

Conformal moments of the two-loop coefficient functions in DVCS

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We develop a new technique and calculate conformal (Gegenbauer) moments of the two-loop coefficient functions in deeply virtual Compton scattering. These results are necessary for the extraction of the generalized parton distributions from the experimental data to the next-to-next-to leading order accuracy within the Mellin-Barnes approach.

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I. INTRODUCTION

The multidimensional composition of the nucleon in terms of quarks and gluons has drawn growing interest in the past decades and became an important frontier for the study of strong interactions. Generalized parton distributions (GPDs) [1] encode the three-dimensional distributions of partons in the momentum fraction and the impact parameter space. Their study has been one of the main physics motivations for the future upgrades at JLAB [2], the Electron-Ion Collider (EIC) [3] in the U.S. and the EIC project [4] in China.

The deep inelastic Compton scattering (DVCS) [5,6] plays an exceptional role in this endeavor. This is the simplest process that involves GPDs. The underlying formalism is well understood [7–9] and in the last years considerable effort was invested to advance the theory description to the next-to-next-to leading order (NNLO) accuracy. The two-loop DVCS coefficient functions (CFs) were calculated in Refs. [10–14] for all cases, and the three-loop evolution equation for the flavor-nonsinglet GPDs was derived in Refs. [15,16]. The flavor-singlet three-loop evolution equation is not known yet. However, the phenomenological impact of the renormalization group improvement at three loops is expected to be minor since the range of photon virtualities accessible in current and future DVCS experiments is rather limited.

Implementation of these new theory results in the data analysis aiming at the GPD extraction is by itself a non-trivial problem. It has been suggested and extensively discussed in the literature [17–22] that this analysis can

be facilitated by going over from the momentum fractions to the conformal moments space and evaluating the convolutions using the Mellin-Barnes techniques. The original motivation to use this representation was that in this way the one-loop evolution equations become diagonalized and can be solved easily. A fast NLO evolution code using Mellin-Barnes formalism also exists [23]. It was realized that going over to conformal moments is also beneficial in another respect: it allows one to utilize various physical constraints on the GPDs, and provides one with a natural platform to construct GPD parametrizations. In particular, the Kumerički-Müller model was proposed and used successfully in fitting the DVCS and deeply virtual meson production measurements [21,24,25]. The same framework is employed in the GUMP initiative [26–29], which aims at the global GPD extraction analysis from extensive experimental inputs, see also [30–32].

Conformal moments of the DVCS CFs that are needed in this approach are given by (analytically continued) integrals of the momentum fraction space CFs with Gegenbauer polynomials, e.g., for quarks

$$C_q(j) \sim \int_{-1}^1 dx (1-x^2) C_q(x) C_j^{3/2}(x). \quad (1)$$

Such integrals become nontrivial in higher orders, as the momentum fraction space CFs are given in terms of generalized polylogarithms of increasing transcendental-ity. In this paper we present a systematic approach to how the Gegenbauer moments can be evaluated with moderate computational effort, and present explicit expressions for the conformal moments of all two-loop DVCS CFs [10–14]—vector and axial vector, flavor-singlet, and flavor nonsinglet, and also for gluon transversity.

To explain the basic idea on an intuitive level, it is convenient to use Dirac bra-ket notation $C_j^{3/2} \leftrightarrow |j\rangle$. The conformal moments (1) can be thought of as scalar products $\langle C_q | j \rangle$. The main observation is that such “states”

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(Gegenbauer polynomials) are eigenfunctions of $SL(2)$ -invariant operators $\mathbb{H}_\alpha$, schematically

$$\mathbb{H}_\alpha|j\rangle = E_\alpha(j)|j\rangle, \quad (2)$$

where α is a certain index used to enumerate such operators. For a (more or less) arbitrary function $f(x)$ obviously

$$\langle \mathbb{H}_\alpha^\dagger f | j \rangle = \langle f | \mathbb{H}_\alpha | j \rangle = E_\alpha(j) \langle f | j \rangle, \quad (3)$$

so that if $\langle f | j \rangle$ [conformal moment of the function $f(x)$] is known and the eigenvalue $E_\alpha(j)$ can be calculated, one obtains from this relation the conformal moment of a (potentially more complicated) function $\mathbb{H}_\alpha^\dagger f$. Using such relations for a sufficiently large set of independent $SL(2)$ -invariant operators (at two loops we will need 16 operators) one can calculate conformal moments for a complete basis of functions with a given weight, from which the results for the DVCS CFs are obtained by solving a system of linear equations. An important technical issue that will be explained in what follows is that it proves to be sufficient to use position space expressions for the invariant operators, which have a simple form. The calculation was done using the HyperInt package [33] and we also used PolyLogTools [34] to simplify the final expressions.

II. DEFINITIONS AND NOTATIONS

The DVCS amplitudes $\gamma^*(q) + N(p) \rightarrow \gamma(q') + N(p')$ are defined by the nucleon matrix element of the time-ordered product of two electromagnetic currents and can be split in the vector V and axial-vector A parts that are symmetric and antisymmetric in the Lorenz indices, respectively,

$$\begin{aligned} T_{\mu\nu} &= i \int d^4x e^{-iqx} \langle p' | T \{ j^\mu(x) j^\nu(0) \} | p \rangle \\ &= -g_{\mu\nu}^{\perp} V + \epsilon_{\mu\nu}^{\perp} A + \text{power corrections}. \end{aligned} \quad (4)$$

The amplitudes V and A , in turn, are usually expressed in terms of the so-called Compton form factors (CFFs), see e.g., Ref. [35],

$$\begin{aligned} V &= \frac{1}{2P_+} \bar{u}(p') \left[\gamma^+ \mathcal{H}(\xi, Q, t) + \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} \mathcal{E}(\xi, Q, t) \right] u(p), \\ A &= \frac{1}{2P_+} \bar{u}(p') \left[\gamma^+ \tilde{\mathcal{H}}(\xi, Q, t) + \frac{\Delta_+}{2M} \tilde{\mathcal{E}}(\xi, Q, t) \right] \gamma_5 u(p), \end{aligned} \quad (5)$$

where $P = (p + p')/2$, $\Delta = q' - q$, $Q^2 = -q^2$, and M is the nucleon mass. The skewedness parameter ξ can be taken as

$$\xi = \frac{x_B}{2 - x_B} + \mathcal{O}(1/Q^2), \quad (6)$$

where $x_B = Q^2/(2pq)$ is the Bjorken variable.

To the leading twist accuracy, the CFFs $\mathcal{H}, \mathcal{E}, \tilde{\mathcal{H}}, \tilde{\mathcal{E}}$ can be written in the factorized form [7–9], e.g.,

$$\begin{aligned} \mathcal{H} &= \sum_q \int_{-1}^1 \frac{dx}{\xi} C_q(x/\xi, \mu^2/Q^2, \alpha_s(\mu)) H_q(x, \xi, t, \mu) \\ &+ \int_{-1}^1 \frac{dx}{\xi^2} C_g(x/\xi, \mu^2/Q^2, \alpha_s(\mu)) H_g(x, \xi, t, \mu), \end{aligned} \quad (7)$$

where H_q, H_g are the quark (gluon) GPDs and C_q, C_g are the corresponding CFs, and similarly for the other CFFs. The CFs for the vector and axial-vector cases are different—we will use the notation $\tilde{C}_q, \tilde{C}_g$ for the axial vector ones—but are the same for the pairs $\mathcal{H}, \mathcal{E}$ and $\tilde{\mathcal{H}}, \tilde{\mathcal{E}}$, respectively. The CFs $C_{q,g}(x/\xi), \tilde{C}_{q,g}(x/\xi)$ are real functions at $|x/\xi| < 1$ and can be continued analytically beyond this region using the $\xi \rightarrow \xi - i\epsilon$ prescription [5,6]. The CFs C_q and $\tilde{C}_g$ are odd functions of x , while C_g and $\tilde{C}_q$ are even functions.

It has been suggested and extensively discussed in the literature [17–22] that going over to the Mellin representation in convolution integrals in (7) can have advantages for the data analysis (see Introduction). These integrals can be brought to the form, schematically

$$\mathcal{F} = \int_C \frac{dj (i\xi)^{-j}}{2\xi \cos \frac{\pi j}{2}} \mathbb{C}_j^{(F)}(Q^2/\mu^2, a_s(\mu)) F_j(\xi, t, \mu) \quad (8)$$

where $\mathcal{F} \in \{\mathcal{H}, \mathcal{E}, \tilde{\mathcal{H}}, \tilde{\mathcal{E}}\}$, F_j are the conformal moments of the corresponding GPDs, and $\mathbb{C}_j^{(F)}$ are conformal moments of the CFs which are given, up to an overall normalization, by the analytically continued integrals of the momentum-space CFs with Gegenbauer polynomials

$$\begin{aligned} C_q(N) &= \frac{1}{4} \int_{-1}^1 dx (1-x^2) C_{N-1}^{(3/2)}(x) C_q(x), \\ C_g(N) &= \frac{3}{16} \int_{-1}^1 dx (1-x^2)^2 C_{N-2}^{(5/2)}(x) C_g(x) \end{aligned} \quad (9)$$

and similar for $\tilde{C}_j^q, \tilde{C}_j^g$. Here $N = j - 1$ is the Lorentz spin of the corresponding local operator. The integral in (8) is taken along a line parallel to the imaginary axis, such that all singularities of the integrand are located to the left of this line. A thorough discussion and technical details can be found in Refs. [20–22, 28, 36, 37].

The CFs have a regular perturbative expansion

$$\begin{aligned} C_q &= C_q^{(0)} + a_s C_q^{(1)} + a_s^2 C_q^{(2)} + \mathcal{O}(\alpha_s^3), \\ C_g &= a_s C_g^{(1)} + a_s^2 C_g^{(2)} + \mathcal{O}(\alpha_s^3), \end{aligned} \quad (10)$$

where $a_s = \alpha_s/4\pi$, and similar for $\tilde{C}_q, \tilde{C}_g$. The tree-level quark CFs take the form [5,6]

$$\begin{aligned} C_q^{(0)}(x) &= e_q^2 \left(\frac{1}{1-x} - \frac{1}{1+x} \right), \\ \tilde{C}_q^{(0)}(x) &= e_q^2 \left(\frac{1}{1-x} + \frac{1}{1+x} \right). \end{aligned} \quad (11)$$

The one-loop CFs $C_q^{(1)}, C_g^{(1)}, \tilde{C}_q^{(1)}, \tilde{C}_g^{(1)}$ in the $\overline{\text{MS}}$ scheme have been known for a long time [8,38–42] and the two-loop CFs have been calculated recently [10–14]. Explicit expressions for all CFs in momentum fraction space are collected in Refs. [13,14]. However, the results for the flavor-nonsinglet axial-vector CF quoted in [14] correspond to Larin's renormalization scheme. For this case, we use instead the expressions from [11,12] in the $\overline{\text{MS}}$ scheme. In this scheme, by definition, the anomalous dimensions of flavor-nonsinglet vector and axial-vector operators coincide to all orders.

As the final remark, the CFs in [10–14] are written as functions of the variable

$$z = (1-x)/2, \quad \bar{z} = 1-z = (1+x)/2, \quad (12)$$

which is more convenient for the calculations. In terms of this variable, the tree level CFs are given by

$$\begin{aligned} C_q^{(0)}(z) &= \frac{e_q^2}{2} \left(\frac{1}{z} - \frac{1}{\bar{z}} \right), \\ \tilde{C}_q^{(0)}(z) &= \frac{e_q^2}{2} \left(\frac{1}{z} + \frac{1}{\bar{z}} \right), \end{aligned} \quad (13)$$

and the Gegenbauer moments (9) take the form

$$\begin{aligned} C_q(N) &= 2 \int_0^1 dz z \bar{z} C_{N-1}^{(3/2)}(1-2z) C_q(z), \\ C_g(N) &= 6 \int_0^1 dz z^2 \bar{z}^2 C_{N-2}^{(5/2)}(1-2z) C_g(z). \end{aligned} \quad (14)$$

III. KERNELS AND MOMENTS

In our normalization, Eq. (14), moments of the tree-level vector and axial-vector quark CFs are equal to

$$C_q(N) = \frac{1 + (-1)^N}{2} e_q^2, \quad \tilde{C}_q(N) = \frac{1 - (-1)^N}{2} e_q^2. \quad (15)$$

In what follows we will tacitly assume that the expressions for vector and axial-vector CFs are written for even and odd N , respectively, and suppress $(1 \pm (-1)^N)/2$ factors.

Beyond treelevel, the CFs are given by linear combinations of the harmonic polylogarithms [43] with simple prefactors,

$$z^{-a} H_m(z), \quad (16)$$

where m is a multiindex, $m = \{m_1, \dots, m_p\}$ with the entries $m_k \in \{0, 1\}$. The power a takes integer values, $a = 0, 1$ for quarks and $a = 0, 1, 2$ for gluons. Gegenbauer moments of the one-loop CFs have been calculated in Refs. [20,44] by first computing the Mellin moments and then performing a resummation. We will describe a different approach and use it for the two-loop CFs. This possibility was also mentioned in [44].

A. General procedure

For definiteness, we consider quark distributions. The starting observation is that the polynomials

$$G_N^{(3/2)}(z) = 2z\bar{z} C_{N-1}^{(3/2)}(1-2z) \quad (17)$$

are eigenfunctions of $\text{SL}(2, R)$ invariant operators $\mathbb{H}$,¹

$$\int_0^1 dz' \mathbb{H}(z, z') G_N^{(3/2)}(z') = E_N G_N^{(3/2)}(z). \quad (18)$$

Thus if $M_N[f]$ is the Gegenbauer moment of a function $f(z)$,

$$M_N[f] = \int_0^1 dz f(z) G_N^{(3/2)}(z), \quad (19)$$

then the Gegenbauer moment of the function

$$f^{\mathbb{H}}(z) = \int_0^1 dz' f(z') \mathbb{H}(z', z) \quad (20)$$

is given by the product of $M_N[f]$ and E_N ,

$$M_N[f^{\mathbb{H}}] = M_N[f] E_N. \quad (21)$$

This equation can be used to construct a basis of functions $f^{\mathbb{H}}(z)$ for which the Gegenbauer moments are known and the DVCS CFs can then be written as linear combinations of these functions. The problem is that explicit expressions for the invariant operators $\mathbb{H}(z', z)$ in momentum fraction representation can be rather involved. Going over to the position space proves to be crucial to make this technique feasible in practice.

The action of the $\text{SL}(2, R)$ invariant operator on a function of two variables $\varphi(w_1, w_2)$ in position space can be written as (see, e.g., [15])

$$[\mathbb{H}^{(h)} \varphi](w_1, w_2) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta h(\tau) \varphi(w_{12}^\alpha, w_{21}^\beta), \quad (22)$$

¹By definition, $\text{SL}(2, R)$ -invariant operators commute with the generators of collinear conformal transformations, which depend on the conformal spin: $j = 1$ for quarks and $j = 3/2$ for gluons.

where

$$w_{12}^\alpha = w_1 \bar{\alpha} + w_2 \alpha, \quad \bar{\alpha} = 1 - \alpha, \quad (23)$$

and $\tau = \alpha\beta/(\bar{\alpha}\bar{\beta})$ is the so-called conformal ratio. Eigenfunctions of such operators,

$$[\mathbb{H}^{(h)}\Psi_N^p](w_1, w_2) = E_N^{(h)}\Psi_N^p(w_1, w_2), \quad (24)$$

can be chosen as

$$\Psi_N^p(w_1, w_2) = \int_0^1 dz e^{-ip(w_1 z + w_2 \bar{z})} G_N^{(3/2)}(z) \quad (25)$$

for an arbitrary $h(\tau)$. The corresponding eigenvalues are given by

$$E_N^{(h)} = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta h(\tau) (1 - \alpha - \beta)^{N-1}. \quad (26)$$

Since the eigenvalues do not depend on p , integrating Eq. (25) over p with a certain weight function one obtains the eigenfunctions of the form

$$\Psi_N^{(f)}(w_1, w_2) = \int_0^1 dz f(w_1 z + w_2 \bar{z}) G_N^{(3/2)}(z), \quad (27)$$

where the function $f(z)$ is more or less arbitrary. Applying the operator $\mathbb{H}^{(h)}$ to (27) and sending $w_1 \rightarrow 1$ and $w_2 \rightarrow 0$ at the end, one immediately finds that the Gegenbauer moments of the functions $f(z)$ and

$$f^h(z) = [\mathbb{H}^{(h)}f](z) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta h(\tau) f(z\bar{\alpha} + \bar{z}\beta) \quad (28)$$

are related to each other as follows

$$M_N[f^h] = E_N^{(h)} M_N[f], \quad (29)$$

cf. (21). Thus the mapping (28), $T_h: f \mapsto f^h$, allows one to construct functions with known Gegenbauer moments using position-space kernels $h(\tau)$. In this way construction of the (much more complicated) invariant operators in momentum fraction space is avoided. Note that any two mappings commute,

$$T_{h_1} T_{h_2} = T_{h_2} T_{h_1} = T_{h_1 \otimes h_2}, \quad (30)$$

so that this construction can be applied recursively. It turns out to be possible to generate all necessary functions (16) starting from $f(z) = 1/z$ and using sufficiently simple kernels $h(\tau)$.

B. Quark CFs

Here we explain in detail how this approach works for the case of quark CFs. Our aim is to find the Gegenbauer moments (19) of the functions

$$\begin{aligned} F_m^\pm(z) &= \frac{1}{z} H_m(z) \pm \frac{1}{\bar{z}} H_m(\bar{z}), \\ \bar{F}_m^\pm(z) &= \frac{1}{\bar{z}} H_m(z) \pm \frac{1}{z} H_m(\bar{z}), \end{aligned} \quad (31)$$

where $m = \{m_1, \dots, m_t\}$, with m_k taking two values, zero and one. Since

$$H_m(\bar{z}) = (-1)^t H_{1-m}(z) + \text{lower weight functions} \quad (32)$$

one can always assume that the rightmost index is zero, $m_t = 0$. For uniformity of notation we define $H_\emptyset$ for the empty set $m = \{\}$ as

$$H_\emptyset = 1. \quad (33)$$

Let also

$$M_m^\pm(N) \equiv M_N[F_m^\pm], \quad \bar{M}_m^\pm(N) \equiv M_N[\bar{F}_m^\pm]. \quad (34)$$

The moments $M_m^\pm(N)$, $\bar{M}_m^\pm(N)$ are nonzero for odd (even) N , respectively. For the empty set obviously

$$\begin{aligned} M_\emptyset^\pm(N) &= 1, \quad N = 2, 4, \dots \\ \bar{M}_\emptyset^\pm(N) &= \pm 1, \quad N = 1, 3, \dots \end{aligned} \quad (35)$$

cf. (15).

As the next step, let us calculate Gegenbauer moments for the functions of weight one, $m = \{0\}$. To this end, first apply the operator $\mathbb{H}^{(1)}$, i.e., with the kernel $h(\tau) = 1$ to the function $F_\emptyset^\pm(z) = 1/z \pm 1/\bar{z}$. One easily finds

$$\mathbb{H}^{(1)} F_\emptyset^\pm(z) = -\left(\frac{\ln z}{\bar{z}} \pm \frac{\ln \bar{z}}{z}\right) = -\bar{F}_{\{0\}}^\pm(z). \quad (36)$$

The eigenvalues of $\mathbb{H}^{(1)}$ (26) are $E_N^{(1)} = 1/N(N+1)$. Thus, using Eq. (29), one gets

$$\bar{M}_{\{0\}}^\pm(N) = -\frac{1}{N(N+1)}. \quad (37)$$

A special invariant operator with the eigenvalues given by $2S_1(N)$ takes the form [45]

$$\begin{aligned} [\hat{\mathbb{H}}f](w_1, w_2) &= \int_0^1 \frac{d\alpha}{\alpha} [2f(w_1, w_2) \\ &\quad - \bar{\alpha}(f(w_{12}^\alpha, w_2) + f(w_1, w_{21}^\alpha))]. \end{aligned} \quad (38)$$

It can be written in the form (22) with the kernel $h(\tau) = \delta_+(\tau)$ [15] (up to a constant), but the above explicit expression is more convenient for applications. One easily obtains

$$[\hat{\mathbb{H}}F_{\{\}}^{\pm}](z) = -\frac{\ln z \pm \ln \bar{z}}{z\bar{z}} = -F_{\{0\}}^{\pm}(z) - \bar{F}_{\{0\}}^{\pm}(z) \quad (39)$$

that results in the following relation for the moments, see Eq. (29)

$$M_{\{0\}}^{\pm}(N) + \bar{M}_{\{0\}}^{\pm}(N) = -2S_1(N). \quad (40)$$

Taking into account (37) one derives

$$M_{\{0\}}^{\pm}(N) = -2S_1(N) + \frac{1}{N(N+1)}. \quad (41)$$

Thus we have obtained moments of the weight-one functions (31) avoiding a direct calculation of the integral (19).

In order to proceed to the next level we apply the operators $\mathbb{H}^{(1)}$ and $\hat{\mathbb{H}}$ to the functions of the weight one, which Gegenbauer moments we know already. These give us three new functions of weight two with known moments, not four, because the operators $\mathbb{H}^{(1)}$ and $\hat{\mathbb{H}}$ commute. Thus we end up with three equations for the four existing weight-two functions, and need to add one more operator to our basis to compensate for this mismatch. A large list of invariant operators and the corresponding eigenvalues up to weight five can be found in [16]. We need to choose one of them with the eigenvalue given by harmonic sums of weight two. One finds by inspection that $\mathbb{H}^{(\bar{\tau})}$, i.e., the invariant operator with the kernel $h(\tau) = \bar{\tau}$, has the required complexity, with the eigenvalues $(-1)^N[2S_{-2}(N) + \zeta_2]$. Calculating the integral (28) with $f = F_{\{\}}^{\pm}$ and the kernel $h = \bar{\tau}$ we obtain the necessary fourth equation.

This procedure can be continued iteratively to the functions of higher weight. At weight four, eight independent harmonic polylogarithms exist if the last index is fixed to be zero, and we need two combinations as in Eq. (31). Thus we need to construct 16 independent basis functions, alias 16 invariant operators using which such functions can be generated. We collect these operators and their eigenvalues in the Appendix.

The feasibility of this technique relies heavily on the fact that the integrals (28) with functions f defined in Eq. (31) and the kernels $h(\tau)$ of the form

$$h(\tau) = \{\bar{\tau}H_m(\tau), (\bar{\tau}/\tau)H_m(\tau)\} \quad (42)$$

can be calculated easily with the HyperInt package [33]. The result of the integration is given in a general case by a linear combination of the functions defined in Eq. (28),

$$I_h[f] = \sum_m (C_m^h F_m^{\pm} + \bar{C}_m^h \bar{F}_m^{\pm}). \quad (43)$$

The Gegenbauer moment of this combination is given by the product of the Gegenbauer moment of the function f and the eigenvalue of the kernel $\mathbb{H}^{(h)}$. The eigenvalues can

be calculated using recurrence relations in N , up to the weight five they can be found in [16]. In this way one can generate a sufficient number of equations in order to determine the Gegenbauer moments $M_m^{\pm}(N), \bar{M}_m^{\pm}(N)$ of the functions in Eq. (31), up to weight four (and, in principle, also higher).

In addition to the functions defined in (31), the two-loop DVCS CFs contain harmonic polylogarithms H_m without the z or $\bar{z}$ factor in the denominator. Such contributions can be handled using a recurrence relation for the Gegenbauer polynomials that results in

$$M_{N-1}[(\bar{z}-z)f] = \frac{NM_N[f] + (N+1)M_{N-2}[f]}{2N+1}. \quad (44)$$

C. Gluon CFs

Extension of the technique described above to the gluon CFs is straightforward. To this end we need to calculate moments

$$M_N[f] = \int_0^1 dz f(z) G_N^{(5/2)}(z) \quad (45)$$

of the functions defined in Eq. (31) and, in addition,

$$\begin{aligned} G_m^{\pm}(z) &= \frac{1}{z^2} H_m(z) \pm \frac{1}{\bar{z}^2} H_m(\bar{z}), \\ \bar{G}_m^{\pm}(z) &= \frac{1}{\bar{z}^2} H_m(z) \pm \frac{1}{z^2} H_m(\bar{z}), \end{aligned} \quad (46)$$

with respect to the polynomials

$$G_N^{(5/2)}(z) = 6(z\bar{z})^2 C_{N-2}^{(5/2)}(1-2z). \quad (47)$$

In this case invariant kernels can be written in the form $(1-\alpha-\beta)h(\tau)$,² so that invariant operators and the corresponding eigenfunctions are written as

$$\begin{aligned} [\mathbb{H}\varphi](w_1, w_2) &= \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta (1-\alpha-\beta) h(\tau) \varphi(w_{12}^{\alpha}, w_{21}^{\beta}), \\ \Psi_N^p(w_1, w_2) &= \int_0^1 dz e^{-ip(w_1 z + w_2 \bar{z})} G_N^{(5/2)}(z), \end{aligned} \quad (48)$$

cf. (22). The $\hat{\mathbb{H}}$ operator for gluons has the same form as for quarks (38), with $\bar{\alpha}$ factors replaced by $\bar{\alpha}^2$, see [[45] Eq. (4.1)]. The eigenvalues are given by the same expression as for quarks, Eq. (26), but the relation (28) acquires an extra $(1-\alpha-\beta)$ factor as well,

²For the general case of a two-particle operator with conformal spins j_1 and j_2 , the invariant kernels have the form $\bar{\alpha}^{2j_1-2}\bar{\beta}^{2j_2-2}h(\tau)$, see, e.g., in [[45], Eq. (2.40)].

$$f^h(z) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta (1 - \alpha - \beta) h(\tau) f(w_{12}^\alpha z + w_{21}^\beta \bar{z}). \quad (49)$$

With these modifications, the relation between the Gegenbauer moments of $f^h(z)$ and $f(z)$ remains the same, Eq. (29).

In our normalization, the Gegenbauer moments of the lowest weight functions are

$$\begin{aligned} M_N[F_{\{\}}^\pm] &= 1, \\ M_N[G_{\{\}}^\pm] &= (N-1)(N+2). \end{aligned} \quad (50)$$

Note that “+” moments are nonzero for even N , and “−” moments for odd N . Proceeding exactly as in the quark case we have calculated the Gegenbauer moments (47) of the functions $F(\bar{F})_m^\pm, G(\bar{G})_m^\pm$ up to weight four.

The final results for the CFs are collected in the next section and also in *Mathematica* format as Supplemental Material. [46].

IV. RESULTS

In this section we present explicit expressions for the Gegenbauer moments (9) of the DVCS CFs (7), (10). The quark CFs can conveniently be separated into nonsinglet and singlet contributions with a different dependence on electric charges

$$\begin{aligned} C_q &= e_q^2 C_{q,\text{NS}} + \Sigma_Q C_{q,\text{PS}}, \\ \tilde{C}_q &= e_q^2 \tilde{C}_{q,\text{NS}} + \Sigma_Q \tilde{C}_{q,\text{PS}}, \end{aligned} \quad (51)$$

where $\Sigma_Q = \frac{1}{2} \sum_{q'} e_{q'}^2$. They can further be separated into the contributions of different orders of perturbation theory and color structures,

$$\begin{aligned} C_{q,\text{NS}} &= C_{q,\text{NS}}^{(0)} + a_s C_F C_{q,\text{NS}}^{(1)} \\ &\quad + a_s^2 C_F \left(C_F C_{q,\text{NS}}^{(2)\text{F}} + \frac{1}{N_c} C_{q,\text{NS}}^{(2)\text{NP}} + \beta_0 C_{q,\text{NS}}^{(2)\beta} \right) + \dots, \\ C_{q,\text{PS}} &= a_s^2 C_F C_{q,\text{PS}}^{(2)} + \dots, \end{aligned} \quad (52)$$

where $C_F = (N_c^2 - 1)/(2N_c)$, $a_s = \alpha_s/(4\pi)$ and $\beta_0 = 11/3N_c - 2/3n_f$.

The gluon CFs start at one loop and can be decomposed in different contributions as

$$C_g = \Sigma_Q \{ a_s C_g^{(1)} + a_s^2 (C_F C_g^{(2)\text{F}} + C_A C_g^{(2)\text{A}}) + \dots \}. \quad (53)$$

The decomposition of the axial-vector CFs (with a “tilde”) is the same.

The expressions presented below for Gegenbauer moments (9) of the vector CFs are for even N , and for the axial-vector CFs for odd N , respectively. We use the notation $L = \ln Q^2/\mu^2$ and

$$H = \frac{1}{N(N+1)}. \quad (54)$$

All harmonic sums are assumed to have the argument N , e.g., $S_1 \equiv S_1(N)$.

The tree-level expressions in the chosen normalization (9) are

$$C_{q,\text{NS}}^{(0)}(N) = 1, \quad \tilde{C}_{q,\text{NS}}^{(0)}(N) = 1. \quad (55)$$

At one loop one obtains [20,36]

$$\begin{aligned} C_{q,\text{NS}}^{(1)} &= 4S_1^2 - 4S_1 H + 2H^2 + 5H - 9 - L(4S_1 - 2H - 3), \\ \tilde{C}_{q,\text{NS}}^{(1)} &= 4S_1^2 - 4S_1 H + 2H^2 + 3H - 9 - L(4S_1 - 2H - 3), \\ C_g^{(1)} &= -(2H + 1) \left(S_1 - \frac{1}{2} H \right) - \frac{1}{2} + L \left(\frac{1}{2} + H \right), \\ \tilde{C}_g^{(1)} &= +(2H - 1) \left(S_1 - \frac{1}{2} H + \frac{1}{2} \right) + L \left(\frac{1}{2} - H \right). \end{aligned} \quad (56)$$

The coefficient of the logarithm for the quark distributions is of course related to the one-loop anomalous dimension

$$\gamma_{qq}^{(1)}(N) = 2C_F(4S_1 - 2H - 3). \quad (57)$$

The two-loop moments are considerably more complicated. We obtain, for quarks

$$\begin{aligned} C_{q,\text{NS}}^{(2)\beta} &= \frac{4}{3} S_3 + \frac{8}{3} S_1^3 + \frac{20}{3} S_1^2 + 4S_{-2} + \frac{38}{9} S_1 - \zeta_3 - \frac{8}{3} \zeta_2 - 4S_1^2 H + 2S_{-2} H + \frac{10}{3} S_1 H + 8S_1 H^2 + \zeta_2 H - 5H^3 \\ &\quad - 5H^2 + \frac{121}{18} H - \frac{457}{24} + L \left(-4S_1^2 + 4S_1 H - \frac{20}{3} S_1 - 4H^2 - \frac{5}{3} H + 2\zeta_2 + \frac{19}{2} \right) + \frac{1}{2} L^2 (4S_1 - 2H - 3), \\ \tilde{C}_{q,\text{NS}}^{(2)\beta} &= \frac{4}{3} S_3 + \frac{8}{3} S_1^3 + \frac{20}{3} S_1^2 + \frac{38}{9} S_1 - \zeta_3 - \frac{14}{3} \zeta_2 - 4S_1^2 H - 2S_{-2} H - \frac{2}{3} S_1 H + 8S_1 H^2 - \zeta_2 H - 5H^3 - H^2 \\ &\quad + \frac{115}{18} H - \frac{457}{24} + L \left(-4S_1^2 + 4S_1 H - \frac{20}{3} S_1 - 4H^2 + \frac{1}{3} H + 2\zeta_2 + \frac{19}{2} \right) + \frac{1}{2} L^2 (4S_1 - 2H - 3), \end{aligned} \quad (58)$$

$$\begin{aligned}
C_{q,NS}^{(2)F} = & 8S_1^4 - 8(2S_{1,3} - S_4) + 32S_3S_1 - 4S_{-2}^2 + 8(2S_{-2,1} - S_{-3}) - 8H(S_3 + S_{-2}S_1 + 2S_1^3 - 4S_1^2) \\
& + 4H^2(2S_{-2} + 7S_1^2) - \frac{76}{3}S_1^2 - 12S_{-2} - S_1 \left(32H^3 + 92H^2 + \frac{86}{3}H - \frac{128}{9} \right) - 4\zeta_2(S_{-2} + 4S_1^2 - 3S_1H + 3S_1) \\
& - 64\zeta_3S_1 - 2\zeta_2H(H + 3) + 32\zeta_3H + 10H^4 + 50H^3 + 49H^2 - \frac{815}{18}H - \frac{14}{5}\zeta_2^2 + 43\zeta_3 + \frac{701}{24} + \frac{16}{3}\zeta_2 \\
& + L(-16S_1^3 - 8S_3 + 12S_1^2(2H + 1) + 8HS_{-2} + 4S_1 \left(\frac{19}{3} - 10H - 6H^2 + 4\zeta_2 \right) + 8H^3 + 32H^2 + \frac{67}{3}H - \frac{47}{2} \\
& - 4\zeta_2H - 6\zeta_2 + 4\zeta_3) + \frac{1}{2}L^2(4S_1 - 2H - 3)^2, \\
\tilde{C}_{q,NS}^{(2)F} = & 8S_1^4 - 8(2S_{1,3} - S_4) + 32S_3S_1 - 4S_{-2}^2 - 8H(S_3 - S_{-2}S_1 + 2S_1^3 - 3S_1^2) + 4H^2(-2S_{-2} + 7S_1^2) \\
& - \frac{76}{3}S_1^2 - S_1 \left(32H^3 + 76H^2 + \frac{26}{3}H - \frac{128}{9} \right) - 4\zeta_2(S_{-2} + 4S_1^2 - 5S_1H + 3S_1) - 64\zeta_3S_1 \\
& - 2\zeta_2H(5H + 1) + 32\zeta_3H + 10H^4 + 42H^3 + 21H^2 - \frac{1001}{18}H + \frac{701}{24} - \frac{14}{5}\zeta_2^2 + 39\zeta_3 + \frac{34}{3}\zeta_2 \\
& + L \left(-16S_1^3 - 8S_3 + 12S_1^2(1 + 2H) + \frac{76}{3}S_1 + 16\zeta_2S_1 + 4\zeta_3 - 6\zeta_2 - \frac{47}{2} + H \left(-32S_1 - 8S_{-2} - 12\zeta_2 + \frac{49}{3} \right) \right. \\
& \left. + H^2(28 - 24S_1) + 8H^3 \right) + \frac{1}{2}L^2(4S_1 - 2H - 3)^2, \tag{59}
\end{aligned}$$

and

$$\begin{aligned}
C_{q,NS}^{(2)NP} = & \frac{6(4S_{-2} + 3)}{(N - 2)(N + 3)} - 4(2S_{1,3} - S_4) - 4S_{-4} - 8S_{-2}^2 + 16(2S_{1,-2} - S_{-3})S_1 + 24S_1S_3 + 4\zeta_2S_{-2} \\
& + 8\zeta_2S_1^2 - 52\zeta_3S_1 + 8S_{-2} + \frac{16}{3}S_1^2 + \frac{64}{9}S_1 + \frac{2}{5}\zeta_2^2 + 54\zeta_3 - \frac{10}{3}\zeta_2 - \frac{73}{12} + H^2(32S_{-2} + 10) \\
& + H \left(8(2S_{-2,1} - S_{-3}) - 40S_{-2}S_1 - 12S_3 + 36\zeta_3 - 8\zeta_2S_1 + 52S_{-2} - \frac{76}{3}S_1 + \frac{1}{9} \right) \\
& + L \left(-8(2S_{1,-2} - S_{-3}) - 8S_3 - \frac{16}{3}S_1 + 8HS_{-2} + \frac{20}{3}H + 1 \right), \\
\tilde{C}_{q,NS}^{(2)NP} = & \frac{24(S_{-2} + 1)}{(N - 1)(N + 2)} + 2 \left\{ 12S_1S_3 - 2S_{-4} - 2(2S_{1,3} - S_4) - 4S_{-2}^2 + 8(2S_{1,-2} - S_{-3})S_1 + 2(\zeta_2 + 2)S_{-2} + 4\zeta_2S_1^2 \right. \\
& + \frac{8}{3}S_1^2 - 26\zeta_3S_1 + \frac{32}{9}S_1 + \frac{1}{5}\zeta_2^2 + 27\zeta_3 - \frac{5}{3}\zeta_2 - \frac{73}{24} + 6H^4 + 12H^3 - 8S_1H^3 + 2H^2(2S_{-2} - 8S_1 + \zeta_2 - 5) \\
& + H \left(-2S_3 + 4(2S_{-2,1} - S_{-3}) - 12S_{-2}S_1 - 2S_{-2} - 4\zeta_2S_1 - \frac{2}{3}S_1 + 8\zeta_3 + 2\zeta_2 - \frac{113}{18} \right) \left. \right\} \\
& + L \left(-8(2S_{1,-2} - S_{-3}) - 8S_3 - \frac{16}{3}S_1 + 8HS_{-2} + 8H^3 + 16H^2 + \frac{20}{3}H + 1 \right). \tag{60}
\end{aligned}$$

For the pure-singlet quark CFs we obtain,

$$\begin{aligned}
C_{q,PS}^{(2)} &= \frac{32}{(N-1)(N+2)} (2S_1^2 - S_{-2} - 4S_1 - \zeta_2 + 3) + H(-32S_1S_{-2} - 16(2S_{-2,1} - S_{-3}) - 48S_1^2 - 24S_{-2} \\
&\quad + 144S_1 - 16\zeta_2S_1 - 8\zeta_3 + 4\zeta_2 - 84 - H(32S_1^2 - 16S_{-2} - 160S_1 - 8\zeta_2 + 140) + 4H^2(16S_1 - 33) - 40H^3) \\
&\quad + 16L \left(-\frac{4(S_1 - 1)}{(N-1)(N+2)} + H \left(2S_{-2} + (2H+3)S_1 + \zeta_2 - 2H^2 - 5H - \frac{9}{2} \right) \right) \\
&\quad + 4L^2 \left(\frac{4}{(N-1)(N+2)} - 2H^2 - 3H \right), \\
\tilde{C}_{q,PS}^{(2)} &= \frac{32(S_{-2} + 1)}{(N-1)(N+2)} - 4H(8S_1S_{-2} + 4(2S_{-2,1} - S_{-3}) + 4S_1^2 + 18S_{-2} + 8S_1 + 4\zeta_2S_1 + 2\zeta_3 + 5\zeta_2 + 14) \\
&\quad + 8H^2(4S_1^2 + 2S_{-2} - 4S_1 + \zeta_2 - 8) + 4H^3(3 - 16S_1) + 40H^4 \\
&\quad + 16LH(2S_{-2} + S_1(1 - 2H) + \zeta_2 + 1 + H + 2H^2) + 4L^2H(2H - 1). \tag{61}
\end{aligned}$$

Gegenbauer moments of the two-loop gluon CFs are given by the following expressions:

$$\begin{aligned}
C_g^{(2),F} &= -L^2(2H+1) \left(S_1 - \frac{1}{4}(3+2H) \right) + L \{ (2H+1)(4S_1^2 - \zeta_2 + 2H^2 + 4H - 2) - 8H(H+1)S_1 + 6H \} \\
&\quad - \frac{2(4S_{-2} + 3)}{(N-2)(N+3)} + (1-8H)(2S_{-2,1} - S_{-3}) + 16S_1S_{-2} - \frac{5}{3}(1-4H)S_3 - \frac{10}{3}S_1^3 + (10H^2 + 5H - 3)S_1^2 \\
&\quad - H(16 + 21H)S_{-2} + \zeta_2(2H+1)S_1 + \left(\frac{5}{2} - 3H - 24H^2 - 12H^3 \right) S_1 + (7-10H)\zeta_3 + \zeta_2 \left(\frac{1}{4} - H - H^2 \right) \\
&\quad + \frac{1}{4}(-2 - 52H + 49H^2 + 68H^3 + 20H^4), \\
\tilde{C}_g^{(2),F} &= L^2(2H-1) \left(S_1 - \frac{1}{4}(3+2H) \right) - L \{ (2H-1)(4S_1^2 - 4HS_1 + 2H^2 + H - \zeta_2 - 2) \} \\
&\quad + \frac{24(S_{-2} + 1)}{(N-1)(N+2)} + (1+8H)(2S_{-2,1} - S_{-3}) - \frac{5}{3}(1+4H)S_3 - 16HS_{-2}S_1 + 2(3H-1)S_{-2} \\
&\quad + (2H-1) \left(\frac{10}{3}S_1 - 5H + 3 \right) S_1^2 + \left(\frac{5}{2} - 9H + 12H^3 + (1-2H)\zeta_2 \right) S_1 + \zeta_2 \left(H^2 + H - \frac{3}{4} \right) \\
&\quad + \zeta_3(10H+7) + \frac{1}{4}(-2 - 100H + 33H^2 + 8H^3 - 20H^4), \tag{62}
\end{aligned}$$

and

$$\begin{aligned}
C_g^{(2),A} &= L^2 \left(\frac{6}{(N-1)(N+2)} - (2H+1)S_1 - 2H(H+2) \right) - 2L \left\{ (N-1)(N+2) \left(2S_{-2,1} - S_{-3} + \frac{1}{2}\zeta_3 \right) \right. \\
&\quad - (2H+1)(S_1^2 + \zeta_2) - (1+4H)S_{-2} + \frac{12(S_1-1)}{(N-1)(N+2)} - (2+7H+2H^2)S_1 + 4H^2(H+3) + \frac{25}{2}H - \frac{1}{2} \Big\} \\
&\quad + \frac{12(2S_1^2 - S_{-2} - \zeta_2 - 4S_1 + 3)}{(N-1)(N+2)} + 6(N-1)(N+2) \left\{ 2S_{-2,1,1} - S_{-3,1} - S_{-2,2} + \frac{2}{3}(2S_{1,-2,1} - S_{-3,1} - S_{1,-3}) \right. \\
&\quad + \frac{1}{2}S_{-4} + \frac{1}{3} \left(S_{-2,1} - \frac{1}{2}S_{-3} \right) + \frac{1}{3}\zeta_2S_{-2} + \frac{2}{15}\zeta_2^2 + \frac{1}{3}\zeta_3S_1 + \frac{1}{12}\zeta_3 \Big\} - (1+2H)S_1^3 - \frac{1}{3}(1+14H)S_3 \\
&\quad - 8HS_{-2}S_1 - 6 \left(S_{-2,1} - \frac{1}{2}S_{-3} \right) - (4H^2 + 14H + 5)S_1^2 + 2 \left(H^2 + H - \frac{5}{4} \right) \zeta_2 + 2(3H-2)\zeta_3 + 4\zeta_2HS_1 \\
&\quad + \left(8H^2 - 2H - \frac{1}{2} \right) S_{-2} + S_1 \left(12H^3 + 42H^2 + 46H - \frac{11}{2} \right) - 10H^4 - 38H^3 - \frac{95}{2}H^2 - 29H - 2,
\end{aligned}$$

$$\begin{aligned}
\tilde{C}_g^{(2)A} = & L^2(2H-1)(S_1-2H) - 2L\{(N-1)(N+2)(2S_{-2,1}-S_{-3} + \frac{1}{2}\zeta_3) + (2H-1)(S_1^2 + \zeta_2) \\
& + (4H-1)S_{-2} + H(3-10H)S_1 - \frac{1}{2} + \frac{5}{2}H + 2H^2 + 6H^3\} + 2(N-1)(N+2)\left\{-(2S_{-2,1}-S_{-3})S_1 \right. \\
& + \left(2S_{1,1,-2} - S_{2,-2} - S_{1,-3} + \frac{1}{2}S_{-4}\right) - S_{-2}S_1^2 + \frac{1}{2}\left(S_{-2,1} - \frac{1}{2}S_{-3}\right) + \zeta_2S_{-2} - \zeta_3S_1 + \frac{2}{5}\zeta_2^2 + \frac{\zeta_3}{4}\left\} \right. \\
& - \frac{12(S_{-2}+1)}{(N-1)(N+2)} + (8H-5)(2S_{-2,1}-S_{-3}) + 8HS_{-2}S_1 + \frac{1}{3}(14H-1)S_3 + \frac{2}{3}(2H-1)S_1^3 \\
& + \left(22H - \frac{9}{2}\right)S_{-2} + (2H-20H^2+1)S_1^2 + 2S_1\left(14H^3 + 7H^2 + 5H - \frac{7}{4} - 2H\zeta_2\right) - \zeta_3(5+2H) \\
& + \zeta_2\left(2H^2 + 8H - \frac{5}{2}\right) - 14H^4 - 6H^3 + \frac{41}{2}H^2 + 17H - 2.
\end{aligned} \tag{63}$$

For completeness, we have calculated also the moments for the two-loop CFs for gluon transversity as defined in Ref. [14],

$$\begin{aligned}
C_{g,T}(N) = & 2\Sigma\{a_s + a_s^2(-2LC_A S_1 + C_F(-2 + S_1) \\
& + C_A(1 + 3S_1 - 2S_2 + 4S_{1,1}))\}.
\end{aligned} \tag{64}$$

This expression holds for even $N \geq 2$.

We stress that the utility of the provided expressions is the possibility of analytic continuation to a complex N plane. For fixed integer values of N , analytic expressions for the Gegenbauer moments (9) can be obtained by direct integration using the `HyperInt` Maple package [33]. We have checked that the expressions obtained in this way agree identically with our results for all CFs for $N \leq 20$. A numerical comparison using the provided *Mathematica* expressions was done as well, in which case we found agreement to machine precision.

As the most interesting property, all Gegenbauer moments satisfy the reciprocity relation [47–49]: their asymptotic expansion at $N \rightarrow \infty$ is symmetric under the interchange $N \leftrightarrow -1 - N$. This feature is nontrivial and provides additional confidence in the results. It can also be useful in applications.

V. SUMMARY AND CONCLUSIONS

We have developed a technique that facilitates calculation of conformal moments of the CFs in DVCS and used it to calculate moments of the two-loop CFs. The moments are given by linear combinations of the reciprocity respecting harmonic sums [47–49] and can easily be continued to complex N . The CFs in this representation are needed for NNLO calculations of the amplitudes using Mellin-Barnes techniques, which have several advantages compared to the traditional approach using convolutions in momentum

fractions. The NNLO contributions in this form can be implemented relatively easily in the existing analysis code [23] and are expected to be substantial [13] for gluon GPD contributions in the EIC kinematic range. Since GPDs are normalized to the conventional PDFs in the forward limit, NNLO corrections are also necessary for consistency with modern PDF determinations where the NNLO accuracy has become the standard of the field.

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DATA AVAILABILITY

No data were created or analyzed in this study.

APPENDIX: INVARIANT KERNELS

The kernels $h(\tau)$ (22) used in our calculation are collected in Table I. As already mentioned in the text, a product of two invariant operators with the eigenvalues $E_N^{(h_1)}$ and $E_N^{(h_2)}$ is an invariant operator with the eigenvalues $E_N^{(h_1 \otimes h_2)} = E_N^{(h_1)} E_N^{(h_2)}$. Such products are naturally used to produce operators with the eigenvalues with higher weights, so that at each step only a few “new” operators have to be added.

Note that the same set of functions $h(\tau)$ can be used for gluon operators. The invariant kernels in this case take the form $(1 - \alpha - \beta)h(\tau)$ but the equation for the eigenvalues, Eq. (26), retains its form. The invariant operator $\hat{H}$ has a slightly different form for quarks and gluons, see [45], Eq. (4.1)], but in the both cases the eigenvalues are the same, $2S_1(N)$.

TABLE I. Invariant kernels and their eigenvalues. All harmonic sums have argument N .

$h(\tau)$	$E_N^{(h)}$
$\mathbb{H}$	$2S_1$
1	$1/N/(N+1)$
$1-\tau$	$(-1)^N(2S_{-2} + \zeta_2)$
$\frac{1}{2}(1-\tau)\ln(\frac{1-\tau}{\tau})$	$(-1)^N(2S_{-2,1} - S_{-3} + \frac{1}{2}\zeta_3)$
$\frac{1}{2}\frac{1-\tau}{\tau}\ln(1-\tau)$	$S_3 - \zeta_3$
$\frac{1}{2}(1-\tau)(\text{Li}_2(\tau) + \frac{1}{2}\ln^2(\tau))$	$(-1)^N(S_{-4} + \frac{7}{20}\zeta_2^2)$
$\frac{1}{4}\frac{1-\tau}{\tau}(\text{Li}_2(\tau) + \frac{1}{2}\ln^2(\tau))$	$S_{1,3} - \frac{1}{2}S_4 - \zeta_3S_1 + \frac{3}{10}\zeta_2^2$
$\frac{1}{2}(1-\tau)\ln^2(1-\tau)$	$(-1)^N 2(4S_{1,1,-2} - 2S_{2,-2} - 2S_{1,-3} + S_{-4} - \zeta_2S_2 - \zeta_3S_1 + 2\zeta_2S_{1,1} + \frac{1}{5}\zeta_2^2)$

- [1] D. Müller, D. Robaschik, B. Geyer, F.M. Dittes, and J. Hořejši, *Fortschr. Phys.* **42**, 101 (1994).
- [2] A. Accardi *et al.*, *Eur. Phys. J. A* **60**, 173 (2024).
- [3] A. Accardi *et al.*, *Eur. Phys. J. A* **52**, 268 (2016).
- [4] D.P. Anderle *et al.*, *Front. Phys.* **16**, 64701 (2021).
- [5] X.-D. Ji, *Phys. Rev. Lett.* **78**, 610 (1997).
- [6] A. V. Radyushkin, *Phys. Lett. B* **380**, 417 (1996).
- [7] A. V. Radyushkin, *Phys. Rev. D* **56**, 5524 (1997).
- [8] X.-D. Ji and J. Osborne, *Phys. Rev. D* **58**, 094018 (1998).
- [9] J.C. Collins and A. Freund, *Phys. Rev. D* **59**, 074009 (1999).
- [10] V.M. Braun, A.N. Manashov, S. Moch, and J. Schoenleber, *J. High Energy Phys.* 09 (2020) 117; 02 (2022) 115(E).
- [11] V.M. Braun, A.N. Manashov, S. Moch, and J. Schoenleber, *Phys. Rev. D* **104**, 094007 (2021).
- [12] J. Gao, T. Huber, Y. Ji, and Y.-M. Wang, *Phys. Rev. Lett.* **128**, 062003 (2022).
- [13] V.M. Braun, Y. Ji, and J. Schoenleber, *Phys. Rev. Lett.* **129**, 172001 (2022).
- [14] Y. Ji and J. Schoenleber, *J. High Energy Phys.* 01 (2024) 053.
- [15] V.M. Braun, A.N. Manashov, S. Moch, and M. Strohmaier, *J. High Energy Phys.* 06 (2017) 037.
- [16] Y. Ji, A. Manashov, and S.-O. Moch, *Phys. Rev. D* **108**, 054009 (2023).
- [17] A. Manashov, M. Kirch, and A. Schafer, *Phys. Rev. Lett.* **95**, 012002 (2005).
- [18] D. Mueller and A. Schafer, *Nucl. Phys.* **B739**, 1 (2006).
- [19] M. Kirch, A. Manashov, and A. Schafer, *Phys. Rev. D* **72**, 114006 (2005).
- [20] K. Kumericki, D. Mueller, and K. Passek-Kumericki, *Nucl. Phys.* **B794**, 244 (2008).
- [21] K. Kumericki and D. Mueller, *Nucl. Phys.* **B841**, 1 (2010).
- [22] H.-C. Zhang and X. Ji, *Nucl. Phys.* **B1010**, 116762 (2025).
- [23] K. Kumericki, Gepard: Tool for studying the 3D quark and gluon distributions in the nucleon, <https://gepard.phy.hr>.
- [24] D. Müller, T. Lautenschlager, K. Passek-Kumericki, and A. Schaefer, *Nucl. Phys.* **B884**, 438 (2014).
- [25] M. Čuić, G. Duplančić, K. Kumericki, and K. Passek-K., *J. High Energy Phys.* 12 (2023) 192; 02 (2024) 225(E).
- [26] Y. Guo, X. Ji, and K. Shiells, *J. High Energy Phys.* 09 (2022) 215.
- [27] Y. Guo *et al.*, GUMP GPD Global Analysis, <https://github.com/yuxunguo/GUMP-Global-GPDs> (2022).
- [28] Y. Guo, X. Ji, M. G. Santiago, J. Yang, and H.-C. Zhang, *Phys. Rev. D* **112**, 054036 (2025).
- [29] Y. Guo, F.P. Aslan, X. Ji, and M. G. Santiago, *Phys. Rev. Lett.* **135**, 261903 (2025).
- [30] K. A. Mamo and I. Zahed, *Phys. Rev. Lett.* **133**, 241901 (2024).
- [31] K. A. Mamo and I. Zahed, *Phys. Rev. D* **110**, 114016 (2024).
- [32] F. Hechenberger, K. A. Mamo, and I. Zahed, *Phys. Rev. D* **112**, 074018 (2025).
- [33] E. Panzer, *Comput. Phys. Commun.* **188**, 148 (2015).
- [34] C. Duhr and F. Dulat, *J. High Energy Phys.* 08 (2019) 135.
- [35] A. V. Belitsky and A. V. Radyushkin, *Phys. Rep.* **418**, 1 (2005).
- [36] D. Mueller, *Phys. Lett. B* **634**, 227 (2006).
- [37] K. Kumericki, D. Mueller, K. Passek-Kumericki, and A. Schafer, *Phys. Lett. B* **648**, 186 (2007).
- [38] X.-D. Ji and J. Osborne, *Phys. Rev. D* **57**, R1337 (1998).
- [39] L. Mankiewicz, G. Piller, E. Stein, M. Vanttinen, and T. Weigl, *Phys. Lett. B* **425**, 186 (1998); **461**, 423(E) (1999).
- [40] A. V. Belitsky and D. Mueller, *Phys. Lett. B* **417**, 129 (1998).
- [41] P. Hoodbhoy and X.-D. Ji, *Phys. Rev. D* **58**, 054006 (1998).
- [42] A. V. Belitsky and D. Mueller, *Phys. Lett. B* **486**, 369 (2000).
- [43] E. Remiddi and J. A. M. Vermaseren, *Int. J. Mod. Phys. A* **15**, 725 (2000).

- [44] B. Melic, D. Mueller, and K. Passek-Kumericki, *Phys. Rev. D* **68**, 014013 (2003).
- [45] V. M. Braun, A. N. Manashov, and J. Rohrwild, *Nucl. Phys. B* **826**, 235 (2010).
- [46] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/p5rk-497z> for the Mathematica expressions.
- [47] Y. L. Dokshitzer, G. Marchesini, and G. P. Salam, *Phys. Lett. B* **634**, 504 (2006).
- [48] Y. L. Dokshitzer and G. Marchesini, *Phys. Lett. B* **646**, 189 (2007).
- [49] M. Beccaria, V. Forini, T. Lukowski, and S. Zieme, *J. High Energy Phys.* 03 (2009) 129.