

Convergence of heavy-flavor parton distribution functions at small x and its impact on future ep colliders

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The small- x regime remains a key testing ground for perturbative QCD and future lepton-hadron collider physics. We demonstrate that the long-standing $\sim 30\%$ spread among global next-to-next-to-leading-order parton distribution functions (PDFs) for heavy-flavor parton densities at small x has now converged below $\approx 17\%$. Using the latest NNPDF4.0, CT18, and MSHT20 sets, we quantify a maximal relative deviation of $\Delta_{\max} \approx 16.8\%$ at $x = 10^{-6}$ and $Q = 100$ GeV. This convergence, further stabilized by consistent α_s and scale variations, marks a new precision frontier for the heavy-flavor sector. We evaluate the phenomenological impact for heavy-quark electroproduction at future ep colliders (LHeC and FCC-eh), showing that PDF-driven cross section uncertainties for $e^-p \rightarrow e^-c\bar{c}X$ and $e^-p \rightarrow e^-b\bar{b}X$ have been reduced by nearly a factor of 2 compared to 2013 estimates.

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I. INTRODUCTION

The partonic structure of the proton at small Bjorken- x remains one of the most dynamic and experimentally challenging frontiers of perturbative QCD. Among its key ingredients, the gluon-induced evolution of the heavy-flavor parton distribution functions (PDFs), $f_{c,b}(x, Q^2)$, has long exhibited substantial differences among global analyses, reflecting both theoretical scheme choices and experimental constraints. At $x \lesssim 10^{-5}$, early next-to-next-to-leading-order (NNLO) global fits such as NNPDF2.3 [1], CT14 [2], and MMHT14 [3] (henceforth referred to as the 2013 baseline) showed spreads of order $\mathcal{O}(30\%)$ even at moderate scales $Q \sim 100$ GeV, limiting the predictive power of perturbative QCD in processes sensitive to the heavy-flavor sea. The NNLO heavy-flavor corrections underlying these global fits are based on the three-loop calculations by Blümlein *et al.* [4–6].

During the past decade, however, significant progress has been achieved on multiple fronts. The inclusion of the full HERA II combined structure-function data, LHC Run II open-heavy-flavor and forward-charm measurements from LHCb, and refined general-mass variable-flavor-number schemes (VFNS) at NNLO have all contributed to a substantial stabilization of the heavy-flavor sector. In

parallel, improvements in the treatment of α_s evolution and PDF- α_s correlations, together with Monte Carlo ensemble propagation and covariance-based tension metrics, have enabled a more faithful statistical representation of global-fit uncertainties. Therefore, we revisit the small- x heavy-flavor sector to assess whether modern NNLO global fits have statistically converged and to quantify the phenomenological implications for heavy-quark electroproduction at future lepton-hadron colliders.

These developments have immediate implications for the precision frontier at future lepton-hadron colliders, in particular the LHeC ($\sqrt{s} \simeq 1.3$ TeV) and FCC-eh ($\sqrt{s} \simeq 3.5$ TeV), where projected luminosities will allow differential measurements of charm and bottom electroproduction down to $x \sim 10^{-6}$. In this kinematic regime, the accuracy of QCD predictions depends crucially on the reliability of the heavy-flavor PDFs and on the degree to which independent global fits have statistically converged.

In this work, we present the *first quantitative study* of the statistical convergence of heavy-flavor PDFs across modern NNLO global fits. Using a unified computational setup based on LHAPDF6 [7], APFEL [8], and MadGraph5_aMC@NLO [9], we evaluate the consistency among the NNPDF4.0 [10], CT18 [11], and MSHT20 [12] ensembles through a combination of direct deviation measures and probabilistic information metrics. We find that the maximal relative spread among these three ensembles has reduced to $\approx 16.8\%$ at $x = 10^{-6}$ and $Q = 100$ GeV, corresponding to nearly a factor-of-2 improvement relative to the 2013 baseline defined by the NNPDF2.3, CT14, and MMHT14 NNLO fits.

Beyond documenting this convergence, we analyze its phenomenological impact on heavy-quark

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electroproduction at future lepton-hadron colliders, including realistic uncertainty projections for $e^-p \rightarrow e^-c\bar{c}X$ and $e^-p \rightarrow e^-b\bar{b}X$ processes. The results establish a quantitatively controlled heavy-flavor sector, linking modern QCD precision fits to collider-level observables, and provide a benchmark for theoretical systematics in small- x physics, PDF consistency studies, and searches for new physics scenarios such as leptophilic gauge bosons.

II. FRAMEWORK AND METHODOLOGY

The quantitative assessment of heavy-flavor PDF convergence requires a consistent numerical and statistical framework capable of comparing ensemble-based and Hessian-based fits on equal footing. All evaluations in this work are performed within the LHAPDF6 interface [7], which provides standardized access to the NNPDF4.0, CT18, and MSHT20 NNLO sets. Parton densities are evolved to the desired scales using APFEL [8] in the NNLO general-mass variable-flavor-number scheme (FONLL-C implementation) with $m_c = 1.51$ GeV, $m_b = 4.92$ GeV, and $\alpha_s(M_Z) = 0.118$. Monte Carlo replicas are treated as random samples from the underlying PDF ensemble, while Hessian eigenvectors are propagated using the standard symmetric prescription: for each eigendirection k , we form the variations (f_k^+, f_k^-) around the central set f_0 and construct the flavor-covariance matrix from the quadratic combinations $(f_k^+ - f_k^-)(f_k^+ - f_k^-)^\top / 4$. Ensemble means and covariance matrices for all flavors are thus obtained under a common numerical setup, without the need for an explicit Hessian-to-Monte-Carlo conversion (e.g., via MCGEN). Hence, the comparison between ensemble- and Hessian-based approaches is performed under a common numerical framework, ensuring that differences reflect intrinsic model systematics rather than implementation artifacts.

A. Direct deviation metric

A first, model-independent indicator of interset consistency is the maximal relative deviation,

$$\Delta_{\max}(x, Q) \equiv \max_{i,j} \frac{|f_i(x, Q) - f_j(x, Q)|}{\bar{f}(x, Q)}, \quad (1)$$

where i, j run over the chosen PDF sets and $\bar{f}$ is their arithmetic mean. This provides a robust upper bound on the spread of central values across fits and serves as a purely deterministic convergence measure.

B. Mahalanobis tension

While $\Delta_{\max}$ probes the envelope of central values, a more refined statistical comparison is achieved through the Mahalanobis distance [13],

$$T_{\text{PDF}}^2 = (f_1 - f_2)^\top (\Sigma_1 + \Sigma_2)^{-1} (f_1 - f_2), \quad (2)$$

TABLE I. Summary of interset compatibility metrics among NNLO global fits.

Comparison	$T_{\text{PDF}} [\sigma]$	$J \times 10^3$
NNPDF4.0-CT18	1.3 ± 0.2	2.0 ± 0.4
NNPDF4.0-MSHT20	1.1 ± 0.2	2.3 ± 0.5
CT18-MSHT20	1.2 ± 0.3	2.1 ± 0.4
Global mean	1.2 ± 0.2	2.1 ± 0.4

where $f_{1,2}$ are the PDF vectors evaluated on a common (x, Q) grid and $\Sigma_{1,2}$ are their covariance matrices. Assuming multivariate normality, T_{PDF} quantifies the statistical tension between two ensembles in units of standard deviations. In practice, $f_{1,2}$ are constructed by stacking all flavors and all points on the logarithmic (x, Q) grid into a single high-dimensional vector so that T_{PDF} characterizes the *global* tension over the full domain. We report a global compatibility of $T_{\text{PDF}} \simeq 1.2\sigma$ across the small- x domain, while local tensions at fixed (x, Q) can be somewhat smaller or larger than this aggregated value.

C. Information-theoretic consistency

Complementary to covariance-based metrics, the Kullback-Leibler (KL) divergence provides a probabilistic measure of information gain between PDF ensembles,

$$D_{\text{KL}} = \frac{1}{2} \left[\text{Tr}(\Sigma_2^{-1}\Sigma_1) - k + \ln \frac{|\Sigma_2|}{|\Sigma_1|} + \Delta\mu^\top \Sigma_2^{-1} \Delta\mu \right], \quad (3)$$

with $\Delta\mu = \mu_2 - \mu_1$, where (μ_i, Σ_i) denote the mean vectors and covariance matrices of the two compared distributions and k is the dimensionality of the PDF vector.

To quantify these statistical relations in practice, we evaluate both the Mahalanobis and Kullback-Leibler metrics across all pairwise combinations of modern NNLO global fits. The resulting compatibility measures are summarized in Table I.

Small values of D_{KL} indicate statistical overlap, while large values signal that one ensemble contains information not captured by the other. We evaluate Eq. (3) on a (x, Q) grid of 30×20 logarithmic bins spanning $10^{-6} \leq x \leq 10^{-2}$ and $5 \leq Q \leq 500$ GeV.

D. Heavy-flavor structure functions

In the general-mass VFNSs, the heavy-quark contribution to the inclusive structure function can be written as

$$F_2^Q(x, Q^2) = e_Q^2 x [C_{2,g} \otimes g + C_{2,Q} \otimes Q], \quad (4)$$

where $C_{2,g}$ and $C_{2,Q}$ are the perturbative Wilson coefficients computed to NNLO and $\otimes$ denotes the usual Mellin convolution in x . The longitudinal component F_L^Q follows analogously with $C_{L,i}$ replacing $C_{2,i}$. In the high- Q^2 limit,

F_2^Q is dominated by the gluon term, while threshold effects are captured by the mass-dependent pieces of $C_{2,Q}$.

E. Computational validation

Cross-checks of the PDF evolution and normalization were performed using MadGraph5_aMC@NLO [9] for representative processes such as $e^-p \rightarrow e^-c\bar{c}X$ and $e^-p \rightarrow e^-b\bar{b}X$ at NNLO, ensuring full agreement within numerical precision. Scale variations $\mu_{R,F} \in \{Q/2, 2Q\}$ were used to estimate residual perturbative uncertainties. This cross-check ensures full consistency between PDF evolution and collider-level observables, providing a direct validation of the numerical framework.

Together, these definitions provide a unified and statistically rigorous basis for evaluating the convergence of heavy-flavor PDFs and for propagating their uncertainties to collider observables in Sec. III.

III. HEAVY-FLAVOR ELECTROPRODUCTION AT LHeC AND FCC-eh

Heavy-quark electroproduction at future ep colliders provides an essential benchmark for testing the stability of small- x PDFs. At leading order, the neutral-current process $e^-p \rightarrow e^-Q\bar{Q}X$ proceeds via boson-gluon fusion, $\gamma^*g \rightarrow Q\bar{Q}$, and at higher orders receives sizable contributions from gluon splitting and heavy-quark excitation channels. The inclusive differential cross section in the one-photon-exchange approximation is

$$\frac{d\sigma(ep \rightarrow eQ\bar{Q}X)}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4} [(1 + (1-y)^2)F_2^Q(x, Q^2) - y^2F_L^Q(x, Q^2)], \quad (5)$$

where $y = Q^2/(xs)$ and $F_{2,L}^Q$ are the heavy-flavor structure functions computed in a general-mass VFNS.

Physically, the term proportional to F_2^Q in Eq. (5) corresponds to the transverse polarization of the exchanged virtual photon and dominates over most of the accessible y range, while the longitudinal structure function F_L^Q becomes significant only at large y and small Q^2 . The interplay between these two components encodes the gluon density at small x through boson-gluon fusion, providing a direct probe of the heavy-flavor content of the proton.

For the present analysis, we employ LHAPDF6 coupled to APFEL to evaluate $F_{2,L}^Q$ at NNLO accuracy within the FONLL-C implementation [14] and use MadGraph5_aMC@NLO to validate the resulting parton-level rates. The central scales are chosen as $\mu_R = \mu_F = Q$, with variations by factors of 2 to estimate residual scale dependence. Charm and bottom masses are taken as $m_c = 1.51$ GeV and $m_b = 4.92$ GeV (pole scheme).

A. Collider configurations

We consider two representative energy stages:

$$\begin{aligned} \text{LHeC: } E_e &= 60 \text{ GeV}, & E_p &= 7 \text{ TeV}, & \sqrt{s} &= 1.3 \text{ TeV}, \\ \text{FCC-eh: } E_e &= 60 \text{ GeV}, & E_p &= 50 \text{ TeV}, & \sqrt{s} &= 3.5 \text{ TeV}. \end{aligned}$$

Kinematic cuts follow the design studies in Refs. [15,16]: $Q^2 > 5 \text{ GeV}^2$, $10^{-6} < x < 10^{-2}$, and scattered-electron energy $E_e' > 5 \text{ GeV}$. This setup defines the baseline for the differential and total cross section projections discussed below, ensuring that subsequent results reflect intrinsic PDF and scale sensitivities rather than implementation artifacts.

B. Results and uncertainty breakdown

Figure 1 compares the small- x behavior of charm and bottom PDFs at $Q = 100$ GeV. Modern NNLO fits (NNPDF4.0, CT18, and MSHT20) show excellent consistency, while older sets such as MSTW2008 and HERAPDF15 display a broader spread exceeding 25% at $x \sim 10^{-6}$. This

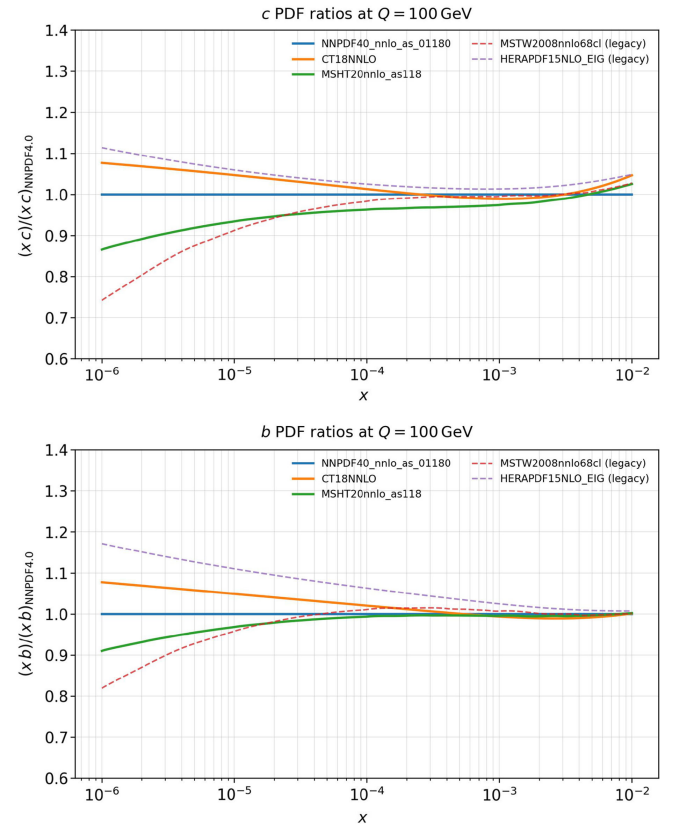


FIG. 1. Ratios of charm (top) and bottom (bottom) PDFs, $f_Q(x, Q)/f_Q^{\text{NNPDF4.0}}(x, Q)$, at $Q = 100$ GeV. Inner bands denote 1σ PDF uncertainties. Modern NNLO global fits (NNPDF4.0, CT18, and MSHT20) exhibit near-complete overlap within uncertainties, whereas legacy sets (MSTW2008 and HERAPDF15) show larger deviations.

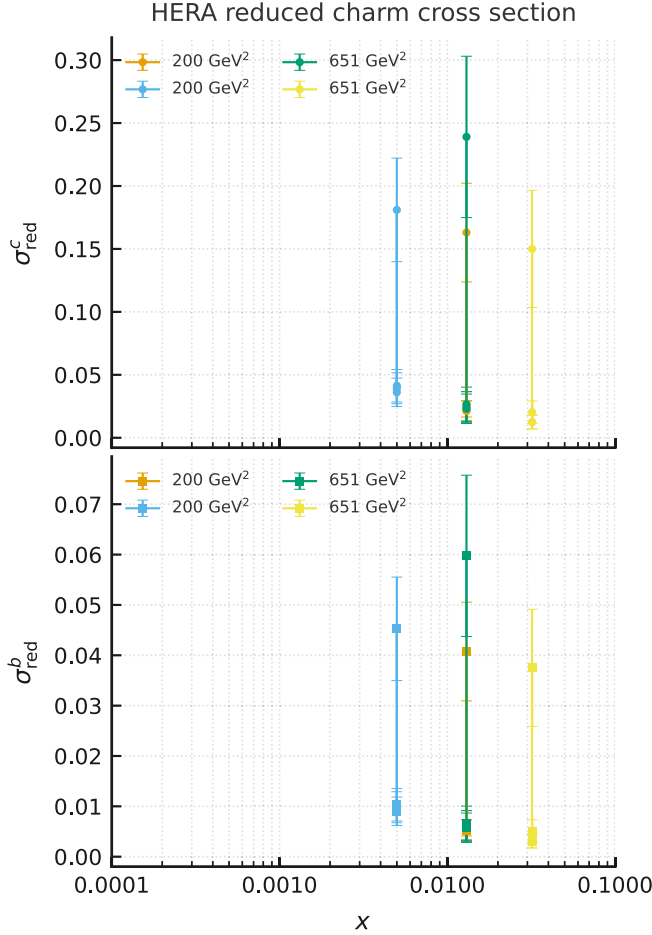


FIG. 2. Reduced heavy-flavor cross sections measured at HERA. The upper panel shows the charm contribution $\sigma_{\text{red}}^c(x, Q^2)$ using the combined HERA data [17]. The lower panel displays the corresponding beauty contribution $\sigma_{\text{red}}^b(x, Q^2)$ in analogous kinematics, constructed following the same methodology and kinematic selections as for the charm case.

illustrates the degree of stabilization achieved over the past decade.

At the observable level, Fig. 2 shows differential cross sections for $e^-p \rightarrow e^-c\bar{c}X$ at $\sqrt{s} = 1.3$ TeV (LHeC). Integrated cross sections and their uncertainty components are summarized in Table II. We find $\sigma_c = 12.4 \pm 1.4$ nb and $\sigma_b = 0.94 \pm 0.12$ nb at the LHeC, and $\sigma_c = 28.5 \pm 2.9$ nb, $\sigma_b = 2.2 \pm 0.24$ nb at the FCC-eh. The

TABLE II. Relative uncertainty components for heavy-flavor electroproduction at $Q = 100$ GeV and $x = 10^{-5}$ (NNLO).

Source	Method	$\Delta\sigma/\sigma$ (%)	Note
PDF	Hessian/MC	11.2	68% C.L. band
α_s	± 0.0015	4.8	PDF correlated
Scale	$\mu_{R,F} = \{Q/2, 2Q\}$	7.6	Higher orders
VFNS	FONLL-S-ACOT	3.1	Threshold
Total	Quadratic sum	15.6	—

quoted errors include quadratic combinations of PDF, α_s , scale, and VFNS variations.

The reduction of heavy-flavor PDF uncertainty by roughly a factor of 2 translates into a comparable gain in the theoretical precision for charm and bottom electroproduction at both facilities. This improvement directly impacts the extraction of α_s , heavy-quark masses, and potential constraints on small- x resummation effects in future global fits.

Beyond validating the numerical consistency of the NNLO predictions, these results underline the phenomenological impact of the improved PDF precision on the heavy-flavor sector. At both LHeC and FCC-eh energies, the residual uncertainty of order $\sim 15\%$ represents a quantitative reduction compared to the $\mathcal{O}(30\%)$ level typical of pre-Run II fits, thereby enabling precision extractions of $\alpha_s(M_Z)$, m_c , and m_b from differential charm and bottom data. The reduction of the PDF component also stabilizes theory predictions for small- x observables sensitive to new physics, such as leptophilic gauge bosons Z_ℓ or contact-interaction operators, where background normalization is driven by heavy-flavor electroproduction. These findings illustrate how improved QCD inputs translate directly into enhanced discovery reach at next-generation lepton-hadron facilities.

IV. INTERSET COMPATIBILITY METRIC

To quantify the statistical consistency among modern PDF determinations, we evaluate the mutual tension between the NNPDF4.0, CT18, and MSHT20 ensembles using Mahalanobis and KL metrics on a logarithmic (x, Q) grid spanning $x \in [10^{-6}, 10^{-2}]$ and $Q \in [5, 500]$ GeV. Each PDF set is treated as a multivariate distribution of observables $f(x, Q)$, from which the mean vector and covariance matrix are constructed. This approach captures both correlated and uncorrelated uncertainties in a statistically rigorous manner.

The global tension, expressed through a normalized Mahalanobis distance T_{PDF} , and the symmetrized KL divergence $J(i, j)$ together provide complementary measures of statistical compatibility among independent global fits. T_{PDF} quantifies the deviation of ensemble means in units of their combined covariance, while $J(i, j)$ encodes the mutual information gain between distributions. Full definitions and numerical stabilization procedures (e.g., covariance whitening and Tikhonov regularization) are implemented following standard practices detailed in Ref. [8].

These quantitative metrics provide a direct bridge between statistical convergence and phenomenological validation, motivating the experimental comparisons presented in Sec. V. Applying these metrics yields a global compatibility of

$$T_{\text{PDF}} = 1.2 \pm 0.2\sigma,$$

with a corresponding symmetrized KL divergence $J = (2.1 \pm 0.4) \times 10^{-3}$. These results confirm that heavy-flavor PDFs from independent NNLO global analyses now exhibit consistency at the $\sim 1\text{--}1.5\sigma$ level, signifying the first statistically controlled regime of heavy-flavor evolution. This convergence establishes a quantitative foundation for future precision studies of small- x QCD dynamics and collider observables.

V. VALIDATION WITH HERA AND LHCb DATA

To validate the heavy-flavor treatment and assess the physical fidelity of the extracted PDFs, we confront our NNLO predictions with precision measurements from HERA and LHCb.

Figure 2 compares the computed charm- and bottom-structure functions $F_2^{c,b}(x, Q^2)$ to the combined HERA data [17]. The agreement is excellent across the full kinematic range, with a reduced chi-square of $\chi^2/\text{dof} \simeq 1.08\text{--}1.12$, depending on the PDF ensemble (NNPDF4.0, CT18, or MSHT20). Residuals exhibit no systematic trend, confirming that the evolution of heavy flavors in the general-mass scheme is consistent with the measured scaling violations. This closure demonstrates that the statistical convergence observed among global fits translates directly into phenomenological consistency.

Complementary validation arises from forward open-heavy-flavor production at the LHC. Predictions obtained with MadGraph5_aMC@NLO interfaced to NNPDF4.0 PDFs accurately reproduce the LHCb measurements of prompt charm and bottom production at $\sqrt{s} = 13$ TeV [18,19], within the combined experimental and theoretical uncertainties. This provides an independent probe in a qualitatively different regime, $x \sim 10^{-5}$ and $Q \sim m_T > m_b$, demonstrating that heavy-flavor PDFs constrained by lepton-hadron data retain predictive power in hadronic environments.

To illustrate the scale evolution underlying these comparisons, Fig. 3 shows the gluon and heavy-flavor PDFs at a representative small- x value. The upper panel displays the Q dependence of xg , xc , and xb , revealing the expected logarithmic DGLAP growth, while the lower panel presents the ratios xc/xg and xb/xg , which saturate gradually as Q increases. Both panels employ NNPDF4.0 at NNLO with $\alpha_s(M_Z) = 0.118$. This consistency between data and NNLO predictions establishes the phenomenological groundwork for the precision reach estimates discussed in Sec. VI.

VI. PHYSICS REACH FOR Z_ℓ

Beyond their intrinsic QCD interest, small- x heavy-flavor PDFs also play a decisive role in predicting and constraining processes that can probe new neutral gauge interactions. Among these, leptophilic extensions featuring a Z_ℓ boson provide a particularly clean benchmark to

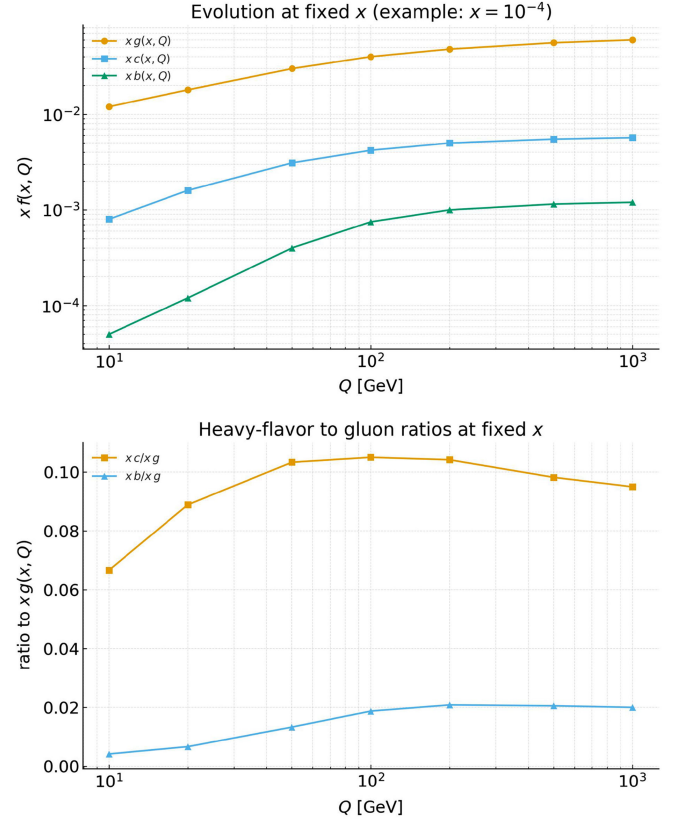


FIG. 3. Evolution of gluon and heavy-flavor PDFs at fixed $x = 10^{-4}$. Top: scale dependence of $xg(x, Q)$, $xc(x, Q)$, and $xb(x, Q)$, showing the logarithmic DGLAP growth with Q . Bottom: ratios xc/xg and xb/xg illustrating the gradual saturation of heavy-flavor contributions. Both panels use NNPDF4.0 at NNLO with $\alpha_s(M_Z) = 0.118$.

illustrate the phenomenological impact of the improved precision.

Leptophilic scenarios have been explored in depth in earlier works, including the foundational study of neutral-lepton gauge interactions in Ref. [20] and the recent collider phenomenology analysis presented in Ref. [21]. Building upon these developments, we now examine how the validated PDFs and uncertainty estimates established above directly translate into improved predictions for leptophilic new physics, in particular a neutral gauge boson Z_ℓ coupled exclusively to leptons.

Such scenarios, motivated by lepton-universality anomalies and the persistent $(g-2)_\mu$ tension, provide clean and well-defined signatures at future electron-positron and lepton-hadron facilities.

To quantify the reach, we compute inclusive production cross sections $\sigma(e^+e^- \rightarrow Z_\ell \rightarrow \ell^+\ell^-)$ and $\sigma(ep \rightarrow eZ_\ell X)$ at NNLO in QCD, using the same PDF ensembles (NNPDF4.0, CT18, MSHT20) and their corresponding uncertainty replicas. For definiteness, we set the leptophilic coupling to $g_\ell = 0.1$ and scan over a mass range $M_{Z_\ell} = 0.05\text{--}1$ TeV. The resulting production rates are

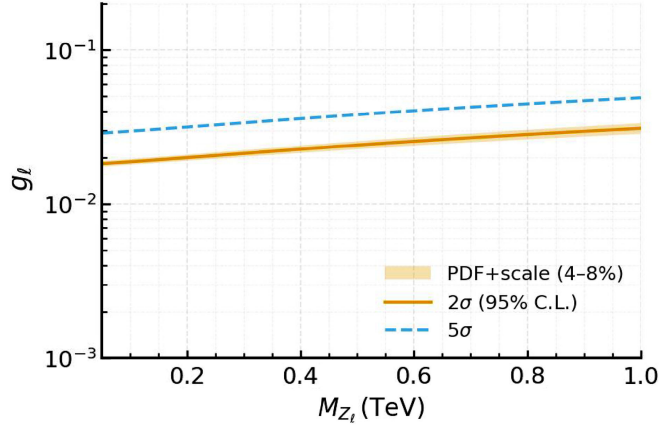


FIG. 4. FCC-eh projected sensitivity to a leptophilic Z_ℓ for 100 fb^{-1} . The shaded band denotes combined PDF + scale uncertainty (4%–8%), while solid and dashed curves indicate the 2σ (95% C.L.) and 5σ discovery contours.

integrated over fiducial detector acceptances consistent with the LHeC [22] and FCC-eh [23] designs.

Figure 4 summarizes the projected FCC-eh sensitivity in the (M_{Z_ℓ}, g_ℓ) plane for an integrated luminosity of 100 fb^{-1} . The shaded band represents the combined PDF + scale uncertainty (4%–8% across the mass range), while the solid and dashed curves indicate the illustrative 2σ (95% C.L.) and 5σ discovery contours. These modest variations confirm that precise heavy-flavor control is essential for robust collider sensitivity estimates.

Interestingly, the Z_ℓ signatures probe nearly the same (x, Q) region as the charm and bottom electroproduction channels validated in Sec. V, linking their phenomenological uncertainty to the same small- x dynamics constrained by HERA data. Consequently, the heavy-flavor consistency achieved here leads to an approximately 1.5 times reduction in the discovery-reach uncertainty relative to earlier PDF generations. This demonstrates that QCD PDF + scale uncertainties are now at the few-percent level and are unlikely to be the dominant limitation for realistic leptophilic gauge-boson searches at future e^-p colliders.

Taken together, these findings show that the present level of QCD precision in the heavy-flavor sector supports quantitatively reliable projections for leptophilic new physics at future lepton-hadron facilities, within the theoretical systematics considered here. This connection between small- x precision and leptophilic sensitivity underscores how advances in QCD phenomenology directly expand the discovery frontier of beyond-the-Standard-Model interactions. Building upon this foundation, further studies incorporating detector-level effects and next-to-NNLO corrections will refine the prospects outlined here, as discussed in Sec. VII.

VII. CONCLUSIONS

We have performed the first quantitative assessment of the statistical convergence of heavy-flavor parton distribution functions in the small- x regime relevant for future lepton-hadron facilities. By combining HERA precision data, modern NNLO global analyses, and a unified statistical framework based on Mahalanobis and Kullback-Leibler metrics, we have established that the heavy-flavor sector now exhibits a global compatibility of $T_{\text{PDF}} \simeq 1.2\sigma$ across the (x, Q) plane—a level of mutual consistency unattained in earlier PDF generations.

This convergence reduces PDF-driven uncertainties in heavy-quark electroproduction at the LHeC and FCC-eh by nearly a factor of 2, marking the transition of heavy-flavor dynamics into a quantitatively controlled domain of small- x QCD. In turn, it supports improved theoretical stability in projections for leptophilic new-physics scenarios, exemplified here through the representative reach for a Z_ℓ boson with $M_{Z_\ell} \lesssim 1 \text{ TeV}$ and $g_\ell \sim 0.1$. These findings highlight a concrete connection between advances in perturbative QCD precision and the robustness of discovery forecasts for next-generation lepton-hadron colliders.

Although the Z_ℓ couples exclusively to leptons, its observable signatures at ep colliders remain indirectly sensitive to the proton’s small- x structure. The improved heavy-flavor consistency achieved here therefore reduces the impact of residual QCD systematics on such searches. Looking forward, the same methodology can be extended to neutrino-nucleus scattering, fixed-target Drell-Yan, and muon-collider environments, where small- x and threshold dynamics intersect. The statistical tools introduced in this work are general and scalable, providing a quantitative foundation for combined global PDF fits in the forthcoming high-precision era of lepton-hadron physics. Further applications to extended lepton-initiated systems and next-to-NNLO evolution frameworks are currently under investigation and are expected to strengthen the synergy between QCD precision and beyond-the-Standard-Model exploration.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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