

Dark bubble cosmology and the equivalence principle

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The main goal of string phenomenology is to find realistic models of particle physics and cosmology within string theory. Dark bubble cosmology is an alternative to string compactifications, where our Universe lives on a bubble expanding in a higher-dimensional spacetime. This construction is inherently nonsupersymmetric and can yield a four-dimensional realistic cosmology, where radiation behaves as expected due to its coupling to higher-dimensional fields. We study the coupling of the electroweak and strong sectors to the induced braneworld gravity via the same mechanism. While the electroweak sector is unaffected, the gravitational and inertial masses of the proton differ significantly, severely violating measurements of the equivalence principle.

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I. INTRODUCTION

The dark bubble model [1–12] offers an alternative to compactification attempts to obtain (quasi-)de Sitter vacua in string theory [13]. According to this proposal, our universe lives on a 3-brane on the wall of a bubble which interpolates between two bulk 5d nonsupersymmetric anti-de Sitter (AdS) vacua with slightly different energy density. As the bubble expands, 4D observers confined to the bubble effectively perceive an expanding Friedmann-Lemaître-Robertson-Walker (FLRW) universe.

Two string embeddings of the dark bubble scenario have been proposed [7,12,15]. The former employs rotating branes in the $\text{AdS}_5 \times S^5$ geometry of type IIB string theory, breaking supersymmetry and generating a positive vacuum energy. The latter uses type 0'B string theory [16,17] but realizes a dynamical dark energy which contradicts observations [12].

Although the dark bubble scenario is not a standard 4D effective field theory [18], gravitational waves [8] and Abelian gauge fields [9,12] behave correctly due to their coupling to bulk fields. We study the same mechanism non-Abelian gauge fields, finding a different low-energy coupling to braneworld gravity which violates the equivalence

between inertial and gravitational mass. We show that for the electroweak sector of the Standard Model there is no violation at tree level, while the strong sector exhibits *significant deviations*. Comparing with data, we conclude that the dark bubble scenario, at least as currently formulated, is incompatible with the Standard Model.

In section “The dark bubble scenario” we present the model and its induced gravity. In section “Non-Abelian fields in the dark bubble” we introduce gauge fields, extending previous work to the non-Abelian case. In contrast with the Abelian case, non-Abelian fields *do not couple to braneworld gravity as in 4D effective field theory*. In section “Electroweak gauge bosons” we show that the tree-level masses of W and Z bosons are unaffected, while in section “Gluons and hadrons” we show that gluons lead to large violations of the equivalence principle for the proton mass.

II. THE DARK BUBBLE SCENARIO

In contrast with Randall-Sundrum, the dark bubble is an inside-outside construction where two AdS_5 vacua have slightly different cosmological constants. At distances longer than the AdS radii, the 4D Einstein equations follow from the junction condition across the brane via the Gauss-Codazzi relation [1,20]. The induced geometry is sourced by the energy-momentum tensor of the brane (which for empty branes acts as a cosmological constant) and by contributions from the AdS bulk. Adding matter to the brane, the backreacted solution leads to a net *positive* energy density, taking into account the extrinsic curvature through the junction condition. Stretched strings pulling on the brane look like particles in 4D, whose mass is related to

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the string tension by the bulk geometry [21]. Similarly, radiation is implemented by 5D black holes.

As outlined above, the energy-momentum tensor of the brane sources a jump in the extrinsic curvature. In terms of the proper time τ on the brane located at $r = a(\tau)$ in global AdS coordinates, the induced metric on the brane has the FLRW form

$$ds_{\text{brane}}^2 = -d\tau^2 + a(\tau)^2 d\Omega_3^2. \quad (1)$$

The Israel junction conditions [22] impose continuity of the induced metric across the brane, and fix the discontinuity of the extrinsic curvature across the brane in terms of its stress tensor. When the latter is dominated by the brane tension σ , the junction conditions imply

$$\sigma = \frac{3}{8\pi G_5} \left(\sqrt{k_-^2 + \frac{1+\dot{a}^2}{a^2}} - \sqrt{k_+^2 + \frac{1+\dot{a}^2}{a^2}} \right), \quad (2)$$

where the cosmological constants inside and outside the shell are $\Lambda_{\pm} = -6k_{\pm}^2 = -6/L_{\pm}^2$, and satisfy $\Lambda_- < \Lambda_+ < 0$ (or $k_- > k_+$). The vacuum with higher energy can decay nucleating a spherical bubble, which may be a Coleman-de Luccia or Brown-Teitelboim bubble; its wall is the 3-brane.

When $k_- - k_+ \ll k_{\pm}$ the tension is slightly below the critical value $\sigma_{\text{cr}} \equiv \frac{3}{8\pi G_5} (k_- - k_+)$, and the scale factor satisfies

$$\frac{\dot{a}^2}{a^2} \sim -\frac{1}{a^2} + \frac{8\pi G_4}{3} \Lambda_4, \quad (3)$$

where

$$\Lambda_4 = \frac{3(k_- - k_+)}{8\pi G_5} - \sigma, \quad G_4 = \frac{2k_- k_+}{k_- - k_+} G_5 \quad (4)$$

are the 4D (positive) cosmological and Newton constants. Since the former is proportional to $\sigma_{\text{cr}} - \sigma$, we see how a slightly subcritical brane is essential to realize scale separation of the braneworld physics with respect to the bulk. This induced 4D cosmology is not a standard 4D effective field theory. In other words, the physics is inherently higher-dimensional, and the separation of scales follows from the interplay of various higher-dimensional ingredients, in contrast with compactifications.

The above considerations can be formulated covariantly as follows. The difference in bulk metrics across the bubble wall must match the energy-momentum tensor S_b^a on the brane. The second Israel junction condition implies

$$\kappa_5 S_{ab} = [K_{ab} - K h_{ab}]|_{+}^{-}, \quad (5)$$

where $[A]_{+}^{-} = A_- - A_+$ and $K_{ab} = \nabla_{\beta} n_{\alpha} e_a^{\alpha} e_b^{\beta}$, with n_{α} a unit normal vector to the bubble wall and e_a^{α} its tangent

vector. h_{ab} is the induced metric on the wall. K_{ab} (with trace K) represents the extrinsic curvature [23]. The Gauss-Codazzi equations read

$$R_{\alpha\beta\gamma\delta}^{(5)} e_a^{\alpha} e_b^{\beta} e_c^{\gamma} e_d^{\delta} = R_{abcd}^{(4)} + K_{ad} K_{bc} - K_{ac} K_{bd}. \quad (6)$$

Inserting this equation (and its contractions) into the thin-shell junction condition yields, at leading order in $\frac{k_- - k_+}{k_+}$ [3],

$$G_{ab}^{(4)} = h_{ab} \left[2\kappa_5 \sigma \left(\frac{k_+ k_-}{k_- - k_+} \right) - 3k_+ k_- \right] - 3\kappa_4 (T_{\text{brane}})_{ab} + \left(\frac{k_+ k_-}{k_- - k_+} \right) \left[\left(\frac{\mathcal{J}_{ab}^+}{k_+} - \frac{\mathcal{J}_{ab}^-}{k_-} \right) - \frac{1}{2} h_{ab} \left(\frac{\mathcal{J}^+}{k_+} - \frac{\mathcal{J}^-}{k_-} \right) \right], \quad (7)$$

where T_{brane} corresponds to matter on the brane and contributes with a negative sign to the energy-momentum tensor in 4D with $\mathcal{J}_{ab}$ defined as

$$\mathcal{J}_{ab} = R_{\alpha\beta\gamma\delta}^{(5)} e_a^{\alpha} e_b^{\beta} e_c^{\gamma} e_d^{\delta} h^{cd}. \quad (8)$$

This means that, in addition to the expected contributions to the induced Einstein equations coming from the stress tensor on the brane, we have to include contributions from the bulk built from the $\mathcal{J}$ tensor.

Turning on world volume fields, their stress-energy contribution to the Einstein equations induced by the Gauss-Codazzi relation appears with the *opposite sign* relative to standard minimal coupling [2,9]. For Abelian gauge fields, this negative sign is compensated by the extrinsic curvature terms [9]. As we shall discuss shortly, this is not the case for non-Abelian gauge fields.

III. NON-ABELIAN FIELDS IN THE DARK BUBBLE

We now discuss how gauge fields couple to braneworld gravity. Stringy realizations of the dark bubble contain 2-form fields B and C , which couple to the world volume of the D3-branes on the wall of the dark bubble. This induces a backreaction in the bulk, in turn modifying the induced geometry on the brane.

In the Abelian case [9,12] the B -field couples to the Maxwell field F on the world volume according to the (Dirac-)Born-Infeld (DBI) action

$$S_{\text{D3}} = -T_3 \int d^4x \sqrt{-\det(g_4 + \tau \mathcal{F})}, \quad (9)$$

where $\tau = 2\pi\alpha'$ with $\alpha' \equiv l_s^2$ the (reciprocal) string tension. The D3-brane tension is $T_3 = \frac{1}{(2\pi)^3 \alpha'^2 g_s}$ in terms of the string coupling g_s , and the combination $\tau \mathcal{F} = \tau F + B$. At low curvatures

$$S_{D3} \sim - \int d^4x \sqrt{-\det(g_4)} \left(T_3 + \frac{1}{4g^2} \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} \right), \quad (10)$$

where $g^2 \equiv 2\pi g_s$ is the gauge coupling. The bulk-brane dynamics are encoded in the action

$$S_5 = \frac{1}{2\kappa_5} \int d^5x \sqrt{-\det(g_5)} \left(R - \frac{1}{12g_s} H^2 - 2\Lambda_{\text{AdS}} \right) + S_{D3}, \quad (11)$$

where $H = dB$ is the curvature of B and Λ_{AdS} is the cosmological constant of AdS_5 [24]. It follows that the B -field induces a discontinuity across the brane [9,25]

$$\Delta H^{r\mu\nu}|_{r=a} = \frac{8G_5 k a}{\pi\alpha'} \mathcal{F}_{\mu\nu}|_{r=a}, \quad (12)$$

which means that electromagnetic waves on the brane source the B -field in the bulk. The Israel junction conditions then yield the 4D Einstein equations with the correct Maxwell stress tensor [9].

Ramond-Ramond (R-R) potentials C_n also couple to the world volume gauge field via the Chern-Simons term. For a Dp -brane sweeping a world volume W , it reads [26]

$$S_{\text{CS}} = \mu_p \sum_n \int_W C_n \wedge \sqrt{\frac{\hat{A}(TW)}{\hat{A}(NW)}} \wedge e^{\tau\mathcal{F}}, \quad (13)$$

where the minimal charge $\mu_p = \frac{1}{(2\pi)^p l_s^{p+1}}$. For a D3-brane, the relevant couplings are

$$S_{\text{CS}} = \mu_3 \int_W (C_4 + \tau\mathcal{F} \wedge C_2), \quad (14)$$

since the $\hat{A}$ -genus contributes terms in degree four which do not couple to the R-R fields. Including R-R fields, the field equations become [12]

$$\begin{aligned} \partial_r H^{r\mu\nu} &= \frac{2G_5 k r}{\pi^2 \alpha'^2} [\tilde{C}^{\mu\nu} + 2\tau \mathcal{F}^{\mu\nu}] \delta(r - a(\eta)) \\ \partial_r F^{r\rho\sigma} &= \frac{2G_5 k r}{\pi\alpha'} \tilde{\mathcal{F}}^{\rho\sigma} \delta(r - a(\eta)), \end{aligned} \quad (15)$$

where $\tilde{\mathcal{F}}$ is the dual electromagnetic field-strength. Since $\tilde{C}$ vanishes on the brane, we obtain the discontinuities

$$\Delta H \sim \mathcal{F}, \quad \Delta F_3 \sim \tilde{\mathcal{F}}. \quad (16)$$

We now generalize this construction to $SU(N)$ world volume gauge fields $\tau\mathcal{F} = B\mathbb{I} + \tau F_a T^a$. We follow the symmetrized trace prescription [27,28] in the absence of scalar fields, which produces a valid action up to $\mathcal{O}(\mathcal{F}^4)$ [29,30]. Then, Eq. (10) becomes

$$S_{D3} \sim - \int d^4x \text{STr} \sqrt{-\det(g_{\mu\nu} + \tau\mathcal{F}_{\mu\nu})}. \quad (17)$$

Up to quartic order,

$$\begin{aligned} \text{STr} \left[\mathcal{F}_{\mu\nu}^2 - \frac{1}{2} \tau^2 \left(\mathcal{F}^4 - \frac{1}{4} (\mathcal{F}^2)^2 \right) \right] &\sim \text{Tr}[\mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu}] \\ &- \frac{\tau^2}{3} \text{Tr} \left[\mathcal{F}_{\mu\nu} \mathcal{F}^{\alpha\nu} \mathcal{F}^{\mu\beta} \mathcal{F}_{\alpha\beta} + \frac{1}{2} \mathcal{F}_{\mu\nu} \mathcal{F}^{\alpha\nu} \mathcal{F}^{\alpha\beta} \mathcal{F}_{\mu\beta} \right. \\ &\left. - \frac{1}{4} \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} \mathcal{F}_{\alpha\beta} \mathcal{F}^{\alpha\beta} - \frac{1}{8} \mathcal{F}_{\mu\nu} \mathcal{F}_{\alpha\beta} \mathcal{F}^{\mu\nu} \mathcal{F}^{\alpha\beta} \right]. \end{aligned} \quad (18)$$

Since the B -field vanishes on the brane, the resulting discontinuity in the B -field is proportional to a cubic term, schematically $\Delta H \propto \text{Tr}(F^3)$. This is because $\text{Tr}F = 0$ in the non-Abelian sector. Even without a specific prescription for a non-Abelian DBI action, one can see that ΔH cannot be sourced by a term linear in F .

This difference has profound implications. The discontinuity corresponds to the induced energy-momentum tensor in the 4D Einstein equations, whose brane contribution has the wrong sign. In the Abelian case, the backreaction from the bulk B -field contributes the correct sign with a factor of two, resulting in $2T_{\mu\nu} - T_{\mu\nu} = +T_{\mu\nu}$. In the non-Abelian sector, the minus sign from the brane contribution cannot be compensated by the higher-order terms, which are subleading in α' .

Analogous considerations hold for R-R fields, including the axion $F_1 = dC_0$. The non-Abelian CS term yields [12],

$$S_{D3} \sim \text{Tr} \left(\mu_3 \int_W C_4 + C_2 \wedge \tau\mathcal{F} + \frac{1}{2} C_0 \wedge \tau^2 \mathcal{F} \wedge \mathcal{F} \right), \quad (19)$$

from which $\Delta F_1 \propto \tau^2 \text{Tr}(F \wedge F)$.

IV. ELECTROWEAK GAUGE BOSONS

According to the above results, in a dark bubble realization of the $SU(2)_L \times U(1)_Y$ electroweak sector of the Standard Model, the energy-momentum tensor reads

$$T^{\mu\nu} = T_{\phi}^{\mu\nu} - T_{SU(2)}^{\mu\nu} + T_{U(1)}^{\mu\nu}, \quad (20)$$

where the first term comes from the Higgs. We remain agnostic about its origin, but we optimistically assume that it contributes positively to the stress tensor; were this not the case, the equivalence principle would be violated already in the Higgs sector. We can diagonalize the mass contributions to T^{00} to find gravitational masses at tree level. The non-Abelian covariant derivative is

$$D_\mu = \partial_\mu - ig A_\mu^a \tau^a - \frac{i}{2} g' B_\mu, \quad (21)$$

where A_μ^a and B_μ are the $SU(2)$ and $U(1)$ gauge fields with corresponding couplings g, g' . In the Higgs phase, ϕ

acquires an expectation value of the form

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \quad (22)$$

The energy density of perturbations around the vacuum in the temporal gauge $A^0 = 0$ then simplifies to

$$T_{00} = -\frac{v^2}{32} (g^2 (A_\alpha^1)^2 + g^2 (A_\alpha^2)^2 + (-gA_\alpha^3 + g'B_\alpha)^2). \quad (23)$$

To compute gravitational masses, we diagonalize the matrix

$$\begin{pmatrix} \frac{v^2 g^2}{4} & 0 & 0 & 0 \\ 0 & \frac{v^2 g^2}{4} & 0 & 0 \\ 0 & 0 & \frac{v^2 g^2}{4} & -gg' \\ 0 & 0 & -gg' & \frac{v^2 g'^2}{4} \end{pmatrix}. \quad (24)$$

The resulting spectrum is

- (i) $W_\alpha^\pm = \frac{1}{\sqrt{2}} (A_\alpha^1 \mp iA_\alpha^2)$, with $m_W = g \frac{v}{2}$,
- (ii) $Z_\mu^0 = \frac{1}{\sqrt{g^2 + g'^2}} (gA_\mu^3 - g'B_\mu)$, with $m_Z = \sqrt{g^2 + g'^2} \frac{v}{2}$,
- (iii) $A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (g'A_\mu^3 + gB_\mu)$, with $m_A = 0$.

Thus, electroweak gauge bosons respect the equivalence principle; we now turn to the strong sector to find deviations.

V. GLUONS AND HADRONS

Since the strong sector confines and is strongly coupled at low energies, we first consider the quark-gluon plasma (QGP) phase present at temperatures above the hadronization scale (~ 150 MeV). If the negative contribution of gluons to the energy density dominates the total energy density ρ of the plasma, this could radically modify the known thermal history. Indeed, during thermal equilibrium the Hubble parameter is $H^2 \simeq \rho$. If the total energy density became negative, the thermal history would radically differ, even breaking the underlying assumption of FLRW. At thermal equilibrium, the relative contribution of gluons can be determined by the number of degrees of freedom $g_g = 8 \times 2 = 16$ which comes from the $SU(3)$ sector. Just before the QCD phase transition we also have quarks (u, d, s), photons, charged leptons (e, μ) and neutrinos, which contribute

$$g_q = 36, \quad g_\gamma = 2, \quad g_l = 8, \quad g_\nu = 6. \quad (25)$$

Since fermions contribute to the total number of relativistic degrees of freedom with a $7/8$ factor compared to bosons,

$$g_\star = \sum_i g_{i,b} + \frac{7}{8} \sum_i g_{i,f}, \quad (26)$$

the total before the QCD phase transition is $g_\star = 61.5$. Then, each particle in thermal equilibrium contributes as

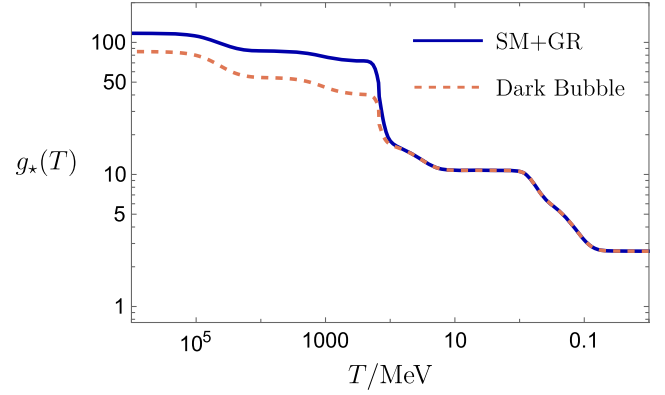


FIG. 1. Number of effective relativistic degrees of freedom according to the standard thermal history of the universe (blue) and to the dark bubble scenario (orange).

$$\rho_i = \frac{g_i}{g_\star} \rho \quad (27)$$

of the total budget ρ , and in particular gluons contribute

$$\rho_g = \frac{g_g}{g_\star} \rho \simeq 0.26 \rho. \quad (28)$$

During isotropic thermal evolution, we have a FLRW geometry with Hubble scale

$$H^2 \simeq \frac{\rho}{M_{\text{Pl}}^2} \simeq \frac{T^4}{M_{\text{Pl}}^2}. \quad (29)$$

If gluons coupled to gravity with the opposite sign, they would effectively contribute as negative degrees of freedom, replacing $g_\star$ with

$$\tilde{g}_\star = g_\star - 2g_g = 29.5. \quad (30)$$

This is depicted schematically in Fig. 1. Had gluons dominated the degrees of freedom during the early universe, this would have led to a radically different thermal history (in particular giving a negative total energy density). It is however much harder to experimentally probe a decreased number of degrees of freedom *only* before QCD phase transition, since very strongly constrained observables come from big bang nucleosynthesis and neutrino decoupling. The fact that the universe was opaque before photons decoupled to produce the cosmic microwave background (CMB) also means that we cannot directly observe this epoch, and we would require further assumptions on its imprint on the CMB. Therefore, we now turn to the low-energy regime of QCD. We will use lattice computations and data to show that gluons coupling with a negative sign to gravity necessarily lead to large violations of the equivalence principle.

A. Tests of the weak equivalence principle

The weak equivalence principle (WEP) states that all test bodies fall at the same rate in a gravitational field, independently of their mass or composition. This implies the universality of free fall and leads to the idea that gravity is a manifestation of spacetime curvature. In particular, inertial and gravitational masses are equal.

The Micro-Satellite à traînée Compensée pour l'Observation du Principe d'Équivalence (MICROSCOPE) mission [31,32] has provided the most stringent empirical constraints to date on violations of the WEP, testing the universality of free fall for test masses in Earth's gravitational field. The result bounds any differential acceleration between test bodies to a precision of

$$\frac{m_g - m_i}{m_i} \lesssim 10^{-15}, \quad (31)$$

where m_g , m_i denote gravitational and inertial mass, respectively. This places severe limits on WEP-violating physics. The WEP can also be tested for individual protons. In [33], the inertial mass of the proton was determined experimentally using high-precision Penning trap mass spectrometry. By calibrating the measured cyclotron frequency of a single proton to that of a ^{12}C nucleus in the same trap, the proton-to-carbon mass ratio was obtained with an extremely high precision, leading to a proton mass of

$$m_p = 0.93827198496 \pm 3 \times 10^{-11} \text{ GeV}. \quad (32)$$

The gravitational mass of a nucleon, averaged over the proton and neutron masses, was obtained indirectly in [34]. The number of atomic nuclei was obtained to high precision on a ~ 1 kg sample of ^{28}Si , which was then used to determine Avogadro's number. One can turn the logic around to determine the nucleon mass using the calibrated test masses and volumes, along with the molar mass measured by x-ray interferometry. In fact, one can obtain the precise mass of the ^{28}Si isotope, but the mass of the nucleons can only be obtained up to the unaccounted binding energy. One has

$$m_{^{28}\text{Si}} = 26.0993547 \pm 6 \times 10^{-7} \text{ GeV}, \quad (33)$$

so that the upper bound $m_p \geq m_{^{28}\text{Si}}/28$ [35] entails

$$m_p \geq 0.9321198 \pm 2 \times 10^{-8} \text{ GeV}. \quad (34)$$

Combining Eqs. (32) and (34), the WEP for the proton must hold up to

$$\frac{m_p^{(g)} - m_p^{(i)}}{m_p^{(i)}} \leq 7 \times 10^{-3}. \quad (35)$$

To estimate the gravitational and inertial masses of the proton in the dark bubble scenario we can rely on results

from lattice QCD (LQCD), which is particularly effective for studying low-energy QCD phenomena, such as hadron formation and confinement. In LQCD it is possible to compute gravitational form factors (GFFs), matrix elements of the QCD energy-momentum tensor, $T^{\mu\nu}$. Analogously to electromagnetic form factors, GFFs encode information about the spatial distribution of mass, pressure, and shear forces within hadrons [36,37]. The decomposition takes the form

$$\begin{aligned} \langle N(\vec{p}', s') | \hat{T}^{\mu\nu} | N(\vec{p}, s) \rangle \\ = \frac{1}{m} \bar{u}(\vec{p}', s') \left[A(t) P^\mu P^\nu + J(t) i \sigma^{\rho\{\mu} P^{\nu\}} \Delta_\rho \right. \\ \left. + D(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4} \right] u(\vec{p}, s). \end{aligned} \quad (36)$$

Recent advancements in LQCD allow for the decomposition of the proton's GFFs into contributions from up, down, strange quarks, and gluons. In [37], this flavor decomposition was performed over the range of momentum transfer $0 \leq -t \leq 2 \text{ GeV}^2$. For our purposes, up to a negligible renormalization-group mixing [36], the energy-momentum tensor admits a decomposition into quark and gluon contributions according to

$$\hat{T}^{\mu\nu} = \hat{T}_q^{\mu\nu} + \hat{T}_g^{\mu\nu}, \quad (37)$$

with

$$\begin{aligned} \hat{T}_g^{\mu\nu} &= 2\text{Tr} \left[F^{\mu\alpha} F_\alpha^\nu - \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} \right], \\ \hat{T}_q^{\mu\nu} &= \sum_{f=u,d,s} \left[\frac{i}{2} \bar{\psi}_f (\gamma^\mu D^\nu + \gamma^\nu D^\mu) \psi_f \right]. \end{aligned} \quad (38)$$

The respective GFF contributions are denoted $A_{g,q}(t)$, $J_{g,q}(t)$ and $D_{g,q}(t)$, as in Eq. (36). The forward limit $t=0$ is particularly significant, since it describes proton structure at rest. LQCD results confirm that quark and gluon contributions to the total momentum fraction satisfy the expected momentum sum rule (up to uncertainties), as shown in Fig. 2. These lattice computations have been compared to experiment for the case of diffractive photoproduction of J/ψ [38,39], which is sensitive to the GFFs of nuclei. The GFFs of our interest from LQCD are given by [37]

$$A_q(0) = 0.510 \pm 0.025, \quad A_g(0) = 0.501 \pm 0.027, \quad (39)$$

which indeed are compatible with the sum rule $A_q(0) + A_g(0) = 1$. Likewise, the total angular momentum contributions obey the spin sum rule $J_q(0) + J_g(0) = \frac{1}{2}$. This shows that quarks and gluons contribute approximately equally to the proton's momentum and spin. These results are also consistent with earlier LQCD studies [40].

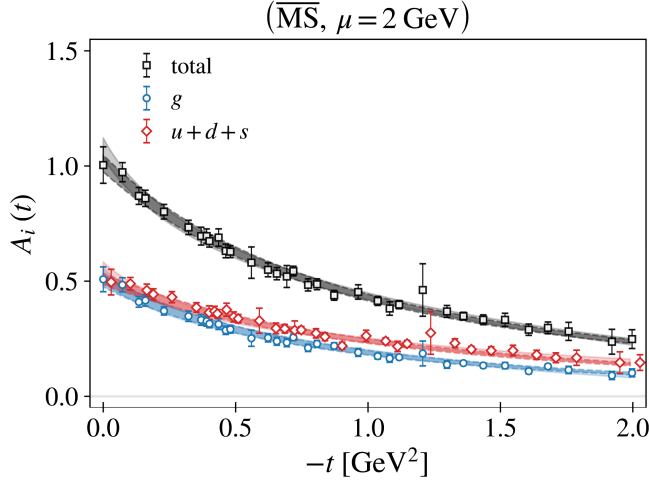


FIG. 2. The gluonic and quarkonic GFF $A(t)$, describing the momentum distribution inside the proton. At rest the form factor sum up to 1, reproducing the expected momentum sum rule. Adapted from [37].

In standard effective field theory the gravitational mass of the proton is given by

$$\langle N | \hat{T}^{00} | N \rangle = A_g(0)m_p + A_q(0)m_p = m_p. \quad (40)$$

However, as we discussed, in the dark bubble scenario gluons contribute with the opposite sign to the energy-momentum tensor up to subleading α' corrections, leading to

$$\langle N | \hat{T}_q^{00} - \hat{T}_g^{00} | N \rangle = (0.009 \pm 0.037)m_p. \quad (41)$$

The deviation between inertial and gravitational masses is then

$$\frac{m_p^{(g)} - m_p^{(i)}}{m_p^{(i)}} \simeq 0.99, \quad (42)$$

which leads to a *significant violation of the WEP* even for the laxest bounds in Eq. (35). Extrapolating this deviation would also violate the macroscopic measurements in Eq. (31) by 13 orders of magnitude.

VI. CONCLUSIONS

The dark bubble scenario [7,41] is an alternative to 4D effective field theory with several attractive features. It avoids many arguments and no-go theorems that apply to standard compactifications. In particular, swampland conjectures are statements about effective field theories coupled to gravity (in the same dimensions), and they generally do not apply to the dark bubble. To wit, this scenario can easily realize (quasi-)de Sitter cosmologies, in contrast with the conjecture of [42,43], while also naturally accommodating chiral fermions and unitary gauge groups.

However, regardless of possible issues due to delocalized gravity [3,44], non-Abelian gauge fields coupling to the induced gravity lead to large violations of the WEP for hadrons, while the QGP thermal history and electroweak physics remain largely unaffected. Reconciling these predictions with existing data is extremely challenging: achieving compatibility would require fine-tuning effects of different orders in α' , while maintaining their separation for all other observables. Given the empirical evidence constraining such deviations and the theoretical issues we discussed, we are led to conclude that the current formulation of the dark bubble scenario is incompatible with the Standard Model. To avoid this conclusion, a different mechanism without B and C fields must couple world volume fields and matter to the induced braneworld gravity. Presumably this would involve specific boundary conditions that realize the effective 4D graviton mode [3,10].

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DATA AVAILABILITY

No data were created or analyzed in this study.

- [1] S. Banerjee, U. Danielsson, G. Dibitetto, S. Giri, and M. Schillo, *Phys. Rev. Lett.* **121**, 261301 (2018).
- [2] S. Banerjee, U. Danielsson, G. Dibitetto, S. Giri, and M. Schillo, *J. High Energy Phys.* **10** (2019) 164.
- [3] S. Banerjee, U. Danielsson, and S. Giri, *J. High Energy Phys.* **04** (2020) 085.

- [4] S. Banerjee, U. Danielsson, and S. Giri, *Int. J. Mod. Phys. D* **30**, 2142029 (2021).
- [5] U. H. Danielsson, D. Panizo, R. Tielemans, and T. Van Riet, *Phys. Rev. D* **104**, 086015 (2021).
- [6] S. Banerjee, U. Danielsson, and S. Giri, *Phys. Rev. D* **108**, 126009 (2023).

- [7] U. Danielsson, O. Henriksson, and D. Panizo, *Phys. Rev. D* **107**, 026020 (2023).
- [8] U. Danielsson, D. Panizo, and R. Tieleman, *Phys. Rev. D* **106**, 024002 (2022).
- [9] I. Basile, U. Danielsson, S. Giri, and D. Panizo, *J. High Energy Phys.* **02** (2024) 112.
- [10] S. Banerjee, U. Danielsson, and M. Zemsch, *J. High Energy Phys.* **02** (2024) 102.
- [11] U. Danielsson and V. V. Hemelryck, [arXiv:2405.13679](#).
- [12] I. Basile, A. Borys, and J. Masias, [arXiv:2502.20438](#).
- [13] See also [14] for a related proposal with end-of-the-world branes.
- [14] B. Muntz, A. Padilla, and P. M. Saffin, *J. High Energy Phys.* **05** (2025) 006.
- [15] U. Danielsson and D. Panizo, *Phys. Rev. D* **109**, 026003 (2024).
- [16] A. Sagnotti, in *Supersymmetry and unification of fundamental interactions. Proceedings, International Workshop, SUSY 95, Palaiseau, France* (1995), pp. 473–484, [arXiv:hep-th/9509080](#).
- [17] A. Sagnotti, *Nucl. Phys. B, Proc. Suppl.* **56B**, 332 (1997).
- [18] As such, it is expected to violate several swampland conditions [19], although the AdS bulk does not.
- [19] E. Anastasi, R. Angius, J. Huertas, A. Uranga, and C. Wang, [arXiv:2501.03310](#).
- [20] S. Banerjee, U. Danielsson, G. Dibitetto, S. Giri, and M. Schillo, *J. High Energy Phys.* **10** (2019) 164.
- [21] I. Basile and S. Lanza, *J. High Energy Phys.* **10** (2020) 108.
- [22] W. Israel, *Nuovo Cimento Soc. Ital. Fis.* **44S10B**, 1 (1966); **48B**, 463(E) (1967).
- [23] For simplicity, we will consider the case where the wall is a simple empty brane with $S_{ab} = -\sigma h_{ab}$.
- [24] Strictly speaking, Eq. (11) is the result of a consistent truncation rather than a dimensional reduction, since scale separation is absent. This does not affect our following considerations on the B -field.
- [25] In the absence of branes, the ambient AdS geometry has vanishing B -field. In the presence of the brane, the inside geometry still has vanishing B -field, while the outside has a B -field sourced by the worldvolume electromagnetic field [9]. By continuity, the B -field vanishes on the worldvolume of the brane.
- [26] J. F. Morales, C. A. Scrucca, and M. Serone, *Nucl. Phys.* **B552**, 291 (1999).
- [27] A. A. Tseytlin, *Nucl. Phys.* **B501**, 41 (1997).
- [28] R. C. Myers, *J. High Energy Phys.* **12** (1999) 022.
- [29] A. Hashimoto and W. Taylor, *Nucl. Phys.* **B503**, 193 (1997).
- [30] A. Sevrin, J. Troost, and W. Troost, *Nucl. Phys.* **B603**, 389 (2001).
- [31] P. Touboul *et al.* (MICROSCOPE Collaboration), *Phys. Rev. Lett.* **129**, 121102 (2022).
- [32] P. Touboul *et al.*, *Phys. Rev. Lett.* **119**, 231101 (2017).
- [33] F. Heiße, F. Köhler-Langes, S. Rau, J. Hou, S. Junck, A. Kracke, A. Mooser, W. Quint, S. Ulmer, G. Werth *et al.*, *Phys. Rev. Lett.* **119**, 033001 (2017).
- [34] B. Andreas *et al.*, *Metrologia* **48**, S1 (2011).
- [35] The combined neutron-proton mass difference and binding energy is negative.
- [36] D. C. Hackett, P. R. Oare, D. A. Pefkou, and P. E. Shanahan, *Phys. Rev. D* **108**, 114504 (2023).
- [37] D. C. Hackett, D. A. Pefkou, and P. E. Shanahan, *Phys. Rev. Lett.* **132**, 251904 (2024).
- [38] S. Adhikari *et al.* (GlueX Collaboration), *Phys. Rev. C* **108**, 025201 (2023).
- [39] K. A. Mamo and I. Zahed, *Phys. Rev. D* **106**, 086004 (2022).
- [40] P. E. Shanahan and W. Detmold, *Phys. Rev. D* **99**, 014511 (2019).
- [41] U. H. Danielsson, S. S. Haque, G. Shiu, and T. Van Riet, *J. High Energy Phys.* **09** (2009) 114.
- [42] G. Obied, H. Ooguri, L. Spodyneiko, and C. Vafa, [arXiv:1806.08362](#).
- [43] D. Andriot and C. Roupec, *Fortschr. Phys.* **67**, 1800105 (2019).
- [44] S. Banerjee, U. Danielsson, and S. Giri, *J. High Energy Phys.* **03** (2021) 250.