

Structure of heavy quarkonia in a strong magnetic field

Ahmad Jafar Arifi^{1,2,*} and Kei Suzuki^{1,†}

¹*Advanced Science Research Center, Japan Atomic Energy Agency, Tokai 319-1195, Japan*

²*Research Center for Nuclear Physics, The University of Osaka, Ibaraki, Osaka 567-0047, Japan*



(Received 7 August 2025; accepted 6 October 2025; published 10 November 2025)

We investigate the structural modifications of heavy quarkonia in the presence of strong magnetic fields using a constituent quark model. By incorporating the effects of spin mixing and quark Landau levels, we employ a nonrelativistic Hamiltonian that captures the essential features of quark dynamics in a magnetic field. The two-body Schrödinger equation is solved using the cylindrical Gaussian expansion method, which respects the cylindrical symmetry induced by a magnetic field. We extract the corresponding light-front wave function (LFWF) densities and analyze their transverse and longitudinal structures, revealing characteristic features such as transverse momentum broadening. While the longitudinal structure is only slightly modified within the nonrelativistic Hamiltonian, we discuss some corrections that can significantly affect its longitudinal structure. Furthermore, we discuss the structure modifications of excited states and find notable changes in the LFWF densities, and state reshuffling near avoided crossings. These results demonstrate the sensitivity of hadron structure to external magnetic fields and help bridge our understanding to relativistic approaches.

DOI: [10.1103/pwsl-xrq5](https://doi.org/10.1103/pwsl-xrq5)

I. INTRODUCTION

The behavior of quarkonia under extreme conditions—including strong magnetic fields, finite temperatures, and densities—provides crucial insights into the nonperturbative structure of Quantum Chromodynamics (QCD) vacuum. This field has attracted growing interest since the original proposal of J/ψ suppression as a quark–gluon plasma signature [1,2]. The relevance has been further amplified by relativistic heavy-ion collisions, where non-central collisions generate transient but intense magnetic fields reaching $|eB| \sim 0.3 \text{ GeV}^2$ at the LHC [3–5], capable of significantly modifying hadronic properties.

Recent theoretical advances have enabled comprehensive studies of these phenomena through multiple approaches. Lattice QCD simulations now provide first-principles calculations of hadron properties in magnetic fields [6–19] ([20] for a review), while model-based frameworks offer complementary perspectives. This vibrant research area is evidenced by numerous recent reviews covering various aspects of quarkonia in extreme environments [21–31].

The study of heavy hadrons containing charm or bottom quarks under magnetic fields has been approached through several theoretical frameworks. Constituent quark models [32–38] and QCD sum rules [39–42] have provided valuable insights, complemented by holographic QCD approaches [43–47] and the complex-potential picture in thermal medium [48–52]. Lattice QCD simulations have further advanced our understanding by investigating the heavy quark–antiquark potential in magnetic backgrounds [53–55]. Beyond heavy-quark systems, significant attention has been devoted to understanding magnetic field effects on hadrons composed of light (up, down, or strange) quarks. For light mesons, studies (e.g., [56–67]) have revealed characteristic modifications, while light baryons have been examined through several approaches [63,68–77].

The constituent quark model remains a powerful framework for understanding static hadron properties, as detailed in a recent review [29]. Magnetic fields influence hadrons through three main mechanisms: (i) spin mixing via the $-\boldsymbol{\mu}_i \cdot \mathbf{B}$ interaction, (ii) quark kinetic energy modifications induced by the $\mathbf{B} \times \mathbf{r}$ term, and (iii) anisotropic deformation of interquark potentials [78–81], where lattice QCD simulations provide crucial constraints [53–55]. Previous investigations of quarkonia [35,36] have systematically examined these effects, particularly spin mixing and energy shifts, employing potential models. These studies utilize the cylindrical Gaussian expansion method (CGEM) to solve the two-body Schrödinger equation, an adaptation of the conventional Gaussian expansion

*Contact author: aj.arifi01@gmail.com

†Contact author: k.suzuki.2010@th.phys.titech.ac.jp

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International license](https://creativecommons.org/licenses/by/4.0/). Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

method (GEM) [82,83] specifically designed for systems with cylindrical symmetry under magnetic fields.

Building upon previous studies [35,36], we aim to study light-front wave functions (LFWFs) of heavy quarkonia from nonrelativistic quark model solutions under strong magnetic fields and analyze their structural modifications. Unlike the direct light-front Hamiltonian diagonalization method proposed in Ref. [38], our approach establishes a connection between magnetically modified LFWFs and the rest-frame wave functions, which offers clearer physical interpretation in coordinate space. From these LFWFs, one can directly access partonic observables such as parton distribution functions (PDFs), relevant for high-energy processes [84–86]. Connections between rest-frame wave functions and LFWFs, particularly in the absence of a magnetic field, have been extensively utilized in previous studies (e.g., [87–94]), providing a solid foundation for the current work.

The present study has several key findings: (i) These LFWFs exhibit common features also seen in relativistic models, such as transverse momentum broadening. This behavior can be clearly understood from the squeezed wave function in transverse coordinate space [35,36]. (ii) The ground-state PDFs, characterizing the longitudinal momentum fraction distributions, exhibit only minor modifications at leading order. Substantial modifications emerge at next-to-leading order through corrections. In contrast, the PDF for excited states is strongly modified due to nodal structure deformation. While experimental verification remains challenging, our predictions provide useful input for lattice QCD simulations [20], especially with recent developments in Large Momentum Effective Theory (LaMET) [95], which enables direct access to partonic structure.

This paper is organized as follows. Sec. II introduces the two-body Hamiltonian, the CGEM framework, and the construction of LFWFs. Numerical results and their detailed analysis are presented in Sec. III. The conclusions and future outlook are provided in Sec. IV. Additional details concerning relativistic corrections and the explicit forms of matrix elements are given in Appendices A and B, respectively.

II. FORMALISM

In this section, we construct the effective Hamiltonian and introduce the CGEM to solve the Schrödinger with the cylindrical symmetry constraint. The LFWF is then introduced and some related quantities are discussed.

A. Quark model in a magnetic field

In this study, we investigate quarkonia in the non-relativistic quark model. In the presence of an external magnetic field, the two-body Hamiltonian composed of a quark and an antiquark can be expressed as [32,33]

$$H_{q\bar{q}} = \sum_{i=q,\bar{q}} \left[m_i + \frac{(\mathbf{p}_i - q_i \mathbf{A})^2}{2m_i} - \boldsymbol{\mu}_i \cdot \mathbf{B} \right] + V_{q\bar{q}}(r). \quad (1)$$

Here, m_i and $\mathbf{p}_i$ denote the mass and momentum of quark or antiquark labeled by $i = q, \bar{q}$, respectively. The magnetic field is contained in the Hamiltonian through the vector potential $\mathbf{A}$, which alters the kinetic-energy term of the single particle. We adopt the symmetric gauge, $\mathbf{A}(\mathbf{r}_i) = \frac{1}{2} \mathbf{B} \times \mathbf{r}_i$, and include the coupling to the quark magnetic moment $\boldsymbol{\mu}_i$.

For the effective quark–antiquark potential $V_{q\bar{q}}$, we adopt the following form [96]

$$V_{q\bar{q}} = C + \sigma r - \frac{A}{r} + \alpha (\mathbf{S}_q \cdot \mathbf{S}_{\bar{q}}) e^{-\Lambda r^2}, \quad (2)$$

where $r = \sqrt{z^2 + r_\perp^2}$, C is a constant shift, and the magnitudes of the confining, color Coulomb, and spin–spin potentials are characterized by the parameters, σ , A , and α , respectively. We smear the spin–spin interaction as a Gaussian function with a parameter Λ [97], for simplicity. The term $\langle \mathbf{S}_q \cdot \mathbf{S}_{\bar{q}} \rangle$ yields the values of 1/4 and $-3/4$ for the vector (spin–triplet) and pseudoscalar (spin–singlet) mesons, respectively.

In a magnetic field, the kinetic momentum $\mathbf{p}_i$ is not conserved, while one can introduce the so-called pseudomomentum [98]. The two-body pseudomomentum is defined as [99–101]

$$\mathbf{K} = \sum_{i=q,\bar{q}} \left[\mathbf{p}_i + \frac{1}{2} q_i \mathbf{B} \times \mathbf{r}_i \right], \quad (3)$$

which is a conserved quantity. By using the pseudomomentum, the total wave function can be factorized as

$$\Phi(\mathbf{R}, \mathbf{r}) = \exp[i\mathbf{P} \cdot \mathbf{R}] \psi(\mathbf{r}) \quad (4)$$

$$= \exp \left[i \left(\mathbf{K} - \frac{q}{2} \mathbf{B} \times \mathbf{r} \right) \cdot \mathbf{R} \right] \psi(\mathbf{r}), \quad (5)$$

where $\mathbf{R}$ and $\mathbf{r}$ are the center-of-mass and relative coordinate, respectively. $\mathbf{P}$ is the center-of-mass momentum.

Here, we focus on the magnetic field $\mathbf{B}$ along the z direction and meson with $\mathbf{K} = 0$. We also focus on heavy quarkonia with neutral charge, $q_1 = -q_2 = q$. In this case, the two-body Hamiltonian is reduced to the Hamiltonian for the relative motion [33–36,62]

$$H = -\frac{\nabla^2}{2\mu} + \frac{q^2 B^2}{8\mu} r_\perp^2 - \sum_{i=q,\bar{q}} \boldsymbol{\mu}_i \cdot \mathbf{B} + V(r), \quad (6)$$

where $\mu = m_q/2$ is the reduced quark mass. The above Hamiltonian maintains the rotational symmetry on the transverse x - y plane and the reflection symmetry along the z -axis. The second term proportional to $B^2 r_\perp^2$ leads to the so-called quark Landau levels.

We discuss the contribution of the effects induced by $\boldsymbol{\mu}_i \cdot \mathbf{B}$ [32,33], where $\boldsymbol{\mu}_i = g q_i S_i / 2m_i$ is the quark magnetic

moment with the Lande g -factor, $g = 2$. Thus, we obtain

$$-(\boldsymbol{\mu}_1 + \boldsymbol{\mu}_2) \cdot \mathbf{B} = -\frac{gq}{4} \left(\frac{\boldsymbol{\sigma}_1}{m_1} - \frac{\boldsymbol{\sigma}_2}{m_2} \right) \cdot \mathbf{B}, \quad (7)$$

where we used $\mathbf{S}_i = \boldsymbol{\sigma}_i/2$. Then, the eigenstates with the different spin quantum numbers mix with each other:

$$\langle 10 | -(\boldsymbol{\mu}_1 + \boldsymbol{\mu}_2) \cdot \mathbf{B} | 00 \rangle = -\frac{gqB}{4\mu}, \quad (8)$$

$$\langle 00 | -(\boldsymbol{\mu}_1 + \boldsymbol{\mu}_2) \cdot \mathbf{B} | 10 \rangle = -\frac{gqB}{4\mu}, \quad (9)$$

where the spin eigenstates are labeled by $|SS_z\rangle$ with $S = 0, 1$ and $S_z = 0, \pm 1$. Thus, only the longitudinal component $|10\rangle$ of $S = 1$ can mix with the $S = 0$ state whereas the transverse components $|1\pm 1\rangle$ do not mix with other components. Taking into account the off-diagonal interaction, we should solve a coupled-channel Schrödinger equation. We note that in the case of quarkonia, all the diagonal matrix elements, such as $\langle 00 | -(\boldsymbol{\mu}_1 + \boldsymbol{\mu}_2) \cdot \mathbf{B} | 00 \rangle$, are vanishing: there is no hadronic Zeeman shift.

B. Cylindrical gaussian expansion method

To diagonalize the Hamiltonian (6), we use a cylindrical Gaussian expansion method [35,36], which is a generalization of the Gaussian expansion method [82,83]. In this method, we expand the wave function in terms of a set of Gaussian basis functions, ϕ_n^{CG} , with a different Gaussian range parameter as

$$\psi = \sum_{n=1}^{n_{\text{max}}} c_n \phi_n^{\text{CG}}, \quad (10)$$

where c_n represents the expansion coefficient. The Gaussian basis function in position space is given by [35,36]

$$\phi_n^{\text{CG}}(z, \mathbf{r}_\perp) = N_n e^{-\beta_n r_\perp^2} e^{-\gamma_n z^2}, \quad (11)$$

and the basis function in the momentum space, obtained through the Fourier transformation

$$\begin{aligned} \phi_n^{\text{CG}}(p_z, \mathbf{p}_\perp) &= \frac{1}{(2\pi)^{3/2}} \int_0^\infty dr_\perp r_\perp \int_{-\infty}^\infty dz \int_0^{2\pi} d\phi \\ &\times e^{-ip_z z} e^{-ir_\perp \cdot \mathbf{p}_\perp} \phi_n^{\text{CG}}(z, \mathbf{r}_\perp), \end{aligned} \quad (12)$$

is given by

$$\phi_n^{\text{CG}}(p_z, \mathbf{p}_\perp) = \tilde{N}_n e^{-p_\perp^2/4\beta_n} e^{-p_z^2/4\gamma_n}. \quad (13)$$

The normalization factors of the basis function, N_n and $\tilde{N}_n$, are determined by $\langle \phi_n^{\text{CG}} | \phi_n^{\text{CG}} \rangle = 1$.

In the GEM, two sets of parameters must be determined: the expansion coefficients c_n and the Gaussian range

parameters (β_n, γ_n) . The coefficients c_n are obtained by solving the generalized eigenvalue problem,

$$\mathbf{H}_{q\bar{q}} \mathbf{c} = M_{q\bar{q}} \mathbf{S} \mathbf{c}, \quad (14)$$

where the Hamiltonian matrix elements are given by $H_{q\bar{q},nm} = \langle \phi_n^{\text{CG}} | \hat{H} | \phi_m^{\text{CG}} \rangle$, and the elements of the overlap matrix are $S_{nm} = \langle \phi_n^{\text{CG}} | \phi_m^{\text{CG}} \rangle$. The Gaussian range parameters β_n and γ_n are determined by adopting a geometric progression [82,83]

$$\beta_n = \frac{1}{r_{\perp n}^2}, \quad r_{\perp n} = r_{\perp 1} b^{n-1}, \quad b = \left(\frac{r_{\perp \text{max}}}{r_{\perp 1}} \right)^{\frac{1}{n_{\text{max}}-1}}, \quad (15)$$

$$\gamma_n = \frac{1}{z_n^2}, \quad z_n = z_1 c^{n-1}, \quad c = \left(\frac{z_{\text{max}}}{z_1} \right)^{\frac{1}{n_{\text{max}}-1}}, \quad (16)$$

which reduces them to only four parameters $(\beta_1, \beta_{n_{\text{max}}})$ and $(\gamma_1, \gamma_{n_{\text{max}}})$ to be optimized. We note that the basis functions are non-orthogonal $S_{nm} \neq \delta_{nm}$ and the states are normalized as

$$\langle \psi | \psi \rangle = \sum_{n,m} c_n^* S_{nm} c_m = 1. \quad (17)$$

Since the spin singlet $|00\rangle$ and longitudinal component $|10\rangle$ of the spin triplet mix with each other in the magnetic field, the wave function should take into account the coupled channel. The basis function is given as a linear combination of spin-0 and spin-1 components [35,36]

$$\Psi(r_\perp, z) = \sum_{n=1}^N c_n^{S=0} \Phi_n^{S=0} + \sum_{n=N+1}^{2N} c_n^{S=1} \Phi_n^{S=1}, \quad (18)$$

where

$$\Phi_n^{S=0}(r_\perp, z) = N_n^P e^{-\beta_n^P r_\perp^2} e^{-\gamma_n^P z^2}, \quad (19)$$

$$\Phi_n^{S=1}(r_\perp, z) = N_n^V e^{-\beta_n^V r_\perp^2} e^{-\gamma_n^V z^2}. \quad (20)$$

In this case, we have four range parameters

$$(\beta_n^P, \gamma_n^P), \quad (\beta_n^V, \gamma_n^V). \quad (21)$$

Because of the geometric progression applied, in total we have eight parameters to optimize.

C. Light-front wave functions

After diagonalizing the Hamiltonian (6), the resulting wave functions are transformed into LFWFs [87–94]. These LFWFs are expressed in terms of internal variables that are invariant under Lorentz boosts in the longitudinal

direction, defined as

$$x_i = \frac{p_i^+}{P^+}, \quad \mathbf{k}_{\perp i} = \mathbf{p}_{\perp i} - x_i \mathbf{P}_{\perp}, \quad (22)$$

where $P^\mu = (P^+, P^-, \mathbf{P}_{\perp})$ is the meson's total four-momentum and p_i^μ denotes the four-momentum of the i th constituent quark. Here, we denote on the quark's longitudinal momentum fraction $x \equiv x_q$ and transverse momentum $\mathbf{k}_{\perp} \equiv \mathbf{k}_{\perp q}$.

To obtain the LFWFs, we first perform a change of variables from the momentum-space coordinates $(k_z, \mathbf{k}_{\perp})$ to $(x, \mathbf{k}_{\perp})$. The longitudinal momentum component k_z can be expressed as

$$k_z = \left(x - \frac{1}{2}\right)M_0 + \frac{m_q^2 - m_{\bar{q}}^2}{2M_0}, \quad (23)$$

where the invariant meson mass M_0 is given by

$$M_0^2 = \frac{\mathbf{k}_{\perp}^2 + m_q^2}{x} + \frac{\mathbf{k}_{\perp}^2 + m_{\bar{q}}^2}{1-x}. \quad (24)$$

The radial part of the LFWF is defined as

$$\Phi(x, \mathbf{k}_{\perp}) = \sqrt{2(2\pi)^3} \sqrt{\frac{\partial k_z}{\partial x}} \psi(\mathbf{k}), \quad (25)$$

where $\psi(\mathbf{k})$ is the wave function obtained from the Hamiltonian diagonalization. The Jacobian factor associated with the variable transformation is

$$\frac{\partial k_z}{\partial x} = \frac{M_0}{4x(1-x)} \left[1 - \frac{(m_q^2 - m_{\bar{q}}^2)^2}{M_0^4} \right]. \quad (26)$$

The spin and orbital angular momentum part $\mathcal{R}_{\lambda_q \lambda_{\bar{q}}}^{JJ_z}$ of the meson LFWF is obtained [102–104] through the interaction-independent Melosh transformation [105], which relates the instant-form spin states to the light-front helicity basis. Here, J and J_z denote the total spin and its projection along the quantization axis, respectively, while λ_q and $\lambda_{\bar{q}}$ are the light-front helicities of the quark and antiquark. This procedure assumes that the meson is described in terms of a non-interacting quark–antiquark basis, in line with the Bakamjian–Thomas construction [106–109]. This procedure connects the spin-orbital structure of the ordinary rest-frame wave function, which carries definite quantum numbers J^{PC} , to its light-front representation in a consistent way. The effect of the external magnetic field is first included in the rest-frame wave function, by diagonalizing the Hamiltonian (6). We assume that these modifications are then carried over to the LFWFs via the Melosh transformation. Since our goal is to understand how the LFWFs are modified in the nonrelativistic quark model, this approach captures the dominant effects of the magnetic field such as spin-mixing and quark Landau levels.

For the spin-orbital angular momentum part, it is convenient to work with the covariant form as [110,111]

$$\mathcal{R}_{\lambda_q \lambda_{\bar{q}}}^{JJ_z} = \frac{1}{\sqrt{2\tilde{M}_0}} \bar{u}_{\lambda_q}(p_q) \Gamma_M v_{\lambda_{\bar{q}}}(p_{\bar{q}}), \quad (27)$$

where $\tilde{M}_0 \equiv \sqrt{M_0^2 - (m_q - m_{\bar{q}})^2}$, and $u(p_q)$ and $v(p_{\bar{q}})$ are the light-front Dirac spinors for the quark and antiquark, respectively, with the effect of the Melosh transformation implicitly included through the spinor structure [112]. The vertex function Γ_M depends on the meson type, with

$$\Gamma_P = \gamma_5, \quad \Gamma_V = -\not{\epsilon}(J_z) + \frac{\epsilon \cdot (p_q - p_{\bar{q}})}{M_0 + m_q + m_{\bar{q}}}, \quad (28)$$

where Γ_P and Γ_V correspond to pseudoscalar and vector mesons, respectively. The polarization vectors $\epsilon^\mu(J_z) = (\epsilon^+, \epsilon^-, \epsilon_{\perp})$ are defined by

$$\epsilon^\mu(\pm 1) = \left(0, \frac{2\epsilon_{\perp}(\pm) \cdot \mathbf{P}_{\perp}}{P^+}, \epsilon_{\perp}(\pm)\right), \quad (29)$$

$$\epsilon^\mu(0) = \left(\frac{P^+}{M_0}, \frac{-M_0^2 + \mathbf{P}_{\perp}^2}{M_0 P^+}, \frac{\mathbf{P}_{\perp}}{M_0}\right), \quad (30)$$

with the transverse polarization vectors $\epsilon_{\perp}(\pm 1) = (1, \pm i) / \sqrt{2}$. Lastly, the spin-orbit wave function $\mathcal{R}_{\lambda_q \lambda_{\bar{q}}}^{JJ_z}$ satisfies the normalization condition

$$\sum_{\lambda_q, \lambda_{\bar{q}}} \mathcal{R}_{\lambda_q \lambda_{\bar{q}}}^{JJ_z \dagger} \mathcal{R}_{\lambda_q \lambda_{\bar{q}}}^{JJ_z} = \delta_{JJ'} \delta_{J_z J'_z}. \quad (31)$$

The LFWF of heavy quarkonia in momentum space can be expressed as

$$\Psi_{\lambda_q \lambda_{\bar{q}}}^{JJ_z}(x, \mathbf{k}_{\perp}) = \Phi(x, \mathbf{k}_{\perp}) \mathcal{R}_{\lambda_q \lambda_{\bar{q}}}^{JJ_z}(x, \mathbf{k}_{\perp}), \quad (32)$$

where $\Phi(x, \mathbf{k}_{\perp})$ represents the radial component of the wave function, and $\mathcal{R}_{\lambda_q \lambda_{\bar{q}}}^{JJ_z}(x, \mathbf{k}_{\perp})$ accounts for the spin and orbital angular momentum structure. The LFWF density is defined by summing over the quark and antiquark helicities, [38]

$$\rho(x, \mathbf{k}_{\perp}) = \frac{1}{2(2\pi)^3} \sum_{\lambda_q, \lambda_{\bar{q}}} |\Psi_{\lambda_q \lambda_{\bar{q}}}^{JJ_z}(x, \mathbf{k}_{\perp})|^2. \quad (33)$$

The wave function normalization condition is given by

$$\int dx d^2 \mathbf{k}_{\perp} \rho(x, \mathbf{k}_{\perp}) = 1. \quad (34)$$

We can also project the LFWF density to its longitudinal and transverse components, defined as

$$\int dx f(x) = \int dk_{\perp} f(k_{\perp}) = 1. \quad (35)$$

It is worth noting that, in the vacuum, the longitudinal distribution $f(x)$ corresponds to the PDFs and the $\rho(x, \mathbf{k}_{\perp})$ corresponds to the transverse momentum dependent distribution (TMD) [113,114].

As we will see later, the LFWFs become deformed under strong magnetic fields, and it is useful to introduce the anisotropy parameter defined by

$$\epsilon_{\text{LF}} = \frac{\langle k_{\perp}^2 \rangle}{2\langle k_z^2 \rangle} - 1, \quad (36)$$

where $\langle k_{\perp}^2 \rangle = \langle k_x^2 + k_y^2 \rangle$. The factor of 2 in the denominator ensures a consistent comparison between the two-dimensional transverse and one-dimensional longitudinal momentum components. The longitudinal momentum operator k_z is defined in Eq. (23), and the expectation values $\langle k_{\perp}^2 \rangle$ and $\langle k_z^2 \rangle$ are determined by the Gaussian range parameters γ and β of the wave function. For a spherical Gaussian basis, the relation $\langle k_{\perp}^2 \rangle = 2\langle k_z^2 \rangle$ holds exactly, resulting in $\epsilon_{\text{LF}} = 0$ and reflecting spherical symmetry. However, this relation no longer holds when using a cylindrical Gaussian basis, indicating anisotropic deformation of the LFWFs.

D. Numerical setup

For the parameters of the constituent quark model, in this work, we adopt those used in Ref. [36], where some parameters were determined from lattice QCD simulations of charmonia [115,116], as listed in Table I. These parameters well reproduce the experimental masses [117] of heavy quarkonia in vacuum, as shown in Table II.

We note that the model parameters may depend on the magnetic field. For example, we can expect anisotropies in the confinement and Coulomb potentials (see Refs. [78–80,80,81] for early works). Such effects can be

TABLE I. Parameters of constituent quark model [36].

	m_Q	α [GeV]	$\sqrt{\sigma}$ [GeV]	Λ [GeV ²]	A	C [GeV]
$(c\bar{c})$	1.784	0.4778	0.402	1.020	0.713	-0.5693
$(b\bar{b})$	4.808	0.1322	0.402	1.020	0.531	0

TABLE II. Comparison between the predicted masses [36] and corresponding experimental data [117]. All the units are in GeV.

	Meson	Our	Expt.	Meson	Our	Expt.
$(c\bar{c})$	$\eta_c(1S)$	2.984	2.984	J/ψ	3.097	3.097
	$\eta_c(2S)$	3.669	3.638	$\psi(2S)$	3.707	3.686
$(b\bar{b})$	$\eta_b(1S)$	9.398	9.398	$\Upsilon(1S)$	9.460	9.460
	$\eta_b(2S)$	9.999	9.999	$\Upsilon(2S)$	10.013	10.023
	$\eta_b(3S)$	10.330	...	$\Upsilon(3S)$	10.339	10.355

implemented in the potential model [34]. However, since in the current work our aim is to examine the effects of the magnetic field on the LFWFs, we focus on the parameters without such modification.

III. RESULTS AND DISCUSSION

In this section, we investigate how the structures of charmonia and bottomonia are modified in the presence of strong magnetic fields. For each quarkonium, we first present the magnetic-field-induced mass shifts, recalculated using a denser set of data points than in previous studies [36]. In this work, we specifically focus on examining the corresponding LFWF densities. We also discuss how the transverse and longitudinal momentum distributions change and quantify the deformation using momentum expectation values and an anisotropy parameter. Finally, we discuss a possible relativistic correction, which plays an important role in modifying the longitudinal structure of the LFWFs, found using the light-front Hamiltonian approach [38].

A. Charmonia

The mass spectra of charmonia under strong magnetic fields are shown in Fig. 1. These results were previously discussed in Ref. [36]. Here, we recalculate the magnetic field dependence of the mass spectra using a denser set of data points. One crucial aspect is the optimization procedure for the range parameters, $[\beta_1, \beta_{n_{\text{max}}}]$ and $[\gamma_1, \gamma_{n_{\text{max}}}]$. Since the wave function becomes deformed into a cigar-like shape under strong magnetic fields, the optimal range parameters vary with eB . With improved optimization by Optim.jl [118] for each data point, we confirm the magnetic field dependence of the mass spectra reported in Ref. [36]. Although the nonrelativistic approximation works well when the magnetic field is smaller than the quark mass, we plot the mass spectra up to $eB = 10 \text{ GeV}^2$ for theoretical exploration, where the difference from the relativistic approach can be useful to discuss the relativistic effect.

The masses of the transverse vector charmonia (J/ψ_T and $\psi(2S)_T$) are displayed in the left panel of Fig. 1, showing that they are mainly affected by the quark Landau levels. As a result, their mass shifts increase approximately linearly with the magnetic field. The mass shift for the excited state is larger than for the ground state, because the elongation of the spatial wave function in the z direction leads to a larger expectation value of $\langle r_{\perp}^2 \rangle$.

In contrast, the mass shifts of the pseudoscalar states ($\eta_c(1S)$ and $\eta_c(2S)$) and longitudinal vector states (J/ψ_L and $\psi(2S)_L$) are shown in the right panel of Fig. 1. In this case, the pseudoscalar-vector mixing occurs due to the $-\boldsymbol{\mu}_i \cdot \mathbf{B}$ term, so that we label the resulting states as 1st, 2nd, 3rd, and 4th. The 1st and 2nd states decrease and increase, respectively, as a result of level repulsion with increasing magnetic field. Around $eB = 1\text{--}3 \text{ GeV}^2$, we find avoided crossings such as 2nd–3rd and 3rd–4th. We also note that

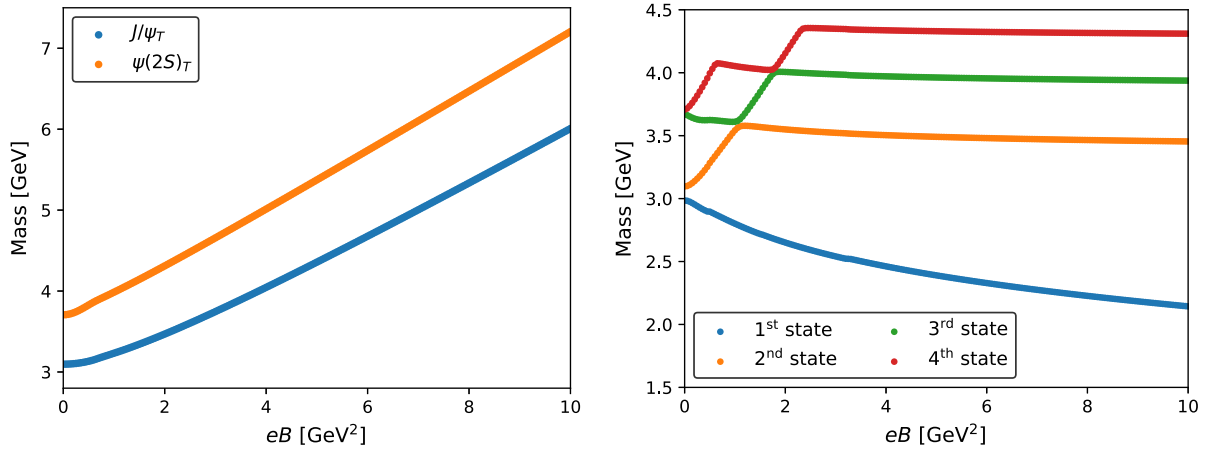


FIG. 1. Magnetic-field dependence of the mass spectra: (left panel) J/ψ_T and $\psi(2S)_T$ states; (right panel) $\eta_c(1S, 2S)$, J/ψ_L , and $\psi(2S)_L$ states. The Landau level primarily affects the masses of the transverse vector charmonia. Level repulsion and avoided crossings appear in the right panel due to state mixing. Note that we plot a dense set of data points so that the curves appear continuous.

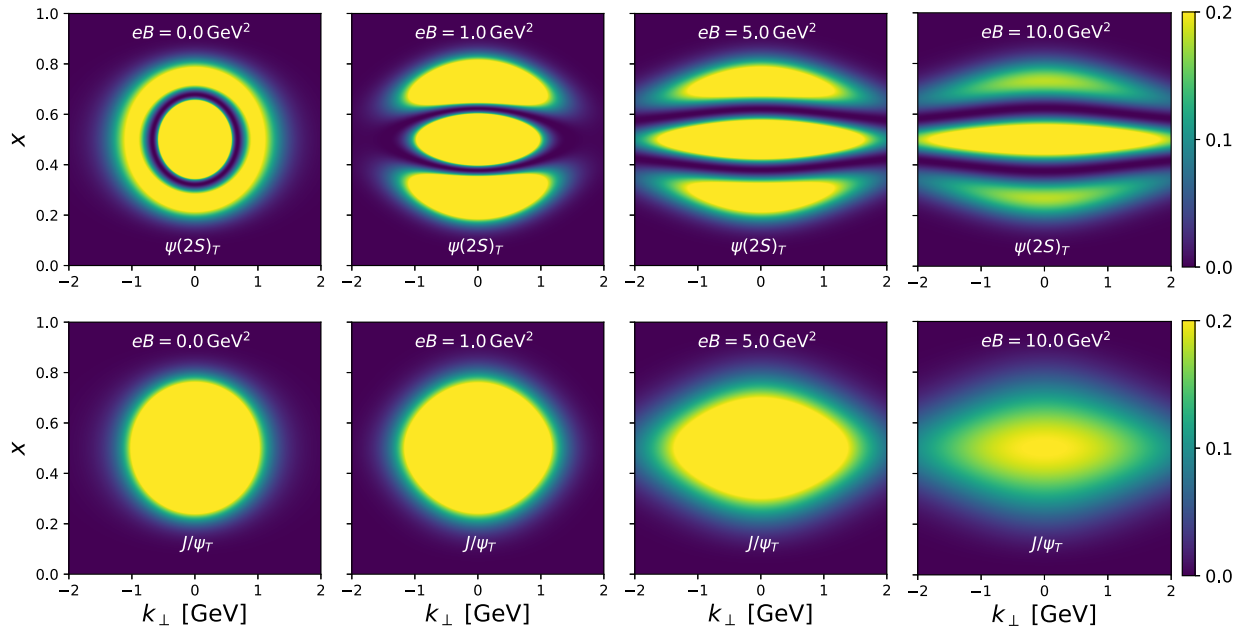


FIG. 2. LFWF densities of the transverse vector charmonia at several different magnetic field strengths: (top panels) J/ψ states and (bottom panels) $\psi(2S)$ states. The densities become more elongated in the transverse direction as the magnetic field increases.

in the strong magnetic field region, the level structure becomes simpler.

While the wave function density in coordinate space was discussed in Ref. [36], here we focus on the LFWF density (33). The LFWF densities for transverse charmonia at various eB values are shown in Fig. 2. Since the LFWF is plotted in momentum space, the transverse momentum distribution becomes more extended with increasing eB , corresponding to the narrowing of the wave function in transverse coordinate space. This transverse momentum broadening originates from the magnetic-field-induced potential proportional to $B^2 r_\perp^2$ in Eq. (6). Importantly,

the nodal structure in the excited states at $eB = 0$ is deformed toward the longitudinal direction. As a result, the density for excited states transforms into three well-separated oblate peaks.

The LFWF densities for the pseudoscalar and longitudinal vector charmonia are shown in Fig. 3, where we present several results for the 1st and 2nd states. The modifications of densities are similar to those in the transverse case, except that a more drastic change appears at certain values of eB . For example, the density of the 2nd state evolves from a single peak into three separated peaks. Such a shape transition indicates that the energy of the

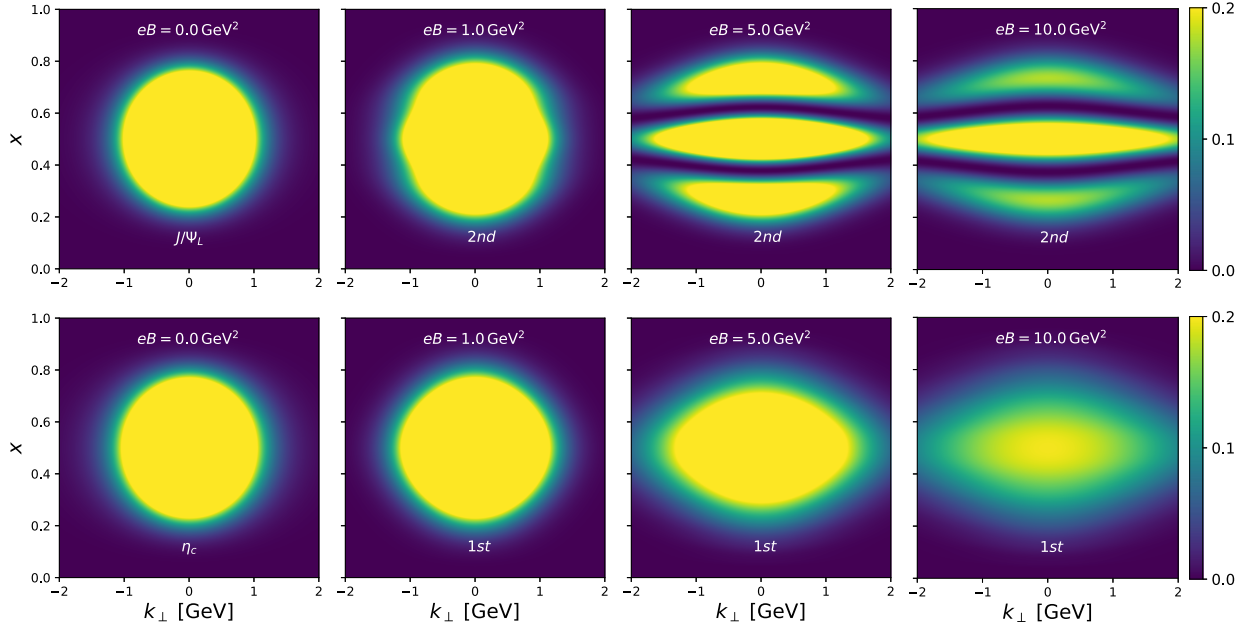


FIG. 3. LFWF densities of pseudoscalar and longitudinal vector charmonia at several different magnetic field strengths: (top panels) 2nd state and (bottom panels) 1st state. The shape of the 2nd state at certain magnetic fields changes significantly due to state mixing and avoided crossing.

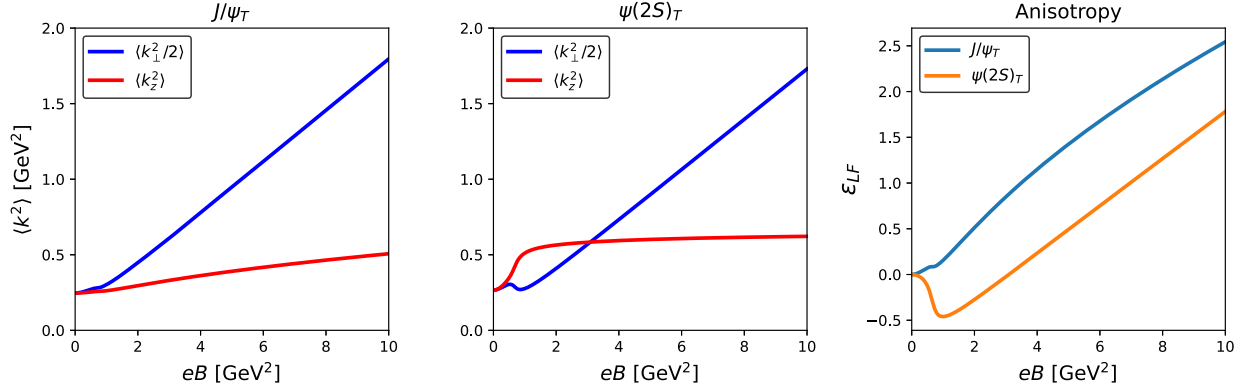


FIG. 4. Magnetic-field dependence of $\langle k^2 \rangle$ and the anisotropy parameter ϵ_{LF} for J/ψ_T and $\psi(2S)_T$ states. Transverse momentum broadening is one of the dominant effects of the magnetic field. The rapid increase in the longitudinal momentum of $\psi(2S)_T$ at low magnetic field arises from the redistribution of the LFWF density.

radially excited state becomes lower than that of the nodeless state, as expected from the mass spectra in Fig. 1.

Since the densities are defined in momentum space, an informative quantity is the expectation value of the momentum squared $\langle k^2 \rangle$, which is similar to the root-mean-square radius $\langle r^2 \rangle$ in coordinate space. In particular, under a magnetic field, it is useful to separate $\langle k^2 \rangle$ into transverse and longitudinal components. The magnetic-field dependence of $\langle \frac{1}{2} k_\perp^2 \rangle$ and $\langle k_z^2 \rangle$ is shown in Fig. 4. Although LFWFs are expressed in terms of the light-front variable x , we can still evaluate $\langle k_z^2 \rangle$ via Eq. (23). At zero magnetic field, $\langle \frac{1}{2} k_\perp^2 \rangle = \langle k_z^2 \rangle$, consistent with the spherical symmetry of the wave function. Under finite magnetic fields, $\langle \frac{1}{2} k_\perp^2 \rangle$ increases

linearly for both J/ψ_T and $\psi(2S)_T$, while $\langle k_z^2 \rangle$ shows only a slight increase. Interestingly, in the weak-field region, nontrivial behavior emerges due to shape transitions. In particular, for $\psi(2S)_T$, $\langle k_z^2 \rangle$ temporarily exceeds $\langle \frac{1}{2} k_\perp^2 \rangle$. The anisotropy parameter ϵ_{LF} , plotted in the right panel of Fig. 4, quantifies the deformation of LFWFs. It clearly shows that the wave functions become more elongated in the transverse direction as the magnetic field increases. This parameter provides a useful measure of rotational symmetry breaking induced by the magnetic field and can serve as a point of comparison with other models. The pseudoscalar and longitudinal vector charmonia exhibit similar trends in their momentum expectation values and anisotropy. However,

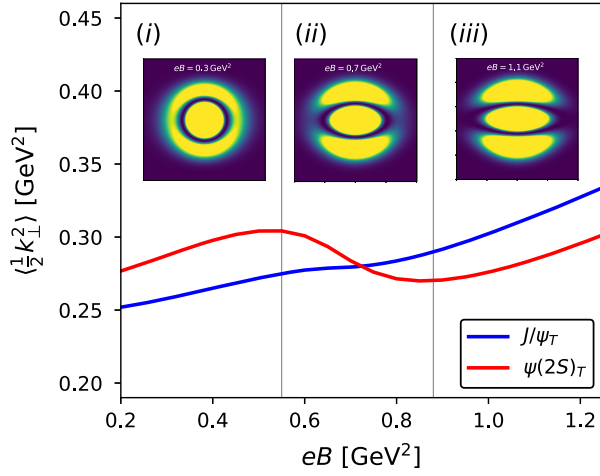


FIG. 5. Close-up view of the weak-field region in Fig. 4, where $\langle \frac{1}{2} k_{\perp}^2 \rangle$ for J/ψ_T and $\psi(2S)_T$ are plotted, and the typical LFWFs of $\psi(2S)_T$ in three different regimes: (i) momentum increase due to elongation of LFWFs with transverse nodes, (ii) momentum decrease due to the node disappearance, and (iii) momentum increase due to elongation of LFWFs with no transverse node.

due to level mixing and state reshuffling, especially in the region of avoided crossings, oscillatory behavior appears in $\langle k_z^2 \rangle$ and ϵ_{LF} . These features reflect the sensitivity of the excited states to the underlying structural reorganization in the presence of the magnetic field.

Figure 5 is a close-up view of the weak-field region of Fig. 4. One of the interesting features is the shape transition

behavior. The magnetic-field dependence of the transverse momentum expectation value, $\langle \frac{1}{2} k_{\perp}^2 \rangle$, for transverse charmonia exhibits three different regimes for excited states: (i) a regime where $\langle \frac{1}{2} k_{\perp}^2 \rangle$ increases due to the elongation of LFWFs (equivalently, the squeezing of spatial wave function) by the Landau level potential, (ii) a regime where the transverse nodes of wave functions gradually disappears, and (iii) a regime where the LFWF is again elongated. Notably, in the regime (ii), the shape transition leads to a pronounced decrease in $\langle \frac{1}{2} k_{\perp}^2 \rangle$, and this effect is more significant for the excited states than for the ground state. This behavior reflects the stronger sensitivity of excited states to the confinement dynamics modified by the magnetic field. We note that this transition behavior can be seen more clearly in bottomonia.

It is also of great importance to examine the projections of the LFWF density onto longitudinal and transverse distributions (35), where the $f(x)$ has important meaning in vacuum as a PDF. Due to the nonrelativistic nature of the quarkonia, the predicted vacuum PDF $f(x)$ from the nonrelativistic quark model [94] is comparable with those other models including the light-front Hamiltonian approach [119].

Figure 6 illustrates the $f(x)$ and $f(k_{\perp})$ distributions for selected charmonium LFWFs at various magnetic field strengths. We find that the $f(x)$ distribution is nearly unchanged for the J/ψ_T ground state and the 1st state. This behavior is expected, as the magnetic-field-dependent potential in the nonrelativistic model does not explicitly involve z or k_z at leading order, whereas the $r_{\perp}^2$ term does, as

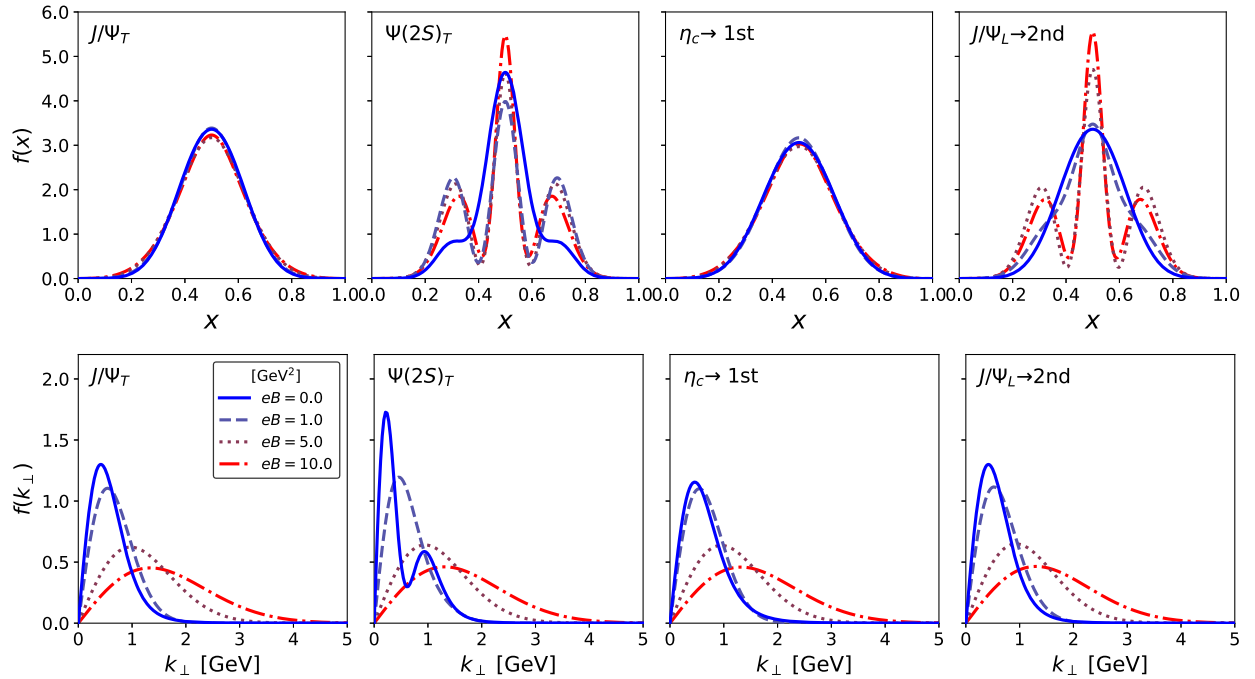


FIG. 6. Longitudinal $f(x)$ and transverse $f(k_{\perp})$ distributions of selected charmonia at several magnetic field strengths. The $f(x)$ is barely modified for the ground states while it is modified for the excited state due to the shape transition. The transition due to avoided crossing changes the distribution significantly. The $f(k_{\perp})$ is generally broaden in the increasing magnetic field.

shown in Eq. (6). In contrast, for the $\psi(2S)_T$ state, the three peaks in the $f(x)$ distribution become more pronounced, while the valleys corresponding to the nodal structure deepen. The 2nd state also exhibits a transition from a single peak to three peaks due to the avoided crossing. These strong modifications of the excited-state $f(x)$ distributions reflect the robustness of their nodal structure.

On the other hand, the $f(k_\perp)$ distribution broadens significantly, as previously mentioned. The distribution extends from $k_\perp = 2$ GeV up to $k_\perp = 4$ GeV, indicating the typical scale of charmonium LFWFs. Moreover, the two-peak structure observed in $\psi(2S)_T$ at low magnetic fields reduces to a single peak at strong fields, marking a clear change in the transverse structure.

Here, we comment on the light-front Hamiltonian approach [38]. The modification of $f(x)$ distribution is particularly important, because the magnetic-field-dependent potential term in the light-front Hamiltonian is written as the form $r_{\perp i}^2/x_i$. This form explicitly depends on x , which is clearly different from the potential with $r_\perp^2$ in the nonrelativistic Hamiltonian (6). To explore this further, we will examine the effect of a relativistic correction to the $f(x)$ distribution in Sec. III C. In addition, in the light-front Hamiltonian approach, the J/ψ density evolves from a single-peak to a two-peak structure resembling a P-wave state. In contrast, the present model shows no such mixing with χ_c states, since we work in relative coordinates and omit center-of-mass couplings to the magnetic field. Identifying the critical magnetic field corresponding to the avoided crossing is essential, for instance, in determining how large the effect of the center-of-mass couplings is, in contrast to the present work.

B. Bottomonia

Following our detailed investigation of charmonium states, we now turn to the bottomonium sector. The bottom

quark has a heavier mass and a smaller electric charge than the case of the charm quark. These features render bottomonia more nonrelativistic and less sensitive to external magnetic fields compared to charmonia. Nonetheless, they provide an important benchmark to test the universality of magnetic-field effects across heavy quarkonia.

The magnetic field dependence of the bottomonium mass spectra is shown in Fig. 7, where we recalculated with a denser set of data points compared with previous works [36]. The transverse vector states $[\Upsilon(1S, 2S, 3S)_T]$ are displayed in the left panel, and the pseudoscalar $[\eta_b(1S, 2S, 3S)]$ and longitudinal vector $[\Upsilon(1S, 2S, 3S)_L]$ states, affected by mixing, are shown in the right panel. In the right panel, we compute the six lowest eigenstates below the $B\bar{B}$ threshold and label them as the 1st–6th states. This allows us to track the avoided crossings and evolution of the eigenstates in the presence of mixing induced by the $-\mu_i \cdot \mathbf{B}$ interaction.

As expected, the transverse states (Υ_T) show linear mass increases with increasing eB , dominated by the quark Landau level effect. The mass shift becomes slightly more pronounced for higher excited states due to their larger transverse size of wave functions. Compared to charmonium, however, the absolute values of shifts are smaller due to the suppression from the larger b -quark mass and smaller charge ($|e_b| = e/3$).

The right panel of Fig. 7 shows the longitudinal sector. In the weak field, we find level repulsions between the $\eta_b(nS)$ and $\Upsilon(nS)_L$ states. The avoided crossings, such as 2nd–3rd and 3rd–4th, occur at slightly higher field strengths compared to charmonium due to the weaker mixing. These crossings are visible across the wide range of magnetic field strengths.

The LFWF densities for the transverse vector bottomonium states are plotted in Fig. 8 for $\Upsilon(1S, 2S, 3S)_T$ across several magnetic field strengths. As in the charmonium

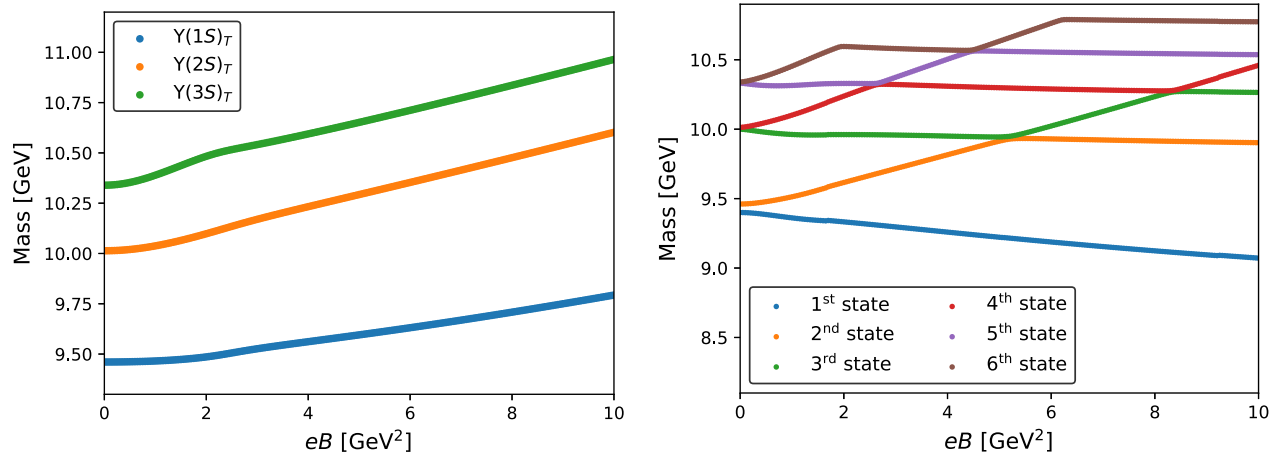


FIG. 7. Magnetic-field dependence of the mass spectra: (left panel) $\Upsilon(1S, 2S, 3S)_T$ states; (right panel) $\eta_b(1S, 2S, 3S)$, $\Upsilon(1S, 2S, 3S)_L$ states. The Landau level primarily affects the masses of the transverse vector bottomonia. Level repulsion and avoided crossings appear in the right panel due to mixing between pseudoscalar and longitudinal vector states. Although the curves appear to overlap near the crossings, they remain distinct.

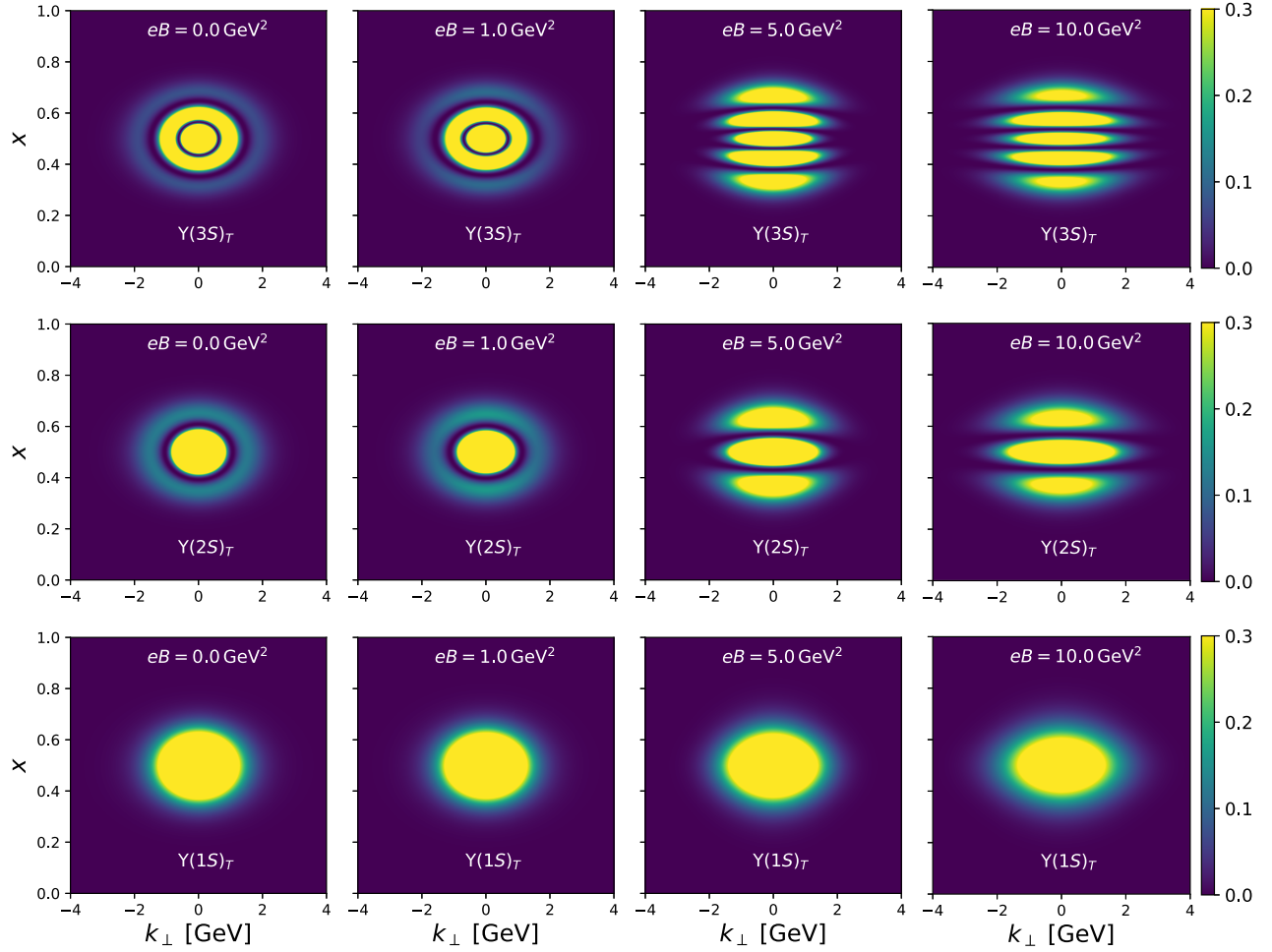


FIG. 8. LFWFs of the transverse vector bottomonia in several different magnetic fields: (top panels) $\Upsilon(3S)_T$, (middle panels) $\Upsilon(2S)_T$, and (bottom panels) $\Upsilon(1S)_T$ states.

case, the densities are shown in momentum space and display transverse broadening due to the magnetic-field-dependent confining potential, which scales as $B^2 r_\perp^2$ and is weighted by q_b^2/m_b , thus making the effect less pronounced. The nodal structures of the excited states change into multiple oblate peaks with increasing magnetic field.

The corresponding LFWF densities for the mixed pseudoscalar and longitudinal vector bottomonia (first and second states) are shown in Fig. 9, where we display only the lowest two states for examples. Their evolution with increasing magnetic field already reveals key features of the magnetic-induced mixing. In particular, the second state develops a three-peak structure at $eB = 10 \text{ GeV}^2$, indicating that the energy of the radial excitation falls below that of the nodeless wave function. This reflects a characteristic reshuffling of state composition: as the magnetic field increases, the eigenstates continuously exchange their internal structure, such as nodal patterns and dominant components. For instance, the third state can be tracked from the mass spectrum evolution as transitioning from a primarily 2S-like

state at low eB , becoming 1S-like near the avoided crossing, and eventually acquiring 3S-like features at higher eB . Such mixing leads to nontrivial deformation patterns in the LFWFs and illustrates how magnetic fields reorganize the internal structure of heavy quarkonia.

To quantify the deformation in momentum space, we compute $\langle \frac{1}{2} k_\perp^2 \rangle$ and $\langle k_z^2 \rangle$ in Fig. 10 for $\Upsilon(1S, 2S, 3S)_T$. We find that the transverse momentum for all three states first increases, then decreases in the weak-field region, and again increases around $eB = 3 \text{ GeV}^2$ as previously discussed for charmonia in Fig. 5. On the other hand, the longitudinal momentum rises rapidly at first, but its increase becomes more gradual as eB continues to grow. Interestingly, the anisotropy parameter ϵ_{LF} shows a negative value up to $eB = 10 \text{ GeV}^2$ for the excited states, indicating that the longitudinal momentum always takes a larger value. This is due to the robustness of their nodal structure. Compared to the charmonium system, it is clear that the deformation is less extreme due to weaker magnetic responses. This provides further evidence of the robustness of the bottomonium system.

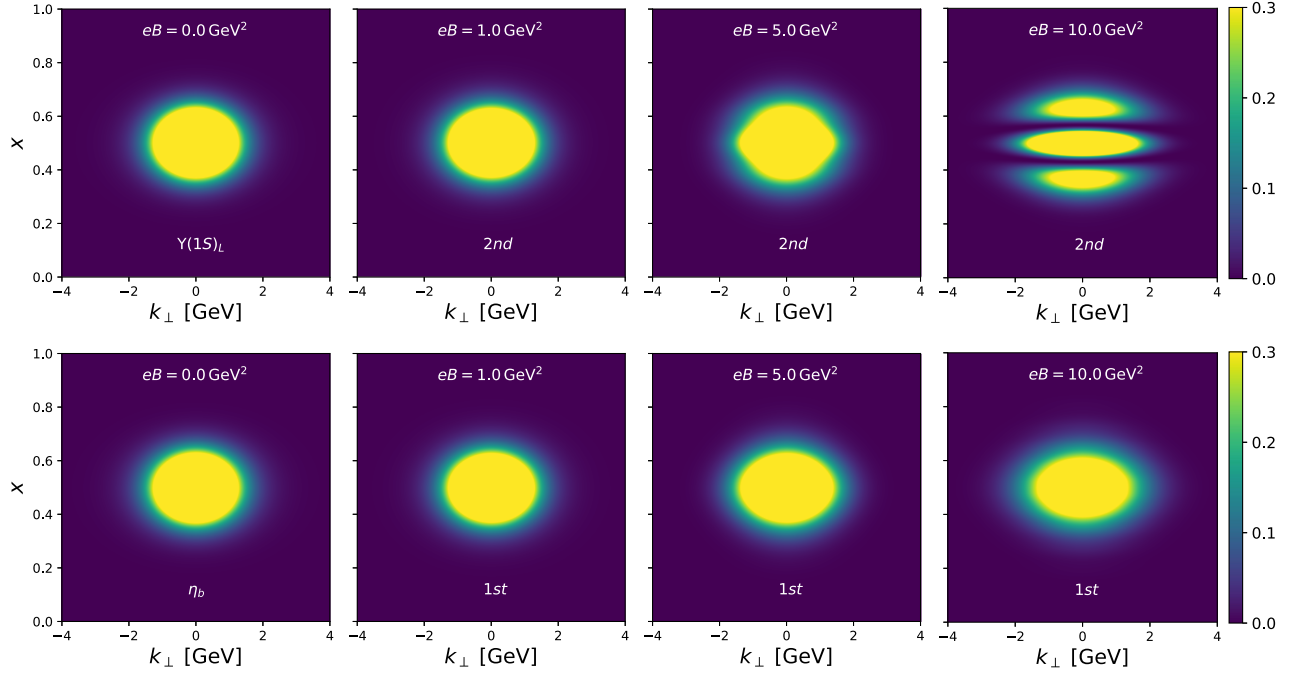


FIG. 9. LFWFs of pseudoscalar and longitudinal vector bottomonia in several different magnetic field strengths: (top panels) the 2nd state and (bottom panels) the 1st state.

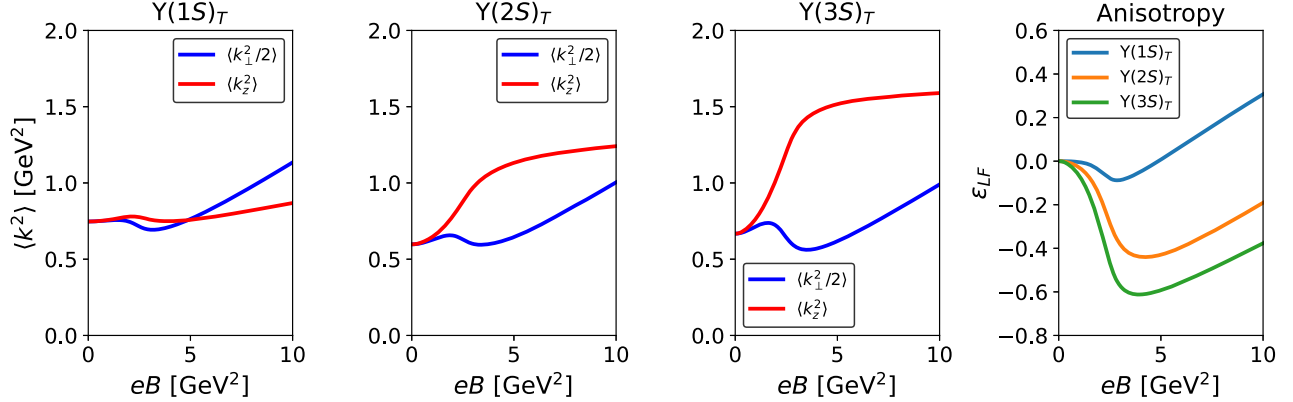


FIG. 10. Magnetic field dependence of $\langle k^2 \rangle$ and the anisotropy parameter ϵ_{LF} for $\Upsilon(1S)_T$, $\Upsilon(2S)_T$, and $\Upsilon(3S)_T$ states.

Lastly, the longitudinal $f(x)$ and transverse $f(k_\perp)$ distributions for bottomonia at various magnetic field strengths are shown in Fig. 11. For the ground state $\Upsilon(1S)_T$, the $f(x)$ distribution remains largely unaffected by the magnetic field. For excited states such as $\Upsilon(2S)_T$, the $f(x)$ distribution becomes narrower in x , consistent with the increase of $\langle k_z^2 \rangle$, and the valleys become deepen and the peaks of the shoulders enhanced. The 2nd state in the longitudinal sector also display evolving multipeak structures due to nodal features and mixing-induced reshaping. The $f(k_\perp)$ distribution extends up to around 3 GeV and stays the same, but the width of the peak is broadened. The two-peak structure for $\Upsilon(2S)_T$ is reduced to a one-peak structure due to the disappearance of the transverse nodes.

C. Relativistic correction

So far, we have demonstrated the deformation of the LFWFs under magnetic field using the nonrelativistic Hamiltonian model as input. One of the dominant effects observed is transverse momentum broadening. However, within this setup, the longitudinal distribution remains largely unmodified for the ground states, as seen in Figs. 6 and 11.

The recent result based on the light-front Hamiltonian approach [38] has shown that the PDFs can also be significantly altered. However, in the nonrelativistic Hamiltonian (6), since the magnetic-field-dependent potential (i.e., the Landau level term) is proportional to $r_\perp^2$, this term leads to the transverse momentum broadening but

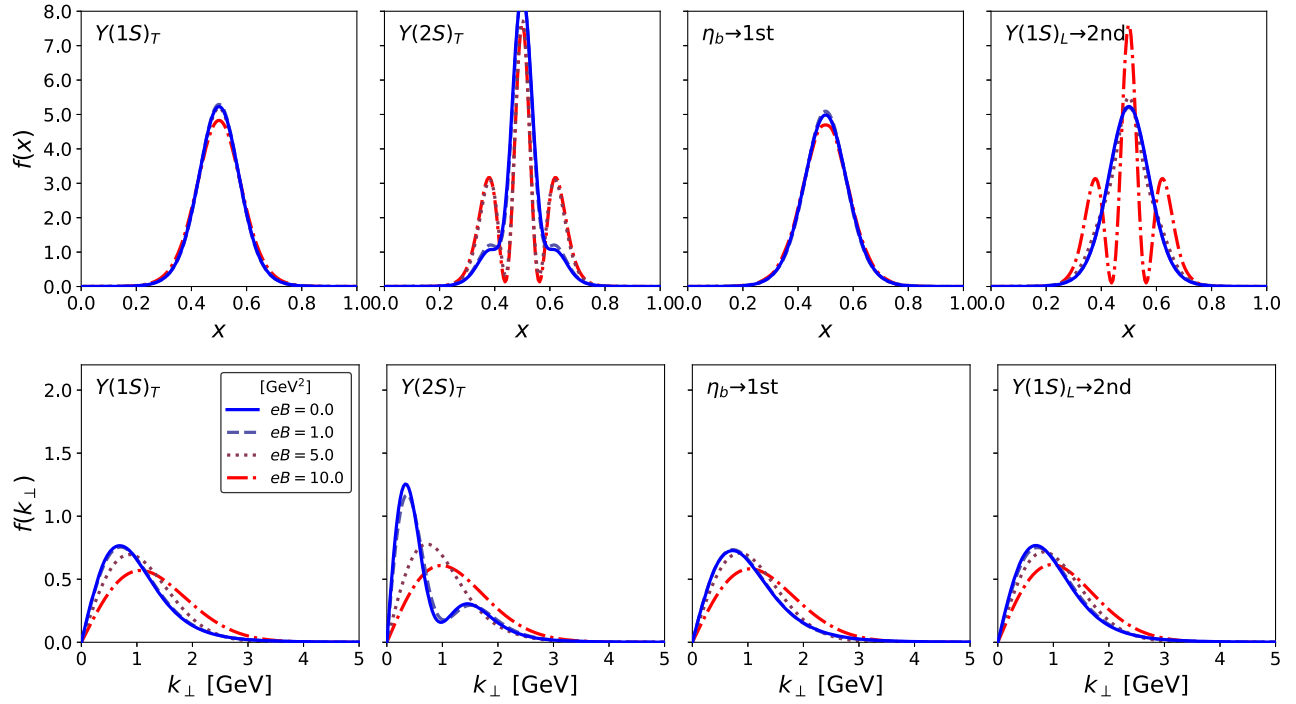


FIG. 11. Longitudinal $f(x)$ and transverse $f(k_\perp)$ distributions of selected bottomonia at several magnetic field strengths.

does not directly contribute to the longitudinal momentum distribution. In this sense, while our numerical results are reasonable, it might be important to discuss possible scenarios of the PDF modifications within our model.

In this work, we phenomenologically introduce a k_z -dependent Landau level term as a relativistic correction (i.e., a higher-order term in the $1/m$ expansion):

$$V_L^\pm = \frac{q^2 B^2}{8\mu} r_\perp^2 \left(1 \pm \frac{k_z^2}{2\mu^2} \right). \quad (37)$$

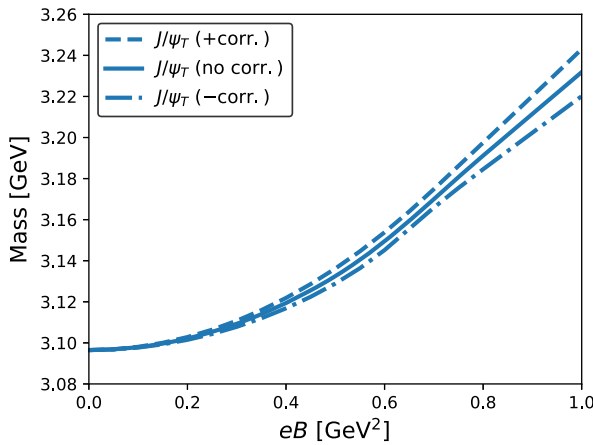


FIG. 12. Magnetic-field dependence of the mass spectrum for J/ψ_T with and without the k_z^2 correction in the Landau level term (37). The correction with a positive (negative) sign slightly increases (decreases) the mass.

This term is not exactly derived from the two-body Hamiltonian, but a possible origin from a one-body Hamiltonian is explained in Appendix A. Although the sign of this correction is suggested to be negative in Appendix A, we treat the sign as a free parameter to explore its impact on the PDFs. We then diagonalize the Hamiltonian with the correction term (37) and focus on only the J/ψ_T as a simple example where the pseudoscalar-vector meson mixing is absent. We note that the correction may be valid only for values of $\sqrt{qB}$ smaller than the quark mass because of the $\sqrt{qB}/m$ expansion.

The impact of the additional k_z^2 term (37) on the mass spectrum is illustrated in Fig. 12. While this term slightly increases or decreases the mass of the J/ψ_T depending on the sign, the linear trajectory remains unaffected. The corresponding LFWFs, shown in Fig. 13, with a positive (negative) correction are now more squeezed (elongated) in the longitudinal direction. One important implication is that the PDFs become slightly narrower or broader, as illustrated in Fig. 14. For comparison, in the light-front Hamiltonian approach [38], the PDFs for charmonium states corresponding to $\eta_c \rightarrow 1\text{st}$ and J/ψ_T^1 at $eB = 1 \text{ GeV}^2$ are slightly broader and narrower, respectively (see Fig. 9 of Ref. [38]). These behaviors may reflect complex relativistic dynamics but might be roughly interpreted as the k_z^2 correction (37). Furthermore, Fig. 15 shows the magnetic-field dependence of $\langle \frac{1}{2} k_\perp^2 \rangle$ and $\langle k_z^2 \rangle$. Notably, the inclusion of the k_z^2 correction

¹In Ref. [38], these states are labeled by a special quantum number $m_{PC} = -1$ and $+1$.

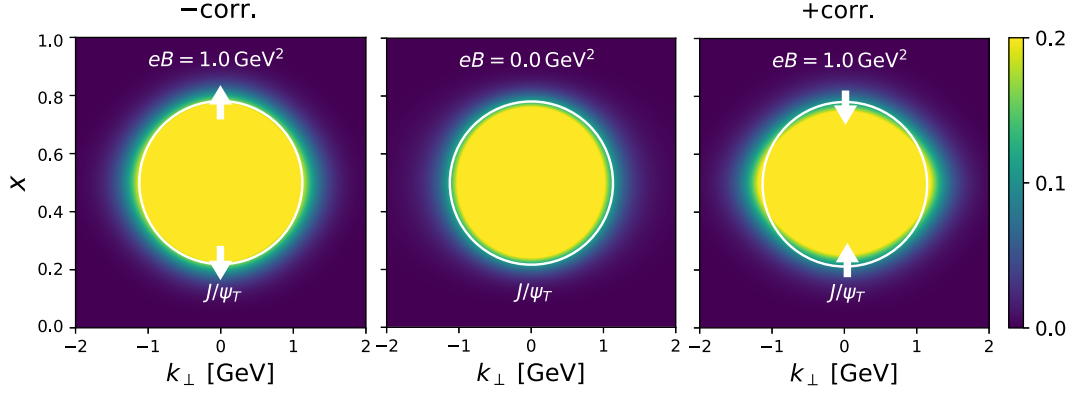


FIG. 13. LFWFs of the J/ψ_T state at $eB = 0$ and 1.0 GeV^2 with a positive or negative correction (37). The longitudinal momentum fraction distribution with a positive (negative) correction becomes more squeezed (elongated).

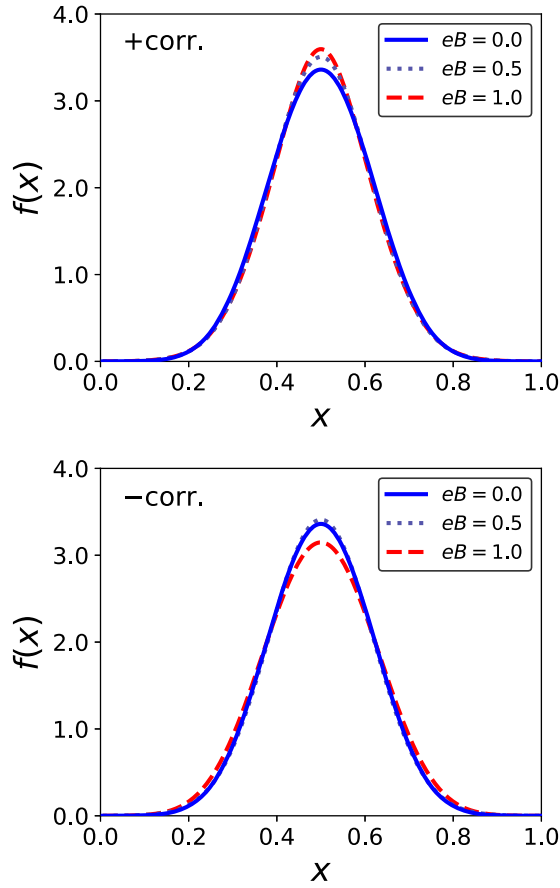


FIG. 14. PDFs of the J/ψ_T states at various magnetic field strengths with a positive or negative correction (37) included. Up to $eB = 1 \text{ GeV}^2$, the PDFs with the positive (negative) correction become slightly narrower (broader) as the magnetic field increases.

with a plus sign reverses the behavior of $\langle k_z^2 \rangle$, changing from an increasing to a decreasing trend with growing magnetic field strength. This reversal is directly reflected in the modification of the PDFs.

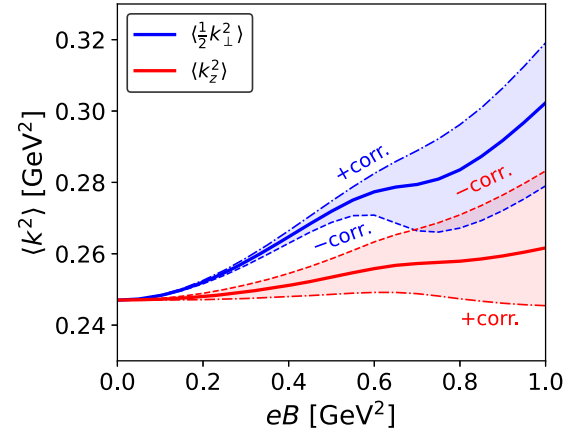


FIG. 15. Magnetic-field dependence of the momentum expectation values, $\langle \frac{1}{2} k_\perp^2 \rangle$ and $\langle k_z^2 \rangle$ for the J/ψ_T state with positive and negative corrections. The correction term (37) significantly modifies $\langle k_z^2 \rangle$ with increasing magnetic field.

IV. CONCLUSION AND OUTLOOK

In this work, we have investigated the structural modifications of heavy quarkonia, charmonia and bottomonia, under strong magnetic fields in the constituent quark model. By solving the two-body Schrödinger equation with the cylindrical Gaussian expansion method [35,36], we obtained precise wave functions that respect the cylindrical symmetry imposed by the magnetic field. Here, these wave functions were then transformed into LFWFs to reveal their momentum-space structure.

Our results show that the presence of a strong magnetic field leads to a clear transverse momentum broadening of the LFWFs, driven by the magnetic-field-induced potential in the nonrelativistic Hamiltonian [35,36]. In contrast, the longitudinal momentum fraction distribution for the ground states remains largely unchanged within this framework. For excited states, however, we find characteristic shape transitions and nodal restructuring near avoided crossings,

which result in significant modifications to the longitudinal momentum distributions. Notably, these avoided crossings play a key role in reorganizing the level structure and internal dynamics of quarkonia under external magnetic fields. To explore the effect beyond this framework, we have introduced a phenomenological relativistic correction to the Landau level term that significantly modifies the longitudinal structure. Furthermore, we expect that bottomonia behave more nonrelativistically and are less sensitive to external magnetic fields compared to charmonia. It is worth noting that the anisotropy parameter may be useful in discussing the structural modifications. This can be compared with relativistic approaches to further explore the role of relativistic effects.

These findings provide a qualitative understanding of how quarkonia structure is modified by magnetic fields within a nonrelativistic Hamiltonian framework [35,36] and help bridge this understanding with results obtained from the light-front Hamiltonian approach [38]. The insights gained here can guide future lattice QCD studies aimed at quantifying magnetic-field effects on hadron structure. Further extensions of this work could include anisotropy of the confining or Coulomb potential observed in lattice QCD simulations [53–55] and explicit time-dependent magnetic fields [120–125] to better approximate the conditions of heavy-ion collisions. Additional effects and phenomena related to quarkonia in strong magnetic fields also remain to be explored for future studies.

ACKNOWLEDGMENTS

A. J. A was partly supported by the RCNP Collaboration Research Network program under project number COREnet 057. This work was supported by the Japan Society for the Promotion of Science (JSPS) KAKENHI (Grant No. JP24K07034).

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: RELATIVISTIC CORRECTIONS

In the main text, we have investigated the impact of a relativistic correction (37). In this Appendix, we explain the origin of this term.

From the Dirac equation in magnetic fields, we can derive the single-particle energy [98,126] [also see, e.g., Eq. (83) of Ref. [30]]:

$$\epsilon_n = \pm \sqrt{p_z^2 + 2n|qB| + m^2}, \quad (\text{A1})$$

where the transverse momentum $p_\perp$ is quantized, and $n = 0, 1, 2, \dots$ is the index of Landau levels. Now, we

consider a nonrelativistic expansion of the single-particle energy as²

$$\epsilon_n = m \sqrt{1 + \frac{p_z^2 + 2n|qB|}{m^2}} \quad (\text{A2})$$

$$= m + \frac{p_z^2 + 2n|qB|}{2m} - \frac{(p_z^2 + 2n|qB|)^2}{8m^3} + \dots \quad (\text{A3})$$

In the last form, the first and second terms are consistent with the mass, kinetic energy, and nonrelativistic Landau levels with the cyclotron frequency $|qB|/m$, which are used in the usual nonrelativistic framework. On the other hand, the third term includes the higher-order corrections proportional to $1/m^3$, which includes the cross term between p_z^2 and $2n|qB|$. Thus, the momentum p_z and the Landau level effect can be correlated as a higher-order relativistic correction. The energy shift from this term is

$$\Delta\epsilon_n = -\frac{4n|qB|p_z^2}{8m^3} = -\frac{p_z^2}{2m^2} \times \frac{n|qB|}{m}. \quad (\text{A4})$$

The factor of $-p_z^2/2m^2$ may be regarded as a correction to the original energy shift of Landau levels.

However, the rigorous derivation of the two-body potential from such a single-particle energy is technically difficult. Thus, the study toward such a direction is limited so far. Under such a situation, to reproduce the narrowing of PDF [found in Ref. [38]], we have phenomenologically introduced the relativistic correction (37). Note that we can also consider the corrections proportional to p_z^4/m^3 or $|qB|^2/m^3$, but they are not relevant for our purpose.

APPENDIX B: POTENTIAL MATRIX ELEMENTS

While the potential matrix elements have been shown in Ref. [36], here we present the analytical expressions for the matrix elements relevant to the Landau-level potential and its relativistic correction. Using the cylindrical Gaussian basis ϕ_n^{CG} , the matrix element, relevant for the Landau term, is given by

$$\begin{aligned} \langle r_\perp^2 \rangle &= \langle \phi_n^{\text{CG}} | r_\perp^2 | \phi_m^{\text{CG}} \rangle \\ &= \frac{N_n N_m \pi^{3/2}}{\beta_{nm}^2 \sqrt{\gamma_{nm}}}, \end{aligned} \quad (\text{B1})$$

where $\beta_{nm} = \beta_n + \beta_m$, $\gamma_{nm} = \gamma_n + \gamma_m$, and the normalization factor

²This expansion holds in the condition of $\sqrt{p_z^2 + 2n|qB|} \ll m$. If $n|qB|$ is large enough, this expansion cannot be used, but it is valid as long as we focus on the lower Landau levels and weak magnetic fields.

$$N_n = \frac{2^{3/4} \gamma_n^{1/4} \sqrt{\beta_n}}{\pi^{3/4}}. \quad (\text{B2})$$

The correction term (37), proportional to $r_\perp^2 k_z^2$, can be computed as

$$\begin{aligned} \langle r_\perp^2 k_z^2 \rangle &= - \left\langle \phi_n^{\text{CG}} \left| r_\perp^2 \frac{\partial^2}{\partial z^2} \right| \phi_m^{\text{CG}} \right\rangle \\ &= \frac{2\gamma_n \gamma_m}{\gamma_{nm}} \left[\frac{N_n N_m \pi^{3/2}}{\beta_{nm}^2 \sqrt{\gamma_{nm}}} \right], \end{aligned} \quad (\text{B3})$$

where k_z^2 corresponds to $-\partial^2/\partial z^2$ in coordinate space.

-
- [1] T. Matsui and H. Satz, J/ψ suppression by quark-gluon plasma formation, *Phys. Lett. B* **178**, 416 (1986).
 - [2] T. Hashimoto, K. Hirose, T. Kanki, and O. Miyamura, Mass shift of charmonium near deconfining temperature and possible detection in lepton pair production, *Phys. Rev. Lett.* **57**, 2123 (1986).
 - [3] D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, The effects of topological charge change in heavy ion collisions: 'Event by event P and CP violation', *Nucl. Phys. A* **803**, 227 (2008).
 - [4] V. Skokov, A. Y. Illarionov, and V. Toneev, Estimate of the magnetic field strength in heavy-ion collisions, *Int. J. Mod. Phys. A* **24**, 5925 (2009).
 - [5] V. Voronyuk, V. D. Toneev, W. Cassing, E. L. Bratkovskaya, V. P. Konchakovski, and S. A. Voloshin, Electromagnetic field evolution in relativistic heavy-ion collisions, *Phys. Rev. C* **83**, 054911 (2011).
 - [6] G. S. Bali, F. Bruckmann, G. Endrodi, Z. Fodor, S. D. Katz, S. Krieg, A. Schafer, and K. K. Szabo, The QCD phase diagram for external magnetic fields, *J. High Energy Phys.* **02** (2012) 044.
 - [7] E. V. Luschevskaya and O. V. Larina, The ρ and A mesons in a strong abelian magnetic field in $SU(2)$ lattice gauge theory, *Nucl. Phys. B* **884**, 1 (2014).
 - [8] Y. Hidaka and A. Yamamoto, Charged vector mesons in a strong magnetic field, *Phys. Rev. D* **87**, 094502 (2013).
 - [9] E. V. Luschevskaya, O. E. Solovjeva, O. A. Kochetkov, and O. V. Teryaev, Magnetic polarizabilities of light mesons in $SU(3)$ lattice gauge theory, *Nucl. Phys. B* **898**, 627 (2015).
 - [10] E. V. Luschevskaya, O. E. Solovjeva, and O. V. Teryaev, Magnetic polarizability of pion, *Phys. Lett. B* **761**, 393 (2016).
 - [11] G. S. Bali, B. B. Brandt, G. Endrődi, and B. Gläsel, Meson masses in electromagnetic fields with Wilson fermions, *Phys. Rev. D* **97**, 034505 (2018).
 - [12] G. S. Bali, B. B. Brandt, G. Endrődi, and B. Gläsel, Weak decay of magnetized pions, *Phys. Rev. Lett.* **121**, 072001 (2018).
 - [13] E. V. Luschevskaya, O. V. Teryaev, D. Y. Golubkov, O. V. Solovjeva, and R. A. Ishkuvatov, Tensor polarizability of the vector mesons from $SU(3)$ lattice gauge theory, *J. High Energy Phys.* **11** (2018) 186.
 - [14] K. Hattori and A. Yamamoto, Meson deformation by magnetic fields in lattice QCD, *Prog. Theor. Exp. Phys.* **2019**, 043B04 (2019).
 - [15] R. Bignell, W. Kamleh, and D. Leinweber, Pion in a uniform background magnetic field with clover fermions, *Phys. Rev. D* **100**, 114518 (2019).
 - [16] R. Bignell, W. Kamleh, and D. Leinweber, Pion magnetic polarisability using the background field method, *Phys. Lett. B* **811**, 135853 (2020).
 - [17] H.-T. Ding, S.-T. Li, A. Tomiya, X.-D. Wang, and Y. Zhang, Chiral properties of $(2+1)$ -flavor QCD in strong magnetic fields at zero temperature, *Phys. Rev. D* **104**, 014505 (2021).
 - [18] H.-T. Ding, S.-T. Li, J.-H. Liu, and X.-D. Wang, Chiral condensates and screening masses of neutral pseudoscalar mesons in thermomagnetic QCD medium, *Phys. Rev. D* **105**, 034514 (2022).
 - [19] H.-T. Ding, J.-B. Gu, S.-T. Li, and R. Thakkar, Chiral condensates and screening masses of neutral pseudoscalar mesons from lattice QCD at physical quark masses, *Phys. Rev. D* **111**, 074513 (2025).
 - [20] G. Endrődi, QCD with background electromagnetic fields on the lattice: A review, *Prog. Part. Nucl. Phys.* **141**, 104153 (2025).
 - [21] D. E. Kharzeev, K. Landsteiner, A. Schmitt, and H.-U. Yee, Strongly interacting matter in magnetic fields: An overview, *Lect. Notes Phys.* **871**, 1 (2013).
 - [22] Á. Mócsy, P. Petreczky, and M. Strickland, Quarkonia in the quark gluon plasma, *Int. J. Mod. Phys. A* **28**, 1340012 (2013).
 - [23] J. O. Andersen, W. R. Naylor, and A. Tranberg, Phase diagram of QCD in a magnetic field, *Rev. Mod. Phys.* **88**, 025001 (2016).
 - [24] X.-G. Huang, Electromagnetic fields and anomalous transports in heavy-ion collisions—A pedagogical review, *Rep. Prog. Phys.* **79**, 076302 (2016).
 - [25] K. Hattori and X.-G. Huang, Novel quantum phenomena induced by strong magnetic fields in heavy-ion collisions, *Nucl. React. Tech.* **28**, 26 (2017).
 - [26] K. Fukushima, Extreme matter in electromagnetic fields and rotation, *Prog. Part. Nucl. Phys.* **107**, 167 (2019).
 - [27] A. Rothkopf, Heavy quarkonium in extreme conditions, *Phys. Rep.* **858**, 1 (2020).
 - [28] J. Zhao, K. Zhou, S. Chen, and P. Zhuang, Heavy flavors under extreme conditions in high energy nuclear collisions, *Prog. Part. Nucl. Phys.* **114**, 103801 (2020).
 - [29] S. Iwasaki, M. Oka, and K. Suzuki, A review of quarkonia under strong magnetic fields, *Eur. Phys. J. A* **57**, 222 (2021).

- [30] K. Hattori, K. Itakura, and S. Ozaki, Strong-field physics in QED and QCD: From fundamentals to applications, *Prog. Part. Nucl. Phys.* **133**, 104068 (2023).
- [31] P. Adhikari *et al.*, Strongly interacting matter in extreme magnetic fields, *Prog. Part. Nucl. Phys.* **146**, 104199 (2026).
- [32] C. S. Machado, F. S. Navarra, E. G. de Oliveira, J. Noronha, and M. Strickland, Heavy quarkonium production in a strong magnetic field, *Phys. Rev. D* **88**, 034009 (2013).
- [33] J. Alford and M. Strickland, Charmonia and bottomonia in a magnetic field, *Phys. Rev. D* **88**, 105017 (2013).
- [34] C. Bonati, M. D'Elia, and A. Rucci, Heavy quarkonia in strong magnetic fields, *Phys. Rev. D* **92**, 054014 (2015).
- [35] K. Suzuki and T. Yoshida, Cigar-shaped quarkonia under strong magnetic field, *Phys. Rev. D* **93**, 051502 (2016).
- [36] T. Yoshida and K. Suzuki, Heavy meson spectroscopy under strong magnetic field, *Phys. Rev. D* **94**, 074043 (2016).
- [37] S. Iwasaki, M. Oka, K. Suzuki, and T. Yoshida, Hadronic Paschen–Back effect, *Phys. Lett. B* **790**, 71 (2019).
- [38] L. Wen, M. Li, Y. Zhou, Y. Li, and J. P. Vary, Relativistic dynamics of charmonia in strong magnetic fields, *Phys. Rev. D* **112**, 014019 (2025).
- [39] S. Cho, K. Hattori, S. H. Lee, K. Morita, and S. Ozaki, QCD sum rules for magnetically induced mixing between η_c and J/ψ , *Phys. Rev. Lett.* **113**, 172301 (2014).
- [40] S. Cho, K. Hattori, S. H. Lee, K. Morita, and S. Ozaki, Charmonium spectroscopy in strong magnetic fields by QCD sum rules: S -wave ground states, *Phys. Rev. D* **91**, 045025 (2015).
- [41] P. Gubler, K. Hattori, S. H. Lee, M. Oka, S. Ozaki, and K. Suzuki, D mesons in a magnetic field, *Phys. Rev. D* **93**, 054026 (2016).
- [42] P. Parui, S. De, A. Kumar, and A. Mishra, QCD sum rule analysis of heavy quarkonium states in magnetized matter: Effects of magnetic and inverse magnetic catalysis, *Phys. Rev. D* **106**, 114033 (2022).
- [43] D. Dudal and T. G. Mertens, Melting of charmonium in a magnetic field from an effective AdS/QCD model, *Phys. Rev. D* **91**, 086002 (2015).
- [44] N. R. F. Braga and L. F. Ferreira, Heavy meson dissociation in a plasma with magnetic fields, *Phys. Lett. B* **783**, 186 (2018).
- [45] N. R. F. Braga and L. F. Ferreira, Quasinormal modes for quarkonium in a plasma with magnetic fields, *Phys. Lett. B* **795**, 462 (2019).
- [46] N. R. F. Braga and R. da Mata, Configuration entropy description of charmonium dissociation under the influence of magnetic fields, *Phys. Lett. B* **811**, 135918 (2020).
- [47] S. S. Jena, B. Shukla, D. Dudal, and S. Mahapatra, Entropic force and real-time dynamics of holographic quarkonium in a magnetic field, *Phys. Rev. D* **105**, 086011 (2022).
- [48] M. Hasan, B. Chatterjee, and B. K. Patra, Heavy quark potential in a static and strong homogeneous magnetic field, *Eur. Phys. J. C* **77**, 767 (2017).
- [49] B. Singh, L. Thakur, and H. Mishra, Heavy quark complex potential in a strongly magnetized hot QGP medium, *Phys. Rev. D* **97**, 096011 (2018).
- [50] M. Hasan and B. K. Patra, Dissociation of heavy quarkonia in a weak magnetic field, *Phys. Rev. D* **102**, 036020 (2020).
- [51] R. Ghosh, A. Bandyopadhyay, I. Nilima, and S. Ghosh, Anisotropic tomography of heavy quark dissociation by using the general propagator structure in a finite magnetic field, *Phys. Rev. D* **106**, 054010 (2022).
- [52] J. Sebastian, L. Thakur, H. Mishra, and N. Haque, Heavy quarkonia in QGP medium in an arbitrary magnetic field, *Phys. Rev. D* **108**, 094001 (2023).
- [53] C. Bonati, M. D'Elia, M. Mariti, M. Mesiti, F. Negro, and F. Sanfilippo, Anisotropy of the quark-antiquark potential in a magnetic field, *Phys. Rev. D* **89**, 114502 (2014).
- [54] C. Bonati, M. D'Elia, M. Mariti, M. Mesiti, F. Negro, A. Rucci, and F. Sanfilippo, Magnetic field effects on the static quark potential at zero and finite temperature, *Phys. Rev. D* **94**, 094007 (2016).
- [55] M. D'Elia, L. Maio, F. Sanfilippo, and A. Stanzione, Confining and chiral properties of QCD in extremely strong magnetic fields, *Phys. Rev. D* **104**, 114512 (2021).
- [56] M. N. Chernodub, Superconductivity of QCD vacuum in strong magnetic field, *Phys. Rev. D* **82**, 085011 (2010).
- [57] M. N. Chernodub, Spontaneous electromagnetic superconductivity of vacuum in strong magnetic field: Evidence from the Nambu–Jona-Lasinio model, *Phys. Rev. Lett.* **106**, 142003 (2011).
- [58] N. Callebaut, D. Dudal, and H. Verschelde, Holographic rho mesons in an external magnetic field, *J. High Energy Phys.* **03** (2013) 033.
- [59] M. Ammon, J. Erdmenger, P. Kerner, and M. Strydom, Black hole instability induced by a magnetic field, *Phys. Lett. B* **706**, 94 (2011).
- [60] M. N. Chernodub, J. Van Doorselaere, and H. Verschelde, Electromagnetically superconducting phase of vacuum in strong magnetic field: Structure of superconductor and superfluid vortex lattices in the ground state, *Phys. Rev. D* **85**, 045002 (2012).
- [61] T. Kojo and N. Su, The quark mass gap in a magnetic field, *Phys. Lett. B* **720**, 192 (2013).
- [62] M. A. Andreichikov, B. O. Kerbikov, V. D. Orlovsky, and Y. A. Simonov, Meson spectrum in strong magnetic fields, *Phys. Rev. D* **87**, 094029 (2013).
- [63] H. Taya, Hadron masses in strong magnetic fields, *Phys. Rev. D* **92**, 014038 (2015).
- [64] K. Hattori, T. Kojo, and N. Su, Mesons in strong magnetic fields: (I) General analyses, *Nucl. Phys.* **A951**, 1 (2016).
- [65] D. Li, M. Huang, Y. Yang, and P.-H. Yuan, Inverse magnetic catalysis in the soft-wall model of AdS/QCD, *J. High Energy Phys.* **02** (2017) 30.
- [66] M. Coppola, D. Gómez Dumm, and N. N. Scoccola, Charged pion masses under strong magnetic fields in the NJL model, *Phys. Lett. B* **782**, 155 (2018).
- [67] T. Kojo, Neutral and charged mesons in magnetic fields: A resonance gas in a non-relativistic quark model, *Eur. Phys. J. A* **57**, 317 (2021).
- [68] M. A. Andreichikov, B. O. Kerbikov, V. D. Orlovsky, and Y. A. Simonov, Neutron in strong magnetic fields, *Phys. Rev. D* **89**, 074033 (2014).
- [69] A. Haber, F. Preis, and A. Schmitt, Magnetic catalysis in nuclear matter, *Phys. Rev. D* **90**, 125036 (2014).

- [70] B.-R. He, Magnetic response of baryon properties in a Skyrme model, *Phys. Rev. D* **92**, 111503 (2015).
- [71] B.-R. He, Skyrme model study of proton and neutron properties in a strong magnetic field, *Phys. Lett. B* **765**, 109 (2017).
- [72] B.-R. He, Magnetic field dependence of delta isobars properties in a Skyrme model, *Phys. Rev. D* **99**, 034019 (2019).
- [73] U. Yakhshiev, H.-C. Kim, and M. Oka, Nucleon and Δ isobar in a strong magnetic field, *Phys. Rev. D* **99**, 054027 (2019).
- [74] M. Coppola, D. Gomez Dumm, and N.N. Scoccola, Diquarks and nucleons under strong magnetic fields in the NJL model, *Phys. Rev. D* **102**, 094020 (2020).
- [75] Y. Chen and W. Jiang, Nucleons and Δ isobars under strong magnetic fields in Skyrme models, *Phys. Rev. C* **107**, 025201 (2023).
- [76] S. Chen, K. Fukushima, and Z. Qiu, Magnetic enhancement of baryon confinement modeled via a deformed Skyrmon, *Phys. Lett. B* **843**, 137992 (2023).
- [77] R. Samanta and W. Broniowski, Magnetic properties of the hadron resonance gas with physical magnetic moments, *Phys. Rev. C* **112**, 045202 (2025).
- [78] V. A. Miransky and I. A. Shovkovy, Magnetic catalysis and anisotropic confinement in QCD, *Phys. Rev. D* **66**, 045006 (2002).
- [79] M. A. Andreichikov, V. D. Orlovsky, and Yu. A. Simonov, Asymptotic freedom in strong magnetic fields, *Phys. Rev. Lett.* **110**, 162002 (2013).
- [80] M. N. Chernodub, QCD string breaking in strong magnetic field, *Mod. Phys. Lett. A* **29**, 1450162 (2014).
- [81] R. Rougemont, R. Critelli, and J. Noronha, Anisotropic heavy quark potential in strongly-coupled $\mathcal{N} = 4$ SYM in a magnetic field, *Phys. Rev. D* **91**, 066001 (2015).
- [82] M. Kamimura, Nonadiabatic coupled-rearrangement-channel approach to muonic molecules, *Phys. Rev. A* **38**, 621 (1988).
- [83] E. Hiyama, Y. Kino, and M. Kamimura, Gaussian expansion method for few-body systems, *Prog. Part. Nucl. Phys.* **51**, 223 (2003).
- [84] G. P. Lepage and S. J. Brodsky, Exclusive processes in perturbative quantum chromodynamics, *Phys. Rev. D* **22**, 2157 (1980).
- [85] S. J. Brodsky, H.-C. Pauli, and S. S. Pinsky, Quantum chromodynamics and other field theories on the light cone, *Phys. Rep.* **301**, 299 (1998).
- [86] M. Diehl, Generalized parton distributions, *Phys. Rep.* **388**, 41 (2003).
- [87] F. Cardarelli, E. Pace, G. Salmè, and S. Simula, Nucleon and pion electromagnetic form-factors in a light front constituent quark model, *Phys. Lett. B* **357**, 267 (1995).
- [88] H.-M. Choi and C.-R. Ji, Mixing angles and electromagnetic properties of ground state pseudoscalar and vector meson nonets in the light-cone quark model, *Phys. Rev. D* **59**, 074015 (1999).
- [89] H.-M. Choi, C.-R. Ji, Z. Li, and H.-Y. Ryu, Variational analysis of mass spectra and decay constants for ground state pseudoscalar and vector mesons in the light-front quark model, *Phys. Rev. C* **92**, 055203 (2015).
- [90] N. Dhiman, H. Dahiya, C.-R. Ji, and H.-M. Choi, Twist-2 pseudoscalar and vector meson distribution amplitudes in light-front quark model with exponential-type confining potential, *Phys. Rev. D* **100**, 014026 (2019).
- [91] A. J. Arifi, H.-M. Choi, C.-R. Ji, and Y. Oh, Mixing effects on $1S$ and $2S$ state heavy mesons in the light-front quark model, *Phys. Rev. D* **106**, 014009 (2022).
- [92] M. Ridwan, A. J. Arifi, and T. Mart, Self-consistent $M1$ radiative transitions of excited B_c and heavy quarkonia with different polarizations in the light-front quark model, *Phys. Rev. D* **111**, 016011 (2025).
- [93] A. J. Arifi, L. Happ, S. Ohno, and M. Oka, Structure of heavy mesons in the light-front quark model, *Phys. Rev. D* **110**, 014020 (2024).
- [94] Q. Wu, Z.-F. Cui, and J. Segovia, Parton distribution functions of ground state mesons composed of c or b quarks, *Phys. Rev. D* **111**, 116023 (2025).
- [95] X. Ji, Parton physics on a Euclidean lattice, *Phys. Rev. Lett.* **110**, 262002 (2013).
- [96] E. Eichten, K. Gottfried, T. Kinoshita, J. Kogut, K. D. Lane, and T.-M. Yan, Spectrum of charmed quark-antiquark bound states, *Phys. Rev. Lett.* **34**, 369 (1975); **36**, 1276(E) (1976).
- [97] T. Barnes, S. Godfrey, and E. S. Swanson, Higher charmonia, *Phys. Rev. D* **72**, 054026 (2005).
- [98] M. H. Johnson and B. A. Lippmann, Motion in a constant magnetic field, *Phys. Rev.* **76**, 828 (1949).
- [99] L. P. Gor'kov and I. E. Dzyaloshinskii, Contribution to the theory of the Mott exciton in a strong magnetic field, *Sov. Phys. JETP* **63**, 449 (1968).
- [100] B. P. Carter, Partial separation of the Schrödinger equation for two charged particles in a magnetic field, *J. Math. Phys. (N.Y.)* **10**, 788 (1969).
- [101] J. E. Avron, I. W. Herbst, and B. Simon, Separation of center of mass in homogeneous magnetic fields, *Ann. Phys. (N.Y.)* **114**, 431 (1978).
- [102] M. V. Terent'ev, On the structure of wave functions of mesons as bound states of relativistic quarks, *Sov. J. Nucl. Phys.* **24**, 106 (1976).
- [103] V. B. Berestetsky and M. V. Terent'ev, Light front dynamics and nucleons from relativistic quarks, *Sov. J. Nucl. Phys.* **24**, 547 (1976).
- [104] V. B. Berestetsky and M. V. Terentev, Nucleon form-factors and dynamics of the light front, *Sov. J. Nucl. Phys.* **25**, 347 (1977).
- [105] H. J. Melosh, Quarks: Currents and constituents, *Phys. Rev. D* **9**, 1095 (1974).
- [106] B. Bakamjian and L. H. Thomas, Relativistic particle dynamics. II, *Phys. Rev.* **92**, 1300 (1953).
- [107] A. J. Arifi, H.-M. Choi, C.-R. Ji, and Y. Oh, Independence of current components, polarization vectors, and reference frames in the light-front quark model analysis of meson decay constants, *Phys. Rev. D* **107**, 053003 (2023).
- [108] A. J. Arifi, H.-M. Choi, and C.-R. Ji, Pseudoscalar meson decay constants and distribution amplitudes up to the twist-4 in the light-front quark model, *Phys. Rev. D* **108**, 013006 (2023).
- [109] A. J. Arifi, H.-M. Choi, and C.-R. Ji, Beyond leading twist: ρ meson decay constants and distribution amplitudes in a

- self-consistent light-front quark model, *Phys. Rev. D* **112**, 033009 (2025).
- [110] W. Jaus, Semileptonic decays of B and D mesons in the light front formalism, *Phys. Rev. D* **41**, 3394 (1990).
- [111] W. Jaus, Relativistic constituent-quark model of electroweak properties of light mesons, *Phys. Rev. D* **44**, 2851 (1991).
- [112] C.-R. Ji, P. L. Chung, and S. R. Cotanch, Light-cone quark-model axial-vector-meson wave function, *Phys. Rev. D* **45**, 4214 (1992).
- [113] C. Lorcé, B. Pasquini, and P. Schweitzer, Unpolarized transverse momentum dependent parton distribution functions beyond leading twist in quark models, *J. High Energy Phys.* **01** (2015) 103.
- [114] H.-M. Choi and C.-R. Ji, Consistency of the pion form factor and unpolarized transverse momentum dependent parton distributions beyond leading twist in the light-front quark model, *Phys. Rev. D* **110**, 014006 (2024).
- [115] T. Kawanai and S. Sasaki, Charmonium potential from full lattice QCD, *Phys. Rev. D* **85**, 091503 (2012).
- [116] T. Kawanai and S. Sasaki, Potential description of charmonium and charmed-strange mesons from lattice QCD, *Phys. Rev. D* **92**, 094503 (2015).
- [117] S. Navas *et al.* (Particle Data Group), Review of particle physics, *Phys. Rev. D* **110**, 030001 (2024).
- [118] P. K. Mogensen and A. N. Riseth, Optim: A mathematical optimization package for Julia, *J. Open Source Software* **3**, 615 (2018).
- [119] J. Lan, C. Mondal, M. Li, Y. Li, S. Tang, X. Zhao, and J. P. Vary, Parton distribution functions of heavy mesons on the light front, *Phys. Rev. D* **102**, 014020 (2020).
- [120] X. Guo, S. Shi, N. Xu, Z. Xu, and P. Zhuang, Magnetic field effect on charmonium production in high energy nuclear collisions, *Phys. Lett. B* **751**, 215 (2015).
- [121] K. Suzuki and S. H. Lee, Delayed versus accelerated quarkonium formation in a magnetic field, *Phys. Rev. C* **96**, 035203 (2017).
- [122] N. Dutta and S. Mazumder, Majorana flipping of quarkonium spin states in transient magnetic field, *Eur. Phys. J. C* **78**, 525 (2018).
- [123] J. Hoelck and G. Wolschin, Electromagnetic field effects on Υ -meson dissociation in PbPb collisions at LHC energies, *Eur. Phys. J. A* **53**, 241 (2017).
- [124] P. Bagchi, N. Dutta, B. Chatterjee, and S. P. Adhya, Dissociation of heavy quarkonium states in a rapidly varying strong magnetic field, *Mod. Phys. Lett. A* **38**, 2350035 (2023).
- [125] S. Iwasaki, D. Jido, M. Oka, and K. Suzuki, Survival probabilities of charmonia as a clue to measure transient magnetic fields, *Phys. Lett. B* **820**, 136498 (2021).
- [126] I. I. Rabi, Das freie Elektron im homogenen Magnetfeld nach der Diracschen Theorie, *Z. Phys.* **49**, 507 (1928).