

Does finetuning make D -term contributions smaller in natural grand unified theories with spontaneous supersymmetry breaking?

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In this paper, we explore the natural grand unified theory (GUT) with spontaneous supersymmetry (SUSY) breaking, focusing on the contribution to sfermion and Higgs masses. Natural GUTs solve various problems in SUSY GUTs with the natural assumption that all terms allowed by $SO(10) \times U(1)_A$ symmetry are introduced with $O(1)$ coefficients. It is also possible to introduce spontaneous SUSY breaking in the natural GUT. This scenario predicts that D -term contribution to sfermion and Higgs masses dominates the F -term contribution, which potentially leads to the SUSY flavor problem. Fortunately, it also predicts high-scale SUSY, which avoids the SUSY flavor problem but leads to the instability of the electroweak scale. The D -term domination results in the superheavy Higgsino, unless the D -term contribution becomes comparable with the F -term contribution. We discuss whether it is possible to suppress the D -term contribution while maintaining the F -term contribution through the tuning of $O(1)$ coefficients in the natural GUT with spontaneous SUSY breaking. Our results indicate that it is impossible. This also suggests that the mass spectrum of the sfermions can be predicted by the D -term contributions and that the Higgsino is not a candidate for dark matter in the scenario.

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I. INTRODUCTION

The grand unified theory (GUT) [1] is one of the most promising candidates for a model beyond the Standard Model (SM). In the GUT, the three gauge interactions in the SM, $SU(3)_C \times SU(2)_L \times U(1)_Y$, are unified into one gauge interaction, for example, $SU(5)$, $SO(10)$, and E_6 . It also realizes the unification of quarks and leptons, for example, one generation of quarks and leptons can be unified into **10** and $\bar{\mathbf{5}}$ of $SU(5)$ or **16** of $SO(10)$. It is surprising that we have already known the experimental supports for both unifications. Regarding the unification of matter, the various mass hierarchies observed in experiments can be explained only by one assumption that **10** fields of $SU(5)$ induce larger hierarchies in Yukawa couplings than $\bar{\mathbf{5}}$ fields of $SU(5)$. As for the unification of gauge interactions, it has been shown that by introducing supersymmetry (SUSY) [2,3], the three gauge coupling

constants of the minimal SUSY SM (MSSM) coincide at the grand unified scale $\Lambda_G \sim 2 \times 10^{16}$ GeV [4].

However, there are two problems in simple SUSY GUTs [3]. The first problem is that, due to the unification of Standard Model particles, simple SUSY GUTs lead to the unrealistic result that their masses are equal. The second is that when unifying the SM Higgs into the **5** representation of $SU(5)$, the mass of its partner (called colored Higgs or triplet Higgs) must be significantly larger than that of the SM Higgs to ensure stable protons. The latter is known as the doublet-triplet (DT) splitting problem [5].

The natural GUTs [6,7] have a natural assumption that all terms allowed by the symmetry are introduced with $O(1)$ coefficients, thereby solving the above problems in SUSY GUT. Furthermore, the natural GUT can naturally introduce the mechanism for spontaneous SUSY breaking [8–11] and predicts high scale SUSY at around 1000 TeV. This has the advantage of suppressing flavor changing neutral current (FCNC) processes and CP -violating processes mediated by SUSY particles, but it also has the disadvantage that the electroweak scale is unstable [12].

As discussed in detail in this paper, there are two types of contributions to the sfermion and Higgs masses, and in the simplest model with only two fields, S and Θ , the D -term contribution becomes one order of magnitude larger than the F -term contribution. If the dominance of the D -term contribution in the simplest model applies to the natural

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TABLE I. Field content of natural $SO(10)$ GUT with $U(1)_A$ charges. $\pm$ labels the Z_2 parity.

$SO(10)$	Negatively charged fields	Positively charged fields	Matter fields
45	$A(a = -1, -)$	$A'(a' = 3, -)$	$\Psi_i(\psi_1 = \frac{9}{2}, \psi_2 = \frac{7}{2}, \psi_3 = \frac{3}{2}, +)$
16	$C(c = -4, +)$	$C'(c' = 3, -)$	
$\overline{16}$	$\bar{C}(\bar{c} = -1, +)$	$\bar{C}'(\bar{c}' = 7, -)$	
10	$H(h = -3, +)$	$H'(h' = 4, -)$	$T(t = \frac{5}{2}, +)$
1	$\Theta(\theta = -1, +), Z(z = -2, -), \bar{Z}(\bar{z} = -2, -)$	$S(s = 5, +)$	

GUT, sfermion masses and the Higgs mass can be predicted only by the quantum numbers of gauge groups $SO(10) \times U(1)_A$. It is interesting to note that the natural GUT can be tested by measuring the sfermion mass spectrum, while a large Higgsino mass μ is required to cancel the large negative D -term contribution to the squared mass of Higgs, which predicts that the Higgsino is a superheavy particle, and therefore the dark matter must be another particle, for example, neutral gaugino, gravitino, axino, or axion. However, the dominance of the D -term contribution may change in the natural GUT because the natural GUT is more complex. (For example, many fields exist.) If the D -term contribution becomes comparable or smaller than the F -term contribution, the above results completely change. Actually, Ref. [11] has discussed the sfermion mass spectrum in the above two cases.

In this paper, we discuss whether it is possible to overturn the dominance of the D -term contribution in more complex models. Specifically, we explore the possibility of suppressing only the D -term contribution while maintaining the F -term contribution by tuning several $O(1)$ coefficients. Our conclusion is that the D -term dominance is quite a general result in this scenario. Therefore, the sfermion mass spectrum is fixed by the D -terms, i.e., the quantum numbers of quarks, leptons, and (GUT) Higgses. Thus, observing the sfermion mass spectrum becomes an interesting test for various natural GUTs while the Higgsino cannot be the dark matter.

After this introduction, we review the natural GUT and the mechanism for spontaneous SUSY breaking in Sec. II. In Sec. III, we use a simple model to concretely demonstrate the relationship between F -term contribution and the D -term contribution. We conclude that the D -term contribution cannot be comparable with the F -term contribution by the cancellation between parameters. In a more general model, the conclusion does not change. Section IV is devoted to discussions and summary.

II. NATURAL GUT AND SUSY BREAKING

In this section, we review the natural $SO(10)$ GUT [6] and the spontaneous SUSY breaking [11].

The anomalous $U(1)_A$ gauge symmetry plays an essential role in restricting the interactions in the superpotential. One of the most important features in the natural GUT is that all terms allowed by the symmetry are introduced with

$O(1)$ coefficients. The typical quantum numbers for the field content under the symmetry $SO(10) \times U(1)_A \times Z_2$ are summarized in Table I. The typical model can solve the DT splitting problem and obtain realistic quark and lepton masses and mixings. Here, Z_2 symmetry is introduced to realize the DT splitting, as discussed in Ref. [6].

In this paper, we use large characters for superfields or operators and small characters for their $U(1)_A$ charges. The $U(1)_A$ gauge symmetry [13] has gauge anomalies, but they can be cancelled by the Green-Schwarz mechanism [14]. Furthermore, the Fayet-Iliopoulos (FI) term [8] $\xi^2 \int d^2\theta V_A$ is assumed, with a vector multiplet V_A of $U(1)_A$ and a small constant parameter $\xi \sim 0.22\Lambda$, where Λ is a cutoff.

A. Anomalous $U(1)_A$ gauge symmetry

We consider a simpler model in which we have only three matter fields, Ψ_i , and two negatively charged fields, H and Θ , as shown in Table I for simplicity. The superpotential invariant under $U(1)_A$ is given as

$$W = y_{ij} \sum_{i,j=1,2,3} \left(\frac{\Theta}{\Lambda} \right)^{\psi_i + \psi_j + h} \Psi_i \Psi_j H, \quad (1)$$

where y_{ij} are the $O(1)$ coefficients. The cutoff scale is determined by the gauge coupling unification (GCU) condition and becomes the usual GUT scale $\Lambda_G \sim 2 \times 10^{16}$ GeV. (Here, we do not explain in detail, please look at [6]). The GCU conditions also require $h \sim 0$. We assume that only Θ has a nonvanishing vacuum expectation value (VEV) here. It is determined as $\langle \Theta \rangle = \xi \equiv \lambda \Lambda$ by the D -flatness condition of $U(1)_A$. Then, the interaction terms in the above superpotential become the Yukawa interactions as

$$W = y_{ij} \lambda^{\psi_i + \psi_j + h} \Psi_i \Psi_j H. \quad (2)$$

In this paper, we take $\lambda \sim 0.22$, which is approximately the Cabibbo angle. Unless otherwise noted, the $O(1)$ coefficients like y_{ij} are omitted, and we take $\Lambda = 1$ for simplicity in this paper. The mechanism that realizes the hierarchical structure of Yukawa couplings by a field that obtains a VEV is called the Froggatt-Nielsen mechanism [15] in the literature.

Furthermore, the holomorphic feature of the superpotential forbids some higher dimensional terms. This mechanism is called the SUSY zero mechanism, and it

has the property that only interactions with non-negative powers of λ are allowed.

B. Vacuum expectation values

We explain how the VEVs of the Higgs fields are determined by the $U(1)_A$ symmetry. For simplicity, we introduce only gauge singlet fields $Z_i^\pm (i = 1, 2, \dots, n_\pm)$ with $U(1)_A$ charges $z_i^\pm$ ($z_i^+ > 0$ and $z_i^- < 0$). The VEVs of the Higgs fields can be determined by $n_+ + n_-$ F -flatness conditions and one D -flatness condition,

$$F_i^\pm = -\left(\frac{\delta W}{\delta Z_i^\pm}\right)^* = 0, \quad D_A = g_A \left(\sum_{i,\pm} z_i^\pm |Z_i^\pm|^2 + \xi^2 \right) = 0, \quad (3)$$

where g_A is the gauge coupling of $U(1)_A$. Due to the gauge invariance of the superpotential, the F -flatness conditions are no longer independent, and one condition is reduced. With generic interactions, we always have a SUSY vacuum with $O(1)$ VEVs. However, in case $n_+ < n_-$, a completely different vacuum, where all positively charged fields have vanishing VEVs and only negatively charged fields have small VEVs, appear. Since each term in the superpotential contains at least one positively charged field, giving the positively charged fields vanishing VEVs automatically satisfies the F -flatness conditions of negatively charged fields. Also, the VEVs of the negatively charged fields can be determined by the n_+ F -flatness conditions of the positively charged fields and a D -flatness condition.

If $n_+ + 1 < n_-$, i.e., the number of constraints is smaller than that of the variables, it is impossible to determine the VEVs of all negatively charged fields, leading to the appearance of massless fields. When $n_+ + 1 = n_-$, the number of constraints are balanced with that of the variables, and all negatively charged fields are determined as [6]

$$\langle Z \rangle \sim \begin{cases} \lambda^{-z} & (z \leq 0) \\ 0 & (z > 0) \end{cases}. \quad (4)$$

On the other hand, in the case of $n_+ + 1 > n_-$, the number of constraints becomes larger than that of the variables, we have no SUSY vacuum with the vanishing VEVs of positively charged fields, and therefore, SUSY is spontaneously broken [9]. The potential energy of the SUSY breaking vacuum becomes $V_{SB} \sim |F_{S_{\max}}|^2 \sim \lambda^{2s_{\max}} \Lambda^4$, where the field $S_{\max}$ has the maximal $U(1)_A$ charge. Since the V_{SB} is much smaller than the potential barrier between this SUSY breaking vacuum and the SUSY vacuum with $O(1)$ VEVs when $s_{\max}$ is sufficiently large, the meta-stable vacuum can have a longer lifetime than the age of the universe.

Note that terms that include more than one positively charge field can be ignored in determining the vacuum

because of the vanishing VEVs of positively charge fields. Therefore, the superpotential W_{Z^+} , which includes only one positively charged field Z^+ , is sufficient to be considered when the VEVs are determined. This is important not only in determining the VEVs but also in considering SUSY breaking. Assigning a $U(1)_R$ charge of 2 to positively charged fields and 0 to negatively charged fields ensures that the superpotential determining the VEV respects the $U(1)_R$ symmetry. Moreover, the nonvanishing VEVs of the negatively charged fields and the F of the positively charge fields do not break the $U(1)_R$. Therefore, the SUSY breaking parameters, which break $U(1)_R$, the gaugino masses, and the A parameters become much smaller than the SUSY breaking parameters, which do not break $U(1)_R$, and the sfermion masses.

C. Spontaneous SUSY breaking

In the natural GUT, the VEVs of all the negatively charged fields except for a pair of the Higgs doublet fields are determined by the F -flatness conditions of the positively charged fields. If we add one positively charged field or omit one negatively charged field, the SUSY is spontaneously broken. The easiest way to build such a model is to omit one negatively charged field $\bar{Z}$ from the natural GUT in Table I [11]. Thus, one of the F -flatness conditions of C' and $\bar{C}'$, which are obtained by the superpotentials, is

$$\begin{aligned} W_{\bar{C}'} &= \bar{C}'(\lambda^{\bar{c}'+c+a}A + \lambda^{\bar{c}'+c+z}Z)C, \\ W_{C'} &= \bar{C}(\lambda^{c'+\bar{c}+a}A + \lambda^{c'+\bar{c}+z}Z)C', \end{aligned} \quad (5)$$

cannot be satisfied, and therefore, the SUSY is spontaneously broken. Here, the VEV of A is fixed by $F_{A'} = 0$. Therefore, choosing the VEV of Z cannot satisfy both $F_{C'} = 0$ and $F_{\bar{C}'} = 0$. When we take larger $\bar{c}'$, the SUSY breaking scale can be obtained as

$$m_{\text{SUSY}} = \frac{F_{\bar{C}'}}{\Lambda} \sim \lambda^{\bar{c}'+\frac{1}{2}(c-\bar{c})}\Lambda, \quad (6)$$

because $\langle A \rangle \sim \lambda^{-a}$, $\langle Z \rangle \sim \lambda^{-z}$, and $|\langle C \rangle| = |\langle \bar{C} \rangle| \sim \lambda^{-\frac{1}{2}(c+\bar{c})}$, where $|\langle C \rangle| = |\langle \bar{C} \rangle|$ is required by the D -flatness condition of $SO(10)$.¹

If supergravity (SUGRA) effects are not taken into account, the gaugino masses become $m_{1/2} \sim m_{\text{SUSY}}^2/\Lambda$, which is much smaller than m_{SUSY} , because for non-vanishing gaugino masses, small $U(1)_R$ breaking must be picked up. Fortunately, the SUGRA effect [10] gives larger, but still small, gaugino masses, which are comparable to the gravitino masses as

¹As discussed in Ref. [10], the term $\bar{C}'AH^2C$ must be forbidden by some discrete symmetry because AH^2 can play the same role as $\bar{Z}$, and therefore, the SUSY vacuum appears.

$$m_{1/2} \sim m_{3/2} = \frac{F_{\tilde{C}'}}{M_{\text{Pl}}} \sim m_{\text{SUSY}} \frac{\Lambda}{M_{\text{Pl}}} \sim 10^{-2} m_{\text{SUSY}}, \quad (7)$$

where $M_{\text{Pl}} \sim 2 \times 10^{18}$ GeV is a reduced Planck scale.

For the sfermion masses, this model has two contributions: F -term contributions through the higher dimensional term, such as $\int d^4\theta |\tilde{C}'|^2 \Psi_i^\dagger \Psi_i$, and the D -term contribution from $\int d^4\theta \Psi_i^\dagger e^{\psi_i g_A V_A} \Psi_i$. As discussed in the following, the D -term contribution is 10 times larger than the F -term contribution in the simplest model.

We will show the result with the simplest model discussed in [9]. We introduce one positively charged field S and one negatively charged field Θ . The generic superpotential and scalar potential are given as

$$W = f(S\Theta^s), \quad (8)$$

$$V = |F_S|^2 + |F_\Theta|^2 + \frac{1}{2} D_A^2, \quad (9)$$

where $f(x)$ is a function of x , $F_S^* = -f'(S\Theta^s)\Theta^s$, $F_\Theta^* = -sf'(S\Theta^s)S\Theta^{s-1}$, and $D_A = g_A(s|S|^2 - |\Theta|^2 + \xi^2)$. This model has SUSY vacua, where $f'(S\Theta^s) = 0$ and $D_A = 0$ are satisfied. But if $s \gg 1$, the meta-stable vacuum, where $\langle S \rangle \sim 0$ and $\langle \Theta \rangle \sim \xi\Lambda$, appears. Basically, the meta-stable vacuum is understood by solving the stationary conditions $\frac{\partial V}{\partial S} = 0$ and $\frac{\partial V}{\partial \Theta} = 0$. $\frac{\partial V}{\partial S} = 0$ determines the VEV of S , and $\frac{\partial V}{\partial \Theta} = 0$ determines the VEV of Θ . The VEV of Θ is approximately determined by $D_A = 0$, but the correct value is determined by fixing the D -term by the stationary condition as

$$\frac{\partial V}{\partial \Theta} = F_S \frac{\partial F_S^*}{\partial \Theta} + F_\Theta \frac{\partial F_\Theta^*}{\partial \Theta} + D_A \frac{\partial D_A}{\partial \Theta} = 0. \quad (10)$$

We can ignore the second term because F_Θ includes S , which has a small VEV. Thus, we obtain the relation $g_A D_A = s|F_S|^2 |\Theta|^{-2}$ because $\frac{\partial F_S^*}{\partial \Theta} \sim s F_S^* \Theta^{-1}$ and $\frac{\partial D_A}{\partial \Theta} \sim -g_A \Theta^*$. Therefore, we conclude that the D -term contribution is 10 times larger than the F -term contribution since $s \sim O(10)$ and $|\Theta|^{-2} \sim O(10)$. Incidentally, $\frac{\partial V}{\partial S} = 0$ determines the VEV of S as $\langle S \rangle \sim \lambda^{s+2}/s^2$, which is quite small.

When the above results in the simplest model is applied to the natural GUT with spontaneous SUSY breaking, we have arrived at the following scenario A:

$$m_{1/2} \sim m_{3/2} \sim O(1 \text{ TeV}), \quad (11)$$

$$m_0^2 \sim \psi g_A D_A \sim (10 m_{\text{SUSY}})^2 \sim O((1000 \text{ TeV})^2), \quad (12)$$

$$m_h^2 \sim h g_A D_A \sim O((1000 \text{ TeV})^2), \quad (13)$$

where m_0^2 and m_h^2 are the mass squares of the sfermion with charge ψ and the SUSY breaking Higgs mass square,

respectively. We have assumed $m_{1/2} \sim 1$ TeV based on the experimental bounds, and thus the sfermion masses and the SUSY breaking Higgs mass become $O(1000)$ TeV. The lightest MSSM SUSY particle (LMSP) becomes the bino, although the thermal relic abundance becomes much larger than the observed DM abundance. Thus, we require some solutions, for example, large entropy production or an axino as the lightest SUSY particle (LSP), which may also solve the gravitino problem, as discussed in [11].

If the D -term contribution is comparable or smaller than the F -term contribution by some reason, for example, tuning between parameters, we have obtained scenario B,

$$m_{1/2} \sim m_{3/2} \sim O(1 \text{ TeV}), \quad (14)$$

$$m_0^2 \sim \tilde{m}_f^2 + \psi g_A D_A \sim (m_{\text{SUSY}})^2 \sim O((100 \text{ TeV})^2), \quad (15)$$

$$m_h^2 \sim m_H^2 + h g_A D_A \sim O((100 \text{ TeV})^2), \quad (16)$$

where $\tilde{m}_f^2$ and m_H^2 are the F -contribution to the mass squares of the sfermions and Higgs. Note that the Higgs mass square can be the weak scale if the F -term contribution and the D -term contribution are canceled out. Thus, in scenario B, the Higgsino can be the LMSP, and it has no problem for overproduction of the LMSP.

For both scenarios, the gravitino mass is $O(1)$ TeV, and thus they are suffering from the gravitino problem [16], in contrast to the high scale SUSY scenarios discussed in Ref. [12]. We need to solve this problem, and a possible way is, for example, to introduce the entropy production [17] or the axino LSP [18], as discussed in Ref. [11].

One of the interesting results in the natural GUT with the spontaneous SUSY breaking is the existence of a long-lived charged particle with $O(1)$ TeV mass. The lifetime of this particle depends on which particle is the LMSP [11]. For both scenarios, since the sfermion masses are large, the SUSY contributions to the FCNC processes and that CP -violating processes are strongly suppressed although the electroweak scale becomes unstable. Note that the sfermion mass spectrum respects the $SU(5)$ GUT symmetry. The natural GUT can be tested by observing the sfermion mass spectrum.

D. Higgsino mass

In a D -term dominant scenario A, it is predicted that the Higgsino becomes much heavier than the weak scale, and therefore, it cannot be the dark matter. This is because in the natural GUT, the SUSY Higgs mass μ , which is nothing but the Higgsino mass, must be taken to be large enough to cancel the large negative D -term contribution to m_h^2 . Note that the Higgs $U(1)_A$ charge h must not be zero. Actually, we have two reasons for having negative h in the natural GUT. One of them is that the tree Higgs mass term $\lambda^{2h} H^2$ must be forbidden to solve the DT splitting problem. When

$h < 0$, it is forbidden by the SUSY zero mechanism. The other is that $h \sim -3^2$ is required to obtain the correct neutrino mass without any tuning of the $O(1)$ coefficients [19].

When the SUSY is broken, the Higgsino mass μ , which is forbidden by the SUSY zero mechanism, is induced, as discussed in Ref. [20]. The induced μ is proportional to the gravitino mass $m_{3/2}$, and the proportional constant, which depends on the charges of some fields, can be sufficiently large enough to obtain $\mu \sim O(1000)$ TeV, although a technical issue, which can be solved in principle, has been discussed in Ref. [11].

If the F -term contribution becomes comparable with the D -term contribution, the Higgsino can be the dark matter when the F and D terms contributions to m_h^2 are cancelled out. Is it possible that the D -term contribution becomes comparable with the F -term contribution? That is nothing but our main question to be answered in this paper.

III. F -TERM OR D -TERM?

In this section, we discuss the possibility of avoiding the dominance of D -term contributions to the sfermion masses. Since the D -term dominance is a result in the simplest model, in this section, we consider more complicated models, in which the number of fields increases, to discuss the possibility that the D -term dominance can be avoided. Under the meta-stable vacuum, where the positively charged fields have quite small VEVs, we have no room to reduce the D -term contribution in the simplest model, which is determined by Eq. (10). However, if the number of positively charged fields increases, the number of non-negligible terms in the stationary condition as in Eq. (10) increases, and it may be possible to reduce the D -term contribution by a cancellation between the non-negligible terms. We are interested in the possibility of making the F -term contribution and the D -term contribution comparable. To achieve this, D_A must be reduced by two orders of magnitude. Throughout this section, we do not assume $U(1)_R$ symmetry. However, we assume that the VEVs of the positively charged fields are small and often neglect them. Whether this approximation is valid will be discussed at the end of this section.

A. Demonstration with two positively charged fields

We consider a simple model which has two positively charged fields and two negatively charged fields, as shown in the Table II. We take $s > t$ and $z < -1 = \theta$ without loss of generality.

Since the number of the negatively charged fields and that of the positively charged fields are equal, the SUSY is spontaneously broken. In this model, the superpotential is

TABLE II. Field content of simple model.

Negatively charged fields	Positively charged fields
$\Theta(\theta = -1)$	$S(s > 0)$
$Z(z < -1)$	$T(t > 0)$

given as

$$W = a_0 S \Theta^s + a_1 S Z \Theta^{s+z} + b_0 T \Theta^t + b_1 T Z \Theta^{t+z}, \quad (17)$$

where a_0 , a_1 , b_0 , and b_1 are $O(1)$ coefficients. Here, we neglected various terms for simplicity. For example, the terms which break $U(1)_R$ symmetry are neglected because in the meta-stable vacuum the positively charged fields have quite small VEVs. Furthermore, we neglect the terms which include more than one Z field for simplicity since these terms are not expected to play an important role in our argument because of $\lambda^z \langle Z \rangle = O(1)$. The potential is given as

$$V = |F_S|^2 + |F_T|^2 + |F_Z|^2 + |F_\Theta|^2 + \frac{1}{2} D_A^2, \quad (18)$$

where

$$F_S^* = -a_0 \Theta^s - a_1 Z \Theta^{s+z}, \quad (19)$$

$$F_T^* = -b_0 \Theta^t - b_1 Z \Theta^{t+z}, \quad (20)$$

$$F_Z^* = -a_1 Z \Theta^{s+z} - b_1 T \Theta^{t+z}, \quad (21)$$

$$F_\Theta^* = -s a_0 S \Theta^{s-1} - (s+z) a_1 S Z \Theta^{s+z-1} - t b_0 T \Theta^{t-1} - (t+z) b_1 T Z \Theta^{t+z-1}, \quad (22)$$

$$D_A = g_A (\xi^2 + s|S|^2 + t|T|^2 + z|Z|^2 - |\Theta|^2). \quad (23)$$

Here, in the meta-stable vacuum, SUSY breaking is caused by $F_S \sim \lambda^s$ because λ^s is smaller than λ^t . We can understand the meta-stable vacuum by solving the stationary conditions $\frac{\partial V}{\partial S} = 0$, $\frac{\partial V}{\partial T} = 0$, $\frac{\partial V}{\partial Z} = 0$, and $\frac{\partial V}{\partial \Theta} = 0$. Basically, the first two stationary conditions, which are the derivative of the potential V by the positively charged fields, determine the VEVs of the positively charged fields, which are quite small. The last two stationary conditions, which are the derivative of the potential by the negatively charged fields, determine the VEVs of the negatively charged fields, which are not so small. Therefore, in order to estimate the D -term, it is sufficient to consider the last two stationary conditions. Moreover, in the meta-stable vacuum, $F_T \sim 0$ because $\lambda^s \ll \lambda^t$, and therefore, $F_T \sim 0$ fixes approximately the VEV of Z . We estimate the D -term by the following steps:

- (1) Approximate $\langle \Theta \rangle$ is determined by $D_A = 0$ as $\langle \Theta \rangle \sim \xi$ because $\langle \Theta \rangle$ is the largest VEV.

²Although it differs from the requirements of the GCU conditions, GCU can be achieved through fine-tuning of $O(1)$ coefficients.

- (2) Calculate approximate $\langle Z \rangle \equiv \langle Z \rangle_0$ by F-flatness condition $F_T = 0$ as $\langle Z \rangle_0 \sim \lambda^{-z}$.
- (3) Since $\langle Z \rangle$ should be shifted due to SUSY breaking as $\langle Z \rangle = \langle Z \rangle_0 + \Delta$, the shift $\Delta \ll \langle Z \rangle_0$ is determined from the stationary condition $\frac{\partial V}{\partial Z} = 0$.
- (4) Substitute $\langle Z \rangle$ into $\frac{\partial V}{\partial \Theta} = 0$ and determine the order of D_A (and the correct $\langle \Theta \rangle$ is fixed).

Now, we perform actual calculations. The approximate VEV of negatively charged field is determined as $\langle Z \rangle_0 = -\frac{b_0}{b_1} \Theta^{-z}$ by $F_T = 0$. Therefore, we can write $\langle Z \rangle$, F_S , and F_T as

$$\langle Z \rangle = \left(-\frac{b_0}{b_1} + \delta \right) \Theta^{-z}, \quad (24)$$

$$F_T^* = -b_1 \delta \Theta^t, \quad (25)$$

$$F_S^* = \left(-a_0 + \frac{a_1 b_0}{b_1} - a_1 \delta \right) \Theta^s. \quad (26)$$

Here, $\delta \equiv \Delta \lambda^z$, where Δ is defined as the shift of $\langle Z \rangle$ due to SUSY breaking, and is expected to be smaller than 1. We consider the following two stationary conditions:

$$\begin{aligned} \frac{\partial V}{\partial Z} &= F_S \frac{\partial F_S^*}{\partial Z} + F_T \frac{\partial F_T^*}{\partial Z} + D_A \frac{\partial D_A}{\partial Z} \\ &= (a_0 \Theta^s + a_1 Z \Theta^{s+z})^* a_1 \Theta^{s+z} \\ &\quad + (b_0 \Theta^t + b_1 Z \Theta^{t+z})^* b_1 \Theta^{t+z} + z g_A D_A Z \\ &= 0. \end{aligned} \quad (27)$$

$$\begin{aligned} \frac{\partial V}{\partial \Theta} &= F_S \frac{\partial F_S^*}{\partial \Theta} + F_T \frac{\partial F_T^*}{\partial \Theta} + D_A \frac{\partial D_A}{\partial \Theta} \\ &= (a_0 \Theta^s + a_1 Z \Theta^{s+z})^* (s a_0 \Theta^{s-1} + (s+z) a_1 Z \Theta^{s+z-1}) \\ &\quad + (b_0 \Theta^t + b_1 Z \Theta^{t+z})^* (t b_0 \Theta^{t-1} + (t+z) b_1 Z \Theta^{t+z-1}) \\ &\quad - g_A D_A \Theta \\ &= 0. \end{aligned} \quad (28)$$

Here, we neglect terms which include F_Z and F_Θ because they contain very small VEVs of S and T . The justification for neglecting this will be discussed in the Sec. III C, where the VEVs of the positively charged fields are determined.

Note that a cancellation between $F_S \frac{\partial F_S^*}{\partial \Theta}$ and $F_T \frac{\partial F_T^*}{\partial \Theta}$ by tuning the $O(1)$ coefficients a_0 , a_1 , b_0 , and b_1 might make D_A smaller than expected because these terms give the same order of contribution to D_A , which will be shown below. This cancellation cannot be expected in the simplest model discussed in the previous section because Eq. (10) has no second term in Eq. (28). The shift parameter δ is determined by solving the stationary condition (27) as

$$\delta = -\frac{a_0^* a_1 - |a_1|^2 b_0 / b_1}{|b_1|^2} \Theta^{2s-2t}, \quad (29)$$

by straightforward calculations. Here, we can neglect the last term in (27) when $z < -1$. Note that the first two terms in Eq. (28) become the same order by substituting $\delta \sim \lambda^{2s-2t} \ll 1$, and therefore, the cancellation can be expected. Finally, D_A can be determined by solving Eq. (28) as

$$D_A = \frac{s}{g_A} \left| a_0 - \frac{a_1 b_0}{b_1} \right|^2 \Theta^{2s-2}, \quad (30)$$

where we use $\delta \ll 1$. Actually, if a relation

$$a_0 b_1 \sim a_1 b_0 \quad (31)$$

is satisfied, D_A can be smaller than the expected value $s \lambda^{2s-2}$. Surprisingly, even with the tuning condition (31), $D_A \sim s |F_S|^2 \lambda^{-2}$ is still satisfied because this relation (31) also gives a smaller F_S , which is seen in Eq. (26). This looks to be a quite nontrivial result because the two conditions for smaller F_S and a smaller D_A look to be independent in this simple model. This result in this simple model suggests that the D -term contribution to sfermion mass squares cannot be smaller than the F -term contribution by tuning the $O(1)$ coefficients.

B. General models

From the discussion so far, we have found that in a simple model, the D -term is proportional to the square of the F -term. Here, we will discuss whether this behavior can be confirmed in a more generalized model and the conditions for such confirmation. In this time, we consider a model which has numerous positively charged fields $S, T_1, T_2, \dots, T_n$ and negatively charged fields $\Theta, Z_1, Z_2, \dots, Z_n$. As before, we assume $s > t_i, \theta > z_i (i = 1, 2, \dots, n)$, and F_S causes SUSY breaking. To obtain the D -term, it is sufficient to consider an $n+1$ stationary condition,

$$\frac{\partial V}{\partial Z_i} = F_S \frac{\partial F_S^*}{\partial Z_i} + \sum_k F_{T_k} \frac{\partial F_{T_k}^*}{\partial Z_i} = 0, \quad (32)$$

$$\frac{\partial V}{\partial \Theta} = F_S \frac{\partial F_S^*}{\partial \Theta} + \sum_k F_{T_k} \frac{\partial F_{T_k}^*}{\partial \Theta} - g_A D_A \Theta = 0, \quad (33)$$

where the contributions from F_Θ and F_{Z_i} are neglected because of the small VEVs of the positively charged fields, which will be discussed in the next subsection. Note that since in Eq. (33) all terms $F_{T_k} \frac{\partial F_{T_k}^*}{\partial \Theta}$ are comparable with the first term, the condition for the smaller D_A looks to be independent of the condition for the smaller F_S . However, this is not the case. To perform the calculations, let us represent these in matrix form. They are written as

$$F_S \begin{pmatrix} \frac{\partial F_S^*}{\partial Z_1} \\ \vdots \\ \frac{\partial F_S^*}{\partial Z_n} \end{pmatrix} + \begin{pmatrix} \frac{\partial F_{T_1}^*}{\partial Z_1} & \cdots & \frac{\partial F_{T_n}^*}{\partial Z_1} \\ \vdots & & \vdots \\ \frac{\partial F_{T_1}^*}{\partial Z_n} & \cdots & \frac{\partial F_{T_n}^*}{\partial Z_n} \end{pmatrix} \begin{pmatrix} F_{T_1} \\ \vdots \\ F_{T_n} \end{pmatrix} = 0, \quad (34)$$

$$F_S \frac{\partial F_S^*}{\partial \Theta} + \left(\frac{\partial F_{T_1}^*}{\partial \Theta} \cdots \frac{\partial F_{T_n}^*}{\partial \Theta} \right) \begin{pmatrix} F_{T_1} \\ \vdots \\ F_{T_n} \end{pmatrix} - g_A D_A \Theta = 0, \quad (35)$$

respectively. From Eq. (34), we can write the F -terms as

$$\begin{pmatrix} F_{T_1} \\ \vdots \\ F_{T_n} \end{pmatrix} = -F_S \begin{pmatrix} \frac{\partial F_{T_1}^*}{\partial Z_1} & \cdots & \frac{\partial F_{T_n}^*}{\partial Z_1} \\ \vdots & & \vdots \\ \frac{\partial F_{T_1}^*}{\partial Z_n} & \cdots & \frac{\partial F_{T_n}^*}{\partial Z_n} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial F_S^*}{\partial Z_1} \\ \vdots \\ \frac{\partial F_S^*}{\partial Z_n} \end{pmatrix}. \quad (36)$$

Furthermore, by substituting Eq. (36) into Eq. (35), we obtain

$$F_S \left[\frac{\partial F_S^*}{\partial \Theta} - \left(\frac{\partial F_{T_1}^*}{\partial \Theta} \cdots \frac{\partial F_{T_n}^*}{\partial \Theta} \right) \begin{pmatrix} \frac{\partial F_{T_1}^*}{\partial Z_1} & \cdots & \frac{\partial F_{T_n}^*}{\partial Z_1} \\ \vdots & & \vdots \\ \frac{\partial F_{T_1}^*}{\partial Z_n} & \cdots & \frac{\partial F_{T_n}^*}{\partial Z_n} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial F_S^*}{\partial Z_1} \\ \vdots \\ \frac{\partial F_S^*}{\partial Z_n} \end{pmatrix} \right] - g_A D_A \Theta = 0. \quad (37)$$

We consider a chain rule, $\frac{dF_{T_i}^*}{d\Theta} = \frac{\partial F_{T_i}^*}{\partial \Theta} + \sum_k \frac{\partial F_{T_i}^*}{\partial Z_k} \frac{dZ_k}{d\Theta}$, where we take all the VEVs of the positively charged fields to be vanishing because they are expected to be very small. Since $F_{T_i}^* \sim 0$ are basically satisfied, $dF_{T_i}^*/d\Theta \sim 0$ are also satisfied. Note that this is not the case for $\partial F_{T_i}^*/\partial \Theta$, which can be large. Thus, the above equation obtained by the chain rule becomes

$$\left(\frac{\partial F_{T_1}^*}{\partial \Theta} \cdots \frac{\partial F_{T_n}^*}{\partial \Theta} \right) = - \left(\frac{dZ_1}{d\Theta} \cdots \frac{dZ_n}{d\Theta} \right) \begin{pmatrix} \frac{\partial F_{T_1}^*}{\partial Z_1} & \cdots & \frac{\partial F_{T_n}^*}{\partial Z_1} \\ \vdots & & \vdots \\ \frac{\partial F_{T_1}^*}{\partial Z_n} & \cdots & \frac{\partial F_{T_n}^*}{\partial Z_n} \end{pmatrix}. \quad (38)$$

Therefore, Eq. (37) is rewritten as

$$\begin{aligned} 0 &= F_S \left[\frac{\partial F_S^*}{\partial \Theta} + \left(\frac{dZ_1}{d\Theta} \cdots \frac{dZ_n}{d\Theta} \right) \begin{pmatrix} \frac{\partial F_S^*}{\partial Z_1} \\ \vdots \\ \frac{\partial F_S^*}{\partial Z_n} \end{pmatrix} \right] - g_A D_A \Theta \\ &= F_S \frac{dF_S^*}{d\Theta} - g_A D_A \Theta, \end{aligned} \quad (39)$$

and from this, it immediately follows that

$$D_A = \frac{1}{g_A} F_S \frac{dF_S^*}{d\Theta} \Theta^{-1}, \quad (40)$$

where $F_S^* = \alpha \Theta^s$ with a coefficient α . For any coefficient α , the above relation shows that $g_A D_A \sim s |F_S|^2 \lambda^{-2}$. Therefore, we conclude that D -term dominance is a quite general

result in the natural GUT with spontaneous SUSY breaking.

Here, we have assumed that the VEVs of S and T are small and have been omitted in the F_{Z_i} terms and F_Θ term in the stationary conditions. We study whether this assumption is correct or not in the next subsection.

C. VEVs of positively charged fields

In the above discussion, we assumed that the VEVs of S and T , induced by SUSY breaking, are extremely small. In this section, we use the model introduced in Sec. III A to demonstrate that such an assumption is valid.

First, we calculate the VEVs of the positively charged fields. To do so, the second-order terms of the positively charged fields are required. We set a superpotential,

$$\begin{aligned} W &= S\Theta^s + SZ\Theta^{s+z} + SZ^2\Theta^{s+2z} \\ &\quad + T\Theta^t + TZ\Theta^{t+z} + TZ^2\Theta^{t+2z} \\ &\quad + ST\Theta^{s+t} + S^2\Theta^{2s} + T^2\Theta^{2t}. \end{aligned} \quad (41)$$

In this subsection, we omit the $O(1)$ coefficients. Higher-order terms of S , T , and Z were omitted as their contributions do not change the order of the VEVs. The F -terms are written as

$$F_S^* = -\Theta^s - Z\Theta^{s+z} - Z^2\Theta^{s+2z} - T\Theta^{s+t} - 2S\Theta^{2s}, \quad (42)$$

$$F_T^* = -\Theta^t - Z\Theta^{t+z} - Z^2\Theta^{t+2z} - S\Theta^{s+t} - 2T\Theta^{2t}, \quad (43)$$

$$F_Z^* = -S\Theta^{s+z} - 2SZ\Theta^{s+2z} - T\Theta^{t+z} + 2TZ\Theta^{t+2z}, \quad (44)$$

$$\begin{aligned}
F_\Theta^* = & -sS\Theta^{s-1} - (s+z)SZ\Theta^{s+z-1} - (s+2z)SZ^2\Theta^{s+2z-1} \\
& -tT\Theta^{t-1} - (t+z)TZ\Theta^{t+z-1} - (t+2z)TZ^2\Theta^{t+2z-1} \\
& - (s+t)ST\Theta^{s+t-1} - 2sS^2\Theta^{2s-1} + 2tT^2\Theta^{2t-1}. \quad (45)
\end{aligned}$$

The possible largest-order terms of each are as follows:

$$\begin{aligned}
F_S : \Theta^s, \quad F_T : \delta\Theta^t \sim \Theta^{2s-t}, \\
F_Z : S\Theta^{s+z} + T\Theta^{t+z}, \quad F_\Theta : S\Theta^{s-1} + T\Theta^{t-1}. \quad (46)
\end{aligned}$$

The VEVs of the positively charged fields S and T are determined by the stationary conditions

$$\frac{\partial V}{\partial S} = F_S \frac{\partial F_S^*}{\partial S} + F_T \frac{\partial F_T^*}{\partial S} + F_Z \frac{\partial F_Z^*}{\partial S} + F_\Theta \frac{\partial F_\Theta^*}{\partial S} + D_A \frac{\partial D_A}{\partial S} = 0, \quad (47)$$

$$\frac{\partial V}{\partial T} = F_S \frac{\partial F_S^*}{\partial T} + F_T \frac{\partial F_T^*}{\partial T} + F_Z \frac{\partial F_Z^*}{\partial T} + F_\Theta \frac{\partial F_\Theta^*}{\partial T} + D_A \frac{\partial D_A}{\partial T} = 0. \quad (48)$$

We can neglect the fourth and fifth terms in the above equations because these terms give smaller contributions than the third term, which can be obtained from (30). Therefore, the two stationary conditions can be rewritten as the same relation,

$$\Theta^{3s} + S\Theta^{2s+2z} + T\Theta^{s+t+2z} = 0, \quad (49)$$

excluding the $O(1)$ coefficients. Finally, we can estimate the VEVs of the positively charged fields as

$$S \sim \Theta^{s-2z}, \quad (50)$$

$$T \sim \Theta^{2s-t-2z}. \quad (51)$$

Second, we justify neglecting F_Z and F_Θ in Eqs. (27) and (28). The stationary condition for Z which determines the shift of $\langle Z \rangle$ is

$$\begin{aligned}
\frac{\partial V}{\partial Z} = & F_S \frac{\partial F_S^*}{\partial Z} + F_T \frac{\partial F_T^*}{\partial Z} + F_Z \frac{\partial F_Z^*}{\partial Z} + F_\Theta \frac{\partial F_\Theta^*}{\partial Z} + D_A \frac{\partial D_A}{\partial Z} \\
= & \Theta^{2s+z} + \Theta^{2s+z} + \Theta^{4s-z} + \Theta^{4s-3z-2} + \Theta^{2s-z-2}. \quad (52)
\end{aligned}$$

It is obvious that the first two terms dominate. This justifies the assumption made in Secs. III A and III B. Furthermore, a similar assumption for the stationary condition for Θ , which determines D_A , is also justified in the same way.

Finally, we can conclude that the D -term dominance is a fairly common result in the natural GUT with spontaneous SUSY breaking.

IV. DISCUSSIONS AND SUMMARY

The natural GUTs solve the various problems in the SUSY GUT by assuming that all terms allowed by $SO(10) \times U(1)_A$ symmetry are introduced with the $O(1)$ coefficients. The natural GUT also can naturally introduce the mechanism for spontaneous SUSY breaking. Therefore, the natural GUT with spontaneous SUSY breaking is one of the most attractive scenarios in the natural GUT. Unfortunately, this scenario predicts high-scale SUSY, which causes the need for fine-tuning to realize the electroweak scale. This is mainly because the cutoff scale becomes smaller than the Planck scale. In Ref. [11], two scenarios have been examined. In scenario A, the D -term dominance, which is the result in the simplest model for SUSY breaking with an FI-term without $U(1)_R$ symmetry, is assumed. As a result, the sfermion mass spectrum is determined only by the quantum number under $SO(10) \times U(1)_A$, although the LMSP becomes the bino, whose thermal relics becomes larger than the observed DM abundance. On the other hand, in scenario B, it is assumed that the D -term contribution is comparable with the F -term contribution. As a result, the Higgsino can be the LMSP, and therefore, scenario B has no overproduction problem.

In this paper, we discussed whether it is possible to construct models, in which the D -term contribution becomes comparable with the F -term contribution through the fine-tuning of $O(1)$ coefficients. As a result, it was shown quite generally that the D -term becomes larger than the square of the F -terms, making it difficult to fine-tune in a way that suppresses only the D -term. We conclude that scenario B is not reasonable.

In our calculation, we do not take into account the SUGRA effects, which may change our result. By introducing the SUGRA effect, the $U(1)_R$ symmetry breaking is larger, and as a result, the VEVs of S and T become larger than those presented in this paper. This study may be important, but we think it is beyond the scope of this paper.

The dominance of the D -term suggests that the mass spectrum of the sfermions is fixed solely by the D -term contributions, as in Ref. [11]. This means that the Yukawa matrices for quarks and leptons are strongly related to the sfermion mass spectrum, which respects $SU(5)$ symmetry. We can test the natural GUT by measuring the mass spectrum of the sfermions.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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