

# Quadrupole forces between quark and gluon subsystems inside higher-spin particles

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We generalize the mechanical interpretation of the forces between quark and gluon subsystems, previously studied for the nucleon, to arbitrary higher-spin particles. For spin-0 and spin-1/2 particles, this force is characterized by the nonconserved  $\bar{c}(t)$  form factor. However, such an interpretation has not yet been established for higher-spin particles due to the intricate structure of the nonconserved energy-momentum tensor (EMT) form factors. By performing a multipole expansion, we identify the physically meaningful combinations of the nonconserved covariant EMT form factors and provide them with a clear mechanical interpretation.

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## I. INTRODUCTION

The energy momentum tensor (EMT) has long been regarded as a valuable tool for investigating the quark and gluon structure of hadrons. In the 1960s [1,2], it was considered largely an academic subject, despite containing rich physical information. With the realization that the EMT can be accessed through hard exclusive scattering processes, however, it has since become an important focus of research. Its interpretation in terms of energy, angular momentum, and mechanical distributions has been extensively developed [3–11]. The EMT structure has also been studied for higher-spin particles [12–16] and in transitions [17–22], opening a novel avenue for accessing tensor-polarized (quadrupole) structures in hadrons with  $J \geq 1$ .

Unlike the electromagnetic structure, the EMT provides separate information on quark and gluon contributions, thereby emphasizing the role of gluons and motivating the physics program of the Electron Ion Collider (EIC). In this context, understanding the interplay between quarks and gluons is essential, as it is characterized by the non-conserved EMT form factors [23]. These form factors have

been interpreted in terms of the forces between quark and gluon subsystems [23], and this concept was subsequently extended to the forces between different quark flavors [24]. Subsequent studies for spin-1/2 particles have also been reported [25–27].

For spin-0 and spin-1/2 particles, information on these forces is encoded in a single  $\bar{c}(t)$  form factor, making their interpretation straightforward and simple. However, for higher-spin particles, a corresponding interpretation of the nonconserved EMT form factors is still lacking. Moreover, the structure of higher-spin particles is far more intricate, requiring a systematic organization with clear physical meaning. In this work, we aim to complete this mechanical interpretation for arbitrary-spin particles by employing a multipole expansion and establishing its connection to the nonconserved EMT form factors.

The current work is organized as follows. In Sec. II, we introduce the definition of the EMT operator in QCD and interpret its hadronic matrix element in terms of three-dimensional (3D) EMT distributions. The method of multipole expansion is outlined in Sec. III and applied to the stress tensor in Sec. IV. In Sec. V, we extend the mechanical interpretation of the forces between the quark and gluon subsystems to higher-spin particles within the framework of the multipole expansion. Their connection to the nonconserved EMT form factors is established for particles with  $J < 2$  in Sec. VI. Using a toy model, we then visualize the forces for a spin-1 particle in both longitudinally and transversely polarized states in Sec. VII. Finally, we present our conclusions and discuss future outlook.

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## II. ENERGY-MOMENTUM TENSOR

The quark ( $Q$ ) and gluon ( $g$ ) parts of the EMT operator in QCD are given by [5]

$$\hat{T}_Q^{\mu\nu}(x) = \frac{1}{4} \bar{\psi}_Q(x) \gamma^{\{\mu} i \overleftrightarrow{\nabla}^{\nu\}} \psi_Q(x), \quad (1a)$$

$$\hat{T}_g^{\mu\nu}(x) = -F^{c\mu\lambda}(x) F^{c\nu}_{\lambda}(x) + \frac{1}{4} g^{\mu\nu} F^2(x), \quad (1b)$$

where  $a^{\{\mu} b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$ , and  $\overleftrightarrow{\nabla} = \overrightarrow{\nabla} - \overleftarrow{\nabla}$  denotes the covariant derivative. Here,  $F$  represents the field strength tensor. Note that the EMT current is conserved only when both quark and gluon contributions are combined,

$$\partial_\mu \hat{T}^{\mu\nu}(x) = 0, \quad \text{with} \quad \hat{T}^{\mu\nu}(x) = \sum_{a=Q,g} \hat{T}_a^{\mu\nu}(x). \quad (2)$$

The matrix element of the QCD EMT operator (1) between any hadron states encodes information about the internal structure and is parametrized by the EMT form factors. It is defined as

$$\langle p', s' | \hat{T}^{\mu\nu}(0) | p, s \rangle, \quad (3)$$

where the initial  $|p, s\rangle$  and final  $\langle p', s'|$  hadron states are characterized by their momenta  $p$  and  $p'$  and spin polarizations  $s$  and  $s'$ , respectively. The states are normalized as  $\langle p', s' | p, s \rangle = 2E(2\pi)^3 \delta^{(3)}(\mathbf{p}' - \mathbf{p}) \delta_{s's}$  where  $E$  is the average energy of the initial and final states.

The momentum variables can be expressed in terms of the average and difference of the initial and final momenta as

$$P = \frac{p' + p}{2}, \quad \Delta = p' - p. \quad (4)$$

The on shell condition for the initial and final states,  $p'^2 = p^2 = m^2$ , can then be rewritten in terms of the variables in Eq. (4) as

$$P \cdot \Delta = 0, \quad P^2 + \frac{\Delta^2}{4} = m^2, \quad (5)$$

where  $m$  denotes the hadron mass.

The matrix element of the EMT operator between hadronic states (3) conveys information on both the motion of a hadron and its internal structure. By performing a Fourier transform, one obtains the corresponding distribution in position space, which reveals the mechanical properties inside the hadron. However, to achieve a strict definition [9] of this spatial distribution that reflects only the intrinsic structure, one must consider relativistic two-dimensional (2D) densities defined within light-front quantization and in the corresponding frames. These densities

are defined in the transverse plane, and thus one may lose information about the longitudinal direction.

On the other hand, the standard frame for studying hadronic matrix elements and their spatial distributions is the three-dimensional (3D) Breit frame (BF) [3]. The spatial distribution defined in this frame is contaminated by relativistic motion of the hadron so that it does not represent purely internal structure and is often regarded as an ambiguous one. Nevertheless, the 3D interpretation provides intuitive and transparent understanding of the corresponding internal structure. It also retains information along the longitudinal direction. In particular, it is conceptually useful to interpret this 3D distribution as an unambiguous relativistic density in the Wigner sense [28,29]. Moreover, it becomes exact in the large- $N_c$  limit [30] or for non-relativistic hadronic motion.

The connection between the 3D distributions and 2D densities has been explored in terms of the Abel transformation [7,8,31]. In particular, the physical interpretation of the stress part of the EMT for the nucleon remains intact, with the difference being purely geometric and only minimally affected by the Wigner rotation. Of course, for higher-spin particles this feature becomes more intricate [32], but in principle the 2D densities can be reconstructed from the 3D distributions in terms of the Abel transformation. Therefore, we will analyze the matrix element in the 3D BF in the current work.

The 3D BF is defined through the following symmetric frame:

$$P = (E, \mathbf{0}), \quad \Delta = (0, \mathbf{\Delta}), \quad (6)$$

where the on shell condition (5) gives  $E = \sqrt{m^2 - t/4}$  with  $\Delta^2 = t$ . In the Wigner sense [28], the 3D EMT distributions are defined as

$$T_a^{\mu\nu}(\mathbf{r}, s', s) = \int \frac{d^3\Delta}{(2\pi)^3 2E} e^{-i\mathbf{\Delta} \cdot \mathbf{r}} \langle p', s' | \hat{T}_a^{\mu\nu}(0) | p, s \rangle. \quad (7)$$

In particular, covariant current conservation (2) ensures the conservation of momentum ( $T^{i0}$ ) and the stress tensor ( $T^{ij}$ ) in the BF,

$$\partial_i T^{i0}(\mathbf{r}, s', s) = 0, \quad \partial_i T^{ij}(\mathbf{r}, s', s) = 0. \quad (8)$$

## III. MULTIPOLE EXPANSION

The basic idea of the multipole expansion is as follows. The 3D angular momentum of the operator (7) is constructed from a rank- $n$  irreducible tensor of the 3D position vector  $\mathbf{r}$  and the polarization operator  $T_{\text{LM}}$  [33], which conveys information on the spin-polarization dependence  $s'$  and  $s$ . In addition, the parity of the structure is controlled by the antisymmetric tensor  $\epsilon^{ijk}$ . Time-reversal symmetry and Hermiticity further constrain the allowed structures. For a covariant multipole expansion, see Ref. [34].

To carry this out in the basis of 3D angular momentum, we first decompose the covariant  $\mu\nu$ -components of the 3D EMT distribution (7) into 00-, 0*i*-, and *ij*-components according to their angular momenta,

$$T^{00} \quad (L = 0), \quad (9a)$$

$$T^{0i} \quad (L = 1), \quad (9b)$$

$$T^{ij} \quad (L = 0, 2). \quad (9c)$$

These components are interpreted as the distributions of energy, angular momentum, and stress tensor, respectively.

Next, depending on the spin quantum number of the hadron considered, the matrix element of the polarization operator is determined by

$$(T_{\text{LM}})_{s's} \propto C_{J\text{LM}}^{J's'}, \quad (10)$$

where the maximal rank of the polarization operator matrix elements is dictated by the triangular selection rule of the Clebsch-Gordan (CG) coefficient  $C_{J\text{LM}}^{J's'}$ . For example, the maximally allowed multipole spin structures for spin-1 and spin-3/2 particles correspond to quadrupole ( $L = 2$ ) and octupole ( $L = 3$ ) spin operators, respectively. For an arbitrary hadron with spin  $J$ , the multipole spin structure extends up to  $L \leq 2J$  as follows:

$$\delta_{s's} \equiv \mathbf{1} \quad (L = 0), \quad (11a)$$

$$\hat{J}_{s's}^i \equiv J^i \quad (L = 1), \quad (11b)$$

$$\hat{Q}_{s's}^{ij} \equiv Q^{ij} \quad (L = 2), \quad (11c)$$

$\vdots$

$$\hat{M}_{s's}^{i_1 \dots i_L} \equiv M^{i_1 \dots i_L} \quad (L = 2J), \quad (11d)$$

In this work, we explicitly present terms up to the quadrupole structure. The matrix element of the dipole spin operator ( $L = 1$ ) is defined in the spherical basis as

$$\hat{J}_{s's}^M = \sqrt{J(J+1)} C_{J\text{LM}}^{J's'} \quad (M = 0, \pm 1). \quad (12)$$

The quadrupole spin operator can be defined in terms of the spin operator (12) as

$$\begin{aligned} T_a^{ij}(\mathbf{r}, s', s) = & \mathbf{1} \delta^{ij} p_0^a(r) + \mathbf{1} Y_2^{ij} s_0^a(r) + Q^{ij} \left( p_2^a(r) + \frac{1}{3} p_3^a(r) - \frac{1}{9} s_3^a(r) \right) + 2 [Q^{ip} Y_2^{pj} + Q^{jp} Y_2^{pi} - \delta^{ij} Q^{pq} Y_2^{pq}] \\ & \times \left( s_2^a(r) - \frac{1}{2} p_3^a(r) + \frac{1}{6} s_3^a(r) \right) + Q^{pq} Y_2^{pq} \left[ \left( \frac{2}{3} p_3^a(r) + \frac{1}{9} s_3^a(r) \right) \delta^{ij} + \left( \frac{1}{2} p_3^a(r) + \frac{5}{6} s_3^a(r) \right) Y_2^{ij} \right] + \dots, \end{aligned} \quad (17)$$

$$\hat{Q}_{s's}^{ij} = \frac{1}{2} \left( \hat{J}^i \hat{J}^j + \hat{J}^j \hat{J}^i - \frac{2}{3} J(J+1) \right)_{s's}, \quad (13)$$

with  $i, j = 1, 2, 3$ , which is manifestly traceless and symmetric,

$$\hat{Q}_{s's}^{ij} = \hat{Q}_{s's}^{ji}, \quad \hat{Q}_{s's}^{ii} = 0. \quad (14)$$

According to the allowed maximal rank of the spin operator (11) and the angular momentum of the operator in Eq. (9), the maximal rank of the irreducible tensor of the 3D position vector  $\mathbf{r}$  (orbital angular momentum) is determined. This is referred to as the angular momentum selection rule. The rank- $n$  irreducible tensors for the position vector are defined as

$$Y_n^{i_1 \dots i_n} = \frac{(-1)^n}{(2n+1)!!} r^{n+1} \partial^{i_1} \dots \partial^{i_n} \frac{1}{r}, \quad (15)$$

with  $r = |\mathbf{r}|$ . More explicit expressions are given by

$$Y_0 = 1, \quad (L = 0), \quad (16a)$$

$$Y_1^i = \hat{r}^i \quad (L = 1), \quad (16b)$$

$$Y_2^{ij} = \hat{r}^i \hat{r}^j - \frac{1}{3} \delta^{ij} \quad (L = 2), \quad (16c)$$

$$\begin{aligned} Y_3^{ijk} = & \hat{r}^i \hat{r}^j \hat{r}^k \\ & - \frac{1}{5} (\delta^{ij} \hat{r}^k + \delta^{ik} \hat{r}^j + \delta^{jk} \hat{r}^i) \quad (L = 3), \end{aligned} \quad (16d)$$

with  $\hat{\mathbf{r}} = \mathbf{r}/|\mathbf{r}|$ . By employing the angular momentum selection rules and discrete symmetries (parity, Hermiticity, time-reversal), one can perform the multipole expansion of Eq. (9) in the basis of Eqs. (11) and (16).

#### IV. MULTIPOLE EXPANSION FOR STRESS TENSOR

In this work, we focus on the stress tensor component of the EMT for a particle with positive intrinsic parity. After performing the multipole expansion, the stress tensor for an arbitrary-spin particle can be expressed in the multipole basis [35] as

where the ellipsis denotes contributions from spin operators beyond the quadrupole, such as the hexadecapole (16-pole), which become relevant for particles with  $J \geq 2$ . As a consequence of discrete symmetries, only even-rank multipole spin operators and orbital angular momentum structures contribute to  $T^{ij}$  for spin-1 and spin-3/2 particles. The triangular condition (10), together with the angular momentum selection rule, implies that the maximal ranks of the spin operator and orbital angular momentum are  $L \leq 2$  and  $L \leq 4$ , respectively.

The dynamical information is carried by the sets of pressure  $p_n(r)$  and shear-force  $s_n(r)$  distributions. The functions  $p_0(r)$  and  $s_0(r)$  describe the pressure and shear-force distributions in spherically symmetric hadrons. Since higher-order sets appear as additional structures for higher-spin particles, they are referred to as the quadrupole (or higher-multipole) pressure and shear-force distributions [35].

These higher-order sets are constructed under the constraint of current conservation (8) and naturally preserve the relation between pressure and shear forces that holds for spherically symmetric hadrons (spin-0 and spin-1/2 particles). For example, by taking derivatives of the stress tensor (17), one finds that the sets of quadrupole pressure and shear-force distributions also satisfy the following equilibrium equations [35], which are likewise valid in the monopole sector:

$$\sum_{a=Q,g} \left[ \frac{2}{3} \frac{ds_n^a(r)}{dr} + 2 \frac{s_n^a(r)}{r} + \frac{dp_n^a(r)}{dr} \right] = 0, \quad (18)$$

with  $n = 0, 2, 3$ .

These equations allow us to define the pressure and shear-force distributions,  $p_n(r)$  and  $s_n(r)$ , as the Fourier transforms of the generalized  $D$ -term form factors [35],

$$p_n(r) = \sum_{a=Q,g} p_n^a(r) = \frac{1}{6m} \partial^2 \mathcal{D}_n(r), \quad (19a)$$

$$s_n(r) = \sum_{a=Q,g} s_n^a(r) = -\frac{1}{4m} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} \mathcal{D}_n(r), \quad (19b)$$

which immediately imply the von Laue condition for the pressure,

$$\int d^3r p_n(r) = \frac{1}{6m} \int d^3r \partial^2 \mathcal{D}_n(r) = 0. \quad (20)$$

Similarly, the total  $r^2$ -weighted pressure and shear-force distributions are related to the generalized  $D$ -term [35],

$$\begin{aligned} D_n &\equiv \int d^3r \mathcal{D}_n(r) \\ &= m \int d^3r r^2 p_n(r) = -\frac{4}{15} m \int d^3r r^2 s_n(r). \end{aligned} \quad (21)$$

## V. FORCES BETWEEN THE QUARK/GLUON SUBSYSTEMS

Once the quark and gluon contributions to the pressure and shear-force distributions are separated, the equilibrium equations (18) are no longer satisfied for quarks and gluons individually. A balanced system emerges only when both contributions are combined. From the viewpoint of the quark subsystem, the gluon contribution may be regarded as an external force. This external force field  $-\mathbf{f}$  can be defined as [23,36]

$$\partial_i T_Q^{ij}(\mathbf{r}, s', s) = f^j(\mathbf{r}, s', s). \quad (22)$$

In classical continuum mechanics, this corresponds to the Cauchy momentum equation for the static stress tensor. For higher-spin particles, however, quantum effects become evident in the spin-polarization dependence of the distributions (22), which may be considered as a natural generalization to quantum systems.

This external force field can be expanded in terms of the multipole structures using the bases of Eqs. (11) and (16). More explicitly, by applying derivatives to the stress tensor (17), following multipole structures appear:

$$\begin{aligned} f^j(\mathbf{r}, s', s) &= \mathbf{1} Y_1^j f_0(r) + Q^{jk} Y_1^k f_2(r) \\ &\quad + Q^{kl} Y_3^{klj} f_3(r) + \cdots, \end{aligned} \quad (23)$$

which are induced from Eq. (17). Consequently, the number of multipole structures in Eq. (23) must coincide with the number of independent equilibrium equations in Eq. (18). The function  $f_0(r)$  originates from the monopole sector of the pressure and shear-force distributions and can therefore be regarded as the monopole force distribution. In contrast,  $f_2(r)$  and  $f_3(r)$  stem from the quadrupole sector of the pressure and shear-force distributions and are thus identified as quadrupole force distributions. Similar to the construction of the pressure and shear-force distributions (19), the multipole distributions for Eq. (23) can be derived as

$$f_0(r) = -m \frac{d}{dr} \bar{\mathcal{C}}_0(r), \quad (24a)$$

$$f_2(r) = -m \frac{d}{dr} \bar{\mathcal{C}}_2(r), \quad (24b)$$

$$f_3(r) = \frac{1}{m} r^3 \left( \frac{1}{r} \frac{d}{dr} \right)^3 \bar{\mathcal{C}}_3(r), \quad (24c)$$

where  $\bar{\mathcal{C}}_n(r)$  denotes the Fourier transform of the non-conserved multipole EMT form factors  $\bar{\mathcal{C}}_n^Q(t)$  (generalized  $\bar{\mathcal{C}}(t)$  form factor) parametrizing the hadronic matrix element (3),

$$\bar{C}_n(r) = \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\Delta \cdot r} \bar{C}_n^Q(t) \quad (n = 0, 2, 3). \quad (25)$$

The conservation of the total EMT (8) implies that the sum of the quark and gluon contributions to the nonconserved multipole EMT form factors vanishes, leading to

$$\bar{C}_n^Q(t) = -\bar{C}_n^g(t). \quad (26)$$

More importantly, the definitions of the separate quark and gluon parts of the nonconserved form factors depend on the chosen decomposition of the QCD EMT operator [37–41]; in particular, these choices are renormalization-scheme dependent [40,42]. To avoid this issue, one may decompose the EMT into quark, gluon, and trace-anomaly parts [37,38]. Among these decompositions, the definition of the quark  $\bar{C}^Q$  form factor is less ambiguous, so that we will adopt this part in what follows. The superscript  $Q$  on the nonconserved form factors will be omitted for brevity, unless explicitly required.

From Eq. (24), the multipole moments of the force distributions can be related to the  $\bar{C}_n(0)$  form factors as

$$\int d^3r r f_{0,2}(r) = 3m \bar{C}_{0,2}(0), \quad (27a)$$

$$\int d^3r r^3 f_3(r) = -\frac{105}{m} \bar{C}_3(0). \quad (27b)$$

The ellipses in Eq. (23) correspond to higher-multipole structures relevant for particles with even larger spin ( $J \geq 2$ ). Incorporating Eq. (24), we express these higher-order multipole distributions generically as

$$f(r) = \frac{i^{N+1}}{m^{N-2}} r^N \left( \frac{1}{r} \frac{d}{dr} \right)^N \bar{C}(r) \quad (N = \text{odd}), \quad (28)$$

where  $f(r)$  designates the multipole distribution associated with a rank- $N$  irreducible tensor in the expansion (23). The corresponding multipole moment is then given by

$$\int d^3r r^N f(r) = -\frac{(2N+1)!!}{m^{N-2}} i^{N+1} \bar{C}(0) \quad (N = \text{odd}). \quad (29)$$

By projecting  $f^j$  in Eq. (23) onto spherical components, the force can be decomposed into normal and tangential parts,

$$\mathbf{f}(\mathbf{r}, s', s) = f^r(\mathbf{r}, s', s) \hat{\mathbf{r}} + f^\theta(\mathbf{r}, s', s) \hat{\boldsymbol{\theta}} + f^\phi(\mathbf{r}, s', s) \hat{\boldsymbol{\phi}}. \quad (30)$$

The explicit forms of the normal and tangential distributions are

$$f^r(\mathbf{r}, s', s) = \mathbf{1} f_0(r) + Q^{rr} \left[ f_2(r) + \frac{3}{5} f_3(r) \right] + \dots, \quad (31a)$$

$$f^\theta(\mathbf{r}, s', s) = Q^{r\theta} f_2(r) + \dots \rightarrow Q^{r\theta} \left[ f_2(r) - \frac{2}{5} f_3(r) \right] + \dots, \quad (31b)$$

$$f^\phi(\mathbf{r}, s', s) = Q^{r\phi} f_2(r) + \dots \rightarrow Q^{r\phi} \left[ f_2(r) - \frac{2}{5} f_3(r) \right] + \dots, \quad (31c)$$

with  $Q^{r\theta} \equiv Q^{ij} \hat{r}^i \hat{\theta}^j$ . The total force is then defined by integrating Eq. (30) contracted with the spherical vectors  $\hat{\mathbf{v}}$  over 3D space [23],

$$F^v(s', s) = \int d^3r \hat{\mathbf{v}} \cdot \mathbf{f}(\mathbf{r}, s', s) \quad (v = r, \theta, \phi). \quad (32)$$

For  $v = \theta, \phi$ , the angular integration over  $\theta$  and  $\phi$  causes the averaged force to vanish. For  $v = r$ , the angular integration eliminates contributions from higher-multipole structures, e.g.,  $Q^{rr}$ , so that the monopole component alone determines the averaged force (32).

In classical continuum mechanics, or for spherically symmetric hadrons, the external force manifests solely as a normal component. By contrast, for higher-spin particles ( $J > 1/2$ ), tangential components of the force also appear due to spin-polarization dependence, reflecting the more intricate interplay between the quark and gluon quantum subsystems.

## VI. CONNECTION TO THE COVARIANT FORM FACTORS

The dynamical information of the multipole force distributions  $f_{0,2,3}$  is carried by the nonconserved EMT form factors. In several works [5,12,13,34,43,44], these form factors are defined through the covariant parametrization of the hadronic matrix element. In this section, we relate these covariant form factors to our multipole form factors  $\bar{C}_n(t)$  for particles with  $J < 2$ . Making use of the relation between the force distribution and the hadronic matrix element, we establish the following relations:

$$f^j(\mathbf{r}, s', s) = -i \int \frac{d^3\Delta}{(2\pi)^3 2E} e^{-i\Delta \cdot r} \Delta^i \langle p', s' | \hat{T}_{ij}^Q(0) | p, s \rangle. \quad (33)$$

*Spin-0 particle.* Due to the triangular condition in Eq. (10), the only allowed multipole spin operator is  $L = 0$ , which implies that all quadrupole contributions vanish. The monopole form factor  $\bar{C}_0(t)$  can then be expressed in terms of the covariant form factors, following the notation of Refs. [5,43]:

$$\bar{C}_0(t) = \frac{m}{2E} \bar{c}(t), \quad (34a)$$

$$\bar{C}_2(t) = \bar{C}_3(t) = 0. \quad (34b)$$

*Spin-1/2 particle.* Similar to the spin-0 case, only even-rank multipole spin operators are permitted by discrete symmetries. Therefore, in the stress tensor, only the monopole spin operator contributes, while the quadrupole structure is absent. Using the parametrization of Ref. [5], the nonconserved monopole form factor  $\bar{C}_0(0)$  is related to the covariant form factor as follows:

$$\bar{C}_0(t) = \bar{c}(t), \quad (35a)$$

$$\bar{C}_2(t) = \bar{C}_3(t) = 0. \quad (35b)$$

*Spin-1 particle.* The quadrupole spin operator first arises in the case of spin-1 particles, offering a novel perspective on quadrupole structures in the force distribution. In the notation of the covariant form factors of Ref. [12], the multipole form factors  $\bar{C}_n(t)$  are related to the covariant form factors as follows:

$$\begin{aligned} \bar{C}_0(t) = \frac{m}{2E} & \left[ \left(1 - \frac{t}{6m^2}\right) \bar{c}_0(t) - \frac{t}{12m^2} \left(1 - \frac{t}{4m^2}\right) \bar{c}_1(t) \right. \\ & \left. - \frac{1}{6} \left(1 - \frac{t}{2m^2}\right) \bar{f}(t) \right], \end{aligned} \quad (36a)$$

$$\begin{aligned} \bar{C}_2(t) = \frac{m}{2E} & \left[ \frac{t}{5m^2} \bar{c}_0(t) + \frac{t}{10m^2} \left(1 - \frac{t}{4m^2}\right) \bar{c}_1(t) \right. \\ & + \left\{ \frac{20m(m+E) - 7t}{10m(m+E)} \right. \\ & \left. + \frac{t}{10m^2} \left(1 + \frac{t}{2(m+E)^2}\right) \right\} \bar{f}(t) \Big], \end{aligned} \quad (36b)$$

$$\begin{aligned} \bar{C}_3(t) = -\frac{m}{2E} & \left[ \frac{1}{2} \bar{c}_0(t) + \frac{1}{4} \left(1 - \frac{t}{4m^2}\right) \bar{c}_1(t) \right. \\ & \left. + \frac{2E^2 - 2m^2 + t}{8(m+E)^2} \bar{f}(t) \right]. \end{aligned} \quad (36c)$$

*Spin-3/2 particle.* As in the spin-1 case, only even-rank multipole spin operators are allowed by discrete symmetries. Thus, the stress tensor exhibits the same multipole structure as that of the spin-1 particle. The nonconserved multipole EMT form factors can be expressed in terms of the notation of Refs. [13,34],

$$\begin{aligned} \bar{C}_0(t) = & \left(1 - \frac{t}{6m^2}\right) F_{3,0}(t) - \frac{t}{6m^2} \left(1 - \frac{t}{4m^2}\right) F_{3,1}(t), \\ & + \frac{2}{3} \left(1 - \frac{t}{4m^2}\right) F_{6,0}(t) \end{aligned} \quad (37a)$$

$$\begin{aligned} \bar{C}_2(t) = & \frac{t}{15m^2} F_{3,0}(t) + \frac{t}{15m^2} \left(1 - \frac{t}{4m^2}\right) F_{3,1}(t), \\ & - \frac{2}{3} \left(1 + \frac{t\{t - 14m(m+E)\}}{40m^2(m+E)^2}\right) F_{6,0}(t), \end{aligned} \quad (37b)$$

$$\begin{aligned} \bar{C}_3(t) = & -\frac{1}{6} F_{3,0}(t) - \frac{1}{6} \left(1 - \frac{t}{4m^2}\right) F_{3,1}(t) \\ & - \frac{1}{6} \left(\frac{m}{m+E} - \frac{t}{4(m+E)^2}\right) F_{6,0}(t). \end{aligned} \quad (37c)$$

*Spin-2 particle.* For a spin-2 particle, the covariant form factors can be related to multipole form factors; however, this requires going beyond the quadrupole spin operator to, for example, the hexadecapole (rank-4) operator. This implies that orbital angular momenta up to rank-6 may contribute to the stress tensor due to the angular momentum selection rules. Five nonconserved form factors appear in the parametrization of the matrix element [34], which may additionally give rise to two more multipole form factors,  $\bar{C}_n(t)$  ( $n = 4, 5$ ), compared to spin-1 and spin-3/2 particles. Such a structure is particularly interesting for probing the internal mechanical properties of spin-2 particles such as tensor mesons.

## VII. NUMERICAL RESULTS AND VISUALIZATION

To visualize the force distributions characterized by the nonconserved EMT form factors, we perform a numerical estimation. Specifically, we consider a spin-1 particle polarized both longitudinally and transversely. For a spin-3/2 particle, the quadrupole pattern is the same as that of the spin-1 particle, but its strength depends entirely on dynamical information. Since this information is currently unknown, we focus on the spin-1 particle as a representative example.

For the numerical analysis, we take the multipole form factors  $\bar{C}_n(t)$  as input and parametrize them by a tripole form,

$$\bar{C}_n^Q(t) = \frac{\bar{C}_n^Q(0)}{(1 - \frac{t}{\Lambda^2})^3}, \quad (38)$$

with  $n = 0, 2, 3$ . The mass of the spin-1 hadron and the tripole mass are set to  $m = \Lambda = 1$  GeV, and the normalizations of the nonconserved form factors are taken to be

$$\bar{C}_{0,2,3}^Q(0) = 1.4 \times 10^{-2}. \quad (39)$$

The value of  $\bar{C}_0^Q(0) = \bar{c}^Q(0)$  [see Eq. (35)] for the nucleon was predicted from the QCD instanton vacuum [23], and we conjecture that this small and positive behavior persists for the higher multipole form factors as well. As discussed in Eqs. (26), (39) represents the partial form factor (force

distribution) from the quark part. In the combined quark-gluon system, the force distribution is balanced and reaches equilibrium by considering the gluon contribution  $\bar{C}_n^g$ , which has the opposite sign to the quark contribution.

To estimate the typical magnitude of the force distribution, we compute the averaged force  $F^r$ . After integrating the radial force distribution  $f^r$  over  $r$ , we obtain

$$F^r(s', s) = \mathbf{1} \frac{3}{4} \Lambda m \bar{C}_0^Q(0) \approx \mathbf{1} \times 5.3 \times 10^{-2} \frac{\text{GeV}}{\text{fm}}, \quad (40)$$

which is proportional to both the tripole mass  $\Lambda$  and the hadron mass  $m$ . As discussed in Eq. (32), this total force is determined solely by the monopole distribution  $f_0(r)$  [or equivalently, by  $\bar{C}_0^Q(0)$ ].

In Fig. 1, we illustrate the cross-sectional view ( $\theta = \pi/2$ ) of the force distribution (30) for a spin-1 particle when its spin is polarized longitudinally (upper panel) and

transversely (lower panel). For the longitudinally polarized spin-1 particle, the quadrupole polarization (see Ref. [33] for the explicit representation) is given by

$$Q^{rr} = -\frac{1}{6}(1 + 3 \cos 2\theta), \quad (41a)$$

$$Q^{r\theta} = \cos \theta \sin \theta, \quad (41b)$$

$$Q^{r\phi} = 0, \\ [\text{for } s'_z = s_z = 0]. \quad (41c)$$

Inserting these results (41) into Eq. (31), we derive the force distribution,

$$f(r, s'_z = 0, s_z = 0) \Big|_{\theta=\frac{\pi}{2}} = \left[ f_0(r) + \frac{1}{3} \left\{ f_2(r) + \frac{3}{5} f_3(r) \right\} \right] \hat{r}. \quad (42)$$

For the cross-sectional view of this force, all  $\phi$ -dependent terms in Eq. (41) vanish, implying that the quadrupole pattern disappears in Eq. (42). Nevertheless, the quadrupole contribution is still present but merges with the monopole force distribution. Moreover, the tangential forces proportional to  $Q^{r\theta}$  and  $Q^{r\phi}$  vanish at  $\theta = \pi/2$ . As a result, only the normal force appears in the upper panel of Fig. 1.

For the transversely polarized spin-1 particle, i.e.,  $|s_x = 0\rangle = (|s_z = 1\rangle - |s_z = -1\rangle)/\sqrt{2}$  [45], the quadrupole polarization is given by [33]

$$Q^{rr} = -\frac{1}{6}(1 + 3 \cos 2\phi), \quad (43a)$$

$$Q^{r\theta} = 0, \quad (43b)$$

$$Q^{r\phi} = \cos \phi \sin \phi, \\ [\text{for } s'_x = s_x = 0]. \quad (43c)$$

Substituting these results (43) into Eq. (31), we derive the force distribution. In contrast to the longitudinally polarized case, all  $\theta$ -dependent terms vanish, leaving only  $\phi$ -dependence. This gives rise to quadrupole deformation in the force distribution on the cross-sectional plane, and generates nonvanishing tangential forces. A more intricate force distribution is evident in the lower panel of Fig. 1, reflecting that hadron shape formation arises from a more complex interplay between the quark and gluon subsystems compared to the spherically symmetric case of hadrons with  $J < 1$ .

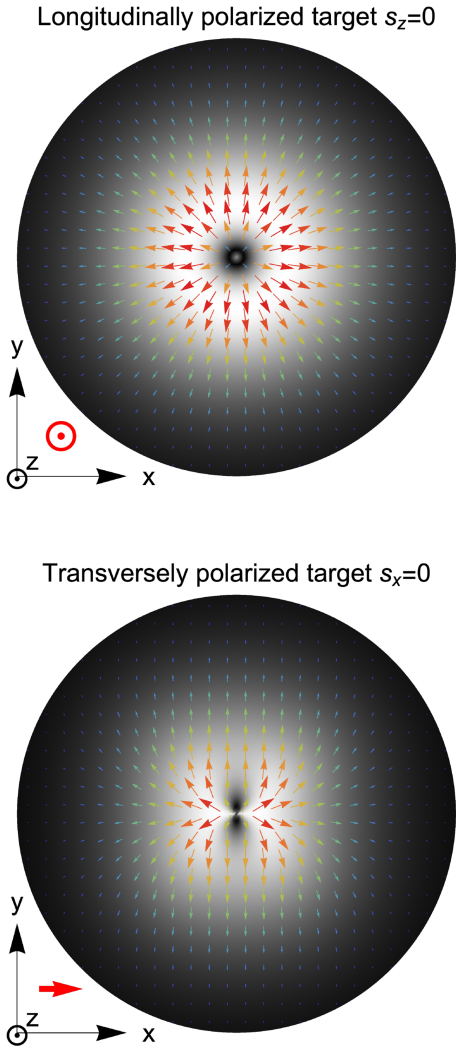


FIG. 1. Visualization of the force distribution for a spin-1 target polarized longitudinally (upper panel) and transversely (lower panel).

## VIII. SUMMARY AND OUTLOOK

In this work, we generalized the mechanical interpretation of the nonconserved  $\bar{c}(t)$  form factor, originally formulated for spherically symmetric hadrons, to higher-spin particles. To provide a clear physical interpretation, we performed a three-dimensional multipole expansion of the stress tensor in the Breit frame under the constraint of current conservation. Separating the quark and gluon contributions to the stress tensor isolates the nonconserved component, which can be interpreted as the force distribution between the quark and gluon subsystems and is connected to the nonconserved multipole EMT form factors  $\bar{C}_n(t)$ . Furthermore, these form factors were related to the nonconserved covariant EMT form factors for particles with  $J < 2$ . Using a toy model, we illustrated the force distribution for a spin-1 particle polarized longitudinally and transversely. We found that, for higher-spin particles, hadron shape formation results from a more intricate balance between the quark and gluon subsystems compared to the spherically symmetric case of hadrons with  $J < 1$ .

A more conclusive visualization of the force distribution in higher-spin particles requires an explicit determination of the nonconserved EMT form factors, which can then serve as input for the analysis. (For example, the chiral quark-

soliton model can be used to compute the EMT form factors etc.)

Finally, for a more precise interpretation of relativistic densities, the present formalism should be reformulated in terms of a two-dimensional light-front multipole expansion, which can be carried out straightforwardly within the present framework. The physical conclusion will remain intact in the two-dimensional transverse plane.

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## DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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*Correction:* Errors in Eqs. (31b) and (31c) have been corrected.