt{\omega_-(\mathfrak{Z}_1, \mathfrak{Z}_1) \omega_+(\mathfrak{Z}_2, \mathfrak{Z}_2)}}. \quad (5.43)$$

Choosing $\mathfrak{Z}_1 = \mathfrak{Z}$ and $\mathfrak{Z}_2 = \mathfrak{Z} + d\mathfrak{Z}$ allows us to obtain the AdS supersymmetric interval defined by

$$ds^2 = \tilde{\omega}(\mathfrak{Z}, \mathfrak{Z} + d\mathfrak{Z}). \quad (5.44)$$

Let us evaluate $\tilde{\omega}(\mathfrak{Z}, \mathfrak{Z} + d\mathfrak{Z})$ in the north chart. We find

$$\begin{aligned} \mathcal{E}(\mathfrak{Z}, \mathfrak{Z} + d\mathfrak{Z}) &= -i|\lambda|^2 \Pi^a \tilde{\sigma}_a, \\ \Pi^a &= dx^a + i(\theta \sigma^a d\bar{\theta} - d\theta \sigma^a \bar{\theta}), \end{aligned} \quad (5.45)$$

where Π^a is the Volkov-Akulov one-form [51,52]. We end up with the supermetric

$$ds^2 = \lambda^2 \bar{\lambda}^2 \Pi^a \Pi^b \eta_{ab}. \quad (5.46)$$

which is AdS invariant.

VI. THE SUPERTWISTOR REALIZATION OF $\text{ADS}^{4|8} \times \mathbb{F}_1(2)$

In this section we will develop a supertwistor realization for the flag superspace (1.5a). Such a realization necessarily makes use of odd supertwistors, for which our conventions are described in Sec. IV. The supertwistor realization for the flag superspace $\tilde{\mathbb{M}}^{4|8} \times \mathbb{F}_1(2)$ was given in [14], and here we will build on that construction.

Our starting point is the space of quadruples $\{T^\mu, \Xi^+, \Xi^-\}$ consisting of two even supertwistors T^μ and two odd supertwistors $\Xi^\pm$ such that (i) the bodies of T^μ are linearly independent four-vectors, and (ii) the bodies of $\Xi^\pm$ are linearly independent two-vectors. These supertwistors are further required to obey the relations

$$\langle T^\mu | T^\nu \rangle_\Omega = 0, \quad \langle T^\mu | \Xi^\pm \rangle_\Omega = 0, \quad (6.1a)$$

$$\langle T^\mu | T^\nu \rangle_{\mathfrak{F}} = \ell \varepsilon^{\mu\nu}, \quad (6.1b)$$

and are defined modulo the equivalence relation

$$\begin{aligned} (\Xi^-, \Xi^+, T^\mu) &\sim (\Xi^-, \Xi^+, T^\nu) \left(\begin{array}{cc|c} a & 0 & 0 \\ b & c & 0 \\ \hline \rho^- & \rho^+ & R^\mu \end{array} \right), \\ \left(\begin{array}{cc|c} a & 0 & 0 \\ b & c & 0 \\ \hline \rho^- & \rho^+ & R \end{array} \right) &\in \text{GL}(2|2; \mathbb{C}), \end{aligned} \quad (6.2)$$

with $\rho^\pm$ anticommuting complex parameters. No symplectic condition is imposed on the odd supertwistors $\Xi^\pm$. As above, one can work with normalized two-planes by fixing a particular value of ℓ . Then, the gauge freedom (6.2) is reduced such that the 2×2 matrix $R \in \text{SL}(2, \mathbb{C})$. In what follows we will set $\ell = 1$.

The even and odd supertwistors can be represented as

$$\mathfrak{Z} = \begin{pmatrix} F \\ G \\ \varphi \end{pmatrix}, \quad \Xi^\pm = \begin{pmatrix} \xi^\pm \\ \psi^\pm \\ V^\pm \end{pmatrix}. \quad (6.3)$$

Then, the conditions (6.1) imply the following:

$$F^\dagger \psi + G^\dagger \xi - \varphi^\dagger V = 0, \quad (6.4)$$

in addition to those we encountered in the previous section, Eq. (5.15).

In the north chart, where $\det F \neq 0$, the supertwistors can be chosen to take the form

$$\mathfrak{T} = \lambda \begin{pmatrix} \mathbb{1}_2 \\ -i\tilde{x}_+ \\ 2\theta \end{pmatrix}, \quad \Xi^\pm = \begin{pmatrix} 0 \\ \frac{2\bar{\theta}^\pm}{v^\pm} \end{pmatrix}, \quad (6.5)$$

with

$$v^\pm = (v_i^\pm), \quad \bar{\theta}^\pm = (\bar{\theta}^{\pm\dot{\alpha}}), \quad (6.6a)$$

and

$$\det(v_i^-, v_i^+) = v^{+i}v_i^- \neq 0, \quad v^{+i} = \varepsilon^{ij}v_j^+. \quad (6.6b)$$

The orthogonality conditions $\langle T^\mu | \Xi^\pm \rangle_\Omega = 0$ imply

$$\bar{\theta}^{\pm\dot{\alpha}} = \bar{\theta}^{\dot{\alpha}i}v_i^\pm. \quad (6.7)$$

The complex harmonic variables $v_i^\pm$ in (6.5) are still defined modulo arbitrary transformations of the form

$$(v_i^-, v_i^+) \rightarrow (v_i^-, v_i^+)\tilde{r}, \quad \tilde{r} = \begin{pmatrix} a & 0 \\ b & c \end{pmatrix} \in \text{GL}(2, \mathbb{C}). \quad (6.8)$$

We see that the complex harmonic variables $v^\pm$ parametrize $\mathbb{F}_1(2)$ as described in Sec. II. It follows that the set $\{T^\mu, \Xi^-, \Xi^+\}$ constitutes a supertwistor realization of the AdS flag superspace (1.5).

Let us make use of the equivalence relation (6.8) to impose the condition

$$v^{+i}v_i^- = 1. \quad (6.9a)$$

The harmonics then obey the identity

$$v_i^+v_j^- - v_j^+v_i^- = \varepsilon_{ij}. \quad (6.9b)$$

As explained in Sec. II, the gauge freedom (6.8) allows one to represent any infinitesimal transformation of the harmonics in the form (2.7).

To determine the action of the AdS supergroup on the harmonic variable v^+ , it is useful to begin with an infinitesimal superconformal transformation (5.21). It can be shown that

$$\delta v_i^+ = -\tilde{\Lambda}^{++}v_i^-, \quad (6.10a)$$

where $\tilde{\Lambda}^{++}$ is expressed as

$$\tilde{\Lambda}^{++} = \Lambda^{ij}v_i^+v_j^+ - 4i\theta^+b\bar{\theta}^+ - 4(\theta^+\eta^+ - \bar{\theta}^+\bar{\eta}^+); \quad (6.10b)$$

see [14] for the derivation. Making use of the derivatives D_α^+ and $\bar{D}_\alpha^+$ defined by Eq. (3.7), we can see that

$$D_\alpha^+\tilde{\Lambda}^{++} = \bar{D}_\alpha^+\tilde{\Lambda}^{++} = 0. \quad (6.11)$$

The variations of $\theta^{+\alpha}$ and $\bar{\theta}^{+\dot{\alpha}}$ are given by

$$\begin{aligned} \delta\theta^{+\alpha} &= \delta\theta^{\alpha i}v_i^+ - \tilde{\Lambda}^{++}\theta^{\alpha i}v_i^-, \\ \delta\bar{\theta}^{+\dot{\alpha}} &= \delta\bar{\theta}^{\dot{\alpha} i}v_i^+ - \tilde{\Lambda}^{++}\bar{\theta}^{\dot{\alpha} i}v_i^-, \end{aligned} \quad (6.12)$$

where $\delta\theta^{\alpha i}$ is given by (5.23). Further, they satisfy the property

$$D_\beta^+\delta\theta_\alpha^+ = \bar{D}_\beta^+\delta\theta_\alpha^+ = 0. \quad (6.13)$$

Now, one can single out the AdS transformations by imposing the constraints (5.24) and (5.25).

Finally, we comment on the analytic bosonic coordinates

$$y^a = x^a - 2i\theta^{(i}\sigma^a\bar{\theta}^{j)}v_i^+v_j^-, \quad D_\alpha^+y^a = \bar{D}_\alpha^+y^a = 0. \quad (6.14)$$

It can be shown that, under an infinitesimal transformation of the AdS supergroup, the variation δy^a satisfies

$$D_\alpha^+\delta y^a = D_\alpha^+\delta(x_-^a - i\theta^+\sigma^a\bar{\theta}^-) = 0, \quad (6.15a)$$

$$\bar{D}_\alpha^+\delta y^a = \bar{D}_\alpha^+\delta(x_+^a - i\theta^-\sigma^a\bar{\theta}^+) = 0. \quad (6.15b)$$

In order to prove the relations (6.15), we make use of the identities (6.9b) and (6.13), as well as the fact that δx_+^a , defined by Eq. (5.23) subject to the conditions (5.24), is chiral, $\bar{D}_\alpha^i\delta x_+^a = 0$. It follows that the analytic subspace parametrized by the variables

$$\zeta = (y^a, \theta^{+\alpha}, \bar{\theta}_\alpha^+, v_i^+, v_i^-), \quad D_\alpha^+\zeta = \bar{D}_\alpha^+\zeta = 0 \quad (6.16)$$

is invariant under the AdS supergroup.

VII. CONCLUSION

When dealing with off-shell $\mathcal{N} = 1$ and $\mathcal{N} = 2$ supersymmetric field theories in AdS_4 (see [25, 28, 53–58] for an incomplete list of references) one usually makes use of *local* superspace differential geometry. In the $\mathcal{N}$ -extended case, the algebra of covariant derivatives for $\text{AdS}^{4|\mathcal{N}}$ is given by the graded commutation relations [38, 39],

$$\{\mathcal{D}_\alpha^i, \mathcal{D}_\beta^j\} = 4S^{ij}M_{\alpha\beta} - 4\varepsilon_{\alpha\beta}S^{k[i}\mathbb{J}^{j]}_k, \quad (7.1a)$$

$$\{\bar{\mathcal{D}}_i^{\dot{\alpha}}, \bar{\mathcal{D}}_j^{\dot{\beta}}\} = -4\bar{S}_{ij}\bar{M}_{\dot{\alpha}\dot{\beta}} + 4\varepsilon^{\dot{\alpha}\dot{\beta}}\bar{S}_{k[i}\mathbb{J}^{j]}_k, \quad (7.1b)$$

$$\{\mathcal{D}_\alpha^i, \bar{\mathcal{D}}_j^{\dot{\beta}}\} = -2i\delta_j^i\mathcal{D}_\alpha^{\dot{\beta}}, \quad (7.1c)$$

$$[\mathcal{D}_\alpha^i, \mathcal{D}_{\beta\dot{\beta}}] = -i\varepsilon_{\alpha\beta}S^{ij}\bar{\mathcal{D}}_{\dot{\beta}j}, \quad [\bar{\mathcal{D}}_i^{\dot{\alpha}}, \mathcal{D}_{\beta\dot{\beta}}] = i\delta_{\dot{\beta}}^{\dot{\alpha}}\bar{S}_{ij}\mathcal{D}_\beta^j, \quad (7.1d)$$

$$[\mathcal{D}_{\alpha\dot{\alpha}}, \mathcal{D}_{\beta\dot{\beta}}] = -2|S|^2(\varepsilon_{\alpha\beta}\bar{M}_{\dot{\alpha}\dot{\beta}} + \varepsilon_{\dot{\alpha}\dot{\beta}}M_{\alpha\beta}),$$

$$|S|^2 := \frac{1}{\mathcal{N}} S^{ij} \bar{S}_{ij} > 0. \quad (7.1e)$$

Here $\mathbb{J}_j^i$ denotes the $\mathrm{SU}(\mathcal{N})$ generator, $S^{ij} = S^{ji}$ is a covariantly constant curvature tensor, and $\bar{S}_{ij} := \overline{S^{ij}}$. When $\mathcal{N} > 1$, the constraint $\mathcal{D}_A S^{jk} = 0$ implies the following integrability condition:

$$\delta_{(k}^{[i} S^{j]m} \bar{S}_{l)m} = 0 \Rightarrow S^{ik} \bar{S}_{jk} = \frac{1}{\mathcal{N}} \delta_j^i S^{kl} \bar{S}_{kl}. \quad (7.2)$$

As demonstrated in [39], performing a local $\mathrm{U}(\mathcal{N})_R$ transformation allows one to bring S^{ij} to the form

$$S^{ij} = \delta^{ij} S. \quad (7.3)$$

Now, the condition $\mathcal{D}_A S^{jk} = 0$ tells us the $\mathrm{SU}(\mathcal{N})_R$ connection involves only the operators

$$\mathcal{J}^{ij} := -2\delta^{k[i} \mathbb{J}^{j]}_k = -\mathcal{J}^{ji}, \quad (7.4)$$

which generate the group $SO(\mathcal{N})$. The algebra of covariant derivatives then takes the form:

$$\{\mathcal{D}_\alpha^i, \mathcal{D}_\beta^j\} = 4S\delta^{ij}M_{\alpha\beta} + 2\varepsilon_{\alpha\beta}S\mathcal{J}^{ij}, \quad (7.5a)$$

$$\{\bar{\mathcal{D}}_i^\alpha, \bar{\mathcal{D}}_j^\beta\} = -4\bar{S}\delta_{ij}\bar{M}^{\alpha\beta} - 2\varepsilon^{\alpha\beta}\bar{S}\mathcal{J}_{ij}, \quad (7.5b)$$

$$\{\mathcal{D}_\alpha^i, \bar{\mathcal{D}}_j^\beta\} = -2i\delta_j^i \mathcal{D}_\alpha^\beta, \quad (7.5c)$$

$$[\mathcal{D}_\alpha^i, \mathcal{D}_{\beta\dot{\beta}}] = -i\varepsilon_{\alpha\beta}S\bar{\mathcal{D}}_{\dot{\beta}}^i, \quad [\bar{\mathcal{D}}_i^\alpha, \mathcal{D}_{\beta\dot{\beta}}] = i\delta_{\beta\dot{\beta}}^{\alpha\dot{\beta}}S\mathcal{D}_{\beta j}, \quad (7.5d)$$

$$[\mathcal{D}_{\alpha\dot{\alpha}}, \mathcal{D}_{\beta\dot{\beta}}] = -2|S|^2(\varepsilon_{\alpha\beta}\bar{M}_{\dot{\alpha}\dot{\beta}} + \varepsilon_{\dot{\alpha}\dot{\beta}}M_{\alpha\beta}). \quad (7.5e)$$

In a previous series of papers [37–39], we have developed the global approach to the $\mathrm{AdS}^{4|\mathcal{N}}$ supergeometry. The novelty of the present paper is that we have reformulated the global realization of $\mathrm{AdS}^{4|\mathcal{N}}$ by explicitly embedding the AdS supergroup in the superconformal group. The virtue of this approach is that (i) it shows how the AdS superspace arises as a certain open domain of $\bar{\mathbb{M}}^{4|\mathcal{N}}$, (ii) it allows us to read off the superconformal and isometry transformation rules for $\mathrm{AdS}^{4|\mathcal{N}}$ in terms of those known for $\bar{\mathbb{M}}^{4|\mathcal{N}}$, and (iii) it proves most suitable for developing a global realization of the flag superspace (1.5).

In a recent work [43] Ivanov and Zagraev derived the AdS isometry transformations on the analytic subspace of the $\mathcal{N} = 2$ AdS harmonic superspace. Their construction was based on the following two inputs: (i) the known $\mathcal{N} = 2$ superconformal transformations in the analytic subspace of harmonic superspace [36], and (ii) the known embedding of the $\mathcal{N}$ -extended AdS superalgebra $\mathfrak{osp}(\mathcal{N}|4; \mathbb{R})$ in the

$\mathcal{N}$ -extended superconformal algebra $\mathfrak{su}(2, 2|\mathcal{N})$ [41,42]. Their analysis was limited to an open domain of the $\mathcal{N} = 2$ AdS harmonic superspace, and no discussion of its global structure was given. In our approach, the AdS isometry transformations acting on the analytic subspace of the $\mathcal{N} = 2$ AdS harmonic superspace are readily derived from the supertwistor formulation given. The two approaches are related to each other as follows. In the notation of Sec. VC, the north chart of $\mathrm{AdS}^{4|\mathcal{N}}$ corresponds to those two-planes (5.14) and (5.15) which are characterized by the condition $\mathrm{def} F \neq 0$. In the $\mathcal{N} = 2$ case, this is exactly the open domain of $\mathrm{AdS}^{4|8}$ which was dealt with in [43]. As concerns the complement of the north chart in $\mathrm{AdS}^{4|\mathcal{N}}$, which is singled out by requiring $\mathrm{def} F = 0$, it is a closed subset of $\mathrm{AdS}^{4|\mathcal{N}}$. The latter subset has measure zero, and therefore it does not contribute to the supersymmetric actions (such as the action over the analytic subspace of $\mathcal{N} = 2$ AdS harmonic superspace [43]) provided the dynamical superfields obey natural boundary conditions. It would be useful to re-formulate the harmonic superspace action for AdS_4 as an integral over a closed super six-form, similar to the flat-superspace case studied in [59]. To achieve such a goal, our global approach to $\mathrm{AdS}^{4|8} \times \mathbb{F}_1(2)$ is expected to be advantageous.

Invariance of the projective-superspace action under the transformations (2.1b) in five-dimensional $\mathcal{N} = 1$ supergravity [18] and four-dimensional $\mathcal{N} = 2$ supergravity [60] was essential for the explicit construction of the action functional. We believe that the harmonic-superspace action principle in supergravity (in the biharmonic setting) should be compatible with invariance under (2.3b). This issue will be discussed elsewhere.

Let us now consider the algebra (7.5) in the $\mathcal{N} = 2$ case. The presence of the $\mathrm{SO}(2)$ generator suggests that it is quite natural to consider an AdS superspace of the form $\mathrm{AdS}^{4|8} \times S^1$, although this superspace does not allow the action of the superconformal group. Making use of our construction in the present paper, one can readily derive a supertwistor realization of this superspace.¹³ To do so, we introduce the space of triples $\{T^\mu, \Xi\}$ consisting of two even supertwistors T^μ and a single real odd supertwistor $\Xi = \star\Xi$ such that (i) the bodies of T^μ are linearly independent four-vectors, and (ii) the body of Ξ is non-vanishing. These supertwistors are required to obey the relations

$$\langle T^\mu | T^\nu \rangle_\Omega = 0, \quad \langle T^\mu | \Xi \rangle_\Omega = 0, \quad (7.6a)$$

$$\langle T^\mu | T^\nu \rangle_{\mathfrak{F}} = \ell \varepsilon^{\mu\nu}, \quad (7.6b)$$

and are defined modulo the equivalence relation

¹³See [40] for a similar story in five dimensions.

$$(\Xi, T^\mu) \sim (\Xi, T^\nu) \left(\frac{a}{0} \middle| \frac{0}{R_\nu^\mu} \right),$$

$$a \in \mathbb{R} - \{0\}, \quad R \in \text{GL}(2, \mathbb{C}). \quad (7.7)$$

The superspace obtained is seen to be

$$\text{AdS}^{4|8} \times \mathbb{R}P^1 \simeq \text{AdS}^{4|8} \times S^1. \quad (7.8)$$

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX A: $\mathcal{N}=2$ CONFORMAL KILLING SUPERVECTOR FIELDS

This appendix is devoted to a brief review of the $\mathcal{N}=2$ conformal Killing supervector fields. Our presentation is inspired by [61] and follows [62].

An infinitesimal superconformal transformation

$$z^A \rightarrow z^A + \delta z^A,$$

$$\delta z^A = \xi z^A = (\xi^a + i(\xi_i \sigma^a \bar{\theta}^i - \theta_i \sigma^a \bar{\xi}^i), \xi_i^\alpha, \bar{\xi}_\alpha^i) \quad (A1)$$

is generated by a *conformal Killing supervector field*

$$\xi = \xi^b \partial_b + \xi_j^\beta D_\beta^j + \bar{\xi}_\beta^j \bar{D}_j^\beta = \bar{\xi}. \quad (A2)$$

The defining property of ξ is

$$[\xi, D_\alpha^i] = -(D_\alpha^i \xi_j^\beta) D_\beta^j. \quad (A3)$$

This condition implies the relations

$$\bar{D}_i^\alpha \xi_j^\beta = 0, \quad \bar{D}_i^\alpha \xi^\beta = 4i \epsilon^{\alpha\beta} \xi_i^\beta \Rightarrow \xi_i^\alpha = -\frac{i}{8} \bar{D}_{\alpha i} \xi^\alpha \quad (A4)$$

and their complex conjugates, and therefore

$$D_{(\alpha i} \xi_{\beta)}^\beta = 0, \quad \bar{D}_{(\alpha}^\beta \xi_{\beta)}^\beta = 0 \Rightarrow \partial_{(\alpha} \xi_{\beta)}^\beta = 0. \quad (A5)$$

It then follows that

$$[\xi, D_\alpha^i] = -K_\alpha^\beta[\xi] D_\beta^i - \frac{1}{2} \bar{\sigma}[\xi] D_\alpha^i - \Lambda_{ij}[\xi] D_\alpha^j. \quad (A6)$$

Here we have introduced the chiral Lorentz $K_{\beta\gamma}[\xi]$ and super-Weyl $\sigma[\xi]$ parameters, as well as the $\text{SU}(2)_R$ parameter $\Lambda^{ij}[\xi]$ defined by

$$K_{\alpha\beta}[\xi] = \frac{1}{2} D_{(\alpha}^i \xi_{\beta)i} = K_{\beta\alpha}[\xi], \quad \bar{D}_i^\alpha K_{\alpha\beta}[\xi] = 0, \quad (A7a)$$

$$\sigma[\xi] = \frac{1}{2} \bar{D}_i^\alpha \bar{\xi}_\alpha^i, \quad \bar{D}_i^\alpha \sigma[\xi] = 0, \quad (A7b)$$

$$\Lambda^{ij}[\xi] = -\frac{i}{16} [D_\alpha^{(i}, \bar{D}_\alpha^{j)}] \xi^{\alpha\dot{\alpha}} = \Lambda^{ji}[\xi],$$

$$\overline{\Lambda^{ij}[\xi]} = \Lambda_{ij}[\xi]. \quad (A7c)$$

We recall that the Lorentz parameters with vector and spinor indices are related to each other as follows: $K^{bc}[\xi] = (\sigma^{bc})_{\beta\gamma} K^{\beta\gamma}[\xi] - (\bar{\sigma}^{bc})_{\dot{\beta}\dot{\gamma}} \bar{K}^{\dot{\beta}\dot{\gamma}}[\xi]$. The parameters in (A7) obey several first-order differential properties:

$$D_\alpha^i \Lambda^{jk}[\xi] = \epsilon^{(j} D_\alpha^k) \sigma[\xi], \quad (A8a)$$

$$D_\alpha^i K_{\beta\gamma}[\xi] = -\epsilon_{\alpha(\beta} D_\gamma^i \sigma[\xi], \quad (A8b)$$

and therefore

$$D_\alpha^{(i} \Lambda^{jk)}[\xi] = \bar{D}_\alpha^{(i} \Lambda^{jk)}[\xi] = 0, \quad (A9a)$$

$$D_\alpha^i D_\beta^j \sigma[\xi] = 0. \quad (A9b)$$

The superconformal transformation law of a primary tensor superfield (with suppressed indices) is

$$\delta_\xi U = \mathcal{K}[\xi] U, \quad (A10a)$$

$$\mathcal{K}[\xi] = \xi + \frac{1}{2} K^{ab}[\xi] M_{ab} + \Lambda^{ij}[\xi] J_{ij} + p\sigma[\xi] + q\bar{\sigma}[\xi]. \quad (A10b)$$

Here the generators M_{ab} and J_{ij} act on the Lorentz and $\text{SU}(2)$ indices of U , respectively. The parameters p and q are related to the dimension (or Weyl weight) w and $\text{U}(1)_R$ charge c of U as $p + q = w$ and $p - q = -\frac{1}{2}c$.

The most general $\mathcal{N}=2$ conformal Killing supervector field has the form

$$\xi_\alpha^\alpha = a^{\alpha\alpha} + \frac{1}{2} (\Delta + \bar{\Delta}) x_+^{\alpha\alpha} + \bar{K}^\alpha_{\beta} x_+^{\beta\alpha} + x_+^{\alpha\beta} K_\beta^\alpha - x_+^{\alpha\beta} b_{\beta\dot{\beta}} x_+^{\dot{\beta}\alpha}$$

$$+ 4i \bar{\epsilon}^{\alpha i} \theta_i^\alpha - 4x_+^{\alpha\beta} \eta_\beta^i \theta_i^\alpha, \quad (A11a)$$

$$\xi_i^\alpha = \epsilon_i^\alpha + \frac{1}{2} \bar{\Delta} \theta_i^\alpha + \theta_i^\beta K_\beta^\alpha + \Lambda_i^j \theta_j^\alpha - \theta_i^\beta b_{\beta\dot{\beta}} x_+^{\dot{\beta}\alpha}$$

$$- i \bar{\eta}_{\dot{\beta} i} x_+^{\dot{\beta}\alpha} - 4\theta_i^\beta \eta_\beta^j \theta_j^\alpha, \quad (A11b)$$

where we have introduced the complex four-vector

$$\xi_+^a = \xi^a + 2i \xi_i \sigma^a \bar{\theta}^i, \quad \bar{\xi}^a = \bar{\xi}^a, \quad (A12)$$

along with the complex bosonic coordinates $x_+^a = x^a + i\theta_i\sigma^a\bar{\theta}^i$ of the chiral subspace of $\mathbb{M}^{4|8}$. The constant bosonic parameters in (A11) correspond to the spacetime translation ($a^{\dot{\alpha}}$), Lorentz transformation ($K_\beta^\alpha, \bar{K}^{\dot{\alpha}}_{\dot{\beta}}$), $SU(2)_R$ transformation ($\Lambda^{ij} = \Lambda^{\dot{j}\dot{i}}$), special conformal transformation ($b_{\alpha\dot{\beta}}$), and combined scale and $U(1)_R$ transformations ($\Delta = \tau - 2i\varphi$). The constant fermionic parameters in (A11) correspond to the Q -supersymmetry (ϵ_i^α) and S -supersymmetry (η_i^α) transformations. The constant parameters $K_{\alpha\dot{\beta}}$, Λ^{ij} and Δ are obtained from $K_{\alpha\dot{\beta}}[\xi]$, $\Lambda^{ij}[\xi]$ and $\sigma[\xi]$, respectively, by setting $z^A = 0$.

In the case of the Q -supersymmetry transformation, when the only nonvanishing parameters in (A11) are ϵ_i^α and its conjugate, it holds that the descendants $K^{ab}[\xi]$, $\Lambda^{ij}[\xi]$, and $\sigma[\xi]$ vanish, and the transformation law (A10) takes the universal form

$$\begin{aligned}\delta_\epsilon U &= (2i(\theta_i\sigma^a\bar{\epsilon}^i - \epsilon_i\sigma^a\bar{\theta}^i)\partial_b + \epsilon_i^\alpha D_\alpha^i + \bar{\epsilon}_{\dot{\alpha}}^i \bar{D}_{\dot{\alpha}}^i)U \\ &=: (\epsilon_i^\alpha Q_\alpha^i + \bar{\epsilon}_{\dot{\alpha}}^i \bar{Q}_{\dot{\alpha}}^i)U.\end{aligned}\quad (\text{A13})$$

APPENDIX B: ANOTHER SIMILARITY TRANSFORMATION FOR THE ADS SUPERGROUP

In Sec. IV C, we described an isomorphic realization of the AdS supergroup, denoted $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})_{\mathcal{U}}$, which was useful for our applications in this paper. It turns out that there is another unitary supermatrix which relates the two realizations of the AdS supergroup $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})$ and $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})_{\mathcal{U}}$. Below, we will describe this supermatrix and how it is related to that of Eq. (4.18).

Let us introduce the supermatrix $\mathfrak{N}$ defined as

$$\mathfrak{N} = \left(\begin{array}{c|c} \mathfrak{n} & 0 \\ \hline 0 & \mathbb{1}_{\mathcal{N}} \end{array} \right), \quad \mathfrak{n} = \frac{e^{-i\pi/4}}{\sqrt{2}} \left(\begin{array}{c|c} \mathbb{1}_2 & -\varepsilon \\ \hline i\varepsilon & -i\mathbb{1}_2 \end{array} \right). \quad (\text{B1})$$

The supermatrix $\mathfrak{N}$ enjoys the properties

$$\mathfrak{N}^\dagger = \mathfrak{N}^{-1}, \quad (\text{B2a})$$

$$\mathfrak{N}^\dagger \mathbb{J} \mathfrak{N} = -i\Omega, \quad (\text{B2b})$$

$$\mathfrak{N}^{sT} \mathbb{J} \mathfrak{N} = \mathfrak{J}. \quad (\text{B2c})$$

It turns out that, for every $f \in \text{OSp}_0(\mathcal{N}|4; \mathbb{R})$, the supermatrix defined by

$$g = \mathfrak{N}^{-1} f \mathfrak{N} \quad (\text{B3})$$

belongs to $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})_{\mathcal{U}}$.

As the supermatrices $\mathfrak{N}$ and $\mathcal{U}$ both take us to the realization $\text{OSp}_0(\mathcal{N}|4; \mathbb{R})_{\mathcal{U}}$, they must be related to each other in the following way:

$$\mathcal{U} = \mathfrak{N} \mathfrak{S}, \quad (\text{B4a})$$

where $\mathfrak{S}$ satisfies the following properties:

$$\mathfrak{S}^\dagger = \mathfrak{S}^{-1}, \quad \mathfrak{S}^\dagger \Omega \mathfrak{S} = \Omega, \quad \mathfrak{S}^{sT} \mathfrak{J} \mathfrak{S} = \mathfrak{J}. \quad (\text{B4b})$$

It can be shown that the solution to Eq. B4 takes the form

$$\mathfrak{S} = \left(\begin{array}{c|c} \mathfrak{s} & 0 \\ \hline 0 & \mathbb{1}_{\mathcal{N}} \end{array} \right), \quad \mathfrak{s} = \frac{1}{\sqrt{2}} \left(\begin{array}{c|c} \varepsilon & i\varepsilon \\ \hline i\varepsilon & \varepsilon \end{array} \right). \quad (\text{B5})$$

APPENDIX C: THE KILLING SUPERVECTORS OF $\text{AdS}^{4|4\mathcal{N}}$

In this appendix we will provide an alternative derivation of the constraints (5.24) and (5.25), making use of the $SU(\mathcal{N})$ superspace formulation for $\text{AdS}^{4|4\mathcal{N}}$ developed in [38,39].

$\text{AdS}^{4|4\mathcal{N}}$ is parametrized by local coordinates $z^M = (x^m, \theta^\mu, \bar{\theta}_{\dot{\mu}})$. Its covariant derivatives $\mathcal{D}_A = (\mathcal{D}_a, D_\alpha^i, \bar{D}_{\dot{\alpha}}^i)$ take the form

$$\mathcal{D}_A = E_A + \frac{1}{2} \Omega_A^{cd} M_{cd} + \Phi_A^i{}_j \mathbb{J}^j{}_i. \quad (\text{C1})$$

Here, $M_{cd} = -M_{dc}$ are the Lorentz generators, and $\mathbb{J}^j{}_i$ are the $SU(\mathcal{N})$ generators. In a conformally flat frame, the covariant derivatives are given by the expressions

$$\mathcal{D}_\alpha^i = e^{\frac{\mathcal{N}-2}{2\mathcal{N}}\sigma + \frac{1}{\mathcal{N}}\bar{\sigma}} (D_\alpha^i + D^{\beta i} \sigma M_{\alpha\beta} + D_\alpha^j \sigma \mathbb{J}^j{}_i), \quad (\text{C2a})$$

$$\bar{\mathcal{D}}_{\dot{\alpha}}^i = e^{\frac{1}{\mathcal{N}}\sigma + \frac{\mathcal{N}-2}{2\mathcal{N}}\bar{\sigma}} (\bar{D}_{\dot{\alpha}}^i - \bar{D}_{\dot{\beta}i} \bar{\sigma} \bar{M}^{\dot{\beta}i} - \bar{D}_{\dot{\alpha}}^j \bar{\sigma} \mathbb{J}^j{}_i), \quad (\text{C2b})$$

$$\begin{aligned}\mathcal{D}_{\alpha\dot{\alpha}} &= e^{\frac{1}{2}\sigma + \frac{1}{2}\bar{\sigma}} \left(\partial_{\alpha\dot{\alpha}} + \frac{i}{2} D_\alpha^i \sigma \bar{D}_{\dot{\alpha}i} + \frac{i}{2} \bar{D}_{\dot{\alpha}i} \bar{\sigma} D_\alpha^i \right. \\ &\quad + \frac{1}{2} \left(\partial_\alpha^\beta (\sigma + \bar{\sigma}) - \frac{i}{2} D^{\beta i} \sigma \bar{D}_{\dot{\alpha}i} \bar{\sigma} \right) M_{\alpha\beta} \\ &\quad + \frac{1}{2} \left(\partial_\alpha^{\dot{\beta}} (\sigma + \bar{\sigma}) + \frac{i}{2} D_\alpha^i \sigma \bar{D}_{\dot{\alpha}i} \bar{\sigma} \right) \\ &\quad \times \bar{M}_{\dot{\alpha}\dot{\beta}} - \frac{i}{2} D_\alpha^i \sigma \bar{D}_{\dot{\alpha}j} \bar{\sigma} \mathbb{J}^j{}_i \Big),\end{aligned} \quad (\text{C2c})$$

where $D_A = (\partial_a, D_\alpha^i, \bar{D}_{\dot{\alpha}}^i)$ are the flat $\mathcal{N}$ -extended covariant derivatives, and σ is a chiral superfield, $\bar{D}_{\dot{\alpha}}^i \sigma = 0$, satisfying the constraints

$$D_{(\alpha}^{[i} D_{\beta]}^j e^\sigma = 0, \quad (\text{C3a})$$

$$[D_\alpha^i, \bar{D}_{\dot{\alpha}i}] e^{\frac{\mathcal{N}}{2}(\sigma + \bar{\sigma})} = 0. \quad (\text{C3b})$$

In the $\mathcal{N} = 1$ case, the constraint (C3a) should be replaced with

$$-\frac{1}{4}e^{2\bar{\sigma}}D^2e^{-\sigma} = \text{const.} \quad (\text{C4})$$

The constraints (C3a), (C3b), and (C4) have been solved in [28,44,57] in the $\mathcal{N}=1$ and $\mathcal{N}=2$ cases, for both stereographic coordinates and Poincaré coordinates. In the $\mathcal{N}$ -extended case, they are solved in [39] for both realizations. For the stereographic solution, $W = e^{-\sigma}$ takes the form

$$W = \left(1 - \frac{s^{ij}\bar{s}_{ij}}{4\mathcal{N}}x_+^2 + s^{ij}\theta_{ij}\right)^{-1}, \quad (\text{C5})$$

where s^{ij} satisfies the properties

$$s^{ij} = s^{ji}, \quad s^{ik}\bar{s}_{kj} = |s|^2\delta_j^i, \quad \bar{s}_{ij} := \overline{s^{ij}}. \quad (\text{C6})$$

The superfield W plays the role of the compensator for $\text{AdS}^{4|4\mathcal{N}}$. It should be pointed out that W had been constructed earlier, in Ref. [42], making use of an alternative approach.

Now we turn to determining the Killing supervectors of $\text{AdS}^{4|4\mathcal{N}}$. An infinitesimal isometry of $\text{AdS}^{4|4\mathcal{N}}$ is generated by a Killing supervector $\xi^A E_A$ which is defined to satisfy the property

$$[\xi^A D_A + \frac{1}{2}\lambda^{cd}M_{cd} + \lambda^i{}_j \mathbb{J}^j{}_i, D_B] = 0 \quad (\text{C7})$$

for a real antisymmetric tensor $\lambda^{cd}(z)$. In the $\mathcal{N}=2$ case, $\lambda^{ij} = \epsilon^{jk}\lambda^i{}_k$ is symmetric, $\lambda^{ij} = \lambda^{ji}$, and (C7) was solved in [28]. Since $\text{AdS}^{4|4\mathcal{N}}$ is conformally related to $\mathcal{N}$ -extended Minkowski superspace $\mathbb{M}^{4|4\mathcal{N}}$, see the relations (C2), the supervector $\xi^A E_A$ can be decomposed with respect to the AdS basis $\{E_A\}$ or the flat basis $\{D_A\}$, as

$$\xi = \xi^A E_A = \xi^A D_A. \quad (\text{C8})$$

Here, ξ^A are the components of a conformal Killing supervector, which generates infinitesimal superconformal transformations in $\mathbb{M}^{4|4\mathcal{N}}$

$$z^A \longrightarrow z^A + \xi^A, \quad (\text{C9})$$

and is defined to satisfy the constraint

$$[\xi, D_\alpha^i] \propto D_\beta^j, \quad (\text{C10})$$

see, e.g., [14,61], for more details. With respect to the basis $\{D_A\}$, the components of ξ are

$$\begin{aligned} \tilde{\xi}_+ = (\xi_+^{\alpha\alpha}) &= \tilde{a} + \frac{1}{2}(\Delta + \bar{\Delta})\tilde{x}_+ + \bar{K}\tilde{x}_+ \\ &+ \tilde{x}_+K - \tilde{x}_+b\tilde{x}_+ + 4i\bar{e}\theta - 4\tilde{x}_+\eta\theta, \end{aligned} \quad (\text{C11a})$$

$$\begin{aligned} (\xi_i^\alpha) &= \epsilon + \frac{1}{2\mathcal{N}}((\mathcal{N}-2)\Delta + 2\bar{\Delta})\theta + \theta K + \Lambda\theta \\ &- \theta b\tilde{x}_+ - i\bar{\eta}\tilde{x}_+ - 4\theta\eta\theta, \end{aligned} \quad (\text{C11b})$$

$$\begin{aligned} \xi^a &= \frac{1}{2}(\xi_+^a + \xi_-^a) + i(\theta_i\sigma^a\bar{\xi}^i - \xi_i\sigma^a\bar{\theta}^i), \\ \xi_+^a &= -\frac{1}{2}\xi_+^{\alpha\alpha}(\sigma^a)_{\alpha\dot{\alpha}} = \bar{\xi}_-^a, \end{aligned} \quad (\text{C11c})$$

where the parameters $\{a, b, K, \bar{K}, \Delta, \bar{\Delta}, \epsilon, \bar{e}, \eta, \bar{\eta}, \Lambda\}$ are identified with those in (5.21).

Given a superconformal transformation, the compensator W transforms as

$$\delta W = \xi W + \sigma[\xi]W, \quad (\text{C12a})$$

$$\sigma[\xi] = \frac{1}{\mathcal{N}(\mathcal{N}-4)}((\mathcal{N}-2)D_\alpha^i\xi_i^\alpha - 2\bar{D}_i^{\dot{\alpha}}\bar{\xi}_{\dot{\alpha}}^i). \quad (\text{C12b})$$

Then, the problem of determining the AdS Killing supervectors proves to be equivalent to determining those conformal Killing supervectors which do not change the compensator (C5),

$$\delta W = 0. \quad (\text{C13})$$

The $\mathcal{N}=1$ and $\mathcal{N}=2$ cases were worked out in [28,44], respectively. It can be shown that Eq. (C13) imposes the following constraints on the transformation parameters in (C11)

$$b^a = -\frac{s^{ij}\bar{s}_{ij}}{4\mathcal{N}}a^a, \quad (\text{C14a})$$

$$\eta_\alpha^i = \frac{1}{2}s^{ij}\epsilon_{aj}, \quad (\text{C14b})$$

$$s^{k(i}\Lambda_k^{j)}) = 0, \quad (\text{C14c})$$

$$\Delta = 0. \quad (\text{C14d})$$

In particular, Eq. (C14c) implies

$$\hat{s}\Lambda + \Lambda^T\hat{s} = 0, \quad \hat{s} := (s^{ij}). \quad (\text{C15})$$

Then, making use of (C8), one can read off the components of the AdS Killing supervector ξ^A .

Finally, comparing the compensator (C5) with the chiral parameter λ given by Eq. (5.20), we find

$$\lambda = e^{-\frac{1}{2}\sigma}. \quad (\text{C16})$$

Further, in the north chart developed in the main body, s^{ij} is given by

$$s^{ij} = 2i\delta^{ij}. \quad (\text{C17})$$

Inserting (C17) into (C14), we find complete agreement with the constraints derived from the supertwistor approach, Eqs. (5.24) and (5.25).

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