

Lee Yang edge singularities of QCD in association with the Roberge-Weiss and chiral phase transitions

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We study the quantum chromodynamics (QCD) phase transitions in the complex chemical potential plane via the Dyson-Schwinger equation approach, incorporating a constant gluonic background field that represents the confining dynamics. We solve the quark gap equation and the background field equation self-consistently, which allows us to directly explore the confinement phase transition and furthermore, evaluate the impact of the back-coupling of confinement on chiral symmetry breaking. Moreover, within such a coupled framework toward the complex chemical potential region, we demonstrate the emergence of Roberge-Weiss (RW) symmetry and investigate the trajectory of Lee-Yang edge singularities (LYESs). Our analysis reveals that the LYESs scaling behavior is similar to our previous findings without the background field condensate. However, a significant difference from our earlier work is that the trajectory of LYESs terminates when the imaginary part of the singularity becomes $1/3\pi T$. We elaborate that this cutoff behavior is caused by the RW symmetry that is symmetric to the imaginary chemical potential $\text{Im}\mu = 1/3\pi T$.

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I. INTRODUCTION

The study of quantum chromodynamics (QCD) phase transitions encompasses two important aspects: the chiral phase transition and the deconfinement phase transition. Considerable efforts have been devoted to elucidating the properties of these transitions and their interrelations [1–4]. For the chiral phase transition, there is clear definition for its order parameter, i.e., the chiral condensate [5–12], while the confinement remains less definitive. Various criteria for confinement have been proposed, see reviews in Refs. [13–16]. Here, we focus on the Polyakov loop, more precisely the gluonic background field [17–19], which stems from the thermal analog of the Wilson loop and serves as a crucial candidate of the order parameter on quark confinement.

The interplay between these two transitions provides valuable insights into the complicated dynamics of strong interactions within the context of QCD. Moreover, a new approach for studying QCD phase transitions has been proposed based on the Lee-Yang edge singularities (LYESs) [20–30], which requires one to extend the calculation into the imaginary chemical potential region. The imaginary chemical potential itself induces a more complicated relation between the chiral and deconfinement phase transition, because a new symmetry, the Roberge-Weiss (RW) symmetry, becomes evident at nonzero imaginary chemical potential. There further exists the Roberge-Weiss transition [31], which has direct impact on the LYESs. Numerous studies on this topic have been conducted by using various methods [32–35], and including investigations into its LYESs [36]. Additionally, the results presented reveal that RW symmetry can partially influence the behavior of the chiral LYESs trajectory discussed in our previous work [37].

To incorporate the confinement aspect into the current study, we utilize the gluonic background field (A_4) approach in functional QCD. Such an approach has been developed both in the Dyson-Schwinger equations (DSE) [18,34,38–40] and in the functional renormalization group [17,18,41–44], which has also been further applied in, e.g., the Curci-Ferrari model [19,45–49]. Specifically, we shall focus on the DSE approach in the present study,

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see [3,13,50–53] for its applications on the chiral symmetry aspect. As for the confinement aspect, the key is to evaluate the quantum equation of motion of A_4 from the effective potential Γ , namely $\frac{\delta\Gamma}{\delta A_4} = 0$, which would naturally yield a nonzero A_4 in the confining regime [18,38]. Further applications on RW symmetry can also be found in Ref. [34]. In fact, recent studies have also revealed the importance of the A_4 condensate on thermodynamic quantities, especially on the conserved charge fluctuations [40,54].

In the present work, we further determine A_4 self-consistently via the coupled solution between the chiral and confinement dynamics in Sec. II. Then in Sec. III, we illustrate the symmetries observed in the background field and the Polyakov loop potential, including the center symmetry and the RW symmetry. In Sec. IV, we show results on QCD phase transitions and the RW transition in the complex chemical potential plane, which also allows us to extract the respective Lee-Yang edge singularities. Finally, in Sec. V we summarize and make conclusions.

II. DSES WITH BACKGROUND FIELD AND POLYAKOV LOOP POTENTIAL

The Polyakov loop, first mentioned by Ref. [55], represents the free energy of a single static quark, given by the expression $L = e^{-F_q/T}$. When $L = 0$, it implies an infinite free energy F_q , suggesting the impossibility of a single quark's existence—a manifestation of confinement. Additionally, $L = 0$ signifies the preservation of center symmetry [56], which can be mathematically expressed through the invariance of L under center transformations: $L = zL$, where z is an element of the center of $SU(N_c)$ gauge group: $Z(N_c)$. Considerable efforts have been directed toward exploring confinement by L using lattice computations [57] and investigating the effects of the chiral order parameter's coupling to the Polyakov loop through models [58,59]. We utilize a derived order parameter from the Polyakov loop, namely the Polyakov loop potential. The potential involves computing Polyakov loop $L(\langle A_4 \rangle)$ by incorporating a constant background gluon field condensate as $A_4 = \bar{A}_4$ [17,43]. Built on these methods, here we would like to incorporate the gluonic background field in the DSEs.

A. Gluonic background field and DSEs

The DSEs involving a constant background field $\bar{A}_4$ have been discussed extensively in Refs. [18,34,38,40]. Based on these studies, we demonstrate for the first time a fully coupled, self-consistent resolution of A_4 within the DSEs truncated up to two-point functions. Such a framework is illustrated below in detail.

With $\bar{A}_\mu$, the total gluon field is given by $A_\mu = a_\mu + \bar{A}_\mu$, where a_μ represents the fluctuating quantum field, serving as the integration variable in the functional integral.

To simplify the Lagrangian's formulation, the original derivative ∂_μ can be replaced by the background-field covariant derivative: $\bar{D}_\mu = \partial_\mu - ig\bar{A}_\mu$, and the corresponding adjoint form $\bar{D}_\mu^{ab} = \partial_\mu \delta^{ab} + gf^{acb}\bar{A}_\mu^c$. Further details can be found in Refs. [60,61]. The final form of the Lagrangian is,

$$\begin{aligned} \bar{\mathcal{L}} = & \bar{\psi}(i\bar{D}_\mu\gamma^\mu + ga_\mu\gamma^\mu - m)\psi - \frac{1}{4}\bar{F}_{\mu\nu}^a\bar{F}_{\mu\nu}^a \\ & - \frac{1}{2\xi}(\bar{D}_\mu^a a_\mu^a)^2 - \bar{c}^a\bar{D}_\mu^{ac}[\bar{D}_\mu^{cd} - gf^{cdf}a_\mu^f]c^d. \end{aligned} \quad (1)$$

We will take $\xi = 0$, the Landau-deWitt gauge, which requests $\bar{D}_\mu^a a_\mu^a = 0$.

After following the derivation process outlined in Ref. [52], the DSE for the quark propagator in the presence of a background field can be derived. Here, we present the DSE at finite temperature T and chemical potential μ , where the $O(4)$ symmetry of the momentum p is broken down to $O(3)$ (with $\vec{p}$ and ω_n). Specifically, $\omega_n = \pi T(2n + 1)$ represents the Matsubara frequency for fermions,

$$\begin{aligned} [\bar{S}(p; \bar{A}_4)]^{-1} = & [\bar{S}_0(p; \bar{A}_4)]^{-1} + \bar{\Sigma}(p), \\ \bar{\Sigma}(p) = & Z_1 g^2 T \sum_n \frac{d^3 \vec{q}}{(2\pi)^3} \gamma_\mu t^a \bar{S}(q) \bar{D}_{\mu\nu}^{ab}(k; \bar{A}_4) \\ & \times \bar{\Gamma}_\nu^b(p, q; k). \end{aligned} \quad (2)$$

The dressed gluon propagator $\bar{D}_{\mu\nu}^{ab}(k; \bar{A}_4)$ and the gluon-quark vertex $\bar{\Gamma}_\nu^b(p, q; k)$ are both defined within the context of a background gluon field. Additionally, the expression for the renormalized bare quark propagator $\bar{S}_0(p; \bar{A}_4)$ is given by,

$$\bar{S}_0(p; \bar{A}_4)^{-1} = iZ_2(\gamma_4 \tilde{\omega}_n + \vec{p} \vec{\gamma}) + Z_2 Z_m m_f, \quad (3)$$

with abbreviation $\tilde{\omega}_n = \omega_n + i\mu + g\bar{A}_4$. Z_2 , Z_1 , and Z_m represent the renormalization constants for the quark wave function, vertex, and quark mass, respectively. And $g^2 = 4\pi\alpha(\zeta)$ denotes the coupling constant, which depends on the renormalization scale ζ . We take the renormalization point at $\zeta = 40$ GeV as in our previous work [37], with the corresponding running coupling taken as $\alpha(\zeta) = 0.1286$, and the average current mass of u , d quark $m_f = (m_u + m_d)/2 = 2.09$ MeV. From Eq. (3), the influence of the background field $\bar{A}_4$ on bare quark propagator can be regarded as a momentum shift in the Matsubara frequency for each color component i, j ,

$$[\bar{S}_0(\omega_n, \vec{p}; \bar{A}_4)^{-1}]_{ij} = [\bar{S}_0(\omega_n + g\bar{A}_4^a t_{ij}^a, \vec{p})^{-1}]_{ij}. \quad (4)$$

The background field $\bar{A}_4$ can be diagonalized into the Cartan subalgebra,

$$\bar{A}_4 = \frac{1}{g} 2\pi T (\phi_3 t^3 + \phi_8 t^8), \quad (5)$$

with $t^a = \lambda^a/2$, λ^a the Gell-Mann Matrix, and the Polyakov loop $L(\bar{A}_4)$ can be derived as,

$$L(\bar{A}_4) = \frac{1}{3} [e^{-i\frac{2\pi\phi_8}{\sqrt{3}}} + 2e^{i\frac{\pi\phi_8}{\sqrt{3}}} \cos(\pi\phi_3)]. \quad (6)$$

When the background field is restricted to the Cartan subalgebra, it can be demonstrated that the dressed quark propagators remain strictly diagonal,¹ thereby expressing the dressed quark propagators as,

$$[\bar{S}(p)]_{ij}^{-1} = \delta_{ij} [\mathbf{i}\tilde{\omega}_n \gamma_4 C_i(\vec{p}, \tilde{\omega}_n) + \mathbf{i}\vec{p} \cdot \vec{\gamma} A_i(\vec{p}, \tilde{\omega}_n) + B_i(\vec{p}, \tilde{\omega}_n)]. \quad (7)$$

Thus, the quark DSEs can be decomposed into coupled equations pertaining to the three color components of the quark. We note that in the quark gap equation in Eq. (2), the $\bar{A}_4$ dependence on the quark propagator dressings A_i , B_i , and C_i in Eq. (7) is fully resolved, which goes beyond the approximation used in previous studies [38,40] where the $\bar{A}_4$ effect is simply captured by shifts in the fermion Matsubara frequencies.

Analogously, considering the background field, the bare gluon propagator can be derived as,

$$[D_0^{-1}]_{ab}^{\mu\nu} = z_a \left[\left(\frac{1}{\zeta} - 1 \right) \bar{D}_\mu^{ca} \bar{D}_\nu^{cb} + \delta_{\mu\nu} \bar{D}_\lambda^{ca} \bar{D}_\lambda^{cb} \right], \quad (8)$$

where z_a represents the gluon wave function renormalization, and $\bar{D}_\mu^{ab}$ is the adjoint covariant derivative: $\partial_\mu \delta_{ab} + g f_{acb} \bar{A}_\mu^c$, as discussed previously. The second term contributes to the nondiagonal components. However, when acting on t^b , this term transforms into $k^\mu t_a + g[t_a, t_c] \bar{A}_4^c$, modifying the original k^μ , the coefficient of nonzero $(t_a)_{ij}$, to $k^\mu + g\bar{A}_4^c(t_c^{jj} - t_c^{ii})$ for $c = 3, 8$. The introduction of the

¹Using a set of basis states $|\rho\rangle$ that diagonalizes the action of t^3 and t^8 in the fundamental representation [62]: $t^j|\rho\rangle = \rho^j|\rho\rangle$, the quark propagator can be written as,

$$\bar{S} = \sum_{\rho,\sigma} S_{\rho,\sigma} |\rho\rangle \langle \sigma|.$$

Consider color rotations: $\exp(i\theta^j t^j)$, for $j = 3, 8$, which do not change the chosen background field $\bar{A}_4$, and are thus symmetries of the theory gauge fixed in the presence of that type of background. One can derive that,

$$S_{\rho,\sigma} = \exp(i\theta^j (\rho^j - \sigma^j)) S_{\rho,\sigma}.$$

The only nonvanishing components of $S_{\rho,\sigma}$ are those such that $\rho = \sigma$, which illustrates that the quark propagators remain diagonal with the current background field.

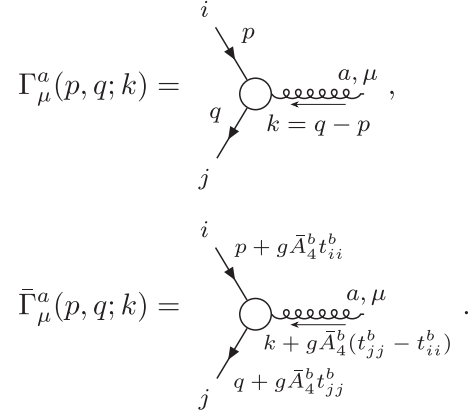


FIG. 1. Upper panel: Feynman diagram depicts the momentum flow for each leg of quark-gluon vertex without the influence of a background field. Lower panel: the diagram showcases the effects of $\bar{A}_4$ on the quark leg, carrying color indices i and j , and the gluon leg, indexed by a , where these effects are treated as momentum translations.

background field does not convert the original zero color component into a nonzero one. For dressed gluon propagators, as discussions in Ref. [63], background fields have complicated effects on propagators and vertices even at one-loop order. However, for simplicity, we adopt an approximation that considers the impact of $\bar{A}_4$ on gluon propagators and the gluon-quark vertex as a momentum translation in solving the quark propagator DSE, as schematized in Fig. 1. Now, the quark DSE can be simplified to the following coupled equations, for $i = \{1, 2, 3\}$:

$$\begin{aligned} \bar{S}_{ii}^{-1}(p + g\bar{A}_{ii}^4) &= \bar{S}_{0,ii}^{-1}(p + g\bar{A}_{ii}^4) + Z_1 g^2 T \sum_n \frac{d^3 \vec{q}}{(2\pi)^3}, \\ &\sum_j \left[\gamma_\mu t_{ij}^a t_{ji}^a \bar{S}(q + g\bar{A}_{jj}^4) D_{\mu\nu}(k + g\bar{A}_{jj}^4 - g\bar{A}_{ii}^4) \Gamma_\nu \right]. \end{aligned} \quad (9)$$

For the dressed gluon propagator $D^{\mu\nu}$ and vertex Γ_ν , in order to compare the influence of the background field on both the chiral phase transition and Lee-Yang edge singularities, we adopt the truncation scheme as in our previous work [37]. Specifically, we employ the “functional-lattice” gluon propagator given by $D_{\mu\nu}(p) = D(p) P_{\mu\nu}$, where $P_{\mu\nu}$ denotes the corresponding Lorentz structure, as in Ref. [64],

$$\begin{aligned} D(p) &= \frac{(a^2 + p^2)/(b^2 + p^2)}{M^2(p^2) + p^2[1 + c \ln(d^2 p^2 + e^2 M_G^2(p^2))]} \gamma, \\ \text{with } M_G^2(p) &= \frac{f^4}{g^2 + p^2}, \end{aligned} \quad (10)$$

and the gluon-quark vertex in Refs. [11,39],

$$\Gamma_\mu(p, q; k) = \gamma_\mu \Gamma(k^2) \left[\delta_{\mu 4} \frac{C(p) + C(q)}{2} + \delta_{\mu i} \frac{A(p) + A(q)}{2} \right],$$

$$\Gamma(k^2) = \frac{d_1}{d_2 + k^2} + \frac{k^2}{\Lambda^2 + k^2} \left(\frac{\beta_0 \alpha(\zeta) \ln[k^2/\Lambda^2 + 1]}{4\pi} \right)^{-2\delta}. \quad (11)$$

The anomalous dimensions are $\gamma = (13 - 4/3N_f)/(22 - 4/3N_f)$, $\beta_0 = (11N_c - 2N_f)/3$, $\delta = 9N_c/(44N_c - 8N_f)$. Here, we take $N_f = 2 + 1$, $N_c = 3$. Because a constant background field does not affect the vacuum solution, the parameters in the truncation scheme determined by vacuum physical values remain the same as listed in Ref. [37]. Therefore, the other parameters are $\{a, b, c, d, e\} = \{1 \text{ GeV}, 0.735 \text{ GeV}, 0.12, 0.0257 \text{ GeV}^{-1}, 0.081 \text{ GeV}^{-1}\}$, together with $\{f, g\} = \{0.65 \text{ GeV}, 0.87 \text{ GeV}\}$, $\alpha(\zeta) = 0.3$, $\Lambda = 1.4 \text{ GeV}$, $d_1 = 6.8 \text{ GeV}^2$, and $d_2 = 0.5 \text{ GeV}^2$.

Then, we can solve the DSEs with an arbitrary constant background field $\bar{A}_4$, which is abbreviated as A_4 in the subsequent text for simplicity. Figure 2 shows the variation of the third color component of the dressed quark mass $M(\vec{0}, \omega_1)$ with respect to a real ϕ_3 at different temperatures T . Obviously, a nonzero real ϕ_3 enhances the strength of the dressed quark mass. When ϕ_3 reaches $2/3$, where the Polyakov loop $L = 0$, the values of the dressed mass at various temperatures approach close to the vacuum value. The diagram illustrates how confinement effects influence chiral symmetry.

B. Polyakov loop potential

The physical value of the gluonic background field corresponds to the minimum point of the Polyakov loop

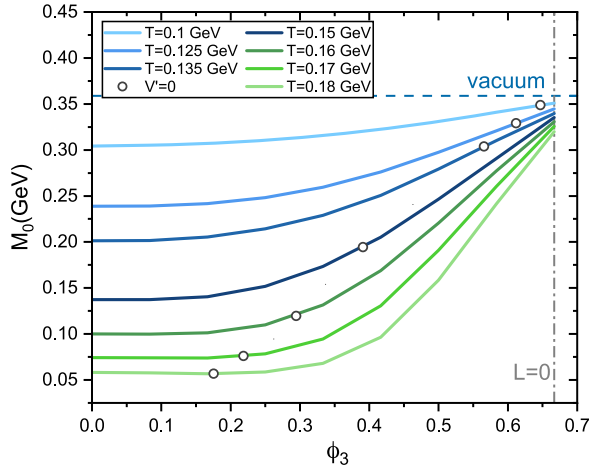


FIG. 2. Third color component of the dressed quark mass $M_0 = M(\vec{0}, \omega_1)$ with respect to real ϕ_3 at various temperatures T . The vacuum value is denoted by a dashed blue line. $L = 0$, specifically when $\phi_3 = 2/3$, is indicated by a dash-dotted gray line. And the open circles signify the points at which the Polyakov loop potential attains its minimum value.

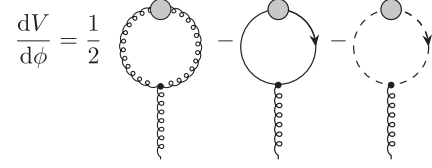


FIG. 3. One loop truncated DSE for the background gluon one-point function.

potential $V(A_4)$. The DSE for the background gluon one-point function, i.e., the derivative dV/dA_4 , has been derived in Refs. [18,38]. Here, we analyze exclusively the one-loop term shown in Fig. 3, whose physical viability is demonstrated in Ref. [18]. The specific formula for this one-loop term of the derivative of the potential with respect to ϕ : $V' = dV/d\phi$, given $A_4 = \frac{2\pi T}{g} \phi \text{diag}(\nu_l, \{l\})$, where ν_l are the eigenvalues of the generating element of A_4 in the fundamental (ν_{ql}) or adjoint ($\tilde{\nu}_{gl}$) representation in QCD, is,

$$V' = \oint_n \left[\frac{1}{2} \sum_{gl} \nu_{gl} \tilde{\omega}_{n,gl} \cdot (z_a G_L(\vec{q}, \tilde{\omega}_{n,gl}) + 2z_a G_T(\vec{q}, \tilde{\omega}_{n,gl}) - 2z_c G_c(\vec{q}, \tilde{\omega}_{n,gl})) - \sum_{ql,f} \nu_{ql} \tilde{\omega}_{n,ql} Z_{2f} S_f(\vec{q}, \tilde{\omega}_{n,ql}) \right] + \frac{1}{2} V'_{\text{Weiss}}, \quad (12)$$

with the abbreviated boson and fermion Mastubara frequency given by:

$$\tilde{\omega}_{n,gl} = 2\pi T(n + \phi \nu_{gl});$$

$$\tilde{\omega}_{n,ql} = \pi T(2n + 1 + 2\phi \nu_{ql}) + i\mu. \quad (13)$$

And $\frac{1}{2} V_{\text{Weiss}}$ is the perturbative one-loop potential [65] in four dimensions,

$$V_{\text{Weiss}} = -\frac{2}{\pi^2} T^4 \sum_l \sum_{n=1}^{N_c-1} \frac{\cos\{2\pi n \nu_l |\phi|\}}{n^4}. \quad (14)$$

The product of the renormalization constant with the propagator, $z_x G_x = z_x \cdot \frac{1}{Z_x(p^2)} p^2$, is renormalization invariant. As the renormalization point ζ changes to ζ' , the transition of the renormalized propagator and renormalization constant is given by,

$$G_x(p; \zeta) = \frac{Z_x(\zeta^2)}{Z_x(p^2) p^2} \quad \text{to} \quad G_x(p; \zeta') = G_x(p; \zeta) \frac{Z_x(\zeta'^2)}{Z_x(\zeta^2)},$$

$$z_x(\zeta) \quad \text{to} \quad z_x(\zeta') = z_x(\zeta) \frac{Z_x(\zeta^2)}{Z_x(\zeta'^2)}. \quad (15)$$

This guarantees the renormalization invariance of the DSE: $G_x^{-1}(\zeta') = z_x(\zeta') G_{0,x}^{-1} + \Sigma(\zeta')$. Consequently, one

can compute Eq. (12) at the same point $\zeta = 40$ GeV as we solve the DSEs. For better convergence of the sum of Matsubara frequencies and to ensure renormalization invariance, choose $Z_{2f} = C(p)$ in Eq. (12). Additionally, It is found in Ref. [9] that the Euclidean $O(4)$ symmetry is effectively restored for high Matsubara frequencies, where the dressing functions converge rapidly toward their vacuum counterparts. To that end, when $\sqrt{\vec{p}^2 + \text{Re}(\tilde{\omega}_n)^2} > 3\pi T$, we take $O(4)$ approximation for $A(\omega_n, \vec{p})$ and $C(\omega_n, \vec{p})$, which are replaced by a (3,3) Padé fit function of the vacuum $A(p)$. We take the same form gluon propagator as Eq. (10) in Polyako-loop computation. As for the ghost propagator $G_c(k^2) = F(k^2)/k^2$, where $F(k^2)$ is the ghost dressing function, it is taken as that in Refs. [66,67].

$$F(k^2) = \frac{(a_1 + b_1\sqrt{k^2} + k^2)/(c_1 + d_1\sqrt{k^2} + k^2)}{[1 + e_1 \ln(f_1^2 k^2 + g_1^2 M_G^2(k^2))]^\delta}, \quad (16)$$

where $M_G^2(p^2)$ has been defined in Eq. (10). And the other fit parameters are $\{a_1, b_1, c_1, d_1, e_1\} = \{0.152 \text{ GeV}^2, 0.697 \text{ GeV}, 0.0055 \text{ GeV}^2, 0.016 \text{ GeV}, 0.045\}$, and $\{f_1, g_1\} = \{0.025 \text{ GeV}^{-2}, 0.0237 \text{ GeV}^{-2}\}$.

The next step is using Eq. (12) to find the minimum points ($A_4, L(A_4)$) and substitute the values back into DSEs.

III. A GENERAL SYMMETRY ANALYSIS OF BACKGROUND FIELD AND POLYAKOV LOOP POTENTIAL

Before presenting further DSE results, we first discuss the symmetry of the background condensate in the (ϕ_3, ϕ_8) plane for a nonzero chemical potential $\mu = \text{Re}\mu - i\pi T\theta$. The symmetry analysis helps to simplify the process of finding the minimum points of the Polyakov loop potential as discussed in Ref. [45], besides, it also provides insights into center symmetry and Roberge-Weiss symmetry.

A. Center symmetry

As mentioned in Sec. II, the effect of A_4 on the quark propagator can be interpreted as a shift of the Matsubara frequencies ω_n ,

$$\begin{aligned} \omega_n &\rightarrow \omega_n + i\text{Re}\mu + \theta\pi T + \phi_3\pi T - \phi_8 \frac{1}{\sqrt{3}}\pi T, \\ \omega_n &\rightarrow \omega_n + i\text{Re}\mu + \theta\pi T - \phi_3\pi T - \phi_8 \frac{1}{\sqrt{3}}\pi T, \\ \omega_n &\rightarrow \omega_n + i\text{Re}\mu + \theta\pi T + \phi_8 \frac{2}{\sqrt{3}}\pi T. \end{aligned} \quad (17)$$

These equations illustrate the symmetries within the (ϕ_3, ϕ_8) plane, indicating that the solution to the DSEs

remains invariant under certain transformations. The symmetries are summarized as follows:

- (a) The real components of ϕ_3 and ϕ_8 exhibit periodic translation symmetry with periods 2 and $2\sqrt{3}$, respectively. This symmetry transformation can be compactly expressed in terms of an arbitrary SU(3) root vector α as given in Ref. [68],

$$(\phi_3, \phi_8) \rightarrow (\phi_3, \phi_8) + 2\alpha, \quad (18)$$

where α belongs to the SU(3) root system.

- (b) The pair (ϕ_3, ϕ_8) possesses color rotation symmetry (Weyl group symmetry), corresponding to the C_{3v} group for SU(3). At $\mu = 0$, this symmetry reduces to discrete rotational symmetry with allowed rotation angles $\theta = n\pi/3$ ($n \in \mathbb{Z}$),

$$(\phi_3, \phi_8) \rightarrow (\phi_3 \cos \theta + \phi_8 \sin \theta, -\phi_3 \sin \theta + \phi_8 \cos \theta). \quad (19)$$

- (c) (ϕ_3, ϕ_8) represent charge conjugation invariance if $\mu = 0$.

$$(\phi_3, \phi_8) \rightarrow (\pm\phi_3, \pm\phi_8). \quad (20)$$

The Polyakov loop L remains invariant under these transformations as well. These symmetry properties are illustrated in Fig. 4(a), which shows the contour plot of QCD potential V at temperature $T = 0.15$ GeV and vanishing chemical potential ($\mu = 0$). The elementary cell and

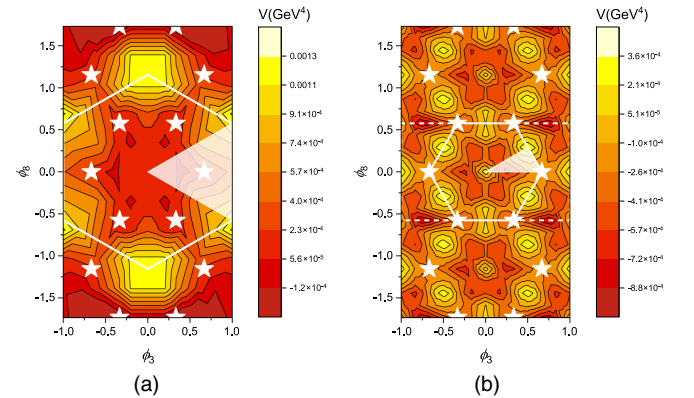


FIG. 4. Two panels display contour plots of the Polyakov loop potential V for the full QCD (left) and gauge field part in Eq. (12) (right), respectively. Dark red regions correspond to the potential minima, while stars denote the degenerate points where Polyakov loop $L = 0$. The hexagonal boundary and the shaded triangular region represent the elementary cell and fundamental domain of the symmetries, under a generic chemical potential. (a) Full Polyakov loop potential of QCD with finite quark mass at $T = 0.15$ GeV. (b) Polyakov loop potential of only the gauge field part as an approximate “Yang-Mills” theory at $T = 0.25$ GeV.

fundamental domain of these three symmetries are represented by a hexagonal boundary and a shaded triangular region, respectively. As shown in the figure, the degenerate minima of V for the QCD system appear as dark red regions, distinctly separated from the white star markers indicating points where $L = 0$ and center symmetry is preserved. This separation between the potential minima and center symmetric points clearly demonstrates the explicit breaking of center symmetry in QCD systems with finite quark masses.

The same theoretical analysis applies equally to pure Yang-Mills theory, which shares all the aforementioned symmetry properties. Nevertheless, several crucial differences emerge. Notably, pure Yang-Mills theory consistently exhibits charge conjugation symmetry, which leads to the fundamental domain being reduced to half that of QCD theory. In addition, the translation period of ϕ_8 is shortened to $2\sqrt{3}/3$. This additional symmetry is referred to as the center symmetry. At low temperature, the minima of the Polyakov loop potential for pure Yang-Mills theory are located at the (ϕ_3, ϕ_8) plane where the Polyakov loop $L = 0$. As the temperature increases till the first-order phase transition region, the minima become the points with nonzero L , and consequently, there exist three degenerate minima, corresponding to $L \in \{l, zl, z^2l\}$ where $l \in \mathbb{R}$ and $z = e^{2\pi i/3}$. In other words, such a triplet of minima signals the spontaneous breaking of the center symmetry in pure Yang-Mills theory. The characteristic contour plot at $T = 0.25$ GeV for the gauge field part of QCD, where this phase transition occurs, is presented in Fig. 4(b). The gauge field part has the same symmetry as the pure Yang-Mills theory, but the location of the phase transition may differ.

B. Roberge-Weiss symmetry

The imaginary chemical potential is more than just an analytic extension of the real one; it carries abundant

symmetry. The center transformation z acting on the Polyakov loop L is equivalent to some specific values of imaginary chemical potential given by,

$$\text{Im } \mu = -\pi T \theta = \frac{2\pi T}{3} n, \quad n \in \mathbb{Z}. \quad (21)$$

In this context, the minima of (ϕ_3, ϕ_8) shift to another center sector, like $\phi_8 = \phi'_8 + \sqrt{3}\theta$. Consequently, Eq. (17) transforms into the following forms:

$$\begin{aligned} \omega_n + i\text{Re}\mu + \phi_3\pi T - \phi'_8 \frac{1}{\sqrt{3}}\pi T, \\ \omega_n + i\text{Re}\mu - \phi_3\pi T - \phi'_8 \frac{1}{\sqrt{3}}\pi T, \\ \omega_n + i\text{Re}\mu + 3\theta\pi T + \phi'_8 \frac{2}{\sqrt{3}}\pi T. \end{aligned} \quad (22)$$

Obviously, the quark propagator remains invariant if θ takes a value in Eq. (21), leading to the invariance symmetry in thermodynamic quantities, known as the Roberge-Weiss symmetry [31]. However, the minima of Polyakov loop potential shift is illustrated in Fig. 5. Additionally, as $\text{Im } \mu' = \text{Im } \mu - 2/3\pi T$ the Polyakov loop acquires a phase factor, $L \rightarrow zL, z = e^{2/3\pi i}$. According to Ref. [31], L does not always undergo a crossover at the midpoint of one Roberge-Weiss period, given by $\text{Im } \mu = (1 + 2n)/3\pi T$ for $n \in \mathbb{Z}$. In fact, at high temperatures, when L is sufficiently large, it undergoes a first-order phase transition. The endpoint of the RW line, which has been extensively discussed in various works [35,69,70] remains a topic of debate concerning whether it represents a second-order transition or a tricritical point. These transitions are referred to as Roberge-Weiss transitions, and the midpoint line is known as the RW transition line.

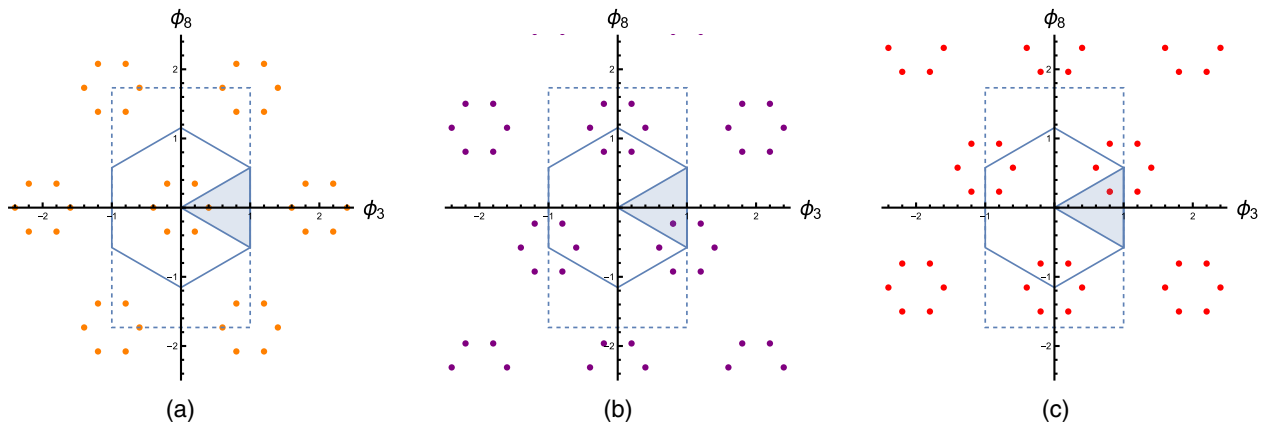


FIG. 5. Minima of Polyakov loop potential at specific imaginary chemical potential $\mu = -\frac{2n\pi T}{3}i, n \in \mathbb{Z}$. With a shift of $\text{Im } \mu$ by $-\frac{2}{3}\pi T$, the minima switch to the nearby center sector, and the Polyakov loop transforms as $L \rightarrow zL$, where $z = e^{2\pi i/3}$. (a) $\text{Im } \mu = 0$. (b) $\text{Im } \mu = -\frac{2}{3}\pi T, \phi_8 = \phi'_8 + \frac{2\sqrt{3}}{3}$. (c) $\text{Im } \mu = -\frac{4}{3}\pi T, \phi_8 = \phi'_8 + \frac{4\sqrt{3}}{3}$.

TABLE I. For specific chemical potential μ , we find the minima of the Polyakov loop potential in the complex (ϕ_3, ϕ_8) plane. Within the fundamental domain shown in Fig. 4(a), only the ϕ variables marked with $\checkmark$ need to be determined. We show all other variables are zero.

	Re ϕ_3	Im ϕ_3	Re ϕ_8	Im ϕ_8
$\mu = 0$	$\checkmark$	0	0	0
Re $\mu \neq 0$, Im $\mu = 0$	$\checkmark$	0	0	$\checkmark$
Im $\mu \neq 0$, Re $\mu = 0$	$\checkmark$	0	$\checkmark$	0
Im $\mu \neq 0$, Re $\mu \neq 0$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$

IV. RESULTS AND DISCUSSIONS

Below, we present our findings concerning the QCD phase transition, the RW transition, and LYEs under varying chemical potential conditions. Our computations reveal that, when $\mu = 0$, the minimum of Polyakov loop potential in the fundamental domain resides on the line spanned by a real ϕ_3 . Conversely, for $\mu \neq 0$, Table I summarizes the variables that require scanning. Further details are provided in the subsequent subsections.

A. At finite temperature and real chemical potential

At zero chemical potential, considering real ϕ_3 alone is sufficient to pinpoint the minima of the system. Several points are marked with green dots in Fig. 2. It is evident that, within the temperature range $T \in (0.1, 0.17)$ GeV, ϕ_3 significantly impacts the chiral symmetry, exacerbating the breaking of this symmetry.

Furthermore, we direct our attention to the QCD phase transition at finite temperatures. We proceed to compute the chiral condensate $\langle \bar{\psi}\psi \rangle$ from the quark propagator using the following equation:

$$\langle \bar{\psi}\psi \rangle = -Z_2 Z_m N_c T \sum_n \int \frac{d^3 p}{(2\pi)^3} \text{tr}[S(\tilde{\omega}_n, p)]. \quad (23)$$

With a finite quark mass, the chiral condensate needs to be regularized; we make use of the reduced condensate $\langle \bar{\psi}\psi \rangle_{\text{reg}}$ in Ref. [64],

$$\langle \bar{\psi}\psi \rangle_{\text{reg}} = \langle \bar{\psi}\psi \rangle_l - m \frac{\partial}{\partial m} \langle \bar{\psi}\psi \rangle_l. \quad (24)$$

And the light-quark chiral susceptibility χ_m^l in Ref. [39] is

$$\chi_m^l = \frac{\partial \langle \bar{\psi}\psi \rangle_l}{\partial m_l}. \quad (25)$$

To determine the pseudocritical temperatures associated with chiral symmetry restoration and the transition to deconfinement, we use the two order parameters: the maximum value of the light-quark chiral susceptibility χ_m^l , and the inflection point of the derivative of the Polyakov

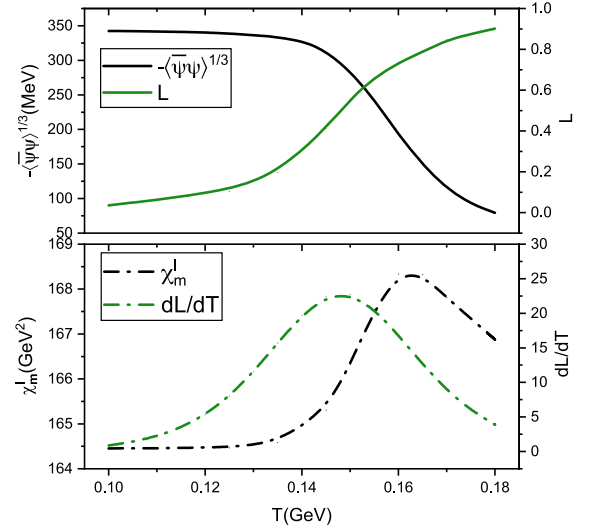


FIG. 6. Upper panel: chiral condensate $\langle \bar{\psi}\psi \rangle$ and the Polyakov loop L in the presence of gluonic background field at $\mu = 0$. Lower panel: the respective chiral susceptibility χ_m^l and thermal susceptibility of the Polyakov loop $\frac{dL}{dT}$.

loop $\frac{dL}{dT}$. In Fig. 6, we show the chiral condensate and the Polyakov loop at vanishing chemical potential together with the respective χ_m^l and $\frac{dL}{dT}$ as functions of temperature. We obtain the pseudocritical temperature for chiral symmetry restoration is $T_\chi = 0.162$ GeV, and for confinement transition is $T_{\text{conf}} = 0.146$ GeV. It is noteworthy that the presence of a nonzero value of ϕ_3 raises T_χ from the previously reported value of 0.155 GeV in Ref. [37].

At nonzero real chemical potential, the behavior of A_4 condensate is similar, however, due to the analytic property and Cauchy-Riemann theorem, the imaginary of ϕ_8 is required. The minima of the system are found to lie within the $(\text{Re}\phi_3, \text{Im}\phi_8)$ plane. Figure 7 illustrates the behavior of $\text{Im}\phi_8$ at a temperature $T = 0.125$ GeV as a function of $\text{Re}\mu$. Initially, $\text{Im}\phi_8$ starts at zero and decreases to negative values as $\text{Re}\mu$ increases. Subsequently, as $\text{Re}\mu$ continues to rise, $\text{Im}\phi_8$ increases back to zero. As in Ref. [62], nonzero values of ϕ_8 are closely linked with the broken charge conjugation induced by chemical potential. More specifically, their relationship can be inferred from the expressions involving the Polyakov loop L and its conjugate $\bar{L}$ as follows:

$$\begin{aligned} L(A_4) &= e^{i \int A_4 dx^4} = e^{-F_q/T}, \\ \bar{L}(A_4) &= e^{-i \int A_4 dx^4} = e^{-F_{\bar{q}}/T}. \end{aligned} \quad (26)$$

For L is an even function of ϕ_3 shown in Eq. (6), the difference between L and $\bar{L}$ is manifested through nonzero values of ϕ_8 . This ϕ_8 represents the disparity in free energy between static quarks and antiquarks, which manifests the degree of charge conjugation asymmetry as illustrated in

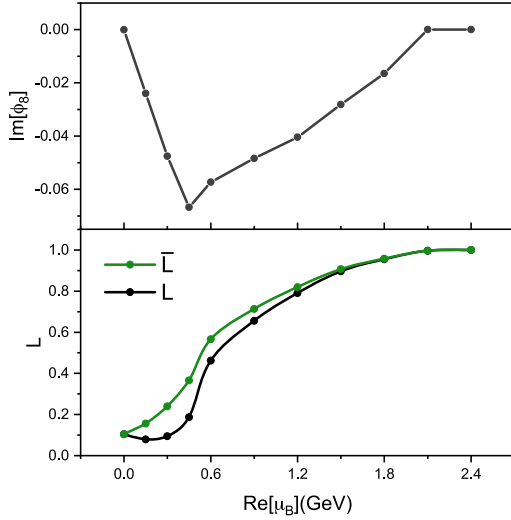


FIG. 7. $\text{Im}\phi_8$ and $L, \tilde{L}$ with respect to real baryon chemical potential $\mu_B = 3\mu$ at $T = 0.125$ GeV.

Sec. III A. Initially, the absolute value of ϕ_8 increases as $\text{Re}\mu$ rises, introducing a distinction between quarks and antiquarks. However, as $\text{Re}\mu$ continues to ascend, the chiral symmetry starts to restore, leading both quarks and antiquarks to exhibit *asymptotic freedom*. Consequently, the discrepancy in their free energies gradually diminishes. According to Eq. (6), a negative value of $\text{Im}\phi_8$ implies that $\tilde{L} > L$, as illustrated in Fig. 7. This result is in agreement with those presented in Ref. [71], which state that in a quark medium, the color screening effect on test antiquarks is more pronounced, yielding a larger value of $\tilde{L}$.

B. The Lee Yang edge singularities of both RW and Chiral phase transitions

The Lee-Yang zeros, defined as the complex roots of the partition function in the complex plane of external control parameters, have sophisticated connections to the analytic and critical properties of phase transitions. These zeros merge near critical points, and their accumulation at the edge of the distribution generates the LYESs, universal signatures of phase transitions characterized by the non-analysis of the free energy. These singularities govern the critical exponents through scaling relations rooted in Lee-Yang theory. In QCD, LYESs manifest in distinctive phase transitions: The chiral phase transition, where dynamical chiral symmetry breaking occurs, exhibits LYESs in the complex chemical potential that dictate the critical behavior. Similarly, the RW transition at imaginary chemical potential $\text{Im}\mu = -\pi T\theta$ features the $\mathbb{Z}_3$ -symmetric LYESs in the complex θ plane. Here, singularities emerge at RW endpoints where the first-order phase transition lines terminate, controlling the universal criticality of this deconfinement transition. Both cases exemplify how LYESs encode fundamental aspects of QCD thermodynamics beyond conventional symmetry-breaking paradigms.

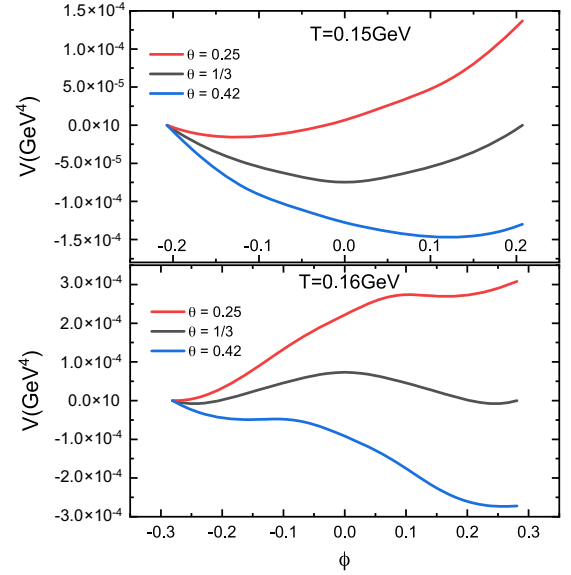


FIG. 8. Polyakov loop potential at temperatures $T = 0.15, 0.16$ GeV and different imaginary chemical potentials $\text{Im}\mu = -\pi T\theta$, along the respective direction defined in Eq. (31) with the control parameter ϕ . In particular for $\theta = 1/3$, $\phi = 0$ corresponds to the background field value $(\phi_{3,0}, \phi_{8,0})$ for the extreme point of the potential, which corresponds to Eqs. (29) and (30).

First of all, the Polyakov loop becomes complex for complex chemical potential, and hence, it is not appropriate to regard it as the usual deconfinement order parameter. The behavior of Polyakov loop at complex chemical potential together with the relation between deconfinement and the RW phase transition is beyond the scope of this paper, and here we directly consider the minimum of the Polyakov loop potential in the (ϕ_3, ϕ_8) plane as the order parameter of the RW phase transition.

In our computation, we observe the RW phase transition by generalizing the system into imaginary chemical potential. In particular, at zero chemical potential, the critical point of the RW phase transition takes place at temperature $T = 155$ MeV. As depicted in Fig. 8, for the temperature above, a first order phase transition occurs at the imaginary chemical potential $\text{Im}\mu = -\frac{1}{3}\pi T$, while for the temperature below, the minimum in (ϕ_3, ϕ_8) plane changes smoothly with respect to the imaginary chemical potential.

To elucidate the connection between the RW transition and the center symmetry, let us analyze the QCD Lagrangian under an imaginary chemical potential $\text{Im}\mu = -\frac{1}{3}\pi T$,

$$\mathcal{L} = \bar{\psi} \left(i\partial + gA_\mu \gamma^\mu + \text{Re}\mu \gamma^0 - i\frac{\pi T}{3} \gamma^0 - m \right) \psi - \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a. \quad (27)$$

Applying a center transformation $z \in \mathbb{Z}_3$ to the Lagrangian, it transforms into its conjugate form,

$$\mathcal{L}' = \bar{\psi} \left(i\partial + gA_\mu \gamma^\mu + \text{Re}\mu \gamma^0 + i\frac{\pi T}{3} \gamma^0 - m \right) \psi - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu}. \quad (28)$$

If this symmetry is preserved in the physical states, the Polyakov loop L must satisfy,

$$L^* = z^{-1} L = e^{-2\pi i/3} L, \quad (29)$$

implying $L = e^{\pi i/3} |L|$. This corresponds to,

$$\phi_8 = \tan \left(-\frac{2\pi}{3} \right) \left(\phi_3 - \frac{2}{3} \right), \quad \phi_3 \in [1/2, 2/3], \quad (30)$$

in the fundamental domain of Fig. 4(a). Following this idea, we further check directly from the physical point position of the Polyakov potential, whether the symmetry presented by Eq. (29) is broken or not. To do this, we first determine the extreme point of the potential along Eq. (30) at $\theta = 1/3$, which we denote as $(\phi_{3,0}, \phi_{8,0})$. Then, we extract the shape of the potential at different T and θ , along a direction perpendicular to Eq. (30),

$$(\phi_3, \phi_8) = \left(\phi_{3,0} + \frac{\sqrt{3}}{2} \phi, \phi_{8,0} - \frac{1}{2} \phi \right), \quad (31)$$

with ϕ as a control parameter. Finally, we check whether the physical point, i.e., minimum of the potential indeed lies at $\phi = 0$. This is illustrated in Fig. 8. We found that below the critical temperature $T_c = 0.155$ GeV at $\theta = \frac{1}{3}$, the minimum of the potential resides at $\arg(L) = \pi/3$, signaling unbroken symmetry behavior. However, when $T > T_c$, the minimum shifts to $\arg(L) \neq \pi/3$, indicating a spontaneous breaking behavior. The complete temperature dependence of the imaginary part of the Polyakov loop with a $e^{-\pi i/3}$ phase rotation: $\text{Im}(e^{-\pi i/3} L)$ is shown in Fig. 9.

Therefore, for every temperature and chemical potential where the RW phase transition can happen, it happens at a certain imaginary chemical potential as,

$$\text{Im}\mu = (1 + 2n)/3\pi T, \quad n \in \mathbb{Z}. \quad (32)$$

Such a transition brings in additional singularities in the imaginary chemical potential region, which affects the behavior of the LYESs in QCD. We then replicate the methodology outlined in Ref. [37] within the current framework to investigate the impact of the A_4 on the Lee-Yang edge singularities (LYESs). The trajectories of the LYESs for both schemes are depicted in Fig. 10 for comparison.

One significant outcome of incorporating A_4 is that the LYESs has represented a cutoff temperature, at $T_e = 0.23$ GeV. This is because at this temperature, the imaginary part of the LYESs approaches the Roberge-Weiss

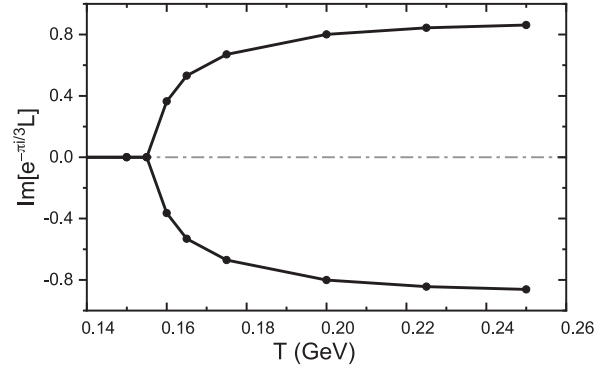


FIG. 9. Temperature dependence of the imaginary part of Polyakov loop L under $e^{-\pi i/3}$ phase rotation.

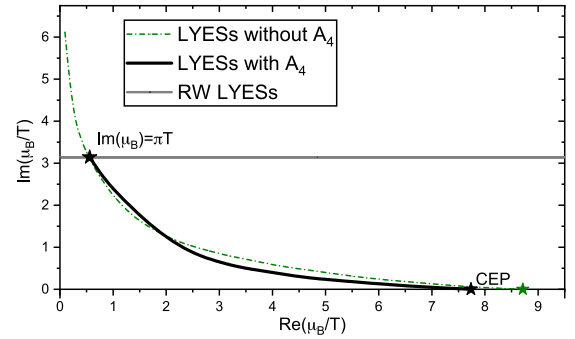


FIG. 10. RW LYESs trajectory (gray-solid) and the chiral LYESs trajectory with (black-solid) and without A_4 (green-dash-dotted) in the $\text{Re}\mu_B/T$ — $\text{Im}\mu_B/T$ plane. The background field generates a cutoff temperature at $T_e = 0.23$ GeV that above this temperature there exists no chiral LYESs due to the RW phase transition.

transition line, given by $\text{Im}(\mu_{\text{LYE}}) = \frac{1}{3}\pi T$. As the temperature increases, in the original scheme, $\text{Im}\mu_{\text{LYE}}$ would grow larger. However, in the current scheme, the RW symmetry entails that A_4^a is symmetric to the axis $\text{Im}\mu = \frac{1}{3}\pi T$. Therefore, the values of A_4 and $\text{Im}\mu$ at $\text{Im}\mu > \frac{1}{3}\pi T$ lead to a singularity that is located at $\text{Im}\mu < \frac{1}{3}\pi T$, and hence, it will never reach the singularity in the correlation function at $\text{Im}\mu > \frac{1}{3}\pi T$. Consequently, the LYESs at higher temperatures terminate within the branch cut of the Roberge-Weiss first-order phase transition.

As depicted in Fig. 10, the consideration of the gluonic background A_4 condensate does not change greatly the behavior of LYESs below $\text{Im}\mu = \frac{1}{3}\pi T$ and $T_e = 0.23$ GeV. For increasing temperature above critical end point (CEP): $(T_{\text{CEP}}, \mu_{B,\text{CEP}}) = (0.095, 0.735)$ GeV, the location of LYESs is monotonously changing to smaller $\text{Re}\mu$ and larger $\text{Im}\mu$. However, the inclusion of the A_4 condensate prevents the LYESs for higher temperature, and makes the further LYESs fixed at the RW phase transition line. The termination of LYESs at T_e also makes the extrapolation of the

CEP location invalid with the LYESs near the temperature $T \approx 0.23$ GeV.

Our study confirms the RW LYESs emerging from the RW point with increasing $\text{Re}\mu$. We observe that the inclusion of the imaginary part of $\phi_{3,8}$ is critical for obtaining accurate scaling behavior. However, the relationship between V' (the derivative of the Polyakov potential with respect to ϕ) and $\text{Im}\phi_{3,8}$ is complex, particularly at nonzero $\text{Re}\mu$. The explicit locations of RW LYESs at various $\text{Re}\mu$ are left to the future work. What we can confirm is that the RW LYESs consistently lie at $\text{Im}\mu = \pi T/3$. Therefore, in Fig. 10, the RW LYESs trajectory is precisely the horizontal line at $\text{Im}\mu_B/T = 3 \text{Im}\mu/T = \pi$.

Furthermore, we check the scaling behavior around CEP for our current results, finding that the scaling exponent s , representing the relationship between the imaginary part of the LYESs and the temperature

$$\text{Im}\mu_{\text{LYE}} = ic_2(T - T_{\text{CEP}})^s, \quad (33)$$

remains unchanged with the existence of A_4 . The points of $(\text{Im}\mu_{\text{LYE}})^{1/s}$ with respect to $T - T_{\text{CEP}}$ of our previous study [37] and current one are shown in Fig. 11.

Within the scaling region $T - T_{\text{CEP}} \lesssim 0.04$ GeV, taking $s = 1.38$, both datasets exhibit linear behavior, demonstrating that the scaling behavior of the LYESs is not sensitive to the presence of a confining gluon background field. It is then intuitive to think that the chiral dynamics dominates in the respective region. Note that here the scaling behavior is not necessarily the critical behavior of the CEP with $s = \beta\delta = 1.58$. There exists a trivial scaling behavior around the CEP that is caused by the soft modes of QCD as

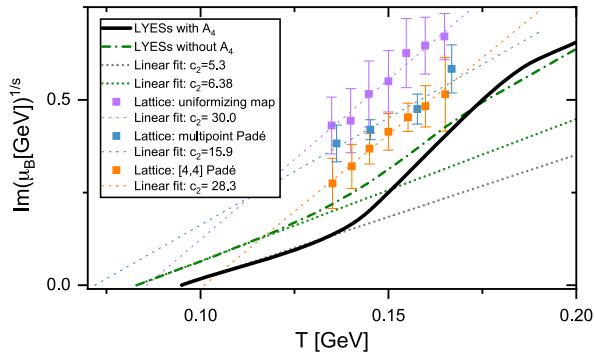


FIG. 11. Scaling analysis on the trajectories with (black-solid) and without A_4 (green-dash-dotted) in the vicinity of the CEP location: the linear fits (gray and green dotted lines) on the trajectory with and without A_4 are performed according to Eq. (33), which yields $s = 1.38$, $c_2 = 5.3$, and 6.38 (GeV units), respectively. We also show the LYESs extracted from lattice QCD simulations using the uniformizing map [29], multipoint Padé [36] and the $[4,4]$ Padé [72] approaches, together with their scaling fits (purple, blue, and orange dashed lines) to the extrapolated CEP locations. The value of s for lattice data is taken as the Z_2 scaling exponents $s = \beta\delta = 1.58$.

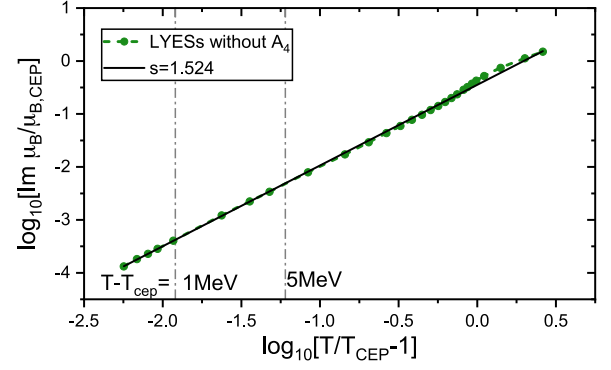


FIG. 12. Detailed analysis of the power law behavior of LYESs without A_4 . The scaling exponent s extracted from the scaling relation (33) yields $s = 1.524 \pm 0.02$ within $T - T_{\text{CEP}} < 5$ MeV.

explained in Ref. [12]. A further detailed calculation for the LYESs without the gluonic background field at $T - T_{\text{CEP}} \lesssim 0.005$ GeV suggests that the scaling exponent is $s = 1.524 \pm 0.02$ as depicted in Fig. 12. It again confirms that the trivial scaling exponent is compatible with the critical scaling of the respective universality class, but with a much larger scaling region as found in Ref. [12]. In our work, the large scaling region of such a soft-modes induced scaling behavior here is helpful for the extrapolation of the CEP location from LYESs at smaller chemical potential region, where the extrapolation interval remains relatively small in lattice.

We also obtain $c_2 = 5.3 \text{ GeV}^{1-s}$ in Eq. (33) and the dimensionless slope parameter $\bar{c}_2 = c_2 \frac{(T_{\text{CEP}})^s}{\mu_{B,\text{CEP}}} = 0.28$, which is consistent with our previous result for the case without A_4 . Following the same analysis of the scaling behavior in Ref. [37], we adopt the $Z(2)$ scaling analysis in combination with the benchmark results of the phase transition line curvature $\kappa = 0.016$ and the chiral pseudocritical temperature at zero chemical potential $T_\chi(0) = 0.156$ GeV. This amounts to an extracted CEP location from the LYESs trajectory at

$$(T, \mu_B) = (118, 608) \text{ MeV}. \quad (34)$$

Such an extracted CEP location is an improved prediction compared with the one from the direct computation at real μ_B , because within the present truncation scheme, a direct computation predicts a curvature $\kappa = 0.015$ and the chiral pseudocritical temperature $T_\chi(0) = 0.162$ GeV, which are slightly different from the results given by the first-principles QCD approaches. This small deviation induced by the truncation scheme may also have an impact on the errors of the CEP location from a direct computation at real μ_B . Moreover, the present truncation scheme yields a mean-field scaling instead of the expected $Z(2)$ scaling for the CEP of QCD, which also asks for further improvements on the truncation scheme at high μ_B for the direct

computation. In short, the CEP extraction from the LYESs trajectory reduces the errors and uncertainties of the truncation scheme and offers a correct critical scaling behavior obtained from the direct computation at high μ_B , resulting in a more reliable prediction on the CEP location. In particular, the agreement of this extracted CEP location to the state-of-the-art estimates from Refs. [9–11] suggests that the dimensionless parameter $\bar{c}_2$ is a characteristic quantity for the CEP location, and its determination based on first-principles QCD computation with precision might offer a reliable prediction on CEP as the onset of new physics.

V. SUMMARY

In this paper, we have implemented the DSE approach for QCD to investigate the quark propagator at complex chemical potential in the presence of a background gluon field condensate. Our primary focus has been on studying the QCD phase transition. In the context of a background gluon field, the behavior of the chiral condensate, denoted as $\langle\bar{\psi}\psi\rangle$, and the Polyakov loop, L , exhibits intriguing characteristics. The chiral condensate, which serves as an order parameter for chiral symmetry breaking, undergoes significant modifications due to interactions with the gluon field. Concurrently, the Polyakov loop, which is indicative of deconfinement phases, also displays a notable response to the presence of the gluon field background.

We summarized the main results concerning the DSEs with A_4 condensate, illustrating how the inclusion of such a field affects the momentum translation of Matsubara frequencies for different color components. This analysis laid the groundwork for deriving the quark propagator and bare gluon propagator in the presence of the background field. Such a scheme allowed for a comparative assessment of the background field's impact on both the chiral phase transition and LYESs. We presented the results for the Roberge-Weiss critical point for light quarks and its

associated LYESs in the low-temperature region. Furthermore, we explored the chiral phase transition LYESs while accounting for a constant background gluon field, specifically the A_4 component, which is crucial for understanding confinement and center symmetry. We examined the trajectories of LYESs, finding that the inclusion of the background field affects the location of these singularities but does not significantly alter the critical exponent.

We also delved into the symmetries of the (ϕ_3, ϕ_8) plane, which parametrizes the diagonal components of the gauge field A_4 . This study provided insights into the periodic and rotational symmetries of the Polyakov loop potential. And these symmetries were crucial in simplifying the search for minimum points of the potential and understanding the behavior of the Polyakov loop under various conditions.

Overall, this study contributes to a deeper understanding of the QCD phase transition at complex chemical potential, highlighting the importance of considering background gluon fields and their impact on chiral symmetry breaking and confinement.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [74].

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