

Entanglement asymmetry in gauge theories: Chiral anomaly in the finite temperature massless Schwinger model

Adrien Florio^{1,*} and Sara Murciano^{2,†}

¹*Fakultät für Physik, Universität Bielefeld, D-33615 Bielefeld, Germany*

²*Université Paris-Saclay, CNRS, LPTMS, 91405 Orsay, France*



(Received 30 November 2025; accepted 27 March 2026; published 5 May 2026)

Entanglement asymmetry has emerged in recent years as a practical quantity to study phases of matter. We present the first study of entanglement asymmetry in gauge theories by considering the chiral anomaly of the analytically solvable massless Schwinger model at both zero and finite temperatures. At zero temperature, we find the asymmetry exhibits logarithmic growth with system size. At finite temperature, we show that it is parametrically more sensitive to chiral symmetry breaking than the corresponding local order parameter: While the chiral condensate decays exponentially, the asymmetry decreases only logarithmically. This establishes the entanglement asymmetry as a promising tool to probe (finite-temperature) phase transitions in gauge theories.

DOI: [10.1103/q386-v4ch](https://doi.org/10.1103/q386-v4ch)

Introduction. The understanding of phases of matter is constantly evolving. The original Landau paradigm classifying gapped and gapless matter according to their symmetries and their symmetry-breaking pattern is now understood in terms of generalized symmetries. As such, it encompasses, for example, topological phases of matter that lack conventional local order parameters as well as the symmetry origin of the masslessness of photons; see [1–8] for reviews.

While progress has been made at the conceptual level, the determination of the nature of the phases of given theories remains a challenge. A particularly acute example is quantum chromodynamics, the non-Abelian gauge theory describing the strong interaction of the Standard Model of particle physics. The deconfinement transition from massless quarks to massive composite particles (hadrons) remains to be understood in terms of symmetries. In the presence of a net density of baryons (for instance, neutrons, such as in a neutron star), a majority of the phase diagram remains elusive; see [9] for a review.

In parallel, efforts have been made to devise nonlocal yet practical order parameters to address these difficulties. Building on information-theoretic quantities from resource theory [10,11] and on the intuition that entanglement behaves differently across phases, Ref. [12] introduced “entanglement asymmetry” as an efficient diagnostic of symmetry breaking

in many-body quantum systems. In the presence of a symmetry, the reduced density matrix can be decomposed into symmetry sectors, allowing us to resolve the contribution of each sector to the entanglement entropy. When the charge operator does not correspond to a symmetry of the state, attempting such a decomposition quantifies the degree of symmetry breaking via the entanglement asymmetry.

Given that the comprehension of entanglement in quantum field theory has rapidly evolved, starting from the pioneering works on the area law of entanglement [13,14], the entanglement in conformal field theories [15,16] and in holography [17], to a growing number of more applied topics including but not limited to the role of entanglement generation in high-energy scattering processes and pair production [18–33], its impact on the construction of effective field theories [34–37], and how it contributes to thermalization and hydrodynamization [38–50], understanding the behavior of the entanglement asymmetry in more complicated theories, such as gauge theories, promises both conceptual and practical progress. We take the first steps in this direction by studying the breaking of the chiral charge in the massless Schwinger model [51], arguably the simplest gauge theory and a model that can be solved analytically. We first perform the computation in the zero-temperature ground state, and we find a logarithmic dependence on the system size. We also generalize the computation to the finite-temperature Gibbs state. At high temperature, we find the asymmetry to be parametrically more sensitive than the chiral condensate, a local order parameter for the breaking of the chiral symmetry. Consequently, and similarly to quantum phase transitions, it promises to be a generic tool that does not rely on a local order parameter. Together, these results pave the way to more extensive analytical and numerical analyses in gauge theories in $1+1$ and higher dimensions.

*Contact author: aflorio@physik.uni-bielefeld.de

†Contact author: sara.murciano@universite-paris-saclay.fr

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The Schwinger model and its chiral charge. We study the massless Schwinger model, the theory that describes the physics of a massless Dirac fermion coupled to a $U(1)$ gauge field in $1 + 1$ dimensions:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - g\gamma^\mu A_\mu)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \quad (1)$$

Here, ψ is a two-component Dirac spinor, $\bar{\psi} = \psi^\dagger \gamma^0$, where γ^μ are 2×2 gamma matrices, A_μ is a $U(1)$ gauge field, $F_{\mu\nu}$ is its field strength tensor, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and the gauge coupling g is the electric charge. The classical action is invariant under both a vector $U(1)$ symmetry $\psi \rightarrow e^{i\alpha}\psi$ and a chiral $U(1)$ symmetry $\psi \rightarrow e^{i\gamma^5\alpha}\psi$. However, at the quantum level, the chiral symmetry is lost. One way to see this is by observing that the vector current $j_\mu = \bar{\psi}\gamma^\mu\psi$ is conserved:

$$\partial^\mu j_\mu = 0, \quad (2)$$

while the axial current $j_\mu^5 = \bar{\psi}\gamma^\mu\gamma^5\psi$ is related to the electric field as [51]

$$\partial^\mu j_\mu^5 = \frac{g}{2\pi} \epsilon_{\mu\nu} F^{\mu\nu}. \quad (3)$$

This is an apparent violation of Noether's theorem and is known as the chiral anomaly. We can write an equivalent bosonic theory to describe the dynamics of these currents ("bosonization" is an exact duality in $1 + 1$ dimensions). Taking into account that the gauge field has no propagating degrees of freedom and can be eliminated by using Gauss law $\partial_1 F^{10} = gJ^0$ and using the bosonization dictionary $j_\mu \leftrightarrow \epsilon_{\mu\nu} \partial^\nu \phi$, $j_\mu^5 \leftrightarrow \partial_\mu \phi$, the massless Schwinger model is equivalent to a massive free boson ϕ of mass $m = \frac{g}{\sqrt{\pi}}$ [52]:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2. \quad (4)$$

When computing the asymmetry, we will use the Hamiltonian lattice discretization of the free boson. In momentum space, it takes the form

$$\mathcal{H} = \sum_l \left(\frac{1}{2} \Pi_l \Pi_{-l} + \frac{\omega(l)^2}{2} \Phi_l \Phi_{-l} \right), \quad (5)$$

with $\omega(l)^2 = m^2 + (2\pi l/L)^2$ the finite volume dispersion relation and L the total system size. In this convention, the fields have units of $[\Phi_l] \sim \sqrt{L}$, $[\Pi_l] \sim 1/\sqrt{L}$ and the operators are canonical conjugates $[\Phi_l, \Pi_m] = i\delta_{l,m}$. In particular, the chiral charge becomes

$$Q_5 = \int dx \pi(x) \rightarrow \sqrt{L} \Pi_0. \quad (6)$$

The fact that an *a priori* discrete fermionic charge is mapped onto a continuous bosonic one may seem puzzling. The naive

fermionic chiral charge needs to be regularized. This mapping corresponds to a specific regularization choice; see [53] for a detailed explanation.

Entanglement asymmetry. Another central notion to our work is entanglement asymmetry. It was originally introduced in the context of quantum resource theory: Asymmetric states are thought of as a resource to access the effect of operations that are not invariant under symmetry [10,11,54–56]. In this sense, the concept has its origins in information theory. Similar quantities were also studied in the context of algebraic quantum field theory [57,58]. However, it has also been used to assess symmetry breaking arising from explicit symmetry-breaking terms, for instance, in many-body Hamiltonians or random circuits [59–73]. For concreteness, we specialize to a $U(1)$ symmetry, for which the entanglement asymmetry is defined as

$$\Delta S = S(\rho_{\text{sym}}) - S(\rho), \quad (7)$$

where $S(\rho) = -\text{Tr}[\rho \log(\rho)]$ is the von Neumann entropy, ρ is the density matrix of the system, and ρ_{sym} is its symmetrized version. Despite originally being introduced to describe the asymmetry of the full system, its definition has also been extended to study the symmetry breaking or the symmetry restoration within a subsystem, and, in this case, ρ is replaced by the reduced density matrix restricted to the subsystem [12]. For instance, if we consider a bipartition $A \cup B$, the reduced density matrix over A can be computed as $\rho_A = \text{Tr}_B \rho$, where we have performed a partial trace over the degrees of freedom inside B . This framework has enabled the investigation of intriguing anomalous relaxation behaviors, such as a quantum analogs of the Mpemba effect. For a comprehensive overview, we refer the reader to Refs. [74,75]. In this work, we restrict ourselves to a setup in which A describes the full system.

A further extension of Eq. (7), which is particularly convenient both for performing analytical computations and for experimental investigations, is the Rényi entanglement asymmetry, a replica version of Eq. (7):

$$\Delta S^{(n)} = S^{(n)}(\rho_{\text{sym}}) - S^{(n)}(\rho), \quad (8)$$

where $S^{(n)}(\rho) = 1/(1-n) \log \text{Tr} \rho^n$ is the Rényi entropy. The replica limit $n \rightarrow 1$ yields Eq. (7).

Equations (7) and (8) will be our probe of the chiral anomaly in the massless Schwinger model. Unlike prior studies of asymmetry, Eq. (1) contains no explicit chiral symmetry-breaking terms; our analysis studies the asymmetry in a setting where symmetry breaking arises intrinsically at a quantum level rather than from external sources.

Zero temperature. We start with the zero-temperature computation, which will serve as a reference point for

the finite-temperature case and potentially future studies of the massive Schwinger model. We aim to compute the asymmetry associated with the chiral charge in the ground state $|\Omega\rangle$ of the Schwinger model. To this end, we need to compute the von Neumann entanglement entropy of the symmetrized density matrix

$$\rho_5 = \int_{-\pi}^{\pi} \frac{d\alpha}{2\pi} e^{-i\alpha Q_5} |\Omega\rangle \langle \Omega| e^{i\alpha Q_5}. \quad (9)$$

Since the total system is in a pure state, $S(|\Omega\rangle \langle \Omega|) = 0$. In terms of the lattice operators introduced above, $Q_5 = \sqrt{L}\Pi_0$. In the absence of the anomaly, i.e., when the charge is conserved, Q_5 has a discrete spectrum. This justifies the use of a discrete projector (with integration limits $\alpha \in [-\pi, \pi]$) in Eq. (9).

Using the replica trick, the starting point is the trace of the n th power of the symmetrized density matrix:

$$\text{Tr}(\rho_5^n) = \int_{-\pi}^{\pi} \frac{d\alpha_1 d\alpha_2 \dots d\alpha_n}{(2\pi)^n} Z_n(\vec{\alpha}), \quad (10)$$

where $Z_n(\vec{\alpha}) = \prod_{j=1}^n \langle \Omega | e^{i\alpha_{j,j+1} Q_5} | \Omega \rangle$, $\alpha_{j,j+1} = \alpha_j - \alpha_{j+1}$, and $\alpha_{j+n} \equiv \alpha_j$. The quantity $Z_n(\vec{\alpha})$ is called the charged moment, and, in this case, its computation reduces to the knowledge of the two-point correlator

$$\langle \Omega | e^{i\alpha \sqrt{L}\Pi_0} | \Omega \rangle. \quad (11)$$

Inserting a resolution of the identity in the momentum basis, we can rewrite the correlator above as¹

$$\int_{-\infty}^{\infty} dp \langle \Omega | e^{i\alpha \sqrt{L}\Pi_0} | p \rangle \langle p | \Omega \rangle. \quad (12)$$

At this point, the mapping of the massless Schwinger model to a massive bosonic field gives

$$\begin{aligned} e^{i\alpha \sqrt{L}\Pi_0} | p \rangle &= e^{i\alpha \sqrt{L}p} | p \rangle, \\ \langle p | \Omega \rangle &= \frac{e^{-p^2/(2m)}}{(m\pi)^{1/4}}, \end{aligned} \quad (13)$$

where we have simply used that $\Pi_0 | p \rangle = p | p \rangle$. Plugging these relations into the integral (12) yields

$$\langle \Omega | e^{i\alpha \sqrt{L}\Pi_0} | \Omega \rangle = e^{-\alpha^2 m L / 4}. \quad (14)$$

Therefore, we have to solve the n -fold integral

$$\int_{-\pi}^{\pi} \frac{d\alpha_1 d\alpha_2 \dots d\alpha_n}{(2\pi)^n} \prod_{j=1}^n e^{-mL/4\alpha_{j,j+1}^2}. \quad (15)$$

¹To be fully consistent with the discrete projection, we should, in principle, restrict ourselves to summing over a discrete subset of momenta. As this does not matter in the thermodynamics limit, we ignore here this subtlety; see also Supplemental Material [76] for more details.

By doing a change of variables $\alpha_{j,j+1} \rightarrow \gamma_j / \sqrt{L}$ and using $\sum_j \alpha_{j,j+1} = 0$, we obtain in the large L limit

$$\int_{-\infty}^{\infty} \frac{d\gamma_1 d\gamma_2 \dots d\gamma_n}{(2\pi)^{n-1} L^{n/2}} \prod_{j=1}^n e^{-m/4\gamma_{j,j+1}^2} \delta\left(\sum_i \gamma_i / \sqrt{L}\right). \quad (16)$$

Using the integral representation of the δ function leads to

$$\text{Tr}(\rho_5^n) = \sum_{k=-\infty}^{\infty} J_k^n, \quad J_k = \int_{-\infty}^{\infty} d\gamma \frac{e^{i\frac{k}{\sqrt{L}}\gamma - \gamma^2 m/4}}{2\pi\sqrt{L}}. \quad (17)$$

Evaluating the Gaussian integral and taking the replica limit $n \rightarrow 1$, the entanglement asymmetry behaves as

$$\Delta S = -\sum_k \text{Re}[J_k \log J_k] \simeq \frac{1}{2} \log(mL\pi) + \frac{1}{2}, \quad (18)$$

where, in the last step, we have approximated the discrete sum over k by a continuum integral.

We see that the entanglement asymmetry shows the typical logarithmic scaling $1/2 \log L$, which has also been found in different contexts like Gaussian [77], Haar random [63], and generic translation-invariant matrix product states where the symmetry is explicitly broken [61]. Another interesting feature is that the “length scale” of the problem is mL , and, strictly speaking, if we take the limit $m \rightarrow 0$, the asymmetry diverges. However, if $m \rightarrow 0$, Eq. (3) implies that the chiral symmetry is restored, so we expect the asymmetry to vanish. The reason for this apparent discrepancy is the noncommutation of the limits $m \rightarrow 0$ and $L \rightarrow \infty$, already observed in [12].

The fact that the scaling of the asymmetry for the chiral anomaly matches other very different cases where the symmetry is explicitly broken [70] can be traced back to the fact that the ground state of the Schwinger model satisfies the cluster property; i.e., the variance of the operator Q_5 grows extensively in the system size, since $\langle Q_5^2 \rangle = mL/2$. Therefore, the chiral symmetry breaking is bounded by [70]

$$\Delta S \leq \frac{1}{2} \log \left[2\pi \left(\frac{mL}{2} + \frac{1}{12} \right) \right] + \frac{1}{2}, \quad (19)$$

which is consistent with our result.

Finite temperature. Having the zero-temperature case as a reference, we can now study what happens at finite temperature. While the massless Schwinger model does not exhibit a phase transition at any finite temperature, we expect on general grounds the symmetry to be progressively restored as the temperature increases. Indeed, in the limit where the inverse temperature β goes to zero, the Gibbs ensemble tends to the (normalized) identity, which is symmetric with respect to any arbitrary charge. This is an ideal setting to probe the behavior of the entanglement

asymmetry: We expect it to get smaller as the temperature goes up. In this case, we also explicitly know the relevant order parameter that detects chiral symmetry breaking. This will allow us to compare their temperature dependence.

To do so, we replace in Eq. (10) the ground state $|\Omega\rangle\langle\Omega|$ density matrix by the Gibbs state $e^{-\beta H}/Z$:

$$\text{Tr}(\rho_5^n(\beta)) = \int_{-\pi}^{\pi} \frac{d\alpha_1 d\alpha_2 \dots d\alpha_n}{(2\pi)^n} \cdot \text{Tr} \left(\prod_{j=1}^n e^{-\alpha_{j,j+1} Q_5} e^{-\beta H} \right). \quad (20)$$

Once again, the expression above can be rewritten in a more convenient form working in the momentum basis. Since the steps are rather technical, we report them in Supplemental Material [76] and move directly to the main results.

The expression of the charged moments to be plugged in Eq. (20) is

$$\frac{\text{Tr}(\rho_5^n(\beta))}{\text{Tr}(\rho^n(\beta))} = \int_{-\pi}^{\pi} \frac{d\alpha_1 d\alpha_2 \dots d\alpha_n}{(2\pi)^n} \exp \left(-\frac{1}{2} \vec{A}^T \mathbf{M}^{-1} \vec{A} L \right),$$

where

$$A_i = \alpha_{i,i+1}, \quad M_{ii} = \frac{2 \coth(m\beta)}{m}, \quad (21)$$

$$M_{i,i+1} = M_{i-1,i} = -\frac{1}{m \sinh(m\beta)} (1 + \delta_{n2}), \quad (22)$$

with the identification $0 \equiv n$ and $n+1 \equiv 1$. After computing explicitly the integral above, we get

$$\frac{\text{Tr}(\rho_5^n(\beta))}{\text{Tr}(\rho^n(\beta))} = \frac{2 \sinh(\frac{\beta m n}{2})}{(2\pi L m \sinh(m\beta))^{n/2}} \sum_{l=-\infty}^{\infty} \exp \left(-\frac{n l^2}{m L} \tanh \left(\frac{\beta m}{2} \right) \right), \quad (23)$$

which leads to the following expression for the entanglement asymmetry:

$$\begin{aligned} \Delta S = & \frac{\sinh(m\beta/2)}{\sqrt{2\pi m L \sinh(m\beta)}} \\ & \cdot \left(-m\beta \coth(m\beta/2) \theta_3(e^{-\tanh(m\beta/2)/(mL)}) \right. \\ & + \theta_3(e^{-\tanh(m\beta/2)/(mL)}) \log(2\pi m L \sinh(m\beta)) \\ & \left. + \frac{2 \tanh(m\beta/2)}{m L} \sum_l l^2 e^{-l^2 \tanh(m\beta/2)/(mL)} \right). \end{aligned} \quad (24)$$

Here, θ_3 denotes the Jacobi theta function. Even though this expression is rather involved, it admits simple asymptotes in two regimes. The first one is the zero-temperature limit, $m\beta \gg 1$, where Eq. (24) reduces to

$$\Delta S \approx \frac{1}{2} (1 + \log(mL\pi)). \quad (25)$$

This is a good cross-check, since we recover the result (18). The other interesting limit is the high-temperature regime, which is valid as far as $1/(mL) < m\beta \ll mL$, and reads

$$\Delta S \approx \frac{1}{2} (-1 + \log(2\pi m^2 \beta L)). \quad (26)$$

As in the zero-temperature case, we can also verify the upper bounds on the asymmetry coming from clustering. We compute explicitly the variance of Q_5 at finite temperature $\langle Q_5^2 \rangle_\beta = \text{Tr}(e^{-\beta H} Q_5^2)/Z$ and find that

$$\langle Q_5^2 \rangle_\beta = \frac{mL}{2} \coth \left(\frac{\beta m}{2} \right). \quad (27)$$

If we use the expression for the entanglement asymmetry in Eq. (24), we can check that the bound

$$\Delta S \leq \frac{1}{2} \log \left[2\pi \left(\frac{mL}{2} \coth \left(\frac{\beta m}{2} \right) + \frac{1}{12} \right) \right] + \frac{1}{2} \quad (28)$$

is satisfied for any value of β .

A natural question to ask now is what we learn about the chiral anomaly from the entanglement asymmetry that we cannot learn from the study of the order parameter. We report here the finite-temperature result of the order parameter found in [78]:

$$\langle \bar{\psi} \psi \rangle = -\frac{2}{\beta} e^{-\frac{\pi}{\beta m} \coth(\frac{Lm}{2})} e^{F(\beta m) - 2H(\beta m, L/\beta)}, \quad (29)$$

where

$$\begin{aligned} F(x) &= \sum_{n>0} \left(\frac{1}{n} - \frac{1}{\sqrt{n^2 + (x/2\pi)^2}} \right), \\ H(x, \tau) &= \sum_{n>0} \frac{1}{\sqrt{n^2 + (x/2\pi)^2}} \frac{1}{e^{\tau \sqrt{(2\pi n)^2 + x^2}} - 1}. \end{aligned} \quad (30)$$

We compare in Fig. 1 the behavior of the order parameter and of the entanglement asymmetry as a function of β for sufficiently large L . To have dimensionless quantities, we rescale the chiral condensate density $\bar{\psi} \psi$ by the lattice volume L . Both quantities detect that the Gibbs state is not symmetric under chiral rotation. The main difference is their scaling with volume and how this interplays with temperature. As expected, both go to zero at high temperature. However, in the regime $1/(mL) < m\beta < 1$, Fig. 1 shows a striking difference: The asymmetry exhibits logarithmic behavior (26), while the condensate decays exponentially as $\langle \bar{\psi} \psi \rangle \sim e^{-\pi/m\beta}$. The presence of quantum superpositions between different symmetry sectors captures the symmetry breaking. This global property of the

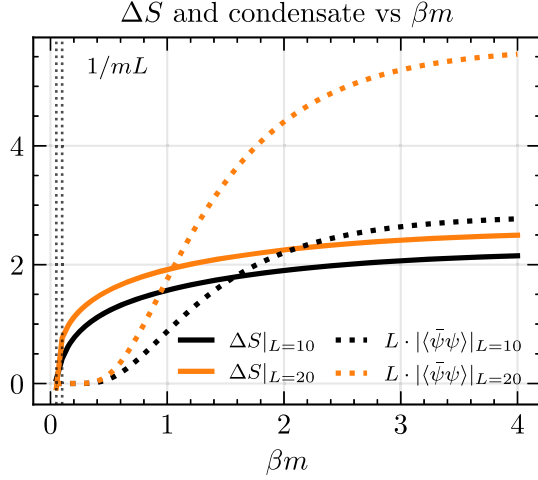


FIG. 1. Entanglement asymmetry (solid lines) and rescaled chiral condensate density $\langle\bar{\psi}\psi\rangle \cdot L$ (dashed lines) as functions of inverse temperature β for different system sizes L . The logarithmic behavior of the asymmetry contrasts with the exponential suppression of the order parameter at high temperature. It also contrasts with the extensive behavior of the order parameter as a function of mL . The vertical dotted lines show the values of $1/mL$; we expect the thermodynamics limit to be reliable for $\beta m > 1/mL$.

state is measured only by the asymmetry and not by the local order parameter. It leads to the parametric enhancement of the former over the latter. Even though, strictly speaking, we do not know how to compute analytically the asymmetry for $\beta \ll 1/(mL)$, we can, however, extend our analysis to small $m\beta$ by directly computing the Rényi asymmetries numerically using Eqs. (20) and (21). We show the result for the second Rényi $\Delta S^{(2)}$ in Fig. 2. Black dots correspond to the numerical computation. The plain blue curve comes from Eq. (23) and is the thermodynamic limit. We first see that both indeed agree very well for $\beta m > 1/mL$. As a result, the parametric enhancement is also visible in the Rényi asymmetries. Furthermore, it persists in the finite size regime $\beta m < 1/mL$, as we can show that there $\Delta S^{(2)} \sim (\pi^2/12)Lm^2\beta$ (green dashed line) is linear in β while the condensate Eq. (29) still exponentially decays. We further support this behavior for larger Rényi indices in Supplemental Material [76]. This confirms that the striking parametric enhancement of the asymmetry is not an artifact of how we computed the thermodynamic limit.

Discussion. The massless Schwinger model is analytically solvable and, through its chiral anomaly, exhibits a non-trivial symmetry breaking even in $1+1$ dimensions. We showed that this is detected by the entanglement asymmetry. Its dependence is logarithmic in the system size. We were also able to compute its temperature dependence and compare it to that of the chiral condensate, the relevant local order parameter. While both quantities go to zero at

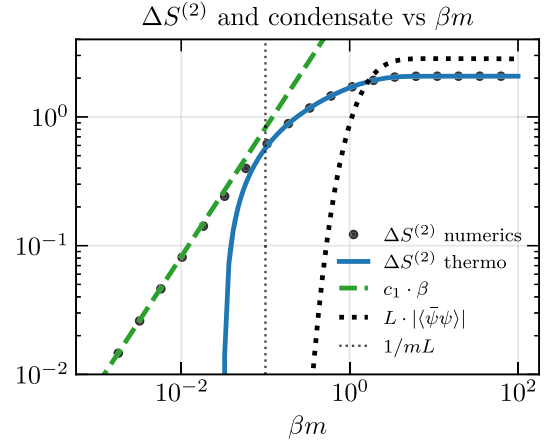


FIG. 2. Second Rényi asymmetry $\Delta S^{(2)}$ and rescaled chiral condensate density $\langle\bar{\psi}\psi\rangle \cdot L$ as functions of β for $mL = 10$. Black dots show numerical results at finite L , while the blue curve shows the thermodynamic limit expression from Eq. (23). The green dashed line indicates the small- β asymptotic behavior $\Delta S^{(2)} \sim (\pi^2/12)Lm^2\beta$. The agreement between finite- L and thermodynamic results is excellent for $\beta m > 1/(mL)$.

infinite temperature, we showed that the asymmetry is parametrically more sensitive to symmetry breaking at large temperatures $m\beta < 1$. In particular, while the order parameter is exponentially suppressed at high temperature, the Rényi entanglement asymmetry only decreases polynomially in $m\beta$. It is reasonable to expect this property to be rather ubiquitous. It is inherited from the logarithmic dependence of the entropy on system size, which is generic for systems obeying the cluster decomposition principle. This provides strong incentives to develop the asymmetry as a more sensitive tool for studying (finite-temperature) phase transitions. Rather than being yet another measure of entanglement, it is a sensitive order parameter that can be studied for different *specific* symmetries. As such, it can, for instance, discern between different symmetry-breaking patterns. This is particularly true as recent advances have shown that Rényi entropies can be extracted from Euclidean Monte Carlo simulations of realistic theories in higher dimensions [79–83]. Generalizing these methods to compute Rényi asymmetries is a natural outlook. The Rényi entropies computed, for instance, in Fig. 2 provide crucial benchmarks to the development of such techniques. Beyond static properties, we have already mentioned that the entanglement asymmetry is also useful to quantify how dynamics can restore a broken symmetry locally and show anomalous symmetry restoration phenomena, like a quantum version of the Mpemba effect in closed systems. It would be interesting to consider whether this also happens in gauge theories. A simple thought experiment is to start from the ground state of the massless Schwinger model ($m \neq 0$) and perform a quench with $m = 0$. In this case, as the chiral anomaly cannot be distinguished from an explicit symmetry breaking, we would expect a quantum version of

the Mpemba effect to persist. Confirming this and investigating it in realistic finite-temperature phase transitions is an exciting outlook. More generally, this work opens up new possibilities to study symmetry breaking or symmetry restoration in Hamiltonian lattice gauge theories. Given the success that Gaussian techniques have had in the study of the entanglement asymmetry, exploiting the Gaussian variational ansatz [84] could be a first step in this direction.

Another direction more specific to gauge theories is understanding better the exact relation between such entanglement measures and anomalous symmetries in higher dimensions. In $3+1$ dimensions, the breaking of the chiral charge can be understood as one aspect of a conserved noninvertible symmetry [85,86]. The presence of a chiral condensate is not automatic, and when present it does not simply amount to an explicit symmetry breaking as in the Schwinger model but leads to the existence of a massless Goldstone mode [87]. Recently, Refs. [72,73] initiated the study of the entanglement asymmetry for noninvertible symmetries. Extending this and our work

in the context of gauge theories and anomalies and understanding the entanglement structure of such theories as well as how it rearranges across phases is of great interest.

Acknowledgments. The authors thank SwissMAP for funding the ALPS Summer School series, without which this work would not have existed. A. F. acknowledges interesting discussions with G. Cuomo and D. Kharzeev. S. M. acknowledges F. Ares, P. Calabrese, G. Di Giulio, and P. Sala. A. F. is supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through the Emmy Noether Programme Project No. 545261797 and the CRC-TR 211 “Strong-interaction matter under extreme conditions,” Project No. 315477589-TRR 211.

Data availability. The data are not publicly available. The data are available from the authors upon reasonable request.

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