

Topological strings in SU(3) gauge theory at finite temperature

Sanatan Digal^{*} and Sumit Shaw[†]

*The Institute of Mathematical Sciences, Chennai, 600113, India
and Homi Bhabha National Institute, Training School Complex, Anushaktinagar,
Mumbai 400094, India*

Vinod Mamale[‡]

*The Institute of Mathematical Sciences, Chennai, 600113, India
and National Institute for Theoretical and Computational Sciences, School of Physics,
and Mandelstam Institute for Theoretical Physics, University of the Witwatersrand,
Johannesburg, Wits 2050, South Africa*



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We investigate string configurations in the deconfined phase of SU(3) gauge theory, which arise from the spontaneous breaking of the Z_3 center symmetry. These configurations form at the junctions of domain walls of the theory. The complex phase of the Polyakov loop changes by multiples of 2π on large spatial loops around the string, rendering them topologically stable. Using the Monte Carlo simulations of the partition function, we compute the free energy associated with these configurations. The simulations are performed on lattices with spatial dimensions $N_{x,y} = 60, N_z = 4$, and temporal extent $N_\tau = 2, 4$. Our results show that the free energy of the Z_3 strings is dominated by the domain walls. Further near the transition point, thermal fluctuations cause the decay of domain walls as well as the Z_3 strings into confined-deconfined interfaces.

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I. INTRODUCTION

The nonperturbative study of quantum chromodynamics (QCD) plays a crucial role in understanding matter under extreme conditions. In particular, it provides key insights into the phase diagram and the transition from the hadronic phase to the quark-gluon plasma (QGP). The thermodynamics of QCD remains an active field of research, driven by relativistic heavy-ion collision (HIC) experiments that probe different regions of the phase diagram. While the chiral and heavy-quark limits do not correspond to physical mass parameters, investigating these limits has significantly enhanced our understanding of the role and possible effects of chiral symmetry breaking and the confinement-deconfinement aspects in realistic QCD [1–24].

QCD in the heavy-quark limit, i.e., pure SU(3) gauge theory, has been extensively studied using lattice simulations, which offer a powerful approach to explore the

nonperturbative regime. In finite-temperature studies of this theory [25–30], it has been firmly established that the system undergoes a first-order confinement-deconfinement (CD) phase transition, at the critical temperature (T_c), between the hadronic phase and the QGP phase. The hadronic and the QGP phases can be interpreted in terms of glueballs and a thermalized medium of gluons, respectively. In pure SU(N) gauge theories, the Polyakov loop expectation value(L), which transforms nontrivially under the $Z(N)$ gauge transformations, serves as the order parameter for the CD phase transition. In the deconfined phase, the Polyakov loop, acquires a nonzero expectation value, i.e., $L \neq 0$, leading to spontaneous breaking of the $Z(N)$ symmetry [31]. Consequently there are N degenerate states. For $N \geq 3$, the Polyakov loop is complex valued and these states are characterized by complex phases of the Polyakov loop, given by,

$$\theta_k = \frac{2\pi k}{N}, \quad k = 0, 1, \dots, N-1. \quad (1)$$

In contrast, for $N = 2$, the Polyakov loop is real valued, and the states are distinguished by its sign. These vacua of discrete states give rise to static domain walls (interfaces) that interpolate between a pair of vacuum sectors, i.e., θ_i 's [32].

^{*}Contact author: digal@imsc.res.in

[†]Contact author: sumitshaw@imsc.res.in

[‡]Contact author: vinod.mamale@wits.ac.za

The domain walls in Pure $SU(3)$ gauge theory, have been studied using lattice simulations, specifically for $N_\tau = 2$ [33–35] and for $N_\tau = 4$ [36]. A more recent study on the calculation of interface tension for $N_\tau \geq 2$ has been conducted using different numerical algorithms and employing the t’Hooft loop dual operator, with significant findings reported in [37–40]. Details on the $2+1D$ simulation of the Z_3 interfaces can be found in [41–43]; these studies provide a comprehensive numerical framework that captures the complex behavior of these interfaces under various conditions. In earlier work, the interface tension was calculated within a perturbative framework for $SU(N)$ gauge theories at finite temperatures, and this was achieved by establishing a connection between the $Z(N)$ vacua and the instanton solutions of an effective action that incorporates both classical and quantum fluctuations [44,45].

For $SU(N)$ gauge theory in the deconfined phase, the interface tension, α_{ij} , of the domain walls depends on the difference in the phases of the Polyakov loop, $|\theta_i - \theta_j|$, apart from temperature (T) [39,46]. Further, $\alpha_{ij} + \alpha_{jk} \geq \alpha_{ik}$, thus the interaction between these domain walls is attractive. The Polyakov loop takes on different thermal expectation values on either side of the domain walls. Its magnitude decreases as one approaches the center from either side of the domain wall, where it reaches a nonzero minimum value (see Sec. II). As a result, in a configuration of *multiple* domain walls, the phase of the Polyakov loop remains well defined in most of the physical space. This suggests the possibility of an intriguing configuration in the deconfined phase, at the intersection of all the N vacua. The domain walls can be oriented such that the topological considerations impose a linelike intersection [47,48], where the value of the Polyakov loop $L(\vec{x})$ becomes exactly zero. As $L(\vec{x})$ becomes zero, locally there is a restoration of Z_N symmetry and confining behavior in an overall deconfined environment. Thus, at high temperatures, the vacuum structure is such that the multiple degenerate $Z(N)$ states meet to form a defect whose core resembles the confined phase. This picture has been further elucidated by studies that employ lattice simulations and effective models to explore nonperturbative features of the quark-gluon plasma. In particular, for $N = 3$, string configurations were investigated [47] using an effective potential for the Polyakov loop with Z_3 symmetry [49], providing valuable insights into how the order parameter deviates sharply from the value corresponding to the deconfined phase, due to the underlying topological structure. These results, which are highly dependent on the effective potential, can only be validated through first-principles calculations, such as lattice simulations of the thermal partition function.

In this paper, we explore the topological string configurations in $SU(3)$ gauge theory using lattice simulations, investigating their intricate role in elucidating the non-perturbative phenomena inherent in QCD. The free energy for these configurations is computed by integrating the

action difference between systems with and without the string, a standard technique in statistical physics. In our study we also compute the interface tension, which is in good agreement with previous results [33], reinforcing the consistency of our approach with earlier studies and highlighting the relevance of these methods in probing non-trivial topological configurations in gauge theories.

The Z_3 topological string studied in this paper and flux tubes in QCD are fundamentally different. In the case of the Z_3 string, it is a deconfined phase everywhere except at the core. The situation reverses for flux tubes, i.e., the phase outside is confined and is deconfined inside. The QCD strings that describe the confinement of static quark–antiquark pairs manifest as flux tubes connecting the pairs, or as glueball states in a purely gluonic theory—a picture that continues to be refined through both numerical and analytical studies [17,50]. Unlike the QCD strings, which are prone to breaking via the spontaneous creation of quark–antiquark pairs [13,51–53], the strings we examine are topologically stable, which allows them to persist and interact in the deconfined phase.

In this study, the effects of the dynamical quarks on the string configurations are neglected. However, previous research provides substantial evidence suggesting that quarks can influence such configurations. Both perturbative and mean-field studies indicate that the presence of quarks explicitly breaks the Z_3 symmetry [54–57]. The explicit breaking weakens the CD transition from a pure gauge first order transition to a crossover with decrease in the quark masses in the heavy quark regime [54–56]. The explicit breaking lifts the degeneracy between the vacua. Since the three domain walls, in other words three vacua, join to form the string configuration, the lifting of degeneracy between the Z_3 vacua renders the string nonstatic. It will move toward the region of metastable states so as to reduce the free energy of the system [58]. In the presence of dynamical quarks, near T_c , only the $\theta = 0$ state is observed in the deconfined phase. The states with $\theta = \pm 2\pi/3$ appear as metastable states only above a certain temperature T_m [59]. This suggests that the string configuration will become unstable and melt as the temperature drops below T_m . Analysis within the framework of the Polyakov–Nambu–Jona-Lasinio (PNJL) model at zero baryon chemical potential shows that if metastable states were to form in HIC experiments, they would survive until the temperature fell below T_m [60]. This suggests that string configurations may potentially affect the dynamics of QGP above T_m in HIC.

This paper is organized as follows. In Sec. II, we will briefly describe the Z_3 symmetry in the continuum and emergence of the string solution using the Polyakov loop effective potential. The lattice gauge action and the lattice Polyakov loop operator, emphasizing its role, are presented in Sec. III. Further, we describe the methodology for calculating the free energy via an indirect approach, since

directly estimating the partition function is computationally challenging. Section IV outlines the numerical setup for studying string configurations and provides the technical details required to estimate the free energy of a string configuration. The conclusions are presented in Sec. V.

II. Z_3 STRING IN PURE $SU(3)$ GAUGE THEORY

In this Section, we briefly discuss the Z_3 symmetry in pure $SU(3)$ gauge theory at finite temperature and how topological defects follow from the Polyakov loop effective potential. In pure $SU(3)$, the gauge fields A_μ at any point in space time, are matrices in color space, i.e., $A_\mu = A_\mu^a T^a$, where $T^a, a = 1, 2, \dots, 8$, are generators of $SU(3)$ [61]. The partition function in the path-integral formulation is given by,

$$\mathcal{Z} = \int \prod_\mu DA_\mu e^{-S_g}. \quad (2)$$

S_g is the Euclidean gauge action, given by,

$$S_g = \frac{1}{4} \int_0^{1/T} d\tau \int d^3x F_{\mu\nu} F^{\mu\nu},$$

$$F^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g[A_\mu, A_\nu]. \quad (3)$$

T is the temperature and g is the gauge coupling. The above action remains invariant under gauge transformations, $\Lambda(x) \equiv \Lambda(\mathbf{x}, \tau)$, i.e.,

$$A_\mu(x) \rightarrow A_\mu^g(x) = \Lambda(x) A_\mu(x) \Lambda^{-1}(x) + i\Lambda(x) \partial_\mu \Lambda^{-1}(x).$$

The path integration in Eq. (2) is carried out over gauge fields, $A_\mu(x) \equiv A_\mu(\mathbf{x}, \tau)$, that are periodic in the temporal direction, i.e., $A_\mu(\mathbf{x}, 0) = A_\mu(\mathbf{x}, 1/T)$. This condition requires the gauge transformations, $\Lambda(x)$, to be periodic in τ , up to a factor z , i.e.,

$$\Lambda(\mathbf{x}, 0) = z\Lambda(\mathbf{x}, 1/T),$$

where $z \in Z_3$ and Z_3 is the center of $SU(3)$. Consequently, all the allowed gauge transformations are classified by the Z_3 group. Under these gauge transformations, the Polyakov loop ($L(\mathbf{x})$),

$$L(\mathbf{x}) = \frac{1}{3} \text{Tr} \left\{ P \left(\exp \left[ig \int_0^{1/T} d\tau A_0(\mathbf{x}, \tau) \right] \right) \right\}, \quad (4)$$

transforms as $L(\mathbf{x}) \rightarrow zL(\mathbf{x})$, similar to how Z_3 spins transform. For temperatures $T \geq T_c$, in the deconfined phase, the thermal and the volume (V_s) average of the Polyakov loop,

$$L(T) = \frac{1}{\mathcal{Z}} \int \prod_\mu DA_\mu \left[\frac{1}{V_s} \int d^3x L(\mathbf{x}) \right] e^{-S_g}, \quad (5)$$

acquires a nonzero value, which leads to the spontaneous

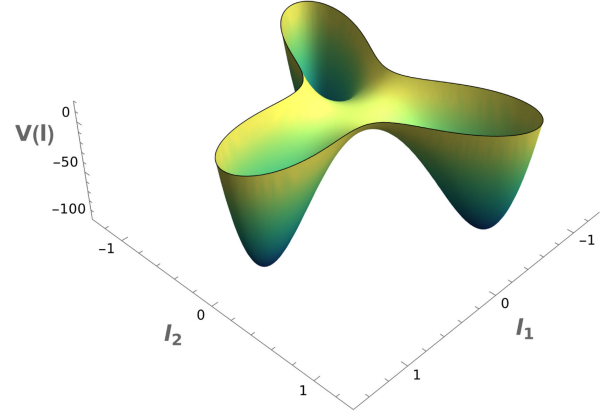


FIG. 1. $V(L)$ vs L , in the complex L plane for $T > T_c$.

breaking of the Z_3 symmetry. Consequently, the effective potential has three degenerate minima, as illustrated in Fig. 1. In the complex L plane, the point $L = 0$ corresponds to a saddle point of the effective potential, while the minima occur for $|L| \neq 0$ with the polar coordinate (phase) $\theta = 0, 2\pi/3$, and $-2\pi/3$ [44,62], which we denote by L_0, L_1 , and L_2 , respectively. Given this, there are three possible domain walls, i.e., L_{ij} interpolating L_i and L_j . A static domain wall solution, on the y - z plane, is obtained by solving the following field equations,

$$\frac{d^2 l_i}{dx^2} = \frac{\partial V}{\partial l_i}, \quad i = 1, 2, \quad (6)$$

where $l_i, i = 1, 2$ are the real and imaginary components of the Polyakov loop. A domain wall solution $[L(x)]$, L_{ij} , interpolating L_i and L_j will satisfy the following boundary condition,

$$\lim_{x \rightarrow -\infty} L(x) = L_i, \quad \lim_{x \rightarrow +\infty} L(x) = L_j.$$

If we replace the x coordinate by time t , then the solution to

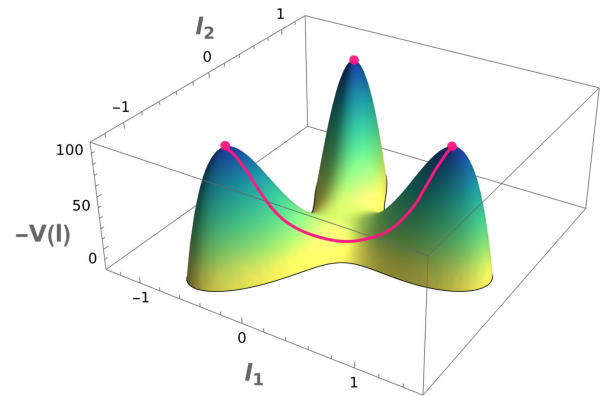


FIG. 2. Inverted potential, $-V(L)$ vs L , in the complex L plane for $T > T_c$.

Eq. (6) with the above boundary conditions corresponds to the trajectory of a particle under the potential, $-V(L)$, shown in Fig. 2. The trajectory starts out at L_i at $t = -\infty$ and approaches L_j at $t = +\infty$.

As the particle departs from the point L_i , the magnitude ($|L|$) initially decreases, reaching a minimum at the midpoint of the trajectory (i.e., at $t = 0$), and then increases again until it arrives at L_j . The midpoint represents the core of the domain wall. Along the path, the phase of L undergoes rapid variation near the midpoint. It is evident that the trajectory must be unique as any deviation from it would cause $|L| \rightarrow \infty$ due to the potential, $-V(L)$. For instance, $L(x) \neq 0$ for any x . Any trajectory originating from L_i that does pass through $L = 0$ would have constant $\arg(L)$, causing the particle to continue in a straight line and inevitably escape to infinity. In standard terminology, this domain wall solution is called a bounce solution.

In the deconfined phase, let us consider a junction of L_{01} , L_{12} , and L_{20} along the z axis. As argued above, except near the z axis, in most of the physical space $|L|$ will take the value corresponding to the minimum of $V(L)$, though near the domain wall core, $|L|$ will reduce to a finite nonzero value. Thus, the phase of L , i.e., $\arg(L)$, is well defined everywhere except near the z axis. Across each of the domain walls, $\arg(L)$ varies by $2\pi/3$. So, the three domain walls can be oriented such that the total variation of phase along a hypothetical loop around the z axis is $\pm 2\pi$. Note that when the phase of a complex-valued field is well defined along a loop in physical space, its total variation along the loop will always be, $2\pi n$, where n is an integer also known as the winding number. In the current scenario, assuming the Polyakov loop configuration is continuous, the total phase variation remains constant as the loop shrinks toward the z axis. If $n \neq 0$, below a certain size, the phase variation will lead to an increase in free energy unless $|L|$ decreases. Consequently, as the loop shrinks to a point, $|L|$ must vanish. Such a configuration is topologically stable, as small deformations cannot change the winding number.

Note that conventionally winding numbers, which classify topological strings, are defined when the order parameter space (OPS), consisting of the minima of the effective potential, is a circle (S^1). The winding numbers correspond to the mappings from loops in physical space to the OPS, S^1 , and are classified by the first homotopy group, i.e., $\pi_1(S^1)$, which are characterized by a set of integers, $\mathbb{Z}$. In the present case, it is useful to define a manifold (M) consisting of all the possible values of the Polyakov loop, taking into account the presence of domain walls. According to this definition, M takes the shape of a deformed equilateral triangle as shown in Fig. 3. This deformation is due to the profile of the domain walls. Now, one can consider mappings from loops in physical space to

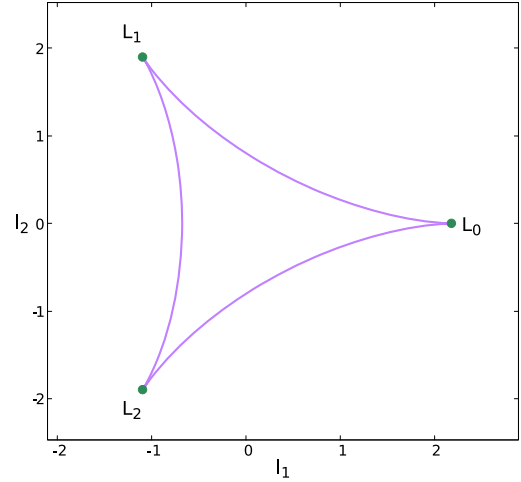


FIG. 3. Order parameter space in the presence of domain walls.

M . Since, M is homeomorphic to S^1 , the corresponding homotopy group will also be $\pi_1(M) = \mathbb{Z}$.

As mentioned earlier, the string configuration has been previously studied using the Polyakov loop effective potential. To validate its existence and determine its properties more accurately, it is necessary to simulate the exact partition function with a string in the background, which we undertake in this work. This approach accounts for all possible thermal fluctuations, ensuring more reliable results. In the following, we describe the lattice formulation of the $SU(3)$ gauge theory at finite temperature.

III. $SU(3)$ GAUGE THEORY ON THE LATTICE

We consider the standard Wilson action [27], as the discrete formulation of the action, Eq. (2), on a four-dimensional Euclidian lattice,

$$S = \beta \sum_{n, \mu > \nu} \text{Tr} \left[1 - \frac{1}{3} \text{Re} U_{\mu\nu} \right], \quad (7)$$

where $U_{\mu\nu} = U_\mu(n)U_\nu(n + \hat{\mu})U_\mu^\dagger(n + \hat{\nu})U_\nu^\dagger(n)$ is the standard Wilson plaquette with $\mu, \nu = 1, 2, 3, 4$, $n = (\vec{n}, n_4)$. $\hat{\mu}$ is a vector of length equal to the lattice spacing(a) in the μ^{th} direction. $U_\mu(n) = \exp[igaA_\mu(n)]$ is the link variable, residing on the link connecting the lattice points n and $n + \hat{\mu}$. $\beta = 6/g^2$ is the lattice gauge coupling that regulates the lattice constant (a) and the temperature scale through the β function.

On the lattice, the trace of the product of links in the temporal direction gives the Polyakov loop operator,

$$L(\vec{n}) = \frac{1}{3} \text{Tr} \prod_{n_4=1}^{N_\tau} U_4(\vec{n}, n_4). \quad (8)$$

For the realization of the string configurations, we use the Polyakov loop as a function of two spatial directions (x, y), which is given by,

$$L(n_1, n_2) = \frac{1}{N_z} \sum_{n_3=1}^{N_z} \frac{1}{3} \text{Tr} \prod_{n_4=1}^{N_\tau} U_4(n_1, n_2, n_3, n_4). \quad (9)$$

Under gauge transformation, the gauge links transform as: $U_\mu(n) \rightarrow \Lambda(n) U_\mu(n) \Lambda^\dagger(n + \hat{\mu})$. As mentioned previously, since the Euclidean time is compactified with a period of N_τ , the gauge transformation is required to be periodic only up to a center element, i.e., $\Lambda(N_\tau, \vec{n}) = z \Lambda(0, \vec{n})$. Accordingly, the Polyakov loop transforms as $L(n) \rightarrow z L(n)$, where $z \in Z_3$ group, i.e., $z \in 1, \exp(2\pi i/3), \exp(4\pi i/3)$. At finite temperatures when the gauge fields undergo CD transition, in the deconfined phase, the Z_3 symmetry is broken, which is characterized by the nonvanishing value of $L(n)$. The possible values of z allow us to have interfaces and string configurations.

String configurations are topological structures that are generally unexpected in an equilibrium system due to their associated free energy cost. They form during phase transitions, evolve, and subsequently annihilate upon encountering their counterparts. For example, a string and an antistring (where the total phase variation is -2π for a loop traversed clockwise in physical space) will annihilate each other.

To compute the free energy of strings, appropriate boundary conditions are imposed on the lattice to induce a specific string configuration, as discussed in the next Section. Specifically, we evaluate the free energy difference between the configurations with and without strings. The free energy can be calculated from the partition function, $F = -T \ln Z$, where Z is the partition function and T is the temperature, but in practice, the direct evaluation of the partition function in lattice simulations is avoided as it is difficult. Instead, we evaluate the derivative of the free energy wrt the lattice gauge coupling β , which is related to the action difference between simulations with and without the string, i.e.,

$$\frac{\partial}{\partial \beta} \left(\frac{F}{T} \right) = \frac{1}{\beta} \langle \Delta S \rangle, \quad (10)$$

where $\Delta S = S_1 - S_0$, and S_1 and S_0 are actions with and without string, respectively. The free energy is obtained by integrating the expectation value,

$$\begin{aligned} \left. \frac{F}{T} \right|_{\beta_c}^\beta &= \int_{\beta_c}^\beta d\beta \left\langle \frac{1}{\beta} \Delta S \right\rangle \\ &= \int_{\beta_c}^\beta d\beta \left\langle \Delta \left(\sum_{n, \mu > \nu} \text{Tr} \left[1 - \frac{1}{3} \text{Re} U_{\mu\nu} \right] \right) \right\rangle. \end{aligned} \quad (11)$$

On a lattice with temporal extension N_τ , the temperature is given by $T = 1/(aN_\tau)$. The string tension, σ , of the string is defined as free energy per unit length,

$$\sigma = \frac{F}{L_z}, \quad (12)$$

where L_z is the spatial extension in the z direction (length of the string). From this, we compute the string tension in units of T^2 , i.e., σ/T^2 , as a function of β . Since three or more domain walls emanate from the string and extend to infinity, the string tension will be dependent on the size of the system.

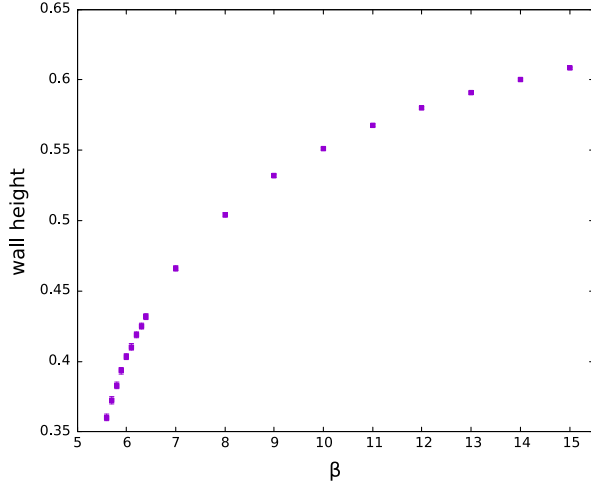
IV. NUMERICAL SETUP AND RESULTS

For the simulations, we use lattices of size $N_x = N_y = 60$, $N_z = 4$ with $N_\tau = 2, 4$. The string is aligned along the z direction. We consider $N_z = 4$ as it suppresses fluctuations in the string length. For $N_\tau = 2$ and 4, the critical couplings for the CD transition are $\beta_c = 5.099$ and 5.6925, respectively. Since string configurations exist only in the deconfined phase, we consider $\beta > \beta_c$ in our simulations.

For the free energy calculations, we carry out simulations with β range [5.2, 15.0] for $N_\tau = 2$ and [5.8, 12.0] for $N_\tau = 4$. Thermal configurations of link variables are generated using the Cabibbo-Marinari algorithm [63]. Each link variable is updated using this heat-bath algorithm, which constitutes a single sweep. Since a new configuration is generated from a previous one, there is always a nonzero autocorrelation between them. To reduce this correlation, we perform five heat-bath sweeps between successive measurements. For each β , we sample 5000 configurations for $N_\tau = 2$ and 10000 for $N_\tau = 4$ to compute physical observables; the larger sample size for $N_\tau = 4$ is required due to stronger fluctuations.

As discussed earlier, in effective models, in the presence of domain walls the manifold M takes the form of a deformed triangle (Fig. 3). To establish that the string configuration remains topological in the exact theory, it is necessary to show that M is a deformed triangle of nonzero size in the complex Polyakov loop plane. The size of M is determined by the bulk equilibrium value of the Polyakov loop (largest) and the value at the core of the domain walls (smallest). As long as the smallest value remains nonzero, the size of M is nonzero and the corresponding string configurations are topological.

We therefore calculate the magnitude of the Polyakov loop at the core of the domain walls. The results for the Polyakov loop value at the center of the domain wall for different β are shown for $N_\tau = 2$ in Fig. 4. We mention here that the Polyakov loop magnitude, at the core of the domain walls, decreases with decrease in temperature faster than the thermal average of the Polyakov loop. This faster

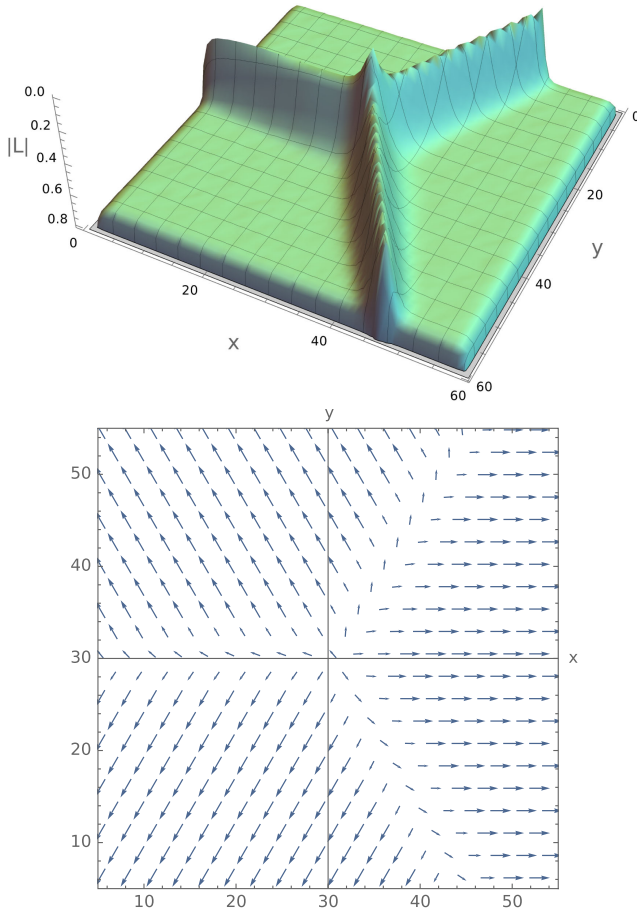
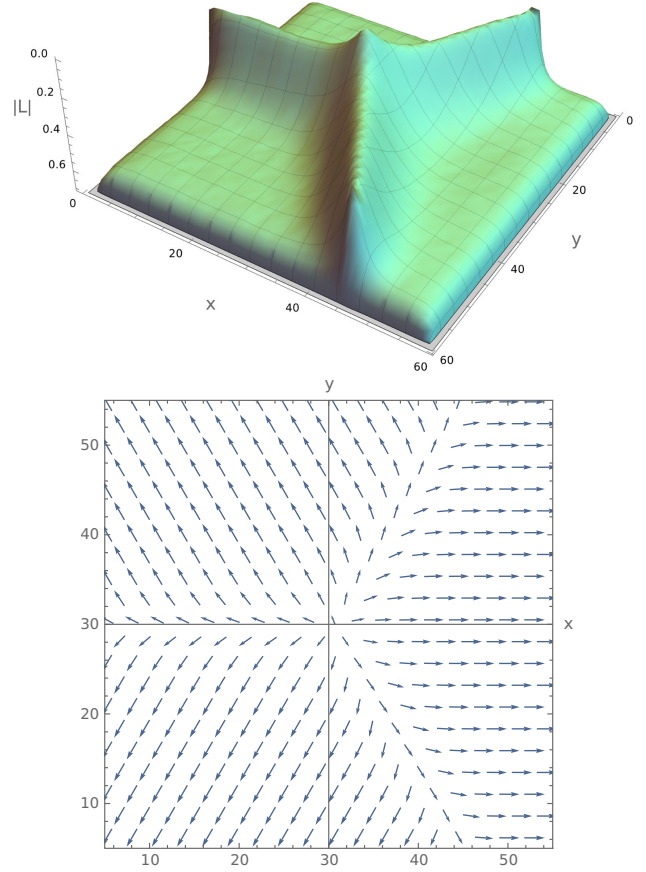
FIG. 4. Interface wall height vs β for $N_\tau = 2$.

decrease especially near T_c could be due to perfect wetting, i.e nucleation of confinement-deconfinement interfaces. The results of Fig. 4 show that the Polyakov loop at the core of domain walls is indeed nonzero. Thus the Z_3 strings

resulting from the junctions of these walls are topological, as they correspond to nontrivial mappings from loops in physical space to M . Consequently, we observe that the Polyakov loop vanishes at the core of the Z_3 strings, see Figs. 5 and 6.

The calculation of the free energy of the string configuration requires simulations both with and without the string. This involves determining the difference in action between the two cases, ΔS . In the simulations with the string, the temporal links are set according to n_4 . For $n_4 < N_\tau$, all the temporal links are set to $U_4(n) = 1$. For $n_4 = N_\tau$, they are set according to the azimuthal angle coordinate (θ_a) corresponding to the position vector $\vec{n}$. $U_4(n) = 1$ for $-\pi/3 \leq \theta_a \leq \pi/3$ and $U_4(n) = z^{-1}(z)$ for $-\pi(\pi/3) < \theta_a < -\pi/3(\pi)$, where $z = \exp(2\pi i/3)$.

Keeping the temporal links at the boundary fixed ensures that the initial configuration thermalizes into a string with domain walls attached. Further, the boundary conditions ensures that the average angular separation between the domain walls is $2\pi/3$, corresponding to the lowest free energy configurations. A typical profile of the string at $\beta = 7.0$ for $N_\tau = 2$ and at $\beta = 10.0$ for $N_\tau = 4$ is shown in

FIG. 5. Typical string attached to Z_3 domain walls for $\beta = 7.0$ for $N_\tau = 2$. The absolute value of the Polyakov loop as a function of (x, y) (top). A vector plot with the real and imaginary part of the Polyakov loop (bottom).FIG. 6. Typical string attached to Z_3 domain walls for $\beta = 10.0$ for $N_\tau = 4$. The absolute value of the Polyakov loop as a function of (x, y) (top). A vector plot with the real and imaginary part of the Polyakov loop (bottom).

Figs. 5 and 6, respectively, where the Polyakov loop is averaged over the z direction. The string configuration shows that the domain walls broaden near the string junction. Our observations indicate that this is due to the presence of other domain walls at the junction. This broadening resembles the broadening observed close to the confinement-deconfinement transition due to perfect wetting. The effect of this boundary condition on the magnitude profile of the Polyakov loop decays within a few lattice points from the boundary wall. To eliminate boundary effects, we exclude the region near the boundary where physical observables deviate from their bulk equilibrium values.

In $SU(N)$ gauge theories with $N > 2$, when the Z_N symmetry is spontaneously broken, the string configuration is always attached to N domain walls. The interface tension of these walls contributes significantly to the total string tension. Therefore before discussing the string configuration, we first present our results for the interface tension of the domain walls connected to the string configurations.

We calculate the interface tension away from the string core. This is because the magnitude of the Polyakov loop vanishes at the core (Figs. 5 and 6), causing the domain walls to deform in that region. To avoid these effects we consider the action difference ΔS within radial annular regions around the string core, defined by $r_a \leq r/a \leq r_b$, with $r_a = 10$ and $r_b = 15, 20$. We then compute the action difference, divide it by three (corresponding to the three

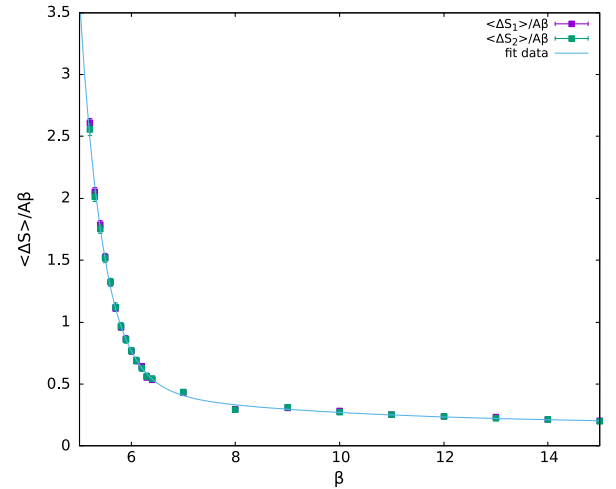
TABLE I. Interface action difference for $N_\tau = 2$.

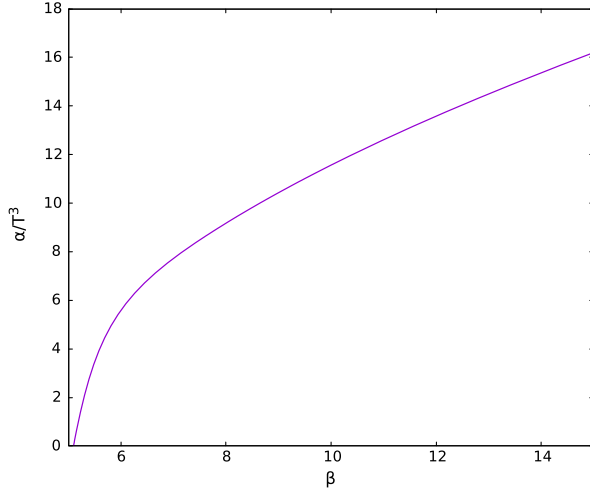
β	Patch 1		Patch 2	
	ΔS_1	Error	ΔS_2	Error
5.20	2.60849	0.03857	2.55569	0.02348
5.30	2.05266	0.03285	2.01550	0.02023
5.40	1.78793	0.03127	1.75478	0.01932
5.50	1.52388	0.02907	1.51606	0.01782
5.60	1.32463	0.02663	1.32239	0.01620
5.70	1.11286	0.02590	1.12360	0.01620
5.80	0.95910	0.02502	0.96622	0.01540
5.90	0.86711	0.02341	0.85801	0.01430
6.00	0.76401	0.02248	0.77086	0.01371
6.10	0.68937	0.02193	0.69040	0.01315
6.20	0.64199	0.02100	0.62903	0.01289
6.30	0.55285	0.02026	0.56255	0.01237
6.40	0.53666	0.01950	0.54253	0.01197
7.00	0.43201	0.01709	0.43442	0.01009
8.00	0.30046	0.01422	0.29513	0.00844
9.00	0.30913	0.01227	0.31334	0.00728
10.00	0.28124	0.01085	0.27210	0.00643
11.00	0.25258	0.00971	0.25486	0.00575
12.00	0.23825	0.00889	0.24045	0.00521
13.00	0.23209	0.00808	0.22244	0.00477
14.00	0.21455	0.00745	0.21222	0.00445
15.00	0.20374	0.00696	0.19966	0.00403

TABLE II. Interface action difference for $N_\tau = 4$.

β	Patch 1		Patch 2	
	ΔS_1	Error	ΔS_2	Error
5.80	0.22362	0.02721	0.26723	0.01704
5.90	0.25909	0.02521	0.28556	0.01566
6.00	0.22189	0.02370	0.23457	0.01473
6.10	0.22350	0.02229	0.21348	0.01389
6.20	0.17070	0.02189	0.16103	0.01359
6.30	0.16071	0.02124	0.15917	0.01315
6.40	0.14759	0.02051	0.15209	0.01284
7.00	0.10863	0.01755	0.11986	0.01088
8.00	0.08822	0.01492	0.08180	0.00925
9.00	0.08280	0.01282	0.09236	0.00801
10.00	0.07491	0.01134	0.07590	0.00701
11.00	0.06360	0.01006	0.07088	0.00622
12.00	0.05636	0.00936	0.06180	0.00576

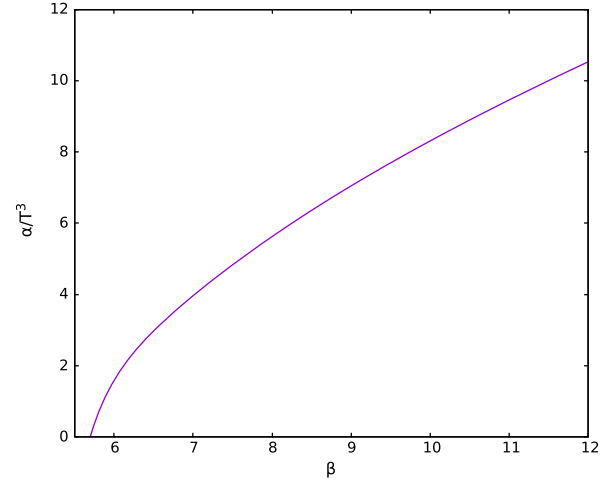
interfaces), and normalize by the area of the annular patches as well as the area of the domain walls. The data for the interface action difference are presented in Tables I and II. The corresponding plots as a function of β are shown in Figs. 7 and 9 for $N_\tau = 2$ and 4, respectively. In the tables, the results for the patch with $r_b = 15$ ($r_b = 20$) are referred to as patch 1 (patch 2). In the plot, ΔS_1 and ΔS_2 correspond to $r_b = 20$ and 15, respectively. The weak coupling analysis suggests that for large β the action difference is expected to behave as $\sim 1/\sqrt{\beta}$, [45]. We find that our measured values of action differences are compatible with the weak coupling results. However, to fit the data in the whole range of β , we consider the following function $f(\beta) = b_1 \exp[-c_1(\beta - d_1)] + b_2 \exp[-c_2(\beta - d_2)] + e_1$. We then integrate this fitted function to calculate the free energy of the interface configuration.

FIG. 7. $\Delta S/A\beta$ vs β for the Z_3 interface with fitted curve, $N_\tau = 2$.

FIG. 8. β dependence of α/T^3 plot for $N_\tau = 2$.

For β close to β_c there are large fluctuations in the location of the string core, leading to larger errors in the action difference. Nevertheless, we obtain a reasonable fit to the data very close to β_c . The results for the interface tension α/T^3 as a function of β are shown in Figs. 8 and 10 corresponding to $N_\tau = 2$ and 4. The results for $N_\tau = 2$ qualitatively agree with previous work [33], confirming the reliability of our method.

It is well known that the pure $SU(3)$ CD transition is weakly first order. At $\beta = \beta_c$ the confined state ($L = 0$) and the three ($L \neq 0$) Z_3 states are degenerate. As a result, in addition to Z_3 domain walls, CD interfaces also emerge. Effective model calculations indicate that near the transition point these domain walls decay into pairs of CD interfaces. This behavior is also observed in the lattice simulations, where due to *perfect wetting* locally CD interfaces can nucleate near the domain wall core, effectively broadening the domain walls and eventually, for

FIG. 10. β dependence of α/T^3 plot for $N_\tau = 4$.

temperatures near T_c , lead to the decay of domain walls into CD interfaces, [39,40]. Thus at $\beta = \beta_c$ the interface tension is expected to be approximately twice that of the CD interface. In the following we present our results for the string tension for $N_\tau = 2$ and $N_\tau = 4$.

A. $N_\tau = 2$ results

For $N_\tau = 2$, we compute the action difference ΔS corresponding to different radial patches around the string core. The results for $\Delta S/\beta$ as a function of β are shown in Fig. 11 and the corresponding data is given in Table III. We mention here that the action difference will constitute the contribution from the domain walls as well as the string core. Unlike the contribution from the domain walls, there are no known results for the string core in the weak coupling limit. However, reasonable fit to the data for the whole range of β considered for simulations was achieved with the function $f(\beta)$ defined earlier. For smaller

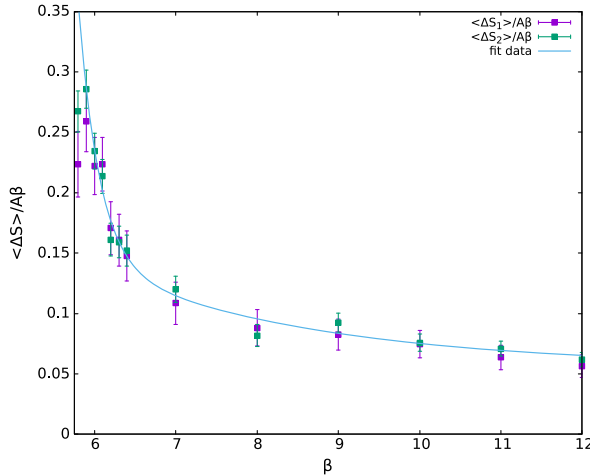
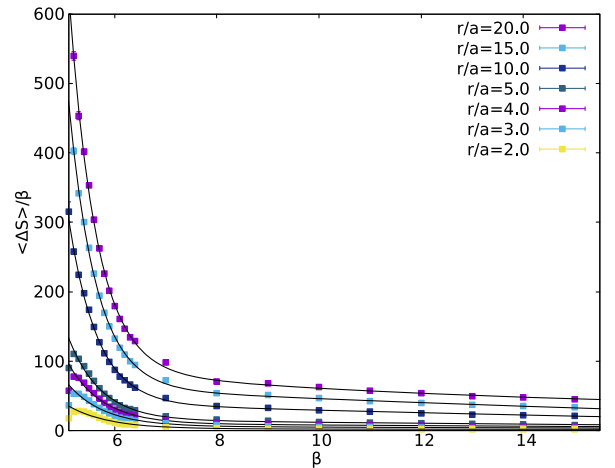
FIG. 9. $\Delta S/A\beta$ vs β for the Z_3 interface with fitted curve, $N_\tau = 4$.FIG. 11. For $N_\tau = 2$, $\Delta S/\beta$ vs β for different radial patches $r/a = 5, 10, 15, 20$ fitted curve with function $f(x)$.

TABLE III. Action difference ΔS and jackknife error for $N_\tau = 2$.

β	$r/a = 5$		$r/a = 10$		$r/a = 15$		$r/a = 20$	
	ΔS	Error	ΔS	Error	ΔS	Error	ΔS	Error
5.2	110.36484	0.60116	257.87405	1.25330	402.49983	1.86822	539.88657	0.41658
5.3	103.59388	0.54066	224.67773	1.07956	341.85150	1.61047	453.54355	0.38919
5.4	92.62027	0.50698	197.82285	1.00640	300.20597	1.50576	401.59460	0.32945
5.5	82.39651	0.47625	174.47203	0.95316	263.79369	1.39979	353.10258	0.30470
5.6	71.70216	0.45873	149.49289	0.90595	226.07948	1.33539	303.65829	0.28723
5.7	61.30492	0.43776	127.88278	0.86314	194.87043	1.26551	262.55875	0.27555
5.8	53.40427	0.40917	111.89299	0.81756	169.56406	1.20557	226.53799	0.26649
5.9	47.13730	0.40463	99.47629	0.79970	150.55162	1.15040	201.96085	0.25441
6.0	41.00473	0.37837	88.08466	0.76249	132.83176	1.12734	179.68027	0.24403
6.1	36.47997	0.37304	78.15178	0.72854	119.75076	1.10030	161.14301	0.23580
6.2	33.01000	0.36098	72.43353	0.70372	109.79351	1.05420	147.18480	0.22791
6.3	31.43271	0.34900	66.88779	0.69024	100.48857	1.02028	134.15992	0.22342
6.4	29.26770	0.34022	62.58853	0.66935	94.97354	0.98328	128.83123	0.21129
7.0	20.97773	0.28890	47.49854	0.58267	72.86676	0.84560	98.35134	0.18141
8.0	16.62055	0.24176	35.60965	0.48075	53.88290	0.70482	71.32716	0.15119
9.0	14.75837	0.21165	33.48775	0.41564	51.24049	0.60717	68.67947	0.12853
10.0	13.04334	0.18806	29.76977	0.37084	46.83962	0.53784	62.64910	0.11453
11.0	11.74472	0.17133	27.68351	0.33805	42.57028	0.49007	57.46369	0.10168
12.0	10.89854	0.15542	25.42321	0.29672	39.76455	0.43152	54.15185	0.09198
13.0	9.85393	0.14322	23.71855	0.27401	37.22653	0.39858	50.06809	0.08413
14.0	9.29583	0.13142	22.70685	0.25704	35.26673	0.37050	48.20444	0.07886
15.0	8.54539	0.11975	21.33430	0.23581	33.56463	0.34129	45.18588	0.07070

patches, as β approaches β_c from above, the rise in ΔS stops and decreases to a smaller nonzero value at β_c . However, this change occurs in a very narrow region close to β_c and does not affect the results qualitatively.

Following this, we integrate $f(\beta)$ using Eq. (11) to obtain the free energy of the string configuration. The resulting string tension σ/T^2 as a function of β for different radial patches is shown in Fig. 12. A steep rise in σ/T^2 is

observed close to β_c , followed by an approximately linear increase for larger β .

B. $N_\tau = 4$ results

We repeat the same analysis for $N_\tau = 4$ lattices. The action difference ΔS is calculated for the radial patches around the string core Fig. 13 and fitted using the same function $f(\beta)$ and the corresponding data is given in Table IV. The qualitative behavior of ΔS as a function

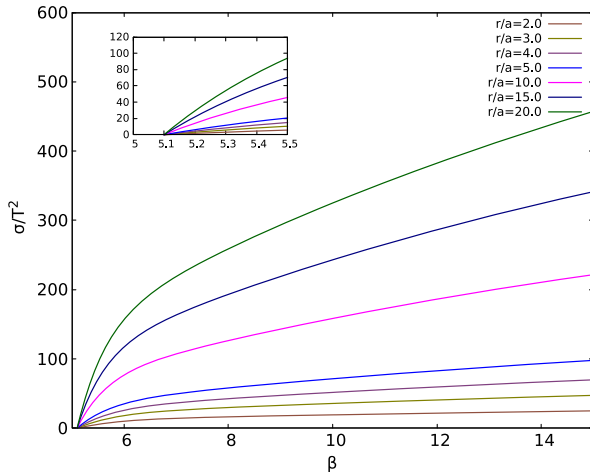


FIG. 12. For $N_\tau = 2$, β dependence of σ/T^2 plot for different radial patches $r/a = 5, 10, 15, 20$.

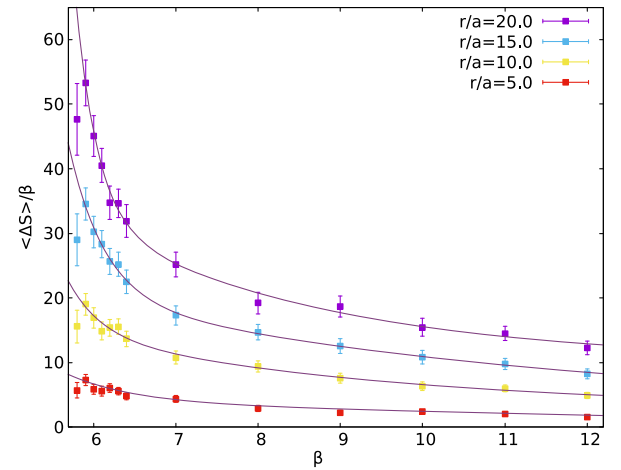


FIG. 13. For $N_\tau = 4$, $\Delta S/\beta$ vs β for different radial patches $r/a = 5, 10, 15, 20$ fitted curve with function $f(x)$.

TABLE IV. Action difference ΔS and jackknife error for $N_\tau = 4$.

β	$r/a = 5$		$r/a = 10$		$r/a = 15$		$r/a = 20$	
	ΔS	Error	ΔS	Error	ΔS	Error	ΔS	Error
5.8	5.68990	1.19789	15.59788	2.50520	29.01490	4.01806	47.66482	5.53609
5.9	7.31175	0.89521	19.02333	1.65565	34.56860	2.45648	53.29115	3.51277
6.0	5.91528	0.76407	16.92231	1.54608	30.23584	2.45368	45.07127	3.11033
6.1	5.58601	0.72473	14.89238	1.40068	28.30254	2.10851	40.51041	2.61986
6.2	6.07489	0.64556	15.42404	1.29691	25.66628	1.98814	34.74756	2.53741
6.3	5.56974	0.57049	15.55875	1.17513	25.20144	1.86186	34.65953	2.21769
6.4	4.78848	0.52888	13.67205	1.16905	22.52737	1.81993	31.92254	2.50208
7.0	4.38683	0.55541	10.79059	1.03709	17.30854	1.51373	25.17335	1.92691
8.0	2.91041	0.43509	9.39564	0.83151	14.68870	1.19849	19.21144	1.65528
9.0	2.28381	0.36117	7.60547	0.72997	12.57344	1.13864	18.68845	1.59952
10.0	2.39689	0.33818	6.36685	0.64099	10.86126	1.05836	15.47505	1.37010
11.0	2.02822	0.28034	6.00958	0.54015	9.82546	0.84510	14.51481	1.09442
12.0	1.59492	0.24851	4.90090	0.48042	8.28244	0.75817	12.31742	1.04125

of β remains similar to that observed for $N_\tau = 2$. Near β_c the fluctuations in the string core position again lead to larger statistical errors, while for larger β the behavior becomes smoother.

Integrating the fitted function $f(\beta)$ yields the free energy of the string configuration and the corresponding string tension. The dependence of σ/T^2 on β is shown in Fig. 14. We see that the free energy for the string as well as the domain walls rises slower near β_c as compared to $N_\tau = 2$, with a rapid rise near β_c and an approximately linear increase at larger β . These results indicate that the qualitative features of the string configurations remain unchanged with the lattice cutoff.

For both $N_\tau = 2$ and $N_\tau = 4$, the rise in σ/T^2 is larger for bigger radial patches. For smaller patches the contribution of the string core dominates the free energy, while for larger patches the contribution from the domain walls

becomes dominant as they extend to the boundary of the system. These observations indicate that the free energy contribution from the core grows more slowly with β compared to the contribution from the domain walls. Moreover for larger patches the string tension and interface tension results agree up to an overall β independent scale factor, confirming that the interface tension dominates the string tension.

V. CONCLUSION

In this work, we have conducted a detailed first-principles nonperturbative study of string configurations in the deconfined phase of $SU(3)$ gauge theory, originating from the spontaneous breaking of Z_3 symmetry. Using the Polyakov loop effective potential model, we argued that these strings are topologically stable. In the lattice simulations, our focus is mainly on the winding one string, which results from the junction of three domain walls. We used specific boundary conditions that lead to the formation of a string with three domain walls connected. Our methodology, based on indirect free energy estimation through action differences, was validated by recovering known results for the Z_3 interface tension, in qualitatively good agreement with previous studies. This suggests that our approach reliably captures the essential physics of the string configurations. Subsequently, we calculated the string tension, σ/T^2 as a function of β . Our results show that the string tension rises steeply near β_c . For large β (weak coupling limit) the string tension for $N_\tau = 2$ and $N_\tau = 4$ approaches $\sqrt{\beta}$ for large patches as the domain walls dominate over the string core. The Z_3 string being a global defect, the string tension rises with the size of the area of the cross section. For a $U(1)$ global string, the tension approaches $\log(r)$, where r is the radius of the cross section. In this present case, the string tension for a larger

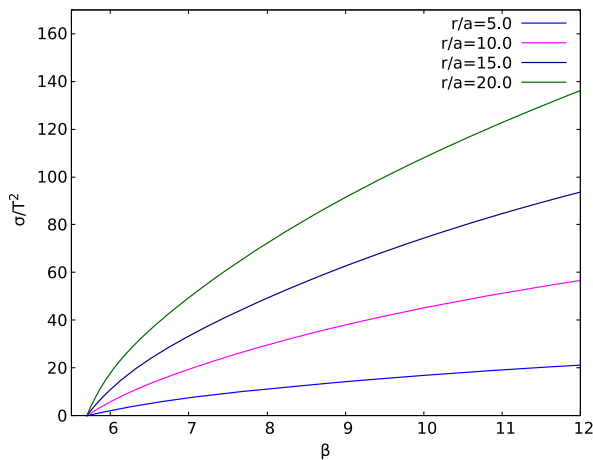


FIG. 14. For $N_\tau = 4$, β dependence of σ/T^2 plot for different radial patches $r/a = 5, 10, 15, 20$.

radius is dominated by the interface tension, hence it rises linearly in r .

Our simulations neglect the effects of dynamical quarks, whose explicit breaking of Z_3 symmetry would likely make these configurations nonstatic. The explicit breaking will make the strings unstable near the critical temperature. However, for temperatures far above T_c they are expected to form and affect the dynamics of the system. In the future, we plan to study the effect of Z_3 explicit breaking on these configurations by incorporating dynamical fermions.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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- [1] H. Satz, Critical behaviour in statistical QCD, *Int. J. Mod. Phys. A* **21**, 672 (2006).
 - [2] Carl G. Kallman and Keith A. Olive, On hot $SU(N)$ gluon matter, *Phys. Lett.* **119B**, 398 (1982).
 - [3] T. Celik, J. Engels, and H. Satz, The latent heat of deconfinement in $SU(3)$ Yang-Mills theory, *Phys. Lett.* **129B**, 323 (1983).
 - [4] Thomas A. DeGrand and Carleton E. DeTar, Thermodynamic properties of the gluon plasma, *Phys. Rev. D* **35**, 742 (1987).
 - [5] Jean Letessier and Johann Rafelski and Ahmed Tounsi, Formation and evolution of the quark—gluon plasma, *Phys. Lett. B* **333**, 484 (1994).
 - [6] G. S. Bali, K. Schilling, A. Hulsebos, A. C. Irving, Christopher Michael, and P. W. Stephenson (UKQCD Collaboration), A comprehensive lattice study of $SU(3)$ glueballs, *Phys. Lett. B* **309**, 378 (1993).
 - [7] M. Caselle, R. Fiore, F. Gliozzi, M. Hasenbusch, K. Pinn, and S. Vinti, Rough interfaces beyond the Gaussian approximation, *Nucl. Phys.* **B432**, 590 (1994).
 - [8] V. M. Belyaev, Order parameter and effective potential, *Phys. Lett. B* **254**, 153 (1991).
 - [9] F. Lenz, The center symmetric phase of QCD, *AIP Conf. Proc.* **494**, 443 (1999).
 - [10] M. Gazdzicki, Evidence for quark gluon plasma from hadron production in high-energy nuclear collisions, *Nucl. Phys.* **A681**, 153 (2001).
 - [11] M. Baldo, P. Castorina, and D. Zappala, Gluon condensation and deconfinement critical density in nuclear matter, *Nucl. Phys.* **A743**, 3 (2004).
 - [12] R. Fiore, P. Giudice, and A. Papa, Numerical test of Polyakov loop models in high temperature $SU(2)$, *Nucl. Phys. B, Proc. Suppl.* **140**, 583 (2005).
 - [13] A. V. Nefediev, Yu. A. Simonov, and M. A. Trusov, Deconfinement and quark-gluon plasma, *Int. J. Mod. Phys. E* **18**, 549 (2009).
 - [14] J. Greensite, The potential of the effective Polyakov line action from the underlying lattice gauge theory, *Phys. Rev. D* **86**, 114507 (2012).
 - [15] S. Sakai, A. Nakamura, and T. Saito, Transport coefficients of quark gluon plasma from lattice gauge theory, *Nucl. Phys.* **A638**, 535 (1998).
 - [16] S. Borsanyi, J. Danzer, Z. Fodor, C. Gatttringer, and A. Schmidt, Coherent center domains from local Polyakov loops, *J. Phys. Conf. Ser.* **312**, 012005 (2011).
 - [17] M. Albanese *et al.* (APE Collaboration), Glueball masses and string tension in lattice QCD, *Phys. Lett. B* **192**, 163 (1987).
 - [18] G. S. Bali, H. Neff, T. Düssel, T. Lippert, and K. Schilling (SESAM Collaboration), Observation of string breaking in QCD, *Phys. Rev. D* **71**, 114513 (2005).
 - [19] J. M. Cornwall and A. Soni, Glueballs as bound states of massive gluons, *Phys. Lett.* **120B**, 431 (1983).
 - [20] A. Yu. Dubin, A. B. Kaidalov, and Yu. A. Simonov, Dynamical regimes of the QCD string with quarks, *Phys. Lett. B* **323**, 41 (1994).
 - [21] P. Petreczky, Lattice QCD at non-zero temperature, *J. Phys. G* **39**, 093002 (2012).
 - [22] C. DeTar and U. M. Heller, QCD thermodynamics from the lattice, *Eur. Phys. J. A* **41**, 405 (2009).
 - [23] M. Baker, P. Cea, V. Chelnokov, L. Cosmai, and A. Papa, Unveiling the flux tube structure in full QCD, *Eur. Phys. J. C* **85**, 29 (2025).
 - [24] P. Bicudo, N. Cardoso, O. Oliveira, and P. J. Silva, String tension at finite temperature lattice QCD, *Proc. Sci. LATTICE2011* (2011) 300.
 - [25] Svetitsky, Benjamin, and Yaffe, Laurence G. Critical behavior at finite temperature confinement transitions, *Nucl. Phys.* **B210**, 423 (1982).
 - [26] T. Celik, J. Engels, and H. Satz, The order of the deconfinement transition in $SU(3)$ Yang-Mills theory, *Phys. Lett.* **125B**, 411 (1983).
 - [27] G. Boyd, J. Engels, F. Karsch, E. Laermann, C. Legelan, M. Lutgemeier, and B. Petersson, Thermodynamics of $SU(3)$ lattice gauge theory, *Nucl. Phys.* **B469**, 419 (1996).
 - [28] Biagio Lucini, Michael Teper, and Urs Wenger, Properties of the deconfining phase transition in $SU(N)$ gauge theories, hep-lat/0502003, *J. High Energy Phys.* **02** (2005) 033.
 - [29] Y. Iwasaki, K. Kanaya, T. Yoshie, T. Hoshino, T. Shirakawa, Y. Oyanagi, S. Ichii, and T. Kawai, Finite temperature phase transition of $SU(3)$ gauge theory on $N(t) = 4$ and 6 lattices, *Phys. Rev. D* **46**, 4657 (1992).

- [30] R. Brower, S. Huang, J. Potvin, and C. Rebbi, The Surface tension of nucleating hadrons using the free energy of an isolated quark, *Phys. Rev. D* **46**, 2703 (1992).
- [31] S. Nadkarni, Large scale structure of the deconfined phase, *Phys. Rev. Lett.* **60**, 491 (1988).
- [32] N. D. Mermin, The topological theory of defects in ordered media, *Rev. Mod. Phys.* **51**, 591 (1979).
- [33] K. Kajantie, Leo Karkkainen, and K. Rummukainen, Tension of the interface between two ordered phases in lattice $SU(3)$ gauge theory, *Nucl. Phys.* **B357**, 693 (1991).
- [34] Yasumichi Aoki and Kazuyuki Kanaya, Interface tension in $SU(3)$ lattice gauge theory at finite temperatures on an $N(\tau) = 2$ lattice, *Phys. Rev. D* **50**, 6921 (1994).
- [35] K. Holland, Confinement in the deconfined phase: A numerical study with a cluster algorithm, *Phys. Rev. D* **60**, 074022 (1999).
- [36] S. Huang, J. Potvin, C. Rebbi, and S. Sanielevici, Surface tension in finite temperature quantum chromodynamics, *Phys. Rev. D* **42**, 2864 (1990); *Phys. Rev. D* **43**, 2056(E) (1991).
- [37] Philippe de Forcrand and David Noth, Precision lattice calculation of $SU(2)$ 't Hooft loops, *Phys. Rev. D* **72**, 114501 (2005).
- [38] Philippe de Forcrand, Biagio Lucini, and David Noth, 't Hooft loops and perturbation theory, *Proc. Sci. LAT2005* (2006) 323 [arXiv:hep-lat/0510081].
- [39] Philippe de Forcrand, Biagio Lucini, and Michele Vettorazzo, Measuring interface tensions in 4d $SU(N)$ lattice gauge theories, *Nucl. Phys. B, Proc. Suppl.* **140**, 647 (2005).
- [40] Z. Frei and A. Patkos, Perfect wetting: An alternative for hadronic matter formation in the cooling universe, *Phys. Lett. B* **229**, 102 (1989).
- [41] S. T. West and J. F. Wheeler, High temperature properties of the $Z(3)$ interface in $(2+1)$ -dimensions $SU(3)$ gauge theory, *Phys. Lett. B* **383**, 205 (1996).
- [42] S. T. West and J. F. Wheeler, Critical properties of the $Z(3)$ interface in $(2+1)$ -dimensions $SU(3)$ gauge theory, *Nucl. Phys.* **B486**, 261 (1997).
- [43] C. Korthals Altes, A. Michels, M. Stephanov, and M. Teper, Gauge theory in $d = 2 + 1$ at high temperature: $Z(N)$ interface, *Nucl. Phys. B, Proc. Suppl.* **42**, 517 (1995).
- [44] Nathan Weiss, The effective potential for the order parameter of gauge theories at finite temperature, *Phys. Rev. D* **24**, 475 (1981).
- [45] Tanmoy Bhattacharya, Andreas Gocksch, Chris Korthals Altes, and Robert D. Pisarski, $Z(N)$ interface tension in a hot $SU(N)$ gauge theory, *Nucl. Phys.* **B383**, 497 (1992).
- [46] P. Giovannangeli and C. P. Korthals Altes, 't Hooft and Wilson loop ratios in the QCD plasma, *Nucl. Phys.* **B608**, 203 (2001).
- [47] Biswanath Layek, Ananta P. Mishra, and Ajit M. Srivastava, Strings with a confining core in a quark-gluon plasma, *Phys. Rev. D* **71**, 074015 (2005).
- [48] A. P. Balachandran and S. Digal, Topological string defect formation during the chiral phase transition, *Int. J. Mod. Phys. A* **17**, 1149 (2002).
- [49] R. D. Pisarski, Quark gluon plasma as a condensate of $SU(3)$ Wilson lines, *Phys. Rev. D* **62**, 111501 (2000).
- [50] T. H. Hansson, K. Johnson, and C. Peterson, The QCD vacuum as a glueball condensate, *Phys. Rev. D* **26**, 2069 (1982).
- [51] G. S. Bali and K. Schilling, Static quark—anti-quark potential: Scaling behavior and finite size effects in $SU(3)$ lattice gauge theory, *Phys. Rev. D* **46**, 2636 (1992).
- [52] Robert W. Johnson and Michael J. Teper, String models of glueballs and the spectrum of $SU(N)$ gauge theories in $(2+1)$ -dimensions, *Phys. Rev. D* **66**, 036006 (2002).
- [53] C. E. Detar, O. Kaczmarek, F. Karsch, and E. Laermann, String breaking in lattice quantum chromodynamics, *Phys. Rev. D* **59**, 031501 (1999).
- [54] F. Green and F. Karsch, Mean field analysis of $SU(N)$ deconfining transitions in the presence of dynamical quarks, *Nucl. Phys.* **B238**, 297 (1984).
- [55] P. Hasenfratz, F. Karsch, and I. O. Stamatescu, The $SU(3)$ deconfinement phase transition in the presence of quarks, *Phys. Lett.* **133B**, 221 (1983).
- [56] H. Satz, Universal deconfinement in $SU(2)$ gauge theory with dynamical quarks, *Phys. Lett.* **157B**, 65 (1985).
- [57] V. M. Belyaev, Ian I. Kogan, and G. W. Semenoff, Nathan Weiss, $Z(N)$ domains in gauge theories with fermions at high temperature, *Phys. Lett. B* **277**, 331 (1992).
- [58] Uma Shankar Gupta, Ranjita K. Mohapatra, Ajit M. Srivastava, and Vivek K. Tiwari, Effects of quarks on the formation and evolution of $Z(3)$ walls and strings in relativistic heavy-ion collisions, *Phys. Rev. D* **86**, 125016 (2012).
- [59] Mridupawan Deka, Sanatan Digal, and Ananta P. Mishra, Metastable states in quark-gluon plasma, *Phys. Rev. D* **85**, 114505 (2012).
- [60] Minati Biswal, Sanatan Digal, and P. S. Saumia, Z_3 metastable states in PNJL model, *Phys. Rev. D* **102**, 074020 (2020).
- [61] Georgi Howard, Lie algebras in particle physics.
- [62] Robert D. Pisarski, Tests of the Polyakov loops model, *Nucl. Phys.* **A702**, 151 (2002).
- [63] N. Cabibbo and E. Marinari, A new method for updating $SU(N)$ matrices in computer simulations of gauge theories, *Phys. Lett.* **119B**, 387 (1982).