

Pion to photon transition form factor: Beyond valence quarks

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We investigate the singly virtual transition form factor (TFF) for the $\pi^0 \rightarrow \gamma^* \gamma$ process in the spacelike region using the hard-scattering formalism within the basis light-front quantization (BLFQ) framework. This form factor is expressed in terms of the perturbatively calculable hard-scattering amplitudes (HSAs) and the light-front wave functions (LFWFs) of the pion. We obtain the pion LFWFs by diagonalizing the light-front QCD Hamiltonian, which is determined for its constituent quark-antiquark and quark-antiquark-gluon Fock sectors with a three-dimensional confinement. We employ the HSAs up to next-to-leading order (NLO) in the quark-antiquark Fock sector and leading order (LO) in the quark-antiquark-gluon Fock sector. The NLO correction to the TFF in the quark-antiquark Fock sector is of the same order as the LO contribution to the TFF in the quark-antiquark-gluon Fock sector. We find that while the quark-antiquark-gluon Fock sector has minimal effect in the large momentum transfer (Q^2) region, it has a noteworthy impact in the low- Q^2 region. Our results show that, after accounting for both Fock sectors, the TFF within the BLFQ framework aligns well with existing experimental data, particularly in the low Q^2 region.

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I. INTRODUCTION

The structure of mesons remains a substantial puzzle in the current field of particle and nuclear physics [1–5]. Because of the difficulty in solving for their nonperturbative structure directly from QCD, it is challenging to gain theoretical insights into the internal structure of mesons. However, the development of the factorization theorem provides a pathway for studying this subject [6–9]. One of the simplest and most significant examples for factorization application in exclusive processes is the pion-photon transition form factor (TFF) with one real and one virtual photon, $F_{\pi\gamma}(Q^2)$. The TFF plays a crucial role in

determining several important observables, for instance, the hadronic light-by-light contribution to the muon anomalous magnetic moment [10–12] and the rates of rare pseudo-scalar decays [13,14].

Based on the factorization theorem, the TFF can be decomposed into the perturbatively calculable hard-scattering amplitudes (HSAs) and the light-front wave functions (LFWFs) which encode the nonperturbative dynamical information of the QCD bound states [15]. In 1980, Lepage and Brodsky first reported the leading-order (LO) HSAs for the TFF [15]. Subsequently, the next-to-leading-order (NLO) correction has been examined in several studies [16–21]. Meanwhile, the next-to-next-to-leading-order (NNLO) HSAs have been evaluated within the conformal scheme [20,22], hard-collinear factorization theorem [23], and principle of maximum commonality [24]. On the other hand, despite the challenges posed by nonperturbative QCD dynamics, various phenomenological models and theoretical frameworks have been devised to compute the distribution amplitudes and pion-photon TFF. These include methodologies like the chiral perturbation theory [25], QCD sum

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rules [26–28], Dyson-Schwinger and Bethe-Salpeter framework [29–34], constituent quark model [35–38], AdS/QCD [39], and lattice QCD [40,41].

Experimentally, the TFF has been extensively measured in spacelike regions up to large transfer momentum, with several collaborations contributing measurements [42–45]. In 2009, the *BABAR* Collaboration issued their data [44] covering the kinematic range $Q^2 \in [4, 40]$ GeV², sparking a heated discussion due to its observation of nonconforming behavior at large Q^2 , which contradicted the well-known asymptotic prediction [15]. However, in 2012, the data from Belle Collaboration [45] showed that $Q^2 F_{\pi\gamma}(Q^2) \rightarrow \text{const}$ for $Q^2 > 15$ GeV², which is consistent with the prediction of perturbative QCD [15]. The discrepancy between these two measurements in the large Q^2 region has consistently drawn the attention of both theoretical and experimental communities.

Our investigation of the TFF relies on the basis light-front quantization (BLFQ) framework, offering a non-perturbative method for solving the relativistic many-body bound states problem within light-front quantum field theory [46]. Recent advancements in the BLFQ framework have led to successful applications in exploring various observables within QCD systems, e.g., the electromagnetic form factors and associated charge radii [47,48], parton distribution functions [47–54], transverse-momentum-dependent parton distributions [52,55–58], and generalized parton distributions [53,59–62]. In an earlier exploration within the BLFQ framework, we reported the TFF only concerning the meson’s valence quarks, where the Hamiltonian includes a three-dimensional confinement potential and the color-singlet Nambu-Jona-Lasinio interaction [63,64]. Our previous results show that valence Fock state alone is insufficient to describe TFF in a finite Q^2 region, suggesting that higher Fock states may have a significant impact.

In this work, we investigate the pion-photon TFF with one real and one virtual photon considering the valence Fock sector with constituent quark-antiquark and an additional Fock sector, encompassing a quark-antiquark pair and a dynamical gluon of the pion. Based on the BLFQ framework [46], we adopt the light-front QCD Hamiltonian that includes a model confining potential and solve it in the first two Fock sectors, $|q\bar{q}\rangle$ and $|q\bar{q}g\rangle$ [47]. We evaluate the TFF through the overlap form of the HSAs and the LFWFs of the pion. The HSAs involving valence and valence-gluon Fock sectors are derived by the light-cone perturbation theory [15,65]. The LFWFs are obtained by diagonalizing the light-front Hamiltonian.

II. MESON LFWFS IN THE BLFQ FRAMEWORK

Following the Lepage-Brodsky convention [15], the light-front variables are defined as $V^\pm \equiv V^0 \pm V^3$ and $\vec{V}_\perp \equiv (V_1, V_2)$. The wave functions of bound states are the eigenvectors of the light-front Hamiltonian equation:

$P^+ P^- |\Psi\rangle = M^2 |\Psi\rangle$, where P^+ and P^- represent the longitudinal momentum and the light-front Hamiltonian of the system, respectively. At the fixed light-front time, $x^+ \equiv x^0 + x^3$, the meson state with mass squared eigenvalue M^2 can be schematically expanded in terms of Fock sectors:

$$|\Psi\rangle = \psi^{q\bar{q}} |q\bar{q}\rangle + \psi^{q\bar{q}g} |q\bar{q}g\rangle + \psi^{q\bar{q}q\bar{q}} |q\bar{q}q\bar{q}\rangle + \cdots, \quad (1)$$

where $\psi^{\cdots}$ are the LFWFs corresponding to the Fock sectors $|\cdots\rangle$. In this work, we truncate the infinite Fock expansion and retain the first two Fock sectors, which indicates that the meson is described by the quark-antiquark $\psi^{q\bar{q}}$ and quark-antiquark-gluon $\psi^{q\bar{q}g}$ LFWFs at the model scale.

We adopt an effective light-front Hamiltonian, $P^- = P_{\text{QCD}}^- + P_{\text{C}}^-$ [47], where P_{QCD}^- is the light-front QCD Hamiltonian, and P_{C}^- represents a model confining potential. In the light-front gauge $A^+ = 0$, the QCD Hamiltonian with one dynamical gluon is given by [47,65]

$$\begin{aligned} P_{\text{QCD}}^- = & \int dx^- d^2x^\perp \left\{ \frac{1}{2} \bar{\psi} \gamma^+ \frac{m_0^2 + (i\partial^\perp)^2}{i\partial^+} \psi \right. \\ & + \frac{1}{2} A_a^i [m_g^2 + (i\partial^\perp)^2] A_a^i + g_s \bar{\psi} \gamma_\mu T^a A_a^\mu \psi \\ & \left. + \frac{1}{2} g_s^2 \bar{\psi} \gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi} \gamma^+ T^a \psi \right\}, \end{aligned} \quad (2)$$

where A_a^μ and ψ represent the gluon and the quark fields, respectively. $T^a = \lambda^a/2$ is half of the Gell-Mann matrix. g_s is the coupling constant. m_g and m_0 stand for the model gluon mass and the bare quark mass, respectively. Though the gluon mass is zero in QCD, we artificially assign it as a phenomenologically motivated mass to fit the mass spectra in the low-energy scale [47]. We introduce a mass counterterm, $\delta m_q = m_0 - m_q$, for the quark in the leading Fock sector to treat quark mass corrections from the higher Fock sectors, where m_q is the renormalized quark mass. Following Refs. [66,67], we allow an independent quark mass m_f in the vertex interaction. Specifically, the vertex mass is determined as part of the model’s parametrization to reproduce the mass splitting between the ground states of the π and ρ mesons. Additionally, it ensures consistency with the observed properties of the pion, including its decay constant and electromagnetic form factor, and aligns with experimental data for related processes [47].

The confining potential consists of the transverse and longitudinal terms. In the valence Fock sector, it is written as [47,68]

$$P_{\text{C}}^- P^+ = \kappa^4 \left\{ x(1-x) \vec{r}_\perp^2 - \frac{\partial_x [x(1-x) \partial_x]}{(m_q + m_{\bar{q}})^2} \right\}, \quad (3)$$

where κ is the strength coefficient of confinement and $\vec{r}_\perp = \sqrt{x(1-x)} (\vec{r}_{q\perp} - \vec{r}_{\bar{q}\perp})$ represents the holographic

variable [69]. We employ a two-dimensional (2D) harmonic oscillator potential in the transverse direction, inspired by light-front holography, along with a complementary longitudinal confining potential that reproduces a one-dimensional harmonic oscillator potential in the non-relativistic limit. Together, these form an effective 3D harmonic oscillator potential. While alternative choices of confining potentials are possible, such as linear or Cornell-like potentials, these often introduce additional complexity in the BLFQ framework due to the nonorthogonality with the harmonic oscillator basis. In contrast, the harmonic oscillator potential ensures compatibility with the BLFQ framework, where we expand the hadronic state using harmonic oscillator basis functions. This choice maintains consistency between the confining potential and the basis functions employed in our calculations. Moreover, the harmonic oscillator potential has been widely validated in prior studies, demonstrating its effectiveness in capturing key hadronic properties. In the valence-gluon Fock sector, the confinement relies solely on the truncation of the BLFQ basis functions (see below).

Using the BLFQ framework [46], we solve the Hamiltonian equation in a chosen basis space. The Fock sectors in Eq. (1) are taken to be direct products of single-particle states $|\alpha\rangle = \otimes_i |\alpha_i\rangle$. We employ the discretized light-cone quantization basis [65] and a two-dimensional harmonic oscillator (2D HO) basis functions to describe the longitudinal and transverse dynamics of single-particle states, respectively. More specifically, in the longitudinal direction, the single particle is confined in a one-dimensional box of length $2L$ with periodic (antiperiodic) boundary conditions for the boson (fermion). For the i th single-particle state, the longitudinal momentum is discretized as $p_i^+ = \frac{2\pi}{L} k_i$, where the longitudinal quantum number k_i is a half-integer (integer) for fermions (bosons). Here, we neglect the zero mode $k_i = 0$ for the boson. In the transverse plane, the 2D HO wave function $\Phi_{n_i, m_i}(\vec{p}_{i\perp}, b)$ carries the radial and the angular quantum numbers denoted by n_i and m_i , respectively. $\vec{p}_{i\perp}$ is the transverse momentum of the i th particle. b is the HO basis scale parameter, which defines the HO energy $E_{n_i, m_i} = (2n_i + |m_i| + 1)b^2$. Each single-particle state $|\alpha_i\rangle$ is characterized by four quantum numbers, $|\alpha_i\rangle = |k_i, n_i, m_i, \lambda_i\rangle$, where λ_i is the light-front helicity. In this work, the first two Fock sectors of the meson both allow for the unique color-singlet state. Therefore, there is no requirement for the color label to identify the color property of the LFWFs.

The projection of total angular momentum M_J consisting of orbital angular momentum projection plus spin angular momentum projection is conserved in light-front field theory. Meanwhile, we respectively introduce truncations $N_{\max}$ and K in transverse and longitudinal directions to perform the numerical calculation. These quantities satisfy the following conditions:

$$\begin{cases} M_J = \sum_i (m_i + \lambda_i), \\ N_{\max} \geq \sum_i (2n_i + |m_i| + 1), \\ K = \sum_i k_i, \end{cases} \quad (4)$$

where the dimensionless variable K is used to parametrize the total longitudinal momentum P^+ . For the i th particle, the longitudinal momentum fraction is defined as $x_i = p_i^+ / P^+ = k_i / K$. The $N_{\max}$ truncation serves as the ultra-violet (UV) and infrared (IR) cutoffs. In momentum space, the UV cutoff $\Lambda_{\text{UV}} \simeq b\sqrt{N_{\max}}$ and the IR cutoff $\Lambda_{\text{IR}} \simeq b/\sqrt{N_{\max}}$ [70].

Through diagonalizing the Hamiltonian, we directly obtain the mass spectra M^2 and the corresponding eigenvectors $\psi_{M_J}^{\mathcal{N}}(\{\alpha_i\})$, which can be converted to the LFWFs in momentum representation:

$$\psi_{M_J, \{\lambda_i\}}^{\mathcal{N}}(\{x_i, \vec{p}_{i\perp}\}) = \sum_{\{n_i, m_i\}} \psi_{M_J}^{\mathcal{N}}(\{\alpha_i\}) \prod_{i=1}^{\mathcal{N}} \Phi_{n_i, m_i}(\vec{p}_{i\perp}, b), \quad (5)$$

where $\mathcal{N}$ is the total number of particles in the Fock sector. α_i represents the four quantum numbers k_i, n_i, m_i , and λ_i , characterizing the i th particle as mentioned above. Note that the meson LFWFs are derived from the light-front effective Hamiltonian at a fixed scale. The model scale of pion, $\mu_0^2 = 0.34 \pm 0.03 \text{ GeV}^2$, is determined by matching evolved results to the total first moments of valence quark and antiquark distributions from global QCD analysis [47].

With the truncation $\{N_{\max}, K\} = \{14, 15\}$, we determine the parameters summarized in Table I by fitting the masses of six representative meson states: the ground and first excited pseudoscalars $[\pi, \pi(1300)]$, a vector $[\rho]$, a scalar $[a_0(980)]$, an axial vector $[a_1(1260)]$, and a hybrid state $[\pi_1(1400)]$ [47]. These states are chosen to represent key features across various quantum number sectors and to effectively constrain the model parameters. In principle, increasing $N_{\max}$ and K could improve the utility of our model but requires substantially more computational resources. Empirically, we set $K = N_{\max} + 1$. We present the mass spectra of light mesons for three different basis truncations, $N_{\max} = \{6, 10, 14\}$, in Fig. 1. We notice that the mass eigenvalues exhibit reasonable convergence as the

TABLE I. The model parameters obtained by fitting the mass eigenvalues of unflavored light mesons with the truncation $\{N_{\max}, K\} = \{14, 15\}$ [47].

m_q [GeV]	m_g [GeV]	b [GeV]	κ [GeV]	m_f [GeV]	g_s
0.39	0.60	0.29	0.65	5.69	1.92

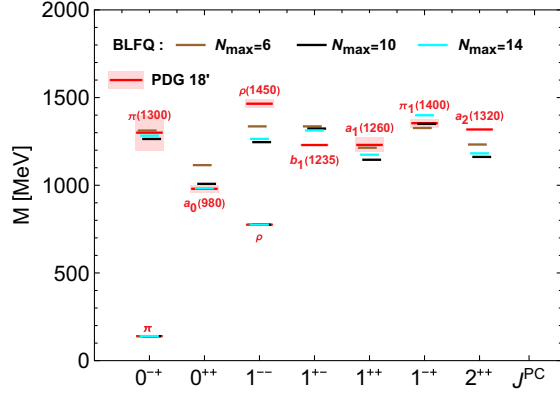


FIG. 1. The mass spectra of unflavored light mesons. Our results are compared with the experimental data taken from PDG (red solid bars) [74].

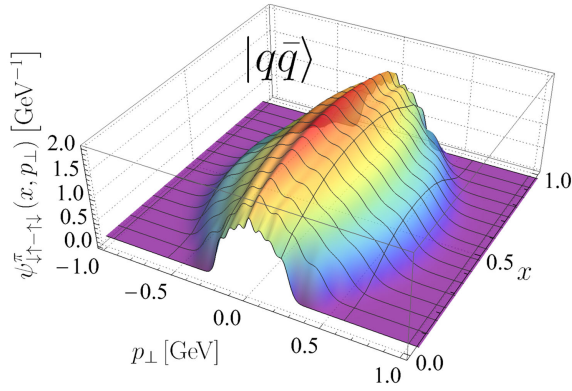


FIG. 2. Leading component of the pion LFWFs in the $|q\bar{q}\rangle$ state.

basis truncation increases. The leading component of the pion LFWFs in the $|q\bar{q}\rangle$ state in momentum space is shown in Fig. 2. We observe oscillations at small transverse momentum, which arise from the harmonic oscillator basis used in our BLFQ approach. Despite these oscillations, the wave functions exhibit general features consistent with various quark models, such as the constituent quark model [71] and holographic light-front QCD [72,73]. Additionally, unlike most phenomenological models that exclude gluonic degrees of freedom and focus solely on valence quarks, our BLFQ approach includes a dynamical gluon through the $|q\bar{q}g\rangle$ Fock state. The LFWFs obtained by our method have simultaneously described the parton distribution functions, decay constant, charge radii, and electromagnetic form factors for the light mesons, as well as the pion-nucleus-induced J/ψ production cross sections [47].

III. PION-PHOTON TRANSITION FORM FACTORS

In general, there are two types of pion-photon transition form factors, specifically, the single and double virtual

transition form factors extracted from the $\pi^0 \rightarrow \gamma\gamma^*$ and $\pi^0 \rightarrow \gamma^*\gamma^*$ processes, respectively [64]. In this work, we limit our research to the single virtual pion-photon transition form factor $F_{\pi\gamma}(Q^2)$. The TFF is defined by the $\pi\gamma\gamma^*$ vertex [15]:

$$\Gamma_\mu = -ie^2 F_{\pi\gamma}(Q^2) \epsilon_{\mu\nu\rho\sigma} p_\pi^\nu \epsilon^{*\rho} q^\sigma, \quad (6)$$

where q and p_π are the momenta of the virtual photon and the pion, respectively. ϵ is the polarization vector of the real photon. Following the prescription by Lepage and Brodsky [15], we consider

$$\begin{cases} \epsilon^* = (0, 0, \vec{\epsilon}_\perp^*), \vec{\epsilon}_\perp^* \cdot \vec{q}_\perp = 0, \\ p_\pi \equiv (p^+, p^-, \vec{p}_\perp) = (p^+, \frac{m_\pi^2}{p^+}, \vec{0}), \\ q = (0, \frac{\vec{q}_\perp^2 - m_\pi^2}{p^+}, \vec{q}_\perp) \Rightarrow q^2 = -\vec{q}_\perp^2 \equiv -Q^2. \end{cases} \quad (7)$$

In principle, the TFF includes the contribution from all Fock sectors. However, the contributions from higher Fock sectors are suppressed by powers of $1/Q$ compared to the valence Fock sector $|q\bar{q}\rangle$ [15], implying that higher Fock sectors may have a noticeable impact in regions where Q is small.

Based on the factorization theorem, the TFF can be factorized into the HSAs and the LFWFs [15,75]. Here, we evaluate the TFF from the convolution of the HSAs with the LFWFs considering our two chosen Fock sectors of the pion. The HSAs are derived by light-cone perturbation theory [15,65], and the LFWFs are obtained by diagonalizing the light-front Hamiltonian [47].

A. TFF in the valence Fock sector

The TFF in the valence Fock sector is expressed as [75]

$$Q^2 F_{\pi\gamma}^{q\bar{q}}(Q^2) = \frac{4}{\sqrt{3}} \int_0^1 dx \int_0^{\vec{x}Q} \frac{d^2 \vec{p}_\perp}{16\pi^3} T^{q\bar{q}}(x, Q^2) \psi_{\lambda_1 \lambda_2}^2(x, \vec{p}_\perp), \quad (8)$$

where $\psi_{\lambda_1 \lambda_2}^2$ is the LFWF of the valence Fock sector with λ_1 and λ_2 being the light-front helicities of the quark and antiquark, respectively. Since the pion is spinless, we omit the subscript M_J . The HSA up to NLO in the valence Fock sector, $T^{q\bar{q}}(x, Q^2)$, is given by [16–20]

$$\begin{aligned} T^{q\bar{q}}(x, Q^2) &= \frac{1}{1-x} + \frac{\alpha_s(\mu_R)}{4\pi} C_F \frac{1}{1-x} \\ &\times \left[-9 - \frac{1-x}{x} \ln(1-x) + \ln^2(1-x) \right. \\ &\left. + \{3 + 2 \ln(1-x)\} \ln\left(\frac{Q^2}{\mu_R^2}\right) \right], \end{aligned} \quad (9)$$

where $\alpha_s(\mu_R) = \frac{4\pi}{\beta_0 \ln(\mu_R^2/\Lambda_{\text{QCD}}^2)}$ is the strong running coupling constant with $\beta_0 = \frac{11}{3}C_A - \frac{2}{3}n_f$ and $\Lambda_{\text{QCD}} = 225$ MeV. Here, the color factors are given by $C_F = \frac{4}{3}$ and $C_A = 3$. The number of active flavors is $n_f = 3$. Although allowing n_f to vary with the scale Q could influence the results, the primary focus of our study is on the dynamics within the light flavor sector. For simplicity, we adopt the regularization scale as $\mu_R^2 = Q^2$ to eliminate the large logarithm term. The HSA follows the naive dimensional regularization scheme. The first term in Eq. (9) is the LO HSA. The term related to α_s is the NLO correction of the HSA in the valence Fock sector. In our calculation, we use the leading-twist hard function in the $\overline{\text{MS}}$ scheme to predict the contribution of the leading Fock state $|q\bar{q}\rangle$ to the TFF, with helicity conservation imposing $\lambda_1 = -\lambda_2$. This ensures that only the twist-2 component contributes, consistent with the leading-twist factorization framework. In our BLFQ framework for the π^0 , the twist-2 component dominates the $|q\bar{q}\rangle$ Fock sector, accounting for 98.7% of the probability. Therefore, we omit the higher-twist contributions from this Fock sector and consider only the case where $\lambda_1 = -\lambda_2$.

B. TFF in the $|q\bar{q}g\rangle$ Fock sector

The NLO correction to the TFF from the valence Fock sector is of the same order as the LO contribution from the valence-gluon Fock sector. The LO contribution from the valence-gluon Fock sector to the TFF can be expressed as

$$\begin{aligned} Q^2 F_{\pi\gamma}^{q\bar{q}g}(Q^2) &= \frac{8m_q \sqrt{2\pi\alpha_s}}{3Q^2} \sum_{i=1}^3 \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \\ &\times \int_0^{f_i(x_1, x_2, Q)} \frac{d^2 \vec{p}_{1\perp}}{16\pi^3} \int_0^{g_i(x_1, x_2, Q)} \frac{d^2 \vec{p}_{2\perp}}{16\pi^3} \\ &\times T_i^{q\bar{q}g}(x_1, x_2) \psi_{\lambda_1 \lambda_2 \lambda_3}^3(x_1, \vec{p}_{1\perp}, x_2, \vec{p}_{2\perp}) \\ &+ (x_1 \leftrightarrow x_2), \end{aligned} \quad (10)$$

where $\psi_{\lambda_1 \lambda_2 \lambda_3}^3$ is the LFWF of the valence-gluon Fock sector with λ_1 , λ_2 , and λ_3 being the light-front helicities of the quark, antiquark, and gluon, respectively. $T_i^{q\bar{q}g}$ denotes the HSA corresponding to the i th diagram among the first three diagrams in Fig. 3. Because of the charge conjugation invariance, the LFWFs of the pion and the HSAs exhibit symmetry under the exchange of the quark and antiquark. Consequently, evaluating the first term in Eq. (10) suffices, implying that the total TFF in the valence-gluon Fock sector is twice the magnitude of the first term. The explicit expressions of LO HSAs in the valence-gluon Fock sector read as

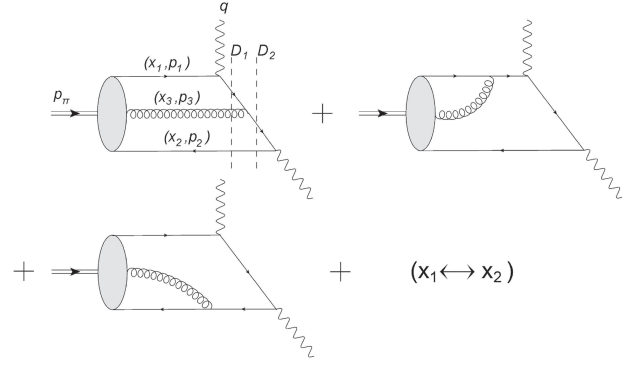


FIG. 3. The leading-order Feynman diagrams in the valence-gluon Fock sector related to $F_{\pi\gamma}(Q^2)$. $(x_1 \leftrightarrow x_2)$ represents the Feynman diagrams after exchanging the quark and antiquark.

$$T_1^{q\bar{q}g}(x_1, x_2) = \frac{x_1}{(1-x_1)(1-x_2)x_2^2} + \frac{1-x_1-x_2}{(1-x_1)(1-x_2)x_2}, \quad (11)$$

$$T_2^{q\bar{q}g}(x_1, x_2) = \frac{x_1 + x_2 - 1}{x_1 x_2 (1-x_2)}, \quad (12)$$

$$T_3^{q\bar{q}g}(x_1, x_2) = \frac{2(x_1 + x_2 - 1)}{x_1 (1-x_2)^2}. \quad (13)$$

The upper limits of the integral over transverse momenta in Eq. (10) are given by

$$f_1 = \min\left(\frac{1-x_1}{2}, \frac{1-x_1-x_2}{1-x_2}\right)Q, \quad (14)$$

$$g_1 = \min\left(x_2, \frac{1-x_1-x_2}{x_1}, \sqrt{\frac{x_2(1-x_1-x_2)}{x_1}}\right)Q, \quad (15)$$

$$f_2 = Q, \quad g_2 = \min\left(x_2, \sqrt{x_2(1-x_2)}\right)Q, \quad (16)$$

$$f_3 = \min\left(1-x_1, \sqrt{x_1(1-x_1)}\right)Q, \quad g_3 = Q. \quad (17)$$

The detailed derivation for the TFF with LO HSAs in $|q\bar{q}g\rangle$ Fock sector is provided in the Appendix.

IV. NUMERICAL RESULTS

For our numerical calculations, we set the truncation limits to $N_{\text{max}} = 14$ for the transverse direction and $K = 15$ for the longitudinal direction. With the model parameters summarized in Table I, we obtain the LFWFs of the pion in the first two Fock sectors with its mass $M = 0.139$ GeV by solving the light-front Hamiltonian equation. At the model scale, the probabilities of the pion being in the Fock states $|q\bar{q}\rangle$ and $|q\bar{q}g\rangle$ are 49.2% and 50.8%, respectively. Subsequently, we compute the pion to photon TFF by

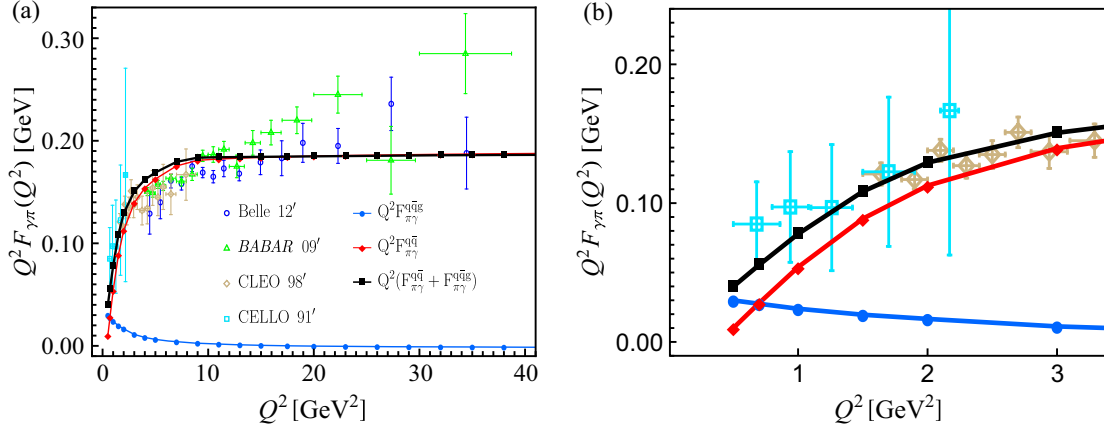


FIG. 4. The BLFQ results for the $\pi^0 \rightarrow \gamma^* \gamma$ transition form factor with truncation parameters $N_{\max} = 14$ and $K = 15$: (a) the TFF in full Q^2 region; (b) the TFF in low- Q^2 region. The blue and red curves correspond to the results with the LO HSA in the $|q\bar{q}g\rangle$ Fock sector and the NLO HSA in the $|q\bar{q}\rangle$ Fock sector, respectively. The black curve represents the sum of them. Data are taken from CELLO [42], CLEO [43], Belle [45], and BABAR [44] Collaborations.

convoluting the HSAs with the pion LFWFs, as described in Eqs. (8) and (10).

Figure 4 shows our results for the pion to photon TFF, $Q^2 F_{\pi\gamma}(Q^2)$, within the BLFQ framework and compares with the available experimental data from the CELLO [42], CLEO [43], Belle [45], and BABAR Collaborations [44]. The red and blue curves represent the contributions from the $|q\bar{q}\rangle$ and $|q\bar{q}g\rangle$ Fock sectors, respectively, with the black curve corresponding to the total TFF. We use χ^2 per degree of freedom to compare our results with the experimental data:

$$\chi^2/\text{d.o.f.} = \frac{1}{N-f} \sum_{i=1}^N \left(\frac{F_i^{\text{data}} - F_i^{\text{BLFQ}}}{F_i^{\text{err}}} \right)^2, \quad (18)$$

where N is the number of experimental data points, f is the number of free parameters in our model, i denotes the index of the experimental data point, F_i^{data} is the observed value of the experimental data point, F_i^{err} is the error of the experimental result, and F_i^{BLFQ} is the calculated value corresponding to the experimental data point in our model. The χ^2 per degree of freedom is 4.1 over all data included in Fig. 4(a). Excluding the BABAR data lowers this to 2.4, while additionally excluding the Belle data, leaving only the CLEO and CELLO data in Fig. 4(b), lowers it to 1.1. We acknowledge that the reduced χ^2 value of 2.4, even after removing the BABAR data, remains relatively large. This discrepancy, particularly at low Q^2 , suggests the need for further refinement in the treatment of higher-twist contributions and subleading Fock states. We also note the tension between the Belle and BABAR data, particularly in the low- Q^2 region, highlighting the necessity of additional experimental measurements to resolve these differences. In future work, we plan to investigate these effects more carefully by including additional higher Fock states and refining our approach to better describe the low- Q^2

regime. Overall, we find a good consistency between our results and the data reported by CELLO [42], CLEO [43], and Belle Collaborations [45]. However, our result deviates from the rapid growth in the large Q^2 region reported by the BABAR Collaboration [44]. Some theoretical studies suggest that the BABAR data are incompatible with QCD calculations [23,24,28,33,64,73,76–79]. Conversely, there are phenomenological studies that reproduce the BABAR data for the pion to photon TFF [80–89]. It has also been demonstrated in Ref. [81] that BABAR's high- Q^2 data cannot be explained using standard QCD frameworks like asymptotic QCD, AdS/QCD, or Chernyak-Zhitnitsky models for the pion DA but can be accommodated by assuming a flat pion DA [80,81]. However, such a flat DA underestimates the TFF at low Q^2 and trends toward violating the Brodsky-Lepage limit $Q^2 F_{\pi\gamma}(Q^2 \rightarrow \infty) = 2f_\pi$ [81]. In contrast, Belle's newer data show lower values at high Q^2 , consistent with QCD expectations. Our results agree with Belle, CLEO, and CELLO but not with BABAR, supporting the view that a reevaluation of conclusions drawn from BABAR data alone is warranted.

While our BLFQ-based calculation of the pion TFF relies on factorization and is not valid in the soft limit ($Q^2 \rightarrow 0$), we can still connect to this regime through the axial anomaly relation. Specifically, we compute the pion decay constant within the BLFQ framework and apply the well-known Schwinger Adler-Bell-Jackiw (SABJ) relation [38,64,72,90,91]

$$F_{\pi\gamma}^{\text{SABJ}}(0) = \frac{1}{2\sqrt{2}\pi^2 f_{\pi^0}} \quad (19)$$

to obtain the transition form factor at zero momentum transfer. Using our result for the decay constant, $f_{\pi^0} = 0.138$ GeV, we find $F_{\pi\gamma}^{\text{SABJ}}(0) = 0.260$ GeV $^{-1}$, which is consistent with expectations from the anomaly relation.

As Q^2 increases, the magnitude of $Q^2 F_{\pi\gamma}^{q\bar{q}}$ gradually increases and eventually saturates, while that of $Q^2 F_{\pi\gamma}^{q\bar{q}g}$ gradually decreases and eventually vanishes.

Although the $|q\bar{q}g\rangle$ Fock sector carries nearly half of the pion's probability in our BLFQ calculation, its contribution to the TFF is dynamically suppressed at high momentum transfer due to the structure of the perturbative expression, which includes an additional $1/Q^2$ factor compared to the leading $|q\bar{q}\rangle$ term. As a result, the gluon-containing Fock sector has a minimal impact on the TFF at large Q^2 , despite its sizable probability. Nevertheless, around $Q^2 \sim 1 \text{ GeV}^2$, the two sectors contribute comparably. With these observations, one sees an example of how Fock sector probabilities do not directly translate to observable dominance in all kinematic regimes.

The behavior of our TFF is consistent with the scaling predicted by pQCD [15,16]. Specifically, the TFF follows that $Q^2 F_{\pi\gamma}(Q^2 \rightarrow \infty) = 2f_\pi$, where f_π is the decay constant [75]. Additionally, contributions to the TFF from higher Fock sectors are suppressed by powers of $1/Q$. Note that the second Fock sector includes a dynamical gluon, introducing an additional propagator at the corresponding HSA. This additional propagator generates a factor of Q^2 in the denominator. Consequently, $F_{\pi\gamma}^{q\bar{q}g}$ exhibits a lower power of Q^2 compared to $F_{\pi\gamma}^{q\bar{q}}$, as indicated by the comparison between Eqs. (8) and (10). The contribution from the $|q\bar{q}g\rangle$ Fock sector decreases more rapidly and has negligible influence in the large- Q^2 region. However, the contribution from the $|q\bar{q}g\rangle$ Fock component has a visible impact in the small- Q^2 region, which improves the theoretical description of experimental data for $Q^2 < 5 \text{ GeV}^2$ as seen in Fig. 4(b). In our current framework, the TFF at low Q^2 is described using the first two Fock states of the pion, as determined from the BLFQ approach. While these states provide a good description of the CELLO data, we acknowledge that higher-twist contributions, significant at small Q^2 , are not fully captured in the present truncation. These effects are incorporated nonperturbatively through the numerical solutions of the light-front Hamiltonian but require the inclusion of additional higher Fock states for a more complete representation. Future work will focus on extending the Fock space and refining the treatment of low- Q^2 dynamics to improve the model's predictive power in this regime.

We estimate the uncertainties in $Q^2 F_{\pi\gamma}^{q\bar{q}}$ by varying the factorization scale μ within the range $\frac{Q^2}{2} < \mu^2 < 2Q^2$. The results, shown in Fig. 5, indicate that the TFF uncertainty from pQCD is minimal.

In this work, we quantify the uncertainties by comparing the results at different quark masses without changing the other parameters. More specifically, we shift the model quark mass from 0.39 to 0.37 GeV and to 0.41 GeV and then solve the light-front stationary Schrödinger equation to obtain the two Fock-sector LFWFs of the pion. Finally, we

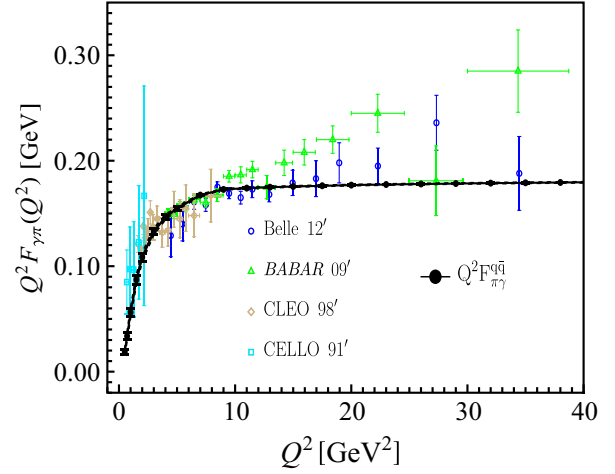


FIG. 5. Uncertainty estimation for the TFF, $Q^2 F_{\pi\gamma}^{q\bar{q}}$. The uncertainties are quantified by varying the factorization scale μ within $\frac{Q^2}{2} < \mu^2 < 2Q^2$.

evaluate the TFF at different quark masses to quantify the uncertainties. We present the result, including the uncertainties, for $Q^2(F_{\pi\gamma}^{q\bar{q}} + F_{\pi\gamma}^{q\bar{q}g})$ in Fig. 6(a). The largest relative uncertainty is 4% at $Q^2 = 0.5 \text{ GeV}^2$, which demonstrates the stability of the model results with respect to slight changes in the input parameters. We also quantify the uncertainties by comparing the results obtained with different truncations, $N_{\text{max}} = \{12, 14, 16\}$, as shown in Fig. 6(b). We find that the maximum relative uncertainty is approximately 9% at $Q^2 = 15 \text{ GeV}^2$.

Note that the axial-vector current and partially conserved axial-vector current (PCAC) [92] do not explicitly enter our formalism. However, our approach reproduces the correct low- Q^2 behavior of the TFF, consistent with the axial anomaly prediction. While we do not evaluate the triangle anomaly diagram directly, the underlying physics of the anomaly is effectively captured in our model. A more explicit treatment of PCAC within the BLFQ framework is an interesting direction for future study.

V. SUMMARY

We evaluated the pion to photon TFF with one on-shell and one off-shell photon using the hard-scattering formalism within the BLFQ framework. The TFF was obtained by convoluting the light-front wave functions of the pion in its first two Fock sectors, $|q\bar{q}\rangle$ and $|q\bar{q}g\rangle$, with the HSAs derived using light-cone perturbation theory. The pion's wave functions were determined from the eigenvectors of the light-front QCD Hamiltonian relevant to constituent quark-antiquark and quark-antiquark-gluon Fock components of the mesons in the light-cone gauge along with a three-dimensional confinement in the leading Fock sector. Notably, our results are purely predictive, with no parameter adjustments made when calculating the TFF. We found that the contribution from the valence Fock sector

dominates in the large- Q^2 region, consistent with the scaling behavior predicted by perturbative QCD. The contribution from the $|q\bar{q}g\rangle$ Fock sector has a modest impact in the low- Q^2 region, enhancing the description of the experimental data compared to considering only the leading Fock sector. Overall, our results are in good agreement with the experimental data, particularly from the Belle, CLEO, and CELLO Collaborations. Combining this success in the TFF with achievements in light meson spectroscopy, decay constants, electromagnetic form factors, and parton distribution functions, the BLFQ framework provides a natural approach to describe the relativistic structure of the pion.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX: TFF WITH LO HSAS IN $|q\bar{q}g\rangle$ FOCK SECTOR

Here, we provide a detailed derivation of the HSAs in the quark-antiquark-gluon Fock sector, as given in Eqs. (11)–(13), and explain the reasons for the upper limit of the integral specified in Eqs. (14)–(17). In our BLFQ framework for the pion, the S wave dominates the LFWFs of the $|q\bar{q}g\rangle$ Fock sector, accounting for 98.4% of the probability in this sector. Consequently, we disregard the contributions of the P and D waves from this Fock sector. For the S wave in this Fock sector, the convolution of the LFWFs with the HSAs is invariant under helicity flip. Considering this symmetry, we need to derive the contribution from only one type of S-wave helicity state.

Taking the first diagram in Fig. 3 as an example, we set the helicity of the quark, antiquark, and gluon as λ_1 , λ_2 , and λ_3 , respectively. Based on the light-cone perturbation theory, the corresponding convolution form of the TFF in the $|q\bar{q}g\rangle$ Fock sector with $\{\lambda_1, \lambda_2, \lambda_3\} = \{\uparrow, \uparrow, \downarrow\}$ can be expressed as

$$F_{\pi\gamma,1st}^{q\bar{q}g} = \frac{\sqrt{4\pi\alpha_s(n_c^2 - 1)}(e_u^2 - e_d^2)}{i\sqrt{2}(\vec{\epsilon}_\perp^* \times \vec{q}_\perp)} \int_0^1 [dx] \int_0^\infty [dp_\perp] \frac{1}{D_1} \frac{1}{D_2} \sum_{\lambda\lambda'} \left[\frac{\bar{v}_{\lambda_2}(x_2, \vec{p}_{2\perp}) \gamma^\rho \epsilon_\rho^* u_{\lambda_1}(x_1 + x_3, \vec{p}_{1\perp} + \vec{p}_{3\perp} + \vec{q}_\perp)}{\sqrt{x_2}\sqrt{x_1 + x_3}} \right. \\ \left. \times \frac{\bar{u}_{\lambda_3}(x_1 + x_3, \vec{p}_{1\perp} + \vec{p}_{3\perp} + \vec{q}_\perp) \gamma^\nu \epsilon_\nu(\lambda_3) u_{\lambda'}(x_1, \vec{p}_{1\perp} + \vec{q}_\perp)}{\sqrt{x_1}\sqrt{x_1 + x_3}} \frac{\bar{u}_{\lambda'}(x_1, \vec{p}_{1\perp} + \vec{q}_\perp) \gamma^+ u_{\lambda_1}(x_1, \vec{p}_{1\perp})}{\sqrt{x_1}\sqrt{x_1}} \right] \psi_{\lambda_1\lambda_2\lambda_3}^3, \quad (A1)$$

where $[dx] \equiv dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3)$ and $[dp_\perp] \equiv \frac{d^2\vec{p}_{1\perp}}{16\pi^3} \frac{d^2\vec{p}_{2\perp}}{16\pi^3}$. ϵ^* and ϵ are the polarization vectors of the final (on-shell) photon and the gluon, respectively. e_u and e_d are the quark charges in units of e , α_s is the strong running coupling constant, and $n_c = 3$ is the number of colors. $\psi_{\lambda_1\lambda_2\lambda_3}^3$ represents the LFWF corresponding to the helicity

state we chose above. Here, we omit the arguments $(x_1, \vec{p}_{1\perp}, x_2, \vec{p}_{2\perp})$ from the LFWF, where the longitudinal momentum fraction and the transverse momentum of the gluon are $x_3 = 1 - x_1 - x_2$ and $\vec{p}_{3\perp} = -\vec{p}_{1\perp} - \vec{p}_{2\perp}$, respectively. D_1 and D_2 are the energy denominators. For the first diagram, they are written as

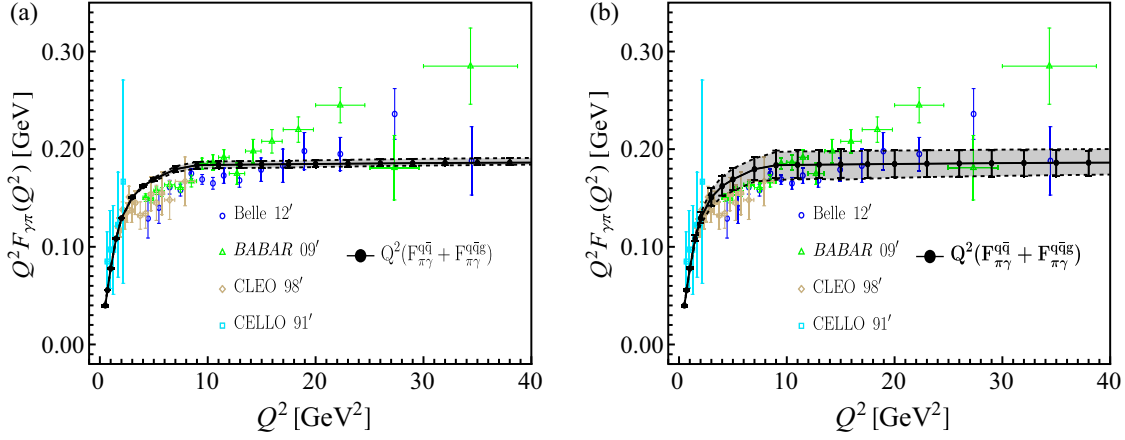


FIG. 6. Uncertainty estimation for the TFF, $Q^2(F_{\pi\gamma}^{q\bar{q}} + F_{\pi\gamma}^{q\bar{q}g})$: (a) the uncertainties are quantified by comparing the results at different model quark masses with truncation parameters $N_{\max} = 14$ and $K = 15$; (b) the uncertainties are quantified by comparing the results at different truncations $N_{\max} = \{12, 14, 16\}$, which the lower and upper lines correspond to $N_{\max} = 12$ and $N_{\max} = 16$, respectively.

$$D_1 = \vec{q}_\perp^2 - \frac{(\vec{p}_{1\perp} + \vec{q}_\perp)^2 + m_q^2}{x_1} - \frac{\vec{p}_{2\perp}^2 + m_q^2}{x_2} - \frac{\vec{p}_{3\perp}^2}{x_3}, \quad D_2 = \vec{q}_\perp^2 - \frac{(\vec{p}_{1\perp} + \vec{p}_{3\perp} + \vec{q}_\perp)^2 + m_q^2}{x_1 + x_3} - \frac{\vec{p}_{2\perp}^2 + m_q^2}{x_2}. \quad (\text{A2})$$

Substituting in the expression of the spinors and the polarization vector of the gluon and neglecting the quark mass m_q relative to Q^2 , the TFF corresponding to the first diagram becomes

$$F_{\pi\gamma,1st}^{q\bar{q}g} = \frac{8m_q\sqrt{2\pi\alpha_s}}{3\vec{e}_\perp^* \times \vec{q}_\perp} \int_0^1 [dx] \int_0^\infty [dp_\perp] \frac{1}{\vec{q}_\perp^2 - \frac{(\vec{p}_{1\perp} + \vec{q}_\perp)^2}{x_1} - \frac{\vec{p}_{2\perp}^2}{x_2} - \frac{\vec{p}_{3\perp}^2}{x_3}} \frac{1}{\vec{q}_\perp^2 - \frac{(\vec{p}_{1\perp} + \vec{p}_{3\perp} + \vec{q}_\perp)^2}{x_1 + x_3} - \frac{\vec{p}_{2\perp}^2}{x_2}} \times \left[\frac{\vec{e}_\perp^* \times ((1-x_1-x_2)\vec{q}_\perp + (1-x_2)\vec{p}_{1\perp} + x_1\vec{p}_{2\perp})}{x_2(1-x_2)^2(x_1+x_2-1)} + \frac{\vec{e}_\perp^* \times (\vec{p}_{2\perp} - x_2\vec{q}_\perp)x_3}{x_1x_2(x_1+x_3)^2} \right] \psi_{\lambda_1=\uparrow, \lambda_2=\uparrow, \lambda_3=\downarrow}^3. \quad (\text{A3})$$

The S-wave component is concentrated in the low transverse momentum region, because a bound state has minimal amplitude when its components move with large transverse momentum. We follow the technique prescribed in Ref. [15] to simplify the energy denominators and the expressions in the square brackets in Eq. (A3). We take the small transverse momentum limit, setting $|\vec{p}_{1\perp}| \leq f_1 = \min(\frac{1-x_1}{2}, \frac{1-x_1-x_2}{1-x_2})Q$ and

$|\vec{p}_{2\perp}| \leq g_1 = \min(x_2, \frac{1-x_1-x_2}{x_1}, \sqrt{\frac{x_2(1-x_1-x_2)}{x_1}})Q$. Consequently, the energy denominators are simplified to $D_1 = D_2 = \vec{q}_\perp^2$, and only the terms related to $\vec{q}_\perp$ in the square brackets survive. Ultimately, by substituting $\vec{q}_\perp^2 = Q^2$, the expression of the TFF is reduced to Eq. (10) with the HSA $T_1^{q\bar{q}g}$ in Eq. (11). The HSAs corresponding to the other diagrams given in Eqs. (12) and (13) can similarly be derived using this approach.

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