

**Erratum: New temperature inversion estimator
to detect CMB patchy screening by large-scale structure
[Phys. Rev. D **109**, 103539 (2024)]**

Theo Schutt , Abhishek S. Maniyar, Emmanuel Schaan, William R. Coulton, and Nishant Mishra



(Received 20 June 2025; published 2 September 2025)

DOI: [10.1103/pps1-zd9m](https://doi.org/10.1103/pps1-zd9m)

**I. THE “TEMPERATURE INVERSION” AND “SIGNED” SCREENING ESTIMATORS
ARE BIASED IN THE PRESENCE OF LENSING**

In Sec. V B, we claim the TI and signed estimators are unbiased in the presence of lensing. In general, this is not true; without adaptations, the estimators are only unbiased in the case of an infinitely small beam and careful filtering of the output reconstructed τ signal. In this erratum, we derive the source of the bias and present a minimal adaptation of the TI estimator to mitigate the lensing bias. The lensing bias to our anisotropic screening estimator was first identified in Refs. [1,2], along with more insights into the lensing bias, a more optimal lens-hardened screening estimator, and associated updated detection signal-to-noise (S/N) forecasts.

II. DERIVING THE LENSING BIAS IN HARMONIC SPACE

As discussed in Sec. V B, the TI and signed estimators are unbiased by lensing in the absence of filtering. However, as also shown in Ref. [3] for kSZ with projected fields, a subtle effect of filtering the temperature fields to form the estimator is that it allows the leakage of lensing into the measured τ signal. In this section, we derive the source of this lensing bias, closely following Ref. [3]. The main difference between the cases of the TI/signed estimators and kSZ with projected fields is that the TI/signed estimators have a different filter for each temperature leg.

We define the unlensed primary CMB temperature fluctuations as T and the lensed temperature fluctuations as $\tilde{T}$:

$$\tilde{T}_\ell^{L/S} = F_\ell^{L/S} b_\ell \tilde{T}_\ell \equiv f_\ell^{L/S} \tilde{T}_\ell, \quad (1)$$

where b_ℓ is the beam and $F_\ell^{L/S}$ is the low- (L) or high-pass (S) filter. F_ℓ^L is a top-hat filter (or smooth variation thereof) defined by a maximum scale ℓ_{cut}^L and F_ℓ^S by a minimum scale ℓ_{cut}^S . We define the product of the beam and each scale filter as $f_\ell^{L/S}$.

The stacked screening estimator at the positions of galaxies is of the form

$$C_L^{(\tilde{T}^L \tilde{T}^S) \times \delta^g} = \int \frac{d^2 \ell}{(2\pi)^2} \langle \tilde{T}_\ell^L \tilde{T}_{L-\ell}^S \delta_{-L}^g \rangle = \int \frac{d^2 \ell}{(2\pi)^2} f_\ell^L f_{|L-\ell|}^S \langle \tilde{T}_\ell \tilde{T}_{L-\ell} \delta_{-L}^g \rangle, \quad (2)$$

where δ^g is the galaxy density field. The expectation value above may have a nonzero contribution due to lensing of the form $\langle T T \phi \delta \rangle = \langle T T \rangle \langle \phi \delta \rangle$, resulting in a lensing bias. More specifically, we Taylor-expand the temperature fields to first order in the gradient of the lensing potential, $\nabla \varphi$:

$$\tilde{T}(\mathbf{x}) = T(\mathbf{x}) + \nabla \varphi(\mathbf{x}) \cdot \nabla T(\mathbf{x}) + \mathcal{O}((\nabla \varphi)^2). \quad (3)$$

In Fourier space, this is

$$\tilde{T}_\ell = T_\ell - \int \frac{d^2 \ell'}{(2\pi)^2} \ell' \cdot (\ell - \ell') \varphi_{\ell'} T_{\ell - \ell'} + \mathcal{O}((\nabla \varphi)^2). \quad (4)$$

We neglect the lensing contribution to T^L but include it in T^S . The lensing bias to Eq. (2) is then

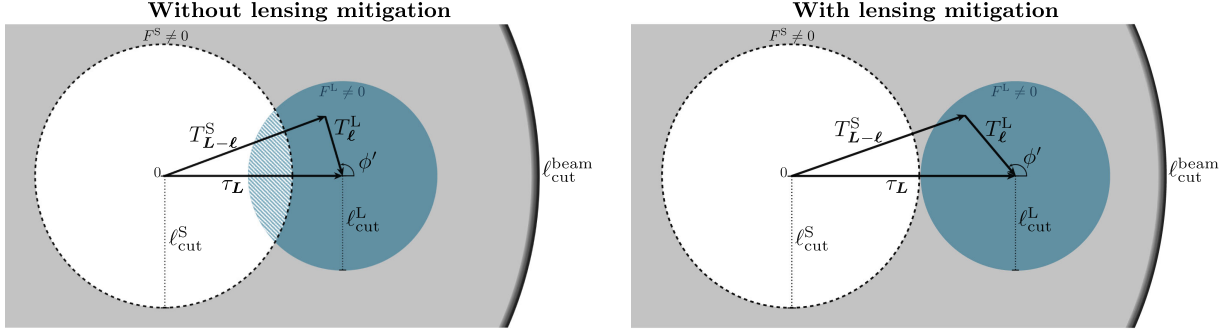


FIG. 1. Harmonic space diagram demonstrating the filtering scheme applied in the TI/signed estimators. The high-pass filter F^S with minimum scale ℓ_{cut}^S imposes $|\mathbf{L} - \boldsymbol{\ell}| > \ell_{\text{cut}}^S$ (lighter gray shaded region), and the low-pass filter F^L with maximum scale ℓ_{cut}^L imposes $|\boldsymbol{\ell}| < \ell_{\text{cut}}^L$ (darker blue shaded region). The beam also imposes an upper limit of $\ell_{\text{cut}}^{\text{beam}}$ (rightmost solid black curve). In order for the ϕ integral in Eq. (5) to equal zero, the integration domain (i.e., where gray and blue overlap) must remain azimuthally symmetric and not cross the F^S or beam cutoff scales. Note that $\phi' = \phi - \pi$ is used in the figures purely for visual clarity. *Left*: Without filtering τ_L , the integration region can overlap with the modes filtered out by F^S (blue striped region), leading to a lensing bias. *Right*: The lensing bias can be mitigated by imposing a filter on τ_L , as proposed in Eq. (6).

$$\begin{aligned}
 \int \frac{d^2 \boldsymbol{\ell}}{(2\pi)^2} f_{\boldsymbol{\ell}}^L f_{|\mathbf{L}-\boldsymbol{\ell}|}^S \langle [\nabla \varphi \cdot \nabla T(\mathbf{x})]_{\mathbf{L}-\boldsymbol{\ell}} T_{\boldsymbol{\ell}} \delta_{-\mathbf{L}}^g \rangle &= - \int \frac{d^2 \boldsymbol{\ell}}{(2\pi)^2} f_{\boldsymbol{\ell}}^L f_{|\mathbf{L}-\boldsymbol{\ell}|}^S \int \frac{d^2 \boldsymbol{\ell}'}{(2\pi)^2} \boldsymbol{\ell}' \cdot (\mathbf{L} - \boldsymbol{\ell} - \boldsymbol{\ell}') \langle \varphi_{\boldsymbol{\ell}'} T_{\mathbf{L}-\boldsymbol{\ell}-\boldsymbol{\ell}'} T_{\boldsymbol{\ell}} \delta_{-\mathbf{L}}^g \rangle \\
 &= - \int \frac{d^2 \boldsymbol{\ell}}{(2\pi)^2} f_{\boldsymbol{\ell}}^L f_{|\mathbf{L}-\boldsymbol{\ell}|}^S (\mathbf{L} \cdot \boldsymbol{\ell}) C_L^{\varphi \delta^g} C_{\boldsymbol{\ell}}^{TT} \\
 &= - \frac{L C_L^{\varphi \delta^g}}{(2\pi)^2} \int d\ell \ell^2 f_{\boldsymbol{\ell}}^L C_{\boldsymbol{\ell}}^{TT} \int d\phi f_{|\mathbf{L}-\boldsymbol{\ell}|}^S \cos \phi,
 \end{aligned} \tag{5}$$

where the second equality uses Wick's theorem, and in the third equality $d^2 \boldsymbol{\ell}$ is expressed in polar coordinates, with ϕ being the angle between $\boldsymbol{\ell}$ and $\mathbf{L}$.

Equation (5) is zero by symmetry of the ϕ integral if the small-scale filter is constant where f^L is nonzero. This is equivalent to the blue shaded region in Fig. 1 remaining a circle—i.e., crossing over neither the minimum high-pass filter scale ℓ_{cut}^S nor the effective beam cutoff scale $\ell_{\text{cut}}^{\text{beam}}$. This can be enforced with a simple filter on the reconstructed $\mathbf{L}$ leg:

$$S'_L = \begin{cases} 1 & \ell_{\text{cut}}^L + \ell_{\text{cut}}^S < |\mathbf{L}| < \ell_{\text{cut}}^{\text{beam}} - \ell_{\text{cut}}^L, \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Implementing the S' filter mitigates lensing to first order at the cost of also filtering out some of the τ signal.

III. REAL-SPACE INTUITION ON THE LENSING BIAS

To gain intuition on the source of the lensing bias, we also derive it here in real space. The real-space analog to Eq. (1) is

$$\tilde{T}^{L/S}(\mathbf{x}) = \int d\mathbf{x}' F^{L/S}(\mathbf{x} - \mathbf{x}') \int d\mathbf{x}'' b(\mathbf{x}' - \mathbf{x}'') \tilde{T}(\mathbf{x}'') \equiv \int d\mathbf{x}' f^{L/S}(\mathbf{x} - \mathbf{x}') \tilde{T}(\mathbf{x}'). \tag{7}$$

As in the previous section, we consider only the lensing contribution to the small-scale $T^S(\mathbf{x})$:

$$T^{S,\text{lens}}(\mathbf{x}) = (\nabla T \cdot \nabla \phi)^S(\mathbf{x}) = \int d\mathbf{x}' f^S(\mathbf{x} - \mathbf{x}') \nabla T(\mathbf{x}') \cdot \nabla \phi(\mathbf{x}'). \tag{8}$$

Consider the lensing contribution to the zero-lag correlation $\langle \hat{\tau}(0) \delta^g(0) \rangle$:

$$\langle \hat{\tau}(0) \delta^g(0) \rangle = \langle T^L(0) (\nabla T \cdot \nabla \phi)^S(0) \delta^g(0) \rangle. \tag{9}$$

In the absence of small-scale filtering, we would get

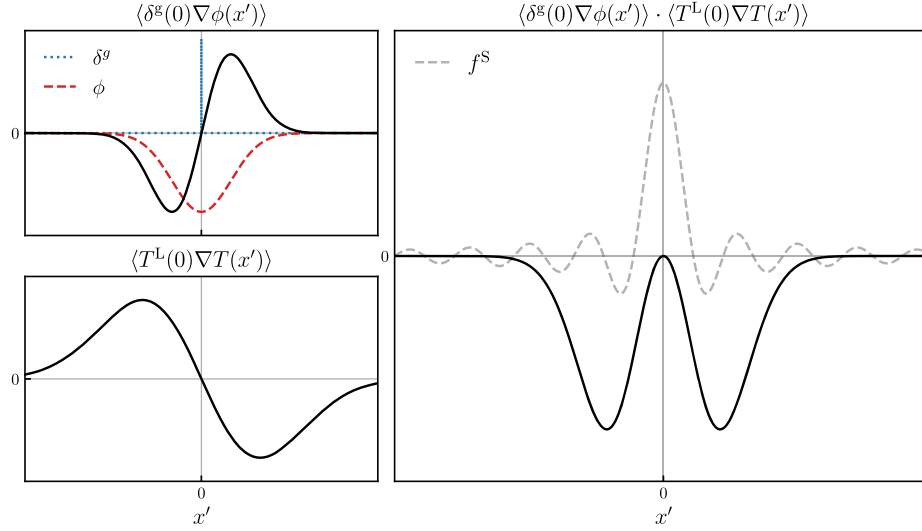


FIG. 2. Schematic of the parity behavior of the quantities in Eq. (12) with respect to the separation x' . *Top left*: At the position of a galaxy at $x' = 0$, δ^g (dotted blue) is a δ function. At the same location, the gravitational potential ϕ (dashed red) has a symmetric feature induced by the galaxy's halo. $\nabla\phi$ (solid black) is subsequently an odd function of x' . *Bottom left*: As shown in Eq. (11), for a Gaussian random field T^L , $\langle T^L(0) \nabla T^L(x') \rangle$ (solid black) is an odd function in x' that is zero everywhere except within the correlation length ($\sim$ degree scale) around $x' = 0$. *Right*: Thus, the product $\langle \delta^g(0) \nabla \phi(x') \rangle \cdot \langle T^L(0) \nabla T(x') \rangle$ (solid black) is even. When convolved with the sinc-like filter f^S (dashed gray), $\langle \hat{\tau}(0) \delta^g(0) \rangle$ will have a nonzero response.

$$\langle \hat{\tau}(0) \delta^g(0) \rangle = \underbrace{\langle T^L(0) \nabla T(0) \rangle}_{=0} \cdot \underbrace{\langle \nabla \phi(0) \delta^g(0) \rangle}_{=0} = 0. \quad (10)$$

However, once we convolve with the small-scale filter f^S , the nonzero-lag correlations must be considered. It can be shown that, in general, $\langle T(0) \nabla T(x') \rangle$ and $\langle \delta^g(0) \nabla \phi(x') \rangle$ are odd functions of the separation x' :

$$\langle T(0) \nabla T(x') \rangle = - \int \frac{d^2 \ell}{(2\pi)^2} \ell C_{\ell}^{TT} \sin \ell x'; \quad \langle \delta^g(0) \nabla \phi(x') \rangle = - \int \frac{d^2 \ell}{(2\pi)^2} \ell C_{\ell}^{\delta^g \phi} \sin \ell x'. \quad (11)$$

Therefore, with the small-scale filtering, we instead get

$$\langle \hat{\tau}(0) \delta^g(0) \rangle = \int dx' \underbrace{f^S(x')}_{\text{even}} \underbrace{\langle T^L(0) \nabla T(x') \rangle}_{\text{odd}} \cdot \underbrace{\langle \nabla \phi(x') \delta^g(0) \rangle}_{\text{odd}} \neq 0. \quad (12)$$

The parity behavior of each function in Eq. (12) as a function of x' is shown in Fig. 2. Essentially, convolution with the f^S filter averages the product of the two odd functions $\langle T^L(0) \nabla T(x') \rangle$ and $\langle \nabla \phi(x') \delta^g(0) \rangle$ over $\sim$ arcmin scales, resulting in a nonzero lensing bias.

IV. MINIMAL METHOD FOR MITIGATING LENSING BIAS

We want to understand how to effectively adapt our TI estimator to incorporate simple filtering that mitigates the present lensing bias. The estimator proposed in the main text is of the form

$$\hat{\tau}_{(R)}^{(1)} = \frac{\sum_{\mathbf{g}} T^L(\mathbf{x}_{\mathbf{g}}) T^{\text{SR}}(\mathbf{x}_{\mathbf{g}})}{\sum_{\mathbf{g}} (T^L(\mathbf{x}_{\mathbf{g}}))^2}, \quad (13)$$

where the summation is over galaxies and $\{\mathbf{x}_{\mathbf{g}}\}$ is the set of galaxy positions. R denotes the application of a ring aperture photometry filter, such that $\hat{\tau}_{(R)}^{(1)}$ is a single number for a given ring aperture inner/outer radius. With the estimator in this form, it is possible but computationally inconvenient to apply an additional filter like Eq. (6) on the L modes for the

reconstructed τ . However, we can implement a very similar alternative method. Here, we form a map $m(\mathbf{x})$ by multiplying the two filtered maps $T^L(\mathbf{x})$ and $T^S(\mathbf{x})$. The S' filter is applied to $m(\mathbf{x})$ to form $m^{S'}(\mathbf{x}) = (T^L(\mathbf{x})T^S(\mathbf{x}))^{S'}$. Before applying any ring aperture filters, stacking on galaxy positions results in a stacked reconstructed τ map:

$$\hat{\tau}^{(2)}(\mathbf{x}) = \frac{\sum_{\mathbf{g}} m^{S'}(\mathbf{x} + \mathbf{x}_{\mathbf{g}})}{\sum_{\mathbf{g}} (T^L(\mathbf{x}_{\mathbf{g}}))^2}. \quad (14)$$

We can then apply the ring aperture filter R to this stacked map to form the estimator

$$\hat{\tau}_{(R)}^{(2)} = \frac{\sum_{\mathbf{g}} m^{S'R}(\mathbf{x}_{\mathbf{g}})}{\sum_{\mathbf{g}} (T^L(\mathbf{x}_{\mathbf{g}}))^2} = \frac{\sum_{\mathbf{g}} [(T^L(\mathbf{x})T^S(\mathbf{x}))^{S'}]^R(\mathbf{x}_{\mathbf{g}})}{\sum_{\mathbf{g}} (T^L(\mathbf{x}_{\mathbf{g}}))^2}. \quad (15)$$

We note that this altered estimator does not reduce to Eq. (13) when S' is removed. However, if T^L does not vary on the scales of the R aperture filter, then $\hat{\tau}_{(R)}^{(2)} \simeq \hat{\tau}_{(R)}^{(1)}$ when $S' = 1$ everywhere (and in any case, $T^L \simeq \text{const.}$ for small scales is an assumption also underlying $\hat{\tau}^{(1)}$).

To confirm the normalization $\sum_{\mathbf{g}} (T^L(\mathbf{x}_{\mathbf{g}}))^2$ remains correct for $\hat{\tau}^{(2)}$ and $\hat{\tau}_{(R)}^{(2)}$, consider the numerator of Eq. (14):

$$\begin{aligned} \langle (T^L T^S)^{S'} \rangle &= \langle [T^L(\tau T^0)^S]^{S'} \rangle = \left\langle W_L^{S'} \int \frac{d^2 \ell}{(2\pi)^2} T_{\ell}^L T_{L-\ell}^S \right\rangle \\ &= \left\langle W_L^{S'} \int \frac{d^2 \ell}{(2\pi)^2} W_{\ell}^L T_{\ell}^0 W_{|L-\ell|}^S \int \frac{d^2 \ell'}{(2\pi)^2} \tau_{\ell'} T_{L-\ell-\ell'}^0 \right\rangle \\ &= W_L^{S'} \int \frac{d^2 \ell}{(2\pi)^2} W_{\ell}^L W_{|L-\ell|}^S \int \frac{d^2 \ell'}{(2\pi)^2} \tau_{\ell'} C_{\ell'}^{TT} (2\pi)^2 \delta_{L-\ell-\ell'}^D = \tau_L W_L^{S'} \int \frac{d^2 \ell}{(2\pi)^2} W_{\ell}^L W_{|L-\ell|}^S C_{\ell}^{TT}. \end{aligned} \quad (16)$$

The W_{ℓ}^L filter imposes an upper limit to $|\ell| < \ell_{\text{cut}}^L$, truncating the integral over harmonic space. In this limit, $|L| \gg |\ell|$, and thus the integral in the final line of Eq. (16) becomes

$$\begin{aligned} \int \frac{d^2 \ell}{(2\pi)^2} W_{\ell}^L W_{|L-\ell|}^S C_{\ell}^{TT} &= \int_{|\ell| < \ell_{\text{cut}}^L} \frac{d^2 \ell}{(2\pi)^2} W_{\ell}^L W_{|L-\ell|}^S C_{\ell}^{TT} \\ &\approx W_L^S \int_{|\ell| < \ell_{\text{cut}}^L} \frac{d^2 \ell}{(2\pi)^2} C_{\ell}^{TT} \\ &\approx W_L^S \langle (T^L)^2 \rangle. \end{aligned} \quad (17)$$

Thus, the normalization remains correct in the regime where τ is reconstructed only on small scales.

Reference [4] v2 presents the updated measurements using this filtering scheme for the signed screening estimator. After lensing mitigation, the measurement significance is no longer sufficient for a detection. As the screening S/N forecasts in the main text were scaled by the original measured signal, the forecasted S/N have been overestimated as well.

- [1] B. Hadzhiyska, N. Sailer, and S. Ferraro, [arXiv:2506.17379](#).
- [2] N. Sailer, B. Hadzhiyska, and S. Ferraro, [arXiv:2506.17217](#).
- [3] S. Ferraro, J. C. Hill, N. Battaglia, J. Liu, and D. N. Spergel, *Phys. Rev. D* **94**, 123526 (2016).
- [4] W. R. Coulton *et al.*, [arXiv:2401.13033](#).