

Investigation of the $\Delta(1232)$ resonance substructure in the $p\gamma^* \rightarrow \Delta(1232)$ process through helicity amplitudes

A. Kaewsnod^{1,*}, K. Xu,¹ T. Sangkhakrit^{1,2}, Z. Zhao,¹ W. Sreethawong,¹ A. Limphirat^{1,†},
K. Khosonthongkee¹ and Y. Yan^{1,‡}

¹*School of Physics and Center of Excellence in High Energy Physics and Astrophysics,
Suranaree University of Technology, Nakhon Ratchasima 30000, Thailand*

²*KOSEN Institute, King Mongkut's Institute of Technology Ladkrabang, Bangkok 10520, Thailand*



(Received 28 May 2025; accepted 21 January 2026; published 10 February 2026)

The substructure of the $\Delta(1232)$ resonance is studied in the $p\gamma^* \rightarrow \Delta(1232)$ transition via helicity amplitudes within a quark-model framework that treats baryons as three-quark systems and includes both quark-core and meson-cloud contributions. In the calculation of the transition amplitudes, the proton wave function is Lorentz boosted from the proton rest frame to the $\Delta(1232)$ resonance rest frame. The comparison of the calculated amplitudes $A_{1/2}$, $A_{3/2}$, and $S_{1/2}$ with experimental data indicates a non-negligible $L = 2$ admixture in the $\Delta(1232)$, about 80% of the $\Delta(1232)$ wave function in the S wave and 20% in the D wave. In particular, the $S_{1/2}$ amplitude is found to be dominated by the $L = 2$ quark-core component rather than the $L = 0$ part. The results challenge the traditional understanding that the $\Delta(1232)$ baryon is dominantly an $L = 0$ state.

DOI: [10.1103/ptgv-jx85](https://doi.org/10.1103/ptgv-jx85)

I. INTRODUCTION

The investigation of baryon mass spectra and internal structure within three-quark models has been active for several decades [1]. Resonances such as $N(1440)$, $N(1520)$, and $N(1535)$ remain of special interest because of persistent tensions between theoretical predictions and experiment [2–13]. These discrepancies point to limitations of simple quark models and motivate the inclusion of additional dynamical components; for example, meson-baryon configurations and multiquark admixtures [14,15].

One promising approach to resolving these discrepancies involves considering pentaquark and meson-cloud contributions in baryonic models. Recent studies suggest that these nonconventional components are critical for understanding the resonance spectrum more comprehensively [16]. In particular, investigations into the helicity transition amplitudes have provided new insights into the structure of resonances like $N(1440)$, $N(1520)$, and $N(1535)$, revealing potential admixtures of higher orbital momentum and exotic quark configurations.

Meson-cloud contributions have been shown to significantly affect both the transition form factors and helicity amplitudes of several nucleon excitations. For example, in [17], the kaon cloud was found to enhance the estimates of radiative decays of strange baryons like $\Sigma(1385)$ and $\Xi(1530)$. This study emphasizes that, while the pion cloud dominates processes like the $\gamma^*N \rightarrow \Delta(1232)$ transition, the kaon cloud plays an essential role in strange baryons. These contributions extend to the Dalitz decays, demonstrating the profound impact of mesonic degrees of freedom on the form factors of baryon resonances.

Further insight into the role of meson cloud in the $\gamma^*N \rightarrow \Delta(1232)$ transition is provided by [18], which presents a global parametrization of the neutron electric form factor and the quadrupole form factors, G_E and G_C . The study highlights the dominance of the pion cloud in these transitions, particularly at low Q^2 , and emphasizes that the quadrupole form factors exhibit large radii due to the extended structure of the pion cloud. These results suggest that meson-cloud effects, especially from the pion, are essential for accurately describing the $\Delta(1232)$ resonance.

A recent comprehensive review [19] summarizes the status of electromagnetic $N \rightarrow N^*$ and $N \rightarrow \Delta$ transitions and notes that both valence-quark structure (including possible D-state components in the Δ wave function) and meson-cloud contributions play roles in reproducing quadrupole form factors. The review further indicates that pion-cloud effects are typically important at low Q^2 , while valence D-wave components are often a smaller contribution to the E2/C2 observables.

*Contact author: a.kaewsnod@gmail.com

†Contact author: ayut@g.sut.ac.th

‡Contact author: yupeng@g.sut.ac.th

While the $\Delta(1232)$ is often described as an S -wave predominant three-quark (q^3) state, experimental data reveal nonzero longitudinal helicity transition amplitudes ($S_{1/2}$) for the $\gamma^* N \rightarrow \Delta(1232)$ process. This observation implies that the $\Delta(1232)$ may not be a pure S -wave state and could involve significant higher orbital (D -wave state) components, as well as contributions from meson-cloud effects and possible multi-quark admixtures [20,21].

The study of D -wave state admixtures in the wave functions of nucleons and $\Delta(1232)$ has provided valuable insight. It has been demonstrated that, while D -wave state components have minimal effects on the elastic form factors of the proton and the magnetic form factor of the neutron, they significantly influence the electric form factor of the neutron [22]. Moreover, in the $\Delta(1232) - N$ transition, the D -wave state admixtures strongly affect the electric form factor, particularly in the ratio R_{EM} , which is highly sensitive to these components. These observations suggest that D -wave state admixtures are essential for understanding the full structure of the $\Delta(1232)$ resonance.

Further, the inclusion of D -wave state components in the $\Delta(1232)$ wave function has been shown necessary for describing the electric quadrupole and magnetic octupole form factors of the Δ resonance. These contributions are key to explaining the $\gamma^* N \rightarrow \Delta$ quadrupole transition and provide a robust framework for analyzing lattice QCD results [23]. At high Q^2 values, the valence quark contributions dominate, while at low Q^2 , pion cloud effects become more relevant.

The present study focuses on the substructure of the $\Delta(1232)$ resonance in the $p\gamma^* \rightarrow \Delta(1232)$ process, using helicity amplitudes as a key probe. We apply a quark model framework that incorporates both the quark-core and meson-cloud contributions, considering baryons as composite three-quark systems.

The structure of this paper is as follows. In Sec. II, we present the wave functions of the proton, which is Lorentz-boosted from the proton rest frame to the $\Delta(1232)$ resonance rest frame, and $\Delta(1232)$ used in this study. We also introduce the helicity amplitudes $A_{1/2}$, $A_{3/2}$, and $S_{1/2}$ for the $p\gamma^* \rightarrow \Delta(1232)$ transition, accompanied by the corresponding quark-core and meson-cloud contributions illustrated in Figs. 1 and 2, respectively. The results of the helicity amplitudes, incorporating contributions from both diagrams in Sec. II, are presented in Sec. III. Finally, conclusions are summarized in Sec. IV.

II. HELICITY AMPLITUDES OF THE $p\gamma^* \rightarrow \Delta(1232)$ PROCESS

The algebraic structure of the proton and $\Delta(1232)$ baryons is governed by the product of standard spin, flavor, and color symmetries: $SU_f(2) \otimes SU_s(2) \otimes SU_c(3)$. We

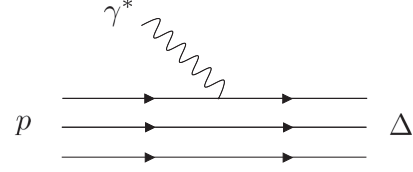


FIG. 1. Photon-quark interaction diagram of the $p\gamma^* \rightarrow \Delta(1232)$ transition, where the photon directly interacts with the quark core of the proton, leading to the production of a $\Delta(1232)$ resonance through the photoproduction process.

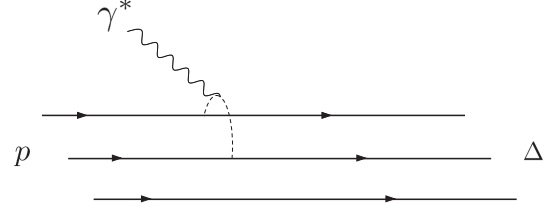


FIG. 2. Photon-meson-cloud interaction diagram of the $p\gamma^* \rightarrow \Delta(1232)$ transition, depicted as a meson-in-flight diagram, where the photon interacts with the virtual mesons which couple with the quarks inside the proton, leading to the photoproduction of the $\Delta(1232)$ resonance.

propose that the spatial wave function of the $\Delta(1232)$ consists of a fully symmetric component in the $L = 0$ state, along with two additional components in the $L = 2$ state, that is, the symmetric (Spin $S = \frac{3}{2}$) and mixed symmetric ($S = \frac{1}{2}$). Within the three-quark model, the full proton and $\Delta(1232)$ wave functions, in the baryon rest frame, are written as

$$\begin{aligned} \Psi_p(\mathbf{k}_1\mu_1, \mathbf{k}_2\mu_2, \mathbf{k}_3\mu_3; \mathbf{P} = \mathbf{0}, S_z) &= \delta^{(3)}\left(\sum_i \mathbf{k}_i\right) \frac{1}{\sqrt{2}} \psi_{p,0}^S (\chi^\lambda \phi^\lambda + \chi^\rho \phi^\rho) \psi^c, \\ \Psi_\Delta(\mathbf{p}_1's_1', \mathbf{p}_2's_2', \mathbf{p}_3's_3'; \mathbf{P}' = \mathbf{0}, S'_z) &= \delta^{(3)}\left(\sum_i \mathbf{p}_i'\right) \left[B\psi_{\Delta,0}^S \chi^S + C\psi_{\Delta,2}^S \otimes \chi^S \right. \\ &\quad \left. + D\frac{1}{\sqrt{2}} (\psi_{\Delta,2}^\lambda \otimes \chi^\lambda + \psi_{\Delta,2}^\rho \otimes \chi^\rho) \right] \phi^S \psi^c, \quad (1) \end{aligned}$$

where ψ^c is the color wave function, and χ^α and ϕ^α are spin and flavor wave functions, respectively, of symmetry type α . $\mathbf{k}_i\mu_i$ and $\mathbf{p}_i's_i'$ are the momenta and spins of quarks in proton and $\Delta(1232)$, respectively. The spatial wave functions $\psi_{X,L}^\alpha$ describe the X baryon with orbital angular momentum L and symmetry α , where $\alpha = S$ (symmetric), λ (lambda type), or ρ (rho type). The $\Delta(1232)$ wave function is normalized with

$$B^2 + C^2 + D^2 = 1. \quad (2)$$

The spatial wave functions for the proton and $\Delta(1232)$ baryons are represented in a harmonic oscillator basis in Jacobi coordinates, as detailed in Refs. [16,24]. We have

$$\begin{aligned} \psi_{p,0}^S &= \sum_{N=0,2,4,\dots} a_N \Psi_{N00}^S, \\ \psi_{\Delta,0}^S &= \sum_{N=0,2,4,\dots} b_N \Psi_{N00}^S, \\ \psi_{\Delta,2}^S &= \sum_{N=2,4,6,\dots} c_N \Psi_{N2M}^S, \\ \psi_{\Delta,2}^{\lambda/\rho} &= \sum_{N=2,4,6,\dots} d_N \Psi_{N2M}^{\lambda/\rho}, \end{aligned} \quad (3)$$

with the normalization conditions

$$\begin{aligned} \sum_{N=0,2,4,\dots} (a_N)^2 &= \sum_{N=0,2,4,\dots} (b_N)^2 = 1, \\ \sum_{N=2,4,6,\dots} (c_N)^2 &= \sum_{N=2,4,6,\dots} (d_N)^2 = 1. \end{aligned} \quad (4)$$

Here, Ψ_{NML}^α represents the harmonic oscillator basis in Jacobi coordinates, with N as the principal quantum number, L as the angular momentum, and M as the angular momentum projection. The symmetry type of the baryon is denoted by α . The spatial wave function of the proton, $\psi_{p,0}^S$, is derived by fitting its electric form factor to experimental data (see Ref. [20] and Appendix for more details). In this work, we fit the $\Delta(1232)$ spatial wave function components, $\psi_{\Delta,0}^S$, $\psi_{\Delta,2}^S$, and $\psi_{\Delta,2}^{\lambda/\rho}$, to experimental data on its helicity amplitudes, $A_{1/2}$, $A_{3/2}$, and $S_{1/2}$ together. The results are presented in Sec. III.

Because the helicity amplitudes are defined in the resonance rest frame while the three-quark proton wave function is specified in the proton rest frame (canonical coordinates), the proton state must be Lorentz boosted when evaluating transition amplitudes. The boost maps quark four-momenta $k_1^\mu + k_2^\mu + k_3^\mu = (M_0, \mathbf{0})$ in the proton rest frame to $p_1^\mu + p_2^\mu + p_3^\mu = (E, -\mathbf{k})$ in the moving proton frame, where $M_0 = \sum_i E_{\mathbf{k}_i}$, $E = \sum_i E_{\mathbf{p}_i}$, and $\mathbf{k}$ is the photon three-momentum. In the canonical basis the boosted proton wave function becomes [25]

$$\begin{aligned} \Psi_p(\mathbf{p}_1 s_1, \mathbf{p}_2 s_2, \mathbf{p}_3 s_3; \mathbf{P} = -\mathbf{k}) \\ = \delta^{(3)}\left(\sum_i \mathbf{p}_i + \mathbf{k}\right) \sqrt{\frac{E_{\mathbf{k}_1} E_{\mathbf{k}_2} E_{\mathbf{k}_3} E}{E_{\mathbf{p}_1} E_{\mathbf{p}_2} E_{\mathbf{p}_3} M_0}} \\ \times D_{s_1 \mu_1}^\dagger[R(\mathbf{k}_1)] D_{s_2 \mu_2}^\dagger[R(\mathbf{k}_2)] D_{s_3 \mu_3}^\dagger[R(\mathbf{k}_3)] \\ \times \Psi_p(\mathbf{k}_1 \mu_1, \mathbf{k}_2 \mu_2, \mathbf{k}_3 \mu_3; \mathbf{P} = 0), \end{aligned} \quad (5)$$

where $D_{s_i \mu_i}[R]$ represent the Wigner rotations acting on the i th quark, accounting for the spin rotation induced by the Lorentz boost. $D_{s_i \mu_i}[R]$ are explicitly given by [26–29]

$$D_{s_i \mu_i}[R(\mathbf{k}_i)] = \cos \frac{\omega_i}{2} - i(\hat{\mathbf{n}}_i \cdot \boldsymbol{\sigma}) \sin \frac{\omega_i}{2}, \quad (6)$$

where ω_i and $\hat{\mathbf{n}}_i$ are the rotation angle and direction, respectively, defined by

$$\tan \frac{\omega_i}{2} = \frac{|\mathbf{k}_i^G \times \mathbf{P}^G|}{1 + \mathbf{k}_i^G \cdot \mathbf{P}^G}, \quad \hat{\mathbf{n}}_i = \frac{\mathbf{k}_i^G \times \mathbf{P}^G}{|\mathbf{k}_i^G \times \mathbf{P}^G|} \quad (7)$$

in terms of the Gibbs vectors,

$$\begin{aligned} \mathbf{k}_i^G &= \frac{\mathbf{k}_i}{E_{\mathbf{k}_i} + m}, \\ \mathbf{P}^G &= \frac{\mathbf{P}}{E + M_0}, \end{aligned} \quad (8)$$

with

$$\begin{aligned} \mathbf{P} &= \sum_{i=1}^3 \mathbf{p}_i, \\ E_{\mathbf{k}_i} &= \sqrt{m^2 + \mathbf{k}_i^2}, \\ E &= \sum_{i=1}^3 \sqrt{m^2 + \mathbf{p}_i^2}, \\ M_0 &= \sum_{i=1}^3 \sqrt{m^2 + \mathbf{k}_i^2}. \end{aligned} \quad (9)$$

We compute the helicity amplitudes $A_{1/2}$, $A_{3/2}$, and $S_{1/2}$ in the resonance rest frame using the quark-core wave functions above. The amplitudes are defined by

$$\begin{aligned} A_{1/2} &= \sum \frac{3}{\sqrt{2K}} \int \prod_{i=1}^3 d\mathbf{p}_i d\mathbf{p}'_i \Psi_\Delta^* \left(\{\mathbf{p}'_i s'_i\}; \mathbf{P}' = \mathbf{0}, S'_z = \frac{1}{2} \right) (\sqrt{Z_p} \sqrt{Z_\Delta} T_{q^3}^+ + T_{MC}^+) \Psi_p \left(\{\mathbf{p}_i s_i\}; \mathbf{P} = -\mathbf{k}, S_z = -\frac{1}{2} \right), \\ A_{3/2} &= \sum \frac{3}{\sqrt{2K}} \int \prod_{i=1}^3 d\mathbf{p}_i d\mathbf{p}'_i \Psi_\Delta^* \left(\{\mathbf{p}'_i s'_i\}; \mathbf{P}' = \mathbf{0}, S'_z = \frac{3}{2} \right) (\sqrt{Z_p} \sqrt{Z_\Delta} T_{q^3}^+ + T_{MC}^+) \Psi_p \left(\{\mathbf{p}_i s_i\}; \mathbf{P} = -\mathbf{k}, S_z = \frac{1}{2} \right), \\ S_{1/2} &= \sum \frac{3}{\sqrt{2K}} \int \prod_{i=1}^3 d\mathbf{p}_i d\mathbf{p}'_i \Psi_\Delta^* \left(\{\mathbf{p}'_i s'_i\}; \mathbf{P}' = \mathbf{0}, S'_z = \frac{1}{2} \right) (\sqrt{Z_p} \sqrt{Z_\Delta} T_{q^3}^0 + T_{MC}^0) \Psi_p \left(\{\mathbf{p}_i s_i\}; \mathbf{P} = -\mathbf{k}, S_z = \frac{1}{2} \right). \end{aligned} \quad (10)$$

Here, the symbol $\sum$ indicates a shorthand for summing over all spin, flavor, and color indices of the three-quark states. The real photon momentum is represented as $K = \frac{M_\Delta^2 - M_p^2}{2M_\Delta}$, where M_Δ and M_p refer to the rest masses of the $\Delta(1232)$ resonance and the proton, respectively. $T_{q^3}^\lambda$ represents the transition corresponding to the interaction between the photon and quark i in the electromagnetic process $\gamma q_i \rightarrow q'_i$, as illustrated by the quark line diagram in Fig. 1. T_{MC}^λ denotes the transition where the photon interacts with the meson cloud, which subsequently couples to quark i , as depicted in the quark line diagram in Fig. 2. The superscript λ refers to the helicity, where $\lambda = 0$ represents the longitudinal helicity amplitude, and $\lambda = +1$ corresponds to the transverse helicity amplitude.

The parameters Z_p and Z_Δ in Eq. (10) represent the wave function renormalization constants for the proton and the Δ , respectively. These constants ensure that the physical electric form factors satisfy the charge normalization condition. This condition is defined as the sum of the quark-core contribution (Fig. 1) and the meson-cloud contribution (Fig. 2), expressed as follows:

$$\begin{aligned} G_E^p(0) &= Z_p G_{QC}^p(0) + G_{MF}^p(0) = 1 \\ G_E^\Delta(0) &= Z_\Delta G_{QC}^\Delta(0) + G_{MF}^\Delta(0) = 1, \end{aligned} \quad (11)$$

with the core contributions as

$$G_{QC}^p(0) = G_{QC}^\Delta(0) = 1. \quad (12)$$

Our calculation yields $Z_p = 0.95$ and $Z_\Delta = 1$, corresponding to $G_{MF}^p(0) = 0.05$ and $G_{MF}^\Delta(0) = 0$. The vanishing of $G_{MF}^\Delta(0)$ arises because the transition matrix of the meson-in-flight operator between two fully symmetric isospin $I = 3/2$ states is zero. For more details, we refer the reader to our previous works about the study of $N(1440)$, $N(1520)$, and $N(1535)$ [20,21].

The quark-core contribution to the electromagnetic transition is calculated in the impulse approximation, where the photon interacts with only one quark, as illustrated in Fig. 1. For instance, when the photon interacts with the third quark, we have

$$T_{q^3}^\lambda = Q_3 \bar{u}_{s'}(p') \gamma^\mu u_s(p) \epsilon_\mu^\lambda(k) \langle q_1 q_2 | q'_1 q'_2 \rangle. \quad (13)$$

Here, Q_3 represents the electric charge of the third quark, $u_s(p)$ is the Dirac spinor with spin s and momentum p , the polarization vector of the photon (ϵ_μ^λ) is defined as $\epsilon_\mu^+ = (0, -\frac{1}{\sqrt{2}}, -\frac{i}{\sqrt{2}}, 0)$ and $\epsilon_\mu^0 = (\frac{K}{Q}, 0, 0, \frac{\omega}{Q})$, where ω is a photon energy. The transition amplitude for the longitudinal helicity component, $S_{1/2}$, is evaluated following the formalism described in Refs. [30–32]. In this work, we adopt a current quark mass for the up (u) and down (d) quarks, setting $m = 5$ MeV.

For the meson-cloud contribution to the $p\gamma^* \rightarrow \Delta(1232)$ transition, we consider only the leading-order process, as illustrated by the meson-in-flight quark line diagram in Fig. 2. Higher-order corrections are assumed to be suppressed and negligible. Within the framework of the chiral quark model, the dynamics of the interaction are governed by the effective Lagrangians [33],

$$\begin{aligned} \mathcal{L}_{qq\pi} &= \frac{1}{2F} \partial_\mu \pi_i \bar{\psi} \gamma^\mu \gamma^5 \tau^i \psi, \\ \mathcal{L}_{\pi\pi\gamma} &= -e \epsilon_{3jk} \pi_j \partial^\nu \pi_k A_\nu, \end{aligned} \quad (14)$$

where $\mathcal{L}_{qq\pi}$ describes the interaction between quarks and pions, with $F = 88$ MeV being the pion decay constant, τ^i the Pauli matrices acting in the isospin space, and π_i the pion fields. $\mathcal{L}_{\pi\pi\gamma}$ describes the interaction between photon and pion fields, where A_ν is the photon field, and ϵ_{3jk} is the Levi-Civita symbol that enforces the symmetry properties of the pion-photon interaction.

III. RESULTS

We fit experimental data of the helicity amplitudes, $A_{1/2}$, $A_{3/2}$, and $S_{1/2}$ of the $p\gamma^* \rightarrow \Delta(1232)$ transition illustrated in Figs. 1 and 2, where the length parameter of the

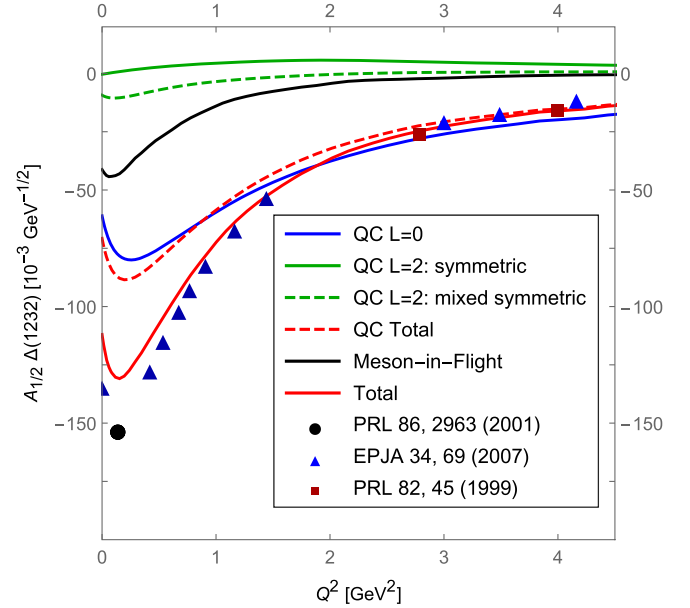


FIG. 3. Transverse helicity amplitude $A_{1/2}$ for the $p\gamma^* \rightarrow \Delta(1232)$ transition compared to experimental data. The blue and green curves represent quark-core (QC) contributions, with the blue (symmetric $L = 0$), green solid (symmetric $L = 2$), and green dashed (mixed-symmetric $L = 2$). The black curve shows the pion cloud contribution. The red dashed curve combines the contributions of the $L = 0$ and $L = 2$ quark-core components, while the red solid curve presents the total result, including both quark-core and pion cloud contributions. The experimental data are taken from [32,34,35].

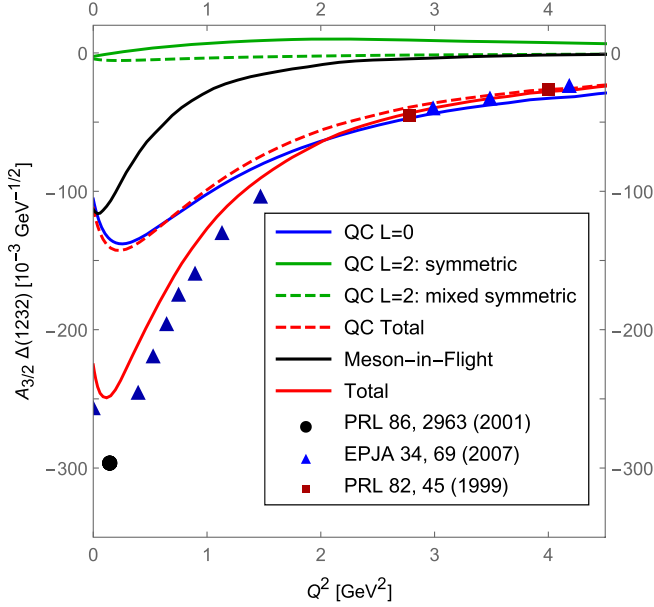


FIG. 4. Transverse helicity amplitude $A_{3/2}$ of the $p\gamma^* \rightarrow \Delta(1232)$ transition compared to experimental data. The notations for the curves are the same as those described in Fig. 3.

harmonic oscillator basis is chosen to be 400 MeV. In the calculation, the wave function of the $\Delta(1232)$ resonance takes both the $L = 0$ and $L = 2$ orbital angular momenta, while the wave function of the proton is extracted from its electric form factor with $L = 0$. For the meson-cloud diagram shown in Fig. 2, the empirical pion charge form factor

$$F_{\pi\pi\gamma}(q^2) = \frac{1}{1 - q^2/\Lambda^2} \quad (15)$$

is applied for the pion-photon interaction, with $\Lambda = 0.732$ GeV, where q is the momentum of the virtual photon. Figures 3–5 present the helicity amplitudes of the $p\gamma^* \rightarrow \Delta(1232)$ transition, contributed by the quark-core diagram and meson-in-flight diagram, compared to the experimental data (shown as colored dots).

The longitudinal amplitude $S_{1/2}$ in Fig. 5 shows a clear signature that the contribution of the $L = 0$ quark core to the $S_{1/2}$ amplitude vanishes, and the amplitude is dominated by the $L = 2$ quark-core contribution. The meson-cloud contribution is significant in low- Q^2 regions.

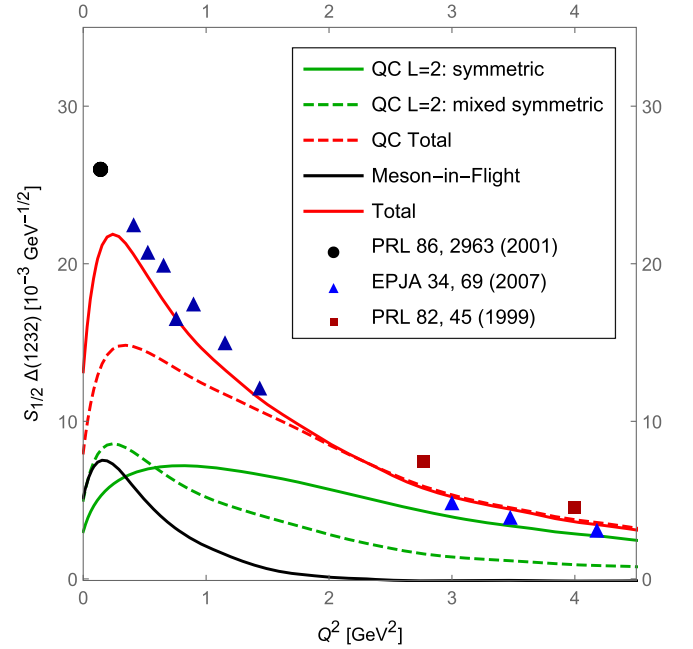


FIG. 5. Longitudinal helicity amplitude $S_{1/2}$ of the $p\gamma^* \rightarrow \Delta(1232)$ transition compared to experimental data. The notations for the curves are the same as those described in Fig. 3.

In the low- Q^2 region (see Figs. 3–5), the meson cloud accounts for roughly 30% of the amplitudes. For the transverse helicity amplitudes $A_{1/2}$ and $A_{3/2}$ the quark-core $L = 0$ and $L = 2$ components contribute approximately 60% and 10%, respectively, while the longitudinal amplitude $S_{1/2}$ gets about 70% of its strength from the quark-core $L = 2$ component. For $Q^2 \gtrsim 1$ GeV², the quark-core $L = 0$ term becomes dominant in $A_{1/2}$ and $A_{3/2}$. The fit of our model to the experimental helicity amplitudes (Figs. 3–5) yields

$$B = 0.892, \quad C = 0.300, \quad D = 0.336,$$

corresponding to a $\Delta(1232)$ composition of approximately 80% in the $L = 0$ state, 9% in the $L = 2$ symmetric state, and 11% in the $L = 2$ mixed-symmetric state.

The expansion coefficients appearing in Eq. (3) are listed below (entries are ordered according to the indicated N values):

$$\begin{aligned} \{a_0, a_2, a_4, \dots, a_{10}\} &= \{0.930, -0.007, 0.254, 0.235, 0.116, -0.004\}, \\ \{b_0, b_2, b_4, \dots, b_{10}\} &= \{0.994, -0.022, 0.049, 0.029, -0.045, -0.065\}, \\ \{c_2, c_4, c_6, \dots, c_{12}\} &= \{0.615, 0.172, 0.526, 0.316, 0.388, 0.252\}, \\ \{d_2, d_4, d_6, \dots, d_{12}\} &= \{0.922, 0.037, 0.145, 0.247, 0.200, 0.156\}. \end{aligned} \quad (16)$$

In the present analysis, the key methodological elements are as follows: (i) The relative weights of the symmetric and mixed-symmetry $L = 2$ components are determined by a simultaneous fit to $A_{1/2}$, $A_{3/2}$, and $S_{1/2}$; (ii) the proton wave function is Lorentz boosted to the resonance rest frame and the associated Wigner-rotation factors are included in the transition matrix elements [see Eq. (5)]; and (iii) the meson-cloud contribution is evaluated within the chiral quark model using an empirical pion charge form factor, with the meson-in-flight diagram included explicitly.

IV. SUMMARY AND DISCUSSION

We have investigated the substructure of the $\Delta(1232)$ resonance in the $p\gamma^* \rightarrow \Delta(1232)$ process using helicity amplitudes. The analysis uses a three-quark model that includes both quark-core and meson-cloud contributions; the proton and $\Delta(1232)$ wave functions are constructed according to the spin, flavor, and color symmetries $SU_f(2) \otimes SU_s(2) \otimes SU_c(3)$. The proton wave function is Lorentz boosted from the proton rest frame to the resonance rest frame in the calculation of the transition amplitudes. Spatial wave functions are expanded in harmonic-oscillator bases in Jacobi coordinates; the $\Delta(1232)$ includes both $L = 0$ and $L = 2$ components while the proton is taken in an $L = 0$ configuration. These spatial components are fitted to the experimental helicity amplitudes $A_{1/2}$, $A_{3/2}$, and $S_{1/2}$. The amplitudes are computed by including both the direct photon-quark interaction and the meson-cloud contributions, where the latter are evaluated in the chiral quark model using the empirical pion charge form factor.

The study confirms that the meson cloud plays a crucial role at low Q^2 , while at higher momentum transfers the $L = 0$ and $L = 2$ quark-core component significantly determines the amplitude shapes. The agreement between the theoretical results and experimental data, as demonstrated in Figs. 3–5, reveals the importance of including both quark-core and meson-cloud contributions to fully describe the helicity amplitudes.

It is found from the fitting that about 80% of the $\Delta(1232)$ wave function resides in the $L = 0$ (S-wave) configuration, with the remaining 20% $L = 2$ admixtures. Particularly for the longitudinal helicity amplitude $S_{1/2}$, the $L = 0$ component of the $\Delta(1232)$ resonance is shown to have no contribution, with only the $L = 2$ component playing a role. These results challenge the traditional understanding that the $\Delta(1232)$ baryon is dominantly an $L = 0$ state.

Earlier quark-model studies have examined the influence of D -wave admixtures on the $\Delta(1232)$ quadrupole strength (see, e.g., Refs. [22,23]). Those works report D -wave probabilities at the level of a few percent and indicate that meson-cloud or other nonvalence mechanisms are important for reproducing the observed E2 and C2 amplitudes.

The observed strong suppression of the $L = 0$ contribution in $S_{1/2}$ highlights the sensitivity of longitudinal transitions to orbital deformations and suggests a fruitful avenue for further model refinement. The presence of a significant $L = 2$ component in the $\Delta(1232)$ resonance structure opens up intriguing possibilities for future studies, particularly in refining models that go beyond the simple quark-core picture.

ACKNOWLEDGMENTS

This research has received funding support from the National Science Research and Innovation Fund (NSRF) via the Program Management Unit for Human Resources & Institutional Development, Research and Innovation (Grant No. B05F640055). A. K. acknowledges the support of Suranaree University of Technology (SUT) (Full-time 66/32/2566). Y. Y. and A. L. acknowledge the support of (i) SUT, (ii) Thailand Science Research and Innovation (TSRI), and (iii) NSRF, Project No. 204211.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available because they are owned by a third party and the terms of use prevent public distribution. The data are available from the authors upon reasonable request.

APPENDIX: SPATIAL WAVE FUNCTION

The spatial wave functions in Eq. (3) are expanded in a harmonic-oscillator basis. In general, the three-quark baryon basis can be obtained as the solution of three equal-mass quarks (m) in a harmonic-oscillator potential. For convenience, the problem is solved in Jacobi coordinates to remove the center-of-mass motion, so the spatial wave function in position space is written as

$$\begin{aligned} \Psi_{N,L,M}^{\alpha}(\lambda, \rho) = & \sum_{n_p, l_p, n_\lambda, l_\lambda} \sum_{m_\rho, m_\lambda} A(n_p, l_p, n_\lambda, l_\lambda) \\ & \times \psi_{n_\lambda, l_\lambda, m_\lambda}(\lambda) \psi_{n_p, l_p, m_\rho}(\rho) \\ & \times C(l_\lambda, m_\lambda; l_\rho, m_\rho; L, M), \end{aligned} \quad (\text{A1})$$

where the single-particle solution of a three-dimensional harmonic oscillator is

$$\psi_{n,l,m}(\mathbf{r}) = \sqrt{\frac{2a^3 n!}{(\frac{1}{2} + l + n)!}} (a^2 \mathbf{r}^2)^{l/2} e^{-\frac{a^2 \mathbf{r}^2}{2}} Y_{l,m}(\hat{\mathbf{r}}). \quad (\text{A2})$$

The coefficients $A(n_p, l_p, n_\lambda, l_\lambda)$ can be determined in the language of the permutation group. The quantum numbers appearing above are related by

$$\begin{aligned}
N &= 2(n_\lambda + n_\rho) + l_\lambda + l_\rho, \\
L &= l_\lambda \otimes l_\rho,
\end{aligned} \tag{A3}$$

and the Jacobi coordinates are defined as

$$\begin{aligned}
\lambda &= \frac{\mathbf{r}_1 + \mathbf{r}_2 - 2\mathbf{r}_3}{\sqrt{6}}, \\
\rho &= \frac{\mathbf{r}_1 - \mathbf{r}_2}{\sqrt{2}}.
\end{aligned} \tag{A4}$$

We have the orthonormality relations

$$\begin{aligned}
\int d\mathbf{r} (\psi_{n,l,m}(\mathbf{r}))^* \psi_{n',l',m'}(\mathbf{r}) &= \delta_{n,n'} \delta_{l,l'} \delta_{m,m'}, \\
\int d\lambda d\rho (\Psi_{N',L',M'}^\alpha(\lambda, \rho))^* \Psi_{N,L,M}^\alpha(\lambda, \rho) &= \delta_{N,N'} \delta_{L,L'} \delta_{M,M'}.
\end{aligned} \tag{A5}$$

The basis states $\Psi_{N,L,M}^\alpha(\lambda, \rho)$ for the symmetric $L = 0$, symmetric $L = 2$, and mixed-symmetry $L = 2$ spatial wave functions are listed in Tables I, II, and III, respectively.

TABLE I. Symmetric $L = 0$.

$\Psi_{0,0,0}^S(\lambda, \rho) =$	$\psi_{0,0,0}(\lambda) \psi_{0,0,0}(\rho)$
$\Psi_{2,0,0}^S(\lambda, \rho) =$	$\frac{1}{\sqrt{2}} [\psi_{1,0,0}(\lambda) \psi_{0,0,0}(\rho) + \psi_{0,0,0}(\lambda) \psi_{1,0,0}(\rho)]$
$\Psi_{4,0,0}^S(\lambda, \rho) =$	$\frac{\sqrt{5}}{4} [\psi_{2,0,0}(\lambda) \psi_{0,0,0}(\rho) + \psi_{0,0,0}(\lambda) \psi_{2,0,0}(\rho)] + \frac{\sqrt{6}}{4} \psi_{1,0,0}(\lambda) \psi_{1,0,0}(\rho)$
$\Psi_{6,0,0}^S(\lambda, \rho) =$	$\frac{3}{4\sqrt{2}} [\psi_{2,0,0}(\lambda) \psi_{1,0,0}(\rho) + \psi_{1,0,0}(\lambda) \psi_{2,0,0}(\rho)] + \frac{\sqrt{7}}{4\sqrt{2}} [\psi_{3,0,0}(\lambda) \psi_{0,0,0}(\rho) + \psi_{0,0,0}(\lambda) \psi_{3,0,0}(\rho)]$
$\Psi_{8,0,0}^S(\lambda, \rho) =$	$\frac{\sqrt{42}}{16} [\psi_{4,0,0}(\lambda) \psi_{0,0,0}(\rho) + \psi_{0,0,0}(\lambda) \psi_{4,0,0}(\rho)] + \frac{\sqrt{14}}{8} [\psi_{3,0,0}(\lambda) \psi_{1,0,0}(\rho) + \psi_{1,0,0}(\lambda) \psi_{3,0,0}(\rho)]$ $+ \frac{\sqrt{15}}{8} \psi_{2,0,0}(\lambda) \psi_{2,0,0}(\rho)$
$\Psi_{10,0,0}^S(\lambda, \rho) =$	$\frac{\sqrt{33}}{16} [\psi_{5,0,0}(\lambda) \psi_{0,0,0}(\rho) + \psi_{0,0,0}(\lambda) \psi_{5,0,0}(\rho)] + \frac{3\sqrt{5}}{16} [\psi_{4,0,0}(\lambda) \psi_{1,0,0}(\rho) + \psi_{1,0,0}(\lambda) \psi_{4,0,0}(\rho)]$ $+ \frac{5}{8\sqrt{2}} [\psi_{3,0,0}(\lambda) \psi_{2,0,0}(\rho) + \psi_{2,0,0}(\lambda) \psi_{3,0,0}(\rho)]$

TABLE II. Symmetric $L = 2$.

$\Psi_{2,2,M}^S(\lambda, \rho) =$	$\frac{1}{\sqrt{2}} [\psi_{0,2,M}(\lambda) \psi_{0,0,0}(\rho) + \psi_{0,0,0}(\lambda) \psi_{0,2,M}(\rho)]$
$\Psi_{4,2,M}^S(\lambda, \rho) =$	$\frac{\sqrt{35}}{10} [\psi_{1,2,M}(\lambda) \psi_{0,0,0}(\rho) + \psi_{0,0,0}(\lambda) \psi_{1,2,M}(\rho)] + \frac{\sqrt{15}}{10} [\psi_{0,2,M}(\lambda) \psi_{1,0,0}(\rho) + \psi_{1,0,0}(\lambda) \psi_{0,2,M}(\rho)]$
$\Psi_{6,2,M}^S(\lambda, \rho) =$	$\frac{\sqrt{105}}{20} [\psi_{2,2,M}(\lambda) \psi_{0,0,0}(\rho) + \psi_{0,0,0}(\lambda) \psi_{2,2,M}(\rho)] + \frac{\sqrt{70}}{20} [\psi_{1,2,M}(\lambda) \psi_{1,0,0}(\rho) + \psi_{1,0,0}(\lambda) \psi_{1,2,M}(\rho)]$ $+ \frac{1}{4} [\psi_{0,2,M}(\lambda) \psi_{2,0,0}(\rho) + \psi_{2,0,0}(\lambda) \psi_{0,2,M}(\rho)]$
$\Psi_{8,2,M}^S(\lambda, \rho) =$	$\frac{\sqrt{330}}{40} [\psi_{3,2,M}(\lambda) \psi_{0,0,0}(\rho) + \psi_{0,0,0}(\lambda) \psi_{3,2,M}(\rho)] + \frac{\sqrt{270}}{40} [\psi_{2,2,M}(\lambda) \psi_{1,0,0}(\rho) + \psi_{1,0,0}(\lambda) \psi_{2,2,M}(\rho)]$ $+ \frac{\sqrt{6}}{8} [\psi_{1,2,M}(\lambda) \psi_{2,0,0}(\rho) + \psi_{2,0,0}(\lambda) \psi_{1,2,M}(\rho)] + \frac{\sqrt{2}}{8} [\psi_{0,2,M}(\lambda) \psi_{3,0,0}(\rho) + \psi_{3,0,0}(\lambda) \psi_{0,2,M}(\rho)]$
$\Psi_{10,2,M}^S(\lambda, \rho) =$	$\frac{\sqrt{4290}}{160} [\psi_{4,2,M}(\lambda) \psi_{0,0,0}(\rho) + \psi_{0,0,0}(\lambda) \psi_{4,2,M}(\rho)] + \frac{\sqrt{990}}{80} [\psi_{3,2,M}(\lambda) \psi_{1,0,0}(\rho) + \psi_{1,0,0}(\lambda) \psi_{3,2,M}(\rho)]$ $+ \frac{\sqrt{27}}{16} [\psi_{2,2,M}(\lambda) \psi_{2,0,0}(\rho) + \psi_{2,0,0}(\lambda) \psi_{2,2,M}(\rho)] + \frac{\sqrt{14}}{16} [\psi_{1,2,M}(\lambda) \psi_{3,0,0}(\rho) + \psi_{3,0,0}(\lambda) \psi_{1,2,M}(\rho)]$ $+ \frac{\sqrt{18}}{32} [\psi_{0,2,M}(\lambda) \psi_{4,0,0}(\rho) + \psi_{4,0,0}(\lambda) \psi_{0,2,M}(\rho)]$
$\Psi_{12,2,M}^S(\lambda, \rho) =$	$\frac{\sqrt{143}}{32} [\psi_{5,2,M}(\lambda) \psi_{0,0,0}(\rho) + \psi_{0,0,0}(\lambda) \psi_{5,2,M}(\rho)] + \frac{\sqrt{143}}{32} [\psi_{4,2,M}(\lambda) \psi_{1,0,0}(\rho) + \psi_{1,0,0}(\lambda) \psi_{4,2,M}(\rho)]$ $+ \frac{\sqrt{110}}{32} [\psi_{3,2,M}(\lambda) \psi_{2,0,0}(\rho) + \psi_{2,0,0}(\lambda) \psi_{3,2,M}(\rho)] + \frac{\sqrt{70}}{32} [\psi_{2,2,M}(\lambda) \psi_{3,0,0}(\rho) + \psi_{3,0,0}(\lambda) \psi_{2,2,M}(\rho)]$ $+ \frac{\sqrt{35}}{32} [\psi_{1,2,M}(\lambda) \psi_{4,0,0}(\rho) + \psi_{4,0,0}(\lambda) \psi_{1,2,M}(\rho)] + \frac{\sqrt{11}}{32} [\psi_{0,2,M}(\lambda) \psi_{5,0,0}(\rho) + \psi_{5,0,0}(\lambda) \psi_{0,2,M}(\rho)]$

TABLE III. Mixed-symmetry $L = 2$.

λ type	
$\Psi_{2,2,M}^\lambda(\lambda, \rho) =$	$\frac{1}{\sqrt{2}} [\psi_{0,0,0}(\lambda)\psi_{0,2,M}(\rho) - \psi_{0,2,M}(\lambda)\psi_{0,0,0}(\rho)]$
$\Psi_{4,2,M}^\lambda(\lambda, \rho) =$	$\frac{\sqrt{35}}{10} [\psi_{1,2,M}(\lambda)\psi_{0,0,0}(\rho) - \psi_{0,0,0}(\lambda)\psi_{1,2,M}(\rho)] + \frac{\sqrt{15}}{10} [\psi_{0,2,M}(\lambda)\psi_{1,0,0}(\rho) - \psi_{1,0,0}(\lambda)\psi_{0,2,M}(\rho)]$
$\Psi_{6,2,M}^\lambda(\lambda, \rho) =$	$\frac{\sqrt{105}}{20} [\psi_{2,2,M}(\lambda)\psi_{0,0,0}(\rho) - \psi_{0,0,0}(\lambda)\psi_{2,2,M}(\rho)] + \frac{\sqrt{70}}{20} [\psi_{1,2,M}(\lambda)\psi_{1,0,0}(\rho) - \psi_{1,0,0}(\lambda)\psi_{1,2,M}(\rho)]$ $+ \frac{1}{4} [\psi_{0,2,M}(\lambda)\psi_{2,0,0}(\rho) - \psi_{2,0,0}(\lambda)\psi_{0,2,M}(\rho)]$
$\Psi_{8,2,M}^\lambda(\lambda, \rho) =$	$\frac{\sqrt{330}}{40} [\psi_{3,2,M}(\lambda)\psi_{0,0,0}(\rho) - \psi_{0,0,0}(\lambda)\psi_{3,2,M}(\rho)] + \frac{\sqrt{270}}{40} [\psi_{2,2,M}(\lambda)\psi_{1,0,0}(\rho) - \psi_{1,0,0}(\lambda)\psi_{2,2,M}(\rho)]$ $+ \frac{\sqrt{6}}{8} [\psi_{1,2,M}(\lambda)\psi_{2,0,0}(\rho) - \psi_{2,0,0}(\lambda)\psi_{1,2,M}(\rho)] + \frac{\sqrt{2}}{8} [\psi_{0,2,M}(\lambda)\psi_{3,0,0}(\rho) - \psi_{3,0,0}(\lambda)\psi_{0,2,M}(\rho)]$
$\Psi_{10,2,M}^\lambda(\lambda, \rho) =$	$\frac{\sqrt{4290}}{160} [\psi_{4,2,M}(\lambda)\psi_{0,0,0}(\rho) - \psi_{0,0,0}(\lambda)\psi_{4,2,M}(\rho)] + \frac{\sqrt{990}}{80} [\psi_{3,2,M}(\lambda)\psi_{1,0,0}(\rho) - \psi_{1,0,0}(\lambda)\psi_{3,2,M}(\rho)]$ $+ \frac{\sqrt{27}}{16} [\psi_{2,2,M}(\lambda)\psi_{2,0,0}(\rho) - \psi_{2,0,0}(\lambda)\psi_{2,2,M}(\rho)] + \frac{\sqrt{14}}{16} [\psi_{1,2,M}(\lambda)\psi_{3,0,0}(\rho) - \psi_{3,0,0}(\lambda)\psi_{1,2,M}(\rho)]$ $+ \frac{\sqrt{18}}{32} [\psi_{0,2,M}(\lambda)\psi_{2,0,0}(\rho) - \psi_{2,0,0}(\lambda)\psi_{0,2,M}(\rho)]$
$\Psi_{12,2,M}^\lambda(\lambda, \rho) =$	$\frac{\sqrt{143}}{32} [\psi_{5,2,M}(\lambda)\psi_{0,0,0}(\rho) - \psi_{0,0,0}(\lambda)\psi_{5,2,M}(\rho)] + \frac{\sqrt{143}}{32} [\psi_{4,2,M}(\lambda)\psi_{1,0,0}(\rho) - \psi_{1,0,0}(\lambda)\psi_{4,2,M}(\rho)]$ $+ \frac{\sqrt{110}}{32} [\psi_{3,2,M}(\lambda)\psi_{2,0,0}(\rho) - \psi_{2,0,0}(\lambda)\psi_{3,2,M}(\rho)] + \frac{\sqrt{70}}{32} [\psi_{2,2,M}(\lambda)\psi_{3,0,0}(\rho) - \psi_{3,0,0}(\lambda)\psi_{2,2,M}(\rho)]$ $+ \frac{\sqrt{35}}{32} [\psi_{1,2,M}(\lambda)\psi_{2,0,0}(\rho) - \psi_{2,0,0}(\lambda)\psi_{1,2,M}(\rho)] + \frac{\sqrt{11}}{32} [\psi_{0,2,M}(\lambda)\psi_{2,0,0}(\rho) - \psi_{2,0,0}(\lambda)\psi_{0,2,M}(\rho)]$
ρ type	
$\Psi_{2,2,M}^\rho(\lambda, \rho) =$	$\psi_{0,1,m_\lambda}(\lambda) \otimes \psi_{0,1,m_\rho}(\rho)$
$\Psi_{4,2,M}^\rho(\lambda, \rho) =$	$\frac{1}{\sqrt{2}} [\psi_{1,1,m_\lambda}(\lambda) \otimes \psi_{0,1,m_\rho}(\rho) + \psi_{0,1,m_\lambda}(\lambda) \otimes \psi_{1,1,m_\rho}(\rho)]$
$\Psi_{6,2,M}^\rho(\lambda, \rho) =$	$\frac{\sqrt{42}}{12} [\psi_{2,1,m_\lambda}(\lambda) \otimes \psi_{0,1,m_\rho}(\rho) + \psi_{0,1,m_\lambda}(\lambda) \otimes \psi_{2,1,m_\rho}(\rho)] + \frac{\sqrt{15}}{6} \psi_{1,1,m_\lambda}(\lambda) \otimes \psi_{1,1,m_\rho}(\rho)$
$\Psi_{8,2,M}^\rho(\lambda, \rho) =$	$\frac{\sqrt{3}}{4} [\psi_{3,1,m_\lambda}(\lambda) \otimes \psi_{0,1,m_\rho}(\rho) + \psi_{0,1,m_\lambda}(\lambda) \otimes \psi_{3,1,m_\rho}(\rho)]$ $+ \frac{\sqrt{5}}{4} [\psi_{2,1,m_\lambda}(\lambda) \otimes \psi_{1,1,m_\rho}(\rho) + \psi_{1,1,m_\lambda}(\lambda) \otimes \psi_{2,1,m_\rho}(\rho)]$
$\Psi_{10,2,M}^\rho(\lambda, \rho) =$	$\frac{\sqrt{33}}{16} [\psi_{4,1,m_\lambda}(\lambda) \otimes \psi_{0,1,m_\rho}(\rho) + \psi_{0,1,m_\lambda}(\lambda) \otimes \psi_{4,1,m_\rho}(\rho)]$ $+ \frac{\sqrt{15}}{8} [\psi_{3,1,m_\lambda}(\lambda) \otimes \psi_{1,1,m_\rho}(\rho) + \psi_{1,1,m_\lambda}(\lambda) \otimes \psi_{3,1,m_\rho}(\rho)]$ $+ \frac{\sqrt{70}}{16} \psi_{2,1,m_\lambda}(\lambda) \otimes \psi_{2,1,m_\rho}(\rho)$
$\Psi_{12,2,M}^\rho(\lambda, \rho) =$	$\frac{\sqrt{858}}{96} [\psi_{5,1,m_\lambda}(\lambda) \otimes \psi_{0,1,m_\rho}(\rho) + \psi_{0,1,m_\lambda}(\lambda) \otimes \psi_{5,1,m_\rho}(\rho)]$ $+ \frac{\sqrt{1650}}{96} [\psi_{4,1,m_\lambda}(\lambda) \otimes \psi_{1,1,m_\rho}(\rho) + \psi_{1,1,m_\lambda}(\lambda) \otimes \psi_{4,1,m_\rho}(\rho)]$ $+ \frac{\sqrt{525}}{48} [\psi_{3,1,m_\lambda}(\lambda) \otimes \psi_{2,1,m_\rho}(\rho) + \psi_{2,1,m_\lambda}(\lambda) \otimes \psi_{3,1,m_\rho}(\rho)]$

Note that the transformation of harmonic-oscillator basis functions from coordinate space to momentum space leads to an additional phase factor $(-i)^N$. That is, for $\Psi_{N,L,M}^\alpha(\lambda, \rho)$, we have $(-i)^N \Psi_{N,L,M}^\alpha(\mathbf{p}_\lambda, \mathbf{p}_\rho)$, where $\mathbf{p}_\lambda$, $\mathbf{p}_\rho$ are the momenta conjugate to λ and ρ .

- [1] A. Thiel, F. Afzal, and Y. Wunderlich, Light baryon spectroscopy, *Prog. Part. Nucl. Phys.* **125**, 103949 (2022).
[2] L. Tiator, D. Drechsel, S. S. Kamalov, and M. Vanderhaeghen, Electromagnetic excitation of nucleon resonances, *Eur. Phys. J. Special Topics* **198**, 141 (2011).
[3] L. Tiator, D. Drechsel, S. S. Kamalov, and M. Vanderhaeghen, Baryon resonance analysis from maid, *Chin. Phys. C* **33**, 1069 (2009).

- [4] I. G. Aznauryan, Electroexcitation of the roper resonance in relativistic quark models, *Phys. Rev. C* **76**, 025212 (2007).
[5] I. G. Aznauryan, V. D. Burkert, W. Kim, K. Park, G. Adams, M. J. Amarian, P. Ambrozewicz, M. Anghinolfi, G. Asryan, H. Avakian *et al.*, Electroexcitation of the roper resonance for $1.7 < q^2 < 4.5$ GeV² in $ep \rightarrow en\pi^+$, *Phys. Rev. C* **78**, 045209 (2008).

- [6] S. Capstick, B. D. Keister, and D. Morel, Nucleon to resonance form factor calculations, *J. Phys. Conf. Ser.* **69**, 012016 (2007).
- [7] G. Ramalho and K. Tsushima, Valence quark contributions for the $\gamma n \rightarrow p_{11}(1440)$ form factors, *Phys. Rev. D* **81**, 074020 (2010).
- [8] B. Golli, S. Sirca, and M. Fiolhais, Pion electro-production in the roper region in chiral quark models, *Eur. Phys. J. A* **42**, 185 (2009).
- [9] V. D. Burkert and C. D. Roberts, Colloquium: Roper resonance: Toward a solution to the fifty year puzzle, *Rev. Mod. Phys.* **91**, 011003 (2019).
- [10] Q. B. Li and D. O. Riska, Role of qq components in the $n(1440)$ resonance, *Phys. Rev. C* **74**, 015202 (2006).
- [11] I. T. Obukhovskiy, A. Faessler, D. K. Fedorov, T. Gutsche, and V. E. Lyubovitskij, Electroproduction of the roper resonance on the proton: The role of the three-quark core and the molecular $n\sigma$ component, *Phys. Rev. D* **84**, 014004 (2011).
- [12] F. Cano and P. Gonzalez, A consistent explanation of the roper phenomenology, *Phys. Lett. B* **431**, 270 (1998).
- [13] S. Štajner, P. Achenbach, T. Beranek, J. Beričič, J. C. Bernauer, D. Bosnar, R. Böhm, L. Correa, A. Denig, M. O. Distler *et al.*, Beam-recoil polarization measurement of π^0 electroproduction on the proton in the region of the roper resonance, *Phys. Rev. Lett.* **119**, 022001 (2017).
- [14] A. V. Sarantsev *et al.*, New results on the roper resonance and the p_{11} partial wave, *Phys. Lett. B* **659**, 94 (2008).
- [15] S. Capstick and W. Roberts, Quark models of baryon masses and decays, *Prog. Part. Nucl. Phys.* **45**, S241 (2000).
- [16] K. Xu, A. Kaewsnod, Z. Zhao, X. Y. Liu, S. Srisuphaphon, A. Limphirat, and Y. Yan, Pentaquark components in low-lying baryon resonances, *Phys. Rev. D* **101**, 076025 (2020).
- [17] G. Ramalho and K. Tsushima, Meson cloud contributions to the dalitz decays of decuplet to octet baryons, *Phys. Rev. D* **108**, 074019 (2023).
- [18] G. Ramalho, Combined parametrization of gen and $\gamma^* n \rightarrow \delta(1232)$ quadrupole form factors, *Eur. Phys. J. A* **55**, 32 (2019).
- [19] G. Ramalho and M. T. Peña, Electromagnetic transition form factors of baryon resonances, *Prog. Part. Nucl. Phys.* **136**, 104097 (2024).
- [20] A. Kaewsnod, K. Xu, Z. Zhao, X. Y. Liu, S. Srisuphaphon, A. Limphirat, and Y. Yan, Study of $n(1440)$ structure via $\gamma^* p \rightarrow n(1440)$ transition, *Phys. Rev. D* **105**, 016008 (2022).
- [21] A. Kaewsnod, K. Xu, Z. Zhao, X. Y. Liu, S. Srisuphaphon, A. Limphirat, and Y. Yan, Study of $n(1520)$ and $n(1535)$ structures via $\gamma^* p \rightarrow n^*$ transitions, *Eur. Phys. J. A* **58**, 185 (2022).
- [22] B. Juliá-Díaz and D. O. Riska, D-state configurations in the electromagnetic form factors of the nucleon and the $\Delta(1232)$ resonance, *Nucl. Phys.* **A757**, 441 (2005).
- [23] G. Ramalho, M. T. Peña, and Franz Gross, Electromagnetic form factors of the Δ with d-waves, *Phys. Rev. D* **81**, 113011 (2010).
- [24] K. Xu, A. Kaewsnod, X. Y. Liu, S. Srisuphaphon, A. Limphirat, and Y. Yan, Complete basis for the pentaquark wave function in a group theory approach, *Phys. Rev. C* **100**, 065207 (2019).
- [25] B. D. Keister and W. N. Polyzou, Relativistic Hamiltonian dynamics in nuclear and particle physics, *Adv. Nucl. Phys.* **20**, 225 (1991).
- [26] Thomas Berry and Matt Visser, Lorentz boosts and Wigner rotations: Self-adjoint complexified quaternions, *Physics* **3**, 352 (2021).
- [27] Thomas Berry and Matt Visser, Relativistic combination of non-collinear 3-velocities using quaternions, *Universe* **6**, 237 (2020).
- [28] Rafael Ferraro and Marc Thibeault, Generic composition of boosts: An elementary derivation of the Wigner rotation, *Eur. J. Phys.* **20**, 143 (1999).
- [29] Kane O'Donnell and Matt Visser, Elementary analysis of the special relativistic combination of velocities, Wigner rotation and thomas precession, *Eur. J. Phys.* **32**, 1033 (2011).
- [30] D. Dreschsel and L. Tiator, Threshold pion photoproduction on nucleons, *J. Phys. G* **18**, 449 (1992).
- [31] D. Drechsel, O. Hanstein, S. S. Kamalov, and L. Tiator, A unitary isobar model for pion photo- and electroproduction on the proton up to 1 gev, *Nucl. Phys.* **A645**, 145 (1999).
- [32] D. Drechsel, S. S. Kamalov, and L. Tiator, Unitary isobar model—maid2007, *Eur. Phys. J. A* **34**, 69 (2007).
- [33] X. Y. Liu, K. Khosonthongkee, A. Limphirat, and Y. Yan, Study of baryon octet electromagnetic form factors in perturbative chiral quark model, *J. Phys. G* **41**, 055008 (2014).
- [34] C. Mertz *et al.*, Search for quadrupole strength in the electroexcitation of the $\Delta^+(1232)$, *Phys. Rev. Lett.* **86**, 2963 (2001).
- [35] V. V. Frolov *et al.*, Electroproduction of the $\Delta(1232)$ resonance at high momentum transfer, *Phys. Rev. Lett.* **82**, 45 (1999).