

Studies of beauty hadron and nonprompt charm hadron production in pp collisions at $\sqrt{s} = 13$ TeV within a transport model approach

Jialin He¹,^{*} Xinye Peng^{1,2}, Xiaoming Zhang², and Liang Zheng^{1,3,2,*}

¹*School of Mathematics and Physics, China University of Geosciences (Wuhan), Wuhan 430074, China*

²*Key Laboratory of Quark and Lepton Physics (MOE) and Institute of Particle Physics, Central China Normal University, Wuhan 430079, China*

³*Shanghai Research Center for Theoretical Nuclear Physics, NSFC and Fudan University, Shanghai 200438, China*



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In high-energy proton-proton (pp) collisions at the LHC, nonprompt charm hadrons, originating from beauty hadron decays, provide a valuable probe for beauty quark dynamics, particularly at low transverse momentum where direct beauty measurements are challenging. We employ a multiphase transport (AMPT) model of string melting version coupled with PYTHIA8 initial conditions to study the beauty hadron and nonprompt charm hadron productions in pp collisions at $\sqrt{s} = 13$ TeV. In this work, the beauty quark mass during the generation stage has been increased to reproduce the measured $b\bar{b}$ cross section, and a beauty flavor-specific coalescence parameter r_{BM}^b is introduced to match LHCb measurements of beauty baryon-to-meson ratios. With these refinements, the AMPT model achieves a reasonable agreement with experimental data on beauty hadron yields and nonprompt charm hadron production from ALICE and LHCb. We present the transverse momentum and multiplicity dependence of nonprompt-to-prompt charm hadron ratios, providing new insights into the interplay between beauty quark production and the hadronization process. We emphasize that the multiplicity dependence of the nonprompt to prompt charm hadron productions can be useful to constrain the flavor dependences of the coalescence dynamics. This work establishes a unified framework for future studies of heavy quark transport and collective flow behavior in small collision systems.

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I. INTRODUCTION

Measurements of hadrons containing charm or beauty quarks in proton-proton (pp) collisions are essential for testing quantum chromodynamics (QCD) calculations [1]. Due to their large masses, heavy quarks are predominantly produced in the initial stages of the collision via hard partonic scatterings, making their inclusive production cross sections calculable within perturbative QCD using a factorization framework [2–5]. State-of-the-art calculations, including fixed-order plus next-to-leading-log (FONLL) [6], general-mass variable-flavour-number scheme (GM-VFNS) [7,8], and next-to-next-to-leading-order QCD [9], generally provide a good description of experimental data by factorizing the process into parton distribution functions, perturbative partonic cross sections, and nonperturbative fragmentation functions.

However, recent experimental results have issued a significant challenge to the assumption of universal hadronization for heavy quarks, which implies that a heavy flavor parton fragments into hadrons independently of the collision system. A growing body of evidence, particularly the enhancement of heavy flavor baryon-to-meson ratios in pp collisions compared to electron positron (e^+e^-) collisions at the LHC energy, suggests that the hadronization process of heavy flavor particles is sensitive to the surrounding environment [10–12]. Several complementary theoretical approaches have been proposed to explain this observed enhancement. Statistical hadronization models assume the thermal production of hadrons at chemical freeze-out and incorporate an augmented spectrum of heavy flavor baryon states, which qualitatively describe the system size dependence of heavy flavor baryon production with the transition from a canonical ensemble in small systems to a grand canonical ensemble in heavy-ion collisions [13–16]. A recent comprehensive comparison of heavy quark recombination models in heavy-ion collisions [17] highlights the importance of hadronization dynamics in shaping final-state observables. It has been conceived that the high-energy pp collisions can create a sufficiently dense parton system in which heavy quarks may undergo scatterings within the

*Contact author: zhengliang@cug.edu.cn

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short-lived evolving parton medium and hadronize by recombining with nearby light quarks via a coalescence mechanism, thereby favoring baryon formation relative to mesons [18–20]. Furthermore, modified string fragmentation models, incorporating color reconnection beyond the leading color approximation to include junction topologies, also improve descriptions of enhanced baryon production [21–23]. Within this broader context, a multiphase transport (AMPT) model, which dynamically simulates partonic scattering and coalescence hadronization for heavy quarks, can provide a unified framework to investigate the interplay between the bulk medium and heavy flavor hadronization [24–27].

Direct measurements of beauty hadrons are experimentally challenging, especially at low transverse momentum (p_T). An effective way to overcome this limitation is through nonprompt charm hadrons, which originate from beauty hadron decays. These particles provide an indirect but powerful probe of beauty quark production and hadronization, extending experimental sensitivity to lower p_T [28–30]. Recent measurements reveal that the nonprompt Λ_c/D^0 ratio is significantly larger in pp collisions than in e^+e^- collisions, and follows a trend similar to that of prompt charm baryons. This observation suggests the presence of a common underlying mechanism for baryon formation across charm and beauty flavors [31]. To further disentangle contributions from initial heavy quark production and from hadronization dynamics, it is essential to investigate these nonprompt-to-prompt charm hadron ratios for both meson and baryon states as a function of event multiplicity [28]. The multiplicity dependence of nonprompt charm production provides further insight into the interplay between hard and soft processes, including multiparton interactions and possible final-state effects in small systems. The AMPT model combines the dynamical parton evolution system with a flavor-dependent spatial coalescence model, providing unique opportunities to link microscopic transport dynamics with the statistical descriptions of hadronization.

In this work, we employ the AMPT model with PYTHIA8 initial conditions to study beauty quark dynamics via nonprompt charm hadron production in $\sqrt{s} = 13$ TeV pp collisions. The string melting version of the AMPT model, incorporating a partonic cascade and coalescence hadronization, has successfully described baryon to meson enhancements and collective flow in small systems [24,25]. We provide a systematic strategy to achieve reasonable descriptions to the $b\bar{b}$ cross section at 13 TeV [32] and beauty baryon yield at forward rapidity [33] in the AMPT framework. Using this improved model, we then analyze the p_T and multiplicity dependence of beauty decayed nonprompt charm hadron to prompt charm hadron ratios. This allows us to disentangle the underlying mechanisms driving the observed multiplicity trends in experiments. This approach not only provides a quantitative baseline to

exclusively interpret the heavy flavor hadron productions in small systems but also offers a foundation for future studies of heavy quark evolution, shedding a light to understand multiplicity dependence in the transition from dilute to dense QCD matter.

The rest of this paper is organized as follows. Section II provides a detailed introduction to the AMPT model and explains the modifications made to its key parameters. Section III presents the primary results, starting with beauty hadron production in Sec. III A and nonprompt c-hadron production in Sec. III B. This is followed by an investigation into the multiplicity dependence of the nonprompt-to-prompt hadron ratio in Sec. III C. Finally, Sec. IV summarizes the findings of this study.

II. THE AMPT MODEL

The AMPT model consists of four main components: initial conditions, partonic interactions, coalescence hadronization, and hadronic rescatterings. The initial conditions are generated using the PYTHIA8 Monte Carlo event generator [34] within the string melting framework [35–37], incorporating subnucleon spatial fluctuations [38]. The subsequent evolution of the resultant partonic matter produced from the PYTHIA8 initial conditions is simulated with Zhang’s parton cascade (ZPC) model [39], which describes the microscopic two-body parton scatterings. A parton rescattering cross section $\sigma = 0.15$ mb is employed, which gives a satisfactory description to the elliptic flow measurements in pp collisions [25]. Hadronization is performed using a naive spatial coalescence scheme, in which quarks combine with their nearest spatial partners to form hadrons [40]. An overall coalescence parameter r_{BM} has been introduced to determine the relative probability for a quark to become a meson or a baryon in this model. The hadrons after coalescence may undergo further hadronic rescatterings implemented in the extended relativistic transport model (ART) [41,42]. In this study, to address this issue, we have made improvements to both the initial conditions and the hadronization process, as detailed below.

It is found that the default parameters within PYTHIA8 produce a $b\bar{b}$ cross section that is higher than what has been measured experimentally at the top LHC energy, as shown in Fig. 1. Figures 1(a) and (b) show the $b\bar{b}$ cross section per rapidity (y) and pseudorapidity (η) in comparison with experimental measurements from ALICE [30] and LHCb [43], respectively. We observe that the default PYTHIA8 calculations with Monash tune indicated by the blue line significantly overestimate $b\bar{b}$ production. This discrepancy highlights that PYTHIA8’s leading-order framework does not inherently capture the full perturbative quantum chromodynamics scale uncertainties and higher-order contributions that FONLL and GM-VFNS handle systematically. To obtain a realistic baseline for beauty production in the AMPT framework, we adjust the effective b quark mass m_b used in the quark production step, as has been done in

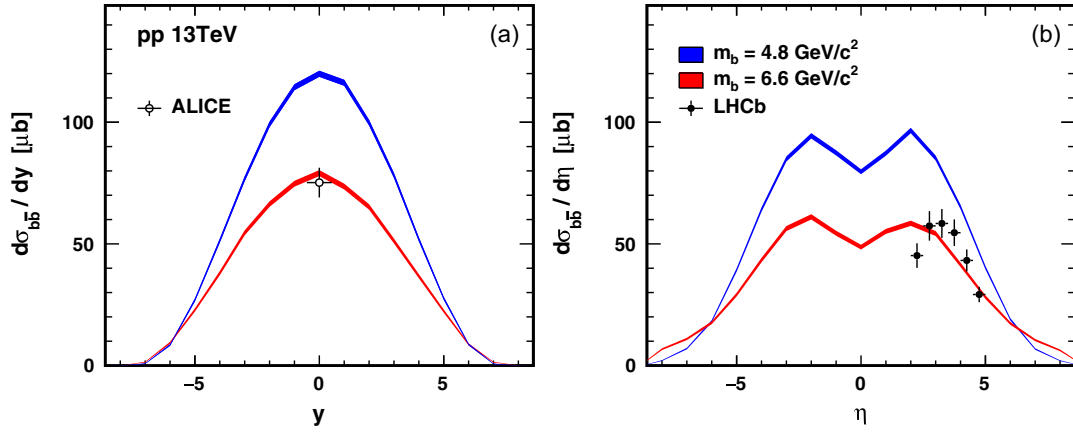


FIG. 1. y (left) and η (right) distribution of the $b\bar{b}$ production cross section in pp collisions at $\sqrt{s} = 13$ TeV. The red and blue curves represent results generated by PYTHIA8 with the b quark effective mass $m_b = 4.8$ and $m_b = 6.6$ GeV/ c^2 , respectively. Experimental data from ALICE [30] and LHCb [43] are shown in black open and solid markers for y and η , respectively.

Refs. [23,44,45]. Through iterative adjustments of m_b and comparisons with experimental data, we determined an optimal value of $m_b = 6.6$ GeV/ c^2 . As shown in Fig. 1, the results show substantial improvement over the default setting with $m_b = 4.8$ GeV/ c^2 and align more closely with experimental data when m_b is increased to 6.6 GeV/ c^2 . We emphasize that the variation of m_b is applied exclusively during the initial production stage implemented in PYTHIA8. After production, the mass is reset to the default value for all subsequent evolution processes, including partonic transport and hadronization. To illustrate the impact of this mass variation and place it in a broader theoretical context, we confront the resulting spectra with the widely established FONLL calculation in Appendix A. FONLL calculations incorporate higher-order perturbative corrections and resummation effects that are not fully captured in leading-order event generators. We find that increasing m_b in PYTHIA8 effectively suppresses the beauty yield, which effectively mimics the higher-order suppression effects characteristic of the FONLL predictions. This comparison demonstrates that the adopted tuning provides a phenomenologically controlled way to benchmark PYTHIA8-based initial conditions against the state-of-the-art perturbative QCD calculations.

In the AMPT coalescence hadronization model, the decision to form a meson or a baryon is based on the relative spatial separations between constituent partons in the partonic center-of-mass frame. For a given quark, the model identifies the closest antiquark, with d_M denoting the relative distance in the quark-antiquark rest frame. Simultaneously, it finds the two closest quarks, with d_B defined as the average of the three pairwise relative distances in the three-quark rest frame. A baryon is formed if the ratio d_B/d_M is less than the coalescence model parameter r_{BM} [40]. A larger r_{BM} value corresponds to a higher baryon production yield. In the present work, we set $r_{BM}^b = 1.2$ for beauty quarks, a value tuned to reproduce the

measured Λ_b production reported by LHCb Collaboration. The detailed comparison with experimental data, along with the systematic study underlying this choice, will be presented later in this paper. We note that $r_{BM}^b = 1.2$ is slightly smaller than the value $r_{BM}^c = 1.4$ used for charm quarks in our previous work, where the latter was found to successfully describe Λ_c^+ production [24]. Compared to the light flavor coalescence parameters [24,27,40,46], the large charm and beauty coalescence parameter is qualitatively consistent with expectations from the statistical hadronization model incorporating an augmented set of heavy flavor baryon states [16].

It should be emphasized that the string melting version of the AMPT model integrates fragmentation and coalescence effects into a unified and internally consistent framework. In this approach, excited strings are first converted into primordial hadrons via string fragmentation, which serves as an intermediate step for implementing the string melting mechanism by decomposing these hadrons into their valence partons. The resulting partons subsequently evolve through the ZPC transport stage before hadronizing via a naive spatial coalescence algorithm that depends solely on spatial proximity. In low-density systems, where partonic scatterings are rare and spatial correlations among partons originating from the same string are largely preserved, the coalescence procedure naturally recombines these partons back into their original primordial hadron configurations. As a result, fragmentation hadronization features are effectively recovered, even though the process is technically realized within the coalescence formalism. In this sense, fragmentation physics is effectively embedded in the modeling of the string melting process itself, and the subsequent parton transport stage provides a smooth transition from recombination-dominated hadronization in dense environments to fragmentationlike behavior in dilute systems within a single dynamical framework.

III. RESULTS

A. Beauty hadron productions

With the AMPT framework incorporating PYTHIA8 initial conditions, the tuned beauty quark mass, and the optimized coalescence parameter r_{BM}^b , the model is now equipped with a consistent set of parameters that provide a realistic description of beauty hadron production. Building on this foundation, we proceed to investigate beauty hadrons and their nonprompt charm decay products in pp collisions at $\sqrt{s} = 13$ TeV. All the hadron cross sections presented in this work are calculated by averaging over particles and their corresponding antiparticles following the experimental analysis procedure outlined in Refs. [30,33,47].

The p_T -differential cross sections of beauty hadrons in pp collisions at $\sqrt{s} = 13$ TeV are presented in Fig. 2. Panels (a) and (b) show the dependence on transverse momentum at mid and forward rapidity, respectively, while panel (c) illustrates the rapidity dependence in the range $0 < p_T < 40$ GeV/c. The AMPT model results for B^0 , B^+ , B_s , and Λ_b^0 are shown as black, red, blue, and green lines, respectively, with solid and open black markers denoting B^+ measurements from LHCb [47] and ALICE [30]. The ALICE data are derived by scaling the $b\bar{b}$ cross section with a fragmentation fraction of 0.399 [48]. For visual clarity, the B^0 results are scaled by a factor of 3/4 in the figure. In Figs. 2(a) and (b), the p_T distributions exhibit a relatively flat profile, consistent with beauty quarks produced predominantly via hard scattering processes involving large transverse momentum transfer. At forward rapidity, the AMPT model predictions reproduce the measured B^+ spectra above $p_T \approx 2$ GeV/c, while a noticeable excess appears at low p_T . This low- p_T excess is attributed to threshold effects in the production of beauty quarks, amplified by the increased beauty quark mass. The large m_b reduces the available phase space for beauty quark production, compressing the spectrum and shifting yield toward the lowest p_T region. The effect is more pronounced at forward rapidity, where the typical partonic center-of-mass energies are closer to threshold, but it is largely absent in the midrapidity spectra. The calculated B^0 and B^+ spectra are nearly identical, as expected from isospin symmetry. In Fig. 2(c), the rapidity distribution of B^+ from the AMPT model is consistent with LHCb data, providing confidence that the tuned model captures the main features of beauty hadron production at forward rapidity.

In pp collisions at the LHC energy [31,49], the Λ_c^+/D^0 production ratio has been found to be significantly enhanced relative to measurements in e^+e^- collisions [50], particularly at low and intermediate transverse momentum. Models incorporating a coalescence hadronization mechanism have been shown to describe this enhancement [49]. Motivated by these findings, it is of interest to investigate whether analogous effects appear in the beauty sector. Figure 3 shows the Λ_b^0/B^0 ratios in pp collisions at $\sqrt{s} = 13$ TeV at

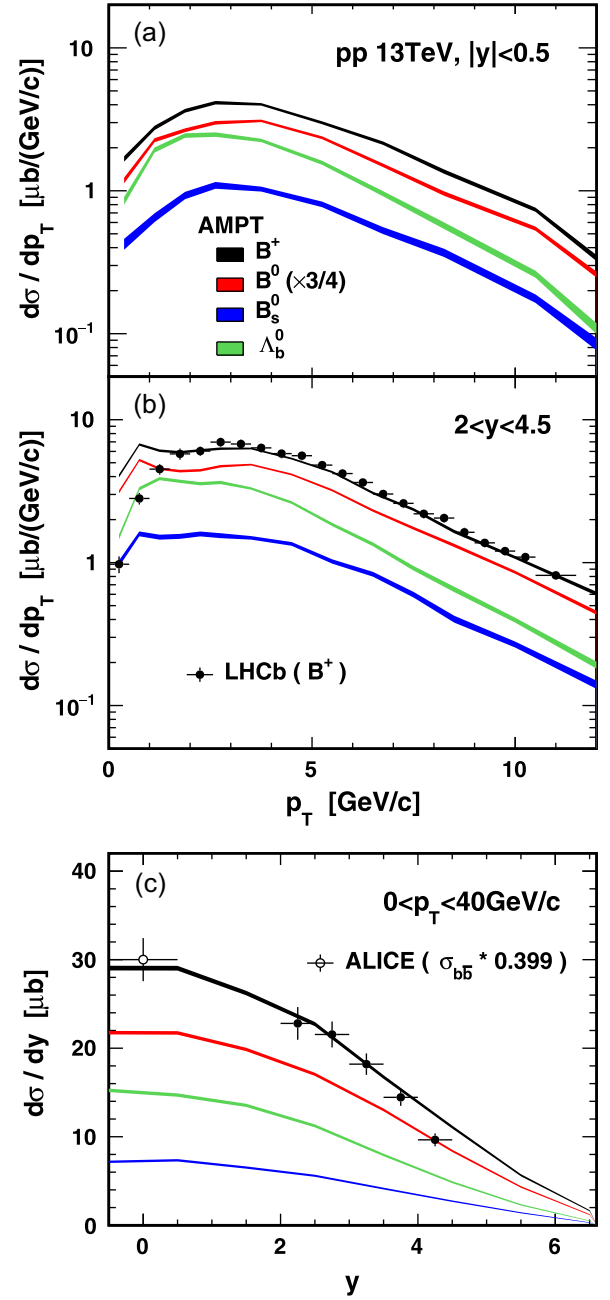


FIG. 2. The p_T -differential cross section of beauty hadrons in pp collisions at $\sqrt{s} = 13$ TeV on midrapidity [$|y| < 0.5$, panel (a)] and forward rapidity [$2 < y < 4.5$, panel (b)], and as a function of y for $0 < p_T < 40$ GeV/c [panel (c)]. The black, red, blue, and green lines show the AMPT results for B^0 , B^+ , B_s , and Λ_b^0 , respectively. Solid and open black markers indicate B^+ results from LHCb [47] and ALICE [30]. The ALICE data are obtained by scaling the measured $b\bar{b}$ cross section with a fragmentation fraction of 0.399 [48].

forward- y as a function of p_T (left) and relative charged particle multiplicity (right). The AMPT calculations are shown by the red line, while the black points correspond to measurements from LHCb Collaboration [33].

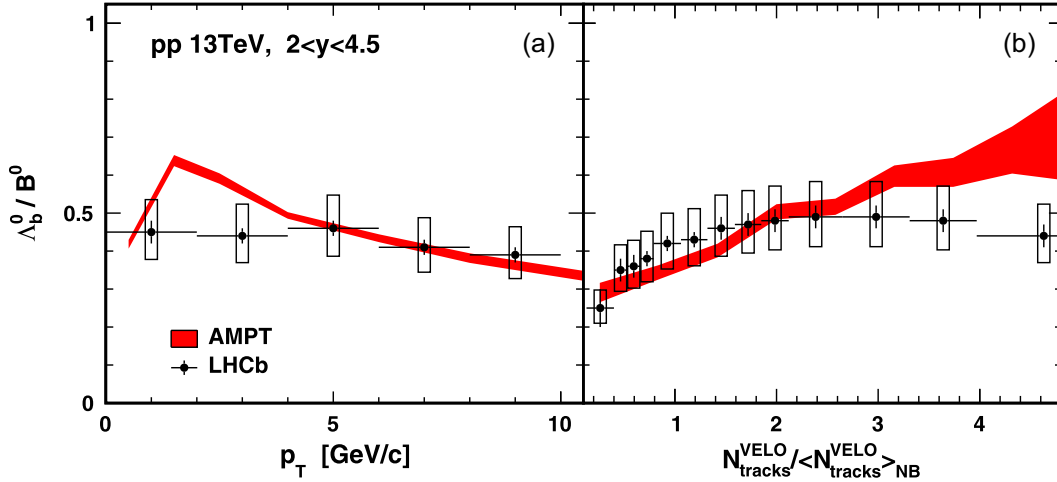


FIG. 3. The Λ_b^0/B^0 ratios in pp collisions at $\sqrt{s} = 13$ TeV on forward- y as a function of p_T (a) and multiplicity (b). The red line and black points represent the AMPT and LHCb results [33], respectively. The vertical error bars on the LHCb points represent the quadratic sum of statistical and systematic uncertainties, and the boxes indicate global uncertainties associated with branching fractions.

In Fig. 3(a), the AMPT calculations show a Λ_b^0/B^0 ratio varying with p_T consistent with the value observed in the LHCb data and broadly compatible within experimental uncertainties. It is found that a mild excess appears around $p_T \approx 2$ GeV/ c in the beauty baryon-to-meson ratio. This enhancement can be interpreted in terms of the kinematic consequences of the coalescence mechanism. The transverse momentum of a Λ_b^0 baryon arises from the vector sum of its three constituent quarks, in contrast to two for a B^0 meson, leading to a shifted baryon p_T spectra and enhanced baryon-to-meson ratio at low p_T . The presence of this feature suggests a sizable contribution of coalescence effects to the beauty quark hadronization at low p_T . A qualitatively similar behavior has also been reported in hydrodynamics-based simulations within the EPOS4HQ framework [51].

In Fig. 3(b), the AMPT calculations reproduce the rising trend of the Λ_b^0/B^0 ratio with charged-particle multiplicity observed by LHCb, especially from the low to intermediate multiplicity region. The horizontal axis represents the event activity, quantified as the number of reconstructed tracks normalized to the mean track multiplicity in the backward pseudorapidity interval $2 < \eta < 5$. The enhancement of baryon production in AMPT increases with multiplicity, consistent with the data. At low multiplicity, the limited parton density favors hadronization through string fragmentation, yielding a relatively small baryon-to-meson ratio. With increasing multiplicity, the relative contribution of coalescence becomes more significant, leading to an enhanced ratio. The experimental measurements also show a tendency toward saturation of the ratio at high multiplicity, whereas this feature is less evident in the model. This saturation behavior aligns with expectations from statistical hadronization models [13,14,16], where the system transitions from a canonical ensemble in low-multiplicity pp

events to a grand canonical ensemble in larger systems. The weaker saturation trend in the AMPT model implies that beauty quarks are less thermalized compared to data indications, with coalescence contributions continuing to grow with multiplicity. Nonetheless, the tuned AMPT model reasonably describes the multiplicity-dependent Λ_b^0/B^0 ratio within experimental uncertainties, providing a robust basis for analyzing nonprompt charm hadron distributions.

B. Nonprompt charm hadron productions

Since nonprompt charm hadrons originate from beauty hadron decays, their spectra directly reflect the kinematic distributions and hadronization mechanisms of beauty quarks, making them a sensitive probe of beauty dynamics in small systems. After adjusting the input $b\bar{b}$ cross section to align with experimental data, the AMPT framework yields an improved description of beauty hadron production, enabling reliable predictions for nonprompt charm hadrons. Figure 4 shows the p_T differential cross sections of nonprompt charm hadrons (D^0 , D^+ , D_s^+ , and Λ_c^+) in pp collisions at $\sqrt{s} = 13$ TeV in the midrapidity region ($|y| < 0.5$). The AMPT model results, shown as black, red, blue, and green lines for D^0 , D^+ , D_s^+ , and Λ_c^+ , respectively, are compared to ALICE measurements [30,31]. The calculations reproduce the experimental trends, with good agreement observed in particular for D^0 , where the model predictions are consistent with the data within uncertainties across most of the measured p_T region. A slight underestimation is seen for nonprompt Λ_c^+ . The comparison demonstrates that the tuned AMPT model provides a reasonable description of nonprompt charm hadron production, establishing a baseline for further studies of charm hadron ratios, baryon-to-meson production, and nonprompt-to-prompt fractions.

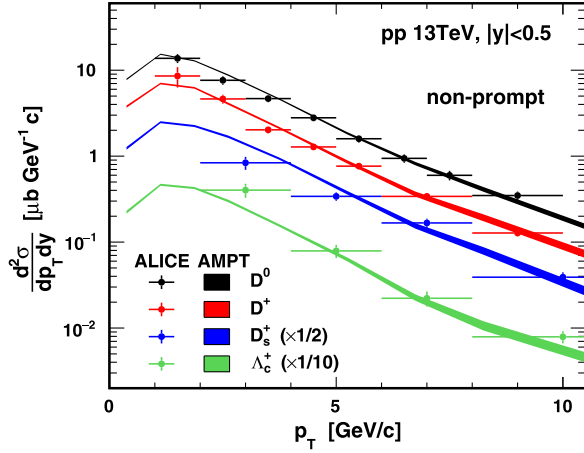


FIG. 4. Double-differential cross section of nonprompt c -hadrons in pp collisions at $\sqrt{s} = 13$ TeV at mid- y , as a function of p_T . The D^0 , D^+ , D_s^+ , and Λ_c^+ are shown in black, red, blue, and green, respectively. The D_s^+ and Λ_c^+ results were scaled by factors of 1/2 and 1/10 to allow better visual comparison. Lines represent AMPT model results, while data points represent ALICE results [30,31].

The relative yields of different charm meson species constrain the probabilities for a charm quark to combine with light or strange quarks, thereby testing isospin symmetry in the D^+/D^0 ratio and possible strangeness enhancement in the $D_s^+/(D^0 + D^+)$ ratio. Separating prompt and nonprompt contributions further provides sensitivity to charm and beauty hadronization mechanisms. Figure 5 shows the ratios of D^+/D^0 (left) and $D_s^+/(D^0 + D^+)$ (right) as a function of p_T in pp collisions at $\sqrt{s} = 13$ TeV at midrapidity. The AMPT model calculations are shown as lines, while the data points correspond to ALICE measurements [10,30]. Prompt and nonprompt contributions are indicated in black and red, respectively.

In Fig. 5(a), the D^+/D^0 ratios from both the AMPT model and ALICE exhibit little dependence on p_T for either prompt or nonprompt mesons, with values close to 0.5 across the measured range. The agreement between prompt and nonprompt results indicates that, within uncertainties, the hadronization of charm and beauty quarks with light u and d quarks proceeds with similar relative probabilities, consistent with expectations from isospin symmetry, and the decay kinematics of beauty hadrons do not alter this behavior. It is shown in Fig. 5(b) that the AMPT calculations for $D_s^+/(D^0 + D^+)$ are compatible with ALICE data for both prompt and nonprompt charm hadrons. While the model reproduces the overall normalization, it predicts an essentially flat p_T dependence, in contrast to the mild enhancement observed experimentally. The nonprompt strange-to-nonstrange meson ratio lies significantly above the prompt one, which can be understood from the decay kinematics of beauty hadrons. Nonprompt D_s^+ mesons receive sizable contributions from both strange and nonstrange B meson decays, whereas nonprompt D^0 and D^+ mesons originate predominantly from nonstrange B mesons.

To complement the study of beauty baryon-to-meson ratios, it is useful to investigate nonprompt charm hadrons, which act as effective proxies for beauty hadrons and allow direct comparison with their prompt counterparts. The relative size of prompt and nonprompt Λ_c^+/D^0 ratios provides sensitivity to the flavor dependence of hadronization. Figure 6 presents the Λ_c^+/D^0 ratios in pp collisions at $\sqrt{s} = 13$ TeV as a function of transverse momentum (left) and charged-particle multiplicity (right). The AMPT model results are shown as lines, while the ALICE measurements [52] are represented by points. Results for prompt and nonprompt charm hadrons are displayed in black and red, respectively.

In Fig. 6(a), the p_T dependence of Λ_c^+/D^0 is presented. For the prompt component, both AMPT and ALICE data

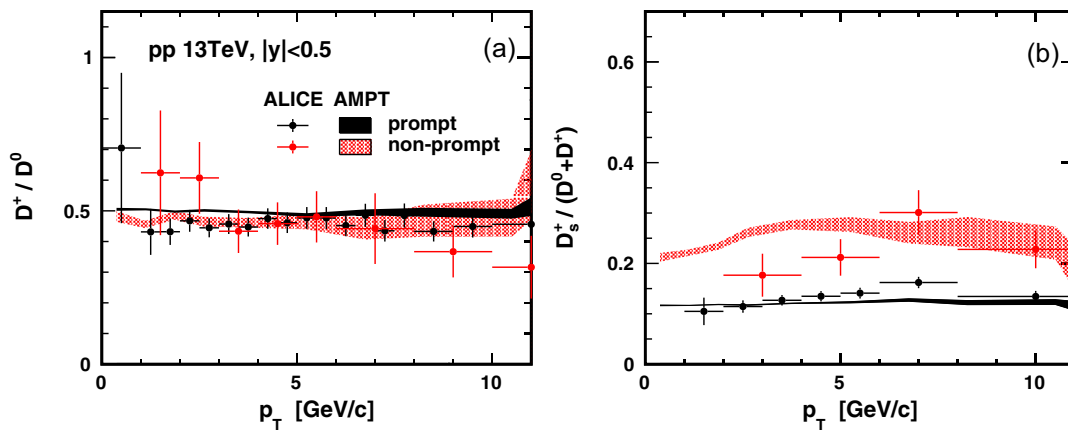


FIG. 5. Ratios of D^+/D^0 (left) and $D_s^+/(D^0 + D^+)$ (right) in pp collisions at $\sqrt{s} = 13$ TeV within $|y| < 0.5$ as a function of p_T . Black and red represent prompt and nonprompt particles, respectively. The lines represent AMPT model results, while the points correspond to ALICE results [10,30].

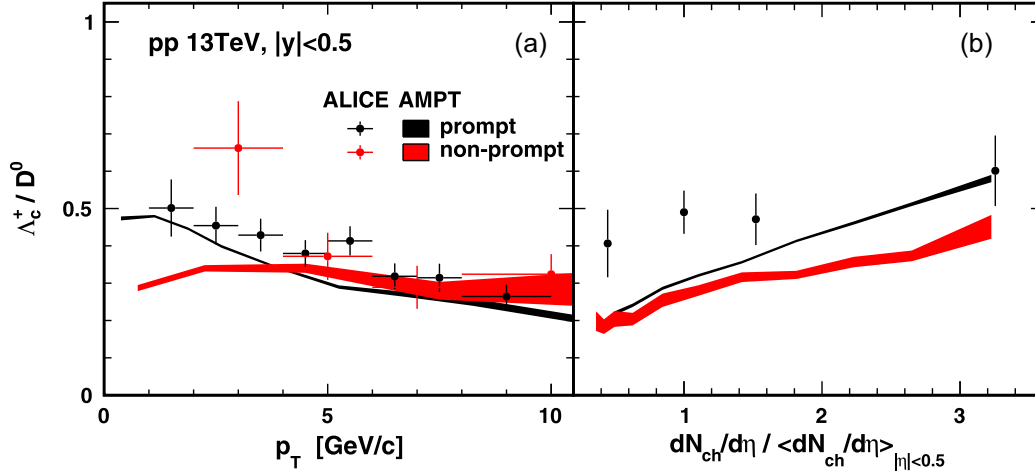


FIG. 6. Λ_c^+/D^0 in pp collisions at $\sqrt{s} = 13$ TeV within $|y| < 0.5$ as a function of p_T (left) and multiplicity (right). Black and red represent prompt and nonprompt particles, respectively. The lines represent AMPT model results, while the points correspond to ALICE results [28,52].

display an enhancement toward low p_T . Although the AMPT predictions lie slightly below the central values of the data, most results remain within experimental uncertainties, indicating that a choice of $r_{BM}^c = 1.4$ provides a reasonable description of the charm baryon-to-meson ratio. For the nonprompt component, the experimental data show a broadly similar trend to the prompt case, while the AMPT model reproduces the data well for $p_T > 4$ GeV/c but falls below the data at lower p_T . The nonprompt Λ_c^+/D^0 in the AMPT model does not exhibit an increase at low p_T . It is also noted that the decay kinematics of beauty hadrons smear the momentum distributions of the daughter charm hadrons, diluting the baryon enhancement at intermediate p_T generated by coalescence shown in Fig. 3(a). In particular, the AMPT model prediction for nonprompt Λ_c^+/D^0 is systematically lower than the prompt ratio at small transverse momentum. This difference suggests distinct manifestations of coalescence dynamics in charm and beauty hadronization. Figure 6(b) shows the multiplicity dependence of Λ_c^+/D^0 for prompt and nonprompt charm hadrons plotted as a function of the self-normalized charged-particle density $dN_{ch}/d\eta/\langle dN_{ch}/d\eta \rangle$. Both components exhibit a clear enhancement with increasing multiplicity, reflecting the growing role of partonic rescatterings and quark recombination in high-activity events. However, the prompt Λ_c^+/D^0 ratio rises more steeply than the nonprompt one, reflecting both the reduced sensitivity of beauty hadronization to event activity and the flavor dependence of the coalescence parameter in the AMPT framework.

C. Nonprompt-to-prompt charm hadron ratios

The nonprompt-to-prompt charm hadron ratios provide a key observable comparing the underlying kinematics of charm and beauty quarks. Figure 7 presents the

nonprompt-to-prompt ratios in pp collisions at $\sqrt{s} = 13$ TeV at midrapidity as a function of p_T . Panels (a)–(d) correspond to D^0 , D^+ , D_s^+ , and Λ_c^+ , respectively. The curves represent the AMPT model results, while the data points denote ALICE measurements. The AMPT calculations reproduce the overall magnitude of the ratios reasonably well. This agreement arises because the large m_b value reduces the yield of beauty hadrons, thereby lowering the nonprompt component. However, the model predictions for the nonprompt-to-prompt ratio show a relatively flat p_T dependence, in contrast to the increasing trend observed in data. This discrepancy arises because the higher beauty quark mass raises the production threshold, shifting a considerable fraction of beauty quarks to lower p_T . As a result, the shapes of the nonprompt and prompt charm hadron spectra become more similar in the low- p_T region after the beauty hadron decay kinematics are included, leading to a weaker rise of the nonprompt-to-prompt ratio with p_T in the AMPT model.

Beyond the transverse momentum dependence, the multiplicity dependence of the nonprompt over prompt charm hadron ratio provides an important probe of heavy flavor quark evolution in high-density small-system environments. Since nonprompt charm hadrons originate from both beauty meson and baryon decays, their relative contributions may vary with event activity. To disentangle these effects, the decay origins are tracked explicitly using the AMPT model. Figure 8 shows the nonprompt-to-prompt charm hadron ratios in pp collisions at $\sqrt{s} = 13$ TeV within $|y| < 0.5$ for $1 < p_T < 24$ GeV/c as a function of self-normalized event multiplicity, with panels (a)–(d) corresponding to D^0 , D^+ , D_s^+ , and Λ_c^+ , respectively. The nonprompt-to-prompt D_s^+ ratios were scaled by 1/2 for better visibility. Solid black, red, and blue points denote the AMPT model predictions for nonprompt charm hadrons

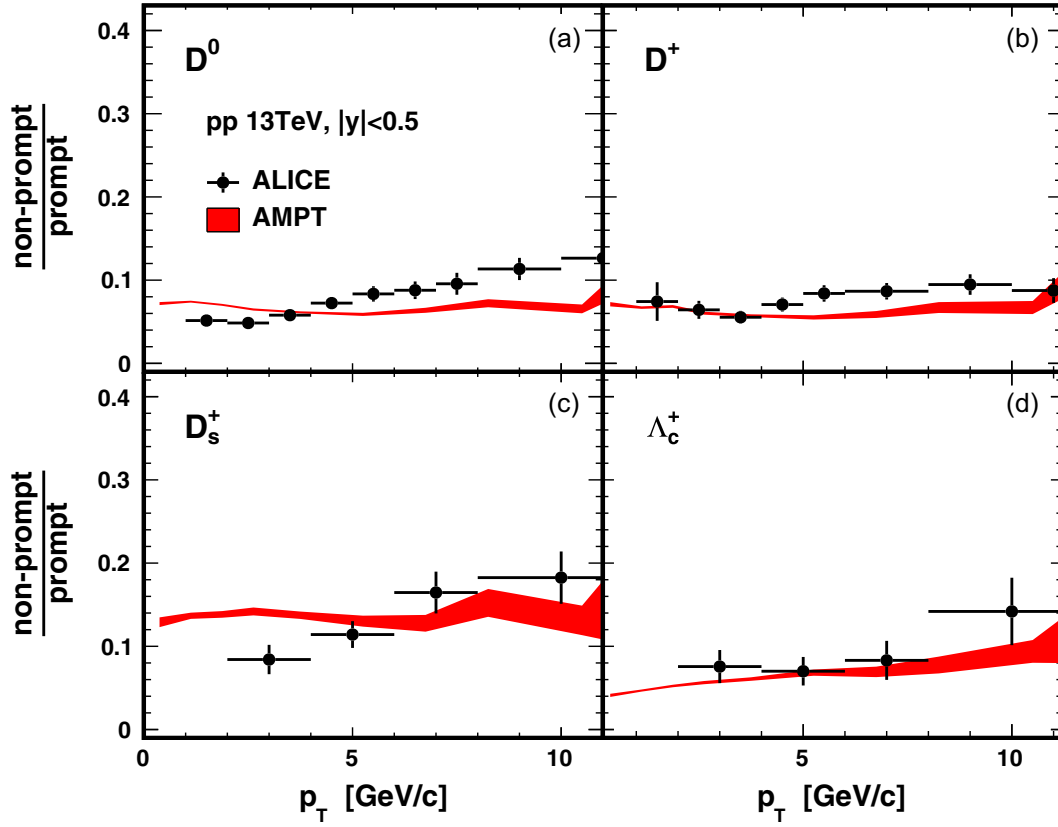


FIG. 7. Nonprompt/prompt ratios in pp collisions at $\sqrt{s} = 13$ TeV within $|y| < 0.5$ as a function of p_T . Panels (a)–(d) correspond to D^0 , D^+ , D_s^+ , and Λ_c^+ , respectively. Lines represent AMPT model results, and points represent ALICE results [30,31,53].

from all beauty hadrons, beauty mesons only, and beauty baryons only, respectively. The solid black points are consistent with the sum of the latter two. Hollow black points indicate ALICE results for nonprompt charm hadrons from inclusive beauty hadron decays [28]. The dashed and dotted curves illustrate the effect of varying the beauty coalescence parameter to $r_{BM}^b = 0.7$ and 2.8 , for the inclusive nonprompt-to-prompt ratios. For D^0 and D^+ from inclusive beauty hadron decay, the AMPT model reproduces the ALICE measurements well [28]. The AMPT calculations demonstrate that nonprompt charm mesons predominantly originate from beauty meson decays, while nonprompt charm baryons are largely produced in beauty baryon decays. This trend is consistent with the decay patterns discussed in the ALICE analysis [28], highlighting the importance of the parent hadron composition in shaping the observed multiplicity dependence. The nonprompt D_s^+ receives a comparatively larger contribution from beauty baryon decays than the D^0 and D^+ mesons, although its overall multiplicity dependence remains similar to the other meson species. A contrast emerges between multiplicity dependences of meson and baryon species. The AMPT model predicts a modest increase of the nonprompt-to-prompt ratio with multiplicity for charm mesons, while for Λ_c^+ this behavior becomes

less compelling. This discrepancy represents the sensitivity of the multiplicity dependence of the nonprompt-to-prompt charm hadron ratio to the relative values of the coalescence parameter r_{BM} for charm and beauty. Varying the r_{BM}^b parameter significantly alters the multiplicity dependence of nonprompt-to-prompt particle ratios, as indicated by the dashed and dotted lines corresponding to r_{BM}^b of 0.7 and 2.8 . A smaller value $r_{BM}^b = 0.7$ suppresses beauty baryon formation, leading to a stronger rise of the D^0 ratios with multiplicity but a marked decrease for Λ_c^+ . Conversely, a larger value $r_{BM}^b = 2.8$ enhances beauty baryon production, flattening the multiplicity trend for mesons while boosting the baryon channel. A more systematic decomposition into primordial hadron production, coalescence probabilities, and decay kinematics is presented in Appendix B. The tuned r_{BM}^b for beauty hadrons naturally leads to a slower growth of the nonprompt-to-prompt ratio varying with multiplicity, consistent with the observations found in experimental data. Future precision measurements of nonprompt baryon-to-meson ratios as a function of multiplicity could thus provide a powerful constraint on flavor-dependent coalescence dynamics, helping to bridge microscopic transport approaches with statistical descriptions of heavy flavor hadronization.

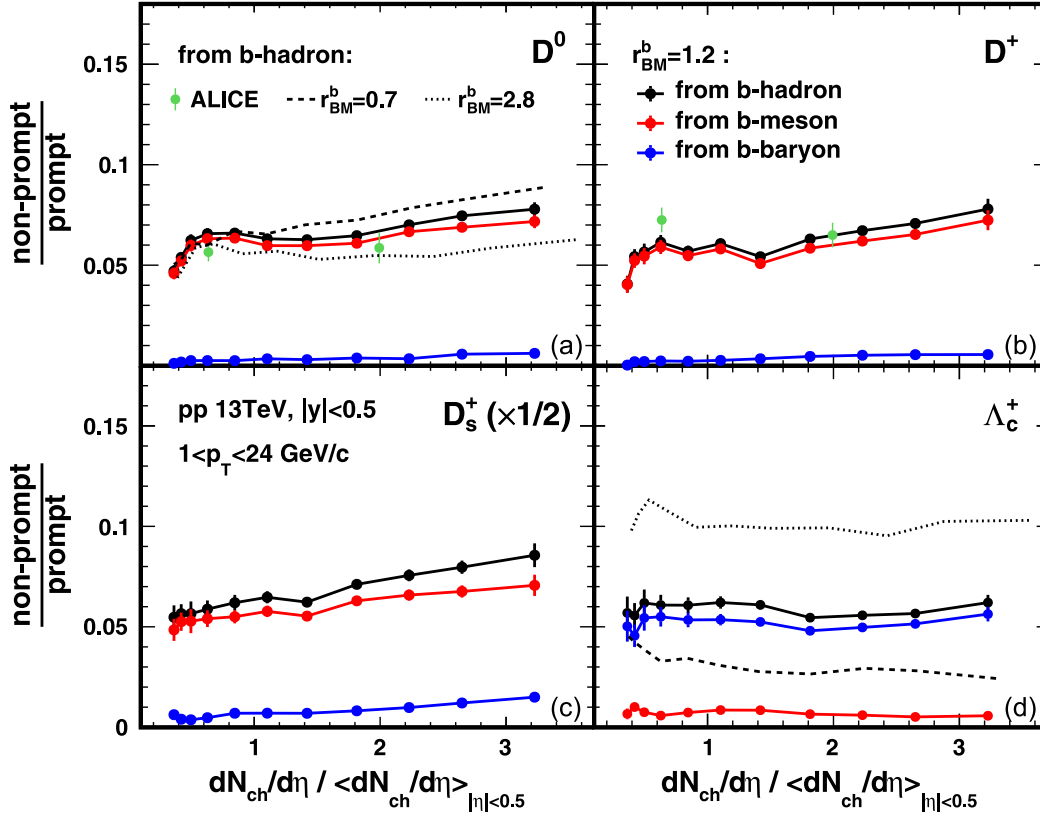


FIG. 8. Nonprompt/prompt charm hadron ratios in pp collisions at $\sqrt{s} = 13$ TeV within $|y| < 0.5$ for $1 < p_T < 24$ GeV/ c as a function of multiplicity. Panels (a)–(d) show results for D^0 , D^+ , D_s^+ , and Λ_c^+ , respectively. Solid black, red, and blue points denote nonprompt charm hadrons from all beauty hadrons, only beauty mesons, and only beauty baryon decays in the AMPT model of $r_{BM}^b = 1.2$. Black dashed, dotted curves and green dots, respectively, represent nonprompt charm hadrons from all beauty hadron decays of $r_{BM}^b = 0.7$, 2.8 and ALICE results [28].

IV. SUMMARY

We present a comprehensive study of beauty hadron and nonprompt charm hadron production in pp collisions at $\sqrt{s} = 13$ TeV within the framework of the AMPT model coupled with PYTHIA8 initial conditions. To improve the description of beauty-related observables within the AMPT model, two targeted modifications are introduced. First, the beauty quark mass is increased to $m_b = 6.6$ GeV/ c^2 in order to reproduce the measured $b\bar{b}$ production cross section, consistent with phenomenological adjustments often applied in FONLL and related QCD calculations [54]. Second, a flavor-dependent coalescence parameter is applied, with $r_{BM}^b = 1.2$ for beauty quarks, guided by LHCb measurements of beauty baryon-to-meson ratios [33,47]. With these refinements, the AMPT model provides an improved description of beauty hadron spectra and nonprompt charm hadron production at both forward and midrapidity, reproducing their transverse momentum and multiplicity dependences.

It is shown in this study that the multiplicity dependence of nonprompt-to-prompt charm hadron ratios can be sensitive to the beauty quark coalescence dynamics. A

reduced beauty flavor coalescence parameter r_{BM}^b suppresses the beauty baryon formation, thereby enhancing the beauty meson decay channels, which leads to a stronger increase with multiplicity for nonprompt-to-prompt charm meson ratios. Conversely, an enlarged r_{BM}^b value promotes beauty baryon production, flattening the meson ratios and amplifying the baryon enhancement. These findings establish nonprompt charm hadrons as powerful observables for probing heavy flavor hadronization in small systems. Their sensitivity to the interplay between quark production, coalescence dynamics, and decay kinematics makes them ideal for disentangling the differences between charm and beauty hadronization mechanisms.

Future high-precision measurements of nonprompt baryon and meson productions as a function of event activity will thus be essential for constraining theoretical models, bridging transport-based approaches like the AMPT model with statistical hadronization descriptions, and advancing our understanding of heavy quark dynamics in the dense environment of high-multiplicity pp collisions. The AMPT framework, by providing a unified treatment of heavy flavor production and bulk medium evolution, offers

a unique platform for investigating heavy quark collectivity phenomena and opens a path toward a deeper understanding of the role of beauty quarks in the evolution of small systems.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

APPENDIX A: COMPARING PYTHIA8 CALCULATIONS WITH m_b VARIATION TO FONLL

In this appendix, we provide a comparative study between beauty quark production cross sections calculated using a modified m_b mass parameter in PYTHIA8 and predictions from the FONLL framework. The modification of the beauty quark mass m_b is applied only during the initial production stage in PYTHIA to regulate the total cross section. It is important to emphasize that the FONLL calculation [54] provides a rigorous theoretical benchmark, as it systematically handles next-to-leading-order corrections and the resummation of large logarithm effects, which are inherently absent in the leading-order event generator of PYTHIA8.

Figure 9 shows the p_T spectra of beauty quarks at midrapidity obtained from PYTHIA8 with different values of m_b , compared to the FONLL calculation [54]. The default PYTHIA8 simulation using $m_b = 4.8$ GeV/c tends to significantly overestimate the cross section compared to FONLL. Increasing m_b significantly suppresses the yield at low p_T and develops a diplike structure due to phase space constraints near threshold. The leading-order perturbative QCD approach employed in PYTHIA8 tends to overestimate the total cross section and yield harder p_T spectra relative to FONLL, which systematically incorporates higher-order corrections and resummation effects. The chosen m_b value brings the PYTHIA results closer to the FONLL baseline, thereby improving agreement with

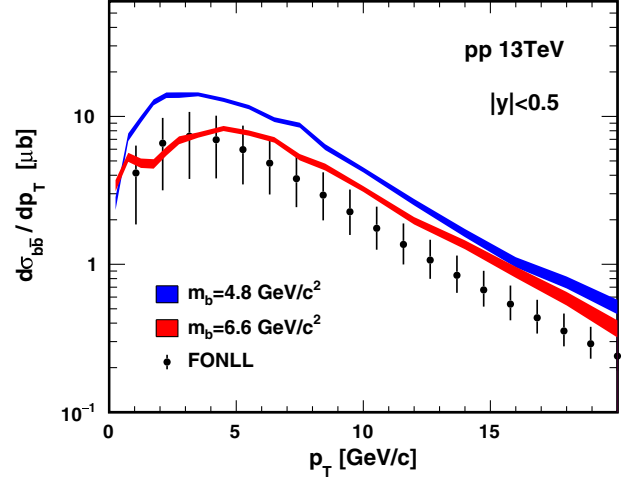


FIG. 9. The p_T -differential spectra of beauty quarks at mid-rapidity from PYTHIA with different m_b , compared with FONLL predictions [54].

experimental data. This tuning effectively mimics the higher-order suppression mechanisms absent in the leading-order PYTHIA calculation.

APPENDIX B: ANALYZING MULTIPLICITY DEPENDENCE FOR NONPROMPT-TO-PROMPT CHARM HADRON RATIOS

To understand the multiplicity dependence of non-prompt-to-prompt charm hadron production ratios, we divide the analysis of this ratio into three factors: heavy quark production, hadronization, and the subsequent weak decays of beauty hadrons. The beauty hadron decay process is expected to be largely insensitive to the event multiplicity and can be regarded as a constant determined by the branching ratio. Consequently, the observed multiplicity dependence of nonprompt-to-prompt production primarily reflects the interplay between the initial heavy quark production and the hadronization dynamics for beauty and charm. For nonprompt charm mesons these dependences can be factorized as follows:

$$\begin{aligned} \frac{NP}{P}(M^c) &= \frac{n_H^b}{n_H^c} \times \frac{1-f^b}{1-f^c} \times BR(M^b \rightarrow M^c) \\ &+ \frac{n_H^b}{n_H^c} \times \frac{f^b}{1-f^c} \times BR(B^b \rightarrow M^c), \end{aligned} \quad (B1)$$

and for nonprompt charm baryons,

$$\begin{aligned} \frac{NP}{P}(B^c) &= \frac{n_H^b}{n_H^c} \times \frac{f^b}{f^c} \times BR(B^b \rightarrow B^c) \\ &+ \frac{n_H^b}{n_H^c} \times \frac{1-f^b}{f^c} \times BR(M^b \rightarrow B^c). \end{aligned} \quad (B2)$$

The superscripts c or b denote charm or beauty flavor. n_H^b and n_H^c represent the numbers of beauty and charm hadrons right after hadronization and before the weak decay process accordingly. M and B represent the meson and baryon states with different flavors. f is the fraction of baryons among hadrons for each flavor without decay, while the meson fraction is $1 - f$. The branching ratios BR specify the decay probabilities from beauty baryons (B^b) and mesons (M^b) into specific final charm baryons (B^c) and mesons (M^c). Since the decay sources of nonprompt charm hadrons can be divided into beauty mesons and baryons, our equation is naturally split into two parts, representing the sum of beauty meson and baryon decay channel contributions. Importantly, in Eq. (B1), the leading contribution to nonprompt charm mesons arises from the $M^b \rightarrow M^c$ decay channel, while in Eq. (B2), the dominant channel is the $B^b \rightarrow B^c$ decay channel. The baryon fraction f is directly influenced by the coalescence parameter r_{BM} and represents the hadronization dynamics within the AMPT model. In the following discussion, we will compare the AMPT model results for three choices of r_{BM}^b (0.7, 1.2, and 2.8) to illustrate how quark production and hadronization jointly shape the multiplicity dependence of nonprompt-to-prompt ratios.

In this factorized formulation, the initial heavy flavor hadron production ratio n_H^b/n_H^c is independent of r_{BM} . We can observe that the multiplicity dependence of the baryon hadronization fraction, f , is largely driven by the

coalescence parameter. The top row of Fig. 10 shows the multiplicity dependence of f in pp collisions at $\sqrt{s} = 13$ TeV, evaluated in full rapidity space with all hadron decay processes turned off. The black and red curves represent the charm (f^c) and beauty (f^b) hadrons, respectively. We observe that the baryon fraction increases with charged particle multiplicity, reflecting the enhanced importance of the coalescence mechanism at higher particle densities. The charm baryon fraction is unchanged in this comparison, as r_{BM}^c is fixed to 1.4 throughout this work. The bottom row of Fig. 10 presents the ratios of beauty-to-charm baryon fractions and beauty-to-charm meson fractions. When the coalescence parameter for beauty quarks r_{BM}^b is smaller than that for charm r_{BM}^c , the growth of the beauty baryon fraction with multiplicity is weaker than that of charm, while the beauty meson fraction increases more strongly. The opposite behavior occurs if $r_{BM}^b > r_{BM}^c$. When the two parameters are comparable, both the baryon and meson fraction ratios approach unity and exhibit little multiplicity dependence. Thus, the relative multiplicity dependence of baryon and meson sectors is directly sensitive to the relative magnitude of the coalescence parameters for beauty and charm, highlighting the role of flavor-dependent hadronization dynamics.

Next, we examine how the initial heavy flavor production, the hadronization process, and subsequent decays combine to determine the multiplicity dependence of the nonprompt-to-prompt ratio. Figure 11 shows the

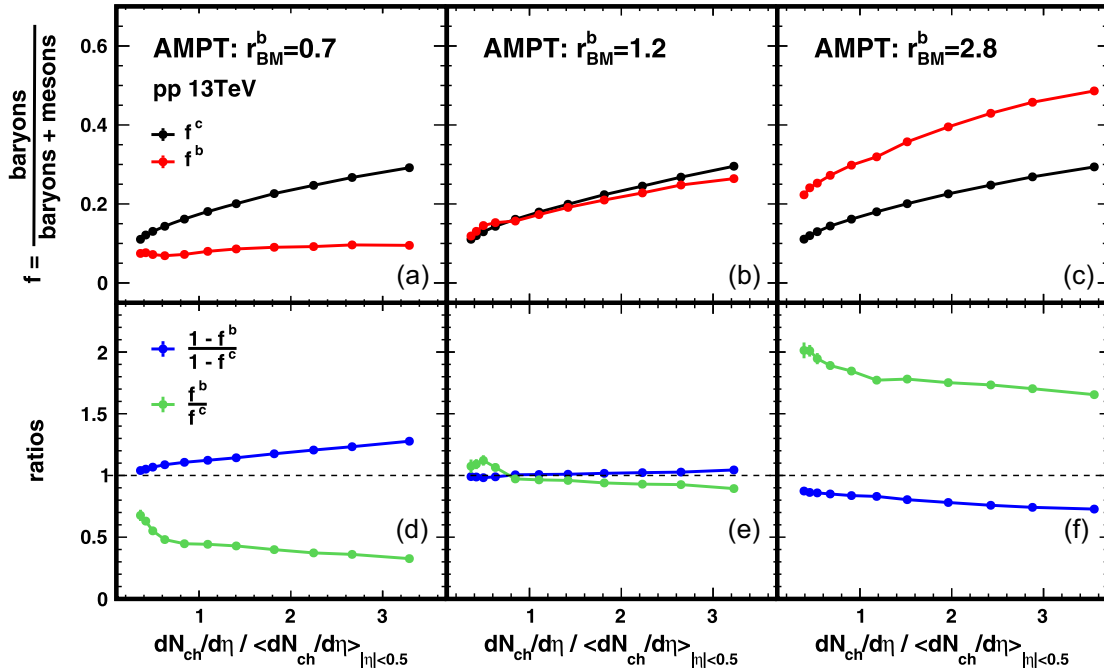


FIG. 10. Top row: multiplicity dependence of f in pp collisions at $\sqrt{s} = 13$ TeV in full rapidity space. The black and red represent hadron flavor charm and beauty (f^c and f^b), respectively, with all hadron decay processes turned off. Bottom row: multiplicity dependence of $(1 - f^b)/(1 - f^c)$ and f^b/f^c , represented by the blue and green markers. From left to right, the panels correspond to r_{BM}^b values in the AMPT model of 0.7, 1.2, and 2.8, respectively.

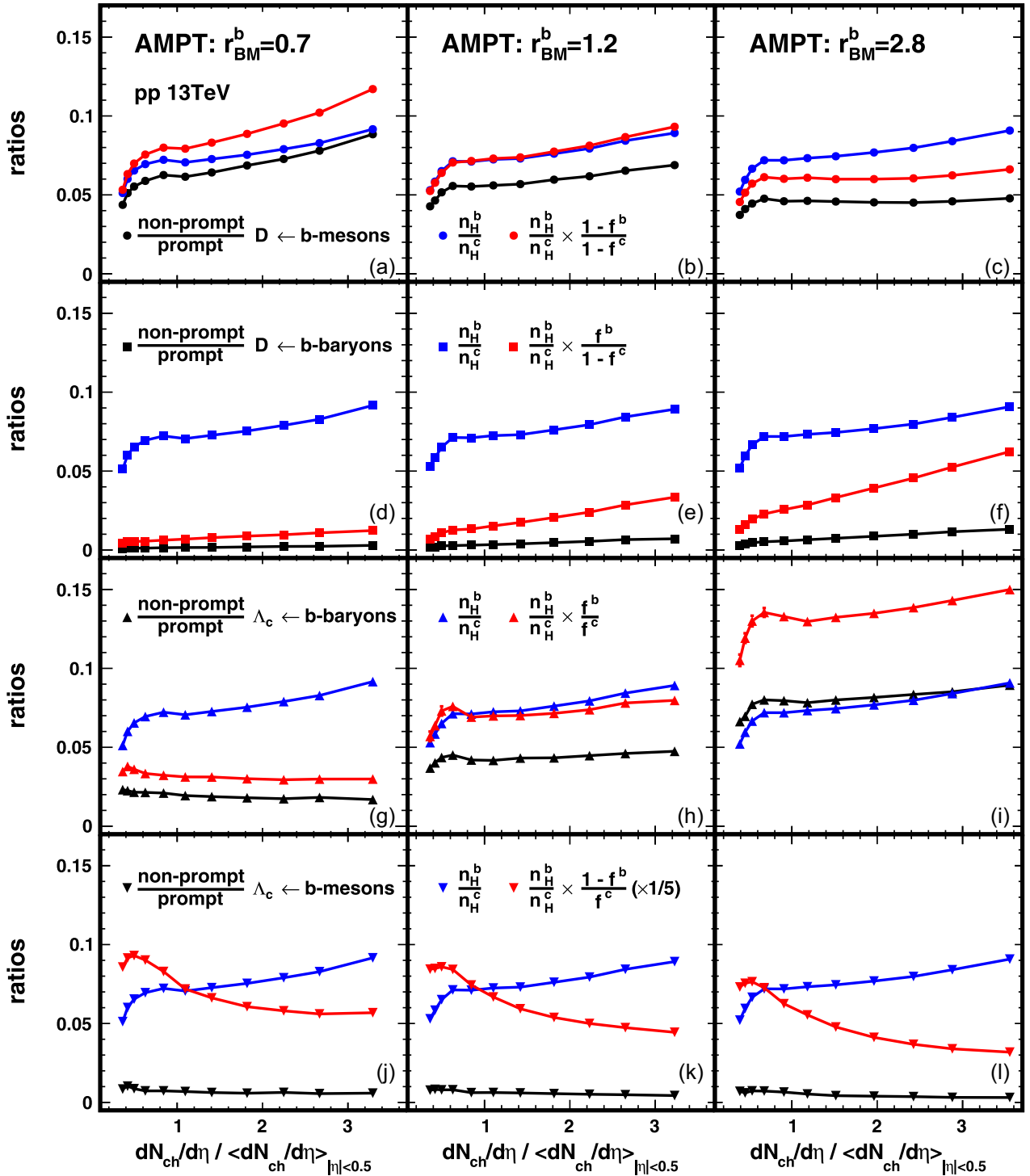


FIG. 11. The multiplicity dependence of $\frac{\text{nonprompt}}{\text{prompt}}$ (black), $\frac{n_H^b}{n_c^b}$ (blue), and $\frac{n_H^b}{n_c^b} \times$ hadronization fraction (red). From left to right, the panels show AMPT model results for $r_{BM}^b = 0.7, 1.2$, and 2.8 . From top to bottom, the panels represent the following studied objects: c -meson from b -meson decays, c -meson from b -baryon decays, b -baryon from b -baryon decays, and b -baryon from b -meson decays, distinguished by different markers, respectively.

multiplicity dependence of three quantities: the nonprompt/prompt ratio (black), the initial hadron production ratio $\frac{n_H^b}{n_c^b}$ (blue), and the product of $\frac{n_H^b}{n_c^b}$ with the hadronization fraction

(red) shown in Fig. 10. From left to right, the panels show the AMPT model results for $r_{BM}^b = 0.7, 1.2$, and 2.8 . From top to bottom, the rows display, respectively, charm mesons from beauty meson decay, charm mesons from beauty

baryon decay, charm baryons from beauty baryon decay, and charm baryons from beauty meson decay. Note that we scale the red curve in the bottom row by a factor of 1/5 for better visibility. The blue curves are nearly identical across all cases and show a mild upward trend, reflecting the fact that the PYTHIA8 initial conditions predict a slightly enhanced beauty quark production probability at higher multiplicities. This indicates differences in the way beauty and charm quarks are produced in PYTHIA8 initial conditions. Beauty production is dominated by harder partonic scatterings, which scale more strongly with event activity, while charm production receives substantial contributions from a softer gluon radiation process. As multiplicity increases, the relative enhancement from hard processes and gluon splitting benefits beauty quarks more than charm, leading to the slow rise of $\frac{n_H^b}{n_H^c}$. By contrast, the red curves ($\frac{n_H^b}{n_H^c} \times$ hadronization fraction) show strong sensitivity to the choice of r_{BM}^b . When r_{BM}^b is significantly smaller than r_{BM}^c , the beauty baryon fraction grows more slowly with multiplicity than the charm baryon fraction, causing the red curves for baryons to rise weakly or even saturate, while the meson channel shows a stronger increase. Conversely, when r_{BM}^b is much larger, the baryon channel steepens considerably with multiplicity, while the meson channel is comparatively suppressed. In the case that r_{BM}^b is similar to r_{BM}^c , both baryon and meson sectors evolve with similar slopes,

yielding nearly flat ratios between beauty and charm fractions. The overall downward shift of the black points (nonprompt-to-prompt ratios) relative to the red curves originates from the branching ratios, which act almost as multiplicative constants smaller than unity. This behavior is consistent with the factorized formulation in Eqs. (B1) and (B2), where the decay contribution enters multiplicatively through the branching ratios, leaving the multiplicity dependence governed primarily by the production and hadronization terms. Finally, it is worth noting that the contributions from beauty baryon to charm meson and beauty meson to charm baryon decays are comparatively small, as seen in the second and fourth rows of Fig. 11. Accordingly, the associated effects on the multiplicity dependence are weak, and the dominant behavior of the nonprompt/prompt ratios is governed by the primary decay channels. Therefore, the contrast across the three columns of Fig. 11 highlights the central role of the coalescence parameter in controlling the flavor-dependent baryon meson competition. Varying r_{BM}^b directly reshapes the multiplicity dependence of nonprompt-to-prompt ratios. This demonstrates that measurements of multiplicity-dependent heavy flavor observables are driven primarily by the baryon-to-baryon and meson-to-meson decay channels, providing particularly sensitive probes of flavor-dependent coalescence dynamics.

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