

Erratum: Supernova production of axionlike particles coupling to electrons, reloaded

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In the published version of this work, we used the wrong squared amplitude for the rate of semi-Compton emission. This was pointed out in Ref. [1]. Therefore, in this Erratum we revise our treatment of semi-Compton emission and present all of our corrected results.

In addition, we also correct the constraints from Axionlike particle (ALP) decay into two photons via loop-induced decay. In our original version, we used the loop-induced coupling for a pseudoderivative axion-electron interaction. On the other hand, as we had argued in Sec. III A, the pseudoderivative coupling also leads to an anomalous contribution to the axion-photon coupling, dominated by fluctuations with arbitrarily large energies. This anomalous axion-photon coupling exists up to arbitrarily large energy scales, and is therefore a feature of the ultraviolet theory. For this reason, in agreement with what we wrote in Sec. III A, we now remove the anomalous contribution from the loop-induced axion-photon coupling, effectively using the same coupling structure as for a pseudoscalar axion-electron interaction.¹

The revised emission rates are public in an updated GitHub repository [2].

Below, we present the two sections whose content is directly affected by these corrections, completely revised. Furthermore, we later show all of our figures with these corrections implemented: all results undergo small quantitative changes, without any qualitative change in our final message.

I. SEMI-COMPTON EMISSION

We revise here the content of Sec. III B of the original paper. The fundamental issue is the sign of the interference term $\mathcal{F}_{us}$, which in the published version of the paper was wrong. This leads to a reduction of the emission rate. To make the material self-consistently readable, we report the entirety of Sec. III B corrected, matching the revision posted on arXiv.

For ALPs, a first channel of emission is semi-Compton scattering, through the process $e\gamma \rightarrow ea$. To determine the spectrum of the emitted ALPs, we must also characterize their energy distribution in the scattering or, equivalently, the angular distribution of the emitted ALPs in the center-of-mass frame. Here, we assume that the scattering occurs in the ultrarelativistic regime, where the center-of-mass energy is much larger than the masses of the particles. In fact, typical center-of-mass energies are on the order of $\sqrt{\mu_e T}$, since the electron has typical energy μ_e and the photon has typical energy T . The electron thermal mass is of the order of $\sqrt{\alpha\mu_e}$. Therefore, we are assuming that $T \gg \alpha\mu_e$ (notice that, in order to consider the photons as relativistic, we were already assuming the more stringent condition $T \gg \omega_{pl} \sim \sqrt{\alpha\mu_e}$) and $\sqrt{\mu_e T} \gg m_a$. The validity of these conditions is again limited only to the regions that dominate the emission. If photons cannot be considered relativistic, the emissivity would be dominated anyway by bremsstrahlung, due to the Boltzmann-suppressed photon population. Similarly, if the ALP mass becomes comparable to the center-of-mass energy, the total cross section is kinematically suppressed, making electron-positron coalescence the dominant contribution. Therefore, we adopt these approximations throughout, with the understanding that they may fail in regions that are not relevant for the overall emission. Additionally, this approximation breaks down when the incoming photon and electron are nearly collinear, as the available center-of-mass energy for scattering becomes small. However, the rate of these scatterings is suppressed by the relative velocity between the particles. As we shall see, their contribution to the total result is negligible in the angular integration we perform.

¹We are grateful to Giuseppe Lucente for pointing out this inconsistency to us.

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To proceed, we write the emitted number of ALPs as

$$\frac{dn_a}{dt dV} = \int \frac{d^3 \mathbf{p}}{(2\pi)^3 2E_{\mathbf{p}}} \frac{d^3 \mathbf{q}}{(2\pi)^3 2E_{\mathbf{q}}} \frac{d^3 \mathbf{k}}{(2\pi)^3 2E_{\mathbf{k}}} \frac{d^3 \mathbf{l}}{(2\pi)^3 2E_{\mathbf{l}}} f_e(p) f_\gamma(q) [1 - f_e(p + q - E_a)] (2\pi)^4 \delta^{(4)}(p^\mu + q^\mu - k^\mu - l^\mu) |\mathcal{M}|^2, \quad (1)$$

where $\mathbf{p}$ and $\mathbf{q}$ are the momenta of the incoming electron and photon respectively, $\mathbf{k}$ and $\mathbf{l}$ are the momenta of the outgoing ALP and electron, and the squared matrix element is summed over all spins and polarizations. We denote by p^μ , q^μ , k^μ , l^μ the four-vectors associated with each three-vector, and with $p = |\mathbf{p}|$ and the same for q , k , and l . The distribution functions $f_e(p)$, $f_\gamma(q)$ are the phase-space distributions for the species (Fermi-Dirac for electrons with temperature T and chemical potential μ_e , Bose-Einstein for photons with temperature T); they are the occupation number per single spin or polarization state. The matrix element $|\mathcal{M}|^2$ is summed over spins and polarizations of all particles.

We work in the lab frame, where we can use the delta function to integrate out the momentum $\mathbf{l}$ and the angle between $\mathbf{k}$ and $\mathbf{p} + \mathbf{q}$. The latter is

$$\cos \theta_{\mathbf{k}, \mathbf{p}+\mathbf{q}} = \frac{2E_{\mathbf{k}}E_{\mathbf{p}} + 2(E_{\mathbf{k}} - E_{\mathbf{p}})q + 2pqX - m_a^2}{2k\sqrt{p^2 + q^2 + 2pqX}}, \quad (2)$$

where X is the cosine of the angle between $\mathbf{p}$ and $\mathbf{q}$, and the integration should be performed only in the region in which this expression satisfies $|\cos \theta_{\mathbf{k}, \mathbf{p}+\mathbf{q}}| < 1$; this imposes the kinematic constraint on the scattering.

In this way, we are led to

$$\frac{dn_a}{dE_a dt dV} = \int_0^{+\infty} p dp \int_0^{+\infty} q dq \int_{-1}^{+1} dX \int_0^{2\pi} d\phi \frac{f_e(p) f_\gamma(q) [1 - f_e(p + q - E_a)]}{512\pi^6 |\mathbf{p} + \mathbf{q}|} |\mathcal{M}|^2, \quad (3)$$

where the ALP energy is denoted explicitly by $E_a \equiv E_{\mathbf{k}}$. Here ϕ is the azimuthal angle of the vector $\mathbf{k}$ around the direction of $\mathbf{p} + \mathbf{q}$.

For the matrix element, we separate out explicitly the contributions coming from the two diagrams corresponding to u channel and s channel, and the interference between the two

$$|\mathcal{M}|^2 = 4\pi\alpha g_{ae}^2 [\mathcal{F}_u + \mathcal{F}_s + \mathcal{F}_{us}], \quad (4)$$

where

$$\begin{aligned} \mathcal{F}_u &= \frac{-\text{Tr}[\gamma^\mu (\not{p} - \not{k}) \gamma^5 \not{p} (-\gamma^5) (\not{p} - \not{k}) \gamma_\mu \not{l}]}{(m_{\text{th}}^2 - u)^2} = \frac{8[2(p - k) \cdot p (p - k) \cdot l - p \cdot l u]}{(m_{\text{th}}^2 - u)^2}, \\ \mathcal{F}_s &= \frac{-\text{Tr}[\gamma^5 (\not{p} + \not{q}) \gamma^\mu \not{p} \gamma_\mu (\not{p} + \not{q}) (-\gamma^5) \not{l}]}{(s - m_{\text{th}}^2)^2} = \frac{8[2(p + q) \cdot p (p + q) \cdot l - s p \cdot l]}{(s - m_{\text{th}}^2)^2}, \\ \mathcal{F}_{us} &= \frac{-2\text{Tr}[\gamma^5 (\not{p} + \not{q}) \gamma^\mu \not{p} (-\gamma^5) (\not{p} - \not{k}) \gamma_\mu \not{l}]}{(s - m_{\text{th}}^2)(u - m_{\text{th}}^2)} = \frac{32p \cdot (p - k) l \cdot (p + q)}{(s - m_{\text{th}}^2)(m_{\text{th}}^2 - u)}. \end{aligned} \quad (5)$$

In our published version, the sign of $\mathcal{F}_{us}$ was opposite.

Finally, we can express all the four-products in the numerator in terms of the kinematic invariants s , u , and $t = m_a^2 - s - u$ (since in the numerator we need only account for the ALP mass), obtaining

$$\begin{aligned} \mathcal{F}_u &= -\frac{4su}{(m_{\text{th}}^2 - u)^2} \simeq -\frac{4s}{m_{\text{th}}^2 - u}, \\ \mathcal{F}_s &= -\frac{4su}{(s - m_{\text{th}}^2)^2} \simeq -\frac{4u}{s - m_{\text{th}}^2}, \\ \mathcal{F}_{us} &= -\frac{8(s - m_a^2)(m_a^2 - u)}{(s - m_{\text{th}}^2)(m_{\text{th}}^2 - u)}. \end{aligned} \quad (6)$$

The corrections due to m_{th}^2 need only be kept in the u -channel denominator $m_{\text{th}}^2 - u$, since for massless electrons and ALPs u could nearly vanish for near-backward scatterings in which the ALP carries most of the electron energy. Even though the phase space integral cures the divergence at $X \simeq 1$, to avoid dealing with small values of the denominator in the numerical integration we also keep m_{th} corrections in s channel denominator. So we have

$$s - m_{\text{th}}^2 = 2q(E_{\mathbf{p}} - pX), \quad (7)$$

$$m_{\text{th}}^2 - u = 2E_{\mathbf{k}}E_{\mathbf{p}} - m_a^2 - \frac{2pk}{\sqrt{p^2 + q^2 + 2pqX}} \left[(p + qX) \cos \theta_{\mathbf{k}, \mathbf{p}+\mathbf{q}} + q\sqrt{1 - X^2} \sin \theta_{\mathbf{k}, \mathbf{p}+\mathbf{q}} \cos \phi \right]. \quad (8)$$

In the numerators, m_{th}^2 can be neglected. The integral over ϕ can now be performed explicitly; it is convenient to use the notation $m_{\text{th}}^2 - u = A - B \cos \phi$, with

$$A = 2E_{\mathbf{k}}E_{\mathbf{p}} - m_a^2 - \frac{2pk(p + qX) \cos \theta_{\mathbf{k}, \mathbf{p}+\mathbf{q}}}{\sqrt{p^2 + q^2 + 2pqX}},$$

$$B = \frac{2pkq\sqrt{1 - X^2} \sin \theta_{\mathbf{k}, \mathbf{p}+\mathbf{q}}}{\sqrt{p^2 + q^2 + 2pqX}}. \quad (9)$$

Then the result reads

$$\frac{dn_a}{dE_a dt dV} = \int_0^{+\infty} p dp \int_0^{+\infty} q dq \int_{-1}^{+1} dX \frac{f_e(p)f_\gamma(q)[1 - f_e(p + q - E_a)]}{512\pi^6 |\mathbf{p} + \mathbf{q}|} 4\pi\alpha g_{ae}^2 I, \quad (10)$$

with

$$I = 8\pi \left[\frac{s}{\sqrt{A^2 - B^2}} + \frac{A}{s - m_{\text{th}}^2} - 2 \left(\frac{m_a^2}{\sqrt{A^2 - B^2}} + 1 \right) \left(1 - \frac{m_a^2}{s - m_{\text{th}}^2} \right) \right]. \quad (11)$$

For later reference, it is useful to report also the expression for I in the massless limit

$$I = 8\pi \left[\frac{\sqrt{p^2 + q^2 + 2pqX}}{|p - k|} + \frac{p(p + qX) - k(p - q)}{p^2 + q^2 + 2pqX} - 2 \right]. \quad (12)$$

The apparent divergence at $p = k$ is caused by the neglect of the electron thermal mass, which becomes not permissible when $|p - k| \sim m_{\text{th}}$, regularizing the divergence.

These expressions provide the emissivity under the only assumptions that $m_{\text{th}}^2 \ll pq(1 - X)$ and $m_\gamma \ll q$, so that the electrons and photons can be taken to be ultrarelativistic. For our numerical computations, we therefore use them. For qualitative understanding, we note here that the matrix element is strongly enhanced when $u \sim m_{\text{th}}^2$, which corresponds to the ALP carrying an energy E_a comparable with the initial electron energy $p \sim \mu_e$. Indeed, if the scattering happened in vacuum, it is well known that Compton scattering of a photon off an electron leads to the final photon carrying most of the electron energy. However, in a medium, there is an opposite tendency due to the Pauli exclusion principle, because of which the electron cannot give away an energy of order μ_e , which would require it to fall deep within the Fermi sea. Thus, in a very degenerate regime, the final ALP energy is rather of the order of $E_a \sim T \ll \mu_e$. In this case, we also have $q \ll p$; assuming also $m_{\text{th}}^2/pq \ll q/p \ll 1$, the denominators can be approximated as

$$s - m_{\text{th}}^2 \simeq m_{\text{th}}^2 - u \simeq 2pq(1 - X), \quad (13)$$

and then

$$|\mathcal{M}|^2 \simeq 8\pi\alpha g_{ae}^2 \frac{2k^2 + 2kq(1 + 3X) - q^2(1 - 3X^2)}{p^2} \quad (14)$$

to lowest order in the small ratios k/p and q/p . In this expression, we may replace p with p_F , the Fermi momentum of the electrons, since the strongly degenerate distribution functions enforce electrons to lie at the Fermi surface. In turn, since the electrons are ultrarelativistic, we have $p_F \simeq \mu_e$. A subtle point here is that the denominator p^{-2} causes a divergence, which is purely due to our assumption $p \gg k, q$. This means that the integral receives in reality also a contribution from the very small electron momenta, but such contribution would be suppressed by the degeneracy factor $1 - f_e(p + q - E_a) \simeq e^{-\mu_e/T}$. The exponential suppression is much stronger than the power-law enhancement $(\mu_e/T)^2$, hence we may safely neglect this range.

We can now perform the integral over p analytically; we also integrate over X , taking into account that the integral must be performed only over the region where $\cos \theta_{\mathbf{k}, \mathbf{p}+\mathbf{q}}$ is in the physical region. From Eq. (2), we find that this requires X to be between $X_{\min}$ and 1, where in the massless limit

$$X_{\min} = \max \left[-1, 1 - \frac{2E_a(p + q - E_a)}{pq} \right]. \quad (15)$$

After performing the integration, we find

$$\frac{dn_a}{dE_a dt dV} = \frac{\alpha g_{ae}^2}{8\pi^4 \mu_e^2} \int_0^{+\infty} dq \frac{E_a q (E_a^2 - q^2)}{(e^{q/T} - 1)(e^{(E_a - q)/T} - 1)}. \quad (16)$$

Thus, in the strongly degenerate limit, ALPs are emitted primarily around $E_a \sim T$. By performing the integral over $E_a dE_a$ numerically, we can now obtain the total energy emitted per unit volume and unit time

$$\begin{aligned} \frac{d\mathcal{E}_a}{dt dV} &\simeq 3.7 \times 10^{-3} g_{ae}^2 T^5 \frac{T^2}{\mu_e^2} \\ &\simeq 1.1 \times 10^{51} g_{ae}^2 \left(\frac{T}{30 \text{ MeV}} \right)^7 \left(\frac{\mu_e}{150 \text{ MeV}} \right)^{-2} \text{ erg/cm}^3 \text{ s}. \end{aligned} \quad (17)$$

The regime of strong degeneracy corresponds to the contribution of electrons with $p \sim \mu_e$ interacting with photons $q \sim T \ll \mu_e$ and producing ALPs with $E_a \sim T$. In principle, ALPs with $E_a \sim T$ can also be produced by photons with momenta $q \sim \mu_e \gg T$. However, the contribution from this region of integration is ultimately subdominant. To see this explicitly, let us consider this region with $E_a \sim T \ll p, q \sim \mu_e$. In this case, the integral I takes a very simple form

$$I = \frac{8\pi q^2}{p(p + q)}, \quad (18)$$

and the range of values of X is very narrow around $X = 1$, with a width $\delta X \simeq 2E_a(p + q)/pq$ [see Eq. (15)]. Thus, we can evaluate the function I for $E_a = 0$ and $X = 1$, replace the integral over dX with δX , and write the emissivity as

$$\frac{dn_a}{dE_a dt dV} = \frac{\alpha g_{ae}^2 E_a}{8\pi^4} \int_{E_a}^{+\infty} dp \int_{E_a}^{+\infty} dq \frac{1}{(e^{(p - \mu_e)/T} + 1)(e^{(\mu_e - p - q)/T} + 1)(e^{q/T} - 1)} \frac{q^2}{p(p + q)}. \quad (19)$$

The integral is cut off at values of the order of $p, q \gtrsim E_a$, below which the approximations used are invalid, and is logarithmically sensitive to the lower cutoff. Due to the Bose-Einstein factor $(e^{q/T} - 1)^{-1}$, the integral is exponentially suppressed if $q \sim \mu_e$, making the contribution from this range subdominant as anticipated.

Finally, for $E_a \sim \mu_e$, all of these approximations are of course invalid, since E_a, p , and q are now all of the same order of magnitude. In this case, when the ALP energy can become comparable with the electron energy, the integral I is greatly enhanced when $p \sim k$. Therefore, keeping only the first term in square brackets of Eq. (12), we may replace everywhere in the integrand $p = k$ except in the denominator; the integral $\int dp/|p - k| \sim 2 \log(k/m_{\text{th}}) = \log(k^2/m_{\text{th}}^2)$ since it must be cut off at $|p - k| \sim m_{\text{th}}$. For $k = p$ the integral over X simply gives 2 [see Eq. (15)] and so we finally find

$$\frac{dn_a}{dE_a dtdV} = \frac{\alpha g_{ae}^2}{8\pi^4} \log \left[\frac{E_a^2}{m_{\text{th}}^2} \right] \frac{E_a}{e^{(E_a - \mu_e)/T} + 1} \int_0^{+\infty} dq \frac{q}{(e^{q/T} - 1)(1 + e^{(\mu_e - q)/T})}. \quad (20)$$

If $\mu_e \gtrsim T$, the dominant contribution to the integral in Eq. (20) comes from $q \sim \mu_e$. We can then approximate the Bose-Einstein distribution with a Boltzmann one, and recognize the result as an integral over a Fermi-Dirac distribution for the electrons. For qualitative understanding, we can replace it with a strongly degenerate distribution, obtaining

$$\frac{dn_a}{dE_a dtdV} = \frac{\alpha g_{ae}^2 \mu_e^2}{16\pi^4} e^{-\mu_e/T} \log \left[\frac{E_a^2}{m_{\text{th}}^2} \right] \frac{E_a}{e^{(E_a - \mu_e)/T} + 1}. \quad (21)$$

We can also obtain an approximate expression for the total energy emitted $\mathcal{E}_a$ using Eq. (21)

$$\frac{d\mathcal{E}_a}{dtdV} = \int dE_a E_a \frac{dn_a}{dE_a dtdV} = \frac{\alpha g_{ae}^2 \mu_e^5}{48\pi^4} \log \left[\frac{4\mu_e^2}{m_{\text{th}}^2} \right] e^{-\mu_e/T}. \quad (22)$$

As μ_e increases to $\mu_e \gg T$, the emissivity drops exponentially, until eventually it must be replaced by Eq. (16). We should also stress that since the photons that are dominating the emission have typical energies of order $q \sim \mu_e$, they are essentially guaranteed to respect our initial approximation $q \gg m_\gamma$.

Our analytical estimates provide a clear justification for neglecting semi-Compton emission from positrons. As we have seen, the emissivity is primarily driven by particles with typical energies of order T in highly degenerate conditions, scaling as T^7/μ_e^2 , or by particles with energies of order μ_e in mildly degenerate conditions, scaling as $\mu_e^5 e^{-\mu_e/T}$. In the case of $e^+ \gamma \rightarrow e^+ a$, however, the relevant particle energies remain of order T , as positrons follow a Maxwell-Boltzmann distribution. However, the number density of positrons is exponentially suppressed as $e^{-\mu_e/T}$. Thus, purely from dimensional analysis, the overall emissivity must scale as $T^5 e^{-\mu_e/T}$, making it generally negligible compared to the semi-Compton scattering on electrons considered here.

II. AXION-PHOTON LOOP-INDUCED COUPLING FOR $a \rightarrow \gamma\gamma$ DECAY

We revise here the content of Sec. VID 1 of the original paper. The effective ALP-photon coupling is now taken to be

$$g_{a\gamma} = \frac{\alpha g_{ae}}{\pi m_e} \left[-\frac{m_e^2}{m_a^2} \left(\pi + i \log \left(\frac{m_a + \sqrt{m_a^2 - 4m_e^2}}{m_a - \sqrt{m_a^2 - 4m_e^2}} \right) \right)^2 \right], \quad (23)$$

without the 1 in square parenthesis that was instead present in the published version. We use the loop-induced coupling for a pseudoscalar, rather than pseudoderivative, axion-electron interaction [3]. This is consistent with our choice, in Sec. III A of the original paper, of removing the anomalous contribution to the axion-photon coupling, since this anomalous contribution survives even in the ultraviolet, receiving contribution mostly from fluctuations at arbitrarily large energies. This is equivalent to considering an axionlike particle coupling through a pseudoderivative interaction, but with a tree-level axion-photon coupling that cancels the anomalous one induced by the triangle anomaly. In any case, as we have found both here and in the previous arXiv version, where we were used the loop-induced coupling for a pseudoderivative interaction, the loop-induced two-photon decay leads to constraints that are not competitive with the other observables considered here.

III. COLLECTED AND UPDATED FIGURES

Finally, in this Section we present all of our figures, revised under the corrections highlighted in this Erratum. All of the semi-Compton emissivities are affected by the corrections, and are therefore lower than in the published version, by roughly a factor 2–3.

Figure 1 in particular shows clearly how the Compton emission behaves compared to our previous estimate. The high-energy part, with axion energies E_a comparable with the chemical potential μ_e , is still well-described by our analytical approximations; this is because at such large axion energies the interference term $\mathcal{F}_{us}$ is basically negligible. However, at low energies $E_a \sim T$, the interference term is significant, and, with the correct sign taken into account, dramatically suppresses the emissivity.

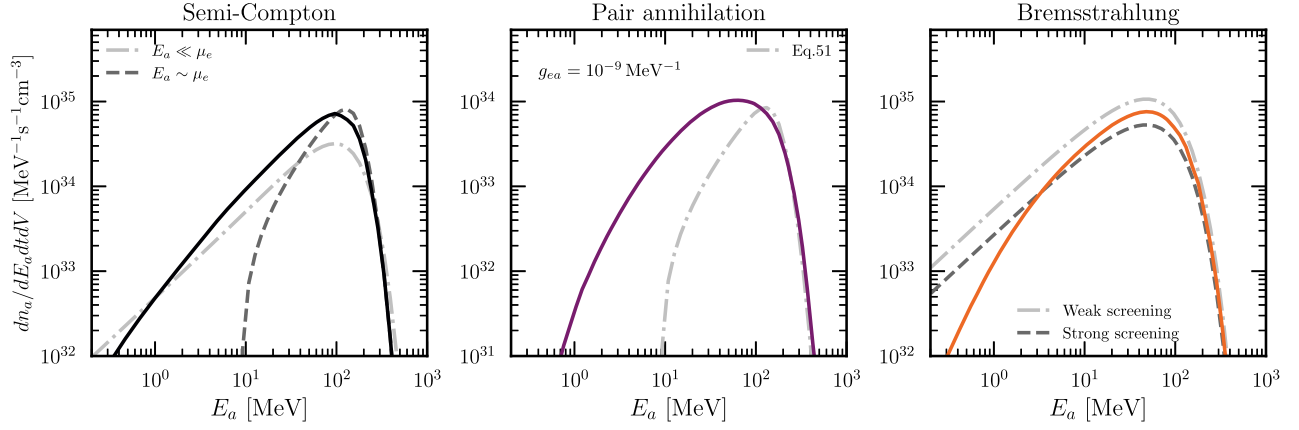


FIG. 1. ALP emissivity for $m_a = 100$ keV, $T = 30$ MeV, $\mu_e = 150$ MeV. We separately show the emissivity for all different processes; numerical results are shown as solid lines. We compare the latter with the analytical approximations: in the *left panel* we show semi-Compton emissivity in the high- E_a [Eq. (21)] and low- E_a [Eq. (16)] limits; in the *central panel* we show pair-annihilation emissivity in the degenerate limit [Eq. (51) in the original paper]; in the *right panel* we show the bremsstrahlung emissivity, in the weak screening [Eq. (69) in the original paper] and in the strong screening [Eq. (80) in the original paper] approximations.

Nevertheless, the role of semi-Compton emission remains very important: as shown in Fig. 2, for typical supernova conditions semi-Compton emission still provides a comparable amount of emission to bremsstrahlung, peaking at somewhat higher axion energies close to the electron chemical potential.

All the subsequent figures containing the axion emissivities from semi-Compton emission (Figs. 3 and 4) contain quantitative differences, but no significant qualitative difference.

Because of the reduced semi-Compton emissivity, the dominance of energy-integrated semi-Compton emission compared to other reactions is not so pronounced as in the published version. Figure 5 shows that at large densities and chemical potential bremsstrahlung in fact dominates over Semi-Compton emission, although at typical supernova (SN) conditions (as in Fig. 2) the energy-integrated emission from semi-Compton still dominates because of the higher typical energy of the axions. At very large chemical potentials, bremsstrahlung emissivity drops because of the suppression due to ion degeneracy: this regime is anyway unphysical, since of course the chemical potential of nucleons cannot be significantly smaller than the electronic one.

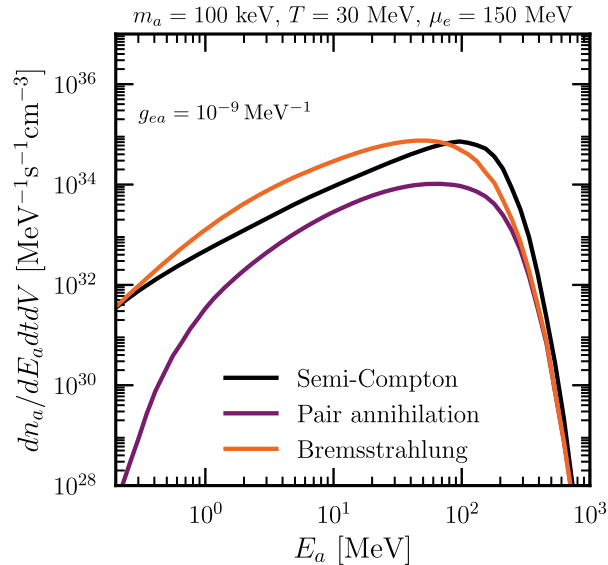


FIG. 2. ALP emissivity for $m_a = 100$ keV, $T = 30$ MeV, $\mu_e = 150$ MeV, separated by components. The coalescence contribution is zero since $m_a < 2m_{\text{th}}$.

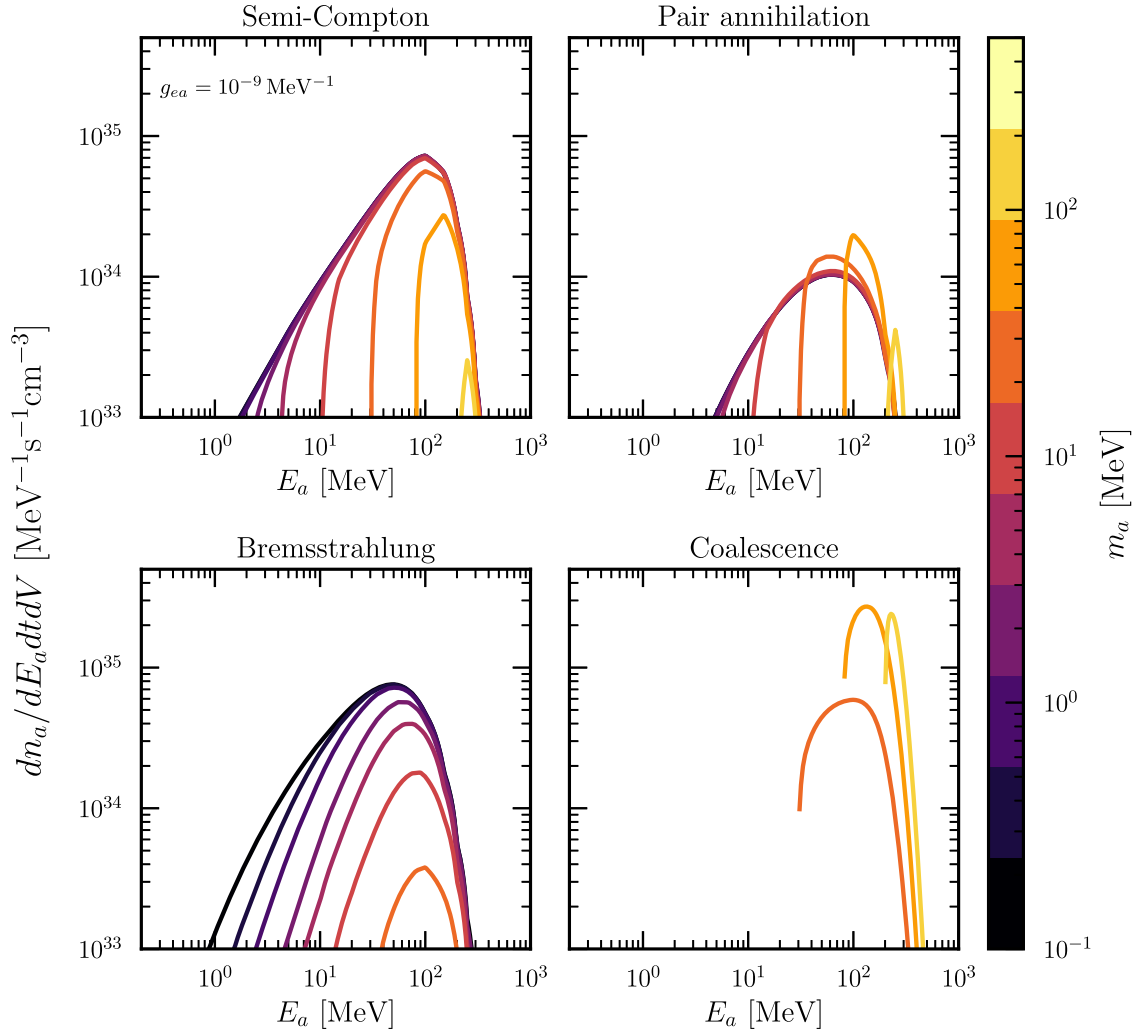


FIG. 3. Emissivities for $T = 30$ MeV, $\mu_e = 150$ MeV, and for varying ALP mass.

For the specific SN models considered here, the results of Figs. 6 and 7 are only mildly affected, and in particular at the peak of the emission the semi-Compton emissivity is basically unchanged, as argued before.

For this reason, our final constraints, shown in Fig. 8, are very similar to the published version. At low axion masses, all bounds are slightly weaker, due to the reduced semi-Compton emissivity. At large axion masses, they are basically identical, because the emissivity is dominated by coalescence, which was already correct. The only bounds that are sizably affected are the ones due to the loop-induced decay $a \rightarrow \gamma\gamma$; the change is driven by the different axion-photon effective coupling as discussed in Sec. II, so that the excluded region is dramatically reduced. However, our qualitative conclusions are still unchanged; we had already concluded in the published version that the dominant constraint comes not from the two-photon decay $a \rightarrow \gamma\gamma$, but rather from the three-particle decay $a \rightarrow e^+e^-\gamma$.

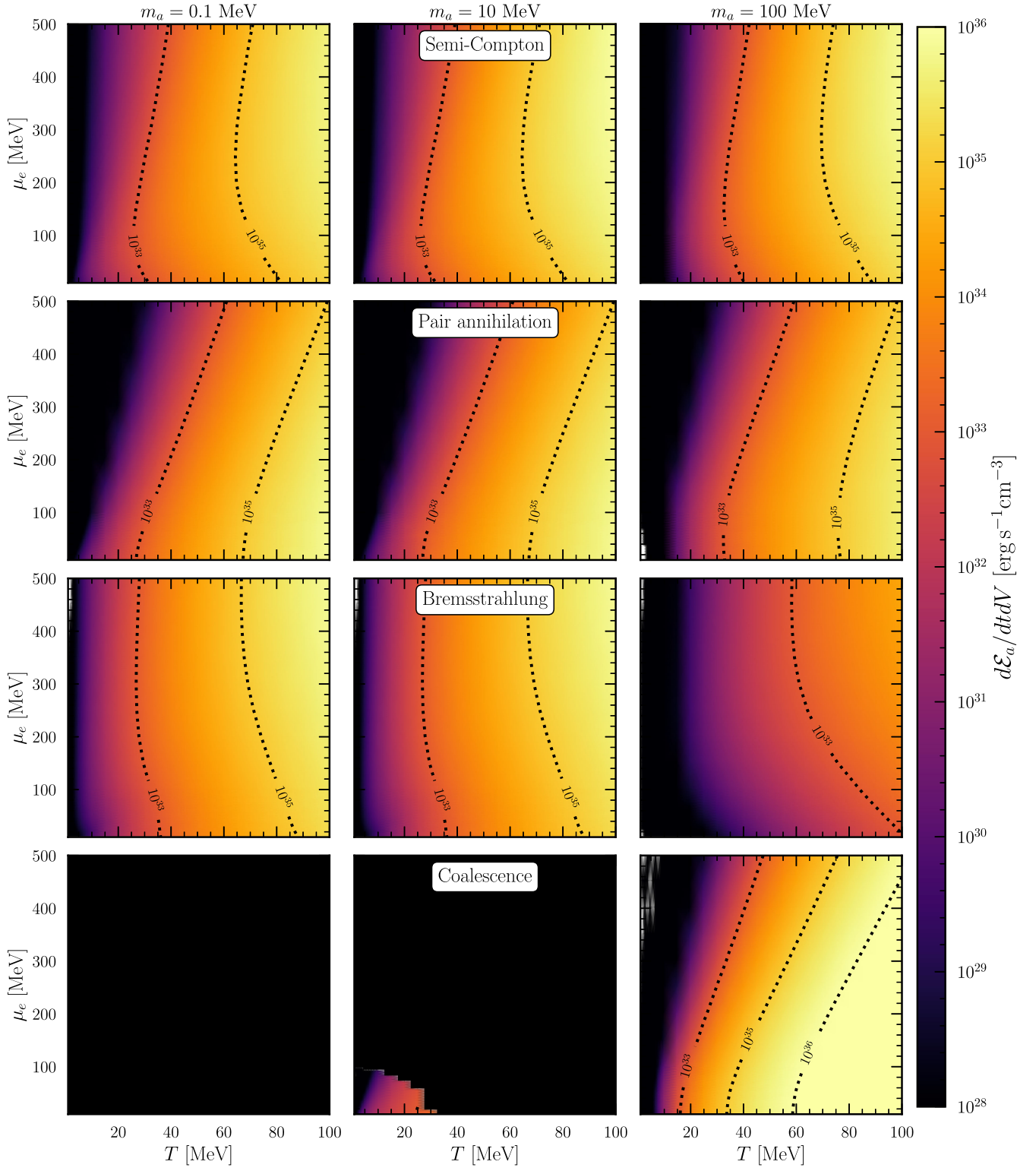


FIG. 4. Emissivities for varying temperature and chemical potential, for the four different emission processes. We show the results for three different benchmark values of the ALP mass. The black regions in the coalescence process indicate parameter space where the emissivity vanishes, as $m_a < 2m_{\text{th}}$.

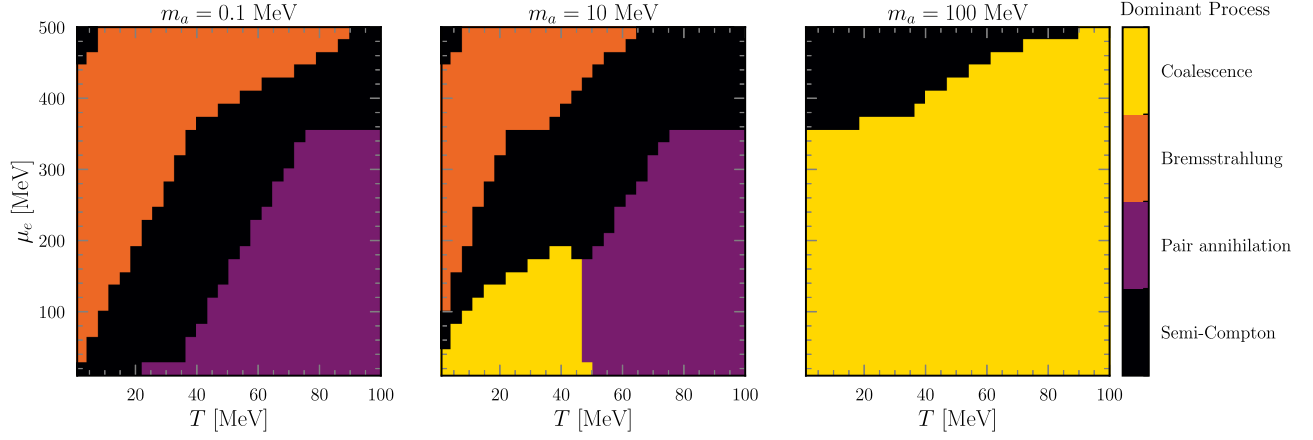


FIG. 5. Dominance of the four ALP production channels across the (T, μ_e) parameter space for different ALP masses. The dominant process at each point is determined by the highest energy production rate among semi-Compton scattering, pair annihilation, bremsstrahlung, and electron-positron coalescence.

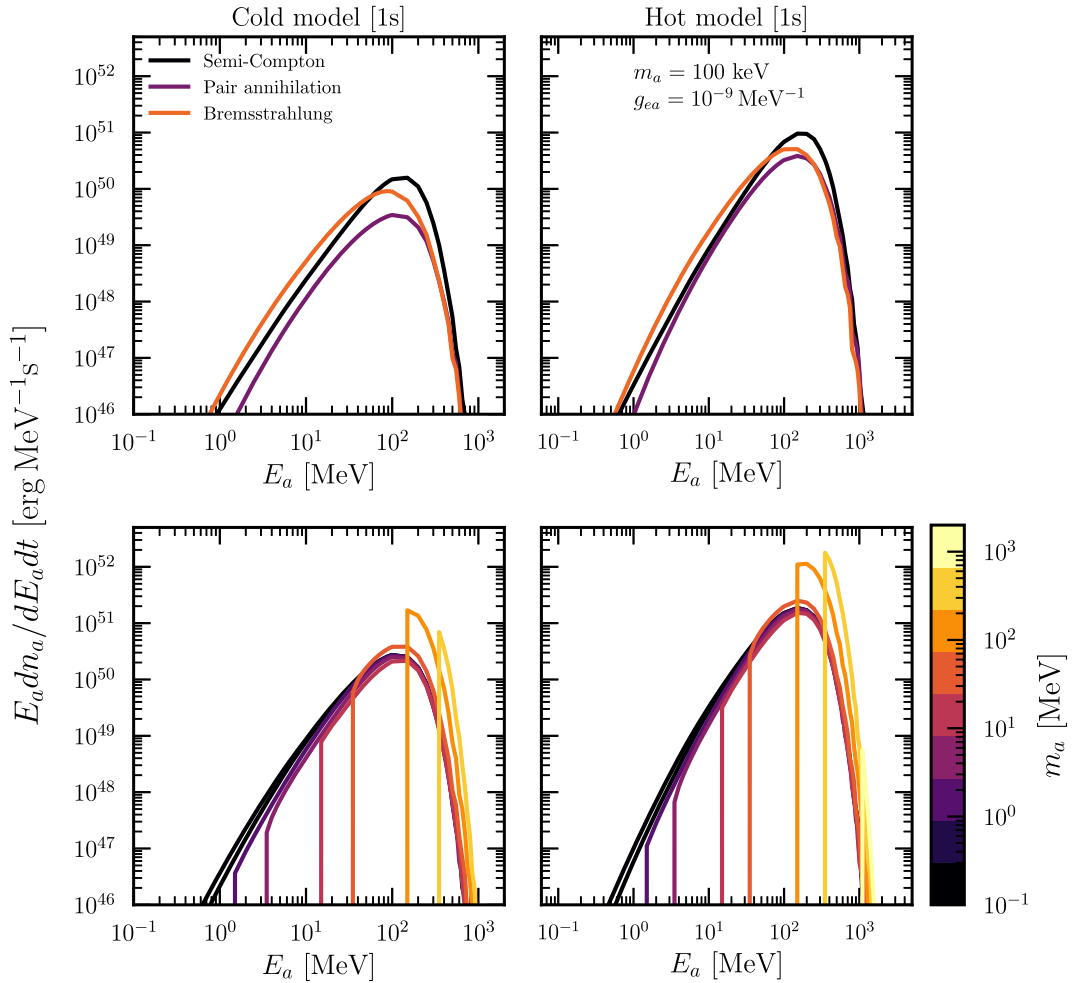


FIG. 6. Upper panel: Volume-integrated ALP emissivity for $m_a = 100$ keV computed at 1s into the simulation from different production channels for both cold and hot SN models. ALP production is enhanced in the hot model due to its higher temperature profile. The coalescence contribution is zero for such small ALP masses and is therefore not shown. Lower panel: Volume-integrated ALP emissivity as a function of ALP mass, for $g_{ea} = 10^{-9} \text{ MeV}^{-1}$. For small ALP masses, the semi-Compton channel dominates the total emitted energy, while at larger m_a , electron-positron coalescence becomes the primary production mechanism.

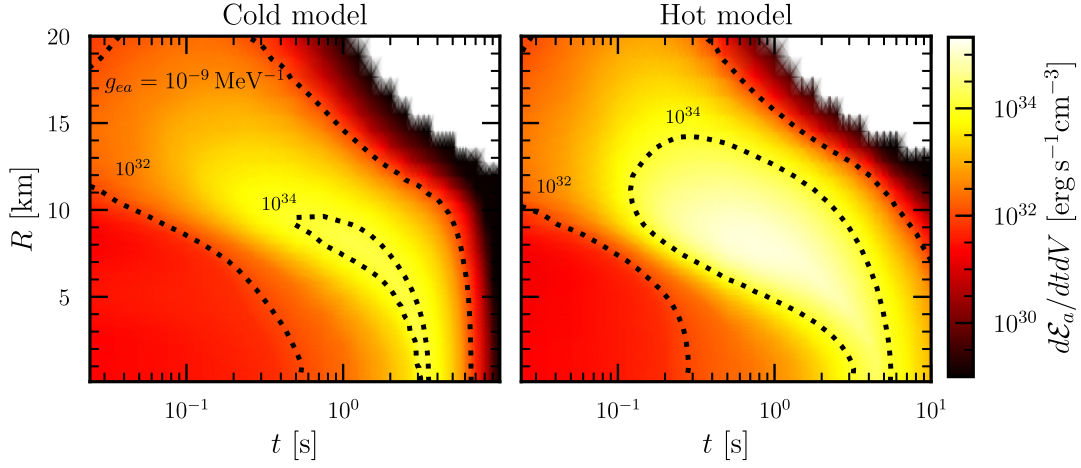


FIG. 7. Contours of the volume and energy integrated emissivity as a function of time and protoneutron star radius for a coupling of $g_{ae} = 10^{-9} \text{ MeV}^{-1}$. The emissivity distribution closely traces the temperature evolution within the protoneutron star. The white region corresponds to a chemical potential below 10 MeV, which lies outside the grid range considered in this work.

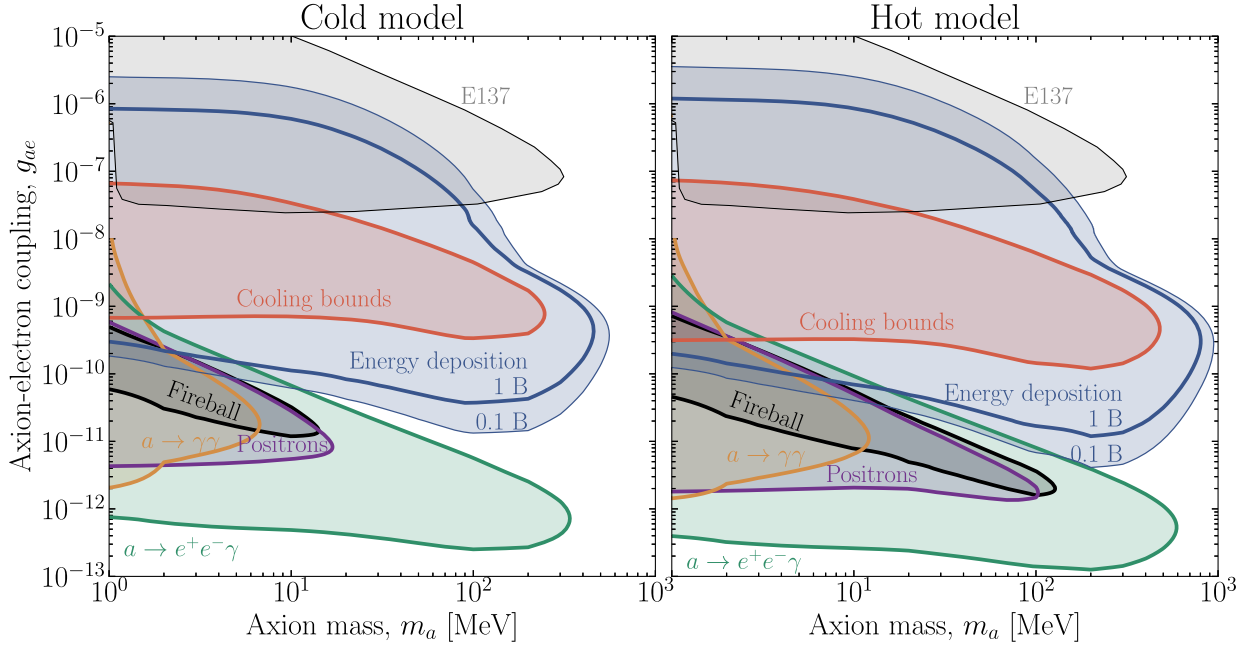


FIG. 8. Collected SN constraints on ALP-electron coupling, for the cold model (left) and the hot model (right). We show in gray the beam dump constraints from E137 [4] (as shown in Fig. 6 of Ref. [5]). We do not show constraints from Orsay [6] (as shown in Fig. 4 of Ref. [5]), which are obtained for a scalar coupling. B = Bethe = 10^{51} erg .

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