

Pulsar kick by the chiral anisotropy conversion

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We discuss a novel mechanism for the proto-neutron star acceleration assisted by the chiral separation effect, which induces an axial vector current in a dense medium. We consider the process of neutrinos scattering off the background axial vector current of electrons. We show that anisotropy of either magnetic field or density in momentum space is essential for nonzero recoil, and we call this mechanism the chiral anisotropy conversion. Assuming a strong magnetic field $B \simeq 10^{12}$ T and anisotropy by $\sim 10\%$, we find that the chiral anisotropy conversion can yield the velocity of order of typical pulsar kicks, i.e., $v_{\text{kick}} \gtrsim 1000$ km/s.

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I. INTRODUCTION

Pulsars are neutron stars (NSs) emitting electromagnetic pulses, and many move at high speed with the mean velocity around $v_{\text{kick}} \simeq 400$ km/s (see Ref. [1] for a statistical study of 233 pulsar proper motions). The top tail $\sim 15\%$ of the velocity distribution reaches $\gtrsim 1000$ km/s, which is orders of magnitude higher than their progenitor, i.e., supernovae [2,3]. This phenomenon, the pulsar kick, is challenging in theory, and it also provides us with valuable information about the properties of neutrinos [4] and dense matter [5,6]. Moreover, the information about the NS velocity distribution is relevant to galaxy evolution [7,8].

There are many models to explain the pulsar-kick origin. The most conventional scenario is that high-speed pulsars originate from the recoil of the anisotropic supernova explosion [9]; see Ref. [10] for a review and Refs. [9,11–13] for some pulsar-kick arguments. The required anisotropy could be generated before the core collapse [14,15] and amplified later [16]. According to 3D hydrodynamic simulation, however, this model still has some limitations such as too small predicted velocity $v_{\text{kick}} \sim 100$ km/s [5,9,11,12,17,18], too strong mass-velocity correlation not seen in observation [9,11] (which also causes a difficulty of explaining the pulsar kick of low-mass NSs with nearly

symmetric ejecta [19]), and the observed bimodal structure in the velocity distribution that cannot be reproduced [3,5].

These limitations hint that the pulsar kicks should presumably be driven by not a single mechanism but multiple complementary mechanisms. Another essential ingredient for kick is the magnetic field $\mathbf{B}$. Charged particles are polarized under $\mathbf{B}$. Because the weak interaction breaks parity symmetry, the neutrino scattering with polarized particles has a preferred direction, and the anisotropic neutrino emission from the proto-neutron star (PNS) kicks the pulsars [20,21]. For this, the neutrino-nucleon scattering is the dominant process [22] than the neutrino-electron scattering [23–25]. See also Ref. [26] for the processes that involve the charged current like neutrino emission and absorption, which are considerably asymmetric under strong $\mathbf{B}$ and contribute to the pulsar kick. In addition, $\mathbf{B}$ may induce vortices in quark matter, which cause beaming of neutrino emission [27].

Recently, moreover, the anomaly-induced pulsar kick has been discussed. The magnetic field gives rise to exotic transport phenomena, such as the chiral magnetic effect (CME) $\mathbf{j} = \mu_5/(2\pi^2)e\mathbf{B}$, the chiral separation effect (CSE) $\mathbf{j}_5 = \mu/(2\pi^2)e\mathbf{B}$, the chiral vortical effect (CVE) $\mathbf{j}_5 = [(\mu^2 + \mu_5^2)/(2\pi^2) + T^2/6]e\boldsymbol{\omega}$, etc. Here, $\mathbf{j}$ and $\mathbf{j}_5$ are the vector and the axial-vector currents of fermions. In the massless limit, chirality is identical to helicity, and these relations at $T = 0$ are understood from the chiral anomaly relation, $\partial_\mu j_5^\mu = (e^2/2\pi^2)\mathbf{B} \cdot \mathbf{E}$. For details about transport phenomena induced by the chiral anomaly in nuclear physics and astrophysics, see Refs. [28,29], respectively. According to Refs. [18,30] the mean free path associated with the induced current could be sufficiently long to penetrate from the core to the PNS surface. Hence, the

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multiple scattering suppresses anisotropy [31,32], but such suppression can be bypassed by the induced current that transfers the momentum to surface neutrinos. Later on, related works along these lines [33–35] have attempted to quantify this mechanism. More recently, an intriguing possibility was pointed out in Ref. [36]. The neutrino transport gives a recoil to electrons, and the resulting charge current is reminiscent of the CME. This effective CME induces the chiral plasma instability [37–40] with simultaneous growth of $\mathbf{B}$ and $\mathbf{j}$, which enhances the pulsar kicks. The predicted kick velocity is $v_{\text{kick}} \simeq B/(10^9\text{--}10^{10}\text{ T})\text{ km/s}$ for some initial condition.

We are proposing a novel mechanism as a result of the combination of anisotropy and chiral transport. As mentioned above, the anisotropy is expected before and during the core collapse, and we show that neutrinos scatter off the anisotropic background axial current $\mathbf{j}_5$, which converts momentum anisotropy to the pulsar kick efficiently; see Fig. 1.

We emphasize the qualitative differences of our mechanism from the preceding works. In comparison to the scattering scenario under $\mathbf{B}$, one might think that the neutrino scattering with $\mathbf{j}_5$ physically corresponds to the scattering with polarized nucleons/electrons. This is partially correct; however, constant $\mathbf{j}_5$ is sensitive to parity-even processes, and this is why external anisotropy should be coupled in our mechanism. Regarding the chiral transport, the previous works [34,39,41] assume a hydrodynamic regime in which neutrinos and electrons are equilibrated, and $\mathbf{j}_5$ involves neutrinos as well. In our case, we consider the microscopic processes that can occur even before the hydrodynamic regime.

In this paper, our goal is to develop a model to estimate the acceleration induced by the mechanism. Based on this model, we plan to show if our mechanism could make a considerable contribution to the pulsar acceleration and hence should be one of the major components included for quantitative analyses of the pulsar kick phenomenon together with others.

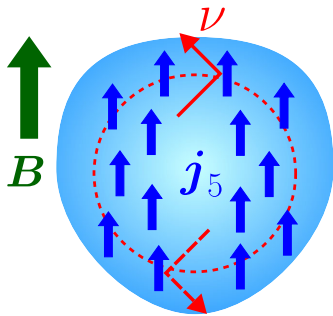


FIG. 1. Schematic figure for the pulsar kick mechanisms. Scattering between neutrinos and the background axial current coupled to momentum anisotropy (probably injected by $\mathbf{B}$ and the density) causes the pulsar kick outside the neutrino sphere.

The paper is organized as follows: Sec. II presents the theoretical formulation, with Sec. II A discussing the underlying assumptions, Sec. II B deriving the axial current distribution, Sec. II C deriving the scattering amplitude between the axial current and the neutrinos, and Sec. II D deriving the analytical expression of acceleration; Sec. III presents and discusses the numerical results, with Sec. III A showing the numerical axial current distribution and Sec. III B showing the time evolution of pulsar kick velocity under different conditions; Finally, Sec. IV summarizes the results.

II. FORMULATION

We shall explain the theoretical formulation of our pulsar kick model. This model consists of three parts: (1) determining the background axial current from PNS simulations, (2) determining the scattering cross section between the neutrinos and the axial current, and (3) computing the acceleration from the recoil of the scattering. We first present the underlying assumptions of this model and then discuss the three parts in (1)–(3) order.

A. Assumptions

For major approximations in the present study, we make the following assumptions on PNS:

- (1) The axial current $\mathbf{j}_5$ arises much earlier than the typical timescale of neutrino emission $\lesssim 10\text{ s}$ due to direct coupling of $\mathbf{B}$ and electrons. We ignore the backreaction of $\mathbf{j}_5 \neq 0$ to the density distribution of electrons and neutrino dynamics other than the scattering with $\mathbf{j}_5$. This means that we assume evolution of the PNSs in the model simulations [42–47] without $\mathbf{j}_5$ and consider the $\mathbf{j}_5$ effect as a perturbation. The back reaction may further enhance anisotropy; see Ref. [48].
- (2) We assume momentum anisotropy of $\mathbf{j}_5$ that could originate from nonspherical $\mathbf{B}$. The magnitude of anisotropy is represented by an overall parameter α_1 . See later discussions around Eq. (7) for details.
- (3) Inside the neutrino sphere, the mean free path of neutrinos is so small that the neutrino kick can only occur by processes outside the neutrino sphere; see Fig. 1. Our formula involves the integration with respect to the momentum provided by $\mathbf{j}_5$; thus, the integrand has a peak at finite momentum. In principle, we should chop off the neutrino sphere region, which would affect the small momentum components of $\mathbf{j}_5$. For order estimate, we assume that the momentum space integrand weighted by the measure is not significantly changed.
- (4) We drop processes inside the neutrino sphere by using the number luminosity of emitted neutrinos in our formula. This is a simple but sufficient recipe for the order estimate instead of solving the kinetic

equations with the collision integral. With this recipe, if we chop off the current in (3), the convoluted effects may result in oversubtraction; in this sense, (3) is a consistent approximation with our recipe in (4).

We furthermore introduce approximations for the neutrino spectrum, the homogeneous $\mathbf{B}$ (apart from the anisotropic component), the unchanged mass of the PNS, the baryon contribution to $\mathbf{j}_5$, etc., and we will explain them later.

B. Axial current

The axial current is the essential ingredient for our pulsar-kick mechanism. To determine the CSE-induced contribution to the axial current, we should estimate the values of parameters appearing in the following formula:

$$\mathbf{j}_{5,e} = \frac{e}{2\pi^2} \mu_{\text{CSE}}(\mu_e, m_e, T) \mathbf{B}. \quad (1)$$

It should be noted that we consider the electron contribution only in the above formula, while the baryons can make finite contributions to the axial current. Due to the baryon mass, which is much larger than the typical temperature in the PNS, $T \sim 10$ MeV, the baryon contributions are suppressed and negligible. For corrections from the baryons, we will discuss the numerical comparison in Sec. III B. We introduced the effective chemical potential, including the mass effect, that is,

$$\mu_{\text{CSE}}(\mu, m, T) = \int_0^\infty dq [f(q; \mu, m, T) - f(q; -\mu, m, T)], \quad (2)$$

where the integrand is the Fermi-Dirac distribution function, i.e.,

$$f(q; \mu, m, T) = \frac{1}{e^{(\sqrt{q^2 + m^2} - \mu)/T} + 1}. \quad (3)$$

It is easy to confirm that $\mu_{\text{CSE}}(\mu, m = 0, T \rightarrow 0) = \mu$.

The electron mass, $m_e = 0.511$ MeV, is a physical parameter, but we need to fix μ_e , T , and $\mathbf{B}$ as functions of the time according to the evolution of the PNS. For this purpose, we must rely on a PNS cooling model, and in this work, we adopt the results by Pons *et al.* reported in Ref. [42]. For related works based on the results from Ref. [42], see Refs. [44,49–52]. From Ref. [42] we take the $M = 1.6M_\odot$ PNS data computed in the models “GM3np,” “GM1np,” “GM3npH,” and “GM1npH,” respectively. These are mean-field models in which protons and neutrons interact via ω and ρ exchange. GM1 and GM3 represent stiff and soft equation-of-state models in Ref. [53], and np and npH represent whether hyperons are included. In the simulation in Ref. [42], the neutrino transport and the

boundary conditions are implemented, and all physical quantities are time-dependent with the typical timescale of the PNS cooling due to neutrino emission. From Figs. 9, 15, 16, 18 in Ref. [42], we take the baryon number density, n_B , the temperature, T , the net electron concentration, $Y_e = n_e/n_B$, and the total neutrino energy luminosity, L_E , as functions of t where the supernova explosion sets $t = 0$.

We shall explain how to fix the electron chemical potential μ_e . Under the assumption that we can treat electrons as a free gas, the following relation should hold:

$$n_e = Y_e n_B = 2 \int \frac{d^3 q}{(2\pi)^3} \frac{1}{e^{(\sqrt{q^2 + m_e^2} - \mu_e)/T} + 1}. \quad (4)$$

Since $Y_e n_B$ is given from Ref. [42], we can numerically solve μ_e from the above equation. Actually, for $T = 0$, the solution is analytically found as $\mu_e|_{T=0} = \sqrt{(3\pi^2 Y_e n_B)^{2/3} + m_e^2}$. At $T \neq 0$, we have numerically solved Eq. (4) and verified that $\mu_e|_{T=0}$ reasonably approximates the results in the typical range of the PNS temperatures.

As for the magnetic field, $\mathbf{B}$, it is known that the distribution of $\mathbf{B}$ is nonuniform, and we should, in principle, take the inhomogeneous profile of $\mathbf{B}$ from the model simulation. As we discuss later, however, our formulation requires the anisotropy only in the combination of μ_e and $\mathbf{B}$. Therefore, without loss of generality, we can treat anisotropic distributions of μ_e only while keeping $\mathbf{B} = B_0 \mathbf{e}_z$ with the magnetic strength B_0 fixed in later discussions.

According to the model simulation, the spatial dependence of μ_e and T is characterized by one parameter, $M_B(r)$, which is the enclosed baryon mass inside the PNS from the center up to the sphere radius r . Since M_B is nothing but a function of r , the axial current distribution obtained from Eq. (1) is isotropic, i.e., the axial current distribution is the same on all longitudes and latitudes of the PNS. In this way, we obtain $\mathbf{j}_{5,e}(r)$, and its Fourier transform, $\tilde{\mathbf{j}}_{5,e}(k)$.

From the obtained axial current, we could deal with the radially inhomogeneous distribution. However, in a real situation, the PNS could acquire anisotropic density and magnetic field profiles from the anisotropic supernova explosions. This anisotropy could be crucial for modeling the pulsar kick accurately. Therefore, we need to figure out a natural way to implement an anisotropic axial current distribution. We consider implementing the anisotropy on $\tilde{\mathbf{j}}_{5,e}(k)$. Using the polar coordinates in momentum space, i.e., $\mathbf{k} = k(\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)^T$, we can employ the spherical harmonics, $Y_l^m(\theta, \varphi)$, as the general basis. The deformed axial current distribution can be expressed as

$$\tilde{\mathbf{j}}_{5,e}(\mathbf{k}) = \tilde{\mathbf{j}}_{5,e}(k) \sum_{l=0}^{\infty} \sum_{m=-l}^l a_{l,m} Y_l^m(\theta, \varphi), \quad (5)$$

Because neutron stars rotate rapidly, the φ dependence of the induced current should be averaged out so that we can simply choose $m = 0$. Thus, $\tilde{j}_{5,e}(\mathbf{k})$ becomes

$$\tilde{j}_{5,e}(\mathbf{k}) = \tilde{j}_{5,e}(k) \sum_{l=0}^{\infty} \alpha_l P_l(\cos \theta). \quad (6)$$

Here, we denote $\alpha_l = \sqrt{(2l+1)/(4\pi)} a_{l,0}$ including the normalization. Without loss of generality, we can choose the normalization such that $\alpha_0 = 1$, and then,

$$\tilde{j}_{5,e}(\mathbf{k}) = \tilde{j}_{5,e}(k)(1 + \alpha_1 \cos \theta + \dots). \quad (7)$$

Since we are interested only in the order of magnitude, we keep the expansion up to α_1 , which is the dipole contribution, and drop higher-order angular dependence.

C. Scattering cross section with neutrinos

We consider the interaction between neutrinos and the background axial current of electrons using Fermi's effective theory of four-fermion interaction [54]. The interacting term in the effective Lagrangian density has two contributions from the neutral and the charged currents. The neutral current represents the process of $\nu_e e \rightarrow \nu_e e$ via the Z exchange as shown in the left of Fig. 2, that is,

$$\mathcal{L}_{\text{NC}} = -[\bar{e}\gamma_\mu(c_L^{\nu_e e} P_L + c_R^{\nu_e e} P_R)e]\bar{\nu}_e\gamma^\mu P_L\nu_e \quad (8)$$

with $P_L = \frac{1}{2}(1 - \gamma_5)$ and $P_R = \frac{1}{2}(1 + \gamma_5)$. Here, $c_L^{\nu_e e} - c_R^{\nu_e e} \simeq \sqrt{2}G_F$. This form has direct coupling to the electron axial current, and the interaction induced by the background current is given by

$$\mathcal{L}_{\text{eff,NC}} = \frac{G_F}{2\sqrt{2}}(j_{5,e})_\mu \bar{\nu}_e\gamma^\mu(1 - \gamma_5)\nu_e. \quad (9)$$

The effective Lagrangian involving the charged current via the $W^\pm$ exchange (see the right of Fig. 2) is identified from the Fierz transformation as

$$\mathcal{L}_{\text{CC}} = \frac{G_F}{\sqrt{2}}\bar{\nu}_e\gamma_\mu P_L e \bar{e}\gamma^\mu P_L \nu_e. \quad (10)$$

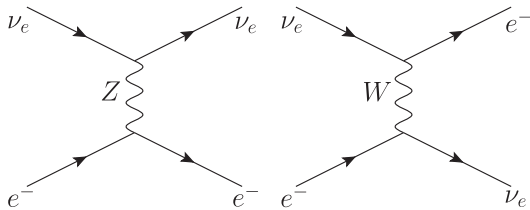


FIG. 2. Two contributions to the scattering mediated by Z (left) and W (right).

We can extract the coupling to the electron axial current by taking the Fierz transformation back, leading to $\mathcal{L}_{\text{eff,CC}} = 2\mathcal{L}_{\text{eff,NC}}$. In total, we find the mean-field effective interaction as

$$\mathcal{L}_{\text{eff}} = \frac{3G_F}{2\sqrt{2}}(j_{5,e})_\mu \bar{\nu}_e\gamma^\mu(1 - \gamma_5)\nu_e. \quad (11)$$

The Fermi coupling is related to the Higgs condensate as $(\sqrt{2}G_F)^{-1/2} = 246$ GeV. As discussed in Sec. II B, the axial currents from protons and neutrons involving u and d quarks are negligible because the nucleon mass is larger than the PNS temperature by one order of magnitude. Thus, we keep only the electron contribution of $j_{5,e}$.

We note that this effective interaction describes the scattering of neutrinos with the background axial current field, i.e., $\nu_e + j_{5,e} \rightarrow \nu_e$ and that of antineutrinos, i.e., $\bar{\nu}_e + j_{5,e} \rightarrow \bar{\nu}_e$. We have neglected the pair production and annihilation processes such as $j_{5,e} \leftrightarrow \nu_e + \bar{\nu}_e$. We can justify this treatment from the typical energy and timescales as follows: PNS cools down within the time of order of seconds, and the energy scale associated with time variation is extremely tiny, that is, $E \sim 1 \text{ s}^{-1} \sim 10^{-21}$ MeV; Therefore, we can regard the axial current as a static background; The typical momentum scale should be characterized by the system size, and we can expect $p \sim 1/(10 \text{ km}) \sim 10^{-17}$ MeV; These scales, $E \ll p$, are both too small as compared to the QCD scale, and the energy-momentum conservation in effect prohibits the pair production and annihilation processes.

In this way, we can just focus on the scattering processes. We can estimate the amplitude for $\nu_e(k_1) + j_{5,e} \rightarrow \nu_e(k_2)$, where k_1 and k_2 are the four-momenta carried by the incoming and the outgoing neutrinos, from the effective interaction strength as

$$i\mathcal{M}(k_1, k_2) = i\frac{3G_F}{\sqrt{2}}\chi_L^\dagger(k_2)\bar{\sigma}^\mu\chi_L(k_1)(\tilde{j}_{5,e}(\mathbf{k}_2 - \mathbf{k}_1))_\mu, \quad (12)$$

where χ_L is a solution of the Weyl equation for the neutrino satisfying $k_\mu\bar{\sigma}^\mu\chi_L = 0$. The solution takes a form of $\chi_L(k) = (\sqrt{E_k - k^z}, e^{i\varphi(k)}\sqrt{E_k + k^z})^T$ with $E_k = |\mathbf{k}|$ and $e^{i\varphi(k)} = (k^x + ik^y)/\sqrt{(k^x)^2 + (k^y)^2}$. Using the polar coordinates in momentum space, we can simplify the expression as $\chi_L(k) = \sqrt{2E_k}(\sin\frac{\theta_k}{2}, e^{i\varphi_k}\cos\frac{\theta_k}{2})$. We choose the z -axis along $\mathbf{B} \parallel \mathbf{j}_{5,e}$. Then, the coupled neutrino current is $\chi_L^\dagger(k_2)\sigma^z\chi_L(k_1) = 2\sqrt{E_1E_2}[\sin\frac{\theta_2}{2}\sin\frac{\theta_1}{2} - e^{i(\varphi_1 - \varphi_2)}\cos\frac{\theta_2}{2}\cos\frac{\theta_1}{2}]$. Here, for notation brevity, we introduced the notation, E_1 , θ_1 , and φ_1 for the energy and the angular variables instead of E_{k_1} , θ_{k_1} , and φ_{k_1} , and E_2 , θ_2 , and φ_2 similarly. The squared quantity is $|\chi_L^\dagger(k_2)\sigma^z\chi_L(k_1)|^2 = 2E_1E_2[1 + \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2\cos(\varphi_1 - \varphi_2)]$. In the same way we get the amplitude for $\bar{\nu}_e(k_1) + j_{5,e} \rightarrow \bar{\nu}_e(k_2)$ as

$$i\bar{\mathcal{M}}(k_1, k_2) = i \frac{3G_F}{\sqrt{2}} \chi_L^\dagger(k_1) \bar{\sigma}^\mu \chi^L(k_2) (\tilde{j}_{5,e}(\mathbf{k}_2 - \mathbf{k}_1))_\mu. \quad (13)$$

Obviously, we find $|\bar{\mathcal{M}}|^2 = |\mathcal{M}|^2$.

After all, the cross section associated with the scattering between (anti)neutrinos and the electron axial current turns out to be

$$\begin{aligned} d\sigma &= d\bar{\sigma} = \frac{1}{2E_1} |\mathcal{M}(k_1, k_2)|^2 2\pi \delta(E_2 - E_1) \frac{d^3\mathbf{k}_2}{2E_2(2\pi)^3} \\ &= \frac{9G_F^2}{16\pi^2} [1 + \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 \cos(\varphi_1 - \varphi_2)] \\ &\quad \times |\tilde{j}_{5,e}^z(\mathbf{k}_2 - \mathbf{k}_1, t)|^2 \delta(E_2 - E_1) d^3\mathbf{k}_2. \end{aligned} \quad (14)$$

D. Acceleration from the recoil

Knowing the scattering cross section, one can integrate over the phase space of incident and scattered neutrinos to compute the total momentum transfer from the axial current to the neutrinos. In this way, one can estimate the pulsar kick acceleration based on momentum conservation.

From the scattering cross section, we express the pulsar kick acceleration originating from the recoil of the scattering as follows:

$$\mathbf{a}(t) = -\frac{1}{M} \int (\mathbf{k}_2 - \mathbf{k}_1) \frac{d\Omega_1}{4\pi} \frac{1}{\pi R^2(t)} \frac{dL(t)}{dE_1} dE_1 d\sigma \quad (15)$$

Here, M and R represent the mass and the radius of the PNS, L is the *observed number luminosity* of neutrinos emitted from the PNS, and $d\Omega_1$ is the angular integration with respect to $\mathbf{k}_1$. It should be remarked that M and R are both time dependent, but we can safely neglect the time dependence in M , while we keep $R(t)$. Plugging the explicit form of $d\sigma$ into this expression, we then reach

$$\begin{aligned} \mathbf{a}(t) &= -\frac{9G_F^2}{64\pi^4 M R(t)^2} \int d\Omega_1 d\Omega_2 dE E^2 \\ &\quad \times [1 + \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 \cos(\varphi_1 - \varphi_2)] \\ &\quad \times (\mathbf{k}_2 - \mathbf{k}_1) |\tilde{j}_{5,e}^z(\mathbf{k}_2 - \mathbf{k}_1, t)|^2 \frac{dL(t)}{dE}, \end{aligned} \quad (16)$$

where we denoted the energy as $E = E_1 = E_2$. Here, we would emphasize that Eq. (16) includes the contribution of the emitted neutrinos only through the *observed number luminosity* L . When neutrinos are rescattered and reabsorbed inside the neutrino sphere, they have no net effect on the acceleration, and our formula has no such contribution by construction.

For the concrete estimate for the luminosity, for simplicity, we assume a simple relation between the number luminosity L and the energy luminosity L_E (the latter can be deduced from Ref. [42]) as

$$\frac{dL(t)}{dE} = \frac{L_E(t)}{\langle E_\nu(t) \rangle} \delta(E - \langle E_\nu(t) \rangle), \quad (17)$$

where $\langle E_\nu(t) \rangle$ is the mean neutrino energy deduced from the temperature at the neutrino sphere.

We substitute Eqs. (7) and (17) into Eq. (16) and find that the acceleration can be separated into the 0th order part and the 1st order part with respect to α_1 as $\mathbf{a}(t) = \mathbf{a}_{(0)}(t) + \mathbf{a}_{(1)}(t)$, where the 0th order part is

$$\begin{aligned} \mathbf{a}_{(0)}(t) &= -\frac{9G_F^2 \langle E_\nu(t) \rangle L_E(t)}{64\pi^4 M R(t)^2} \int d\Omega_1 d\Omega_2 \\ &\quad \times [1 + \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 \cos(\varphi_1 - \varphi_2)] \\ &\quad \times (\mathbf{k}_2 - \mathbf{k}_1) |\tilde{j}_{5,e}^z(|\mathbf{k}_2 - \mathbf{k}_1|, t)|^2, \end{aligned} \quad (18)$$

and the 1st order part is

$$\begin{aligned} \mathbf{a}_{(1)}(t) &= -\alpha_1 \frac{9G_F^2 \langle E_\nu(t) \rangle L_E(t)}{32\pi^4 M R(t)^2} \int d\Omega_1 d\Omega_2 \\ &\quad \times [1 + \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 \cos(\varphi_1 - \varphi_2)] \\ &\quad \times (\mathbf{k}_2 - \mathbf{k}_1) |\tilde{j}_{5,e}^z(|\mathbf{k}_2 - \mathbf{k}_1|, t)|^2 \frac{(\mathbf{k}_2 - \mathbf{k}_1)_z}{|\mathbf{k}_2 - \mathbf{k}_1|}. \end{aligned} \quad (19)$$

Here, it should be noted that $|\mathbf{k}_1| = |\mathbf{k}_2| = k = \langle E_\nu(t) \rangle$. Now it would be more convenient to replace $\mathbf{k}_2$ by $\delta\mathbf{k} = \mathbf{k}_2 - \mathbf{k}_1$. To this end, we can rewrite $d\Omega_2$ into the form of $d^3\mathbf{k}_2$, which can be changed to $d^3\delta\mathbf{k}$. Then, we arrive at

$$\int d\Omega_1 d\Omega_2 = \frac{2}{k} \int d\Omega_1 d^3\delta\mathbf{k} \delta(k^2 - 2\delta\mathbf{k} \cdot \mathbf{k}_1). \quad (20)$$

where $\delta(k^2 - 2\delta\mathbf{k} \cdot \mathbf{k}_1)$ is a Dirac delta function.

To further simplify the above expressions, we introduce an approximation that works for $|\mathbf{k}_2 - \mathbf{k}_1| \sim 10^{-16} \text{ MeV} \ll |\mathbf{k}_1| = |\mathbf{k}_2| = \langle E_\nu(t) \rangle$. When this condition holds, we can drop higher order terms in $\delta k/k$, and approximate the integrand as

$$\begin{aligned} 1 + \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 \cos(\varphi_2 - \varphi_1) \\ = \frac{\delta k^2}{2k^2} + 2 \cos\theta_1 \cos\theta_2 \simeq 2 \cos^2\theta_1. \end{aligned} \quad (21)$$

Also, in Eq. (20), we can evaluate $\delta(k^2 - 2\delta\mathbf{k} \cdot \mathbf{k}_1) \simeq \delta(2\delta\mathbf{k} \cdot \mathbf{k}_1)$ and so $\delta\mathbf{k} \perp \mathbf{k}_1$.

With these preparations, we can express the 0th order acceleration in the z direction as

$$\begin{aligned} a_{(0)}^z(t) &= -\frac{9G_F^2 \langle E_\nu(t) \rangle L_E(t)}{64\pi^4 M R(t)^2} \frac{1}{k^2} \int d\Omega_1 d\delta k d\varphi_\delta \delta k \\ &\quad \times 2 \cos^2\theta_1 \delta k \cos\varphi_\delta \sin\theta_1 |\tilde{j}_{5,e}^z(\delta k, t)|^2, \end{aligned} \quad (22)$$

where φ_δ is the polar angle of $\delta\mathbf{k}$ in the plane perpendicular to $\mathbf{k}_1$. It is straightforward to verify that $a_{(0)}^z(t)$ is vanishing after the integration with respect to φ_δ .

In the same way, we can express the 1st order acceleration in the z -direction as follows:

$$\begin{aligned} a_{(1)}^z(t) &= -\alpha_1 \frac{9G_F^2 \langle E_\nu(t) \rangle L_E(t)}{32\pi^4 MR(t)^2} \frac{1}{k^2} \int d\Omega_1 d\delta k d\varphi_\delta \delta k \\ &\quad \times 2 \cos^2 \theta_1 \delta k \cos^2 \varphi_\delta \sin^2 \theta_1 |\tilde{j}_{5,e}^z(\delta k, t)|^2 \\ &= -\alpha_1 \frac{3G_F^2 \langle E_\nu(t) \rangle L_E(t)}{10\pi^2 MR(t)^2} \frac{1}{k^2} \int d\delta k \delta k^2 |\tilde{j}_{5,e}^z(\delta k, t)|^2. \end{aligned} \quad (23)$$

It is straightforward to confirm $a_{(0)}^{x,y}(t) = a_{(1)}^{x,y}(t) = 0$, which is understood from axial symmetry around the z -axis.

In summary, introducing a dimensionless variable ρ by $\delta k = \langle E_\nu(t) \rangle \rho$, we can write down the total acceleration as

$$\begin{aligned} \mathbf{a}(t) &= \hat{e}_z a_{(1)}^z(t) \\ &= -\hat{e}_z \alpha_1 \frac{3G_F^2 \langle E_\nu(t) \rangle^2 L_E(t)}{10\pi^2 MR(t)^2} \\ &\quad \times \int_0^\infty d\rho \rho^2 |\tilde{j}_{5,e}^z(\langle E_\nu(t) \rangle \rho, t)|^2. \end{aligned} \quad (24)$$

This is the central result in this paper that represents the pulsar kick acceleration.

III. NUMERICAL CALCULATIONS AND RESULTS

We shall perform numerical calculations based on Eq. (24) to estimate the acceleration from our mechanism. First, we discuss radial dependence and anisotropy of the axial current. Then, we present the time evolution of the pulsar kick velocity.

A. Axial current

We start analyzing the radial dependence of the axial current by estimating the magnetic field strength B_0 . We introduced the magnetic field strength B_0 in Sec. II B and in this work we choose $B_0 = 10^{12}$ T. Although this value is relatively high as compared to the standard strength on the neutron star surface, it is rather a conservative estimation in pulsar kick models driven by the magnetic field of PNS; see Refs. [20,21,32,36,55–57] for related works. There are at least three reasons why our choice of $B_0 = 10^{12}$ T can commonly be the case.

First, it is widely accepted that the poloidal component of surface magnetic field could be estimated based on the simple formula, $B_p \approx 3.2 \times 10^{15} \sqrt{P\dot{P}}$ T, where P is the rotation period of neutron stars made dimensionless in units of second and $\dot{P}$ is the dimensionless time derivative of P . From the fact that $B_p \sim 10^{11}$ T could occur in magnetars

with relatively old age (1–10 kyr) and younger neutron stars tend to have stronger magnetic fields (see Fig. 2 in Ref. [58], Fig. 1 in Ref. [59], and Fig. 6 in [60]), the magnetic field strength of order $B_0 = 10^{12}$ T is commonly expected in the PNS with an age of a few seconds.

Second, B_p estimated from $B_p = 3.2 \times 10^{15} \sqrt{P\dot{P}}$ T is the *surface* poloidal magnetic field. To the best of our knowledge, there is no established model for the magnetic field distribution in the neutron star interior. Nevertheless, it is generally the case that the interior magnetic field is stronger than the surface value by several orders of magnitude due to the squeezing and/or instabilities that amplify the magnetic field [61–64].

Third, B_p refers to the poloidal magnetic field only in the outer dipolar distribution. There could be a possibility that the toroidal magnetic field may be hundreds of times stronger than the surface magnetic field; see Ref. [63]. It is thus likely that the relevant magnetic field may well be far stronger than our conservative estimate, $B_0 = 10^{12}$ T.

To perform numerical calculation, we do need the spacetime distribution of the effective chemical potential, $\mu_{\text{CSE}}(r, t)$. As discussed earlier, we can compute $\mu_{\text{CSE}}(r, t)$ from the simulation results of $n_B(r, t)$, $Y_e(r, t)$, and $T(r, t)$ presented in Ref. [42]. One example of the radial distribution is shown in Fig. 3. In this figure and the original paper, the radial dependence is presented in terms of the enclosed baryonic mass defined by

$$\frac{dM_B(r)}{dr} = 4\pi r^2 e^\lambda n_B(M_B) m_N, \quad (25)$$

where m_N is the nucleon mass and $e^{-\lambda} = \sqrt{1 - 2GM(r)/r}$ with the gravitational enclosed mass $M(r)$. Since we are interested in the order estimate in this work, we simplify the analysis by approximating $M(r) \simeq M_B(r)$. Then, Eq. (25) is a closed relation and we can numerically solve $r(M_B)$

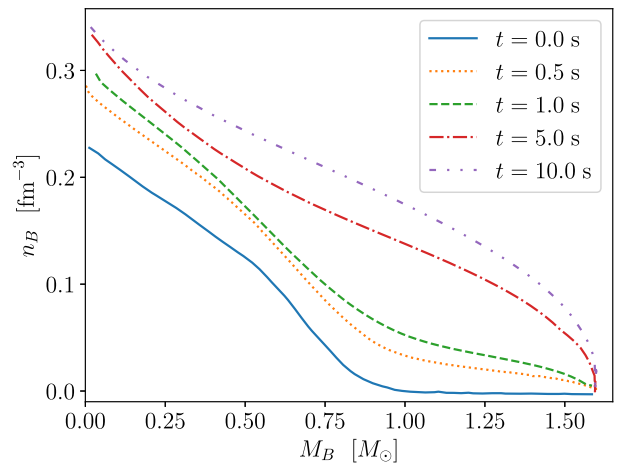


FIG. 3. Baryon density as a function of the enclosed baryonic mass in the GM3 model for $M = 1.6M_\odot$ adapted from Ref. [42].

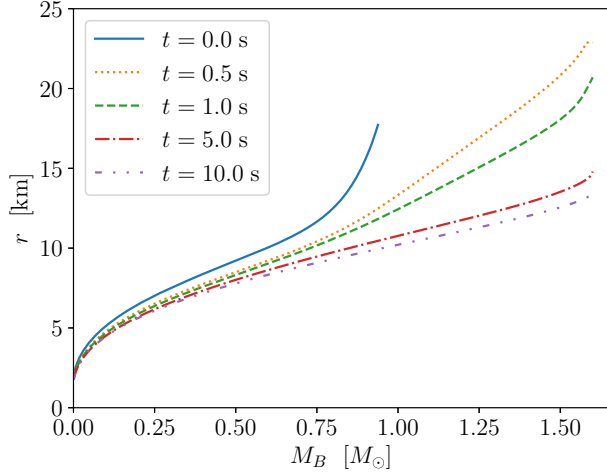


FIG. 4. Relation between the enclosed baryonic mass, M_B , and the radius r corresponding to the baryon density in Fig. 3.

using $n_B(M_B)$ as given in Fig. 3. Figure 4 shows $r(M_B)$ corresponding to the data in Fig. 3.

In this way, M_B can be interpreted as r , and we can read $n_B(r)$. The electron fraction, Y_e , can be deduced from Ref. [42] as well, and the spatial profile of the corresponding chemical potential, $\mu_e(r)$, is obtained from Eq. (4). Since the electron mass is tiny as compared to the typical scale of $\mu_e(r)$, the coefficient in the chiral separation effect, μ_{CSE} in Eq. (2) is almost identical with μ_e at $T = 0$, and the spatial profile including the finite temperature effects can be estimated from Eq. (2) as presented in Fig. 5.

It is then straightforward to perform the Fourier transformation of $\mathbf{j}_{5,e} = (e/2\pi^2)\mu_{\text{CSE}}\mathbf{B}$ and numerically estimate $|\tilde{j}_{5,e}(k)|$ which damps quickly at $k \sim 1/(10 \text{ km}) \sim 10^{-17} \text{ MeV}$ as shown in Fig. 6. Due to phase-space suppression, even smaller- k regions do not contribute to the integrated value. Thus, our proposed mechanism has the contributions peaked

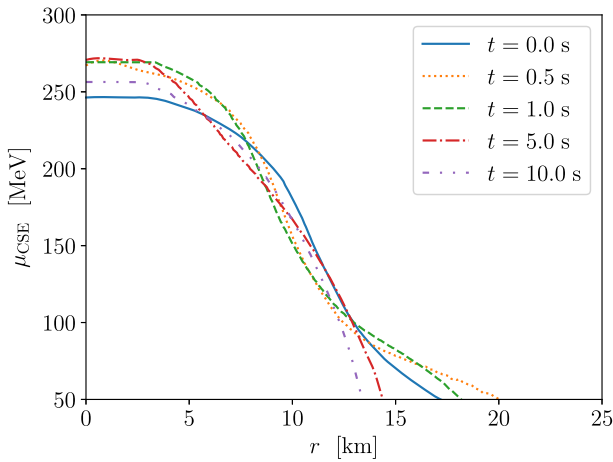


FIG. 5. Coefficient in the chiral separation effect or the effective chemical potential as a function of the radial distance r from the center of the star.

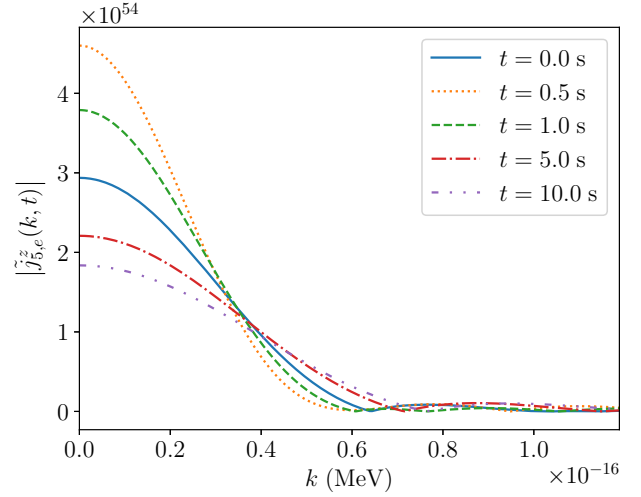


FIG. 6. Fourier component of the induced axial current as a function of the momentum for various time.

in spatial regions near or outside the neutrino sphere. This argument is a rough estimate, but suffices for our present purpose to demonstrate the relevance of the new mechanism by estimating the order of magnitude. The Fourier transformed current is dimensionless, and for such a spatial profile with a typical extension of $\sim 10 \text{ km}$, it turns out that $|\tilde{j}_{5,e}(0)| \sim 10^{54}$. This gigantic number arises from the macroscopic scale of the star's radius. It would be an instructive check to make the order estimate at this point. From Ref. [42] we see that $\langle E_\nu(t) \rangle \sim 10 \text{ MeV}$ and $L_E(t) \sim 50 \times 10^{51} \text{ erg/s}$ in small t regions. Then, let us approximate that $|\tilde{j}_{5,e}^z(\langle E_\nu(t) \rangle \rho)|^2$ has a support for $0 \leq \rho \leq 10^{-18}$ with a constant amplitude $\sim 10^{54}$. In the denominator, $M = 1.6M_\odot$ and $R \sim 10 \text{ km}$ would be a reasonable choice. Under these assumptions Eq. (24) leads to $a \sim \alpha_1 \times 5000 \text{ km/s}^2$.

We still need to fix the value or at least the order of α_1 . To the best of our knowledge, there is no consensus about α_1 in the PNS simulations. This is partially because the anisotropy predicted by the supernova explosion simulation has large ambiguity (see Refs. [11, 17]) and also because the evolution of the anisotropy within the short timescale of less than one minute after the explosion has not been thoroughly studied. Nevertheless, some supernova explosion simulations [9, 13, 65, 66] provide us with useful hints. In these works, the anisotropy of the ejecta at 10% level is reported, and if it scales with the anisotropy of the core, it would be sensible to constrain $\alpha_1 > 0.1$. We note that other studies [55, 67] show highly anisotropic $\mu_e(\mathbf{r}, t)$, $\mathbf{B}(\mathbf{r}, t)$, and $T(\mathbf{r}, t)$, which could persist during the first seconds of the newly born neutron star. The PNSs are held by the supernova remnant at the first seconds of their life (see a review [59] for more details), namely, a cluster of hot and highly inhomogeneous plasma, which could distort the magnetic field distribution. Furthermore, in neutron star cores, some instability mechanisms might amplify the initial anisotropy [36, 49, 61, 62]. All these arguments

suggest an even larger value of α_1 , and in this work, we shall decide to make a conservative choice of $\alpha_1 = 0.1$. The final results are proportional to α_1 , which is the largest uncertainty, but it is also a testable prediction. It is intriguing to make an experimental comparison between the velocity distribution and the distribution of anisotropy in observed supernovae remnants.

B. Time evolution of pulsar kick velocity

We are ready to proceed to the numerical calculations using the data from Ref. [42]. Because we assume constant B_0 and α_1 , the final results are simply proportional to $\alpha_1 B_0^2$, which is obvious from Eqs. (1) and (24). After performing numerical calculations, the acceleration as a function of t is shown in Fig. 7; the velocity as a result of the integrated acceleration as a function of t is shown in Fig. 8.

From these figures, we can see that the pulsar kick behaviors of different models stay close to each other: the velocity of the newly born pulsar first rises rapidly with

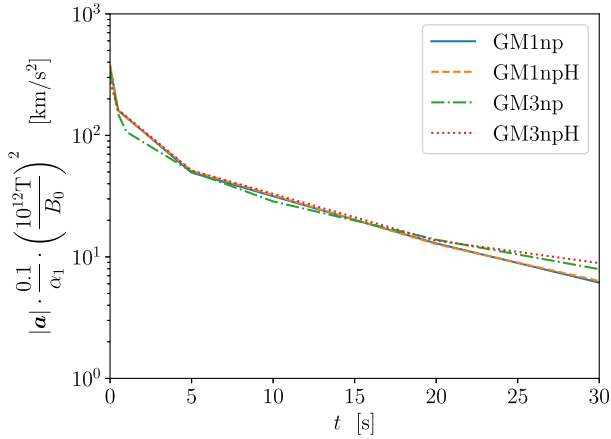


FIG. 7. Acceleration by the chiral pulsar kick as a function of time.

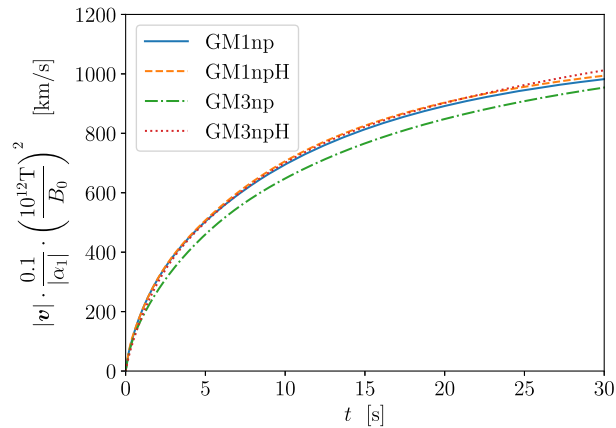


FIG. 8. Velocity as a result of integrated acceleration by the chiral pulsar kick as a function of time.

respect to time; then, at around $t \gtrsim 30$ s, it stabilizes at around

$$|\mathbf{v}_{\text{kick}}| \approx \frac{|\alpha_1|}{0.1} \left(\frac{B_0}{10^{12} \text{ T}} \right)^2 \times 1000 \text{ km/s}. \quad (26)$$

As for the direction of the kick velocity, it depends on both the direction of the magnetic field and the profile of the anisotropic part of the axial current. In the present simple setup, the velocity is positive in the direction antiparallel to the magnetic field for positive α_1 .

This result shows that for pulsars with magnetic field strength $B \sim 10^{11} - 10^{12}$ T and 10% density or magnetic field anisotropy, the anomaly-induced pulsar kick is sufficiently strong to explain the observed pulsar kick velocity (~ 400 km/s) independently. Therefore, the scattering between the axial current and the neutrinos emitted during PNS cooling is a valid mechanism to generate a pulsar kick from the chiral transport. This hints that the quantum anomalies could significantly influence the evolution of the pulsar during its early age.

In the above part of the paper, we only considered the kick generated by the electrons in the pulsar. However, although the kick by baryons is suppressed by their large mass, in principle, baryons may still make some contributions to the final kick velocity.

In our model, the baryon contributions can be included as the $j_{5,u}$ and $j_{5,d}$ in Eq. (11). To obtain $j_{5,u}$ and $j_{5,d}$, we need the up and down quark chemical potentials, i.e., μ_u and μ_d . In the following, we consider the situation that quarks are still confined in protons and neutrons. Also, as a rough estimation, we ignore the contribution from virtual quarks and simply assume μ_u and μ_d can be derived from the effective chemical potentials of protons and neutrons, μ_p , μ_n , with the following relation:

$$\mu_u = \frac{2}{3}\mu_p - \frac{1}{3}\mu_n, \quad \mu_d = \frac{2}{3}\mu_n - \frac{1}{3}\mu_p. \quad (27)$$

Here, the “effective chemical potential” means that μ_p and μ_n contain contributions through the mean fields. We can derive the distribution of μ_p and μ_n by numerically solving the following Fermi-Dirac distributions:

$$Y_e n_B = 2 \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \frac{1}{1 + e^{\beta(\sqrt{k^2 + M^{*2}} - \mu_p)}}, \quad (28)$$

$$(1 - Y_e) n_B = 2 \int \frac{d^3 \mathbf{k}}{(2\pi)^3} \frac{1}{1 + e^{\beta(\sqrt{k^2 + M^{*2}} - \mu_n)}}, \quad (29)$$

with M^* the effective baryon mass including the mean-field effects. Its numerical values for different models are given in Ref. [42].

Then, after taking into account the mass corrections in the same way as the electrons, we can compute the velocity

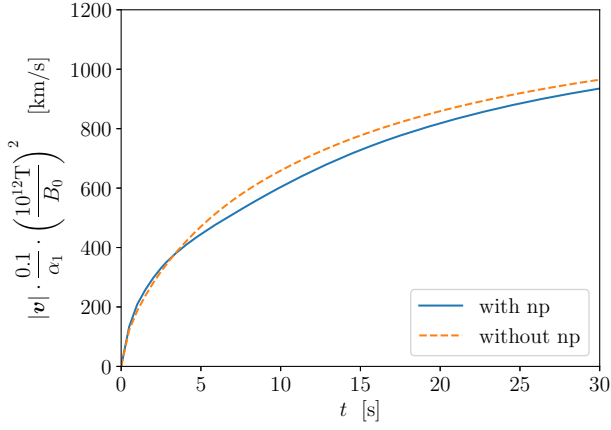


FIG. 9. Velocity with (solid curve) and without (dashed curve) contributions of the neutron and the proton. The underlying EoS is fixed to be GM3np.

of the pulsar kick with baryon contribution, as shown in Fig. 9. This figure shows that the inclusion of baryon contributions only slightly changes the velocity. Hence, this is a justification of our former statement that the mass effect strongly suppresses the neutron and proton axial currents. Nevertheless, the influences of baryons are not completely negligible. As shown in Fig. 9, at later time ($t > 5$ s), as the deleptonization reduces the population of electrons in the neutron star, the kick acceleration of baryons may become more important. We note that the inclusion of the baryon contribution reduces the acceleration. This is because the magnetic moment of protons is positive, while that of electrons and neutrons is negative. Therefore, the proton axial current is directed oppositely to the electron and neutron axial currents. The overall decrease in the acceleration is a result of competing contributions from protons and neutrons.

Finally, we note that our rough approximation of baryon contribution assumes uniform nuclear matter. However, one may need to consider the following possibilities: (1) In the inner crust of neutron stars where nonspherically symmetric pasta phases occur, the antientrapment effect can lead to the reduction of the effective mass of free neutrons [68–71], which could increase the pulsar kick velocity induced by neutrons. (2) In the crust regions, some baryons are bound in the inhomogeneous structures. If they are deeply trapped, they may not contribute to the pulsar kick. (3) It is highly possible that neutron stars could have hybrid compositions of matter in their central cores (e.g., quark matter) [72–74]. The quark mass in these cores could be smaller than the constituent mass, so the axial current can be enhanced in the core regions. According to the mechanism in Ref. [34], the anomalous transport is enhanced toward the regions near the neutrino sphere, and then an interplay between our proposed mechanism and others is expected. In the present scope, we did not consider such an interplay, and this is an interesting direction for future investigations.

IV. CONCLUSION

We proposed a mechanism that efficiently converts anisotropy to propulsion through scattering between neutrinos and the background axial current $\mathbf{j}_{5,e}$ of electrons. The presence of $\mathbf{j}_{5,e}$ is a robust consequence of the chiral separation effect, but constant $\mathbf{j}_{5,e}$ does not discriminate parallel/antiparallel scatterings along the magnetic axis. We find, however, that once momentum space anisotropy in either density or magnetic profile is coupled, the scattering involving $\mathbf{j}_{5,e}$ plays a significant role in the chiral anisotropy conversion that transforms anisotropy to acceleration.

We computed the acceleration of the PNS induced by the scattering between $\mathbf{j}_{5,e}$ and the neutrinos emitted during the cooling. The result shows that the recoil velocity generated by this mechanism scales with the square of the magnetic field strength B_0^2 and the anisotropic factor of the axial current α_1 (where the anisotropy of the axial current is induced by the anisotropy of the density and the magnetic field profile). If the axial current has 10% anisotropy, and the magnetic field reaches 10^{12} T, a 1000 km/s pulsar kick velocity could be generated.

In the past, the anisotropy effect and the magnetic effect were separately considered, and our new mechanism is hybrid. Our estimate results in a pulsar kick that is enough to explain neutron stars with magnetic fields 10^{11} – 10^{12} T. Therefore, our mechanism should be one of the major components included for quantitative analyses together with others. It should be noted that the magnetic dependence $\propto B_0^2$ is quite characteristic in the present scenario, and this hints at a possible distinction from others. Importantly, the acceleration direction in our proposed scenario is determined by both $\mathbf{B}$ and nonuniform α_1 . This could also explain the observation that the velocity is not perfectly aligned to the magnetic direction.

There are several future extensions. Further discussion of the baryon contribution of the pulsar kick is an interesting direction to work on. Another more ambitious direction is to develop the PNS simulation fully taking account of the background $\mathbf{j}_{5,e}$ effect and the neutrino transport processes. Besides, we simply assumed the strength of anisotropy, but the CSE would work in favor of forming anisotropic distributions. To implement such backreactions, we should go beyond the current scope, i.e., we should develop the chiral radiation hydrodynamics in the future.

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