

Examining scalar portal inelastic dark matter with lepton fixed-target experiments

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Inelastic dark matter scenarios have attracted considerable attention in contemporary particle physics. In this study, we investigate the phenomenology of sub-GeV inelastic dark matter interacting via a lepton-specific scalar portal. By solving the Boltzmann equations, we obtain thermal target curves for several inelastic DM mass splittings in the sub-GeV mediator-mass range. We study the discovery potential of lepton fixed-target experiments, particularly NA64e, Light Dark Matter eXperiment (LDMX), and NA64 μ , via their missing-energy signatures. Our analysis focuses on the ϕ -strahlung process, $lN \rightarrow lN\phi$, followed by the invisible decay of the scalar mediator into Majorana dark matter particles $\phi \rightarrow \chi_1\chi_2$. We use this channel to probe the mediator coupling to charged leptons of the Standard Model. For phenomenologically viable parameters of the inelastic dark matter scenario, we derive projected sensitivities for NA64e, LDMX, and NA64 μ , assuming that the boosted state χ_2 decays visibly via $\chi_2 \rightarrow \chi_1 e^+ e^-$ outside the detector acceptance. Our results demonstrate the complementary roles of electron and muon beam experiments in exploring the sub-GeV inelastic dark matter sector interacting via a scalar portal.

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I. INTRODUCTION

In recent decades, astrophysical observations have provided strong evidence for the existence of dark matter (DM) [1,2]. Empirical support for DM arises from a variety of astrophysical phenomena, including galactic rotation curves, anisotropies in the cosmic microwave background, and gravitational lensing observations [3–5]. Current cosmological measurements indicate that DM constitutes approximately 85% of the total matter content of the Universe, a component not explained within the framework of the Standard Model (SM) of particle physics [6,7].

One can assume that thermalization between DM and SM sectors occurs via different portal interactions. Theoretical frameworks typically consider bosonic mediators with different spins: spin-0 (e.g., light hidden Higgs bosons) [8–19], spin-1 (e.g., sub-GeV dark photons) [20–31], and spin-2 (e.g., massive dark gravitons) [32–44].

The search for DM particles at accelerator-based facilities is one of the major efforts of contemporary high-energy physics, aimed at probing physics beyond

the SM [18,45–50]. The portal-mediated light dark matter scenarios predict distinctive experimental signatures characterized by events with missing energy. These signatures originate from bremsstrahlung-like processes in which a charged lepton interacts with the nucleus of a target, producing a mediator that subsequently decays into a pair of DM particles. Fixed-target experiments are particularly effective for probing sub-GeV DM due to their combination of high-energy lepton beams and substantial beam intensities. Current experimental efforts include operations at CERN SPS fixed-target facilities, such as NA64e [51–63] and NA64 μ [64–66]. In our analysis, we also discuss the planned Light Dark Matter eXperiment (LDMX) electron fixed-target experiment at SLAC [67–70].

One of the extended dark sector models is the concept of inelastic dark matter [71–73]. The inelastic nature of the scenario implies a mass splitting between DM states, which gives rise to novel experimental signatures. Originally proposed to explain the anomalous signals observed by DAMA Collaboration [74], inelastic DM scenarios have emerged as a theoretically well-motivated framework for sub-GeV thermal DM interacting through a vector mediator [47,50,75–77]. The key phenomenological feature of these models involves off-diagonal couplings between the DM ground state, m_{χ_1} , and an excited state, m_{χ_2} , with $m_{\chi_2} \gtrsim m_{\chi_1}$. This structure kinematically suppresses the DM-nucleon scattering cross sections for current direct detection experiments.

This work investigates the projected sensitivities of the NA64e, LDMX, and NA64 μ experiments, where we use

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the simplified scenario of the inelastic fermionic dark matter model with a scalar mediator predominantly coupled to the charged leptons of the SM. We focus on a scalar mediator with the mass from $\mathcal{O}(1)$ MeV to $\mathcal{O}(1)$ GeV and the mass splitting of inelastic dark matter, $\Delta_2 \equiv (m_{\chi_2} - m_{\chi_1})/m_{\chi_1}$, in the range from 0.1 to 0.5.

This paper is organized as follows. In Sec. II we discuss a simplified benchmark model for inelastic DM mediated by the scalar portal. In Sec. III we estimate the inelastic DM relic abundance by solving the Boltzmann equations. In Sec. IV we discuss the missing-energy signatures at NA64e, LDMX, and NA64 μ experiments, arising from the process $lN \rightarrow lN\phi(\rightarrow \chi_1\chi_2)$ followed by $\chi_2 \rightarrow \chi_1 e^+ e^-$ decays. In Sec. V we discuss the thermal target curves for inelastic DM and regarding sensitivity at NA64e, LDMX, and NA64 μ fixed-target facilities. Finally, conclusions are drawn in Sec. VI. In Appendices A, B, C, and D we provide some useful formulas and discuss direct detection constraints.

II. SIMPLIFIED BENCHMARK SCENARIOS

In this section, we discuss simplified benchmark scenarios for inelastic dark matter with a focus on the lepton-specific scalar mediator interacting with Majorana dark matter.

The simplified Lagrangian describing a leptonphilic spin-0 mediator reads as follows [67,78]:

$$\mathcal{L}_{\text{eff}}^\phi \supset \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m_\phi^2 \phi^2 - \sum_{l=e,\mu,\tau} c_l^\phi \bar{l}l\phi, \quad (1)$$

and the last term in Eq. (1) can be understood as originating from the effective gauge-invariant dimension-5 operators [79]

$$\mathcal{L} \supset \sum_{i=e,\mu,\tau} \frac{c_i}{\Lambda} \phi \bar{L}_i H E_i. \quad (2)$$

In this framework, Λ represents the characteristic scale of new physics, while c_i denotes the Wilson coefficient corresponding to the flavor i . We adopt the assumption that these couplings remain diagonal in the mass basis. Regarding the relative magnitudes of the Wilson coefficients c_i , a theoretically motivated ansatz suggests proportionality to the respective Yukawa couplings y_i . Consequently, following electroweak symmetry breaking, the effective couplings c_{ll}^ϕ scale with the corresponding lepton masses. This leads us to establish the flavor-dependent ratio

$$c_{ee}^\phi : c_{\mu\mu}^\phi : c_{\tau\tau}^\phi = m_e : m_\mu : m_\tau, \quad (3)$$

which we implement in Eq. (1) and maintain consistently throughout our analysis.

The dark matter sector consists of two Majorana fermion states χ_1 and χ_2 , described by the Lagrangian

$$\mathcal{L}_{\text{kin,term.}}^{\text{DM}} \supset \sum_{i=1,2} \left[\frac{1}{2} \bar{\chi}_i i \gamma^\mu \partial_\mu \chi_i - \frac{1}{2} m_{\chi_i} \bar{\chi}_i \chi_i \right], \quad (4)$$

where m_{χ_i} denotes the physical fermion masses, and we take $m_{\chi_1} \lesssim m_{\chi_2}$ such that χ_1 is the lightest stable DM candidate.

We consider two effective benchmark Lagrangians involving a spin-0 mediator that couples to a pair of Majorana fermions via either a CP -odd or a CP -even coupling, such that the typical interactions are given by [80]

$$\begin{aligned} \text{Bench 1: } \mathcal{L}_{\text{eff}}^{(+)\text{DM}} &\supset \text{Re}(\lambda_{\chi_1\chi_2}^\phi) \bar{\chi}_1 \chi_2 \phi, \\ \text{Im}(\lambda_{\chi_1\chi_2}^\phi) &= 0, \end{aligned} \quad (5)$$

$$\begin{aligned} \text{Bench 2: } \mathcal{L}_{\text{eff}}^{(-)\text{DM}} &\supset -i \text{Im}(\lambda_{\chi_1\chi_2}^\phi) \bar{\chi}_1 \gamma_5 \chi_2 \phi, \\ \text{Re}(\lambda_{\chi_1\chi_2}^\phi) &= 0. \end{aligned} \quad (6)$$

In our simplified scenario, we adopt the conservative assumption that diagonal couplings between the mediator and Majorana DM are suppressed, such that only off-diagonal terms contribute to the effective interaction [81].

III. RELIC ABUNDANCE OF INELASTIC DARK MATTER

For the freeze-out mechanism considered in the present paper, we employ the standard Boltzmann equation technique to compute thermal target curves [67,78,82–85], assuming a kinetic and chemical equilibrium between DM and the SM thermal bath at the early stages of the Universe expansion. The subsequent departure of DM from thermal equilibrium through the depletion processes provides a relic abundance that can account for the observed DM density in the Universe.

Note that DM thermalization through the so-called secluded t -channel reaction $\chi_i \chi_i \rightarrow \phi\phi$ (where $i = 1, 2$) can provide a viable freeze-out mechanism [83] for the DM relic abundance as long as $m_{\chi_i} \gtrsim m_\phi$, for sufficiently large DM coupling to the mediator. However, a drawback of this framework is that the predicted relic density is largely insensitive to the coupling strength between the ϕ field and SM states, c_{ll}^ϕ . This independence presents a challenge for experimental searches, such as those conducted at direct detection facilities or accelerator-based experiments.

In contrast to the well-established freeze-out scenario, the freeze-in mechanism [86] provides an alternative framework to generating the observed relic abundance of dark matter. The assumption of freeze-in is that DM was never in thermal equilibrium with the SM plasma in the early Universe. Instead, the initial abundance of DM is

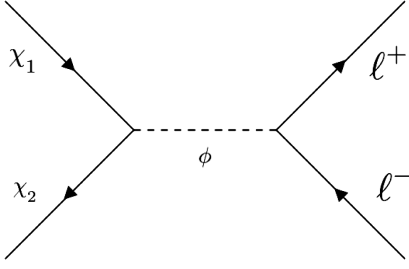


FIG. 1. Feynman diagrams giving rise to $\chi_1\chi_2 \rightarrow \phi \rightarrow l^+l^-$ annihilation in the early Universe.

assumed to be negligible. The observed DM density is then accumulated gradually through the slow, continuous production of DM from interactions with particles in the hot thermal bath [87,88].

For $2m_l \gtrsim m_{\chi_1} + m_{\chi_2}$ and $m_\phi \gtrsim m_{\chi_1} + m_{\chi_2}$, the channels contributing to freeze-in are the annihilation of SM fermions, $l^+l^- \rightarrow \chi_1\chi_2$, or ϕ decays, $\phi \rightarrow \chi_1\chi_2$. The latter channel dominates for sufficiently large DM coupling to the mediator [86]. The drawback of the aforementioned freeze-in scenario via the fast decay $\phi \rightarrow \chi_1\chi_2$ is analogous to that of freeze-out via the secluded channel $\chi_i\chi_i \rightarrow \phi\phi$. Therefore, these scenarios are beyond the scope of the present paper.

Instead, we focus on the freeze-out mechanism via the coannihilation channel $\chi_1\chi_2 \rightarrow l^+l^-$ (see Fig. 1). Specifically, for the parameter space of interest $m_\phi \gtrsim m_{\chi_1} + m_{\chi_2}$, the on-shell inverse decay rate of the process $\chi_1\chi_2 \rightarrow \phi$ is sharply suppressed relative to the s -channel process $\chi_1\chi_2 \rightarrow l^+l^-$ [89], due to the thermal tail of DM velocity distribution. Thus, we do not include the inverse decay into the Boltzmann equation. As a result, the dark matter coannihilation $\chi_1\chi_2 \rightarrow l^+l^-$ becomes the dominant contribution to the DM depletion during the freeze-out mechanism [90].

The current value of the cold DM relic abundance obtained from the Planck 2018 combined analysis is [7,91]

$$\Omega_c h^2 = 0.1200 \pm 0.0012.$$

As a result, the relic density is estimated to be [92]

$$\Omega_c h^2 \propto \left(\int_{x_f}^{\infty} \frac{\langle \sigma_{\text{eff}} v \rangle}{x^2} dx \right)^{-1}, \quad (7)$$

where $\langle \sigma_{\text{eff}} v \rangle$ is a thermally averaged coannihilation cross section and $x = m_{\chi_1}/T$ is a typical ratio of DM mass and temperature. We provide the details for the calculation of (7) in Appendices A, B, and C.

IV. MISSING-ENERGY SIGNATURES

This section presents the experimental configurations of fixed-target facilities capable of probing the invisible decay

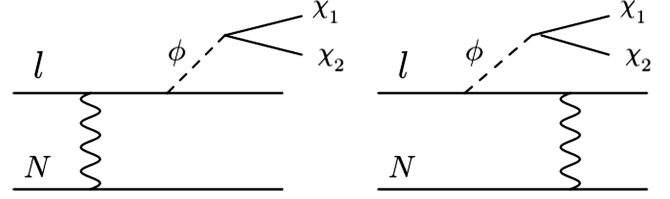


FIG. 2. Feynman diagrams for radiative scalar mediator production $lN \rightarrow lN\phi$ followed by invisible decay $\phi \rightarrow \chi_1\chi_2$.

channel $\phi \rightarrow \chi_1\chi_2$ (see Fig. 2). We discuss two complementary experiments currently operating at the CERN SPS: NA64e [51–63] utilizes an electron beam to investigate the process $eN \rightarrow eN\phi (\rightarrow \chi_1\chi_2)$, and NA64 μ [64–66,93–95] employs a muon beam to study $\mu N \rightarrow \mu N\phi (\rightarrow \chi_1\chi_2)$. For completeness, we also discuss the planned electron fixed-target experiment LDMX at SLAC [67,68,70]. The differential cross section $d\sigma_{2\rightarrow 3}/dx$ of the relevant process is calculated in Ref. [96], where $x = E_\phi/E_l$ is a missing-energy fraction, E_l is the lepton beam energy, E_ϕ is the energy of radiated scalar mediators, and $l = (e, \mu)$ represents the incident lepton (electron or muon).

In an electron fixed-target experiment, the initial energy of the incident beam, E_e , decreases as it traverses the target material. The distribution of beam energies in the electromagnetic shower E_s after propagation through a target thickness t is [97]

$$I_e(E_e, E_s, t) = \frac{1}{E_e} \frac{[\ln(E_e/E_s)]^{\frac{4}{3}t-1}}{\Gamma(\frac{4}{3}t)}, \quad (8)$$

where t is expressed in units of radiation length and E_e denotes the initial beam energy at $t = 0$.

We estimate the number of scalar mediators produced via bremsstrahlung at fixed-target facilities using $x \equiv E_\phi/E_e$ and $z \equiv E_\phi/E_s$ as follows [98,99]:

$$N_\phi^{\text{brem}} = \text{EOT} \times \frac{\rho N_A X_0}{A} \times \int_{x_{\text{cut}}}^{x_{\text{max}}} dx \int_x^1 \frac{E_e}{z} \frac{d\sigma}{dz} dz \int_0^T I_e(E_e, xE_e/z, t) dt, \quad (9)$$

where T is the thickness of the target in units of the radiation length X_0 (cm), x_{cut} and x_{max} are the minimum and maximum fractions of missing energy, respectively, ρ is the density of the target material, N_A is Avogadro's number, A is the atomic mass number, Z is the atomic number, and EOT denotes electrons accumulated on target. Explicit expressions for the differential cross sections and the corresponding discussion are given in Refs. [96,100,101].

In the approximation that mediator radiation occurs effectively in a thin layer of the target, i.e., for $T \ll 1$, the electron energy distribution $I_e(E_e, E_s, t)$ is [102]

TABLE I. The typical parameters governing the total production cross section for scalar mediators via the process $l^\pm N \rightarrow l^\pm N + \phi$ in lepton missing-energy experiments are presented. The parameter $E_\phi^{\text{cut}} = x_{\text{cut}} E_l$ represents the characteristic minimum missing-energy threshold, which is determined by the specific experimental configuration. The planned statistics of 1.5×10^{12} EOT for NA64e and 10^{11} MOT for NA64 μ were chosen in the analysis in order to suppress the expected background, $N_{\text{bckg}} \lesssim 1$. For the LDMX we chose 10^{15} EOT. We assume that upgrades of the experimental electronics and detectors will be implemented to further suppress the background by the final phase of running, achieving $N_{\text{bckg}} \ll 1$.

	NA64e	NA64 μ	LDMX
target material	Pb	Pb	Al
Z, atomic number	82	82	13
A, g · mole ⁻¹	207	207	27
L_T , cm	22.5	22.5	3.56
$x_{\text{cut}} = E_\phi^{\text{cut}}/E_l$	0.5	0.5	0.7
$l^\pm$, primary beam	electron	muon	electron
E_l , GeV, beam energy	100	160	16
Expected statistics:	EOT = 1.5×10^{12}	MOT = 10^{11}	EOT = 10^{15}
Expected background:	$N_{\text{bckg}} \lesssim 0.75$ [103]	$N_{\text{bckg}} \lesssim 0.35$ [95]	$N_{\text{bckg}} \ll 1$ [67]

$$\lim_{t \rightarrow 0} I_e(E_e, E_s, t) \rightarrow \frac{x}{E_e} \delta(z - x). \quad (10)$$

Thus, neglecting electron energy loss, the number of mediators produced via bremsstrahlung is

$$N_\phi^{\text{brem}} \simeq \text{EOT} \times \frac{\rho N_A}{A} L_T \int_{x_{\text{cut}}}^{x_{\text{max}}} dx \frac{d\sigma_{2 \rightarrow 3}(E_e)}{dx}, \quad (11)$$

where $L_T = TX_0$ is the effective thickness in radiation lengths. In our calculations, we employ Eq. (9) for NA64e since its target is sufficiently thick, $T \gg 1$, and Eq. (11) for LDMX due to the thin target employed in the design of its experimental setup, $T \ll 1$. For NA64 μ we also use the thin target approach, Eq. (11), implying the label replacement $E_e \rightarrow E_\mu$ and $\text{EOT} \rightarrow \text{MOT}$, where MOT denotes the typical number of muons accumulated on target. Specifically,

$$N_\phi^{\text{brem}} \simeq \text{MOT} \times \frac{\rho N_A}{A} L_T \int_{x_{\text{cut}}}^{x_{\text{max}}} dx \frac{d\sigma_{2 \rightarrow 3}(E_\mu)}{dx}. \quad (12)$$

In Table I the parameters of the considered experiments are shown.

A. NA64e

The scalar mediator ϕ can be produced through the scattering of ultrarelativistic electrons with energies $E_e \simeq 100$ GeV on the nuclei of a target via the process $eN \rightarrow eN\phi$, followed by the prompt decay $\phi \rightarrow \chi_1 \chi_2$. A fraction of the initial electron energy, $E_{\text{miss}} = xE_e$, may be carried away by the DM pair $\chi_1 \chi_2$, which traverses the NA64e detector without significant energy deposition in calorimeter modules. The remaining energy fraction, $E_e^{\text{rec}} \simeq (1 - x)E_e$, can be measured in the electromagnetic calorimeter (ECAL) of the NA64e experiment through the

detection of recoil electrons. Consequently, the production of the hidden scalar ϕ would manifest as an anomalous accumulation of events featuring a single electromagnetic shower with energy E_e^{rec} in the signal box [52]. In particular, candidate events are selected by requiring $E_e^{\text{rec}} \lesssim 0.5E_e \simeq 50$ GeV, corresponding to an energy fraction threshold of $x_{\text{cut}} \simeq 0.5$ in Eq. (11) for the NA64e experiment.

Furthermore, it should be noted that the ECAL of the NA64e experiment serves as the active target for the incoming electron beam. The ECAL is composed of 6×6 Shashlyk-type modules, each constructed from alternating layers of plastic scintillator (Sc) and lead (Pb) plates. The typical parameters of NA64e are summarized in Table I. The NA64e aims to collect approximately 1.5×10^{12} EOT at the H4 beam line after Long Shutdown 3.

To account for the upstream acceptance effects of the NA64e detector, we rely on the analysis in Ref. [103]. The ECAL is positioned at a 20 mrad angle to the beam line, and the beam electrons are deflected by the upstream magnet to hit it. This beam line deflection was implemented to improve the high-energy electron selection and suppress background from the possible admixture of low-energy electrons, as was suggested in Ref. [104]. Specifically, the NA64e employs a tagging system utilizing the synchrotron radiation from high-energy electrons [105] in a dipole magnet installed upstream of the detector, as illustrated schematically in Fig. 1 of Ref. [103]. Thus, we assume in our analysis that an electron hitting the NA64e active target is properly tagged upstream and passes all event selection criteria [103].

We neglect the effect of signal acceptance related to the electron recoil angle in the active ECAL target, as the typical recoil angle of the outgoing electron in the reaction $eN \rightarrow eN\phi$ is estimated to be sufficiently small, $m_\phi/E_e \simeq \mathcal{O}(10^{-3})$ (see, e.g., Ref. [66] and references therein). As a

result, the dominant energy of signal recoil electrons will be deposited within the sufficiently thick and wide ECAL target, producing a single electromagnetic shower with energy in the signal region, as mentioned above. Detector efficiencies are incorporated into the effective number of electrons accumulated on the target.

The principal background processes in NA64e are [103] (i) losses or decays of dimuons in the target, (ii) decays in-flight along the beam line, (iii) insufficient calorimeter coverage, and (iv) particles flying through the calorimeters.

We rely on the background analysis of Ref. [103], which yields a conservative estimate of $N_{\text{bckg}} \lesssim 0.75$ for the anticipated statistics of $\text{EOT} \simeq 1.5 \times 10^{12}$. However, we expect that upgraded detector electronics will improve background event rejection by a factor of $\mathcal{O}(1)$, which would lead to a sufficiently suppressed number, $N_{\text{bckg}} \ll 1$, of such events.

B. The LDMX

The LDMX is a planned fixed-target facility at SLAC designed to search for new dark sector particles using the missing-momentum technique [67–70]. This experiment employs a high-intensity electron beam and a precise measurement of the missing momentum for each electron, providing sensitivity that is complementary to the missing-energy approach of NA64 [58,103]. The LDMX setup is designed to apply stringent requirements on the missing energy and to exploit a comprehensive system of veto detectors, yielding an experimental configuration with negligible background contamination [67].

A detailed discussion of the LDMX detector concept and its physics reach can be found in Ref. [67]. The LDMX detection system comprises a precision silicon tracking spectrometer with tracking stations located upstream and downstream of a thin aluminum target, followed by a high-granularity sampling electromagnetic calorimeter and a surrounding hadronic calorimeter [67,69]. The selection criteria ensure that dark sector particles typically carry away the majority of the beam energy, while background processes must produce additional particles that can be vetoed calorimetrically. In the analysis considered here, events are required to have reconstructed electron energy $E_e^{\text{rec}} \lesssim 0.3E_e$, which corresponds to a cut $x_{\text{cut}} = 0.7$, and the target thickness is chosen to be $L_T \simeq 0.4X_0$. The benchmark parameters of the LDMX configuration used in this work are summarized in Table I.

According to current projections, residual backgrounds in LDMX arise from several distinct sources [70]: (i) unbiased electron interactions, (ii) photonuclear processes, (iii) electronuclear interactions, and (iv) muon conversion events. To preserve the intended sensitivity at exposures up to $\text{EOT} \lesssim 10^{15}$, a dedicated program of front-end electronics development is planned [70]. The goal of the detector upgrades is to enable an effectively background-free LDMX search in the region of the

parameter space of light dark matter, where the benchmark background is below the single-event level [67,69].

C. NA64μ

NA64μ is a fixed-target facility at the CERN SPS that investigates dark sector particles through the missing-energy channel $\mu N \rightarrow \mu N \phi$, followed by the rapid decay $\phi \rightarrow \chi_1 \chi_2$. This configuration serves as a muon beam counterpart of the electron beam NA64e experiment, providing complementary sensitivity to the dark sector parameter space. The typical scheme of the NA64μ facility can be found in Ref. [95].

The NA64μ detection system employs two magnetic spectrometers to measure the energies of both incoming and outgoing muons. Our analysis implements a kinematic cut on the scattered muon, $E_\mu^{\text{rec}} \lesssim 0.5E_\mu \simeq 80$ GeV, corresponding to $x_{\text{cut}} = 0.5$ in Eq. (11). The typical parameters of NA64μ are summarized in Table I.

For our sensitivity analysis of NA64μ, we consider a muon beam energy of $E_\mu \simeq 160$ GeV and an anticipated muon-on-target statistics of $\text{MOT} \simeq 10^{11}$. The experimental setup utilizes a lead-based Shashlyk electromagnetic calorimeter, which serves simultaneously as the target with an effective thickness of $L_T \simeq 40X_0 \simeq 22.5$ cm.

We emphasize that for muons with energy $E_\mu \simeq 160$ GeV, the energy loss in lead is sufficiently small for traversal of a target medium of $40X_0$ radiation lengths. That permits neglect of the average stopping power $\langle dE_\mu/dx \rangle \simeq 12.7 \times 10^{-3}$ GeV/cm when numerically computing the ϕ -boson production yield at the NA64μ experiment. Specifically, by the end of the ECAL target the muon beam energy decreases to

$$E_\mu^{\text{end}} \simeq E_\mu - L_T \langle dE_\mu/dx \rangle \simeq 159.7 \text{ GeV}.$$

Consequently, this approximation validates the use of Eq. (12) for estimating $N_\phi^{\text{brem.}}$, with the target length parameter $L_T \simeq 40X_0$ appropriate for the NA64μ experimental setup. The typical precision of the muon momentum reconstruction, including multiple scattering in the target, is at the level of $\simeq 3\%$ for the momentum $E_\mu^{\text{rec}} \simeq 80$ GeV, and does not significantly affect the sensitivity of the search [64].

We assume that there is no loss of efficiency in the energy measurement for ϕ particles produced near the end of the ECAL (i.e., for reconstructed outgoing muon energies $E_\mu^{\text{rec}} \lesssim 0.5E_\mu \simeq 80$ GeV). This assumption is justified as follows.

The final-state muons are identified by Micromegas trackers and two large hadronic calorimeters (HCAL) located downstream. Most muons that radiate a scalar mediator ϕ with masses $m_\phi \lesssim 1$ GeV are deflected by a sufficiently small angle $\psi_\mu^{\text{rec}} \lesssim m_\phi/E_\mu \simeq 6.3 \times 10^{-3}$ (see, e.g., Ref. [17]). Such collinear emission of ϕ implies that

outgoing muons, whether deflected in the first layer or at the end of the ECAL, will be detected by the Micromegas trackers and the HCAL. The deflection angles remain within the hermeticity acceptance limits, which are estimated to be $\theta_{\text{ECAL}}^{\text{in}} \lesssim 0.105$ and $\theta_{\text{ECAL}}^{\text{out}} \lesssim 0.12$ for the first layer of ECAL and its end, respectively (see, e.g., Refs. [65,95] and references therein).

For NA64 μ , the main background sources are [95] (i) momentum misreconstruction, (ii) hadron in-flight decays, (iii) single-hadron punch-through, (iv) dimuons' production, and (v) detector nonhermeticity.

The conservative analysis based on [95] yields an upper limit of $N_{\text{bckg}} \lesssim 0.35$ for the anticipated MOT $\simeq 10^{11}$. Future mitigation of this background is foreseen through electronic upgrades, which are expected to improve event rejection by a factor of $\mathcal{O}(1)$ and consequently suppress the number of such events to $N_{\text{bckg}} \ll 1$.

D. Invisible signatures for fixed-target experiment

The sufficiently large benchmark coupling constant of a mediator with DM, $|\lambda_{\chi_1\chi_2}^\phi|^2 \gg (c_{ll}^\phi)^2$, implies that the total decay width of the spin-0 boson is $\Gamma_\phi^{\text{tot}} \simeq \Gamma_{\phi \rightarrow \chi_1\chi_2}$, with visible decays being neglected, $\Gamma_{\phi \rightarrow \chi_1\chi_2} \gg \Gamma_{\phi \rightarrow ll}$. We consider the following typical parameters:

$$\alpha_{\text{iDM}} \simeq 0.5, \quad \Delta_2 \in (0.1, 0.5), \quad m_{\chi_1}/m_\phi \simeq 1/3, \quad (13)$$

where $\alpha_{\text{iDM}} = (\text{Re}[\lambda_{\chi_1\chi_2}^\phi])^2/(4\pi)$ or $\alpha_{\text{iDM}} = (\text{Im}[\lambda_{\chi_1\chi_2}^\phi])^2/(4\pi)$ depending on benchmark scenarios (5) and (6), respectively.

To ensure that the invisible mode dominates over visible decay channel signatures of the lepton-specific mediator, we require that

$$c_{\tau\tau}^\phi \ll \sqrt{4\pi\alpha_{\text{iDM}}},$$

which leads to

$$c_{ee}^\phi \ll (m_e/m_\tau)\sqrt{4\pi\alpha_{\text{iDM}}}.$$

In particular, one can find that $c_{ee}^\phi \ll 7.2 \times 10^{-4}$ for $\alpha_{\text{iDM}} = 0.5$. It is also worth noting that this value is comparable to the *BABAR* constraints. We will show below that the benchmark choice (13) implies invisible decay signatures of the mediator for both NA64e and NA64 μ .

The choice of the benchmark parameter space (13) is motivated by the following considerations. To maintain the thermally averaged cross section $\langle\sigma_{\text{eff}}v\rangle$ at the value required by the relic density constraint (7), a larger α_{iDM} necessitates a smaller c_{ll}^ϕ on the thermal target curve. This dependence arises because $\langle\sigma_{\text{eff}}v\rangle \propto \alpha_{\text{iDM}} \times (c_{ll}^\phi)^2$. Thus, sufficiently small values of α_{iDM} are ruled out by the *BABAR* experiment (see Figs. 4 and 5). As a result, we choose α_{iDM} as large as possible in order to ensure a

perturbative regime, $\alpha_{\text{iDM}} \lesssim 1$ (see, e.g., Refs. [78,83,106] and references therein). This implies that larger values of α_{iDM} shift the thermal target curves downward relative to the constraints, as depicted in Figs. 4 and 5. Furthermore, we set the mass ratio $m_{\chi_1}/m_\phi \lesssim 1/2$ to avoid resonant enhancement [84].

We refer the reader to Refs. [84,107] for the general expression for the three-body decay width $\chi_2 \rightarrow \chi_1 l^+ l^-$. The relevant differential decay widths for benchmarks (5) and (6) read, respectively,

$$\begin{aligned} \frac{d\Gamma_{\chi_2 \rightarrow \chi_1 l^+ l^-}}{ds_a} &= \frac{(c_{ll}^\phi)^2 (\text{Re}[\lambda_{\chi_1\chi_2}^\phi])^2}{(2\pi)^3} \frac{1}{32m_{\chi_2}^3} \frac{1}{2m_\phi^4} \\ &\cdot \frac{4}{\sqrt{s_a}} (s_a - 4m_l^2)^{3/2} ((m_{\chi_2} + m_{\chi_1})^2 - s_a) \\ &\cdot \sqrt{\lambda(s_a, m_{\chi_1}^2, m_{\chi_2}^2)} \theta((m_{\chi_2} - m_{\chi_1})^2 - 4m_l^2), \end{aligned} \quad (14)$$

$$\begin{aligned} \frac{d\Gamma_{\chi_2 \rightarrow \chi_1 l^+ l^-}}{ds_a} &= \frac{(c_{ll}^\phi)^2 (\text{Im}[\lambda_{\chi_1\chi_2}^\phi])^2}{(2\pi)^3} \frac{1}{32m_{\chi_2}^3} \frac{1}{2m_\phi^4} \\ &\cdot \frac{4}{\sqrt{s_a}} (s_a - 4m_l^2)^{3/2} ((m_{\chi_2} - m_{\chi_1})^2 - s_a) \\ &\cdot \sqrt{\lambda(s_a, m_{\chi_1}^2, m_{\chi_2}^2)} \theta((m_{\chi_2} - m_{\chi_1})^2 - 4m_l^2), \end{aligned} \quad (15)$$

where the variable s_a varies in the range from $s_a^- = 4m_l^2$ to $s_a^+ = (m_{\chi_2} - m_{\chi_1})^2$. We also consider the approaches

$$s_a \ll m_\phi^2, \quad \Gamma_{\text{tot}}/m_\phi \ll 1,$$

which leads to

$$(s_a - m_\phi^2)^2 + m_\phi^2 \Gamma_{\text{tot}}^2 \simeq m_\phi^4.$$

Additionally, by taking into account expressions for integration limits as

$$s_a \ll (m_{\chi_2} + m_{\chi_1})^2,$$

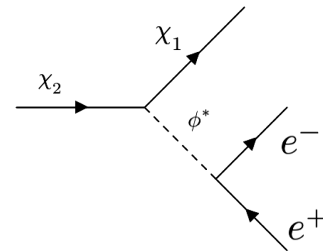


FIG. 3. Feynman diagram describing visible decay of excited state χ_2 to DM particle χ_1 and electron positron pair e^+e^- .

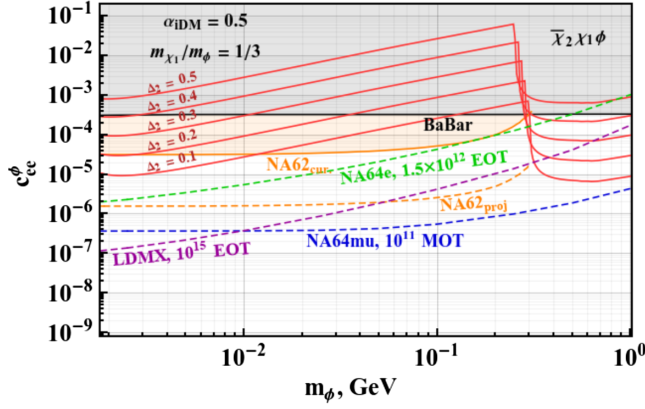


FIG. 4. The projected experimental reach at 90% C.L. as a function of m_ϕ mass for the benchmark scenario (5). The grey shaded region corresponds to the existing BABAR [108] monophoton limit from $e^+e^- \rightarrow \gamma\phi$. The dashed green, blue, and purple lines show expected limits for NA64e, NA64 μ , and LDMX experiments, respectively. The current and projected limits of the NA62 experiment [109,110] are shown by solid and dashed orange lines, respectively. The red lines correspond to the typical thermal target curves for a set of mass splittings Δ_2 .

and the small mass splitting of inelastic dark matter

$$m_l \ll (m_{\chi_2} - m_{\chi_1}), \quad \Delta_2 \ll 1,$$

we get

$$\lambda(s_a, m_{\chi_1}^2, m_{\chi_2}^2) \simeq 2m_{\chi_2}^2 \sqrt{(m_{\chi_2} - m_{\chi_1})^2 - s_a},$$

$$(m_{\chi_2} + m_{\chi_1})^2 - s_a \simeq 4m_{\chi_2}^2, \quad s_a - 4m_l^2 \simeq s_a.$$

Finally, one can obtain the following expressions for the benchmarks (5) and (6), respectively:

$$\Gamma_{\chi_2 \rightarrow \chi_1 l^+ l^-} = \frac{(c_{ll}^\phi)^2 (\text{Re}[\lambda_{\chi_1 \chi_2}^\phi])^2 (m_{\chi_2} - m_{\chi_1})^5}{60\pi^3 m_\phi^4}, \quad (16)$$

$$\Gamma_{\chi_2 \rightarrow \chi_1 l^+ l^-} = \frac{(c_{ll}^\phi)^2 (\text{Im}[\lambda_{\chi_1 \chi_2}^\phi])^2 (m_{\chi_2} - m_{\chi_1})^7}{560\pi^3 m_{\chi_2}^2 m_\phi^4}. \quad (17)$$

The relevant decays imply the kinematic threshold $m_{\chi_1} \Delta_2 > 2m_l$. Thus, one can see that the muon channel $\chi_2 \rightarrow \chi_1 \mu^+ \mu^-$ is allowed for sufficiently large mass splitting $\Delta_2 \gtrsim 6m_\mu/m_\phi \simeq 0.61$ and the masses of interest $r = m_{\chi_1}/m_\phi = 1/3$ with $m_\phi \lesssim 1$ GeV. As a result, for $\Delta_2 \lesssim 0.5$ the dominant decay channel of excited state χ_2 is associated with three-body decay $\chi_2 \rightarrow \chi_1 e^+ e^-$ (see Fig. 3).

The decay length of the heavier inelastic dark matter state in the laboratory frame is

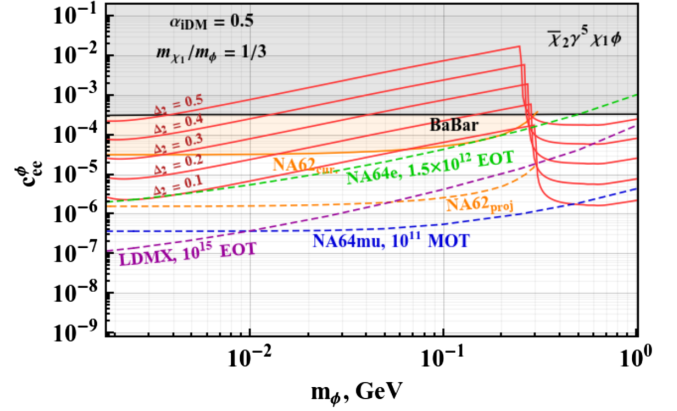


FIG. 5. The same as in Fig. 4 but for the benchmark model (6).

$$l_{\chi_2} = \frac{E_{\chi_2}}{m_{\chi_2}} \frac{1}{\Gamma_{\chi_2}^{\text{tot}}}, \quad (18)$$

where we assume that $\Gamma_{\chi_2}^{\text{tot}} \simeq \Gamma_{\chi_2 \rightarrow \chi_1 e^+ e^-}$ for $\Delta_2 < 0.5$. Using the approximation $E_{\chi_2} \simeq E_\phi/2 \simeq E_l/2$, the decay lengths for (5) and (6) read, respectively,

$$l_{\chi_2} \simeq 8.76 \times 10^6 \text{ cm} \times \left(\frac{0.3}{\Delta_2}\right)^5 \left(\frac{1}{1 + \Delta_2}\right) \left(\frac{1/3}{r}\right)^6 \times \left(\frac{E_l}{10 \text{ GeV}}\right) \left(\frac{0.1 \text{ GeV}}{m_\phi}\right)^2 \left(\frac{10^{-5}}{c_{ee}^\phi}\right)^2 \left(\frac{0.5}{\alpha_{\text{IDM}}}\right), \quad (19)$$

$$l_{\chi_2} \simeq 9.09 \times 10^8 \text{ cm} \times \left(\frac{0.3}{\Delta_2}\right)^7 \left(\frac{1 + \Delta_2}{1}\right) \left(\frac{1/3}{r}\right)^6 \times \left(\frac{E_l}{10 \text{ GeV}}\right) \left(\frac{0.1 \text{ GeV}}{m_\phi}\right)^2 \left(\frac{10^{-5}}{c_{ee}^\phi}\right)^2 \left(\frac{0.5}{\alpha_{\text{IDM}}}\right). \quad (20)$$

Thus, within the relevant parameter space, a rough estimate gives $l_{\chi_2} \gtrsim \mathcal{O}(1)$ km. The typical number of χ_2 decays at the fixed-target facilities can be approximated as

$$N_{\text{sign}} \simeq N_\phi^{\text{brems.}} \exp(-L_{\text{det}}/l_{\chi_2}), \quad (21)$$

where $L_{\text{det.}}$ is the typical length of the experimental facility, which is estimated to be $\lesssim (1-10)$ m for the considered fixed-target experiments. This, however, implies $L_{\text{det}} \ll l_{\chi_2}$ and $N_{\text{sign}} \simeq N_\phi^{\text{brems.}}$. This means that the visible decay of χ_2 occurs outside LDMX, NA64e, and NA64 μ detectors for the parameter space of interest (13). However, that results in the invisible signature at the considered fixed-target facilities, since the decays $\chi_2 \rightarrow \chi_1 e^+ e^-$ evade detection in their calorimetric system.

V. THE EXPERIMENTAL REACH

In this section, we discuss the expected experimental reach of the fixed-target facilities, such as NA64e, LDMX,

and NA64 μ , assuming that the scalar mediator decays into the invisible mode.

We set the 90% C.L. exclusion limit on the coupling constant between the electron and the scalar mediator, using the typical number of signal events $N_{\text{sign}} \gtrsim 2.3$. Also, we imply the suppressed background, $N_{\text{bckg}} \lesssim 1$, and the null results of the observed missing-energy events for experiments, $N_{\text{obs}} = 0$. For the evaluation of experimental limits, we employ the projected statistics of fixed-target facilities that are presented in Table I.

As discussed above, the decay length of the heavier dark sector state χ_2 is much larger than the detector base of the considered experiments. Thus, we focus on the invisible mode at the lepton fixed-target facilities, $lN \rightarrow lN\phi(\rightarrow \chi_1\chi_2)$.

In Figs. 4 and 5, we present the typical thermal target curves for the benchmark couplings (5) and (6), respectively. We also display the expected sensitivities of NA64e and NA64 μ lepton missing-energy experiments for the anticipated statistics of $EOT = 1.5 \times 10^{12}$ and $MOT = 10^{11}$, respectively. We adopt the benchmark relation between couplings of charged leptons and the scalar mediator as (3).

We emphasize that the heavier dark sector state χ_2 predominantly decays into the lightest state and an electron-positron pair $\chi_2 \rightarrow \chi_1 e^+ e^-$ for the considered parameter space (13) of the fixed-target experiments. The muon decay channel $\chi_2 \rightarrow \chi_1 \mu^+ \mu^-$ requires a sufficiently large mass splitting $\Delta_2 \gtrsim 0.61$, which was excluded by the BABAR experiment.

The coupling of a scalar mediator to leptons induces an additional tree-level decay of the charged kaon, $K^+ \rightarrow \mu^+ \nu_\mu \phi$ [111]. In this process, the scalar mediator can be produced invisibly, carrying energy away from the detector. In Ref. [110], the projected reach of the NA62 experiment for the muonphilic scalar coupling $\mathcal{L} \supset c_{\mu\mu}^{\phi} \phi \bar{\mu} \mu$ was obtained. This leads to the typical bound on the coupling at the level of $c_{ee}^{\phi} \lesssim \mathcal{O}(10^{-6})$ for mediator masses $m_\phi \lesssim 300$ MeV.

However, these projected bounds can be recast using the current NA62 data [109] on the invisible branching fraction of charged kaons at the 90% C.L., yielding

$$\mathcal{O}(10^{-7}) \lesssim \text{Br}(K^+ \rightarrow \mu^+ \nu_\mu \phi)_{\text{data}} \lesssim \mathcal{O}(10^{-5}).$$

The data from Ref. [109] imply that for $m_\phi \simeq 50$ MeV,

$$\text{Br}(K^+ \rightarrow \mu^+ \nu_\mu \phi)_{\text{data}} \simeq 2 \times 10^{-6}.$$

On the other hand, the authors of Ref. [110] have shown that for $m_\phi \simeq 50$ MeV the projected luminosity data of NA62 can yield a typical bound on the branching fraction,

$$\text{Br}(K^+ \rightarrow \mu^+ \nu_\mu \phi)_{\text{proj}} \simeq 5 \times 10^{-9}.$$

That can be translated to the current limits on $(c_{ee}^{\phi})_{\text{data}}$ in the following form:

$$(c_{ee}^{\phi})_{\text{data}} \simeq (c_{ee}^{\phi})_{\text{proj}} \left(\frac{\text{Br}(K^+ \rightarrow \mu^+ \nu_\mu \phi)_{\text{data}}}{\text{Br}(K^+ \rightarrow \mu^+ \nu_\mu \phi)_{\text{proj}}} \right)^{1/2},$$

where we use the relation (3) between muon and electron coupling to the mediator, and this yields the recasting coefficient $(c_{ee}^{\phi})_{\text{data}} \simeq 20 \times (c_{ee}^{\phi})_{\text{proj}}$. Consequently, the NA62 limits rule out several of the splittings for the thermal Majorana inelastic dark matter below $m_\phi \lesssim 300$ MeV.

The NA64 μ experiment can be sensitive to couplings at the level of $c_{ee}^{\phi} \gtrsim 5 \times 10^{-7}$, and the NA64e experiment can rule out the coupling in the range $c_{ee}^{\phi} \gtrsim 10^{-6}$ for the small mass region $m_\phi \lesssim 10$ MeV. Moreover, the anticipated number of electrons collected on target $EOT \simeq 1.5 \times 10^{12}$ for NA64e will allow us to constrain the inelastic DM (iDM) model with $\Delta_2 \gtrsim 0.1$, $\alpha_{\text{iDM}} = 0.5$, and $m_{\chi_1}/m_\phi = 1/3$ for the mediator in the mass range $m_\phi \lesssim 300$ MeV. Furthermore, the NA64 μ with $MOT \simeq 10^{11}$ can rule out the relevant parameter space in the wider mass range of the mediator, $2 \text{ MeV} \lesssim m_\phi \lesssim 1 \text{ GeV}$ for the scalar coupling between iDM and the mediator.

In addition, the LDMX can be sensitive to couplings at the level of $c_{ee}^{\phi} \gtrsim 10^{-7}$ and will provide new limits on the interaction coupling for $m_\phi \lesssim 10$ MeV.

To address the limits from the production of heavy-flavor leptons and their decays—specifically, $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau \phi$, $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \phi$, and $\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau \phi$ —we refer the reader to Ref. [78]. Its authors demonstrate that the corresponding bounds are weaker than those from current accelerator-based experiments [108]. This implies a limit of $c_{ee}^{\phi} \simeq 5 \times 10^{-4}$, which has been ruled out by BABAR.

For completeness, we refer the reader to Ref. [16], which addresses the projected bounds from the Charm-Tau factory. This facility implies the potential for heavy lepton production associated with the invisible decay of a scalar mediator, such as in the process $e^+ e^- \rightarrow \tau^+ \tau^- \phi$. The relevant limits could be as small as $c_{ee}^{\phi} \lesssim 10^{-8}$ for a projected luminosity of $\mathcal{L} \simeq 3 \text{ ab}^{-1}$ with collider energy at $\sqrt{s} \simeq 4.2 \text{ GeV}$. As a result, a dominant portion of the benchmark iDM parameter space, $1 \text{ MeV} \lesssim m_\phi \lesssim 1 \text{ GeV}$, can be ruled out by the Charm-Tau factory.

Let us discuss the impact of the mass splitting on the thermal target curves for inelastic dark matter in Figs. 4 and 5. Increasing the mass splitting value shifts the relic curves upward relative to the constraints. That can be explained as follows. By taking into account the expression for the effective thermally averaged cross section (A5) and approximations for the fraction of heavy inelastic dark matter (C6), we can estimate the typical scaling for the coupling $c_{ee}^{\phi} \propto e^{\Delta_2 x_f}$. Thus, the larger Δ_2 , the larger c_{ee}^{ϕ} for the typical thermal target curve. On the other hand, smaller

values, $\Delta_2 x_f \ll 1$, would not impact on the DM relic abundance coupling c_{ee}^ϕ . A sizable contribution of the mass splitting to the thermal target curves occurs for $\Delta_2 \gtrsim 1/x_f \simeq 0.04$.

Regions beneath the red contours are excluded, as they correspond to a cosmological overabundance of DM, $\Omega_c h^2 \gtrsim 0.12$, for the specified mass ratio, $m_\phi/m_{\chi_1} = 3$. The discontinuities observed near $m_\phi \simeq 2m_\mu$ arise from the kinematic opening of the new coannihilation channel $\chi_1 \chi_2 \rightarrow \mu^+ \mu^-$ in addition to the $e^+ e^-$ pair production.

In order to conclude this section we note that the one-loop elastic direct detection signature $\chi_1 e^- \rightarrow \chi_1 e^-$ provides the bound that has been already ruled out by *BABAR* (see Appendix D).

VI. CONCLUSION

In this work, we explore the consequences of sub-GeV inelastic dark matter scenarios mediated by a lepton-specific scalar portal, and by solving the Boltzmann equations, we derive thermal targets for a set of DM mass splittings, $0.1 \lesssim \Delta_2 \lesssim 0.5$, focusing on the sub-GeV regime for the scalar mediator mass. Considering a minimal spin-0 extension of the SM lepton sector, we evaluate the sensitivity of lepton fixed-target experiments, such as NA64e, LDMX, and NA64 μ . In particular, we use the missing-energy signature to obtain the projected constraint on coupling of the scalar mediator and electron. That channel can be represented as ϕ -strahlung processes with invisible decay $\phi \rightarrow \chi_2 \chi_1$. We focus on a scalar mediator with the masses from $\mathcal{O}(1)$ MeV to $\mathcal{O}(1)$ GeV. For mass splitting greater than 0.5, the corresponding models are excluded from the *BABAR* experiment. For the coupling constant relation, $c_{ee}^\phi : c_{\mu\mu}^\phi = m_e : m_\mu$, the NA64 μ experiment can be sensitive to $c_{ee}^\phi \gtrsim 5 \times 10^{-7}$ for the sub-GeV mediator mass window, $m_\phi \lesssim 1$ GeV. In addition, the NA64e experiment can rule out the typical couplings $c_{ee}^\phi \gtrsim 2 \times 10^{-6} - 10^{-4}$ for mediator masses in the range $m_\phi \lesssim 300$ MeV. The LDMX electron fixed-target facility will probe couplings $c_{ee}^\phi \gtrsim 10^{-7} - 10^{-4}$ for sub-GeV mediator masses. We show that current data of the NA62 experiment [109] rule out relatively large splitting, $\Delta_2 \gtrsim 0.2 - 0.3$, for the masses below $m_\phi \lesssim 300$ MeV. We argue that one-loop-induced direct detection signature $\chi_1 e^- \rightarrow \chi_1 e^-$ constraints from XENON1T (2019) data have been ruled out by the *BABAR* experiment.

ACKNOWLEDGMENTS

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from the NA62 experiment and the projected limits from the LDMX was supported by RSF Grant No. 24-72-10110.

DATA AVAILABILITY

The data that support the findings of this article are openly available [112].

APPENDIX A: RELIC DENSITY OF INELASTIC DARK MATTER

In this appendix, we discuss the form of the Boltzmann equation in the case of inelastic dark matter. The relic abundance of the lightest state of dark matter in the freeze-out mechanism can be described by the sum of the densities of all particles of the hidden sector [113,114]. Next, summing the Boltzmann equations for particle densities of the k th type of iDM, n_k , one can get the following expression [113–115]:

$$\dot{n} = -3Hn - \sum_f \sum_{i,j=1}^2 \langle \sigma_{ij} v_{ij} \rangle (n_i n_j - n_i^{\text{eq}} n_j^{\text{eq}}), \quad (\text{A1})$$

where $\sum_f$ is the sum over the final state of SM particles, $n = \sum_{i=1}^2 n_i$ is the total particle density of DM, and $n_{\text{eq}} = \sum_{k=1}^2 n_k^{\text{eq}}$ is the total particle density of DM in equilibrium. The thermally averaged cross section reads [115]

$$\langle \sigma_{ij} v_{ij} \rangle = \frac{g_i g_j}{n_i^{\text{eq}} n_j^{\text{eq}}} \int \sigma_{ij} v_{ij} f_i f_j \frac{d\mathbf{p}_i}{(2\pi)^3} \frac{d\mathbf{p}_j}{(2\pi)^3}, \quad (\text{A2})$$

where $f_i = e^{-E_i/T}$ is the equilibrium distribution function in the Maxwell-Boltzmann approximation for the typical temperature. The Moller velocity has the following form:

$$v_{lk} = I_{lk}/(E_l E_k), \quad (\text{A3})$$

where the Moller invariant is

$$I_{lk} = \sqrt{(p_l, p_k)^2 - m_{\chi_l}^2 m_{\chi_k}^2} = (1/2) \sqrt{\lambda(s, m_{\chi_l}^2, m_{\chi_k}^2)},$$

$\lambda(s, m_{\chi_l}^2, m_{\chi_k}^2) = (s - (m_{\chi_l} + m_{\chi_k})^2)(s - (m_{\chi_l} - m_{\chi_k})^2)$ is the triangular function, and $s = (p_i + p_j)^2$ is the Mandelstam variable.

Taking into account $n_i/n \simeq n_i^{\text{eq}}/n^{\text{eq}}$, one can get the Boltzmann equation for total particle density of DM in the standard form as

$$\dot{n} = -3Hn - \langle \sigma_{\text{eff}} v \rangle (n^2 - n_{\text{eq}}^2), \quad (\text{A4})$$

where the effective thermally averaged cross section is

$$\langle \sigma_{\text{eff}} v \rangle = \sum_f \sum_{i,j=1}^2 \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{\text{eq}} n_j^{\text{eq}}}{n_{\text{eq}}^2}. \quad (\text{A5})$$

The explicit form of the effective thermally averaged cross section is shown in (C7) for the nonrelativistic case.

Let us consider the dependence of the critical dark matter density on the thermally averaged cross section. Taking into account the Friedmann universe, the Hubble parameter is defined as [92]

$$H(x) \simeq \frac{H(m_{\chi_1})}{x^2}, \quad H(m_{\chi_1}) \simeq 0.331 g_{*s}^{1/2}(m_{\chi_1}) \frac{m_{\chi_1}^2}{M_{\text{Pl}}},$$

where $M_{\text{Pl}} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass, $g_{*s}(T)$ is the effective degree of freedom [91], and m_{χ_1} is the lightest state mass of DM. The variable x is the typical ratio, defined as $x = m_{\chi_1}/T$. Assuming that the effective number of entropy degrees of freedom, $g_{*s}(T)$, depends weakly on temperature, one can obtain the expression for the Hubble parameter from the conservation of comoving entropy, $d(a^3(t)s)/dt = 0$, as

$$H(t) = (-1/T) dT/dt,$$

where $a(t)$ is the scale factor, and the total entropy density takes the following forms:

$$s(T) = \frac{2\pi^2}{45} g_{*s}(T) T^3, \quad s(x) = \frac{2\pi^2}{45} g_{*s} \left(T = \frac{m_{\chi_1}}{x} \right) \frac{m_{\chi_1}^3}{x^3}.$$

Thus, time is related to the parameter x as $dt = x dx / H(m_{\chi_1})$.

The Boltzmann Eq. (A4) can be written by defining the term $Y(x) = n(x)/s(x)$ as a ratio of DM particle density $n(x)$ over the total SM entropy density $s(x)$ in the following form:

$$\frac{dY}{dx} = -\frac{s(x)}{xH(x)} \langle \sigma_{\text{eff}} v \rangle (Y^2(x) - Y_{\text{eq}}^2(x)), \quad (\text{A6})$$

where $Y_{\text{eq}}(x) = n_{\text{eq}}(x)/s(x)$.

The decoupling of the considered type of particles is achieved if the following condition is fulfilled [92]:

$$n_{\text{eq}} \langle \sigma_{\text{eff}} v \rangle|_{x=x_f} \simeq H(x_f), \quad (\text{A7})$$

where T_f is the critical temperature and $x_f = m_{\chi_1}/T_f$ is the parameter of freeze-out, which is estimated to be $x_f \simeq 25$.

The current value of variable Y tends to the relic value $Y(\infty)$, which can be estimated on the interval (x_f, ∞) from the Boltzmann Eq. (A6) as

$$Y^{-1}(\infty) = \frac{s(T=m_{\chi_1})}{H(m_{\chi_1})} J(x_f), \quad J(x_f) = \int_{x_f}^{\infty} \frac{\langle \sigma_{\text{eff}} v \rangle}{x^2} dx, \quad (\text{A8})$$

where we take into account that $Y(x) \gg Y_{\text{eq}}(x)$ for $x > x_f$. In case of s-wave annihilation one can exploit $J(x_f) = \langle \sigma v \rangle / x_f$.

The critical density of cold dark matter is

$$\Omega_c = \frac{\epsilon_{\text{DM}}}{\epsilon_{\text{cr}}} = \frac{s_0 \sum_{k=1}^2 m_{\chi_k} Y_k}{\epsilon_{\text{cr}}} \simeq \frac{m_{\chi_1} s_0 Y_1}{\epsilon_{\text{cr}}}, \quad (\text{A9})$$

where $Y_1 \simeq Y(\infty)$, $Y_k = n_k(x)/s(x)$, $\epsilon_{\text{DM}} = \sum_{k=1}^2 m_{\chi_k} n_k$ is the total energy density of cold dark matter and $\epsilon_{\text{cr}} = 3H_0^2 M_{\text{Pl}}^2$ is critical density. We assume for the mass of inelastic dark matter that $m_{\chi_i} = m_{\chi_1} (1 + \Delta_i) \simeq m_{\chi_1}$. Thus, one can get

$$\Omega_c \simeq \frac{m_{\chi_1} s_0}{3M_{\text{Pl}}^2 H_0^2} \frac{H(m_{\chi_1})}{s(T=m_{\chi_1})} \left(\int_{x_f}^{\infty} \frac{\langle \sigma_{\text{eff}} v \rangle}{x^2} dx \right)^{-1}, \quad (\text{A10})$$

where $s_0 = s(T_0)$ is the current total entropy density, and the current temperature of the Universe is

$$T_0 = 2.726 \text{ K} = 2.35 \times 10^{-13} \text{ GeV}.$$

The current values of the Hubble parameter, H_0 , and the dimensionless constant, h , read, respectively [7],

$$H_0 = 2.13h \times 10^{-42} \text{ GeV}, \quad h \simeq 0.674 \pm 0.005.$$

In addition, we take into account that the current value of relic abundance of cold DM obtained from the Planck 2018 combined analysis is [7]

$$\Omega_c h^2 = 0.1200 \pm 0.0012.$$

As a result, the expression of relic density takes the following form [92]:

$$\Omega_c h^2 \simeq \frac{0.85 \times 10^{-10}}{\text{GeV}^2} g_{*s}^{-1/2}(m_{\chi_1}) \left(\int_{x_f}^{\infty} \frac{\langle \sigma_{\text{eff}} v \rangle}{x^2} dx \right)^{-1}, \quad (\text{A11})$$

where $g_{*s}(T)$ and $g_{*e}(T)$ are the energy and entropy effective degrees of freedom [91], respectively, and it is taken into account that $g_{*s}(T_0) \approx 3.91$, and for the temperature $T \gtrsim 1$ MeV we can set $g_{*e}(T) \simeq g_{*s}(T)$.

APPENDIX B: ANNIHILATION CROSS SECTIONS OF INELASTIC DARK MATTER

In this appendix, we provide the cross sections of inelastic DM annihilation into SM particles in the case

of a lepton-specific scalar mediator. For the calculation of both the decay width and cross section, we employ the state-of-the-art FeynCalc package [116,117] for the Wolfram Mathematica routine [118].

The decay widths of the scalar mediator in cases of lepton and considered benchmarks are, respectively,

$$\Gamma_{\phi \rightarrow l^+ l^-}(s) = \frac{(c_{ll}^\phi)^2}{8\pi} \sqrt{s} \beta^3(4m_l^2, s), \quad (\text{B1})$$

$$\Gamma_{\phi \rightarrow \chi_2 \chi_1}(s) = \frac{(\text{Re}[\lambda_{\chi_1 \chi_2}^\phi])^2}{8\pi} \sqrt{s} \beta^3(s_0, s) \beta(\delta_0, s), \quad (\text{B2})$$

$$\Gamma_{\phi \rightarrow \chi_2 \chi_1}(s) = \frac{(\text{Im}[\lambda_{\chi_1 \chi_2}^\phi])^2}{8\pi} \sqrt{s} \beta(s_0, s) \beta^3(\delta_0, s), \quad (\text{B3})$$

where $s_0 = (m_{\chi_2} + m_{\chi_1})^2$ and $\delta_0 = (m_{\chi_2} - m_{\chi_1})^2$, and we denote $\beta(a, b) = (1 - a/b)^{1/2}$. Moreover, the decay widths reduce to well-known expressions [100,101] for the chosen Lagrangian densities in the case of equal masses of dark matter.

In the case of the two-body final-state process, the resonant total cross section for $\chi_i \chi_j \rightarrow \phi \rightarrow l^+ l^-$ can be approximated by the Breit-Wigner (BW) resonant formula [84],

$$\sigma_{\chi_i \chi_j \rightarrow \phi \rightarrow l^+ l^-} = 4\pi N_\sigma^{\text{BW}} \frac{s^2 \Gamma_{\phi \rightarrow \chi_i \chi_j}(s) \Gamma_{\phi \rightarrow l^+ l^-}(s)}{I_{ij}^2 D_{\text{DM}}^2(s)}, \quad (\text{B4})$$

where $N_\sigma^{\text{BW}} = S_{\text{BW}}(2J+1)(2S_i+1)^{-1}(2S_j+1)^{-1}$, Γ_{tot} is the total decay width of resonance, J is the spin of resonance, $S_{\text{BW}} = 1$ for different particles and $S_{\text{BW}} = 2$ for identical particles in the initial state, and S_i and S_j are spins of initial particles. In Eq. (B4) we denote

$$D_{\text{DM}}^2(s) = (s - m_\phi^2)^2 + m_\phi^2 (\Gamma_\phi^{\text{tot}}(m_\phi^2))^2. \quad (\text{B5})$$

The total cross sections for the considered benchmarks are

$$\sigma_{\chi_1 \chi_2 \rightarrow \phi \rightarrow l^+ l^-}(s) = \frac{(\text{Re}[\lambda_{\chi_1 \chi_2}^\phi])^2 (c_{ll}^\phi)^2}{16\pi D_{\text{DM}}^2(s)} s \cdot \beta^3(4m_l^2, s) \beta(s_0, s) \beta^{-1}(\delta_0, s), \quad (\text{B6})$$

$$\sigma_{\chi_1 \chi_2 \rightarrow \phi \rightarrow l^+ l^-}(s) = \frac{(\text{Im}[\lambda_{\chi_1 \chi_2}^\phi])^2 (c_{ll}^\phi)^2}{16\pi D_{\text{DM}}^2(s)} \sqrt{s} \cdot \beta^3(4m_l^2, s) \beta^{-1}(s_0, s) \beta^{-1}(\delta_0, s). \quad (\text{B7})$$

In particular, nonrelativistic leading terms of total cross sections are

$$\sigma_{\chi_1 \chi_2 \rightarrow \phi \rightarrow l^+ l^-}^{\text{lov vel.}}(v_-) = \frac{(\text{Re}[\lambda_{\chi_1 \chi_2}^\phi])^2 (c_{ll}^\phi)^2}{32\pi D_{\text{DM}}^2(s_0)} \cdot s_0 \beta^3(4m_l^2, s_0) v_-, \quad (\text{B8})$$

$$\sigma_{\chi_1 \chi_2 \rightarrow \phi \rightarrow l^+ l^-}^{\text{lov vel.}}(v_-) = \frac{(\text{Im}[\lambda_{\chi_1 \chi_2}^\phi])^2 (c_{ll}^\phi)^2}{8\pi D_{\text{DM}}^2(s_0)} \cdot s_0 \beta^3(4m_l^2, s_0) v_-^{-1}. \quad (\text{B9})$$

This means that coannihilation for the benchmark scenarios (5) and (6) implies a p-wave and s-wave channel, respectively [119].

APPENDIX C: THERMALLY AVERAGED CROSS SECTION IN LOW-VELOCITY APPROACH

In this appendix, we discuss the thermally averaged cross section in the nonrelativistic approach. In particular, the expression of the thermally averaged cross section (A2) in the nonrelativistic limit and the center-of-mass system is [120]

$$\langle \sigma_{ij} v_{ij} \rangle_{\text{non.rel.}} = \frac{\int \sigma_{ij} v_{ij} e^{-\sqrt{s}/T} \delta(\mathbf{V}) d\mathbf{v}_i d\mathbf{v}_j}{\int e^{-\sqrt{s}/T} \delta(\mathbf{V}) d\mathbf{v}_i d\mathbf{v}_j}, \quad (\text{C1})$$

where $\mathbf{V} = (m_{\chi_i} \mathbf{v}_i + m_{\chi_j} \mathbf{v}_j)/(m_{\chi_i} + m_{\chi_j})$ is the center-of-mass velocity. In general, the Mandelstam variable, $s = (p_i + p_j)^2$, expanded in terms of the relative velocity, $\mathbf{v}_- = \mathbf{v}_i - \mathbf{v}_j$, reads as

$$s \simeq s_0 + m_{\chi_i} m_{\chi_j} v_-^2, \quad \sqrt{s} \simeq \sqrt{s_0} + (\mu_{ij}/2) v_-^2, \quad (\text{C2})$$

where $s_0 = (m_{\chi_i} + m_{\chi_j})^2$, $v_- = |\mathbf{v}_i - \mathbf{v}_j|$, and $\mu_{ij} = m_{\chi_i} m_{\chi_j} / \sqrt{s_0}$ is the reduced mass. Accounting for $v_{ij} \approx v_-$ and $d\mathbf{v}_i d\mathbf{v}_j = d\mathbf{V} d\mathbf{v}_-$, the low-velocity limit of the thermally averaged cross section takes the following form:

$$\langle \sigma_{ij} v_{ij} \rangle_{\text{non.rel.}} = \frac{x^{3/2}}{2\sqrt{\pi}} \left(\frac{2\mu_{ij}}{m_{\chi_1}} \right)^{3/2} \cdot \int_0^\infty \sigma(v_-) v_-^3 \exp \left[-\frac{\mu_{ij}}{2m_{\chi_1}} x v_-^2 \right] dv_-. \quad (\text{C3})$$

Thus, in order to obtain a nonrelativistic expansion of the total cross section for the corresponding process, one can use the substitution $(s - s_0)/(m_{\chi_i} m_{\chi_j}) \rightarrow v_-^2$.

By using the nonrelativistic expansion for the effective, thermally averaged cross section (C3) in the low-velocity approach as $\sigma(v_-) v_- = \sum_{k=0}^\infty a_k v_-^{2k}/k!$, one can get

$$\langle \sigma_{ij} v_{ij} \rangle_{\text{non.rel.}} = \frac{4}{2\sqrt{\pi}} \sum_{k=0}^\infty b^k \Gamma(k+3/2) \frac{a_k}{k!} x^{-k} \approx a_0 + (3/2) b a_1 x^{-1} + (15/8) b^2 a_2 x^{-2}, \quad (\text{C4})$$

where $b = (2m_{\chi_1}/\mu_{ij})$, a_k are the expansion coefficients of the cross section for the low-velocity series for each channel, and $\Gamma(k)$ is the Gamma function.

Moreover, for $m_{\chi_1} = m_{\chi_2}$, Eq. (C4) reduces to the well-known expansion of the thermally averaged cross section [115,120,121]. It is worth mentioning that $n = 0$, $n = 1$, and $n = 2$ correspond to the s-wave, p-wave, and d-wave annihilations, respectively [92]. Thus, explicit analytical integrated expressions for the relic density can be obtained by considering a first nonzero term in the low-velocity approach (C4) as $\langle\sigma_{ij}v_{ij}\rangle_{\text{non-rel.}} = \sigma_{ij}^0 x^{-n}$.

In the case of $T \lesssim m_{\chi_k}$ the particle net density in equilibrium reads [122]

$$n_k^{\text{eq}} \approx g_k \left(\frac{m_{\chi_k} T}{2\pi} \right)^{3/2} e^{-m_{\chi_k}/T} \left(1 + \frac{15T}{8m_{\chi_k}} + \dots \right). \quad (\text{C5})$$

Thus, one can estimate [113]

$$\frac{n_i^{\text{eq}}}{n_{\text{eq}}} = \frac{g_i(1 + \Delta_i)^{3/2} e^{-x\Delta_i}}{\sum_{k=1}^2 g_k(1 + \Delta_k)^{3/2} e^{-x\Delta_k}} \sim e^{-x\Delta_i}, \quad (\text{C6})$$

where $\Delta_i = (m_{\chi_i} - m_{\chi_1})/m_{\chi_1}$.

As a result, in the approach of the approximation for density of DM in equilibrium (C5) and the low-velocity approach (C4), one can see that the effective, thermally averaged cross section (A5) takes the following form [113]:

$$\langle\sigma_{\text{eff}}v\rangle = \sum_f \sum_{i,j=1}^2 \sigma_{ij}^0 x^{-n} \frac{n_i^{\text{eq}} n_j^{\text{eq}}}{n_{\text{eq}}^2} = \sum_f \sum_{i,j=1}^2 \sigma_{ij}^0 x^{-n} \frac{g_i g_j (1 + \Delta_i)^{3/2} (1 + \Delta_j)^{3/2} e^{-x(\Delta_i + \Delta_j)}}{(\sum_{l=1}^2 g_l (1 + \Delta_l)^{3/2} e^{-x\Delta_l})^2}. \quad (\text{C7})$$

It is also worth noticing the heavier component the hidden sector can decay into the lightest mass-state particle. The contribution of these decays to the DM relic density is expected to be negligible [113]. This also holds for the upscattering cross section $\chi_i l \rightarrow \chi_j l$ (see, e.g., Ref. [113] and references therein). It means that coannihilation channel $\chi_1 \chi_2 \rightarrow l^+ l^-$ provides a dominant contribution to the observed DM net density.

APPENDIX D: DIRECT DETECTION

For the model under consideration, the inelastic scattering of the lightest state of electrons, $\chi_1 e \rightarrow \chi_2 e$, is kinematically suppressed for relatively large mass splittings, $m_{\chi_1} \Delta \gtrsim 1 - 100$ keV, leading to a weak signal in direct detection experiments (see, e.g., Refs. [123,124] and references therein). On the other hand, the elastic scattering $\chi_1 e^- \rightarrow \chi_1 e^-$ of DM can be induced at the one-loop level via the scalar exchange depicted in Fig. 6.

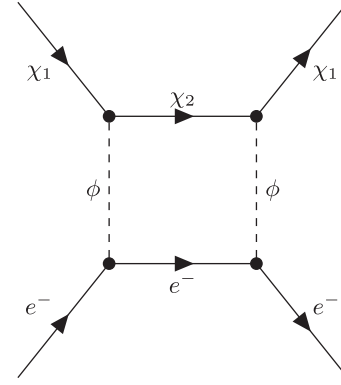


FIG. 6. A Feynman diagram responsible for the one-loop coupling of DM to electrons. We do not show the crossed diagram.

The one-loop contribution to the elastic scattering $\chi_1 f \rightarrow \chi_1 f$ of a SM fermion f , mediated by a light hidden vector, is discussed in Refs. [45,125,126]. Unlike the inelastic channel, these processes are not kinematically suppressed for large mass splittings, but they are suppressed due to the leading nonvanishing quantum correction.

Specifically, the effective low-energy Lagrangian in this case is $\mathcal{L}_{\text{EFT}} \supset C_e^{(0)} \tilde{\chi}_1 \chi_1 m_e \bar{e} e$. The spin-independent scattering cross section is then given by

$$\sigma_{\text{SI}}^{\text{el}} = \frac{4}{\pi} (\mu_{e\chi_1})^2 |\mathcal{M}_{e\chi_1}|^2, \quad (\text{D1})$$

where $\mu_{e\chi_1} = m_e m_{\chi_1} / (m_e + m_{\chi_1})$ is the DM-electron reduced mass and $\mathcal{M}_{e\chi_1}$ is the spin-independent amplitude for electron scattering. This amplitude reads

$$\mathcal{M}_{e\chi_1} = m_e C_e^{(0)}. \quad (\text{D2})$$

We estimate the direct detection sensitivity for the one-loop scalar exchange by adapting the results of Ref. [45] for a vector mediator. For a sufficiently heavy mediator, $m_\phi \gg m_{\chi_1} \gg m_e$, the Wilson coefficient $C_e^{(0)}$ typically scales with the model parameters as [45]

$$C_e^{(0)} \propto \frac{(c_{ee}^\phi)^2 \alpha_{\text{IDM}} m_{\chi_1}}{4\pi (m_\phi)^4}. \quad (\text{D3})$$

Although a detailed calculation of this Wilson coefficient is beyond the scope of this work, we do not expect these estimates to change our final scalar results by more than an order of magnitude.

As a result, the DM scattering cross section is estimated to be

$$\sigma_{\text{SI}}^{\text{el}} \propto \frac{1}{4\pi^3} (c_{ee}^\phi)^4 (\alpha_{\text{IDM}})^2 \frac{m_e^4 m_{\chi_1}^2}{(m_\phi)^8}. \quad (\text{D4})$$

We compare the predicted DM-electron elastic scattering cross section from Eq. (D4) to the limit from XENON1T (2019) data, $\sigma_{\text{SI}}^{\text{el}} \lesssim 10^{-40} \text{ cm}^2$ for $m_\chi \simeq 100 \text{ MeV}$ (see, e.g., Fig. 5.12 of Ref. [3]). This comparison yields a typical

lower limit of $c_{ee}^{\phi} \gtrsim 1.9$, which is already ruled out by the *BABAR* experiment. Hence, current direct detection limits are not competitive with those from electron-positron colliders for the sub-GeV mass range.

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