

Matching HEFT and SMEFT in double and triple Higgs boson production from weak boson fusion

D. Domenech,^{1,*} M. Herrero^{1,†}, R. A. Morales^{2,‡} and A. Salas-Bernárdez^{3,§}

¹*Departamento de Física Teórica and Instituto de Física Teórica, IFT-UAM/CSIC, Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain*

²*IFLP, CONICET—Departamento de Física, Universidad Nacional de La Plata, C.C. 67, 1900 La Plata, Argentina*

³*Departamento de Análisis Matemático y Matemática Aplicada and IPARCOS, Universidad Complutense de Madrid, Plaza de las Ciencias 3, 28040 Madrid, Spain*



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In this work, we study the matching between the two most popular effective field theories for beyond standard model Higgs physics, SMEFT and HEFT. To perform this matching, we follow the approach of identifying the corresponding scattering amplitudes for physical processes in both theories, instead of the most usual approach of relating the corresponding effective Lagrangians or effective actions. In this work, we focus on the physical processes of double and triple Higgs production from weak boson fusion, in particular $WW \rightarrow HH$, $ZZ \rightarrow HH$, $WW \rightarrow HHH$, and $ZZ \rightarrow HHH$ and complement them with the elastic scattering $HH \rightarrow HH$. We present here the analytical solution to this matching in terms of relations among the coefficients in both theories and comment on the most relevant phenomenological implications of such relations for collider physics.

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I. INTRODUCTION

Since the discovery of the Higgs boson [1,2], there has been no clear evidence of new physics beyond the Standard Model (SM) of particle physics. In the absence of any experimental signal of new fundamental particles, resonances, interactions, or distortions from SM interactions, the most appropriate tool for studying Beyond Standard Model (BSM) physics is mostly agreed to be provided by the Effective Field Theory (EFT) approach. In this work we will focus on the EFT approach to describe BSM Higgs physics and will assume that the potential new physics appears exclusively in the bosonic sector, including both the scalar and the gauge boson sectors. Consequently, the fermions and their couplings to the other SM particles are assumed here to be as in the SM. As for the particular EFTs, we will consider here the two most popular ones, nowadays widely employed for tests of BSM Higgs physics at colliders: the so-called Higgs Effective Field Theory

(HEFT) and the Standard Model Effective Field Theory (SMEFT). The advantage of using EFTs is that they allow for a test of the BSM Higgs physics in a model-independent way, i.e., without assuming a particular underlying fundamental theory which operates at the ultraviolet (UV) high-energy scale, this being generically called Λ . At low energies compared to Λ , the information of the new Higgs physics is encoded in an effective Lagrangian in terms of a set of effective operators, these being built with the SM fields (here the bosonic fields) and with the unique requirement of being invariant under the SM gauge symmetry, $SU(3) \times SU(2) \times U(1)$. The coefficients in front of these operators (usually called Wilson coefficients) are generically unknown and encode information of the particular underlying fundamental theory [for reviews, see for instance [3,4]]. For our study here, we focus on the electroweak (EW) part of these two EFTs and restrict ourselves to the CP preserving effective operators.

The values of the EFT coefficients can then be computed by integrating out the heavy modes from the particular assumed UV theory. It is well known that depending on the kind of dynamics involved in the fundamental theory, one EFT or another is more appropriate to use. Usually, SMEFT is more appropriate to describe the low-energy behavior of weakly interacting dynamics, where by construction the heavy modes of the UV theory decouple from low-energy physics and the observable effects left at the colliders go as inverse powers of Λ . In contrast, HEFT is more appropriate to describe strongly interacting underlying UV

*Contact author: daniel.domenech@uam.es

†Contact author: maria.herrero@uam.es

‡Contact author: roberto.morales@fisica.unlp.edu.ar

§Contact author: alex sala@ucm.es

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dynamics where the heavy modes may not decouple from the low-energy physics, and the effects left at the colliders might not all go as inverse powers of Λ . An interesting example of this different behavior of the two EFTs is provided when considering the particular case of the Two Higgs Doublet Model (2HDM) as the UV theory operating at the high-energy Λ . It has been shown that, depending on the particular values assumed on the input 2HDM parameters (basically, the masses m_{H^0} , $m_{H^\pm}$, and m_{A^0} , and the mixing angles $\cos(\beta - \alpha)$ and $\tan \beta$) and after integrating out the heavy scalars H^0 , $H^\pm$, and A^0 , the resulting EFT with the lightest Higgs boson h^0 being identified with the observed Higgs particle H of mass $m_H = 125$ GeV can be either SMEFT [5] (with coefficients indicating decoupling of H^0 , $H^\pm$ and A^0), or HEFT [6] (with coefficients indicating nondecoupling of H^0 , $H^\pm$ and A^0). The final step in disentangling the two approaches is then to test with the present and future colliders, which are the values of their respective EFT coefficients that fit best to experimental data.

At the practical level, the main differences between these two bosonic EFTs are, on the one hand, the representation used for the bosonic fields and, on the other hand, the criteria to order the effective operators built with these bosonic fields. In SMEFT the scalar fields (both the Higgs and the would be Goldstone bosons) are assumed to be in a linear representation of the electroweak symmetry group, specifically in a doublet of $SU(2)$, and the criterion to order the SMEFT effective operators is by increasing their canonical dimension. In the HEFT, the Higgs boson is a singlet under all symmetries [including $SU(2)$] whereas the would-be Goldstone bosons are placed in a nonlinear representation of $SU(2)$, usually by an exponential function. The HEFT effective operators are ordered differently, by their chiral dimension which counts the powers of momenta and masses (m_H , m_W , and m_Z in the present case). Thus, one usually refers to truncations of the respective Lagrangians as: 1) NLO-SMEFT, which includes up to canonical dimension 6 effective operators, and 2) NLO-HEFT, which includes up to chiral dimension 4 effective operators. On the other hand, both EFTs incorporate the gauge particles by the gauge principle based on the same gauge symmetry of the SM, i.e., $SU(3) \times SU(2) \times U(1)$. Both SMEFT and HEFT are renormalizable in the weaker sense of EFTs, and the one-loop renormalization in the generic R_ξ covariant gauges has been provided in the literature for both SMEFT [7] and the HEFT [8–10]. For more details and relevant effective operators regarding multiple Higgs boson production within HEFT, see also [11–14].

The question of how to relate both theories has also been considered in the literature. Most frequently, these two EFTs are related by comparing their respective truncated Lagrangians. For instance, the NLO-HEFT and the NLO-SMEFT can be related by comparing their respective predictions for the so-called anomalous couplings (κ_W , κ_Z , κ_γ , etc.). This is equivalent to comparing their

respective predictions for the interaction vertices, i.e., their corresponding Feynman rules in the momentum space. This is done, for instance, in Ref. [15], where they conclude on the role of the so-called SMEFT at dimension 6 sibling effective operators and also on the decorrelation/correlation among the effective coefficients when comparing HEFT and SMEFT [see also [16]]. The use of coupling modifiers κ_V , κ_{2V} , etc. to disentangle BSM patterns from the two SMEFT and HEFT approaches at colliders has also been considered in the literature, see for instance [17,18]. Another type of comparison of HEFT and SMEFT at the Lagrangian level has been done, in this case for the pure scalar theory (Higgs and GBs), in Refs. [19–22]. They relate the truncated Lagrangians of HEFT and SMEFT by performing a series of transformations of the scalar fields and arrive to particular relations among the HEFT and SMEFT coefficients that are involved in the scalar field interactions. Another different and more complete way to compare these two theories is by their respective effective actions, or equivalently their predictions for the complete set of one-particle irreducible (1PI) functions [see, for instance, the review [3] and references therein]. Other distinct techniques apply on-shell methods in EFTs to calculate tree-level amplitudes with the spinor helicity formalism. These techniques have been used in the literature in the recent years to construct the operator basis, both in HEFT [23,24] and in SMEFT [25]. They have also been applied to compare the two EFTs, HEFT and SMEFT, in some particular cases of two-to-two scattering on-shell amplitudes [see, for instance, Ref. [26]]. Finally, there are also comparisons using arguments based on a geometric description of the scalar field space [27–31]. Within this geometric approach, it is well established that SMEFT can always be cast in HEFT form and not the other way around; therefore, it is concluded that HEFT is more general than SMEFT.

In the present work, we proceed as in Ref. [17] by comparing these two EFTs in a different way, which we believe is better motivated for phenomenological purposes. Instead of comparing their Lagrangian and/or interaction vertices, we compare their respective predictions for scattering and decay amplitudes which are the mathematical objects directly related to physical observables, i.e., to cross sections, differential cross sections, total and partial decay widths, etc. More specifically, we choose a set of physical processes, make analytical predictions for the corresponding amplitudes of these processes from the two truncated NLO-HEFT and NLO-SMEFT at the tree level, identify correspondingly these amplitudes of the two theories, and finally solve analytically for the solution of the coefficients of the two EFTs fulfilling these matching equations for the full set of amplitudes considered. As stated here, the predicted amplitudes in both theories are Lorentz and gauge-invariant functions, and therefore the solutions for the coefficients from the matching equations are unambiguously determined. The set of processes

considered here are motivated by present and future searches of new Higgs physics in multiple Higgs productions at colliders. In particular, we include here the following electroweak processes: single, double, and triple Higgs production from Weak Boson Fusion (WBF), $WW \rightarrow H$, $ZZ \rightarrow H$, $WW \rightarrow HH$, $ZZ \rightarrow HH$, $WW \rightarrow HHH$, $ZZ \rightarrow HHH$, and add, for completeness, the case $HH \rightarrow HH$. Then we solve in this paper the HEFT/SMEFT matching equations for all the corresponding physical amplitudes, including all possible polarizations for the external gauge bosons (longitudinal and transverse). Finally, we conclude by presenting the solutions to all these matching equations in terms of the relations among their corresponding HEFT/SMEFT coefficients and comment on the potential implications for collider phenomenology of those relations. Particular attention will be devoted to the contact interactions $WWHHH$ and $ZZHHH$, which naturally arise at LO in the HEFT. On the experimental side, a recent ATLAS analysis [32] focuses on triple Higgs boson production in the $6b$ -jets final state, as proposed in [33,34] [see also [35,36] for the key challenges in future HHH searches].

The paper is organized as follows. In Sec. II, we review the relevant NLO-HEFT bosonic operators for the processes of most interest here, multiple Higgs production from WBF. In Sec. III, we review the NLO-SMEFT relevant bosonic operators for these same processes. In Sec. IV, we present the analytical predictions for the scattering amplitudes of those processes from both NLO-HEFT and NLO-SMEFT in terms of the selected effective bosonic operators and the corresponding EFTs coefficients. In Sec. V, we discuss the matching equations for the amplitudes of the selected processes and present their solutions in terms of relations among the corresponding HEFT and SMEFT coefficients. Finally, in Sec. VII, we summarize the conclusions and comment on the phenomenological implications at colliders.

II. RELEVANT HEFT OPERATORS FOR DOUBLE AND TRIPLE HIGGS WBF PRODUCTION

The relevant EW effective bosonic operators for the present work are contained in the so-called Next to Leading Order (NLO) HEFT Lagrangian (also called Electroweak Chiral Lagrangian in the literature). Here, we restrict ourselves to the CP invariant subset of effective operators. It is customary to present this NLO Lagrangian separated into two pieces, following the standard conventions for Chiral Lagrangians, where the first piece is assigned to chiral dimension 2, usually named leading order (LO) Lagrangian or $\mathcal{L}_2$, and the second piece $\mathcal{L}_4$ is assigned to chiral dimension 4. This separation in chiral dimension is not really necessary for the present work, but helps in understanding the posterior comparison of the corresponding amplitudes for HEFT and SMEFT. Remember that in the Chiral Lagrangian context it is customary to assign a double role to the $\mathcal{L}_4$ part. On the one hand, it provides the

needed counterterms for the renormalization of the one-loop contributions from $\mathcal{L}_2$. On the other hand, it provides new contributions to the scattering amplitudes that encode BSM physics, which can be checked experimentally. These contributions from $\mathcal{L}_4$ in the scattering amplitudes are considered in momentum space as NLO with respect to the LO contributions from $\mathcal{L}_2$ since they count as $\mathcal{O}(p^4)$, where p refers to either momentum or mass, in contrast to the contributions from $\mathcal{L}_2$ that count as $\mathcal{O}(p^2)$. For the present work, we will compute all the scattering amplitudes at the tree level, and therefore the role played by the $\mathcal{L}_4$ part will be just the second one.

We summarize the HEFT Lagrangian that is relevant for this work in the following:

$$\mathcal{L}_{\text{HEFT}}^{\text{NLO}} = \mathcal{L}_2 + \mathcal{L}_4. \quad (2.1)$$

The LO Lagrangian is given by

$$\begin{aligned} \mathcal{L}_2 = & \frac{v^2}{4} \left(1 + 2a \frac{H}{v} + b \frac{H^2}{v^2} + c \frac{H^3}{v^3} \right) \text{Tr}[D_\mu U^\dagger D^\mu U] \\ & + \frac{1}{2} \partial_\mu H \partial^\mu H - V(H) - \frac{1}{2g^2} \text{Tr}[\hat{W}_{\mu\nu} \hat{W}^{\mu\nu}] \\ & - \frac{1}{2g'^2} \text{Tr}[\hat{B}_{\mu\nu} \hat{B}^{\mu\nu}] + \mathcal{L}_{GF} + \mathcal{L}_{FP}, \end{aligned} \quad (2.2)$$

where H is the Higgs field which is introduced in the HEFT as a singlet field, in contrast to the SM or SMEFT, where it is introduced as a component of the usual doublet Φ . Due to this singlet nature of H within the HEFT, the first term in $\mathcal{L}_2$ contains a polynomial function in powers of (H/v) which, for the present work, we take up to cubic powers. Then there are three coefficients involved in this function, a , b , and c , that will have consequences for the phenomenology of multiple Higgs production from WBF that we will discuss later.

The U field is a 2×2 matrix

$$U = \exp \left(i \frac{\omega_i \tau_i}{v} \right), \quad (2.3)$$

that contains the three GB fields ω_i ($i = 1, 2, 3$) in a nonlinear representation of the $SU(2)$ symmetry group and τ_i ($i = 1, 2, 3$) are the three Pauli matrices.

The EW covariant derivative of this U field is defined as

$$D_\mu U = \partial_\mu U - i \hat{W}_\mu U + i U \hat{B}_\mu, \quad (2.4)$$

that contains the EW gauge fields $\hat{W}_\mu = \frac{g}{2} W_\mu^i \tau^i$ and $\hat{B}_\mu = \frac{g'}{2} B_\mu \tau^3$ and the EW gauge couplings g and g' . The corresponding EW field strength tensors are given by

$$\hat{W}_{\mu\nu} = \partial_\mu \hat{W}_\nu - \partial_\nu \hat{W}_\mu - i [\hat{W}_\mu, \hat{W}_\nu], \quad \hat{B}_{\mu\nu} = \partial_\mu \hat{B}_\nu - \partial_\nu \hat{B}_\mu. \quad (2.5)$$

The physical gauge fields are then given, as usual, by

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2), \quad Z_\mu = c_W W_\mu^3 - s_W B_\mu, \\ A_\mu = s_W W_\mu^3 + c_W B_\mu, \quad (2.6)$$

where we use the short notation $s_W = \sin \theta_W$ and $c_W = \cos \theta_W$, with θ_W being the weak angle. $V(H)$ is the Higgs potential within the HEFT, which includes the triple and quartic Higgs self-interactions:

$$V(H) = \frac{1}{2}m_H^2 H^2 + \kappa_3 \lambda v H^3 + \kappa_4 \frac{\lambda}{4} H^4. \quad (2.7)$$

The relations among couplings and masses within the LO-HEFT are $m_H^2 = 2\lambda v^2$, $m_W = gv/2$, and $m_Z = m_W/c_W$.

The vacuum expectation value v can be written in terms of the Fermi coupling constant G_F by $v = (\frac{1}{\sqrt{2}G_F})^{1/2} = 246$ GeV.

Finally, $\mathcal{L}_{GF}$ and $\mathcal{L}_{FP}$ are the gauge fixing and Faddeev-Popov terms, respectively, whose explicit expressions in the general R_ξ covariant gauges can be found in [8–10], together with more definitions and specifications within the HEFT [see also [13,14]].

The remaining operators in the NLO Lagrangian that are relevant for the present computation of double and triple Higgs production from WBF are contained in $\mathcal{L}_4$. We mention here only those affecting the interactions of the scalar sector with the EW gauge bosons and the self-interactions of the scalar sector [10,13,14] [for a more complete list of NLO effective operators see [15]]:

$$\begin{aligned} \mathcal{L}_4 = & -a_{dd\nu\nu 1} \frac{\partial^\mu H \partial^\nu H}{v^2} \text{Tr}[\mathcal{V}_\mu \mathcal{V}_\nu] - a_{dd\nu\nu 2} \frac{\partial^\mu H \partial_\mu H}{v^2} \text{Tr}[\mathcal{V}^\nu \mathcal{V}_\nu] - \frac{m_H^2}{4} \left(2a_{H\nu\nu} \frac{H}{v} + a_{HH\nu\nu} \frac{H^2}{v^2} \right) \text{Tr}[\mathcal{V}^\mu \mathcal{V}_\mu] \\ & + \left(a_{H0} \frac{H}{v} + a_{HH0} \frac{H^2}{v^2} + a_{HHH0} \frac{H^3}{v^3} \right) (m_Z^2 - m_W^2) \text{Tr}[U \tau^3 U^\dagger \mathcal{V}_\mu] \text{Tr}[U \tau^3 U^\dagger \mathcal{V}^\mu] \\ & + \left(\hat{a}_{Hdd} \frac{H}{v} + \hat{a}_{HHdd} \frac{H^2}{v^2} \right) \partial^\mu H \partial_\mu H + a_{ddd} \frac{1}{v^4} \partial_\mu H \partial^\mu H \partial_\nu H \partial^\nu H \\ & - \left(a_{HWW} \frac{H}{v} + a_{HHWW} \frac{H^2}{v^2} \right) \text{Tr}[\hat{W}_{\mu\nu} \hat{W}^{\mu\nu}] + i \left(a_{d2} + a_{Hd2} \frac{H}{v} \right) \frac{\partial^\nu H}{v} \text{Tr}[\hat{W}_{\mu\nu} \mathcal{V}^\mu] \\ & - \left(a_{HBB} \frac{H}{v} + a_{HHBB} \frac{H^2}{v^2} \right) \text{Tr}[\hat{B}_{\mu\nu} \hat{B}^{\mu\nu}] + i \left(a_{d1} + a_{Hd1} \frac{H}{v} \right) \frac{\partial^\nu H}{v} \text{Tr}[U \hat{B}_{\mu\nu} U^\dagger \mathcal{V}^\mu] \\ & + \left(a_{H1} \frac{H}{v} + a_{HH1} \frac{H^2}{v^2} \right) \text{Tr}[U \hat{B}_{\mu\nu} U^\dagger \hat{W}^{\mu\nu}], \end{aligned} \quad (2.8)$$

where we have used the usual definition $\mathcal{V}_\mu \equiv (D_\mu U)U^\dagger$ and a short notation with respect to [10,13] given by $\hat{a}_{Hdd} = a_{Hdd} \frac{m_W^2}{v^2} + a_{ddW} \frac{m_W^2}{v^2} + a_{ddZ} \frac{m_Z^2}{v^2}$ and $\hat{a}_{HHdd} = a_{HHdd} \frac{m_W^2}{v^2} + a_{HddW} \frac{m_W^2}{v^2} + a_{HddZ} \frac{m_Z^2}{v^2}$. Notice that in the list above, the full set of bosonic operators in the NLO Lagrangian, displayed in Ref. [15], has been reduced to a minimal set by the use of the equations of motion. For further details, and the relation among the various notations, see [10].

It is important to remark that the previous HEFT Lagrangian in Eq. (2.1) is invariant under the EW $SU(2)_L \times U(1)_Y$ gauge symmetry and is renormalizable in the weaker sense of EFTs with a truncated Lagrangian. The one-loop renormalization of the previous truncated HEFT Lagrangian in Eq. (2.1) is possible by the proper renormalization of the HEFT parameters involved, including the particle masses, couplings, fields, and both the LO ($a, b, c, \kappa_3, \kappa_4$) and NLO (a_i 's) HEFT coefficients. The one-loop renormalization of the bosonic HEFT in R_ξ covariant gauges has been performed in a series of works, see Refs. [8–10,13], where the counterterms of the HEFT parameters and fields in Eq. (2.1) have been set.

The Feynman rules of the relevant HEFT vertices are summarized in Fig. 1 and in Appendix B. These relevant vertices include the effective interactions of the Higgs boson H with the electroweak gauge bosons $V = W^\pm, Z$ and the Higgs self-interactions. More specifically, we focus on the following interactions: VVH , $VVHH$, $VVHHH$ with $VV = WW, ZZ$, and HHH , $HHHH$. As can be seen in these Feynman rules, both the LO and NLO HEFT coefficients are involved in these effective interactions. Moreover, as will be shown in the forthcoming sections, we wish to remark that only a subset of these HEFT coefficients will be needed to solve the matching of the HEFT and SMEFT amplitudes that we are aiming to provide in this work. Specifically, we will present analytical results for the scattering amplitudes of the following specific processes including single, double and triple Higgs production: $WW \rightarrow H$, $ZZ \rightarrow H$, $HH \rightarrow HH$, $WW \rightarrow HH$, $ZZ \rightarrow HH$, $WW \rightarrow HHH$, and $ZZ \rightarrow HHH$. These processes have been selected because they participate as possible subprocesses in multiple Higgs production at future lepton and hadron colliders. In particular these subprocesses with initial EW gauge boson pairs are

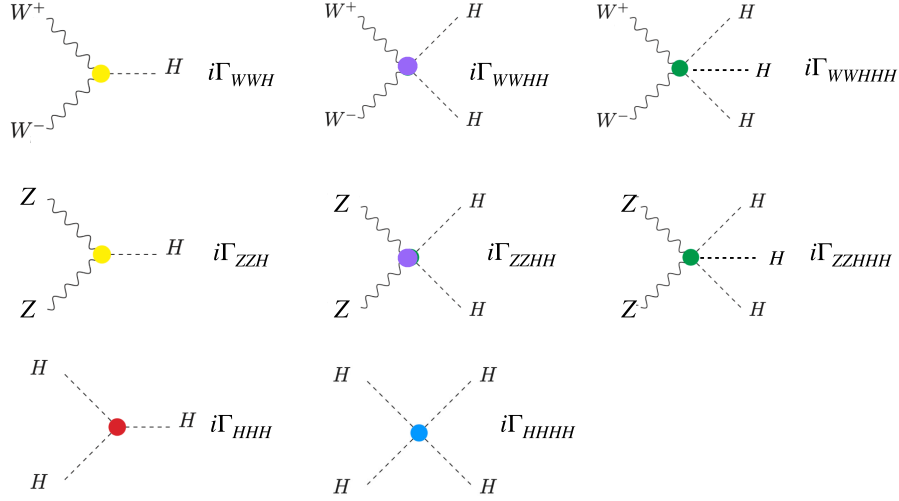


FIG. 1. Relevant effective vertices for double and triple Higgs production. The corresponding values of Γ_{WWH} , Γ_{WWHH} , Γ_{ZZH} , Γ_{ZZHH} , Γ_{HHH} , and Γ_{HHHH} are given in Appendix B for HEFT and in Appendix C for SMEFT. We use different colors in these BSM vertices to visually differentiate them more clearly. For comparison, the corresponding SM Feynman rules are also included in Appendix A.

essential for the production mechanism called single H , double HH , and triple HHH production from weak boson fusion.

For our main objective here, i.e., solving the matching HEFT/SMEFT, we have selected the proper subset of HEFT coefficients. It will be shown later that these HEFT coefficients involved in the selected processes are the following 14 specific coefficients: a , b , c , κ_3 , κ_4 , a_{H0} , a_{HH0} , a_{HHH0} , a_{HWW} , a_{HHWW} , a_{HBB} , a_{HHBB} , a_{H1} , and a_{HH1} .

Regarding the comparison of the HEFT with the SM, it can also be checked that the SM Feynman rules, summarized in Appendix A, can be reached from the HEFT Feynman rules, summarized in Appendix B, by setting $a = b = \kappa_3 = \kappa_4 = 1$, $c = 0$, and $a_i = 0$ for the rest of HEFT coefficients. This is akin to saying that the proper choice for the HEFT parameters that match the SM predictions for the scattering amplitudes are: $a = b = \kappa_3 = \kappa_4 = 1$, and $c = 0$. The rest of HEFT coefficients a_i in $\mathcal{L}_4$ must also be set to zero in order to match the HEFT with the SM. Notice that the contact interactions parameterized by c are absent at the tree level within the SM. Thus, any experimentally detectable effect from $c \neq 0$ and/or $a_i \neq 0$ will provide clear signals of BSM Higgs Physics at colliders [13,14,17,37,38].

Regarding the codes used in this work, for the forthcoming computations we use WOLFRAM MATHEMATICA [39] with our own model file that implements the previous HEFT Lagrangian in the R_ξ gauges, and derive the Feynman rules and scattering amplitudes with the help of FeynRules [40] and FormCalc [41]. We also performed cross checks with the help of FeynCalc [42]. As for the drawing of Feynman diagrams, we use FeynArts [43].

For phenomenological discussions, it is convenient to relate these LO-HEFT coefficients with the so-called κ modifiers which parameterize the coupling deviations with

respect to the SM couplings. In particular,

$$a = \kappa_V, \quad b = \kappa_{2V}, \quad \kappa_3 = \kappa_\lambda. \quad (2.9)$$

Some progress for testing these anomalous couplings by multiple Higgs production from Vector Boson Fusion at LHC is being discussed in the literature, with particular focus on improving the precision tools using the κ -formalism; see, for instance, [44,45].

Generically, as stated above, the LO-HEFT parameters a , b , κ_3 , and κ_4 are set to 1 for the matching to the SM. Then, the deviations of the LO-HEFT predictions from the SM predictions should be parameterized in terms of the differences of these LO-HEFT coefficients with respect to 1. We define here these differences as $\Delta a \equiv 1 - a$, $\Delta b \equiv 1 - b$, $\Delta \kappa_3 \equiv 1 - \kappa_3$, and $\Delta \kappa_4 \equiv 1 - \kappa_4$.

The present data from LHC set important constraints on these κ parameters of Eq. (2.9), which can be translated directly into constraints on the LO-HEFT parameters a , b and κ_3 . The quartic coupling κ_4 is not constrained yet. These constraints can be shortly summarized as follows; see [46–50] for the corresponding analysis' assumptions:

$$\begin{aligned} \text{ATLAS: } \kappa_V &\in (0.96, 1.11)(95\%CL), \\ \kappa_{2V} &\in (0.6, 1.5)(95\%CL), \\ \kappa_\lambda &\in (-0.4, 6.3)(95\%CL), \end{aligned} \quad (2.10)$$

$$\begin{aligned} \text{CMS: } \kappa_V &\in (0.97, 1.09)(95\%CL), \\ \kappa_{2V} &\in (0.6, 1.5)(95\%CL), \\ \kappa_\lambda &\in (-1.4, 7.8)(95\%CL), \end{aligned} \quad (2.11)$$

III. RELEVANT SMEFT OPERATORS FOR DOUBLE AND TRIPLE HIGGS WBF PRODUCTION

Within the SMEFT, the set of EW bosonic effective operators that are relevant for the present work are contained in the following NLO SMEFT Lagrangian:

$$\mathcal{L}_{\text{SMEFT}}^{\text{NLO}} = \mathcal{L}_{\text{SMEFT}}^{(4)} + \mathcal{L}_{\text{SMEFT}}^{(6)}, \quad (3.1)$$

where the LO-SMEFT Lagrangian, with canonical dimension 4, is the SM Lagrangian of the bosonic sector, and $\mathcal{L}_{\text{SMEFT}}^{(6)}$ contains the set of relevant SMEFT operators with canonical dimension 6. We use here the basis of operators and notation as defined in [51]. The LO-SMEFT Lagrangian can then be written as

$$\begin{aligned} \mathcal{L}_{\text{SMEFT}}^{(4)} = \mathcal{L}_{\text{SM}} = & -\frac{1}{4} W_{\mu\nu}^I W^{I\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \mathcal{L}_{GF}^{\text{SM}} + \mathcal{L}_{FP}^{\text{SM}} \\ & + (D_\mu \Phi)^\dagger (D^\mu \Phi) + m^2 \Phi^\dagger \Phi - \frac{1}{2} \lambda (\Phi^\dagger \Phi)^2, \end{aligned} \quad (3.2)$$

where Φ is the SM complex scalar doublet,

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + H + i\phi^0) \end{pmatrix}, \quad (3.3)$$

and the conventions for the covariant derivative and SM electroweak field strengths are set here as in [51]:

$$\begin{aligned} D_\mu &= \partial_\mu + ig' B_\mu Y + ig W_\mu^I T^I, \quad I = 1, 2, 3, \\ W_{\mu\nu}^I &= \partial_\mu W_\nu^I - \partial_\nu W_\mu^I - g\epsilon^{IJK} W_\mu^J W_\nu^K, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu. \end{aligned} \quad (3.4)$$

The definitions of the gauge fixing Lagrangian in the covariant R_ξ gauges, $\mathcal{L}_{GF}^{\text{SM}}$, and the Fadeev-Popov Lagrangian, $\mathcal{L}_{FP}^{\text{SM}}$, are the usual ones in the SM. It should be noticed that the usual normalization in the SMEFT for the Higgs self-coupling in Eq. (3.2) is different than its usual normalization in the HEFT in Eq. (2.7). Specifically, $\lambda_{\text{SMEFT}} = 2\lambda_{\text{HEFT}}$.

The operators of canonical dimension 6 that are relevant for the present work are

$$\begin{aligned} \mathcal{L}_{\text{SMEFT}}^{(6)} = & \frac{c_\Phi}{\Lambda^2} (\Phi^\dagger \Phi)^3 + \frac{c_{\Phi\Box}}{\Lambda^2} (\Phi^\dagger \Phi) \Box (\Phi^\dagger \Phi) + \frac{c_{\Phi D}}{\Lambda^2} (\Phi^\dagger D^\mu \Phi)^* (\Phi^\dagger D_\mu \Phi) \\ & + \frac{c_{\Phi W}}{\Lambda^2} (\Phi^\dagger \Phi) W_{\mu\nu}^I W^{I\mu\nu} + \frac{c_{\Phi B}}{\Lambda^2} (\Phi^\dagger \Phi) B_{\mu\nu} B^{\mu\nu} + \frac{c_{\Phi WB}}{\Lambda^2} (\Phi^\dagger \tau^I \Phi) W_{\mu\nu}^I B^{\mu\nu}. \end{aligned} \quad (3.5)$$

Notice again that we have restricted ourselves to the CP invariant subset of operators. Then, there are in total six Wilson coefficients involved: c_Φ , $c_{\Phi\Box}$, $c_{\Phi D}$, $c_{\Phi W}$, $c_{\Phi B}$, and $c_{\Phi WB}$. All these c_i 's are dimensionless and we keep the prefactor $1/\Lambda^2$ explicitly in the forthcoming computation. Again, it is important to remark that the previous SMEFT Lagrangian in Eq. (3.1) is invariant under the electroweak $SU(2)_L \times U(1)_Y$ gauge symmetry. The one-loop renormalization in R_ξ covariant gauges of the previous SMEFT Lagrangian has been set, and the full list of Feynman rules has also been provided in [7].

The Feynman rules of the relevant SMEFT effective vertices are summarized in Fig. 1 and in Appendix C. For the following computations we use the rules from the `SmeftFR-v3` package [52]. As in the previous HEFT case, the computations of the scattering amplitudes are performed with the help of `FeynRules` [40] and `FormCalc` [41]. We have also performed cross checks with the help of `FeynCalc` [42]. As for the drawing of Feynman diagrams, we use `FeynArts` [43].

IV. HEFT AND SMEFT AMPLITUDES FOR $VV \rightarrow HH$ AND $VV \rightarrow HHH$

Here, we summarize the analytical results for double and triple Higgs production from WBF. The computation is

performed at the tree level. We choose to compute them in the unitary gauge and present here the results separately for all the contributing diagrams. The diagrams for $VV \rightarrow HH$ are displayed in Fig. 2. The diagrams for $VV \rightarrow HHH$ are displayed in Fig. 3.

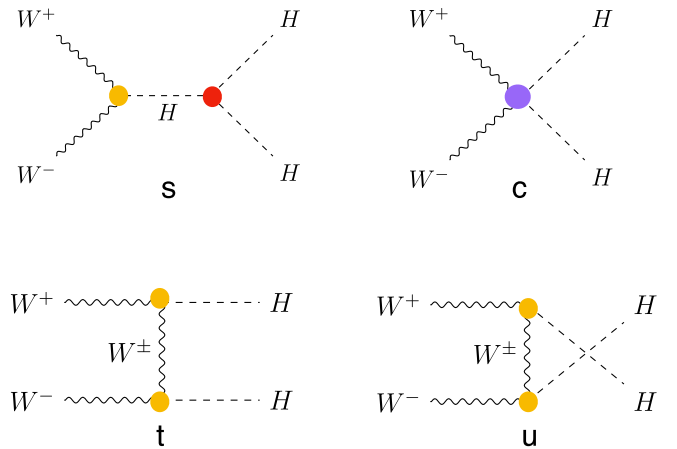


FIG. 2. Feynman diagrams for $WW \rightarrow HH$ scattering in the unitary gauge. The labels s, c, t, and u refer to s-channel, c-channel, t-channel, and u-channel respectively. The diagrams for $ZZ \rightarrow HH$ are similar but changing W by Z .

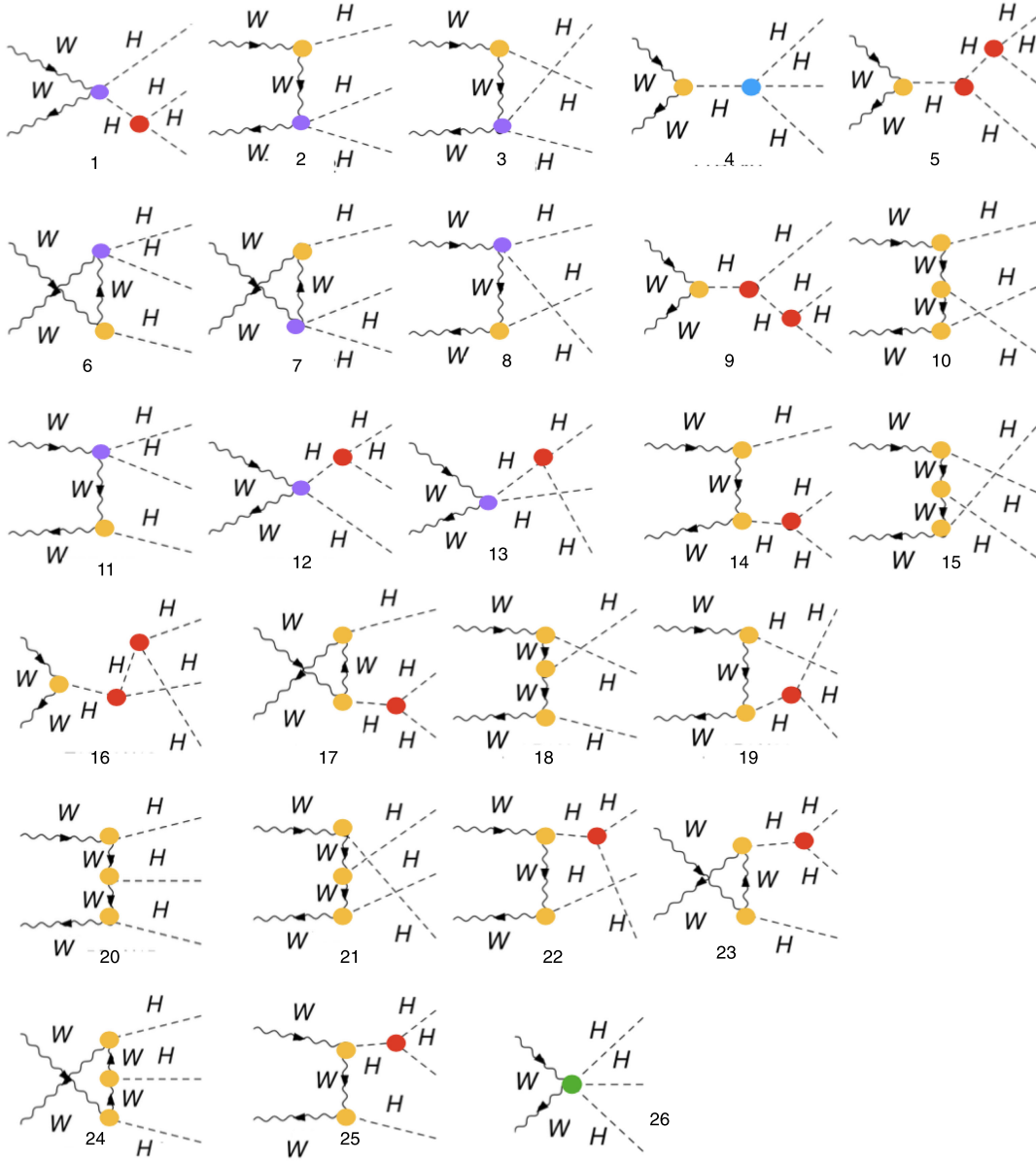


FIG. 3. Feynman diagrams for $WW \rightarrow HHH$ scattering in the unitary gauge. These diagrams are numbered from 1 to 26. The diagrams for $ZZ \rightarrow HHH$ are similar but changing W by Z .

An important point that is worth emphasizing here is the explicit demonstration that all these scattering amplitudes are gauge invariant. In particular, we have checked the full agreement in the predictions of all the scattering amplitudes when computed in the unitary gauge with the results when computed in the R_ξ gauges. Notice that in this later case, the full set of diagrams including also the internal Goldstone bosons (not shown here for shortness) must be considered. This check involves demonstrating explicitly the ξ independence of the resulting amplitude in all the studied cases for $VV \rightarrow HH$ and $VV \rightarrow HHH$ with $VV = WW$ and $VV = ZZ$, and for the two EFT predictions, HEFT and SMEFT. This gauge invariance of all the scattering amplitudes is an important feature for the forthcoming comparison

of the resulting HEFT and SMEFT amplitudes and for solving the corresponding matching equations. By means of the computation of these amplitudes within the SM in the R_ξ gauges (not shown here for brevity), we have also checked the ξ independence of all the SM amplitudes, as expected.

We summarize all these HEFT and SMEFT analytical results in the following subsections. We present all these results in the physical scheme where the amplitudes are written in terms of physical parameters for on-shell external particles, namely the physical masses, m_H , m_W , m_Z and the vacuum expectation value related to the Fermi constant $v = (\frac{1}{\sqrt{2}G_F})^{1/2} = 246$ GeV. In addition, the HEFT and SMEFT results are given in terms of the corresponding EFT coefficients. Notice that the following results are for all the

possible polarizations of the external EW gauge bosons, including both longitudinal and transverse modes. Each particular helicity amplitude can be fixed by setting the proper polarization vectors in our analytical results. Finally, in the cases where the scattering is initiated by ZZ, and for both HEFT and SMEFT results, we have used in addition the convenient short notation for some frequently appearing mass combinations:

$$\begin{aligned} c_W^2 &= \frac{m_W^2}{m_Z^2}, & s_W^2 &= \frac{m_Z^2 - m_W^2}{m_Z^2}, \\ s_W c_W &= \frac{m_W \sqrt{m_Z^2 - m_W^2}}{m_Z^2}. \end{aligned} \quad (4.1)$$

A. $WW \rightarrow HH$

We use the following momenta assignments $W^+(k_1)W^-(k_2) \rightarrow H(k_3)H(k_4)$ with k_1 and k_2 ingoing

and k_3 and k_4 outgoing. The conservation of momentum is, then, $k_1 + k_2 = k_3 + k_4$. To simplify, we also used here $\epsilon_1 \cdot k_1 = 0, \epsilon_2 \cdot k_2 = 0$, where ϵ_1 and ϵ_2 are the polarization vectors of the ingoing W^+ and W^- , respectively.

1. HEFT

The HEFT results for the scattering amplitude $A(WW \rightarrow HH)$ contain two parts corresponding to the contributions from $\mathcal{L}_2$ and $\mathcal{L}_4$, respectively. We present them here separated by channels. The diagrams for these channels are shown in Fig. 2. The part from $\mathcal{L}_2$ coincides with the SM result if one sets $a = b = \kappa_3 = 1$. The part from $\mathcal{L}_4$ contains the a_i 's HEFT coefficients. For brevity, we keep here just the subset of HEFT coefficients that are needed to solve the forthcoming matching of the HEFT and the SMEFT amplitudes. For this amplitude, these are a , b , κ_3 , a_{HWW} , and a_{HHWW} . We find the following results for the HEFT amplitude:

$$\begin{aligned} A^{\text{HEFT}} &= A_c^{\text{HEFT}} + A_s^{\text{HEFT}} + A_t^{\text{HEFT}} + A_u^{\text{HEFT}}, \\ A_c^{\text{HEFT}} &= 2bv^{-2}m_W^2(\epsilon_1 \cdot \epsilon_2) + \frac{2m_W^2}{v^4}(4a_{HHWW}(s - 2m_W^2)\epsilon_1 \cdot \epsilon_2 - 8a_{HHWW}\epsilon_1 \cdot k_2\epsilon_2 \cdot k_1), \\ A_s^{\text{HEFT}} &= \frac{6a\kappa_3 v^{-2}m_H^2 m_W^2}{s - m_H^2}(\epsilon_1 \cdot \epsilon_2) + \frac{2m_W^2}{v^4} \frac{1}{s - m_H^2}(6\kappa_3 a_{HWW}m_H^2((s - 2m_W^2)\epsilon_1 \cdot \epsilon_2 - 2\epsilon_1 \cdot k_2\epsilon_2 \cdot k_1)), \\ A_t^{\text{HEFT}} &= \frac{4a^2 v^{-2}m_W^2}{t - m_W^2}(m_W^2(\epsilon_1 \cdot \epsilon_2) + (\epsilon_1 \cdot k_3)(\epsilon_2 \cdot k_4)) \\ &\quad + \frac{2m_W^2}{v^4} \frac{a}{t - m_W^2}(-8a_{HWW}m_W^2((t + m_W^2 - m_H^2)\epsilon_1 \cdot \epsilon_2 + \epsilon_1 \cdot k_3\epsilon_2 \cdot k_1 + \epsilon_1 \cdot k_2\epsilon_2 \cdot k_4)), \\ A_u^{\text{HEFT}} &= A_t^{\text{HEFT}}(t \leftrightarrow u, k_3 \leftrightarrow k_4). \end{aligned} \quad (4.2)$$

Regarding the important issue of gauge invariance, it is worth commenting at this point that we have checked the gauge invariance of the separated contributions: A_s , A_t , A_u , and A_c . When these contributions by channels are computed in generic R_ξ gauges, the amplitudes A_s and A_c , which are the ones without vector boson propagators, are ξ independent and their values coincide in the unitary and R_ξ covariant gauges. The t-channel amplitude A_t , when computed in the R_ξ gauges, receives contributions from two diagrams, one with the W propagator and the other with the corresponding GB propagator, and when adding these two diagrams the ξ dependence cancels and the result of A_t is identical to the one obtained above in the unitary gauge, where there is just one diagram with the W

propagator. The same happens for A_u . We have checked that this gauge invariance separately by channels happens in both cases, HEFT and the SMEFT, and for both types of scattering with $VV = WW$ and $VV = ZZ$. Notice also that this gauge invariance refers to all helicity amplitudes, both with initial longitudinal and transverse EW gauge bosons.

2. SMEFT

The SMEFT results contain a first contribution that coincides with the SM result and a second contribution from $\mathcal{L}^{(6)}$, where the corresponding coefficients appear explicitly. We also present the SMEFT results separated by channels with the corresponding diagrams specified in Fig. 2:

$$\begin{aligned}
A^{\text{SMEFT}} &= A_c^{\text{SMEFT}} + A_s^{\text{SMEFT}} + A_t^{\text{SMEFT}} + A_u^{\text{SMEFT}}, \\
A_c^{\text{SMEFT}} &= 2v^{-2}m_W^2(\epsilon_1 \cdot \epsilon_2) + \frac{1}{\Lambda^2}(((4c_{\Phi\Box} - c_{\Phi D})m_W^2 - 2c_{\Phi W}(s - 2m_W^2))(\epsilon_1 \cdot \epsilon_2) + 4c_{\Phi W}(\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1)), \\
A_s^{\text{SMEFT}} &= \frac{6v^{-2}m_H^2m_W^2}{s - m_H^2}(\epsilon_1 \cdot \epsilon_2) - \frac{1}{(s - m_H^2)}\frac{1}{\Lambda^2}((12c_{\Phi}m_W^2v^2 \\
&\quad - (4c_{\Phi\Box} - c_{\Phi D})m_W^2(5m_H^2 + s) + 6c_{\Phi W}m_H^2(s - 2m_W^2))(\epsilon_1 \cdot \epsilon_2) - 12c_{\Phi W}m_H^2(\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1)), \\
A_t^{\text{SMEFT}} &= \frac{4v^{-2}m_W^2}{t - m_W^2}(m_W^2(\epsilon_1 \cdot \epsilon_2) + (\epsilon_1 \cdot k_3)(\epsilon_2 \cdot k_4)) \\
&\quad + \frac{2}{t - m_W^2}\frac{1}{\Lambda^2}m_W^2(((4c_{\Phi\Box} - c_{\Phi D})m_W^2 + 4c_{\Phi W}(t - m_H^2 + m_W^2))(\epsilon_1 \cdot \epsilon_2) \\
&\quad + 4c_{\Phi W}(\epsilon_1 \cdot k_2)((\epsilon_2 \cdot k_1) - (\epsilon_2 \cdot k_3)) + (\epsilon_1 \cdot k_3)(4c_{\Phi W}(\epsilon_2 \cdot k_1) + (4c_{\Phi\Box} - c_{\Phi D})(\epsilon_2 \cdot k_4))), \\
A_u^{\text{SMEFT}} &= A_t^{\text{SMEFT}}(t \leftrightarrow u, k_3 \leftrightarrow k_4).
\end{aligned} \tag{4.3}$$

B. $ZZ \rightarrow HH$

We use the following momenta assignments $Z(k_1)Z(k_2) \rightarrow H(k_3)H(k_4)$. Now, ϵ_1 and ϵ_2 are the polarization vectors of the two ingoing Z bosons, respectively. The other conventions and simplifications used here are as in $WW \rightarrow HH$.

1. HEFT

As in the previous case, for brevity, we keep just the coefficients that are needed for the matching of the HEFT with the SMEFT amplitude. We get the following results in this case:

$$\begin{aligned}
A^{\text{HEFT}} &= A_c^{\text{HEFT}} + A_s^{\text{HEFT}} + A_t^{\text{HEFT}} + A_u^{\text{HEFT}}, \\
A_c^{\text{HEFT}} &= 2v^{-2}bm_Z^2(\epsilon_1 \cdot \epsilon_2) + \frac{2m_Z^2}{v^4}(4((a_{HHWW}c_W^4 + a_{HHBB}s_W^4 + a_{HH1}s_W^2c_W^2)(s - 2m_Z^2) - a_{HH0}m_Z^2s_W^2)(\epsilon_1 \cdot \epsilon_2) \\
&\quad - 8(a_{HHWW}c_W^4 + a_{HHBB}s_W^4 + a_{HH1}s_W^2c_W^2)(\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1)), \\
A_s^{\text{HEFT}} &= \frac{6v^{-2}a\kappa_3m_H^2m_Z^2}{s - m_H^2}(\epsilon_1 \cdot \epsilon_2) \\
&\quad + \frac{2m_Z^2}{v^4}\frac{1}{s - m_H^2}6\kappa_3m_H^2(((a_{HHWW}c_W^4 + a_{HHBB}s_W^4 + a_{H1}s_W^2c_W^2)(s - 2m_Z^2) + 2a_{H0}m_Z^2s_W^2)(\epsilon_1 \cdot \epsilon_2) \\
&\quad - 2(a_{HHWW}c_W^4 + a_{HHBB}s_W^4 + a_{H1}s_W^2c_W^2)(\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1)), \\
A_t^{\text{HEFT}} &= \frac{4v^{-2}a^2m_Z^2}{t - m_Z^2}(m_Z^2(\epsilon_1 \cdot \epsilon_2) + (\epsilon_1 \cdot k_3)(\epsilon_2 \cdot k_4)) \\
&\quad + \frac{2m_Z^4}{v^4}\frac{a}{t - m_Z^2}(-8((a_{HHWW}c_W^4 + a_{HHBB}s_W^4 + a_{H1}s_W^2c_W^2)(t + m_Z^2 - m_H^2) + 2a_{H0}m_Z^2s_W^2)(\epsilon_1 \cdot \epsilon_2) \\
&\quad - 8(a_{HHWW}c_W^4 + a_{HHBB}s_W^4 + a_{H1}s_W^2c_W^2)(\epsilon_1 \cdot k_3\epsilon_2 \cdot k_1 + \epsilon_1 \cdot k_2\epsilon_2 \cdot k_4) - 16a_{H0}s_W^2(\epsilon_1 \cdot k_3)(\epsilon_2 \cdot k_4)), \\
A_u^{\text{HEFT}} &= A_t^{\text{HEFT}}(t \leftrightarrow u, k_3 \leftrightarrow k_4).
\end{aligned} \tag{4.4}$$

2. SMEFT

In SMEFT, we get the following results:

$$\begin{aligned}
A^{\text{SMEFT}} &= A_c^{\text{SMEFT}} + A_s^{\text{SMEFT}} + A_t^{\text{SMEFT}} + A_u^{\text{SMEFT}}, \\
A_c^{\text{SMEFT}} &= 2v^{-2}m_Z^2(\epsilon_1 \cdot \epsilon_2) + \frac{1}{\Lambda^2}(4(c_{\Phi\Box} + c_{\Phi D})m_Z^2(\epsilon_1 \cdot \epsilon_2) - 2(c_{\Phi B}s_W^2 + c_{\Phi W}c_W^2 + c_{\Phi WB}s_Wc_W) \\
&\quad \times ((s - 2m_Z^2)(\epsilon_1 \cdot \epsilon_2) - 2(\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1))), \\
A_s^{\text{SMEFT}} &= \frac{6v^{-2}m_H^2m_Z^2}{s - m_H^2}(\epsilon_1 \cdot \epsilon_2) - \frac{1}{(s - m_H^2)}\frac{1}{\Lambda^2}((12c_{\Phi}m_Z^2v^2 - 20c_{\Phi\Box}m_H^2m_Z^2 + 2c_{\Phi D}m_H^2m_Z^2 \\
&\quad + 6m_H^2(c_{\Phi B}s_W^2 + c_{\Phi W}c_W^2 + c_{\Phi WB}s_Wc_W)(s - 2m_Z^2) - (4c_{\Phi\Box} - c_{\Phi D})m_Z^2s)(\epsilon_1 \cdot \epsilon_2) \\
&\quad - 12m_H^2(c_{\Phi B}s_W^2 + c_{\Phi W}c_W^2 + c_{\Phi WB}s_Wc_W)(\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1)), \\
A_t^{\text{SMEFT}} &= \frac{4v^{-2}m_Z^2}{t - m_Z^2}(m_Z^2(\epsilon_1 \cdot \epsilon_2) + (\epsilon_1 \cdot k_3)(\epsilon_2 \cdot k_4)) + \frac{2m_Z^2}{t - m_Z^2}\frac{1}{\Lambda^2}((4c_{\Phi\Box} + c_{\Phi D})m_Z^2(\epsilon_1 \cdot \epsilon_2) \\
&\quad + 4(c_{\Phi B}s_W^2 + c_{\Phi W}c_W^2 + c_{\Phi WB}s_Wc_W)((t - m_H^2 + m_Z^2)(\epsilon_1 \cdot \epsilon_2) + (\epsilon_1 \cdot k_3)(\epsilon_2 \cdot k_1) \\
&\quad + (\epsilon_1 \cdot k_2)((\epsilon_2 \cdot k_1) - (\epsilon_2 \cdot k_3))) + (4c_{\Phi\Box} + c_{\Phi D})(\epsilon_1 \cdot k_3)(\epsilon_2 \cdot k_4)), \\
A_u^{\text{SMEFT}} &= A_t^{\text{SMEFT}}(t \leftrightarrow u, k_3 \leftrightarrow k_4).
\end{aligned} \tag{4.5}$$

C. $WW \rightarrow HHH$

We use the momenta assignments $W^+(k_1)W^-(k_2) \rightarrow H(k_3)H(k_4)H(k_5)$ with k_1 and k_2 ingoing and k_3, k_4 , and k_5 outgoing. The polarization vectors of the two ingoing W are denoted by ϵ_1 and ϵ_2 , respectively. The conservation of momentum used for simplifications is then $k_1 + k_2 = k_3 + k_4 + k_5$. In this case there are 26 contributing diagrams in the unitary gauge; they are depicted in

Fig. 3 with the corresponding label numbers. In this case of triple Higgs production, we use a convenient short notation for the frequently appearing momenta combinations, $k_{ij} = k_i - k_j$ and $q_{ij} = k_i + k_j$.

1. HEFT

Here we show the results of HEFT, making explicit the analytical result of each diagram.

$$\begin{aligned}
A^{\text{HEFT}} &= A_1^{\text{HEFT}} + \dots + A_{26}^{\text{HEFT}}, \\
A_1^{\text{HEFT}} &= 6m_W^2m_H^2v^{-3}\frac{1}{q_{45}^2 - m_H^2}(b\kappa_3)(\epsilon_1 \cdot \epsilon_2) - 48m_W^2m_H^2v^{-5}\frac{1}{q_{45}^2 - m_H^2}(\kappa_3a_{HHWW})((\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1) - (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_2)), \\
A_2^{\text{HEFT}} &= 4m_W^4v^{-3}\frac{1}{k_{13}^2 - m_W^2}(ab)((\epsilon_1 \cdot \epsilon_2) - m_W^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})) \\
&\quad - 16m_W^4v^{-5}\frac{1}{k_{13}^2 - m_W^2}(2aa_{HHWW}((\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_{13}) - (\epsilon_1 \cdot \epsilon_2)(k_2 \cdot k_{13})) \\
&\quad - ba_{HWW}((\epsilon_2 \cdot k_1)(\epsilon_1 \cdot k_{13}) - (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_{13}))), \\
A_3^{\text{HEFT}} &= A_2^{\text{HEFT}}(k_3 \leftrightarrow k_4), \\
A_4^{\text{HEFT}} &= 6m_W^2m_H^2v^{-3}\frac{(a\kappa_4)}{q_{12}^2 - m_H^2}(\epsilon_1 \cdot \epsilon_2) - 24m_W^2m_H^2v^{-5}\frac{a_{HWW}\kappa_4}{q_{12}^2 - m_H^2}((\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1) - (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_2)), \\
A_5^{\text{HEFT}} &= \frac{18m_W^2m_H^4v^{-3}}{(q_{12}^2 - m_H^2)(q_{34}^2 - m_H^2)}((a\kappa_3^2)(\epsilon_1 \cdot \epsilon_2) - 4a_{HWW}\kappa_3^2v^{-2}((\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1) - (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_2))), \\
A_6^{\text{HEFT}} &= A_2^{\text{HEFT}}(k_3 \leftrightarrow k_5), \\
A_7^{\text{HEFT}} &= A_2^{\text{HEFT}}(k_1 \leftrightarrow k_2),
\end{aligned}$$

$$\begin{aligned}
A_8^{\text{HEFT}} &= A_2^{\text{HEFT}}(k_3 \leftrightarrow k_4, k_1 \leftrightarrow k_2), \\
A_9^{\text{HEFT}} &= A_5^{\text{HEFT}}(k_3 \leftrightarrow k_5), \\
A_{10}^{\text{HEFT}} &= A_{20}^{\text{HEFT}}(k_4 \leftrightarrow k_5), \\
A_{11}^{\text{HEFT}} &= A_2^{\text{HEFT}}(k_3 \leftrightarrow k_5, k_1 \leftrightarrow k_2), \\
A_{12}^{\text{HEFT}} &= A_1^{\text{HEFT}}(k_3 \leftrightarrow k_5), \\
A_{13}^{\text{HEFT}} &= A_1^{\text{HEFT}}(k_3 \leftrightarrow k_4), \\
A_{14}^{\text{HEFT}} &= 12m_W^4 m_H^2 v^{-3} \frac{(a^2 \kappa_3)}{(k_{13}^2 - m_W^2)(q_{45}^2 - m_H^2)} ((\epsilon_1 \cdot \epsilon_2) - m_W^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})) \\
&\quad - 48m_W^4 m_H^2 v^{-5} \frac{a\kappa_3 a_{HWW}}{(k_{13}^2 - m_W^2)(q_{45}^2 - m_H^2)} ((\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_{13}) - (\epsilon_1 \cdot \epsilon_2)(k_2 \cdot k_{13})) \\
&\quad + 48m_W^4 m_H^2 v^{-5} \frac{a\kappa_3 a_{HWW}}{(k_{13}^2 - m_W^2)(q_{45}^2 - m_H^2)} ((\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_1) - (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_{13})), \\
A_{15}^{\text{HEFT}} &= A_{20}^{\text{HEFT}}((k_3, k_4, k_5) \rightarrow (k_4, k_5, k_3)), \\
A_{16}^{\text{HEFT}} &= A_5^{\text{HEFT}}(k_4 \leftrightarrow k_5), \\
A_{17}^{\text{HEFT}} &= A_{14}^{\text{HEFT}}(k_1 \leftrightarrow k_2), \\
A_{18}^{\text{HEFT}} &= A_{20}^{\text{HEFT}}(k_3 \leftrightarrow k_4), \\
A_{19}^{\text{HEFT}} &= A_{14}^{\text{HEFT}}(k_3 \leftrightarrow k_4), \\
A_{20}^{\text{HEFT}} &= 8m_W^6 v^{-3} \frac{1}{(k_{13}^2 - m_W^2)(k_{25}^2 - m_W^2)} (a^3)((\epsilon_1 \cdot \epsilon_2) - m_W^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13}) - m_W^{-2}(\epsilon_1 \cdot k_{25})(\epsilon_2 \cdot k_{25})) \\
&\quad + m_W^{-4}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{25})(k_{25} \cdot k_{13})) - 32m_W^6 v^{-5} \frac{a^2 a_{HWW}}{(k_{13}^2 - m_W^2)(k_{25}^2 - m_W^2)} (-(\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_{25}) + (\epsilon_1 \cdot \epsilon_2)(k_2 \cdot k_{25})) \\
&\quad + m_W^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{25})(k_2 \cdot k_{13}) - m_W^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})(k_2 \cdot k_{25}) + (\epsilon_1 \cdot k_{25})(\epsilon_2 \cdot k_{13}) - (\epsilon_1 \cdot \epsilon_2)(k_{13} \cdot k_{25}) \\
&\quad - (\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_1) + (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_{13}) + m_W^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{25})(k_1 \cdot k_{25}) - m_W^{-2}(\epsilon_1 \cdot k_{25})(\epsilon_2 \cdot k_{25})(k_1 \cdot k_{13})), \\
A_{21}^{\text{HEFT}} &= A_{20}^{\text{HEFT}}((k_3, k_4, k_5) \rightarrow (k_5, k_3, k_4)), \\
A_{22}^{\text{HEFT}} &= A_{14}^{\text{HEFT}}(k_1 \leftrightarrow k_2, k_3 \leftrightarrow k_4), \\
A_{23}^{\text{HEFT}} &= A_{14}^{\text{HEFT}}(k_3 \leftrightarrow k_5), \\
A_{24}^{\text{HEFT}} &= A_{20}^{\text{HEFT}}(k_3 \leftrightarrow k_5), \\
A_{25}^{\text{HEFT}} &= A_{14}^{\text{HEFT}}(k_1 \leftrightarrow k_2, k_3 \leftrightarrow k_5), \\
A_{26}^{\text{HEFT}} &= 6m_W^2 v^{-3} (c)(\epsilon_1 \cdot \epsilon_2). \tag{4.6}
\end{aligned}$$

Notice that we have explicitly set some relations among various diagrams having a similar (Lorentz invariant) momenta structure which then, for practical purposes, can be grouped together into the same subset. Within each of these subsets, two diagrams can be related to each other by interchanges of the external momenta. Thus, the 26 diagrams in Fig. 3 can be grouped into seven subsets S1: *diag1*, *diag12*, *diag13*; S2: *diag2*, *diag3*, *diag6*, *diag7*, *diag8*, *diag11*; S3: *diag14*, *diag17*, *diag19*, *diag22*, *diag23*, *diag25*; S4: *diag5*, *diag9*, *diag16*; S5: *diag10*, *diag15*, *diag18*, *diag20*, *diag21*, *diag24*; S6: *diag4*; S7: *diag26*. In each of these subsets, we have marked in *italic* the diagram that has been chosen as the referent in Eq. (4.6). Notice that the diagrams in each subset

can be easily visualized from Fig. 3 because they all have the same pattern of colored balls in the vertices: S1 with 1 purple and 1 red; S2 with 1 purple and 1 yellow; S3 with 2 yellow and 1 red; S4 with 1 yellow and 2 red; S5 with 3 yellow; S6 with 1 yellow and 1 light blue; S7 with 1 green. Correspondingly, in this $WW \rightarrow HHH$ case, all the diagrams in the same subset contain the same combination of LO-HEFT parameters in their first terms of the above analytical results: S1 ($b\kappa_3$); S2 (ab); S3 ($a^2\kappa_3$); S4 ($a\kappa_3^2$); S5 (a^3); S6 ($a\kappa_4$); S7 (c). As will be shown later, this way of grouping the various diagrams will help in identifying the most efficient way to solve the HEFT/SMEFT matching in the forthcoming Sec. V.

Finally, regarding the gauge invariance of the above 26 amplitudes of the unitary gauge in Eq. (4.6), we have checked that, adding together the diagrams with the same number of W propagators, we get an identical result as in the R_ξ gauges when adding to these the corresponding diagrams with the GB propagators, and this sum leads to a ξ independent result. This is equivalent to saying that starting with the previous results for the 26 diagrams in the unitary gauge, the sum of diagrams with 0 W propagators (diag1 + diag4 + diag5 + diag9 + diag12 + diag13 + diag16 + diag26 = S1 + S4 + S6 + S7), the sum of diagrams with 1 W propagator (diag2 + diag3 + diag6 + diag7 + diag8 + diag11 + diag14 + diag17 + diag19 + diag22 + diag23 + diag25 = S2 + S3) and the sum of diagrams with 2 W propagators (diag10 + diag15 + diag18 +

diag20 + diag21 + diag24 = S5) are gauge-invariant quantities separately.

Similar classifications, relations among diagrams and the setting of the subsets of diagrams and gauge-invariant groups can be done in all the following predictions of triple Higgs production from WBF (both in HEFT and SMEFT).

2. SMEFT

As in the previous case, we write explicitly the analytical SMEFT results for each contributing diagram in Fig. 3. Notice that we keep the same format as in the HEFT case in order to facilitate the HEFT/SMEFT comparison that will be performed when solving the matching in the forthcoming Sec. V.

$$A^{\text{SMEFT}} = A_1^{\text{SMEFT}} + \dots + A_{26}^{\text{SMEFT}},$$

$$A_1^{\text{SMEFT}} = 6m_W^2 m_H^2 v^{-3} \frac{1}{q_{45}^2 - m_H^2} (\epsilon_1 \cdot \epsilon_2) - \frac{2m_W^2}{v\Lambda^2} \frac{1}{q_{45}^2 - m_H^2} \left(6c_\Phi v^2 + \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) (7m_H^2 + 2q_{45}^2) \right) (\epsilon_1 \cdot \epsilon_2) \\ + \frac{12m_H^2}{v\Lambda^2} \frac{1}{q_{45}^2 - m_H^2} \left(-m_W^2 \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) (\epsilon_1 \cdot \epsilon_2) + c_{\Phi W} ((\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1) - (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_2)) \right),$$

$$A_2^{\text{SMEFT}} = 4m_W^4 v^{-3} \frac{1}{k_{13}^2 - m_W^2} ((\epsilon_1 \cdot \epsilon_2) - m_W^{-2} (\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})) \\ - 12m_W^4 \frac{1}{v\Lambda^2} \frac{1}{k_{13}^2 - m_W^2} \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) ((\epsilon_1 \cdot \epsilon_2) - m_W^{-2} (\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})) + 8m_W^2 \frac{1}{v\Lambda^2} \frac{1}{k_{13}^2 - m_W^2} (c_{\Phi W}) \\ \times ((\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_{13}) - (\epsilon_1 \cdot \epsilon_2)(k_2 \cdot k_{13}) - (\epsilon_2 \cdot k_1)(\epsilon_1 \cdot k_{13}) + (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_{13})),$$

$$A_3^{\text{SMEFT}} = A_2^{\text{SMEFT}}(k_3 \leftrightarrow k_4),$$

$$A_4^{\text{SMEFT}} = 6m_W^2 m_H^2 v^{-3} \frac{1}{q_{12}^2 - m_H^2} (\epsilon_1 \cdot \epsilon_2) - \frac{2m_W^2}{v\Lambda^2} \frac{1}{q_{12}^2 - m_H^2} \left(36c_\Phi v^2 + \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) (12m_H^2 + 2q_{12}^2) \right) (\epsilon_1 \cdot \epsilon_2) \\ + \frac{6m_H^2}{v\Lambda^2} \frac{1}{q_{12}^2 - m_H^2} \left(-m_W^2 \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) (\epsilon_1 \cdot \epsilon_2) + 2c_{\Phi W} ((\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1) - (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_2)) \right),$$

$$A_5^{\text{SMEFT}} = 18m_W^2 m_H^4 v^{-3} \frac{1}{(q_{12}^2 - m_H^2)(q_{34}^2 - m_H^2)} (\epsilon_1 \cdot \epsilon_2) - 6m_W^2 m_H^2 \frac{1}{v\Lambda^2} \frac{1}{(q_{12}^2 - m_H^2)(q_{34}^2 - m_H^2)} \\ \times \left(12c_\Phi v^2 + \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) (12m_H^2 + 2(q_{12}^2 + 2q_{34}^2)) \right) (\epsilon_1 \cdot \epsilon_2) + \frac{18m_H^4}{v\Lambda^2} \frac{1}{(q_{12}^2 - m_H^2)(q_{34}^2 - m_H^2)} \\ \times \left(-m_W^2 \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) (\epsilon_1 \cdot \epsilon_2) + 2c_{\Phi W} ((\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1) - (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_2)) \right),$$

$$A_6^{\text{SMEFT}} = A_2^{\text{SMEFT}}(k_3 \leftrightarrow k_5),$$

$$A_7^{\text{SMEFT}} = A_2^{\text{SMEFT}}(k_1 \leftrightarrow k_2),$$

$$A_8^{\text{SMEFT}} = A_2^{\text{SMEFT}}(k_3 \leftrightarrow k_4, k_1 \leftrightarrow k_2),$$

$$A_9^{\text{SMEFT}} = A_5^{\text{SMEFT}}(k_3 \leftrightarrow k_5),$$

$$A_{10}^{\text{SMEFT}} = A_{20}^{\text{SMEFT}}(k_4 \leftrightarrow k_5),$$

$$A_{11}^{\text{SMEFT}} = A_2^{\text{SMEFT}}(k_3 \leftrightarrow k_5, k_1 \leftrightarrow k_2),$$

$$A_{12}^{\text{SMEFT}} = A_1^{\text{SMEFT}}(k_3 \leftrightarrow k_5),$$

$$\begin{aligned}
A_{13}^{\text{SMEFT}} &= A_1^{\text{SMEFT}}(k_3 \leftrightarrow k_4), \\
A_{14}^{\text{SMEFT}} &= 12m_W^4 m_H^2 v^{-3} \frac{1}{(k_{13}^2 - m_W^2)(q_{45}^2 - m_H^2)} ((\epsilon_1 \cdot \epsilon_2) - m_W^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})) - \frac{4m_W^4}{v\Lambda^2} \frac{1}{(k_{13}^2 - m_W^2)(q_{45}^2 - m_H^2)} \\
&\quad \times \left(6c_\Phi v^2 + \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) (7m_H^2 + 2q_{45}^2) \right) ((\epsilon_1 \cdot \epsilon_2) - m_W^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})) \\
&\quad + \frac{12m_W^2 m_H^2}{v\Lambda^2} \frac{1}{(k_{13}^2 - m_W^2)(q_{45}^2 - m_H^2)} \left(-2m_W^2 \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) ((\epsilon_1 \cdot \epsilon_2) - m_W^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})) \right. \\
&\quad \left. + 2c_{\Phi W}((\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_{13}) - (\epsilon_1 \cdot \epsilon_2)(k_2 \cdot k_{13}) - (\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_1) + (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_{13})) \right), \\
A_{15}^{\text{SMEFT}} &= A_{20}^{\text{SMEFT}}((k_3, k_4, k_5) \rightarrow (k_4, k_5, k_3)), \\
A_{16}^{\text{SMEFT}} &= A_5^{\text{SMEFT}}(k_4 \leftrightarrow k_5), \\
A_{17}^{\text{SMEFT}} &= A_{14}^{\text{SMEFT}}(k_1 \leftrightarrow k_2), \\
A_{18}^{\text{SMEFT}} &= A_{20}^{\text{SMEFT}}(k_3 \leftrightarrow k_4), \\
A_{19}^{\text{SMEFT}} &= A_{14}^{\text{SMEFT}}(k_3 \leftrightarrow k_4), \\
A_{20}^{\text{SMEFT}} &= 8m_W^6 v^{-3} \frac{1}{(k_{13}^2 - m_W^2)(k_{25}^2 - m_W^2)} ((\epsilon_1 \cdot \epsilon_2) - m_W^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13}) - m_W^{-2}(\epsilon_1 \cdot k_{25})(\epsilon_2 \cdot k_{25}) \\
&\quad + m_W^{-4}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{25})(k_{25} \cdot k_{13})) + \frac{8m_W^4}{v\Lambda^2} \frac{1}{(k_{13}^2 - m_W^2)(k_{25}^2 - m_W^2)} \left(-3m_W^2 \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) ((\epsilon_1 \cdot \epsilon_2) \right. \\
&\quad \left. - m_W^{-2}(\epsilon_1 \cdot k_{25})(\epsilon_2 \cdot k_{25}) - m_W^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13}) - m_W^{-4}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{25})(k_{13} \cdot k_{25})) \right. \\
&\quad \left. + 2c_{\Phi W}(-(\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_{25}) + (\epsilon_1 \cdot \epsilon_2)(k_2 \cdot k_{25}) + m_W^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{25})(k_2 \cdot k_{13}) \right. \\
&\quad \left. - m_W^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})(k_2 \cdot k_{25}) + (\epsilon_1 \cdot k_{25})(\epsilon_2 \cdot k_{13}) - (\epsilon_1 \cdot \epsilon_2)(k_{13} \cdot k_{25}) - (\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_1) + (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_{13}) \right. \\
&\quad \left. + m_W^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{25})(k_1 \cdot k_{25}) - m_W^{-2}(\epsilon_1 \cdot k_{25})(\epsilon_2 \cdot k_{25})(k_1 \cdot k_{13})) \right), \\
A_{21}^{\text{SMEFT}} &= A_{20}^{\text{SMEFT}}((k_3, k_4, k_5) \rightarrow (k_5, k_3, k_4)), \\
A_{22}^{\text{SMEFT}} &= A_{14}^{\text{SMEFT}}(k_1 \leftrightarrow k_2, k_3 \leftrightarrow k_4), \\
A_{23}^{\text{SMEFT}} &= A_{14}^{\text{SMEFT}}(k_3 \leftrightarrow k_5), \\
A_{24}^{\text{SMEFT}} &= A_{20}^{\text{SMEFT}}(k_3 \leftrightarrow k_5), \\
A_{25}^{\text{SMEFT}} &= A_{14}^{\text{SMEFT}}(k_1 \leftrightarrow k_2, k_3 \leftrightarrow k_5), \\
A_{26}^{\text{SMEFT}} &= 0.
\end{aligned} \tag{4.7}$$

D. $ZZ \rightarrow HHH$

We use the momenta assignments $Z(k_1)Z(k_2) \rightarrow H(k_3)H(k_4)H(k_5)$. The polarization vectors of the two ingoing Z are denoted by ϵ_1 and ϵ_2 , respectively. The other conventions and simplifications used here are as in $WW \rightarrow HHH$. In this case of triple Higgs production, we use again the short notation for the frequently appearing momenta combinations, $k_{ij} = k_i - k_j$ and $q_{ij} = k_i + k_j$.

1. HEFT

Here we show the results of HEFT, making explicit the analytical result of each diagram:

$$A^{\text{HEFT}} = A_1^{\text{HEFT}} + \dots + A_{26}^{\text{HEFT}},$$

$$A_1^{\text{HEFT}} = 6m_Z^2 m_H^2 v^{-3} \frac{1}{q_{45}^2 - m_H^2} (b\kappa_3)(\epsilon_1 \cdot \epsilon_2) - 48m_Z^2 m_H^2 v^{-5} \frac{1}{q_{45}^2 - m_H^2} (\kappa_3)(a_{HHWW}c_W^4 + a_{HHBB}s_W^4 + a_{HH1}s_W^2 c_W^2) \\ \times ((\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1) - (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_2)) - 48m_Z^4 m_H^2 v^{-5} \frac{1}{q_{45}^2 - m_H^2} (\kappa_3 a_{HH0}s_W^2)(\epsilon_1 \cdot \epsilon_2),$$

$$A_2^{\text{HEFT}} = 4m_Z^4 v^{-3} \frac{1}{k_{13}^2 - m_Z^2} (ab)((\epsilon_1 \cdot \epsilon_2) - m_Z^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})) \\ - 16m_Z^4 v^{-5} \frac{1}{k_{13}^2 - m_Z^2} (2a(a_{HHWW}c_W^4 + a_{HHBB}s_W^4 + a_{HH1}s_W^2 c_W^2)((\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_{13}) - (\epsilon_1 \cdot \epsilon_2)(k_2 \cdot k_{13})) \\ - b(a_{HWW}c_W^4 + a_{HBB}s_W^4 + a_{H1}s_W^2 c_W^2)((\epsilon_2 \cdot k_1)(\epsilon_1 \cdot k_{13}) - (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_{13}))) \\ - 16m_Z^6 v^{-5} \frac{1}{k_{13}^2 - m_Z^2} (2aa_{HH0}s_W^2 - ba_{H0}s_W^2)((\epsilon_1 \cdot \epsilon_2) - m_Z^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})),$$

$$A_3^{\text{HEFT}} = A_2^{\text{HEFT}}(k_3 \leftrightarrow k_4),$$

$$A_4^{\text{HEFT}} = 6m_Z^2 m_H^2 v^{-3} \frac{a\kappa_4}{q_{12}^2 - m_H^2} (\epsilon_1 \cdot \epsilon_2) - 24m_Z^2 m_H^2 v^{-5} \frac{1}{q_{12}^2 - m_H^2} \\ \times \kappa_4(a_{HWW}c_W^4 + a_{HBB}s_W^4 + a_{H1}s_W^2 c_W^2)((\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1) - (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_2)) - 24m_Z^4 m_H^2 v^{-5} \frac{\kappa_4 a_{H0}s_W^2}{q_{12}^2 - m_H^2} (\epsilon_1 \cdot \epsilon_2),$$

$$A_5^{\text{HEFT}} = \frac{18m_Z^2 m_H^4 v^{-3}}{(q_{12}^2 - m_H^2)(q_{34}^2 - m_H^2)} (a\kappa_3^2(\epsilon_1 \cdot \epsilon_2) - 4\kappa_3^2 v^{-2}(a_{HWW}c_W^4 + a_{HBB}s_W^4 + a_{H1}s_W^2 c_W^2) \\ \times ((\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1) - (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_2)) - 4m_Z^2 v^{-2} \kappa_3^2 a_{H0}s_W^2(\epsilon_1 \cdot \epsilon_2)),$$

$$A_6^{\text{HEFT}} = A_2^{\text{HEFT}}(k_3 \leftrightarrow k_5),$$

$$A_7^{\text{HEFT}} = A_2^{\text{HEFT}}(k_1 \leftrightarrow k_2),$$

$$A_8^{\text{HEFT}} = A_2^{\text{HEFT}}(k_3 \leftrightarrow k_4, k_1 \leftrightarrow k_2),$$

$$A_9^{\text{HEFT}} = A_5^{\text{HEFT}}(k_3 \leftrightarrow k_5),$$

$$A_{10}^{\text{HEFT}} = A_{20}^{\text{HEFT}}(k_4 \leftrightarrow k_5),$$

$$A_{11}^{\text{HEFT}} = A_2^{\text{HEFT}}(k_3 \leftrightarrow k_5, k_1 \leftrightarrow k_2),$$

$$A_{12}^{\text{HEFT}} = A_1^{\text{HEFT}}(k_3 \leftrightarrow k_5),$$

$$A_{13}^{\text{HEFT}} = A_1^{\text{HEFT}}(k_3 \leftrightarrow k_4),$$

$$A_{14}^{\text{HEFT}} = 12m_Z^4 m_H^2 v^{-3} \frac{a^2 \kappa_3}{(k_{13}^2 - m_Z^2)(q_{45}^2 - m_H^2)} ((\epsilon_1 \cdot \epsilon_2) - m_Z^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})) \\ - 48m_Z^4 m_H^2 v^{-5} \frac{a\kappa_3(a_{HWW}c_W^4 + a_{HBB}s_W^4 + a_{H1}s_W^2 c_W^2)}{(k_{13}^2 - m_Z^2)(q_{45}^2 - m_H^2)} ((\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_{13}) - (\epsilon_1 \cdot \epsilon_2)(k_2 \cdot k_{13})) \\ + 48m_Z^4 m_H^2 v^{-5} \frac{a\kappa_3(a_{HWW}c_W^4 + a_{HBB}s_W^4 + a_{H1}s_W^2 c_W^2)}{(k_{13}^2 - m_Z^2)(q_{45}^2 - m_H^2)} ((\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_1) - (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_{13})) \\ - 96m_Z^6 m_H^2 v^{-5} \frac{a\kappa_3 a_{H0}s_W^2}{(k_{13}^2 - m_Z^2)(q_{45}^2 - m_H^2)} ((\epsilon_1 \cdot \epsilon_2) - m_Z^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13}))$$

$$A_{15}^{\text{HEFT}} = A_{20}^{\text{HEFT}}((k_3, k_4, k_5) \rightarrow (k_4, k_5, k_3)),$$

$$A_{16}^{\text{HEFT}} = A_5^{\text{HEFT}}(k_4 \leftrightarrow k_5),$$

$$\begin{aligned}
A_{17}^{\text{HEFT}} &= A_{14}^{\text{HEFT}}(k_1 \leftrightarrow k_2), \\
A_{18}^{\text{HEFT}} &= A_{20}^{\text{HEFT}}(k_3 \leftrightarrow k_4), \\
A_{19}^{\text{HEFT}} &= A_{14}^{\text{HEFT}}(k_3 \leftrightarrow k_4), \\
A_{20}^{\text{HEFT}} &= 8m_Z^6 v^{-3} \frac{1}{(k_{13}^2 - m_Z^2)(k_{25}^2 - m_Z^2)} (a^3)((\epsilon_1 \cdot \epsilon_2) - m_Z^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13}) - m_Z^{-2}(\epsilon_1 \cdot k_{25})(\epsilon_2 \cdot k_{25}) \\
&\quad + m_Z^{-4}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{25})(k_{25} \cdot k_{13})) - 32m_Z^6 v^{-5} \frac{a^2(a_{HWW}c_W^4 + a_{HBB}s_W^4 + a_{H1}s_W^2 c_W^2)}{(k_{13}^2 - m_Z^2)(k_{25}^2 - m_Z^2)} (-(\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_{25}) \\
&\quad + (\epsilon_1 \cdot \epsilon_2)(k_2 \cdot k_{25}) + m_Z^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{25})(k_2 \cdot k_{13}) - m_Z^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})(k_2 \cdot k_{25}) \\
&\quad + (\epsilon_1 \cdot k_{25})(\epsilon_2 \cdot k_{13}) - (\epsilon_1 \cdot \epsilon_2)(k_{13} \cdot k_{25}) - (\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_1) + (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_{13}) \\
&\quad + m_Z^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{25})(k_1 \cdot k_{25}) - m_Z^{-2}(\epsilon_1 \cdot k_{25})(\epsilon_2 \cdot k_{25})(k_1 \cdot k_{13})) \\
&\quad - 96m_Z^8 v^{-5} \frac{a^2 a_{H0}s_W^2}{(k_{13}^2 - m_Z^2)(k_{25}^2 - m_Z^2)} ((\epsilon_1 \cdot \epsilon_2) - m_Z^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13}) \\
&\quad - m_Z^{-2}(\epsilon_1 \cdot k_{25})(\epsilon_2 \cdot k_{25}) + m_Z^{-4}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{25})(k_{25} \cdot k_{13})), \\
A_{21}^{\text{HEFT}} &= A_{20}^{\text{HEFT}}((k_3, k_4, k_5) \rightarrow (k_5, k_3, k_4)), \\
A_{22}^{\text{HEFT}} &= A_{14}^{\text{HEFT}}(k_1 \leftrightarrow k_2, k_3 \leftrightarrow k_4), \\
A_{23}^{\text{HEFT}} &= A_{14}^{\text{HEFT}}(k_3 \leftrightarrow k_5), \\
A_{24}^{\text{HEFT}} &= A_{20}^{\text{HEFT}}(k_3 \leftrightarrow k_5), \\
A_{25}^{\text{HEFT}} &= A_{14}^{\text{HEFT}}(k_1 \leftrightarrow k_2, k_3 \leftrightarrow k_5), \\
A_{26}^{\text{HEFT}} &= 6m_W^2 v^{-3} (c)(\epsilon_1 \cdot \epsilon_2) - 48m_Z^4 v^{-5} a_{HHH0}s_W^2 (\epsilon_1 \cdot \epsilon_2). \tag{4.8}
\end{aligned}$$

2. SMEFT

As in the previous case, we write explicitly the analytical SMEFT results for each contributing diagram in Fig. 3:

$$\begin{aligned}
A^{\text{SMEFT}} &= A_1^{\text{SMEFT}} + \dots + A_{26}^{\text{SMEFT}}, \\
A_1^{\text{SMEFT}} &= 6m_Z^2 m_H^2 v^{-3} \frac{1}{q_{45}^2 - m_H^2} (\epsilon_1 \cdot \epsilon_2) - \frac{2m_Z^2}{v\Lambda^2} \frac{1}{q_{45}^2 - m_H^2} \left(6c_\Phi v^2 + \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) (7m_H^2 + 2q_{45}^2) \right) (\epsilon_1 \cdot \epsilon_2) \\
&\quad + \frac{12m_Z^2}{v\Lambda^2} \frac{1}{q_{45}^2 - m_H^2} \left(-m_Z^2 \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) (\epsilon_1 \cdot \epsilon_2) + (c_{\Phi W}c_W^2 + c_{\Phi B}s_W^2 + c_{\Phi WB}c_W s_W) \right. \\
&\quad \times ((\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1) - (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_2)) \left. \right) + \frac{15m_Z^2 m_H^2}{v\Lambda^2} \frac{1}{q_{45}^2 - m_H^2} c_{\Phi D} (\epsilon_1 \cdot \epsilon_2), \\
A_2^{\text{SMEFT}} &= 4m_Z^4 v^{-3} \frac{1}{k_{13}^2 - m_Z^2} ((\epsilon_1 \cdot \epsilon_2) - m_Z^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})) - 12m_Z^4 \frac{1}{v\Lambda^2} \frac{1}{k_{13}^2 - m_Z^2} \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) ((\epsilon_1 \cdot \epsilon_2) \\
&\quad - m_Z^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})) + 8m_Z^2 \frac{1}{v\Lambda^2} \frac{1}{k_{13}^2 - m_Z^2} (c_{\Phi W}c_W^2 + c_{\Phi B}s_W^2 + c_{\Phi WB}c_W s_W) \\
&\quad \times ((\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_{13}) - (\epsilon_1 \cdot \epsilon_2)(k_2 \cdot k_{13}) - (\epsilon_2 \cdot k_1)(\epsilon_1 \cdot k_{13}) + (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_{13})) \\
&\quad + \frac{12m_Z^4}{v\Lambda^2} \frac{1}{k_{13}^2 - m_Z^2} (c_{\Phi D}) ((\epsilon_1 \cdot \epsilon_2) - m_Z^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})), \\
A_3^{\text{SMEFT}} &= A_2^{\text{SMEFT}}(k_3 \leftrightarrow k_4), \\
A_4^{\text{SMEFT}} &= 6m_Z^2 m_H^2 v^{-3} \frac{1}{q_{12}^2 - m_H^2} (\epsilon_1 \cdot \epsilon_2)
\end{aligned}$$

$$\begin{aligned}
& -\frac{2m_Z^2}{v\Lambda^2} \frac{1}{q_{12}^2 - m_H^2} \left(36c_\Phi v^2 + \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) (12m_H^2 + 2q_{12}^2) \right) (\epsilon_1 \cdot \epsilon_2) \\
& + \frac{6m_H^2}{v\Lambda^2} \frac{1}{q_{12}^2 - m_H^2} \left(-m_Z^2 \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) (\epsilon_1 \cdot \epsilon_2) + 2(c_{\Phi W}c_W^2 + c_{\Phi B}s_W^2 + c_{\Phi WB}c_W s_W) \right. \\
& \times ((\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1) - (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_2)) \left. \right) + \frac{3m_Z^2 m_H^2}{v\Lambda^2} \frac{1}{q_{12}^2 - m_H^2} c_{\Phi D} (\epsilon_1 \cdot \epsilon_2), \\
A_5^{\text{SMEFT}} &= 18m_Z^2 m_H^4 v^{-3} \frac{1}{(q_{12}^2 - m_H^2)(q_{34}^2 - m_H^2)} (\epsilon_1 \cdot \epsilon_2) - 6m_Z^2 m_H^2 \frac{1}{v\Lambda^2} \frac{1}{(q_{12}^2 - m_H^2)(q_{34}^2 - m_H^2)} \\
& \times \left(12c_\Phi v^2 + \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) (12m_H^2 + 2(q_{12}^2 + 2q_{34}^2)) \right) (\epsilon_1 \cdot \epsilon_2) \\
& + \frac{18m_H^4}{v\Lambda^2} \frac{1}{(q_{12}^2 - m_H^2)(q_{34}^2 - m_H^2)} \left(-m_Z^2 \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) (\epsilon_1 \cdot \epsilon_2) \right. \\
& + 2(c_{\Phi W}c_W^2 + c_{\Phi B}s_W^2 + c_{\Phi WB}c_W s_W) ((\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_1) - (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_2)) \left. \right) \\
& + \frac{9m_Z^2 m_H^2}{v\Lambda^2} \frac{1}{(q_{12}^2 - m_H^2)(q_{34}^2 - m_H^2)} c_{\Phi D} (\epsilon_1 \cdot \epsilon_2), \\
A_6^{\text{SMEFT}} &= A_2^{\text{SMEFT}}(k_3 \leftrightarrow k_5), \\
A_7^{\text{SMEFT}} &= A_2^{\text{SMEFT}}(k_1 \leftrightarrow k_2), \\
A_8^{\text{SMEFT}} &= A_2^{\text{SMEFT}}(k_3 \leftrightarrow k_4, k_1 \leftrightarrow k_2), \\
A_9^{\text{SMEFT}} &= A_5^{\text{SMEFT}}(k_3 \leftrightarrow k_5), \\
A_{10}^{\text{SMEFT}} &= A_{20}^{\text{SMEFT}}(k_4 \leftrightarrow k_5), \\
A_{11}^{\text{SMEFT}} &= A_2^{\text{SMEFT}}(k_3 \leftrightarrow k_5, k_1 \leftrightarrow k_2), \\
A_{12}^{\text{SMEFT}} &= A_1^{\text{SMEFT}}(k_3 \leftrightarrow k_5), \\
A_{13}^{\text{SMEFT}} &= A_1^{\text{SMEFT}}(k_3 \leftrightarrow k_4), \\
A_{14}^{\text{SMEFT}} &= 12m_Z^4 m_H^2 v^{-3} \frac{1}{(k_{13}^2 - m_Z^2)(q_{45}^2 - m_H^2)} ((\epsilon_1 \cdot \epsilon_2) - m_Z^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})) \\
& - \frac{4m_Z^4}{v\Lambda^2} \frac{1}{(k_{13}^2 - m_Z^2)(q_{45}^2 - m_H^2)} \\
& \times \left(6c_\Phi v^2 + \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) (7m_H^2 + 2q_{45}^2) \right) ((\epsilon_1 \cdot \epsilon_2) - m_Z^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})) \\
& + \frac{12m_Z^2 m_H^2}{v\Lambda^2} \frac{1}{(k_{13}^2 - m_Z^2)(q_{45}^2 - m_H^2)} \left(-2m_Z^2 \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) ((\epsilon_1 \cdot \epsilon_2) - m_W^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})) \right. \\
& + 2(c_{\Phi W}c_W^2 + c_{\Phi B}s_W^2 + c_{\Phi WB}c_W s_W) \\
& \times ((\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_{13}) - (\epsilon_1 \cdot \epsilon_2)(k_2 \cdot k_{13}) - (\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_1) + (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_{13})) \left. \right) \\
& + \frac{12m_Z^4 m_H^2}{v\Lambda^2} \frac{1}{(k_{13}^2 - m_Z^2)(q_{45}^2 - m_H^2)} c_{\Phi D} ((\epsilon_1 \cdot \epsilon_2) - m_Z^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})), \\
A_{15}^{\text{SMEFT}} &= A_{20}^{\text{SMEFT}}((k_3, k_4, k_5) \rightarrow (k_4, k_5, k_3)), \\
A_{16}^{\text{SMEFT}} &= A_5^{\text{SMEFT}}(k_4 \leftrightarrow k_5), \\
A_{17}^{\text{SMEFT}} &= A_{14}^{\text{SMEFT}}(k_1 \leftrightarrow k_2),
\end{aligned}$$

$$\begin{aligned}
A_{18}^{\text{SMEFT}} &= A_{20}^{\text{SMEFT}}(k_3 \leftrightarrow k_4), \\
A_{19}^{\text{SMEFT}} &= A_{14}^{\text{SMEFT}}(k_3 \leftrightarrow k_4), \\
A_{20}^{\text{SMEFT}} &= 8m_Z^6 v^{-3} \frac{1}{(k_{13}^2 - m_Z^2)(k_{25}^2 - m_Z^2)} ((\epsilon_1 \cdot \epsilon_2) - m_Z^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13}) - m_Z^{-2}(\epsilon_1 \cdot k_{25})(\epsilon_2 \cdot k_{25}) \\
&\quad + m_Z^{-4}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{25})(k_{25} \cdot k_{13})) \\
&\quad + \frac{8m_Z^4}{v\Lambda^2} \frac{1}{(k_{13}^2 - m_Z^2)(k_{25}^2 - m_Z^2)} \left(-3 \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) (m_Z^2(\epsilon_1 \cdot \epsilon_2) - (\epsilon_1 \cdot k_{25})(\epsilon_2 \cdot k_{25}) \right. \\
&\quad - (\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13}) + m_Z^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{25})(k_{13} \cdot k_{25})) \\
&\quad + 2(c_{\Phi W}c_W^2 + c_{\Phi B}S_W^2 + c_{\Phi WB}c_W S_W)((\epsilon_1 \cdot k_{25})(\epsilon_2 \cdot k_{13}) - (\epsilon_1 \cdot \epsilon_2)(k_{13} \cdot k_{25}) \\
&\quad - (\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_1) + (\epsilon_1 \cdot \epsilon_2)(k_1 \cdot k_{13}) + m_Z^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{25})(k_1 \cdot k_{25}) \\
&\quad - m_Z^{-2}(\epsilon_1 \cdot k_{25})(\epsilon_2 \cdot k_{25})(k_1 \cdot k_{13})) - (\epsilon_1 \cdot k_2)(\epsilon_2 \cdot k_{25}) + (\epsilon_1 \cdot \epsilon_2)(k_2 \cdot k_{25}) \\
&\quad \left. + m_Z^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{25})(k_2 \cdot k_{13}) - m_Z^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})(k_2 \cdot k_{25}) \right) \\
&\quad + \frac{12m_Z^6}{v\Lambda^2} \frac{1}{(k_{13}^2 - m_Z^2)(k_{25}^2 - m_Z^2)} c_{\Phi D} ((\epsilon_1 \cdot \epsilon_2) - m_Z^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13}) \\
&\quad - m_Z^{-2}(\epsilon_1 \cdot k_{25})(\epsilon_2 \cdot k_{25}) + m_Z^{-4}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{25})(k_{25} \cdot k_{13})), \\
A_{21}^{\text{SMEFT}} &= A_{20}^{\text{SMEFT}}((k_3, k_4, k_5) \rightarrow (k_5, k_3, k_4)), \\
A_{22}^{\text{SMEFT}} &= A_{14}^{\text{SMEFT}}(k_1 \leftrightarrow k_2, k_3 \leftrightarrow k_4), \\
A_{23}^{\text{SMEFT}} &= A_{14}^{\text{SMEFT}}(k_3 \leftrightarrow k_5), \\
A_{24}^{\text{SMEFT}} &= A_{20}^{\text{SMEFT}}(k_3 \leftrightarrow k_5), \\
A_{25}^{\text{SMEFT}} &= A_{14}^{\text{SMEFT}}(k_1 \leftrightarrow k_2, k_3 \leftrightarrow k_5), \\
A_{26}^{\text{SMEFT}} &= \frac{12m_Z^2}{v\Lambda^2} c_{\Phi D} (\epsilon_1 \cdot \epsilon_2). \tag{4.9}
\end{aligned}$$

V. MATCHING HEFT AND SMEFT AMPLITUDES

In order to clarify the way in which we proceed to perform the matching among HEFT and SMEFT, we start first analyzing the simpler cases of $VV \rightarrow H$ and $HH \rightarrow HH$ amplitudes. After this, we will solve the cases of interest here of $VV \rightarrow HH$ and $VV \rightarrow HHH$.

The amplitude of single Higgs production from WBF, $W(k_1)W(k_2) \rightarrow H(k_3)$, using as input the physical parameters m_W , m_H and v is given in the HEFT and SMEFT respectively by:

$$\begin{aligned}
A(WW \rightarrow H)^{\text{HEFT}} &= \frac{2m_W^2}{v} a(\epsilon_1 \cdot \epsilon_2) \\
&\quad - \frac{8m_W^2}{v^3} a_{HWW}((k_1 \cdot \epsilon_2)(k_2 \cdot \epsilon_1) - (k_1 \cdot k_2)(\epsilon_1 \cdot \epsilon_2)), \tag{5.1}
\end{aligned}$$

$$\begin{aligned}
A(WW \rightarrow H)^{\text{SMEFT}} &= \frac{2m_W^2}{v} (\epsilon_1 \cdot \epsilon_2) \\
&\quad + \frac{2v}{\Lambda^2} \left(m_W^2 \left(c_{\Phi\Box} - \frac{1}{4}c_{\Phi D} \right) (\epsilon_1 \cdot \epsilon_2) \right. \\
&\quad + 2c_{\Phi W}((k_1 \cdot \epsilon_2)(k_2 \cdot \epsilon_1) \\
&\quad \left. - (k_1 \cdot k_2)(\epsilon_1 \cdot \epsilon_2)) \right). \tag{5.2}
\end{aligned}$$

Imposing the matching of these two amplitudes,

$$A(WW \rightarrow H)^{\text{HEFT}} = A(WW \rightarrow H)^{\text{SMEFT}}, \tag{5.3}$$

requires the identification of all the Lorentz and gauge-invariant structures involved. Then, if we write $a = 1 - \Delta a$, the first terms in these two amplitudes coincide with the SM amplitude and this contribution cancels in both sides of the matching Eq. (5.3). Therefore, solving the matching equation in terms of the EFT's coefficients leads to the following

solution:

$$\Delta a = \frac{v^2}{\Lambda^2} \left(-c_{\Phi\Box} + \frac{1}{4} c_{\Phi D} \right), \quad (5.4)$$

$$a_{HWW} = -\frac{v^4}{2m_W^2\Lambda^2} c_{\Phi W}. \quad (5.5)$$

It should be noticed that the other coefficients entering in the Feynman rules of this WWH vertex (see Appendix B), a_{d2} and $a_{H\gamma\gamma}$, are not needed to match the HEFT and SMEFT results at this order. If instead one goes beyond dimension 6 in the SMEFT, i.e., if one introduces dimension 8 operators in

the SMEFT predictions, then there could appear additional structures that might require the presence of these two HEFT coefficients to get the proper matching. In consequence, at the established order in the two predictions, these two coefficients a_{d2} and $a_{H\gamma\gamma}$ can be ignored. It should also be noticed that the same solution in Eqs. (5.4) and (5.5) is obtained if the matching is instead applied to the decay amplitudes of the Higgs boson into two W 's (either real or virtual), as expected.

One can proceed similarly in the case of single Higgs production from the fusion of two Z bosons, $Z(k_1)Z(k_2) \rightarrow H(k_3)$, now using as input parameters m_Z , m_H , and v . We also use here the short notation given in Eq. (4.1). The corresponding amplitudes are:

$$A(ZZ \rightarrow H)^{\text{HEFT}} = \frac{2m_Z^2}{v} a(\epsilon_1 \cdot \epsilon_2) - \frac{8m_Z^2}{v^3} (a_{HWW} c_W^4 + a_{HBB} s_W^4 + a_{H1} s_W^2 c_W^2) ((k_1 \cdot \epsilon_2)(k_2 \cdot \epsilon_1) - (k_1 \cdot k_2)(\epsilon_1 \cdot \epsilon_2)) - 8 \frac{m_Z^2}{v^3} s_W^2 m_Z^2 a_{H0}(\epsilon_1 \cdot \epsilon_2), \quad (5.6)$$

$$A(ZZ \rightarrow H)^{\text{SMEFT}} = \frac{2m_Z^2}{v} (\epsilon_1 \cdot \epsilon_2) + \frac{2v}{\Lambda^2} \left(m_Z^2 \left(c_{\Phi\Box} + \frac{1}{4} c_{\Phi D} \right) (\epsilon_1 \cdot \epsilon_2) + 2(c_{\Phi W} c_W^2 + c_{\Phi B} s_W^2 + c_{\Phi WB} s_W c_W) ((k_1 \cdot \epsilon_2)(k_2 \cdot \epsilon_1) - (k_1 \cdot k_2)(\epsilon_1 \cdot \epsilon_2)) \right). \quad (5.7)$$

The matching equation in this case is

$$A(ZZ \rightarrow H)^{\text{HEFT}} = A(ZZ \rightarrow H)^{\text{SMEFT}}. \quad (5.8)$$

Then the solution to this matching equation, in addition to the previous values for Δa and a_{HWW} in Eqs. (5.4) and (5.5), is

$$s_W^2 a_{H0} = -\frac{v^4}{8m_Z^2\Lambda^2} c_{\Phi D}, \quad (5.9)$$

$$s_W^2 a_{HBB} = -\frac{v^4}{2m_Z^2\Lambda^2} c_{\Phi B}, \quad (5.10)$$

$$s_W c_W a_{H1} = -\frac{v^4}{2m_Z^2\Lambda^2} c_{\Phi WB}. \quad (5.11)$$

It is also illustrative to show the matching equation and its solution for the case of the $HH \rightarrow HH$ scattering amplitude. Here we use the momentum assignments $H(k_1)H(k_2) \rightarrow H(k_3)H(k_4)$. In this case, there are four contributing diagrams corresponding to contact c-channel, s-channel, t-channel, and u-channel. We then write the amplitude by just using the subset of HEFT coefficients that are needed for the required matching. In this case we have found that this subset is given by just κ_3 and κ_4 , and the other HEFT coefficients can be ignored. We then find the following contributions from the various channels, correspondingly:

$$A(HH \rightarrow HH)^{\text{HEFT}} = A_c^{\text{HEFT}} + A_s^{\text{HEFT}} + A_t^{\text{HEFT}} + A_u^{\text{HEFT}} \\ = -\frac{3\kappa_4 m_H^2}{v^2} - \frac{9\kappa_3^2 m_H^4}{(s - m_H^2)v^2} - \frac{9\kappa_3^2 m_H^4}{(t - m_H^2)v^2} - \frac{9\kappa_3^2 m_H^4}{(u - m_H^2)v^2}, \quad (5.12)$$

$$\begin{aligned}
A(HH \rightarrow HH)^{\text{SMEFT}} &= A_c^{\text{SMEFT}} + A_s^{\text{SMEFT}} + A_t^{\text{SMEFT}} + A_u^{\text{SMEFT}} \\
&= -\frac{3m_H^2}{v^2} + \frac{1}{\Lambda^2} \left(\left(-14c_{\Phi\Box} + \frac{7}{2}c_{\Phi D} \right) m_H^2 + 36c_{\Phi} v^2 \right) - \frac{9m_H^4}{(s - m_H^2)v^2} \\
&\quad + \frac{3}{\Lambda^2} \frac{1}{(s - m_H^2)} \left(\left(-14c_{\Phi\Box} + \frac{7}{2}c_{\Phi D} \right) m_H^4 + (-4c_{\Phi\Box} + c_{\Phi D}) m_H^2 s + 12c_{\Phi} m_H^2 v^2 \right) - \frac{9m_H^4}{(t - m_H^2)v^2} \\
&\quad + \frac{3}{\Lambda^2} \frac{1}{(t - m_H^2)} \left(\left(-14c_{\Phi\Box} + \frac{7}{2}c_{\Phi D} \right) m_H^4 + (-4c_{\Phi\Box} + c_{\Phi D}) m_H^2 t + 12c_{\Phi} m_H^2 v^2 \right) - \frac{9m_H^4}{(u - m_H^2)v^2} \\
&\quad + \frac{3}{\Lambda^2} \frac{1}{(u - m_H^2)} \left(\left(-14c_{\Phi\Box} + \frac{7}{2}c_{\Phi D} \right) m_H^4 + (-4c_{\Phi\Box} + c_{\Phi D}) m_H^2 u + 12c_{\Phi} m_H^2 v^2 \right). \quad (5.13)
\end{aligned}$$

The matching equation in this case is:

$$A(HH \rightarrow HH)^{\text{HEFT}} = A(HH \rightarrow HH)^{\text{SMEFT}}. \quad (5.14)$$

To simplify this equation, we first write in A^{HEFT} the κ 's as $\kappa_3 = 1 - \Delta\kappa_3$ and $\kappa_4 = 1 - \Delta\kappa_4$ and keep in the amplitude just the $\mathcal{O}(\Delta^0)$ terms and the linear terms in the Δ 's, of $\mathcal{O}(\Delta^1)$, and we neglect the $\mathcal{O}(\Delta^2)$ terms. Then, the first contributions in A^{HEFT} from these $\mathcal{O}(\Delta^0)$ terms and the LO terms in A^{SMEFT} provide the SM amplitude which cancels in both sides of the matching equation. Solving this matching equation then requires the identification of all Lorentz invariant structures involving s , t , u and m_H^2 in this case. In this way, the total predicted amplitude $A(s, m_H^2, \cos\theta)$ is an identical function of the energy $\sqrt{s}$, mass m_H^2 , and the relevant angle θ between $\vec{k}_3$ and $\vec{k}_1$, in the CM frame for both theories. Finally, we find the following solution to the matching equation:

$$\Delta\kappa_3 = \frac{2v^4}{m_H^2\Lambda^2} c_{\Phi} + \frac{3v^2}{\Lambda^2} \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right), \quad (5.15)$$

$$\Delta\kappa_4 = \frac{12v^4}{m_H^2\Lambda^2} c_{\Phi} + \frac{50v^2}{3\Lambda^2} \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right). \quad (5.16)$$

At this point, it is interesting to compare this solution in Eqs. (5.15) and (5.16) with the one obtained for Δa in Eq. (5.4). We find the following relations:

$$\Delta\kappa_3 = \frac{2v^4}{m_H^2\Lambda^2} c_{\Phi} + 3\Delta a, \quad (5.17)$$

$$\Delta\kappa_4 = \frac{12v^4}{m_H^2\Lambda^2} c_{\Phi} + \frac{50}{3}\Delta a, \quad (5.18)$$

which in turn implies the following interesting relation between $\Delta\kappa_3$, $\Delta\kappa_4$ and Δa :

$$\Delta\kappa_4 = 6\Delta\kappa_3 - \frac{4}{3}\Delta a. \quad (5.19)$$

Notice that when assuming $\Delta a = 0$, we find a direct correlation between κ_4 and κ_3 within the HEFT given by $\Delta\kappa_4 = 6\Delta\kappa_3$, which is interesting to compare with the relation among the corresponding SM vertices $vV_{HHHH}^{\text{SM}} = V_{HHHH}^{\text{SM}}$. This different relation in HEFT as compared to SM could provide different patterns of BSM versus SM signals in the phenomenology of multiple Higgs production, which deserves further study.

It is also remarkable that the above relation between $\Delta\kappa_3$, $\Delta\kappa_4$, and Δa in Eq. (5.19) was also obtained in [19] (in a different notation), using a very different approach. They worked in the pure scalar theory, did not consider $c_{\Phi D}$, and performed the matching of the two theories at the Lagrangian level, by means of a series of scalar field transformations.

Next we proceed to analyze the matching of our greatest interest, i.e., the matching of the amplitudes for the cases $VV \rightarrow HH$ and $VV \rightarrow HHH$. For this purpose, we will use our analytical results for these amplitudes within the HEFT and the SMEFT presented in the previous section.

In the case of $WW \rightarrow HH$, the matching equation is:

$$A(WW \rightarrow HH)^{\text{HEFT}} = A(WW \rightarrow HH)^{\text{SMEFT}}, \quad (5.20)$$

where the results for $A(WW \rightarrow HH)^{\text{HEFT}}$ and $A(WW \rightarrow HH)^{\text{SMEFT}}$, using the momenta assignments $W^+(k_1), W^-(k_2) \rightarrow H(k_3)H(k_4)$, are given in Eqs. (4.2) and (4.3), respectively. Then we proceed as in the previous examples. First, we determine which is the subset of HEFT coefficients that are needed for the matching. Next, we identify all the terms in both amplitudes with the same Lorentz invariant structures that depend on s , t , u and the masses involved, m_W^2 and m_H^2 , in this case. This, in practice, corresponds to identify separately $A_s + A_c$, A_t and A_u . Grouping together the c-channel and the s-channel in solving the matching equation can be understood because terms in the s-channel going as $s/(s - m_H^2) = 1 + m_H^2/(s - m_H^2)$ contain the “1” term that has the same structure as the c-channel. Thus, the full identification of the HEFT and SMEFT total amplitudes means that they are indeed the same analytical function of the energy, masses, and relevant

angle, $A(s, m_W^2, m_H^2, \cos \theta)$ (with θ in this case, the angle between $\vec{k}_3$ and $\vec{k}_1$).

For this case, we find that the needed HEFT coefficients for the matching is the subset: a , κ_3 , b , a_{HWW} , and a_{HHWW} . The other coefficients either appear in combination with these, therefore not producing new structures, or else they provide new structures that only appear in the SMEFT at dimension 8. This is, for instance, the case of $a_{dd\gamma\gamma_1}$ and $a_{dd\gamma\gamma_2}$ that, as shown in [17], must be matched in particular with the dimension 8 SMEFT coefficients $c_{\Phi^4}^{(i)}$ with $i = 1, 2, 3$ (see this reference for details). One could aim for a more complete matching HEFT/SMEFT including the full set of dimension 8 operators in the predictions of the SMEFT [for a very recent SMEFT computation of $VV \rightarrow HH$ including dimension 8 operators, see [53]] and by repeating a similar exercise to the one we are doing here but including also all the additional needed HEFT NLO operators that could match with the contributions to the amplitudes from the dimension 8 SMEFT operators. But this is not our purpose here. Setting then the minimal subset of HEFT coefficients to perform the matching of HEFT-NLO with dimension 6 SMEFT, then, is equivalent to eliminating the other coefficients in the prediction of the scattering amplitude. Specifically, to solve the matching in this case, we use the already simplified HEFT amplitude in Eq. (4.2) which contains just the needed coefficients for this matching with dimension 6 SMEFT.

Next, we write the HEFT amplitudes in terms of $\Delta a = 1 - a$, $\Delta b = 1 - b$, $\Delta \kappa_3 = 1 - \kappa_3$, a_{HWW} , and a_{HHWW} and keep only the $\mathcal{O}(\Delta^0)$ terms, the linear terms in a_i 's coefficients, and the $\mathcal{O}(\Delta^1)$ terms. The first terms in the two amplitudes, HEFT and SMEFT, provide the results of the SM amplitudes that then cancel in both sides of the matching equation. Then we find the solution to the matching equation given by the following predictions of the HEFT coefficients in terms of the SMEFT coefficients. Firstly, we get the same prediction for Δa as in Eq. (5.4), the same prediction for a_{HWW} as in Eq. (5.5), and the same prediction for $\Delta \kappa_3$ as in Eq. (5.15). Secondly, we get in addition the following predictions for the remaining coefficients:

$$\Delta b = \frac{4v^2}{\Lambda^2} \left(-c_{\Phi\Box} + \frac{1}{4} c_{\Phi D} \right) \quad (5.21)$$

$$a_{HHWW} = -\frac{v^4}{4m_W^2 \Lambda^2} c_{\Phi W}. \quad (5.22)$$

At this point, some comments are in order. First, we wish to note that, in practice, Δa and a_{HWW} are obtained from the matching using A_t (similarly with A_u). Once these two effective couplings are set, Δb and a_{HHWW} are obtained from the matching using $A_s + A_c$. This means in particular that Δb should not be extracted by identifying just the

contact vertices, i.e., identifying A_c in the two theories. We wish to note that this apparently intuitive matching of contact vertices among the two EFTs would have led to a wrong result for Δb . This is equivalent to say that the identification between the HEFT and the SMEFT should not be done at the Feynman rules level (nor at the Lagrangian level) but at the amplitude level. It is the amplitude that is directly related with the observable quantity, and doing the matching at the amplitude level provides the proper solution for the relations among HEFT and SMEFT coefficients which in turn have phenomenological implications. Thus, to solve properly for Δb , one must add the two channels, s-channel and c-channel. Second, we get the same solutions for Δa , $\Delta \kappa_3$, and Δa_{HWW} as in the previous examples. The extra predictions now are Δb and Δa_{HHWW} . The above solutions from the matching between $A(WW \rightarrow HH)$ amplitudes were already found previously in [17]. There is only one different result compared to this reference, which is that of $\Delta \kappa_3$. The reason for this difference is that in [17], on one hand, the chosen input parameters were not the same as here and, on the other hand, the coefficient c_Φ was not included. In [17], the input parameters were the Lagrangian parameters, whereas here the input parameters, as already said, are the physical masses and G_F (or equivalently v extracted from G_F). Finally, we notice some interesting relations that are worth to comment. We find:

$$\Delta b = 4\Delta a, \quad (5.23)$$

$$2a_{HHWW} = a_{HWW}. \quad (5.24)$$

The above relation between Δa and Δb was also obtained previously in the context of the pure scalar theory in [20,21]. The previous result for the relations between Δa and Δb and between a_{HHWW} and a_{HWW} should be compared with the corresponding relations among SM vertices, $vV_{HHWW}^{\text{SM}} = V_{HWW}^{\text{SM}}$. These different relations in the HEFT respect to the SM ones should provide again some hints in looking for different patterns in BSM versus SM predictions at double Higgs production.

Next we analyze the $ZZ \rightarrow HH$ case. The matching equation reads, in this case,

$$A(ZZ \rightarrow HH)^{\text{HEFT}} = A(ZZ \rightarrow HH)^{\text{SMEFT}}. \quad (5.25)$$

We have found that for this case the needed subset of HEFT coefficients to perform the matching is given by: a , κ_3 , b , a_{HWW} , a_{HHWW} , a_{HBB} , a_{HHBB} , a_{H0} , a_{HH0} , a_{H1} , and a_{HH1} . We then proceed with these coefficients, as in the previous case, using the linear approach in the Δ 's and a_i 's again, and canceling the SM-like contributions in both sides of the matching equation. To shorten the computation, we can then use the previous solutions for Δa in Eq. (5.4), $\Delta \kappa_3$ in Eq. (5.15), Δb in Eq. (5.21), a_{HWW} , in Eq. (5.5), a_{HHWW} in

Eq. (5.22), a_{HBB} in Eq. (5.10), a_{H0} in Eq. (5.9), and a_{H1} in Eq. (5.11), and then solve for the remaining coefficients. Thus, we get the following additional relations among the HEFT and SMEFT coefficients from the $ZZ \rightarrow HH$ case:

$$s_W^2 a_{HHBB} = -\frac{v^4}{4m_Z^2 \Lambda^2} c_{\Phi B}, \quad (5.26)$$

$$s_W^2 a_{HH0} = -\frac{5v^4}{16m_Z^2 \Lambda^2} c_{\Phi D}, \quad (5.27)$$

$$s_W c_W a_{HH1} = -\frac{v^4}{4m_Z^2 \Lambda^2} c_{\Phi WB}. \quad (5.28)$$

And we find the following additional relations among the EFTs coefficients:

$$2a_{HHBB} = a_{HBB}, \quad (5.29)$$

$$2a_{HH0} = 5a_{H0}, \quad (5.30)$$

$$2a_{HH1} = a_{H1}. \quad (5.31)$$

Finally, we analyze the matching in the triple Higgs boson production amplitudes. For the $WW \rightarrow HHH$ case, the matching equation reads as follows:

$$A(WW \rightarrow HHH)^{\text{HEFT}} = A(WW \rightarrow HHH)^{\text{SMEFT}}, \quad (5.32)$$

where $A(WW \rightarrow HHH)^{\text{HEFT}}$ and $A(WW \rightarrow HHH)^{\text{SMEFT}}$ are given in Eqs. (4.6) and (4.7) respectively. In this case, the HEFT coefficients involved are those of the previous case $WW \rightarrow HH$, i.e., a , κ_3 , b , a_{HWW} , a_{HHWW} , and two more coefficients, κ_4 and c .

To solve the matching equation, we proceed as in the previous cases. First, we write the HEFT amplitudes in terms of $\Delta a = 1 - a$, $\Delta b = 1 - b$, $\Delta \kappa_3 = 1 - \kappa_3$, $\Delta \kappa_4 = 1 - \kappa_4$, c and the involved a_i 's. Then, we keep only the $\mathcal{O}(\Delta^0)$ and $\mathcal{O}(\Delta^1)$ terms, the linear terms in c , and the linear terms in the a_i 's coefficients. The first terms in the two amplitudes, HEFT and SMEFT, provide the results of the SM amplitudes that then cancel in both sides of the matching equation. Next, to simplify again the procedure, we check first the previous solutions for Δa , Δb , $\Delta \kappa_3$, $\Delta \kappa_4$, a_{HWW} , and a_{HHWW} by plugging them into the HEFT scattering amplitude. Finally, with the remaining contributions left from all the diagrams, we then solve the matching equation for the new coefficient c . To understand the behavior of the different contributions we group together in both cases, HEFT and SMEFT, the 26 diagrams into the seven subsets, S1-S7, as explained in Sec. (4.6). This matching in practice can be solved by identifying the HEFT/SMEFT results for the sum of diagrams within these subsets for both EFTs. Concretely, we find that the analytical results for each diagram in S5 (all having two

W propagators) properly match the HEFT and SMEFT when using as input the previously found coefficients. Remember that the sum of diagrams in S5 is a gauge-invariant quantity. Next we consider the diagrams in S2 and S3 that all have one W propagator. Remember that the sum of diagrams in S2 + S3 is another gauge-invariant quantity. Then we find that considering by pairs the sum of one diagram in S2 and another in S3, and using again the input values of the involved coefficients already computed, we get again a good HEFT/SMEFT matching. Concretely, these sums that fully match in HEFT and SMEFT are diag2 + diag14, diag3 + diag19, diag6 + diag23, diag7 + diag17, diag8 + diag22, diag11 + diag25. Finally, we consider the set of eight diagrams left, all of them having zero W propagators: diag1 + diag4 + diag5 + diag9 + diag12 + diag13 + diag16 + diag26 = S1 + S4 + S6 + S7. Remember that this sum is again another gauge-invariant quantity. Then, by plugging into this sum the values of all the previously coefficients found and identifying the two results in the HEFT and the SMEFT, we solve finally for the value of the missing coefficient. In this way we find the following solution for c :

$$c = -\frac{8v^2}{3\Lambda^2} \left(-c_{\Phi\Box} + \frac{1}{4} c_{\Phi D} \right). \quad (5.33)$$

Notice that, similar to what happened with the solution for Δb in the $WW \rightarrow HH$ case, it would be a wrong procedure to compute c by directly matching the amplitudes of the contact diagram, diag26, in the two EFTs. Indeed, since diag26 vanishes in the SMEFT, this would have lead to a wrong prediction of a vanishing c .

Finally, we analyze the $ZZ \rightarrow HHH$ case. For this we follow exactly the same procedure as for the previous $WW \rightarrow HHH$ case. Now the matching equation is

$$A(ZZ \rightarrow HHH)^{\text{HEFT}} = A(ZZ \rightarrow HHH)^{\text{SMEFT}}, \quad (5.34)$$

with $A(ZZ \rightarrow HHH)^{\text{HEFT}}$ and $A(ZZ \rightarrow HHH)^{\text{SMEFT}}$ given in Eqs. (4.8) and (4.9), respectively. Now the HEFT coefficients involved are the previous ones in $WW \rightarrow HHH$, i.e., a , b , c , κ_3 , κ_4 , a_{HWW} , and a_{HHWW} , and in addition, a_{HBB} , a_{HHBB} , a_{H0} , a_{HH0} , a_{HHH0} , a_{H1} , and a_{HH1} . Following the same procedure as before, we first plug in the values already obtained for the already known coefficients, and then solve for the remaining ones. In this case, there is only one coefficient left to be determined by this new matching which is a_{HHH0} . Thus, we finally get a perfect matching of the total amplitudes, HEFT and SMEFT, by setting this new coefficient as follows:

$$s_W^2 a_{HHH0} = -\frac{v^4}{4m_Z^2 \Lambda^2} c_{\Phi D}. \quad (5.35)$$

With this last solution, the full set of coefficients for the matching of HEFT/SMEFT amplitudes is completed.

VI. HIGH-ENERGY BEHAVIOR OF MULTIPLE HIGGS PRODUCTION FROM $V_L V_L$

Finally, in this section we study the high-energy behavior of the scattering amplitudes for double and triple Higgs production in the particular case of longitudinally polarized EW gauge bosons. We also consider the single Higgs production case for comparison. We focus here on the amplitudes $A(W_L W_L \rightarrow HH)$ and $A(W_L W_L \rightarrow HHH)$. We wish to explore the high-energy behavior of these amplitudes by means of an analytical expansion in powers of a small parameter written generically as $m_{EW}/\sqrt{s}$, where $\sqrt{s}$ is the energy of the process and m_{EW} refers to the EW masses involved and v . In the case of $W_L W_L$ fusion this implies $\sqrt{s} \gg m_W, m_H, v$. The interest of this study is twofold: on the one hand, it serves us to test the prediction done in the literature using the Equivalence Theorem, where these amplitudes are approximated by the corresponding amplitudes with external massless GBs and where these later are computed within the pure scalar theory [11,12,22]. On the other hand, this computation will also serve us to show the different energy behavior that the HEFT and the SMEFT amplitudes have, this being, in turn, very relevant for their respective tests at colliders.

For this exercise, we start with our HEFT amplitudes in Sec. IV and particularize them to the case where the $W^\pm$ polarization vectors, ϵ_1 and ϵ_2 are set to the longitudinal ones, ϵ_1^L and ϵ_2^L , respectively. Since we search only for the leading contributions to the amplitudes found in Sec. IV in this large s expansion, which turn out to be linear in s , i.e., of $\mathcal{O}(s)$, we use some starting simplifying assumptions. First, we consider only the terms in the various diagrams that only involve the LO-HEFT coefficients. The other terms can be shown to be subleading, i.e., of $\mathcal{O}(s^0)$. We have already mentioned explicitly those LO-terms in Sec. IV. Second, we approximate the longitudinal polarization vectors respectively by $\epsilon_1^L \simeq k_1/m_W$ and $\epsilon_2^L \simeq k_2/m_W$. Third, we consider all the Lorentz-invariant quantities involving products of two external four-momenta. Generically, products of two different four-momenta are quantities of order s , i.e., $k_i \cdot k_j = \mathcal{O}(s^1)$ for

$i \neq j$, whereas for two equal four-momenta they are $k_i^2 = m_i^2 = \mathcal{O}(s^0)$. Then, using the same notation as in Sec. IV, we get the following results for the leading terms of all the diagrams involved in the high-energy limit.

A. Case HH

In the double Higgs production case, for the first terms in this high-energy expansion, we get:

$$\begin{aligned} A_c^{\text{HEFT}} &\simeq 2bv^{-2}m_W^2(\epsilon_1 \cdot \epsilon_2) = \mathcal{O}(s^1), \\ A_s^{\text{HEFT}} &\simeq \frac{6a\kappa_3 v^{-2}m_H^2 m_W^2}{q_{12}^2 - m_H^2}(\epsilon_1 \cdot \epsilon_2) = \mathcal{O}(s^0), \\ A_t^{\text{HEFT}} &\simeq \frac{4a^2 v^{-2}m_W^2}{k_{13}^2 - m_W^2}(-(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})) = \mathcal{O}(s^1), \\ A_u^{\text{HEFT}} &\simeq \frac{4a^2 v^{-2}m_W^2}{k_{14}^2 - m_W^2}(-(\epsilon_1 \cdot k_{14})(\epsilon_2 \cdot k_{14})) = \mathcal{O}(s^1). \end{aligned} \quad (6.1)$$

Therefore, by adding them and considering $k_1 + k_2 = k_3 + k_4$, we get from the sum of A_c , A_t , and A_u the total result of $\mathcal{O}(s)$, where the angular dependence in A_t and A_u cancels, leading to:

$$A^{\text{HEFT}}(W_L W_L \rightarrow HH) \simeq -\frac{1}{v^2}(a^2 - b)s. \quad (6.2)$$

Similarly, one can conclude on the high-energy behavior result of single Higgs production from WBF of two longitudinal W 's. From the first equation in Sec. V we find, straightforwardly:

$$A^{\text{HEFT}}(W_L W_L \rightarrow H) \simeq \frac{1}{v}as. \quad (6.3)$$

B. Case HHH

In the triple Higgs production case we get, for the seven reference diagrams:

$$\begin{aligned} A_1^{\text{HEFT}} &\simeq 6m_W^2 m_H^2 v^{-3} \frac{1}{q_{45}^2 - m_H^2} (b\kappa_3)(\epsilon_1 \cdot \epsilon_2) = \mathcal{O}(s^0), \\ A_2^{\text{HEFT}} &\simeq 4m_W^4 v^{-3} \frac{1}{k_{13}^2 - m_W^2} (ab)(-m_W^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})) = \mathcal{O}(s^1), \\ A_4^{\text{HEFT}} &\simeq 6m_W^4 m_H^2 v^{-3} \frac{(a\kappa_4)}{q_{12}^2 - m_H^2} (\epsilon_1 \cdot \epsilon_2) = \mathcal{O}(s^0), \\ A_5^{\text{HEFT}} &\simeq \frac{18m_W^2 m_H^4 v^{-3}}{(q_{12}^2 - m_H^2)(q_{34}^2 - m_H^2)} (a\kappa_3^2)(\epsilon_1 \cdot \epsilon_2) = \mathcal{O}(s^{-1}), \end{aligned}$$

$$\begin{aligned}
A_{14}^{\text{HEFT}} &\simeq 12m_W^4 m_H^2 v^{-3} \frac{(a^2 \kappa_3)}{(k_{13}^2 - m_W^2)(q_{45}^2 - m_H^2)} (-m_W^{-2}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{13})) = \mathcal{O}(s^0), \\
A_{20}^{\text{HEFT}} &\simeq 8m_W^6 v^{-3} \frac{1}{(k_{13}^2 - m_W^2)(k_{25}^2 - m_W^2)} (a^3)(+m_W^{-4}(\epsilon_1 \cdot k_{13})(\epsilon_2 \cdot k_{25})(k_{25} \cdot k_{13})) = \mathcal{O}(s^1), \\
A_{26}^{\text{HEFT}} &\simeq 6m_W^2 v^{-3}(c)(\epsilon_1 \cdot \epsilon_2) = \mathcal{O}(s^1),
\end{aligned} \tag{6.4}$$

The other diagrams can be obtained from the above-referenced ones by the proper change in momenta, as defined in Eq. (4.6). Then, by grouping the 26 diagrams by subsets and using $k_1 + k_2 = k_3 + k_4 + k_5$, we get:

$$\begin{aligned}
S1 &= \mathcal{O}(s^0); & S2 &\simeq -\frac{4}{v^3}(ab)s; & S3 &= \mathcal{O}(s^0); & S4 &= \mathcal{O}(s^0), \\
S5 &\simeq \frac{4}{v^3}(a^3)s; & S6 &= \mathcal{O}(s^0); & S7 &\simeq \frac{3}{v^3}(c)s.
\end{aligned} \tag{6.5}$$

Finally, adding the leading subsets S2, S5, and S7 we get the resulting total amplitude of $\mathcal{O}(s)$:

$$A^{\text{HEFT}}(W_L W_L \rightarrow HHH) \simeq \frac{3}{v^3} \left(c + \frac{4}{3} a(a^2 - b) \right) s. \tag{6.6}$$

It is worth noticing that the above results in Eqs. (6.2) and (6.6) coincide with the approximate result found in [12,22], where they used the Equivalence Theorem and computed with external massless GBs in the pure scalar theory.

Finally, to end this section, it is illustrative to show the particular high-energy results of these longitudinal WBF amplitudes in two cases: 1) when comparing HEFT and SM, and 2) when comparing HEFT and SMEFT. With this purpose in mind, we first write the results in Eqs. (6.3), (6.2), and (6.6) in terms of Δa , Δb , and c . This leads to the generic BSM predictions at large energies within the LO-HEFT:

$$\begin{aligned}
A^{\text{HEFT}}(W_L W_L \rightarrow H) &\simeq \frac{s}{v}(1 - \Delta a), \\
A^{\text{HEFT}}(W_L W_L \rightarrow HH) &\simeq \frac{s}{v^2}(2\Delta a - \Delta b), \\
A^{\text{HEFT}}(W_L W_L \rightarrow HHH) &\simeq \frac{s}{v^3} 3 \left(c - 2\frac{8}{3}\Delta a + \frac{4}{3}\Delta b \right).
\end{aligned} \tag{6.7}$$

When setting the HEFT parameters to the SM values, i.e., for $\Delta a = 0$, $\Delta b = 0$, and $c = 0$, we get the well-known SM results:

$$\begin{aligned}
A^{\text{SM}}(W_L W_L \rightarrow H) &\simeq \frac{s}{v}(1), \\
A^{\text{SM}}(W_L W_L \rightarrow HH) &\simeq \frac{s}{v^2}(0) + \mathcal{O}(s^0), \\
A^{\text{SM}}(W_L W_L \rightarrow HHH) &\simeq \frac{s}{v^3}(0) + \mathcal{O}(s^0),
\end{aligned} \tag{6.8}$$

which shows that at high energies compared to the involved boson masses, the double and triple Higgs production in the

SM are suppressed with respect to the BSM prediction, since the SM amplitudes do not grow with the CM squared energy. We can also use the above generic HEFT results to get the SMEFT results. This is done by the simple substitution of the previous values found for the Δa , Δb , and c from the matching, i.e., by the values in Eqs. (5.4), (5.21), and (5.33) respectively. Then, we get for SMEFT the following results:

$$\begin{aligned}
A^{\text{SMEFT}}(W_L W_L \rightarrow H) &\simeq \frac{s}{v} \left(1 - \frac{v^2}{\Lambda^2} \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) \right), \\
A^{\text{SMEFT}}(W_L W_L \rightarrow HH) &\simeq \frac{s}{v^2} \left(-2\frac{v^2}{\Lambda^2} \left(-c_{\Phi\Box} + \frac{1}{4}c_{\Phi D} \right) \right), \\
A^{\text{SMEFT}}(W_L W_L \rightarrow HHH) &\simeq \frac{s}{v^3}(0) + \mathcal{O}(s^0).
\end{aligned} \tag{6.9}$$

The interesting results above show that triple Higgs production from WBF of two longitudinal W 's vanishes within the dimension 6 SMEFT at $\mathcal{O}(s)$, as happens in the SM case. This is in remarkable contrast with the result of the LO-HEFT that grows linearly with s . This may have important implications for the BSM Higgs searches at future colliders. For a recent discussion on some phenomenological implications of these analytical results of the HEFT versus SMEFT within the context of multiple Higgs production at both lepton and hadron colliders, see for instance Ref. [14], where the full bosonic sector (scalar and gauge) is considered. This has also been explored in the simplified scenario of the pure scalar theory in Refs. [11,12,22].

VII. CONCLUSIONS

In this paper, we have explored the issue of matching HEFT and SMEFT by means of the identification of scattering amplitudes in both theories. For this purpose, we have chosen a set of seven specific processes that have direct connection with experimental data. These selected processes are: single, double, and triple Higgs production from WBF, $VV \rightarrow H$, $VV \rightarrow HH$, $VV \rightarrow HHH$ with $VV = WW, ZZ$, and the elastic scattering $HH \rightarrow HH$.

TABLE I. Summary of the results found from the matching of HEFT/SMEFT amplitudes.

Matching dimension 6 SMEFT to HEFT		
LO-HEFT	NLO-HEFT	
$\Delta a = \frac{v^2}{\Lambda^2} \left(-c_{\Phi\Box} + \frac{1}{4} c_{\Phi D} \right)$	$s_W^2 a_{H0} = -\frac{v^4}{8m_Z^2 \Lambda^2} c_{\Phi D}$	$a_{HWW} = -\frac{v^4}{2m_W^2 \Lambda^2} c_{\Phi W}$
$\Delta b = \frac{4v^2}{\Lambda^2} \left(-c_{\Phi\Box} + \frac{1}{4} c_{\Phi D} \right)$	$s_W^2 a_{HH0} = -\frac{5v^4}{16m_Z^2 \Lambda^2} c_{\Phi D}$	$a_{HHWW} = -\frac{v^4}{4m_W^2 \Lambda^2} c_{\Phi W}$
$c = -\frac{8v^2}{3\Lambda^2} \left(-c_{\Phi\Box} + \frac{1}{4} c_{\Phi D} \right)$	$s_W^2 a_{HHH0} = -\frac{v^4}{4m_Z^2 \Lambda^2} c_{\Phi D}$	$s_W^2 a_{HBB} = -\frac{v^4}{2m_Z^2 \Lambda^2} c_{\Phi B}$
$\Delta\kappa_3 = \frac{2v^4}{m_H^2 \Lambda^2} c_{\Phi} + \frac{3v^2}{\Lambda^2} \left(-c_{\Phi\Box} + \frac{1}{4} c_{\Phi D} \right)$		$s_W^2 a_{HHBB} = -\frac{v^4}{4m_Z^2 \Lambda^2} c_{\Phi B}$
$\Delta\kappa_4 = \frac{12v^4}{m_H^2 \Lambda^2} c_{\Phi} + \frac{50v^2}{3\Lambda^2} \left(-c_{\Phi\Box} + \frac{1}{4} c_{\Phi D} \right)$		$s_W c_W a_{H1} = -\frac{v^4}{2m_Z^2 \Lambda^2} c_{\Phi WB}$
		$s_W c_W a_{HH1} = -\frac{v^4}{4m_Z^2 \Lambda^2} c_{\Phi WB}$

For the solving of the matching equations, we have truncated the predictions of the HEFT at NLO, i.e., including effective operators up to chiral dimension 4, and the predictions of the SMEFT at NLO, i.e., including effective operators up to canonical dimension 6. The solutions to the HEFT/SMEFT matching equations, when imposed all together and for all the helicity amplitudes involved, lead to fix in total 14 HEFT coefficients in terms of the six SMEFT coefficients defining the dimension 6 SMEFT. The solutions found in this work are summarized in Table I. These contain the solutions for the five LO-HEFT coefficients, Δa , Δb , c , $\Delta\kappa_3$, and $\Delta\kappa_4$ and for the nine NLO-HEFT coefficients: a_{H0} , a_{HH0} , a_{HHH0} , a_{HWW} , a_{HHWW} , a_{HBB} , a_{HHBB} , a_{H1} , and a_{HH1} . As is well known, the matching of HEFT/SMEFT coefficients occurs in crossed orders of these EFTs. The LO-HEFT coefficients are given in terms of the three dimension 6 Wilson coefficients referring to the doublet Φ , i.e., $c_{\Phi\Box}$, $c_{\Phi D}$, and c_{Φ} . The NLO-HEFT coefficients a_{H0} , a_{HH0} , and a_{HHH0} are given in terms of just one SMEFT coefficient, $c_{\Phi D}$. The remaining NLO-HEFT coefficients in the last column of the table are given in terms of the SMEFT coefficients involving weak gauge bosons, W and B . It is also interesting to notice that when considering the weak isospin limit (also referred to as imposing custodial symmetry) given by $m_Z = m_W$ (or, equivalently, $g' = 0$, or $s_W = 0$) we get an important

reduction in the relevant coefficients among these later involving weak gauge bosons in the third column. These coefficients are a_{HWW} and a_{HHWW} , which are both given in terms of one SMEFT coefficient $c_{\Phi W}$.

It is also worth commenting that in order to fix other NLO-HEFT coefficients of Eq. (2.8), the matching with the SMEFT should go beyond dim6. In particular, focusing in the most relevant operators for BSM Higgs signals at colliders in the TeV energy regime, which are those containing four derivatives, the coefficients in front of those operators, a_{ddVV1} and a_{ddVV2} , have been proven in [17] to match with the SMEFT coefficients in front of the three dim8 operators containing also four derivatives, $C_{\phi^4 D^4}^{(1),(2),(3)}$ in the notation of [52]. Concretely the matching of HEFT/SMEFT amplitudes for $WW \rightarrow HH$ in [17] finds the following relations among these coefficients: $a_{ddVV1} = \frac{v^4}{4\Lambda^4} (C_{\phi^4 D^4}^{(1)} + C_{\phi^4 D^4}^{(2)})$ and $a_{ddVV2} = \frac{v^4}{4\Lambda^4} C_{\phi^4 D^4}^{(3)}$.

Besides the previous predictions for the HEFT coefficients in terms of the SMEFT ones, we have also found a set of interesting relations among EFT coefficients. Concretely, when assuming the SMEFT to be the low-energy theory for BSM Higgs Physics, the following summary in Eq. (7.1) of relations is found among the corresponding HEFT coefficients:

$$\begin{aligned}
 \Delta b &= 4\Delta a; & c &= -\frac{8}{3}\Delta a = -\frac{2}{3}\Delta b; & \Delta\kappa_4 &= 6\Delta\kappa_3 - \frac{4}{3}\Delta a, \\
 a_{HHWW} &= \frac{1}{2}a_{HWW}; & a_{HHBB} &= \frac{1}{2}a_{HBB}; & a_{HHH0} &= \frac{4}{5}a_{HH0} = 2a_{H0}.
 \end{aligned} \tag{7.1}$$

These relations may have important impact for the search of BSM Higgs signals at future colliders. This is because they may lead to correlations among specific observables that

can be measured in the future experiments. In particular, we have investigated here one of those observables that will be relevant for the future colliders exploring the multiple

Higgs production from WBF. Concretely, by an explicit analytical computation with the full bosonic particle content (scalar and gauge), we have explored the behavior of the double and triple Higgs production from longitudinal WBF at high energies compared with the boson masses involved. We have demonstrated analytically in Eq. (6.7) that in the generic HEFT, both the double and triple amplitudes grow with energy as $\mathcal{O}(s)$. In contrast, for the SMEFT, we have found in Eq. (6.9) that the double production also grows with energy as $\mathcal{O}(s)$, but the triple Higgs production does not grow with s , leading to a vanishing result to $\mathcal{O}(s)$ (as in the SM case). We then get, analytically, the suppression of triple Higgs production in SMEFT in contrast to HEFT, where this production will be enhanced (with respect to the SM prediction) at large energies compared to the boson masses. This implies that the search for triple Higgs production and the comparison with double Higgs production at colliders will provide a very powerful tool to disentangle the two EFTs. Most probably, this is nothing else than a potential signal disentangling the singlet/doublet Higgs nature in both HEFT/SMEFT approaches which in our opinion deserves further study.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: FEYNMAN RULES OF SM

Here and in the next appendices, we summarize the relevant Feynman rules for double and triple Higgs production from WBF in the SM, HEFT-NLO, and SMEFT-NLO. In all the vertices, we use the convention of all momenta ingoing and the following specific assignments of momenta and Lorentz indices:

$$\begin{aligned}
 &H(p_1)H(p_2)H(p_3), & H(p_1)H(p_2)H(p_3)H(p_4) \\
 &H(p_1)W^{+\mu_2}(p_2)W^{-\mu_3}(p_3), & H(p_1)H(p_2)W^{+\mu_3}(p_3)W^{-\mu_4}(p_4) \\
 &H(p_1)Z^{\mu_2}(p_2)Z^{\mu_3}(p_3), & H(p_1)H(p_2)Z^{\mu_3}(p_3)Z^{\mu_4}(p_4) \\
 &H(p_1)H(p_2)H(p_3)W^{+\mu_4}(p_4)W^{-\mu_5}(p_5), & H(p_1)H(p_2)H(p_3)Z^{\mu_4}(p_4)Z^{\mu_5}(p_5)
 \end{aligned}$$

In the Feynman rules presented in these Appendices we use the short notation, omitting indices:

$$\begin{aligned}
 \Gamma_{HHH} &\equiv \Gamma_{H_1H_2H_3} \equiv \Gamma(H(p_1), H(p_2), H(p_3)), \\
 \Gamma_{HHHH} &\equiv \Gamma_{H_1H_2H_3H_4} \equiv \Gamma(H(p_1), H(p_2), H(p_3), H(p_4)), \\
 \Gamma_{HVV} &\equiv \Gamma_{H_1V_2V_3}^{\mu_2\mu_3} \equiv \Gamma(H(p_1), V^{\mu_2}(p_2), V^{\mu_3}(p_3)), \\
 \Gamma_{HHVV} &\equiv \Gamma_{H_1H_2V_3V_4}^{\mu_3\mu_4} \equiv \Gamma(H(p_1), H(p_2), V^{\mu_3}(p_3), V^{\mu_4}(p_4)), \\
 \Gamma_{HHHVV} &\equiv \Gamma_{H_1H_2H_3V_4V_5}^{\mu_4\mu_5} \equiv \Gamma(H(p_1), H(p_2), H(p_3), V^{\mu_4}(p_4), V^{\mu_5}(p_5)).
 \end{aligned}$$

We also use a short notation for the frequently appearing W and Z mass combinations:

$$c_W^2 \equiv \frac{m_W^2}{m_Z^2}, \quad s_W^2 \equiv \frac{m_Z^2 - m_W^2}{m_Z^2}, \quad s_W c_W \equiv \frac{m_W \sqrt{m_Z^2 - m_W^2}}{m_Z^2}.$$

For a clear comparison, we remind first the SM Feynman rules for these vertices when expressed in terms of the physical input parameters, i.e., the masses m_H , m_W , m_Z , and the vev related to the Fermi constant, $v = (\frac{1}{\sqrt{2}G_F})^{1/2} = 246$ GeV.

$$\begin{aligned}
i\Gamma_{HHH}^{\text{SM}} &= -3i \frac{m_H^2}{v}, & vV_{HHHH}^{\text{SM}} &= V_{HHH}^{\text{SM}}, \\
i\Gamma_{HHHH}^{\text{SM}} &= -3i \frac{m_H^2}{v^2}, & vV_{HHWW}^{\text{SM}} &= V_{HWW}^{\text{SM}}, \\
i\Gamma_{HWW}^{\text{SM}} &= 2i \frac{m_W^2}{v} g^{\mu_2\mu_3}, & vV_{HHZZ}^{\text{SM}} &= V_{HZZ}^{\text{SM}}. \\
i\Gamma_{HHWW}^{\text{SM}} &= 2i \frac{m_W^2}{v^2} g^{\mu_3\mu_4}, & & \\
i\Gamma_{HZZ}^{\text{SM}} &= 2i \frac{m_Z^2}{v} g^{\mu_2\mu_3}, & & \\
i\Gamma_{HHZZ}^{\text{SM}} &= 2i \frac{m_Z^2}{v^2} g^{\mu_3\mu_4}, & & \\
i\Gamma_{HHHWW}^{\text{SM}} &= 0, & & \\
i\Gamma_{HHHZZ}^{\text{SM}} &= 0. & &
\end{aligned} \tag{A1}$$

It is immediately deducible from the above expressions the well known relations between SM pairs of vertices:

These relations are a consequence of H being a component of a $SU(2)$ doublet within the SM. Within the HEFT and SMEFT, however, these relations do not hold, as shown through this work. In the HEFT case these vertices are not correlated, even at LO, since H is a singlet. In the SMEFT-NLO, they are related but in a different way than in the SM case due to the contributions from the dimension 6 operators.

APPENDIX B: FEYNMAN RULES OF HEFT-NLO

In the following we display the Feynman rules of the HEFT-NLO that we also express in terms of the physical parameters, the masses m_H , m_W , m_Z , and the vev v related to the Fermi constant:

$$\begin{aligned}
i\Gamma_{HHH}^{\text{HEFT}} &= -3i\kappa_3 \frac{m_H^2}{v} + \frac{i}{v^3} (a_{Hdd}m_H^2 + a_{ddW}m_W^2 + a_{ddZ}m_Z^2)(p_1^2 + p_2^2 + p_3^2) + \mathcal{O}(p^4), \\
i\Gamma_{HHHH}^{\text{HEFT}} &= -3i\kappa_4 \frac{m_H^2}{v^2} + \frac{2i}{v^4} (a_{HHdd}m_H^2 + a_{HddW}m_W^2 + a_{HddZ}m_Z^2)(p_1^2 + p_2^2 + p_3^2 + p_4^2) + \mathcal{O}(p^4), \\
i\Gamma_{HWW}^{\text{HEFT}} &= 2ia \frac{m_W^2}{v} g^{\mu_2\mu_3} - \frac{2im_W^2}{v^3} (a_{d2}(p_1^{\mu_2}p_2^{\mu_3} + p_1^{\mu_3}p_3^{\mu_2} + g^{\mu_2\mu_3}p_1^2) - a_{H\gamma\gamma}m_H^2g^{\mu_2\mu_3} + 4a_{HWW}(p_2^{\mu_3}p_3^{\mu_2} - g^{\mu_2\mu_3}p_2 \cdot p_3)), \\
i\Gamma_{HHWW}^{\text{HEFT}} &= 2ib \frac{m_W^2}{v^2} g^{\mu_3\mu_4} - 2i \frac{m_W^2}{v^4} (a_{Hd2}((p_1 + p_2)^{\mu_3}p_3^{\mu_4} + (p_1 + p_2)^{\mu_4}p_4^{\mu_3} + g^{\mu_3\mu_4}(p_1 + p_2)^2) \\
&\quad - a_{HH\gamma\gamma}m_H^2g^{\mu_3\mu_4} + 8a_{HHWW}(p_3^{\mu_4}p_4^{\mu_3} - g^{\mu_3\mu_4}p_3 \cdot p_4) + 2a_{dd\gamma\gamma 1}(p_1^{\mu_3}p_2^{\mu_4} + p_1^{\mu_4}p_2^{\mu_3}) + 4a_{dd\gamma\gamma 2}g^{\mu_3\mu_4}p_1 \cdot p_2), \\
i\Gamma_{HZZ}^{\text{HEFT}} &= 2ia \frac{m_Z^2}{v} g^{\mu_2\mu_3} - 2i \frac{m_Z^2}{v^3} ((a_{d2}c_W^2 - a_{d1}s_W^2)(p_1^{\mu_2}p_2^{\mu_3} + p_1^{\mu_3}p_3^{\mu_2} + g^{\mu_2\mu_3}p_1^2) \\
&\quad - a_{H\gamma\gamma}m_H^2g^{\mu_2\mu_3} + 4(a_{HWW}c_W^4 + a_{HBB}s_W^4 + a_{H1}s_W^2c_W^2)(p_2^{\mu_3}p_3^{\mu_2} - g^{\mu_2\mu_3}p_2 \cdot p_3) + 4a_{H0}s_W^2m_Z^2g^{\mu_2\mu_3}), \\
i\Gamma_{HHZZ}^{\text{HEFT}} &= 2ib \frac{m_Z^2}{v^2} g^{\mu_3\mu_4} - 2i \frac{m_Z^2}{v^4} ((a_{Hd2}c_W^2 - a_{Hd1}s_W^2)((p_1 + p_2)^{\mu_3}p_3^{\mu_4} + (p_1 + p_2)^{\mu_4}p_4^{\mu_3} + g^{\mu_3\mu_4}(p_1 + p_2)^2) \\
&\quad - a_{HH\gamma\gamma}m_H^2g^{\mu_3\mu_4} + 8(a_{HHWW}c_W^4 + a_{HHBB}s_W^4 + a_{HH1}s_W^2c_W^2)(p_3^{\mu_4}p_4^{\mu_3} - g^{\mu_3\mu_4}p_3 \cdot p_4) \\
&\quad + 2a_{dd\gamma\gamma 1}(p_1^{\mu_3}p_2^{\mu_4} + p_1^{\mu_4}p_2^{\mu_3}) + 4a_{dd\gamma\gamma 2}g^{\mu_3\mu_4}p_1 \cdot p_2) + 8a_{HH0}s_W^2m_Z^2g^{\mu_3\mu_4}), \\
\Gamma_{HHHWW}^{\text{HEFT}} &= 6ic \frac{m_W^2}{v^3} g^{\mu_4\mu_5} + \mathcal{O}(m^2p^2) + \mathcal{O}(p^4), \\
i\Gamma_{HHHZZ}^{\text{HEFT}} &= 6ic \frac{m_Z^2}{v^3} g^{\mu_4\mu_5} - 48i \frac{m_Z^4}{v^5} a_{HHH0}s_W^2g^{\mu_4\mu_5} + \mathcal{O}(m^2p^2) + \mathcal{O}(p^4).
\end{aligned} \tag{B1}$$

First, notice that the set of SM Feynman rules summarized in Appendix A can be reached from these HEFT Feynman rules above by fixing $a = b = \kappa_3 = \kappa_4 = 1$, $c = 0$ and setting to zero all the NLO coefficients, $a_i = 0$. Second, these interaction vertices enter in Feynman diagrams of Figs. 2 and 3. Third, some of these HEFT coefficients appear in the matching equations discussed in this work (collected in Table I). Finally, notice that in the above expressions of $\Gamma_{HHH}^{\text{HEFT}}$ and $\Gamma_{HHHH}^{\text{HEFT}}$, we have not specified the $\mathcal{O}(p^4)$ terms, which go as four powers of momentum since they are not needed for the present work. We have not included either the $\mathcal{O}(p^4)$ contributions in the contact interactions, $\Gamma_{HHHVV}^{\text{HEFT}}$, for the same reason.

APPENDIX C: FEYNMAN RULES OF SMEFT-NLO

Here we collect the Feynman Rules of SMEFT-NLO. We use again as input parameters the physical masses m_H , m_W , m_Z , and v related to the Fermi constant:

$$\begin{aligned}
i\Gamma_{HHH}^{\text{SMEFT}} &= -3i\frac{m_H^2}{v} + i\frac{1}{\Lambda^2} \left(6c_\Phi v^3 + v \left(\frac{1}{4}c_{\Phi D} - c_{\Phi\Box} \right) (3m_H^2 + 2(p_1^2 + p_2^2 + p_3^2)) \right), \\
i\Gamma_{HHHH}^{\text{SMEFT}} &= -3i\frac{m_H^2}{v^2} + i\frac{1}{\Lambda^2} \left(36c_\Phi v^2 + \left(\frac{1}{4}c_{\Phi D} - c_{\Phi\Box} \right) (6m_H^2 + 2(p_1^2 + p_2^2 + p_3^2 + p_4^2)) \right), \\
i\Gamma_{HWW}^{\text{SMEFT}} &= 2i\frac{m_W^2}{v} g^{\mu_2\mu_3} + i\frac{v}{\Lambda^2} \left(-2m_W^2 \left(\frac{1}{4}c_{\Phi D} - c_{\Phi\Box} \right) g^{\mu_2\mu_3} + 4c_{\Phi W} (p_2^{\mu_3} p_3^{\mu_2} - g^{\mu_2\mu_3} p_2 \cdot p_3) \right), \\
i\Gamma_{HHWW}^{\text{SMEFT}} &= 2i\frac{m_W^2}{v^2} g^{\mu_3\mu_4} + i\frac{1}{\Lambda^2} \left(-4m_W^2 \left(\frac{1}{4}c_{\Phi D} - c_{\Phi\Box} \right) g^{\mu_3\mu_4} + 4c_{\Phi W} (p_3^{\mu_4} p_4^{\mu_3} - g^{\mu_3\mu_4} p_3 \cdot p_4) \right), \\
i\Gamma_{HZZ}^{\text{SMEFT}} &= 2i\frac{m_Z^2}{v} g^{\mu_2\mu_3} + i\frac{v}{\Lambda^2} \left(2m_Z^2 \left(\frac{1}{4}c_{\Phi D} + c_{\Phi\Box} \right) g^{\mu_2\mu_3} + (4c_{\Phi W} c_W^2 + 4c_{\Phi B} s_W^2 + 4c_{\Phi WB} s_W c_W) (p_2^{\mu_3} p_3^{\mu_2} - g^{\mu_2\mu_3} p_2 \cdot p_3) \right), \\
i\Gamma_{HHZZ}^{\text{SMEFT}} &= 2i\frac{m_Z^2}{v^2} g^{\mu_3\mu_4} + i\frac{1}{\Lambda^2} (4m_Z^2 (c_{\Phi\Box} + c_{\Phi D}) g^{\mu_3\mu_4} + (4c_{\Phi W} c_W^2 + 4c_{\Phi B} s_W^2 + 4c_{\Phi WB} s_W c_W) (p_3^{\mu_4} p_4^{\mu_3} - g^{\mu_3\mu_4} p_3 \cdot p_4)), \\
i\Gamma_{HHHWW}^{\text{SMEFT}} &= 0 + 0 + \mathcal{O}\left(\frac{1}{\Lambda^4}\right), \\
i\Gamma_{HHHZZ}^{\text{SMEFT}} &= 0 + i\frac{1}{\Lambda^2} 12m_Z^2 c_{\Phi D} g^{\mu_4\mu_5}.
\end{aligned} \tag{C1}$$

Notice that the first terms in these SMEFT vertices coincide with the SM values given in Appendix A. The second terms in these vertices are of $\mathcal{O}(\frac{1}{\Lambda^2})$. Then, it is interesting to remark that, in the SMEFT, the $HHHWW$ contact vertex does not appear at this order, $\mathcal{O}(\frac{1}{\Lambda^2})$. In contrast, the $HHHZZ$ contact vertex does appear. None of these two contact vertices appear in the SM case, so they are generic BSM Higgs interactions. This helps to identify in which place of the Feynman diagrams they are contributing to the scattering amplitudes studied here.

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