

F-term hybrid inflation with *T*-model Kähler geometry and beyond

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We analyze *F*-term hybrid inflation (FHI) within various grand unified theories in the presence of a Kähler potential for the inflaton field which parametrizes the Kähler manifolds $SU(1,1)/U(1)$ or $SU(2)/U(1)$. We take into account supergravity, radiative, and soft supersymmetry-breaking corrections to the tree-level potential and find that viable FHI can be realized without extrema along the inflationary trajectory for a broad region of the parameter space. For selected superpotential parameters, the models' predictions are largely influenced by the curvature of the internal space and the magnitude of the tadpole parameter which are constrained so as to achieve compatibility with the current Atacama Cosmology Telescope and the South Pole Telescope data. We also discuss the formation of cosmic strings and their associated gravitational wave signals, potentially detectable by current and upcoming experiments.

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I. INTRODUCTION

Precision measurements of the cosmic microwave background (CMB) have transformed inflation into a quantitatively testable framework—for a review, see, e.g., Ref. [1]. The Planck satellite has played a central role in this progress by providing highly precise measurements of temperature and polarization anisotropies over a wide range of angular scales. Complementary ground-based experiments, such as the Atacama Cosmology Telescope (ACT) and the South Pole Telescope (SPT), have further improved constraints through high-resolution observations of small-scale anisotropies.

While the results of these experiments are broadly consistent, recent analyses reveal a mild tension in the preferred values of the scalar spectral index, n_s . In particular, the combination of *Planck* with BICEP2/Keck Array yields [2,3]

$$n_s \simeq 0.9652 \pm 0.0084 \quad (\text{P-BK-LB data at 95\% C.L.}), \quad (1.1)$$

whereas the ACT data release 6 (DR6), combined with *Planck*, BICEP2/Keck Array and DESI BAO measurements, prefers [4,5]

$$n_s = 0.9743 \pm 0.0068 \Rightarrow 0.967 \lesssim n_s \lesssim 0.981 \quad (\text{P-ACT-LB-BK18 data at 95\% C.L.}), \quad (1.2)$$

where the n_s elevation above is mainly attributed to a tension between CMB data and DESI BAO data [6]. On the other hand, the SPT measurements, when combined with *Planck* and ACT data, yield an intermediate value [7]

$$n_s = 0.9684 \pm 0.006 \Rightarrow 0.962 \lesssim n_s \lesssim 0.974 \quad (\text{P-ACT-SPT data at 95\% C.L.}). \quad (1.3)$$

If the recent results above—mainly that in Eq. (1.2)—will be confirmed by, e.g., future CMB data [8–10], they will place significant pressure on several well-motivated *Planck*-consistent models such as *T*-model inflation (TMI) [11–13]. TMI belongs to a class of cosmological α -attractors [12], which is realized for inflaton values of the order of the reduced Planck mass m_p close to a second-order pole emerging in the inflaton kinetic terms. This kinetic mixing may originate from a Kähler potential of the form

$$K_I = -Nm_p^2 \ln(1 - |S|^2/Nm_p^2), \quad (1.4)$$

where $N > 0$ and S is the gauge-singlet inflaton—for TMI with a gauge nonsinglet inflaton, see Refs. [14,15]. K_I parametrizes the hyperbolic Kähler manifold

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$SU(1,1)/U(1)$ with constant scalar (moduli space) curvature $R_1 = -2/N$. This mechanism leads to a robust and nearly universal prediction $n_s \simeq 0.963$ for number of e-folding $N_{1*} \simeq 55$ [11,16]. Several variants [16–26] have been proposed recently to reconcile TMI with the P-ACT-LB-BK18 data in Eq. (1.2)—for a review, see Ref. [27].

In this work, we wish to emphasize that beyond the strong regime above the geometry of TMI can be also applied in a weak regime for S values well below m_p . This regime is expected to cooperate well with inflationary models which develop a plateau due to the structure of their potential. A prominent representative of such models is F -term hybrid inflation (FHI) [28] which is undoubtedly a well-known model thanks to its observational flexibility and the ease of embedding in particle models [29]. Most notably, FHI is based on a renormalizable superpotential, uniquely determined by a gauge symmetry $\mathbb{G}$ and a global $U(1)_R$ symmetry; it does not require fine-tuned superpotential parameters and transplanckian inflaton values; it can be, moreover, naturally followed by a grand unified theory (GUT) phase transition which may lead to the production of cosmological defects if predicted by the symmetry-breaking scheme—see, e.g., Ref. [30]. Among them, cosmic strings (CSs) attract a fair amount of attention currently since they contribute to the CMB and generate [31] a stochastic background of gravitational waves (GWs) in the nanohertz range, probed by pulsar timing arrays (PTAs) such as *NANOGrav* (15-yr) [32], *EPTA* [33], *PPTA* [34], and *CPTA* [35]. The tension of CSs is directly linked to the symmetry-breaking scale of $\mathbb{G}$, connecting ACT/SPT-preferred inflationary parameters with the spectrum of the GWs observed by PTAs—see, e.g., Refs. [15,36–42].

For a reliable approach to FHI, soft supersymmetry (SUSY)-breaking terms [43–50] and supergravity (SUGRA) corrections [51–55] have to be taken into account together with the radiative corrections employed in the original version of the model [28]. Both of the aforementioned corrections are of crucial importance in order to reconcile the inflationary observables with present data [38,56–58]. In particular, the SUGRA corrections are obviously related to the adopted geometry of the moduli space. In conventional treatments, canonical [43,44,47–50] or quasicanonical Kähler [51–55] potentials are typically introduced to mitigate the η -problem of FHI—for a review, see [59]. However, such constructions often rely on multiple small coefficients, which reduces the predictive power of the framework and lacks a clear connection to known superstring compactifications.

Inspired by the geometry of TMI, we here adopt the Kähler potential in Eq. (1.4) and consider both signs of N . Note that, for $N < 0$, K_1 parametrizes the compact Kähler manifold $SU(2)/U(1)$ with constant scalar curvature as well. This SUGRA setting is governed by just one parameter, N in Eq. (1.4), whereas FHI models based on polynomial Kähler potentials typically require at least two

independent parameters. In this sense, the present construction is more predictive. For $N > 0$, K_1 was first introduced in Refs. [60,61] in the presence of a stabilized modulus to alleviate the η problem of FHI—cf. Refs. [62,63]. It was also shown that N values close to the superconformal limit can support FHI in Ref. [64] in the context of flipped $SU(5)$ for specific values of the superpotential parameters. It was also employed, independently of inflation, to accomplish gravity-mediated SUSY breaking under the assumption of a mildly violated R symmetry [65] or within no-scale SUGRA [66].

We here consider N as an input parameter which is restricted by the inflationary requirements as in the case of TMI [11,14]. We find an ample available parameter space adjusting (as functions of the superpotential parameters) N in conjunction with the tadpole parameter a_S for $N > 0$ or $N < 0$. In the major part of the emergent parameter space, $|N|$ acquires quite large values which do not pose, though, any observational—as in the case of TMI—or theoretical difficulty. Indeed, from the SUGRA perspective, N is not fixed and may take arbitrary values unless a particular ultraviolet completion is specified—see, e.g., Ref. [13]. This situation may be compared with that of nonminimal Higgs inflation [64,67–71], where a very large coupling to Ricci scalar curvature is typically required. In contrast to that case, though, the S values here are much lower than m_p and so S turns out to be essentially canonically normalized. As a consequence, N does not appear in any numerator of the expansion of the inflationary potential for low S values and therefore, no issue with the validity of the effective theory arises [72,73].

The salient features of our models are introduced in Sec. II, and the resulting inflationary potential is derived in Sec. III. In Sec. V, we show that the inflationary observables can be successfully reconciled with a number of constraints listed in Sec. IV. We also examine the formation of CSs in one version of these models and discuss the associated spectrum of GWs in Sec. VI. Our conclusions are summarized in Sec. VII.

II. MODEL SETUP

The simplest version of FHI [28,59] can be implemented by introducing three superfields $\bar{\Phi}$, Φ , and S . The first two are left-handed chiral superfields oppositely charged under a gauge group $\mathbb{G}$, while the latter corresponds to the inflaton and is a $\mathbb{G}$ -singlet left-handed chiral superfield. In this work, we identify $\mathbb{G}$ with three possible gauge groups with different representation dimensions $N_{\mathbb{G}}$ to which $\bar{\Phi}$ and Φ belong—see Table I. Namely, we consider the following $\mathbb{G}$'s:

$$\mathbb{G}_{B-L} := \mathbb{G}_{SM} \times U(1)_{B-L} \quad \text{with} \quad N_{\mathbb{G}} = 1, \quad (2.1a)$$

$$\begin{aligned} \mathbb{G}_{LR} &:= SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ &\text{with} \quad N_{\mathbb{G}} = 2, \end{aligned} \quad (2.1b)$$

TABLE I. Representations and R charges of the superfields involved in FHI for various $\mathbb{G}$'s.

Superfields	Representations under $\mathbb{G}$			R charge
	$\mathbb{G}_{B-L}$	$\mathbb{G}_{LR}$	$\mathbb{G}_{51F}$	
Higgs superfields				
Φ	$(\mathbf{1}, \mathbf{1}, 0, 2)$	$(\mathbf{1}, \mathbf{1}, \mathbf{2}, 1)$	$(\mathbf{10}, 1)$	0
$\bar{\Phi}$	$(\mathbf{1}, \mathbf{1}, 0, -2)$	$(\mathbf{1}, \mathbf{1}, \mathbf{\bar{2}} - 1)$	$(\mathbf{\bar{10}}, -1)$	0
S	$(\mathbf{1}, \mathbf{1}, 0, 0)$	$(\mathbf{1}, \mathbf{1}, \mathbf{1}, 0)$	$(\mathbf{1}, 0)$	2

$$\mathbb{G}_{51F} := SU(5) \times U(1)_X \quad \text{with} \quad N_{\mathbb{G}} = 10. \quad (2.1c)$$

Here $\mathbb{G}_{SM}$ is the well-known gauge group of the Standard Model (SM):

$$\mathbb{G}_{SM} := SU(3)_C \times SU(2)_L \times U(1)_Y, \quad (2.1d)$$

to which $\mathbb{G}$ is broken via the vacuum expectation values of Φ and $\bar{\Phi}$ at the end of FHI. With regard to the cosmological defects, CSs are produced only for $\mathbb{G} = \mathbb{G}_{B-L}$ —see Sec. VI.

Indeed, this scheme can be realized if we adopt the superpotential

$$W = \kappa S(\bar{\Phi}\Phi - M^2), \quad (2.2)$$

where κ and M are free parameters which may be constrained by the inflationary requirements—see Sec. IV. W in Eq. (2.2) is the most general renormalizable superpotential consistent with a continuous R symmetry [28] under which

$$S \rightarrow e^{ir} S, \quad \bar{\Phi}\Phi \rightarrow \bar{\Phi}\Phi \quad \text{and} \quad W \rightarrow e^{ir} W. \quad (2.3)$$

The SUSY potential, V_{susy} , extracted from W in Eq. (2.2) includes contributions from F - and D -terms. The latter contribution vanishes along the direction $|\bar{\Phi}| = |\Phi|$, whereas the former (in the flat limit with $m_P \rightarrow \infty$) reads

$$V_F = \sum_{\alpha} |F_{\alpha}|^2 = \kappa^2((|\Phi|^2 - M^2)^2 + 2|S|^2|\Phi|^2), \quad (2.4)$$

where $F_{\alpha} = \partial_{\phi_{\alpha}} W$ with $\phi_{\alpha} = S, \Phi, \bar{\Phi}$ and the symbol ∂_X denotes the partial derivative with respect to (w.r.t.) X . Consequently, the SUSY minimum is

$$\langle S \rangle = 0 \quad \text{and} \quad |\langle \Phi \rangle| = |\langle \bar{\Phi} \rangle| = M. \quad (2.5)$$

At the vacuum, $\mathbb{G}$ is spontaneously broken to $\mathbb{G}_{SM}$, and SUSY is preserved up to soft SUSY-breaking terms—see, e.g., Ref. [15]. The inflaton system acquires mass $m_I = \sqrt{2}\kappa M$ [29].

As mentioned in Sec. I, W in Eq. (2.2) may cooperate with a variety of Kähler potentials for implementing FHI. We here focus on the quadratic Kähler potential

$$K = K_I + |\Phi|^2 + |\bar{\Phi}|^2, \quad (2.6)$$

with K_I given in Eq. (1.4). K parametrizes the following (curved) Kähler manifold:

$$(SU(1,1)/U(1))_S \times U(1)_{\bar{\Phi}} \times U(1)_{\Phi} \quad \text{for } N > 0$$

or $(SU(2)/U(1))_S \times U(1)_{\bar{\Phi}} \times U(1)_{\Phi} \quad \text{for } N < 0, \quad (2.7)$

where the subscripts indicate the moduli parametrizing the corresponding manifolds.

III. INFLATIONARY POTENTIAL

It is well known [28] that FHI occurs for sufficiently large values of $|S|$ along a F - and D -flat direction of the F -term potential

$$\bar{\Phi} = \Phi = 0, \quad \text{where } V_F(\Phi = 0) := V_{10} = \kappa^2 M^4 \quad \text{and}$$

$$H_I = \sqrt{V_{10}/3m_P^2} \quad (3.1)$$

are the constant potential energy density and corresponding Hubble parameter which drive FHI—the subscript 0 means that this is the tree level value. The interplay of several corrections determines the slope of the inflationary potential which can be written as follows:

$$V_I = V_{10}(1 + C_{\text{SUGRA}} + C_{\text{SSB}} + C_{\text{RC}}). \quad (3.2)$$

The individual contributions are specified below:

- (a) SUGRA corrections. Given that the D -term contributions into the SUGRA potential vanish due to the stabilization of Φ and $\bar{\Phi}$ at the origin, we focus on the F -term SUGRA scalar potential with just one field S ,

$$V_{\text{SUGRA}} = e^{K/m_P^2} (K^{SS*} D_S W D_{S^*} W^* - 3|W|^2/m_P^2),$$

where $D_S = \partial_S + \partial_S K/m_P^2$ (3.3)

is the Kähler-covariant derivative. We also introduce the inverse of the Kähler metric

$$K_{SS^*} = \partial_{S^*} \partial_S K_I = (1 - \sigma^2/2Nm_P^2)^{-2} \quad \text{with}$$

$$S = \sigma e^{i\theta_S}/\sqrt{2} \quad (3.4)$$

using the relation $K^{SS^*} = K_{SS^*}^{-1}$. Taking this result into account and plugging into Eq. (3.3) the expression of K_I in Eq. (1.4), we arrive at the following result:

$$V_{\text{SUGRA}} = V_{10} \left(1 - \frac{\sigma^2}{2Nm_P^2} \right)^{-N}$$

$$\times \left(1 - \frac{(2+N)}{N} \frac{\sigma^2}{2m_P^2} + \left(\frac{N-1}{N} \right)^2 \frac{\sigma^4}{4m_P^4} \right). \quad (3.5)$$

Expanding for $S \ll m_P$ the expression above, we can identify the SUGRA corrections of Eq. (3.2) after subtracting the dominant constant contribution of Eq. (3.1), i.e.,

$$C_{\text{SUGRA}} = -\frac{1}{N} \frac{\sigma^2}{m_P^2} + \frac{2 + N(N-7)}{8N^2} \frac{\sigma^4}{m_P^4} + \frac{1 + N(2N-9)}{24N^2} \frac{\sigma^6}{m_P^6} + \dots \quad (3.6)$$

As can be seen from the expansion above, the largest coefficient of the various terms for $N \gg 1$ is of order unity and so the ultraviolet cutoff scale of our theory is m_P . In the same N limit, we can reproduce the expansion obtained for canonical K_I —see, e.g., Ref. [59].

- (b) Soft SUSY-breaking corrections. Possible coupling of the inflationary sector with the SUSY-breaking sector—cf. Ref. [49]—generates corrections to V_I/V_{I0} arising from soft SUSY-breaking terms. Namely, these corrections can be written as

$$\begin{aligned} V_{\text{SSB}} &= -\kappa M^2 (a_S S + a_S^* S^*) \\ \Rightarrow \langle V_{\text{SSB}} \rangle_I &= V_{I0} C_{\text{SSB}} \quad \text{with} \\ C_{\text{SSB}} &= -\sqrt{2} |a_S| \sigma / V_{I0}, \end{aligned} \quad (3.7)$$

where the minus sign arises from the stabilization of $\theta = \theta_S + \theta_{a_S}$ —see Eq. (3.4). Indeed, for $\theta/m_P = 0 \pmod{2\pi}$, the contribution $-a_S S + \text{c.c.} = -\sqrt{2} |a_S| \sigma \cos(\theta/m_P)$ is minimized. Here we further assume that θ_S remains constant during FHI so that the simple one-field slow-roll approximation is valid. A possible variation of θ_S is investigated in Ref. [48] where it was found that acceptable solutions with $\theta \neq 0$ require significant fine-tuning. We also checked that a possible soft SUSY-breaking mass term $m_S^2 |S|^2$ with m_S of the order of a_S is irrelevant for the values of the parameters considered in our setup.

- (c) Radiative corrections. These corrections originate [28] from a mass splitting in the $\Phi - \bar{\Phi}$ supermultiplets due to SUSY breaking on the inflationary valley. To compute them, we work out the mass spectrum of the fluctuations of the various fields about the inflationary trough in Eq. (3.1). We obtain $2N_G$ Weyl fermions and $2N_G$ pairs of real scalars with mass squared, respectively:

$$m_f^2 = \kappa^2 \sigma^2 / 2 \quad \text{and} \quad m_{\pm}^2 = \kappa^2 (\sigma^2 / 2 \pm M^2). \quad (3.8)$$

Inserting these masses into the well-known Coleman-Weinberg formula, we find the correction

$$V_{\text{RC}} = V_{I0} C_{\text{RC}} \quad \text{with}$$

$$C_{\text{RC}} = \frac{\kappa^2 N_G}{32\pi^2} \left(\sum_{i=\pm} m_i^4 \ln \frac{m_i^2}{Q^2} - 2m_f^4 \ln \frac{m_f^2}{Q^2} \right), \quad (3.9)$$

where Q is a renormalization scale. In the largest part of the parametric space of the model, $m_{\pm}^2$ develops a tachyonic instability for

$$\sigma < \sigma_c = \sqrt{2} M, \quad (3.10)$$

terminating abruptly FHI. SUGRA corrections to the masses above are at most of the order M^4/m_P^2 and can be safely ignored.

IV. INFLATIONARY REQUIREMENTS

The parameters of FHI models can be constrained by imposing a number of observational and theoretical constraints described in Secs. IV A and IV B below.

A. Observational constraints

Below we describe a number of observational constraints considered in our investigation. In the relevant standard formulas, we take into account that σ effectively coincides with the canonically normalized inflaton since $\langle K_{SS^*} \rangle_I \simeq 1$, thanks to the fact that $\sigma \ll m_P$ and $N \gg 1$. Namely,

- (a) The number of e-foldings that the pivot scale $k_{\star} = 0.05/\text{Mpc}$ suffered during FHI have to be enough to resolve the shortcomings of the standard big bang, i.e., Ref. [2]:

$$\begin{aligned} N_{I\star} &= \frac{1}{m_P^2} \int_{\sigma_f}^{\sigma_{\star}} d\sigma \frac{V_I}{V_I'} \simeq \frac{1}{m_P^2} \int_{\sigma_f}^{\sigma_{\star}} \frac{d\sigma}{C'_{\text{RC}} + C'_{\text{SSB}} + C'_{\text{SUGRA}}} \\ &\simeq 19.4 + \frac{2}{3} \ln \frac{V_{I0}^{1/4}}{1 \text{ GeV}} + \frac{1}{3} \ln \frac{T_{\text{rh}}}{1 \text{ GeV}}, \end{aligned} \quad (4.1)$$

where prime denotes derivation w.r.t. σ , $\sigma_{\star}$ is the value of σ when $k_{\star}$ exits the horizon of FHI, and $\sigma_f \simeq \sigma_c$ signals the termination of FHI due to the tachyonic instability encountered in Eq. (3.9). Despite this well-known fact, we also check the validity of slow-roll condition which reads

$$\max\{\epsilon(\sigma_f), |\eta(\sigma_f)|\} = 1, \quad (4.2)$$

where the relevant slow-roll parameters can be found from the relations

$$\begin{aligned} \epsilon &\simeq \frac{m_P^2}{2} \left(\frac{V_I'}{V_I} \right)^2 \simeq \frac{m_P^2}{2} (C'_{\text{RC}} + C'_{\text{SSB}} + C'_{\text{SUGRA}})^2 \quad \text{and} \\ \eta &\simeq m_P^2 \frac{V_I''}{V_I} \simeq m_P^2 (C''_{\text{RC}} + C''_{\text{SUGRA}}). \end{aligned} \quad (4.3)$$

In the last part of Eq. (4.1), we assume that FHI is followed in turn by an oscillatory phase with equation-of-state parameter $w_{\text{rh}} = 0$, radiation, and matter domination. Note that after FHI we obtain two oscillatory systems composed by the complex scalar fields S and $\delta\Phi_+ = (\delta\bar{\Phi} + \delta\Phi)/\sqrt{2}$, where $\delta\bar{\Phi} = \bar{\Phi} - M$ and $\delta\Phi = \Phi - M$. Given that $w_\phi = (p-2)/(p+2)$ for a power-law potential ϕ^p [74], we expect that $w_S \simeq 0$ and $w_\Phi \simeq 1/3$ since V_F in Eq. (2.4) displays a quadratic [quartic] dependence on S [Φ] with Φ [S] close to its value in Eq. (2.5). As shown in the related setting of Ref. [63], the energy density of $\delta\Phi_+$ is overshadowed by that of S which, thereby, dominates the reheating phase yielding $w_{\text{rh}} = w_S \simeq 0$. Lastly, in Eq. (4.1), we adopt for the reheating temperature T_{rh} a value close to $(0.01-1)$ EeV. These values are compatible with nonthermal leptogenesis [75,76] which can be naturally accommodated within FHI—cf. Refs. [46,47,64]. In all cases, we obtain $N_{I\star} \simeq 50$.

- (b) The amplitude A_s of the power spectrum of the curvature perturbation generated by σ during FHI must be appropriately normalized [5], i.e.,

$$\begin{aligned} A_s &= \frac{1}{12\pi^2 m_{\text{P}}^6} \left. \frac{V_I^3}{|V_I'|^2} \right|_{\sigma=\sigma_\star} \\ &\simeq \frac{1}{12\pi^2 m_{\text{P}}^6} \frac{V_{I0}}{(C'_{\text{RC}} + C'_{\text{SSB}} + C'_{\text{SUGRA}})^2} \Big|_{\sigma=\sigma_\star} \\ &\simeq 2.1326 \times 10^{-9}. \end{aligned} \quad (4.4)$$

- (c) The remaining observables—the scalar spectral index n_s , its running α_s , and the scalar-to-tensor ratio r —are calculated by the standard slow-roll formulas:

$$\begin{aligned} n_s &= 1 - 6\epsilon_\star + 2\eta_\star, \\ \alpha_s &= \frac{2}{3}(4\eta_\star^2 - (n_s - 1)^2) - 2\xi_\star, \quad \text{and} \quad r = 16\epsilon_\star, \end{aligned} \quad (4.5)$$

where $\xi_\star \simeq m_{\text{P}}^4 V_I' V_I''' / V_I^2$, and all quantities with the subscript $\star$ are evaluated at $\sigma = \sigma_\star$. These observables must be consistent with current CMB measurements. Using the data from Eqs. (1.2) and (1.3) and considering the $\Lambda\text{CDM} + r$ model, we end up with the following allowed range:

$$0.962 \lesssim n_s \lesssim 0.981 \quad \text{and} \quad r \lesssim 0.038, \quad \text{at } 95\% \text{ C.L.}, \quad (4.6)$$

with a negligible running of n_s , namely, α_s .

B. Theoretical considerations

From a more theoretical point of view, our model can be further refined using the following criteria:

- Boundedness of V_I .** Requiring that V_I is bounded from below, we leave open the possibility that FHI may occur under generic initial conditions set at $\sigma \simeq m_{\text{P}}$. Since the large σ behavior of V_I is predominantly controlled by the positive (for large N values) higher order terms in Eq. (3.6), the boundedness of V_I is naturally ensured in our scheme.
- Convergence of V_I .** The expansion of V_I in Eq. (3.6) is expected to converge at least for $\sigma \sim \sigma_\star$. This property is automatically ensured in our setup thanks to the specific form of K_I in Eq. (1.4) which exhibits a convergent expansion for $\sigma \ll m_{\text{P}}$ and $|N| > 1$. This advantage of our proposal has to be contrasted with the polynomial K_I [54,55,59] which includes multiple coefficients of order unity in K_I , and so the convergence of the expansion of V_{SUGRA} Eq. (3.3) has to be checked in each case separately.
- Monotonicity of V_I .** Depending on the values of k , N , and a_5 in Eq. (3.2), V_I is a monotonic function of σ or develops a local minimum and maximum. The latter case leads to the possibility in which the system gets trapped near the minimum of the inflationary potential and, consequently, no FHI takes place. It is, therefore, crucial to check if we can avoid the minimum-maximum structure of V_I . In such a case, the system can start its slow rolling from any point on the inflationary path without the danger of getting trapped. This can be achieved if we require that V_I is a monotonically increasing function of σ , i.e., $V_I' > 0$ for any σ or, equivalently,

$$V_I'(\bar{\sigma}_{\min}) > 0 \quad \text{with} \quad V_I''(\bar{\sigma}_{\min}) = 0 \quad \text{and} \quad V_I'''(\bar{\sigma}_{\min}) > 0, \quad (4.7)$$

where $\bar{\sigma}_{\min}$ denotes the value of σ at which the minimum of V_I lies.

- Tuning of the initial conditions.** When hilltop FHI occurs with σ rolling from the region of the maximum down to smaller values, a mild tuning of the initial conditions is required [59] in order to obtain acceptable n_s 's. In particular, the smaller the desired value of n_s , the closer we must set $\sigma_\star$ to $\sigma_{\max}$, where $\sigma_{\max}$ is the value of σ at which the maximum of V_I lies. On the other hand, in cases where a_5 plays a crucial role in the inflationary dynamics—especially for $N < 0$ — $\sigma_\star$ must lie close to σ_c , giving rise to another source of possible tuning. To quantify the amount of tuning involved, we define [47,49] the quantities:

$$\Delta_{m\star} = (\sigma_{\max} - \sigma_\star) / \sigma_{\max} \quad \text{and} \quad \Delta_{c\star} = (\sigma_\star - \sigma_c) / \sigma_\star. \quad (4.8)$$

The natural realization of FHI improves as $\Delta_{m\star}$ and $\Delta_{c\star}$ increase.

Let us, finally, note that in (standard) FHI—cf. Ref. [63]—the $\mathbb{G}$ -breaking scale M is constrained by Eq. (4.4) and in most cases it is typically slightly lower than the value dictated by the unification of the gauge coupling constants within minimal SUSY SM (MSSM), i.e., Refs. [59,63]

$$gM \simeq 20 \text{ YeV} \Rightarrow M \simeq 28.6 \text{ YeV}, \text{ with } g \simeq 0.7 \quad (4.9)$$

being the value of the unified gauge coupling constant. In our setup, we do not fulfill Eq. (4.9). This is totally acceptable for $\mathbb{G} = \mathbb{G}_{B-L}$, since the gauge boson associated with the $U(1)_{B-L}$ breaking is neutral under $\mathbb{G}_{\text{SM}}$ and so it does not contribute to the relevant renormalization group running—this case can be further constrained, though, as discussed in Sec. VI. On the other hand, if $\mathbb{G} = \mathbb{G}_{\text{LR}}$ or $\mathbb{G}_{5\text{IF}}$, we may invoke threshold corrections or additional matter supermultiples to restore the gauge coupling unification—for $\mathbb{G} = \mathbb{G}_{\text{LR}}$ see Refs. [15,40], and for $\mathbb{G} = \mathbb{G}_{5\text{IF}}$ see Ref. [77].

V. INFLATION ANALYSIS

We first analyze analytically, in Sec. VA, and then numerically, in Sec. VB, the inflationary dynamics of our model.

A. Analytic results

For $\sigma_\star \gg \sigma_c$, the contributions C_{RC} and C_{SUGRA} to V_I in Eq. (3.2) can be approximated as

$$C_{\text{RC}} \simeq \frac{\kappa^2 \mathbf{N}_{\mathbb{G}}}{16\pi^2} \left(\frac{\kappa \sigma^2}{2Q^2} + \frac{3}{2} \right) \quad \text{and} \quad C_{\text{SUGRA}} \simeq -\frac{1}{N} \frac{\sigma^2}{m_{\text{P}}^2} + \frac{1}{8} \frac{\sigma^4}{m_{\text{P}}^4}. \quad (5.1)$$

The relevant derivatives w.r.t. σ which are involved in the calculation of the various inflationary observables of Sec. IVA read

$$C'_{\text{SUGRA}} \simeq -\frac{2}{N} \frac{\sigma}{m_{\text{P}}^2} + \frac{1}{2} \frac{\sigma^3}{m_{\text{P}}^4} \quad \text{and} \quad C''_{\text{SUGRA}} \simeq -\frac{2}{N m_{\text{P}}^2} + \frac{3}{2} \frac{\sigma^2}{m_{\text{P}}^4}, \quad (5.2a)$$

$$C'_{\text{RC}} \simeq \frac{\kappa^2 \mathbf{N}_{\mathbb{G}}}{8\pi^2} \frac{1}{\sigma} \quad \text{and} \quad C''_{\text{RC}} \simeq -\frac{\kappa^2 \mathbf{N}_{\mathbb{G}}}{8\pi^2} \frac{1}{\sigma^2}, \quad (5.2b)$$

$$C'_{\text{SSB}} \simeq -\sqrt{2} a_S / \sqrt{V_{10}} \quad \text{and} \quad C''_{\text{SSB}} = 0. \quad (5.2c)$$

From the expressions above, we can easily verify that in the limit of minimal SUGRA—where N tends to infinity and so the quadratic term in C_{SUGRA} tends to zero— C''_{RC} is the unique contribution to η in Eq. (4.3), thereby driving n_s to an unacceptably large value [28,56] while keeping V_I monotonic. A careful choice of $|a_S|$ as a function of κ [38,47,48] renders V_I non-monotonic and reduces

n_s to an acceptable level. Taking advantage of the expressions above, we can easily infer that $V_I''' > 0$ for any $\sigma > 0$ and so the monotonicity criterion in Eq. (4.7)—which is closely related to the n_s values—reduces to the following condition:

$$V_I'(\bar{\sigma}_{\min}) > 0 \Rightarrow \frac{\kappa^3 M^2 \mathbf{N}_{\mathbb{G}}}{\pi^2 \bar{\sigma}_{\min}} - 4\kappa M^2 \frac{4m_{\text{P}}^2 \bar{\sigma}_{\min} - N \bar{\sigma}_{\min}^3}{N m_{\text{P}}^4} \geq 8\sqrt{2} a_S, \quad (5.3)$$

where $\bar{\sigma}_{\min}$ can be derived by solving the equation in Eq. (4.7). This task can be simplified, if we analyze separately the situation emerging for the two signs of N .

1. $N > 0$ case

In this case, the reduction of n_s at the level of Eq. (4.6) can be achieved by increasing the negative coefficients of σ^2 in C_{SUGRA} and/or of σ in C_{SSB} . In both cases the monotonicity of V_I in Eq. (3.2) can be investigated in the same manner by applying Eq. (4.7). Namely, the value of $\bar{\sigma}_{\min}$ can be approximately determined by solving the equation

$$V_I''(\bar{\sigma}_{\min}) = 0 \Rightarrow \bar{\sigma}_{\min} \simeq \frac{m_{\text{P}}}{\sqrt{6\pi}} \sqrt{\sqrt{3\kappa^2 \mathbf{N}_{\mathbb{G}} + \frac{16\pi^2}{N^2}} + \frac{4\pi}{N}} \sim \frac{2}{\sqrt{3N}} m_{\text{P}}, \quad (5.4)$$

where we use the rightmost expression in Eq. (5.2a) and neglect the contribution from C''_{RC} in Eq. (5.2) for the derivation of the last result above. Note that the total result is a_S -independent due to the rightmost result in Eq. (5.2c). If V_I is not monotonic, it reaches a maximum at $\sigma = \sigma_{\max}$ which can be accurately approximated as follows:

$$V_I'(\sigma_{\max}) = 0 \Rightarrow \sigma_{\max} = \frac{N m_{\text{P}}^2}{4} \left(-\frac{\sqrt{2}|a_S|}{\kappa M^2} + \sqrt{\frac{2|a_S|^2}{\kappa^2 M^4} + \frac{\kappa^2 \mathbf{N}_{\mathbb{G}}}{N \pi^2 m_{\text{P}}^2}} \right), \quad (5.5)$$

where the term of C_{SUGRA} in Eq. (3.6) proportional to σ^4 has been ignored for simplicity.

The investigation of the remaining inflationary features is simplified if we distinguish two subcases depending on the magnitude of C_{SSB} . Namely, we consider the cases:

- (a) $N > 0$ without sizable C_{SSB} . In this case, C_{SSB} can be totally neglected in Eq. (3.2) and so V_I can be simplified appreciably. When Eq. (5.3) is violated, V_I develops a maximum and minimum for the following values of σ , respectively:

$$\sigma_{\max} \simeq \frac{\kappa \sqrt{N N_{\mathbb{G}}}}{4\pi} m_{\text{P}} \quad \text{and} \quad \sigma_{\min} \simeq \frac{2}{\sqrt{N}} m_{\text{P}}. \quad (5.6)$$

Note that $\sigma_{\max}$ is dominated by C'_{RC} in Eq. (5.2b) and the quadratic term of C_{SUGRA} in Eq. (3.6) with the derived result being consistent with Eq. (5.5) in the limit $|a_{\text{S}}| = 0$. On the other hand, $\sigma_{\min}$ is approximately determined by the vanishing of C'_{SUGRA} since C_{SUGRA} in Eq. (3.6) dominates the large- σ behavior of V_{I} .

Performing the integration in Eq. (4.1) for $\sigma_{\star} \gg \sigma_{\text{f}}$ and expanding the resulting expression in powers of $\sigma_{\star}$, we can reliably estimate $N_{\text{I}\star}$ with the result

$$N_{\text{I}\star} \simeq \frac{4\pi^2 \sigma_{\star}^2}{\kappa^4 m_{\text{P}}^4 N N_{\mathbb{G}}} (\kappa^2 m_{\text{P}}^2 N N_{\mathbb{G}} + 8\pi^2 \sigma_{\star}^2) \Rightarrow \sigma_{\star} \simeq \frac{\kappa m_{\text{P}}}{4\pi} \sqrt{D_N N_{\mathbb{G}}}, \quad (5.7)$$

where $D_N = \sqrt{N(8N_{\text{I}\star} + N)} - N$. If we substitute the expression for $\sigma_{\star}$ into Eq. (4.5), we can obtain sufficiently accurate expressions for the inflationary observables. In particular, we obtain

$$n_{\text{s}} \simeq 1 - \frac{4}{N} - \frac{4}{D_N} + 6\kappa^2 \frac{N_{\mathbb{G}} D_N}{32\pi^2}, \quad (5.8)$$

whereas the expressions for α_{s} and r are more complicated, resulting in sufficiently low values which are comfortably consistent with Eq. (4.6).

- (b) $N > 0$ with sizable C_{SSB} . In this case, C_{SSB} in Eq. (3.7) contributes significantly to V_{I} —see Eq. (3.2)—and cannot be ignored. This effect, though, is restricted to specific κ (or M) values. Its presence reinforces the negative contribution of the first term in C'_{SUGRA} —see Eq. (5.2a)—into V'_{I} . Therefore, $|a_{\text{S}}|$ influences ϵ and consequently $N_{\text{I}\star}$ in Eqs. (4.3) and (4.1), respectively. Its presence in the relevant formulas complicates the analytical computation, and so it is not possible to find specific $\sigma_{\star} = \sigma_{\star}(N_{\text{I}\star})$ and $n_{\text{s}} = n_{\text{s}}(N_{\text{I}\star})$ relations—as in Eqs. (5.7) and (5.8), respectively. Consequently, we are obliged to trust the results of our numerical calculation presented in Sec. VB below. As shown there, the ramifications due to C_{SSB} for selected $|a_{\text{S}}|$ are confined to isolated values of κ .

2. $N < 0$ case

In this case, the presence of C_{SSB} in Eq. (3.2) is crucial for the viability of our inflationary scenario since this is the only negative contribution into V_{I} that may assist in reducing n_{s} to the acceptable range of Eq. (4.6). The determination of $\bar{\sigma}_{\min}$ now requires consideration of all terms in Eq. (3.2) with the approximations in Eq. (5.1), i.e.,

$$V'_{\text{I}}(\bar{\sigma}_{\min}) = 0 \Rightarrow \bar{\sigma}_{\min} \simeq \frac{m_{\text{P}}}{\sqrt{6\pi}} \sqrt{\sqrt{3\kappa^2 N_{\mathbb{G}} + \frac{16\pi^2}{|N|^2}} - \frac{4\pi}{|N|}}. \quad (5.9)$$

As in Eq. (5.4), we note that $\bar{\sigma}_{\min}$ is still independent of $|a_{\text{S}}|$. By setting $N = -|N|$ in Eq. (5.3), we can obtain the condition for which V_{I} remains monotonic. In the opposite case, the extrema of V_{I} can be well approximated if we ignore the quartic term in C_{SUGRA} . Namely, we obtain

$$V'_{\text{I}}(\sigma) = 0 \Rightarrow \sigma_{\max}^{\min} = \frac{|N| m_{\text{P}}^2}{4} \left(\frac{\sqrt{2}|a_{\text{S}}|}{\kappa M^2} \mp \sqrt{\frac{2|a_{\text{S}}|^2}{\kappa^2 M^4} - \frac{\kappa^2 N_{\mathbb{G}}}{|N| \pi^2 m_{\text{P}}^2}} \right). \quad (5.10)$$

For $\sigma > \sigma_{\min}$, the boundedness of V_{I} is assured as explained in Sec. IV B.

The numerical analysis in Sec. VB for $N < 0$ reveals that, in most of the allowed parameter space, the successful realization of FHI requires $\sigma_{\star}$ to lie close to σ_{c} [47]. As a consequence, the expansion of C_{RC} in Eq. (5.1) ceases to be sufficiently accurate. The appropriate behavior of V_{I} , for given $N_{\mathbb{G}}$ and N , can be achieved thanks to the similar magnitudes and opposite signs of the terms $C'_{\text{RC}} + C'_{\text{SUGRA}}$ and C'_{SSB} in Eqs. (5.2a)–(5.2c) which can be achieved through a careful choice of κ and a_{S} . This arrangement accommodates large enough $N_{\text{I}\star}$ —see Eq. (4.1). On the other hand, the establishment of the inequality $|C'_{\text{RC}}| > C'_{\text{SUGRA}}$ helps obtain $\eta < 0$ and so low enough n_{s} —see Eqs. (4.3) and (4.5).

B. Numerical results

We here present a detailed numerical analysis of the parameter space of our model, paying special attention to identifying regions which ensure the monotonicity of V_{I} . Our strategy consists of restricting $\sigma_{\star}$ and N as functions of our free parameters κ (or M) for given a_{S} and $N_{\mathbb{G}}$, enforcing Eqs. (4.1), (4.4), and (4.6). For presentation purposes, we divide the analysis into the various cases introduced in Sec. VA. In particular,

- (a) $N > 0$ without $|a_{\text{S}}|$ dependence. Our results are displayed in Fig. 1 where we depict allowed regions in $\kappa - N$ (left panels) and $M - N$ (right panels) planes as determined by the aforementioned constraints for $|a_{\text{S}}| = 0.5 \text{ TeV}$ and various $\mathbb{G}$'s shown in the legends. The lower (upper) boundary curves of the allowed regions come from the lower (upper) bound on n_{s} in Eq. (4.6). The dark blue (orange) lines included within the allowed regions correspond to the central value of n_{s} in Eq. (1.2) [Eq. (1.3)]. Moreover, the light green region indicates the portion of the allowed parameter space where V_{I} is not monotonic while the unshaded regions correspond to a monotonic V_{I} . The separation of the two regions is indicated by the black

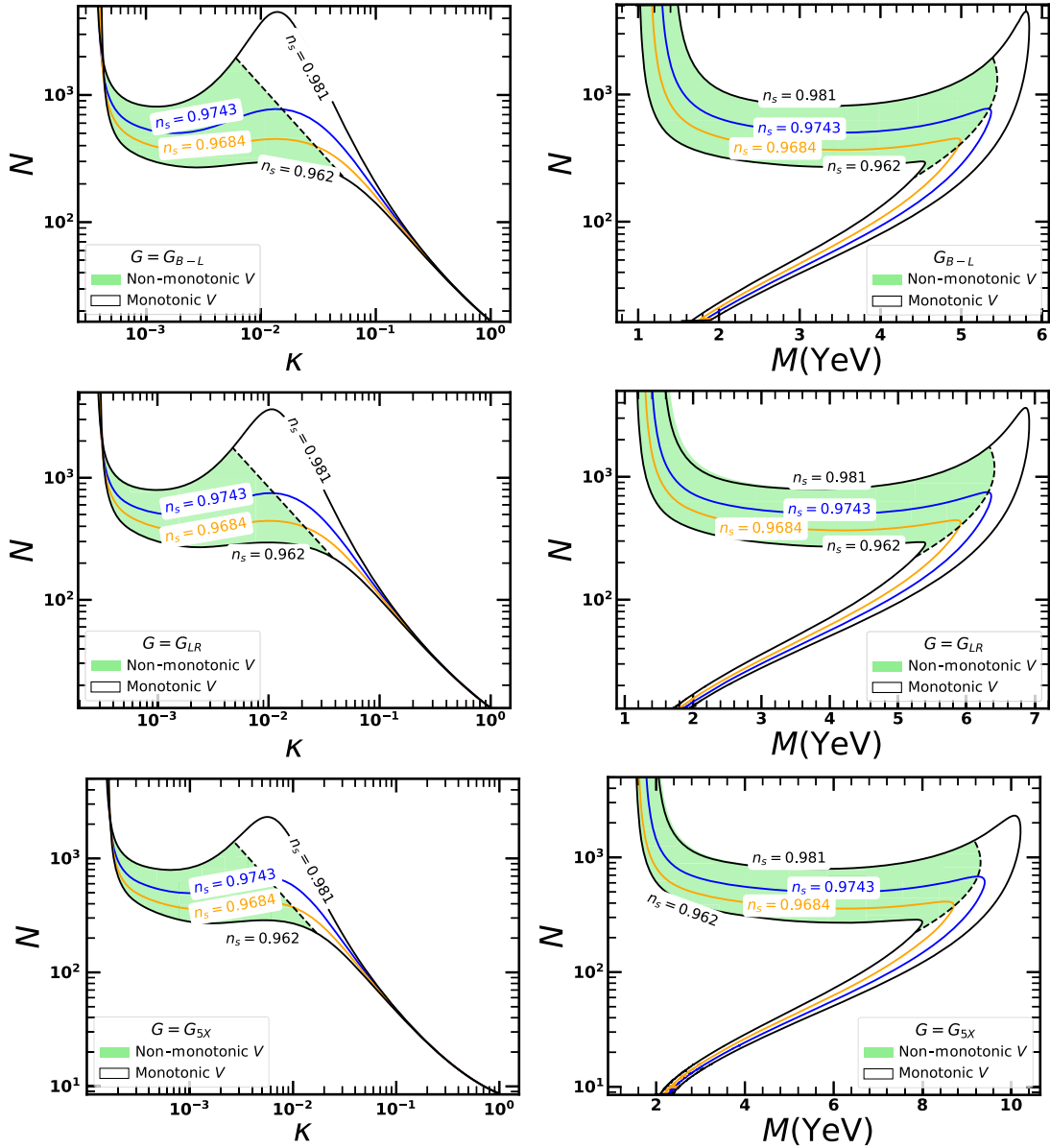


FIG. 1. Allowed regions in $\kappa - N$ (left panels) and $M - N$ (right panels) as determined by Eqs. (4.1), (4.4), and (4.6) for $|a_S| = 0.5$ TeV and various $\mathbb{G}$'s shown in the legends. The light green region indicates the allowed parameter space where V_I is not monotonic whereas it remains monotonic in the white patch. The dark blue [orange] curve yields the n_s central value in Eq. (1.2) [Eq. (1.3)].

dashed line, along which the inequality in Eq. (5.3) is saturated.

Comparing the plots of Fig. 1 with different $N_{\mathbb{G}}$ values, we can appreciate its impact on our results. Namely, we employ, respectively, the $N_{\mathbb{G}}$ values shown in Eqs. (2.1a)–(2.1c) in the upper, middle, and lower row of the plots of Fig. 1. We observe that the allowed κ and N values are similar in all cases but the M values increase with $N_{\mathbb{G}}$ without, though, to approach the value favored by Eq. (4.9). On the other hand, the $\kappa - N$ allowed region is slightly reduced for larger $N_{\mathbb{G}}$ values. Along the dark blue curves, $\Delta_{m\star}$ in Eq. (4.8) varies in the following ranges (0.37 – 0.66) for $\mathbb{G} = \mathbb{G}_{B-L}$, (0.36 – 0.65) for $\mathbb{G} = \mathbb{G}_{LR}$, and (0.34 – 0.63) for

$\mathbb{G} = \mathbb{G}_{51F}$. Therefore, even in the case of the non-monotonic V_I , the tuning in the initial conditions is very low and similar to that needed for quasicanonical K 's—cf. Ref. [59].

To gain further insight into the behavior of the solutions in the green and white regions of the plots of Fig. 1, we list in Table II two representative benchmark points (BMPs) A and B. In both BMPs, we fix $\kappa = 0.01$, $N_{\mathbb{G}} = 2$, and $|a_S| = 1$ TeV. In BMP A, however, we select $N = 600$ and we obtain $V'_I < 0$ —see Eq. (5.3)—indicating the development of a local minimum and maximum of V_I , whereas in BMP B we use $N = 820$ yielding $V'_I > 0$, which signals the monotonic structure of V_I . The variation of V_I for the BMPs A and B is plotted in the left panel of the upper row of Fig. 2

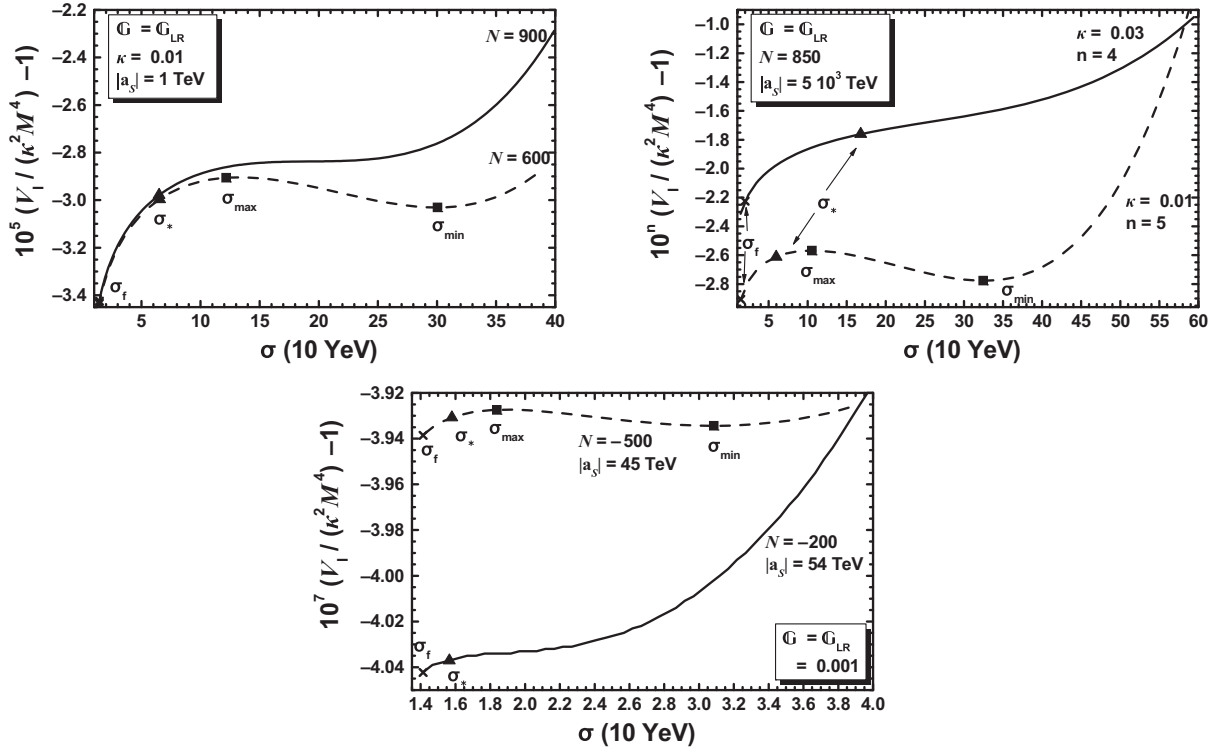


FIG. 2. Variation of V_I in Eq. (3.2) as a function of σ for $\mathbb{G} = \mathbb{G}_{\text{LR}}$ and the other inputs (κ , N , and a_S) of the BMPs in Table II. The values of σ_* , σ_f , σ_{max} , and σ_{min} are also depicted.

where the aforementioned results regarding the monotonicity of V_I are verified. In the same plot, we also depict points σ_* and σ_f for BMPs A and B, and σ_{max} and σ_{min} for BMP B. From the analytic expressions in Eqs. (5.6)–(5.8),

we obtain $\sigma_{\text{max}} \simeq 11.1M$, $\sigma_{\text{min}} \simeq 30.7M$, and $n_s \simeq 0.97$ for BMP A and $n_s \simeq 0.973$ for BMP B in good agreement with the numerical findings listed in Table II. From that table, we can also infer that the inflaton mass m_I —defined below

TABLE II. Input and output parameters of six BMPs compatible with Eqs. (4.1), (4.4), and (4.6) for $N_{\mathbb{G}} = 2$ and $N > 0$ with (without) sizable C_{SSB} [A, B (C, D)] or $N < 0$ (E, F).

BMP	A	B	C	D	E	F
κ	0.01	0.01	0.01	0.03	0.001	0.001
N	600	900	850	850	−500	−200
a_S/TeV	1	1	5×10^3	5×10^3	45	54
M/YeV	6	6.3	5.5	6.5	2.7	2.4
$N_{I\star}$	48.5	48.5	48.5	48.7	47.5	47.4
σ_*/M	6.02	5.9	5.97	16.8	1.58	1.565
σ_f/M	1.43	1.45	1.42	2	1.41	1.41
$\bar{\sigma}_{\text{min}}/M$	21.5	18	21.3	25.7	2.4	1.9
σ_{max}/M	11.9		10.5		1.8	
σ_{min}/M	30.8		32.4		3.1	
$10^9 V'(\bar{\sigma}_{\text{min}})/\kappa^2 M^3$	−136	2.7	−141	855	−0.89	0.46
$\Delta_{m\star}(\%)$	49		43		12	
$\Delta_{c\star}(\%)$	76	76	76	91.5	10.4	9.7
n_s	0.971	0.974	0.968	0.979	0.974	0.974
$-\alpha_s/10^{-4}$	3.9		4.3	5.4	2.9	3.8
$r/10^{-7}$	1.2	1.4	0.8	15	4.8×10^{-4}	2.9×10^{-4}
m_I/ZeV	85.2	89	77.5	277	3.8	3.4

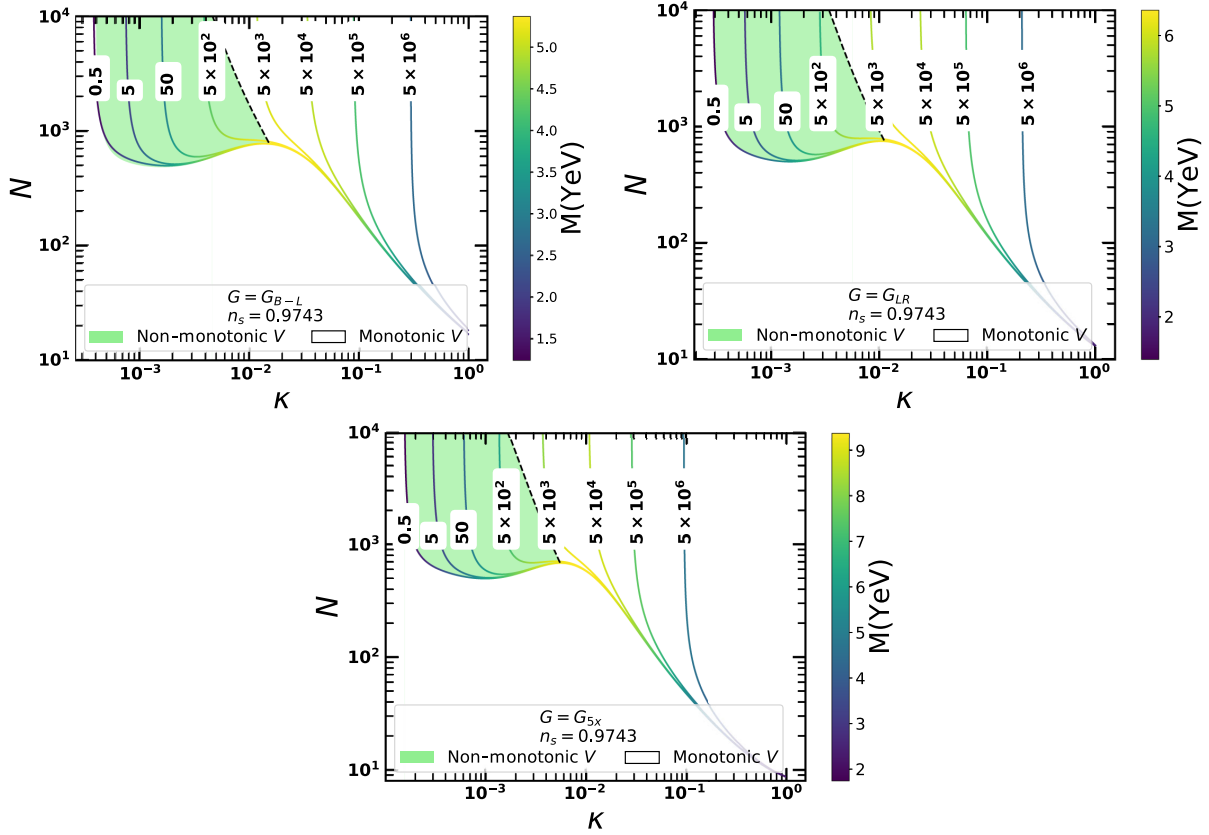


FIG. 3. Curves allowed by Eqs. (4.1) and (4.4) in the $\kappa - N$ plane for central n_s in Eq. (1.2) and various $|a_S|/\text{TeV}$ and G 's shown on the curves and in the legends, respectively. The color coding of the curves indicates the variation of M . The curves within the green shaded region correspond to non-monotonic V_1 .

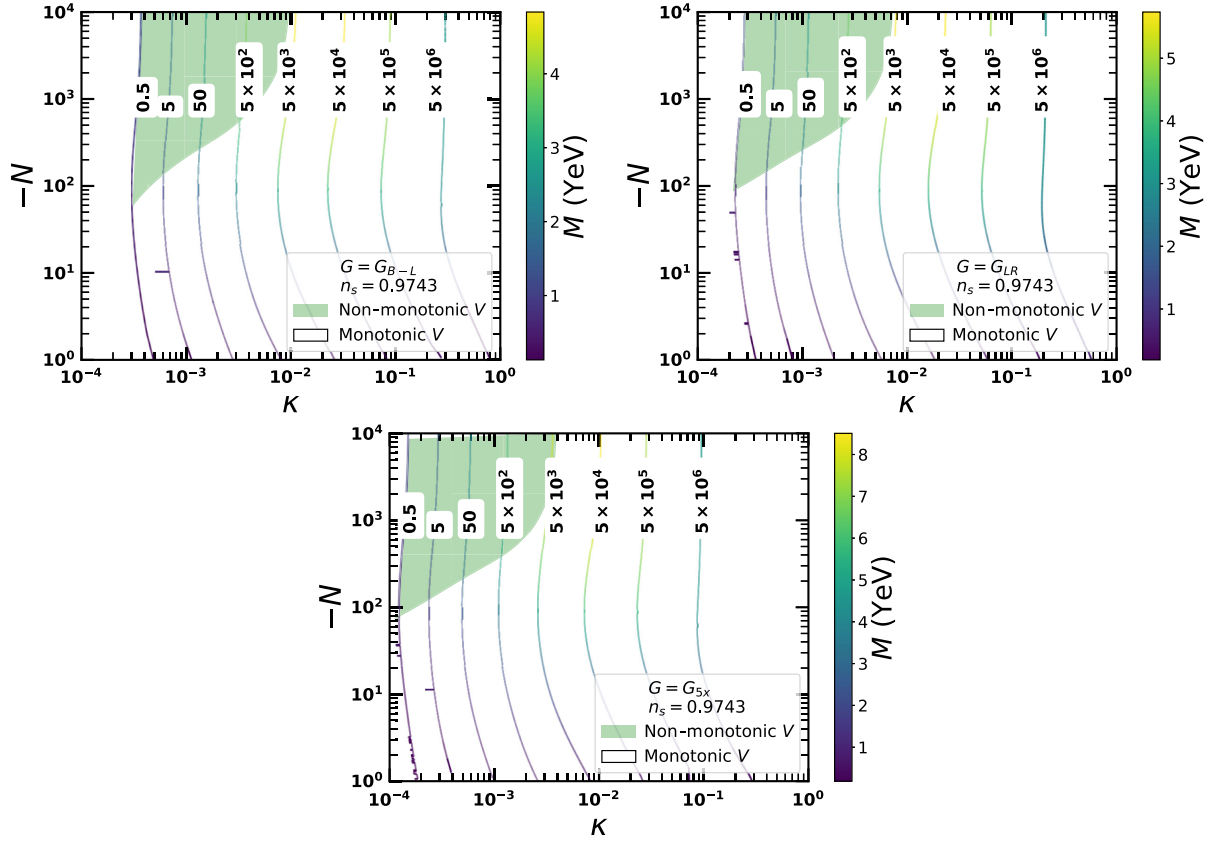
Eq. (2.5)—lies in the ZeV range whereas the quantities in Eq. (4.8) do not signal any disturbing tuning.

(b) $N > 0$ with $|a_S|$ dependence. Related results are presented in Fig. 3, where we depict curves allowed by Eqs. (4.1) and (4.4) in the $\kappa - N$ plane for central n_s in Eq. (1.2). We employ various $|a_S|/\text{TeV}$ values shown on the curves and the three G 's indicated in legends of the plots. We focus on the central n_s value in Eq. (1.2) since the results for other n_s values in Eq. (4.6) are similar as we show in Fig. 1. The color coding of the curves indicates the variation of M . As in Fig. 1, we observe that M increases with N_G but it turns out to be lower than its values in Fig. 1 for fixed N_G . Therefore, it remains lower than its value required by Eq. (4.9) too.

The line for $|a_S|/\text{TeV} = 0.5$ is identical to the dark blue line included in Fig. 1. Also, for low $|a_S|$ values, the various lines share the same segment with the dark blue line in the corresponding plots of Fig. 1 and for a sizable range of κ values. Increasing $|a_S|$, however, the almost vertical part of the allowed curves is shifted to the right and the common part of the lines with that in Fig. 1 is reduced. Interestingly, for $|a_S| > 5$ PeV, the obtained lines lie outside the green shaded region, assuring the monotonicity of V_1 . On the other hand, the curves within the green shaded region

correspond to non-monotonic V_1 . The transition between these regimes is illustrated by a black dashed line derived from the condition in Eq. (5.3). We observe that, for $\kappa \gtrsim 0.01$, monotonic solutions are preferred, particularly for $|a_S|$ in the range $(1 - 10^3)$ PeV. This indicates that, for moderately large couplings, FHI favors V_1 stabilized by substantial soft SUSY-breaking contributions, consistent with either split [78] or high-scale [79] SUSY scenarios.

The salient features of our inflationary solutions in the green and white regions of the plots of Fig. 3 can be further appreciated by BMPs C and D in Table II. In both BMPs, we fix $N = 850$, $N_G = 2$, and $|a_S| = 5$ PeV. In BMP C, however, $\kappa = 0.01$ and so $V_1' < 0$ —see Eq. (5.3)—resulting in non-monotonic V_1 , whereas in BMP D $\kappa = 0.03$ yields $V_1' > 0$, assuring the monotonicity of V_1 . The variation of V_1 for BMPs C and D is plotted in the rightmost panel of the upper row of Fig. 2 where the aforementioned results regarding the monotonicity of V_1 are verified. Due to different κ and M , the two curves differ appreciably, and for this reason we use different normalization for the variation of V_1 . In the same plot, we also depict the points $\sigma_\star$ and σ_f for the BMPs C and D, and $\sigma_{\max}$ and $\sigma_{\min}$ for BMP C. From the analytic expression in Eq. (5.5), we obtain $\sigma_{\max} = 10M$ in accordance with the numerical finding listed in Table II. From the same

FIG. 4. Same as Fig. 3 but for $N < 0$.

table, we also remark that no tuning issue regarding $\Delta_{c\star}$ arises—despite the fact that $|a_S|$ is now sizable—whereas $\Delta_{m\star}$ for BMP C is similar to that for BMP A.

(c) $N < 0$. Our results are presented in Fig. 4 where we illustrate curves compatible with Eqs. (4.1) and (4.4) in the $\kappa - N$ plane for central n_s in Eq. (1.2). We use the same $|a_S|/\text{TeV}$ values—shown on the curves—as in Fig. 3. Also the three panels correspond to the three G 's considered and specified in the legends of each panel. As in the previous figures, within the white region, V_I enjoys monotonicity whereas within the green shaded region V_I turns out to be non-monotonic. The color coding along the curves indicates the corresponding values of M which increase with N_G but remain lower than the corresponding values in Figs. 1 and 3. We remark that now each κ value corresponds, essentially, to one specific $|a_S|$ value almost independently from the N value. This fact signals that the mechanism described in Refs. [47,48] is activated to obtain the correct n_s value. Contrary to those papers, however, here we find regions with monotonic V_I thanks to the presence of the quadratic part of C_{SUGRA} in Eq. (3.6).

To complete our investigation for $N < 0$, we collect two representative BMPs E and F in Table II. In both BMPs, we fix $N_G = 2$ and $\kappa = 0.001$. In BMP E, we set $|N| = 500$ and $|a_S| = 45$ TeV which yield a non-monotonic V_I since

$V_I' < 0$ —see Eq. (5.3)—whereas in BMP F we select $|N| = 200$ and $|a_S| = 54$ TeV leading to monotonic V_I since $V_I' > 0$. The variation of V_I for BMPs E and F is depicted in the plot of the lower row of Fig. 2 where the aforementioned results regarding the monotonicity of V_I are verified. In the same plot, we also indicate the points $\sigma_\star$ and σ_f for the BMPs E and F, and $\sigma_{\max}$ and $\sigma_{\min}$ for BMP E. From the analytic expressions in Eqs. (5.9) and (5.10), we obtain $\bar{\sigma}_{\min} = 2.3M$ ($\bar{\sigma}_{\min} = 1.6M$), $\sigma_{\max} = 1.6M$, and $\sigma_{\min} = 3.12M$ for BMP E (F), in good agreement with the numerical findings shown in Table II. From the displayed values of $\Delta_{m\star}$ and $\Delta_{c\star}$, we notice that $\Delta_{m\star}$ is lower than the ones obtained in BMPs A and C whereas $\Delta_{c\star}$ is much lower than those obtained for $N > 0$. Therefore, this case can be characterized, generally, as less natural regarding the aspect of tuning.

The predicted values of $|\alpha_s|$ and r within our scheme remain very small in all cases analyzed above. In particular, $|\alpha_s| \sim 10^{-4}$ remains essentially unchanged across the range of $|a_S|$. In contrast, r increases noticeably for large $|a_S|$, reflecting the growing influence of C_{SSB} . For $N > 0$, r rises up to $\sim 2.3 \times 10^{-5}$, whereas for $N < 0$, its largest value is 3.3×10^{-6} for large $|a_S|$ values. These results demonstrate that, while N and $|a_S|$ can affect r , it remains even at its maximum well below the sensitivity of current and near-future CMB and gravitational wave experiments.

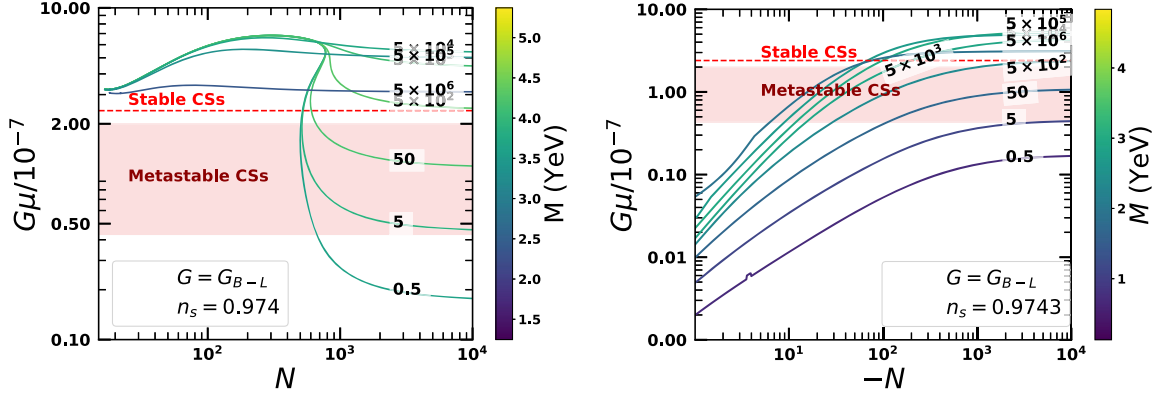


FIG. 5. $G\mu$ as a function of N for $N > 0$ (left panel) and $-N$ for $N < 0$ (right panel). We use $n_s = 0.974$ and various $|a_S|/\text{TeV}$ values shown on the curves. The color coding of the curves indicates the variation of M . The upper bound on $G\mu$ in Eq. (6.2) and the margin of Eq. (6.6) are also depicted by the horizontal red dashed line and the light red shaded region, respectively.

VI. FHI AND COSMIC STRINGS

When $\mathbb{G} = \mathbb{G}_{B-L}$, type I—according the terminology of Refs. [80,81]—CSs may be produced after FHI with dimensionless tension [47]:

$$G\mu \simeq \frac{1}{2} \left(\frac{M}{m_p} \right)^2 \epsilon_{\text{cs}}(r_{\text{cs}}) \quad \text{with} \quad \epsilon_{\text{cs}}(r_{\text{cs}}) = \frac{2.4}{\ln(2/r_{\text{cs}})} \quad \text{and} \quad r_{\text{cs}} = \frac{\kappa^2}{8g^2} \leq 10^{-2}, \quad (6.1)$$

where $G = 1/8\pi m_p^2$ is the Newton gravitational constant and $g \simeq 0.7$ is the gauge coupling constant at a scale close to M . We also take into account that $(B-L)(\Phi) = 2$ in accordance with our assumptions in Table I. The $B-L$ charge of Φ is determined by the superpotential coupling between $\bar{\Phi}$ and the right-handed neutrinos, ν_i^c . We here assume a renormalizable coupling of the form $\bar{\Phi}(\nu_i^c)^2$ as in Ref. [47]—cf. Ref. [38]. The resulting uncertainty in $G\mu$ may induce minor changes to our results.

If the CSs are *stable*, the corresponding parameter space is fully allowed by the level of the CS contribution to the observed anisotropies of CMB which is constrained by *Planck* [82], using field-theory simulations of the Abelian-Higgs action—cf. Ref. [83]—in the range

$$G\mu \lesssim 2.4 \times 10^{-7} \quad \text{at 95\% C.L.} \quad (6.2)$$

On the other hand, the recent PTA bound requires [31]

$$G\mu \lesssim 2 \times 10^{-10} \quad \text{at 95\% C.L.,} \quad (6.3)$$

which excludes the $G\mu$ values predicted within our model—see below. However, if these CSs are *metastable* due to the embedding of $\mathbb{G}_{B-L}$ into a larger gauge group—such as the Pati-Salam Ref. [39], the flipped $SU(5)$ [84] or $SO(10)$ [85–88]—whose spontaneous breaking to $\mathbb{G}_{B-L}$ produces monopoles, an explanation of the PTA data on the GWs is possible if $G\mu$ lies within the range preferred

by the interpretation of the *NANOGrav 15-year data* (NG15) [31]:

$$4.3 \times 10^{-8} \lesssim G\mu \lesssim 2.4 \times 10^{-4} \quad \text{for } 8.2 \gtrsim \sqrt{r_{\text{ms}}} \gtrsim 7.5 \quad \text{at 95\% C.L.} \quad (6.4)$$

Here, the metastability factor r_{ms} is the ratio of the monopole mass squared, M_m^2 , to μ . Given that M_m is related to the symmetry breaking scale of the GUT framework containing $\mathbb{G}_{B-L}$, the rightmost restriction in Eq. (6.4) may constrain the relevant scale close to the M shown in Fig. 5—see below.

The $G\mu$ range in Eq. (6.4) can be more stringently constrained from above from the LIGO-Virgo-KAGRA (LVK) collaboration, which implies [89]

$$G\mu \lesssim 2 \times 10^{-7} \quad \text{at 95\% C.L.} \quad (6.5)$$

Note that larger values of $G\mu$ are fully compatible within nonstandard cosmological scenarios, such as those discussed in Refs. [37,38,42]. Assuming here, however, a conventional postinflationary evolution, as assumed in the derivation of Eq. (4.1), we end up with the following preferred region in the case of metastable CSs:

$$4.3 \times 10^{-8} \lesssim G\mu \lesssim 2 \times 10^{-7}. \quad (6.6)$$

Taking as input the M values for $\mathbb{G} = \mathbb{G}_{B-L}$ and $N > 0$ in Fig. 3 or $N < 0$ in Fig. 4—which are consistent with the inflationary requirements in Eqs. (4.1) and (4.4) and assure $n_s = 0.974$ —we can compute through Eq. (6.1) the predicted $G\mu$. The results are presented in Fig. 5 which shows $G\mu$ as a function of N for $N > 0$ (left panel) and $-N$ for $N < 0$ (right panel). Each curve corresponds to a fixed value of $|a_S|$, ranging from 0.5 to 5×10^5 TeV, and the color coding along each curve indicates the variation of M (in units of 1 YeV). In both panels, we also include the upper bound on $G\mu$ in Eq. (6.2), marked by a horizontal red

dashed line, and the margin of Eq. (6.6) depicted by the light red shaded region. We note that most of the parameter space in Fig. 4 lies above the much stricter stable-CS bound in Eq. (6.3), confirming that stable CSs are ruled out by this limit.

From the left panel, we observe that $G\mu$ decreases monotonically as N increases beyond 500, and therefore, we obtain compatibility with Eqs. (6.2) and (6.6) for low $|a_S|$ values, i.e., for $|a_S| \simeq (0.5\text{--}500)$ TeV. On the contrary, from the right panel, we remark that $G\mu$ increases with $|N|$ and so for the entire displayed range of N we can find $|a_S|$ which provides coverage of the range of Eq. (6.6) favored by NG15, i.e., for $5 \lesssim |a_S|/\text{TeV} \lesssim 5 \times 10^6$. The consistency with NG15 seems to prefer, for $N > 0$, the region in Fig. 3 where V_I becomes non-monotonic. On the contrary, for $N < 0$, regions from Fig. 4 where V_I is monotonic can be compatible with NG15. Thus, Fig. 5 provides a direct link between the internal geometry of the inflationary sector via N , the soft SUSY-breaking scale via $|a_S|$, and the observable GW background from CSs.

VII. CONCLUSIONS

We checked the viability of FHI under the assumption that the gauge-singlet inflaton field obeys hyperbolic or compact Kähler geometry characterized by a single curvature parameter $N > 0$ and $N < 0$, respectively—see Eq. (1.4). Our main findings can be summarized as follows:

- (i) The inflationary potential receives three crucial corrections: SUGRA corrections depending on N , soft SUSY-breaking parametrized by $|a_S|$, and radiative corrections with some dependence on the dimensionality N_G of the representations of the waterfall fields.
- (ii) Using the latest CMB data, we determined the allowed ranges of the W coupling κ and scale M , as a function of N for given N_G and a_S in Figs. 1 and 3 for $N > 0$ and in Fig. 4 for $N < 0$. For all three representative GUT scenarios ($\mathbb{G}_{B-L}$, $\mathbb{G}_{LR}$, $\mathbb{G}_{51F}$) and for $N > 0$, the model admits a wide and natural parameter range of parameters which ensure monotonic V_I . Most notably, for $n_s \simeq 0.974$, we found

$$0.008 \lesssim \kappa \lesssim 0.1, \quad 10 \lesssim N \lesssim 850, \\ 5 \lesssim |a_S|/\text{PeV} \lesssim 5 \times 10^3 \quad \text{and} \quad 1 \lesssim M/\text{TeV} \lesssim 9, \quad (7.1)$$

where M is slightly below than the MSSM unification scale. For $N < 0$, we also obtain a broad range of parameters with monotonic V_I , almost independent of N , which suffers, however, from some tuning associated with the proximity between

$\sigma_\star$ and σ_c —see Eq. (4.8). Moreover, our acceptable solutions require a specific correspondence between $|a_S|$ and κ (or M). For example, for $|a_S| = 5$ PeV, we find

$$0.79 \lesssim \kappa/10^{-2} \lesssim 1.9, \quad 1 \lesssim -N \lesssim 10^4, \quad \text{and} \\ 0.3 \lesssim M/\text{TeV} \lesssim 5.65. \quad (7.2)$$

For both signs of N , the primordial inflationary gravitational waves are essentially undetectable by current and future experiments since r remains below about 10^{-5} .

- (iii) The effective field theory remains valid up to the Planck scale m_P , without issues related to perturbative unitarity or higher-order corrections.
- (iv) When $\mathbb{G} = \mathbb{G}_{B-L}$, CSs are produced at the end of FHI with dimensionless tension in the range $G\mu \sim 10^{-8}\text{--}10^{-7}$. If CSs are stable, the model is ruled out by the NG15 upper bound. On the other hand, if the CSs are metastable, the same $G\mu$ values fall within the window that explains the observed stochastic background of GWs, as shown in Fig. 5. Thus, our model provides a simultaneous explanation of CMB data and the NG15 signal without fine tuning.

Finally, we note that a complete inflationary scenario must also account for the transition to a radiation-dominated universe and the generation of the observed baryon asymmetry. These aspects have been extensively explored in the context of FHI with canonical or quasicanonical Kähler potentials—see, e.g., Refs. [47,64]. Our setup preserves many of the successful features of this postinflationary evolution, which may impose additional constraints on the viable parameter space and help identify the most compelling version of FHI.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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