

Exclusive $J/\psi + \gamma$ production in ultraperipheral ion collisions

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Ultraperipheral collisions (UPCs) of ions provide new opportunities to study the quarkonium production mechanism in photon-photon scattering. In this paper, we investigate the exclusive process $\gamma + \gamma \rightarrow J/\psi + \gamma$ up to $\mathcal{O}(\alpha_s v^2)$ accuracy within the nonrelativistic quantum chromodynamics factorization framework. We evaluate the corresponding cross sections for Pb-Pb and p-p UPCs at the Large Hadron Collider. Numerical results show that the $\mathcal{O}(\alpha_s)$, $\mathcal{O}(v^2)$, and $\mathcal{O}(\alpha_s v^2)$ corrections are about -50% , -33% , and 15% of the leading-order (LO) contribution, respectively, showing reasonable convergence in both α_s and v^2 expansion. Collectively, these corrections suppress the LO cross section by a factor of about $1/3$, which is a crucial effect for reliable phenomenological analysis. Our results suggest that future experimental measurements of this process are feasible.

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I. INTRODUCTION

Heavy quarkonium plays an important role in high energy collider physics, as it provides an ideal laboratory for exploring the interplay between perturbative and non-perturbative quantum chromodynamics (QCD). Owing to the large mass of its constituent quarks, the quarkonium production process is assumed to be factorizable into two stages. First, a heavy quark-antiquark pair with an invariant mass near the bound-state mass is produced perturbatively, then the pair binds nonperturbatively into the quarkonium state. This process can be systematically described by the nonrelativistic QCD (NRQCD) factorization formalism [1], where the nonperturbative hadronization effects are encoded into the long-distance matrix elements (LDMEs). Theoretical predictions within the NRQCD factorization framework takes the form of a double expansion in the strong coupling constant α_s and the heavy quark relative velocity v .

While rooted in rigorous effective field methods, the NRQCD factorization formalism is still required to be verified in various collider experiments. Quarkonium production in photon-photon collisions provides an interesting testing ground. The cross section for inclusive J/ψ production via photon-photon scattering was measured by the Large Electron-Positron Collider II experiment [2], yet the underlying production mechanism remains

incompletely understood. Theoretical studies indicate that the color-singlet contributions alone are inadequate to explain the data, while the predictions to color-octet contributions based on different LDME sets diverge quite a lot, which even lead to opposite conclusions in some cases [3–5]. Investigating specific subprocesses separately may help to clarify the situation [6–12], while experimental data for such channels is currently limited by the scarcity of the events at existing e^+e^- colliders. Fortunately, ultraperipheral collisions (UPCs) at the Large Hadron Collider (LHC) provide new opportunities for the study of photon-photon physics. In UPCs, the ion impact parameter is larger than twice the ion radius, hence the ions are kept unbroken during the collision, and the interaction is purely electromagnetic. This allows photons from the colliding ions to interact at high center-of-mass energy. Furthermore, with the luminosity upgrade of the LHC, both inclusive and exclusive processes can be measured with high precision.

In this paper, we focus on the exclusive process $\gamma + \gamma \rightarrow J/\psi + \gamma$. Compared to inclusive J/ψ production, the production mechanism of $J/\psi + \gamma$ is relatively simpler, which may provide key insight into the quarkonium production process in photon-photon scattering. This process was previously investigated at the next-to-leading order (NLO) in α_s , as a subprocess of $\gamma + \gamma \rightarrow J/\psi + \gamma + X$, in Ref. [10]. While the phenomenological analysis assumed a future e^+e^- linear collider, which is not yet realized. Recently, the authors of Ref. [13] studied this process in Pb-Pb UPCs, at the leading-order (LO) accuracy. Their LO results confirm the feasibility of measuring this process at the High-Luminosity LHC (HL-LHC). Considering that the QCD and relativistic corrections to charmonium production process are normally significant, to make a more reliable prediction, in this work, we perform a comprehensive

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calculation by including the $\mathcal{O}(\alpha_s)$, $\mathcal{O}(v^2)$, and $\mathcal{O}(\alpha_s v^2)$ corrections to the $\gamma + \gamma \rightarrow J/\psi + \gamma$ process.

The rest of the paper is organized as follows. In Sec. II, we present the general formula for $\gamma + \gamma \rightarrow J/\psi + \gamma$ process up to $\mathcal{O}(v^2)$ within the NRQCD factorization framework. In Sec. III, we elucidate some technical details for the perturbative calculation. In Sec. IV, the numerical evaluation for the concerned process is performed. The last section is a summary and conclusions.

II. NRQCD FACTORIZATION FORMULA UP TO $\mathcal{O}(v^2)$

According to the NRQCD factorization formula [1,14], the amplitude for $\gamma + \gamma \rightarrow J/\psi + \gamma$ process up to $\mathcal{O}(v^2)$ can be expressed as

$$\mathcal{M}^{\gamma+\gamma \rightarrow J/\psi+\gamma} = \sqrt{2M} \left[c_0 \langle J/\psi | \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} \chi | 0 \rangle + \frac{c_2}{m^2} \langle J/\psi | \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \right)^2 \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} \chi | 0 \rangle \right], \quad (1)$$

where c_0 and c_2 are short-distance coefficients, and $\langle J/\psi | \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} \chi | 0 \rangle$ and $\langle J/\psi | \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \right)^2 \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} \chi | 0 \rangle$ are NRQCD vacuum-to- J/ψ matrix elements. The quantities m and M represent the masses of charm quark and J/ψ meson, respectively. The prefactor $\sqrt{2M}$ on the right side of Eq. (1) reconciles the nonrelativistic normalization convention of NRQCD matrix elements with the relativistic normalization of the amplitude on the left side. Note, both $\langle J/\psi | \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} \chi | 0 \rangle$ and $\langle J/\psi | \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \right)^2 \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} \chi | 0 \rangle$ are independent of J/ψ helicity, and there is no helicity summation in Eq. (1).

To determine the short-distance coefficients c_0 and c_2 in Eq. (1), we follow the standard matching procedure [1]. Since these coefficients are insensitive to long-distance dynamics, we replace the physical J/ψ state with a free $c\bar{c}$ pair carrying the quantum number $^3S_1^{[1]}$. This allows us to exploit the matching condition between perturbative QCD and perturbative NRQCD,

$$\begin{aligned} \mathcal{M}^{\gamma+\gamma \rightarrow c\bar{c}(^3S_1^{[1]})+\gamma} \Big|_{\text{pert QCD}} &= c_0 \langle c\bar{c}(^3S_1^{[1]}) | \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} \chi | 0 \rangle \Big|_{\text{pert NRQCD}} \\ &+ \frac{c_2}{m^2} \langle c\bar{c}(^3S_1^{[1]}) | \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \right)^2 \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} \chi | 0 \rangle \Big|_{\text{pert NRQCD}}. \end{aligned} \quad (2)$$

Here, the factor $\sqrt{2M}$ is absent because we use the same (relativistic) normalization for the $c\bar{c}$ state on both sides. The calculation details of $\mathcal{M}^{\gamma+\gamma \rightarrow c\bar{c}(^3S_1^{[1]})+\gamma} \Big|_{\text{pert QCD}}$ will be

presented in Sec. III. For now, we express it formally as an expansion series

$$\begin{aligned} \mathcal{M}^{\gamma+\gamma \rightarrow c\bar{c}(^3S_1^{[1]})+\gamma} \Big|_{\text{pert QCD}} &= \mathcal{M}_{00} + \frac{\alpha_s}{\pi} \mathcal{M}_{01} + \frac{\mathbf{q}^2}{m^2} \mathcal{M}_{10} \\ &+ \frac{\mathbf{q}^2}{m^2} \frac{\alpha_s}{\pi} \mathcal{M}_{11}, \end{aligned} \quad (3)$$

where $\mathbf{q}$ is the three-momentum of the c quark in the $c\bar{c}$ rest frame, and therefore the velocity variable $v = |\mathbf{q}|/m$. It should be noted that, in calculating $\mathcal{M}_{01}$ and $\mathcal{M}_{11}$, we expand the amplitude as series in $\mathbf{q}^2$, before performing the Feynman integration. This procedure avoids encountering the Coulomb singularity, which would manifest as $\mathcal{O}(1/v)$ terms if the $\mathbf{q}^2$ expansion were performed after the Feynman integration. In dimensional regularization with spacetime dimension $D = 4 - 2\epsilon$, the NRQCD vacuum-to- $c\bar{c}$ matrix elements up to $\mathcal{O}(\alpha_s v^2)$ take the form [1,15–17]

$$\begin{aligned} \langle c\bar{c}(^3S_1^{[1]}) | \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} \chi | 0 \rangle \Big|_{\text{pert NRQCD}} &= \sqrt{2N_c} 2E_q \left[1 + \frac{2C_F}{3} \frac{\mathbf{q}^2}{m^2} \frac{\alpha_s}{\pi} \left(\frac{\mu_R^2}{\mu_F^2} \right)^\epsilon \right. \\ &\times \left. \left(\frac{1}{\epsilon_{\text{IR}}} - \gamma_E + \ln(4\pi) \right) \right], \end{aligned} \quad (4)$$

$$\langle c\bar{c}(^3S_1^{[1]}) | \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \right)^2 \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} \chi | 0 \rangle \Big|_{\text{pert NRQCD}} = \sqrt{2N_c} 2E_q \mathbf{q}^2, \quad (5)$$

where $N_c = 3$, $C_F = 4/3$ are color factors, $E_q = \sqrt{m^2 + \mathbf{q}^2}$ is the energy of the c (or $\bar{c}$) quark in the $c\bar{c}$ rest frame, and μ_F and μ_R denote the factorization and renormalization scales, respectively. Note that the NRQCD operator in Eq. (4) is renormalized in the modified minimal-subtraction scheme. Substituting Eqs. (3)–(5) into Eq. (2), the short-distance coefficients c_0 and c_2 can be solved order by order in v^2 and α_s expansion.

With the amplitude in Eq. (1), we obtain the cross section for $\gamma + \gamma \rightarrow J/\psi + \gamma$ process

$$\begin{aligned} \hat{\sigma}_{\text{NLO}}^{\gamma+\gamma \rightarrow J/\psi+\gamma} &= \frac{1}{2\hat{s}} \int d\text{PS}_2 \sum |\mathcal{M}^{\gamma+\gamma \rightarrow J/\psi+\gamma}|^2 \\ &= \frac{1}{2\hat{s}} \int d\text{PS}_2 \frac{\langle \mathcal{O} \rangle_{J/\psi}}{6m} \left(\mathcal{S}_{00} + \frac{\alpha_s}{\pi} \mathcal{S}_{01} \right. \\ &\quad \left. + \frac{\langle \mathbf{q}^2 \rangle}{m^2} \mathcal{S}_{10} + \frac{\langle \mathbf{q}^2 \rangle}{m^2} \frac{\alpha_s}{\pi} \mathcal{S}_{11} \right), \end{aligned} \quad (6)$$

where $\hat{s}$ is the center-of-mass energy squared of the colliding photons, $d\text{PS}_2$ denotes the two-body phase-space measure, and $\sum$ indicates summation over final-state spins and

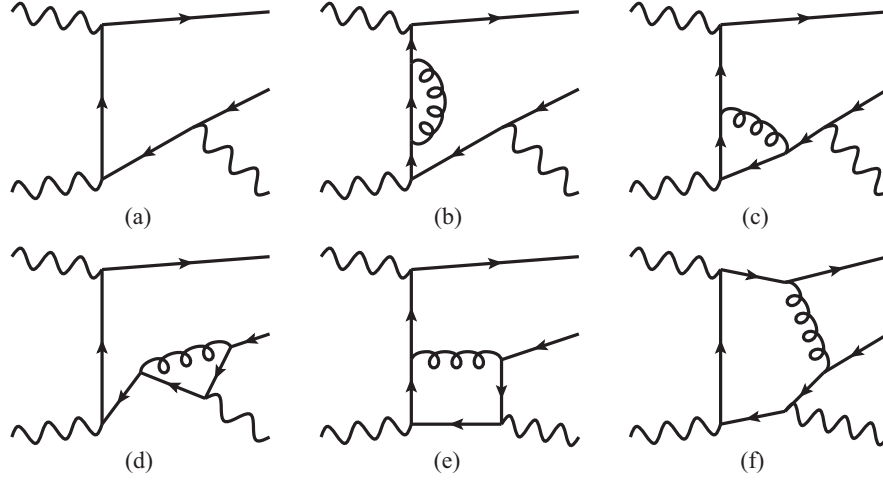


FIG. 1. Typical Feynman diagrams for $\gamma + \gamma \rightarrow J/\psi + \gamma$ process. (a) Tree-level diagram. (b)–(f) One-loop diagrams. Other diagrams can be obtained by exchanging the photon lines or inverting the quark line.

averaging over initial-state spins. The LDME $\langle \mathcal{O}_1 \rangle_{J/\psi} = |\langle J/\psi | \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} \chi | 0 \rangle|^2$ and the ratio $\langle q^2 \rangle = \frac{\langle J/\psi | \psi^\dagger (-i\vec{D})^2 \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} \chi | 0 \rangle}{\langle J/\psi | \psi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\epsilon} \chi | 0 \rangle}$ are taken as input parameters. After the matching procedure, we obtain

$$\mathcal{S}_{00} = \overline{\sum} |\mathcal{M}_{00}|^2, \quad (7)$$

$$\mathcal{S}_{01} = \overline{\sum} 2\text{Re}(\mathcal{M}_{01} \mathcal{M}_{00}^*), \quad (8)$$

$$\mathcal{S}_{10} = \overline{\sum} 2\text{Re}(\mathcal{M}_{10} \mathcal{M}_{00}^*) - \frac{\mathcal{S}_{00}}{2}, \quad (9)$$

$$\begin{aligned} \mathcal{S}_{11} = & \overline{\sum} 2\text{Re}(\mathcal{M}_{10} \mathcal{M}_{01}^* + \mathcal{M}_{11} \mathcal{M}_{00}^*) \\ & - \frac{4C_F}{3} \left(\frac{\mu_R^2}{\mu_F^2} \right)^\epsilon \left(\frac{1}{\epsilon_{\text{IR}}} - \gamma_E + \ln(4\pi) \right) \mathcal{S}_{00} - \frac{\mathcal{S}_{01}}{2}. \end{aligned} \quad (10)$$

Throughout this paper, we designate the contributions from $\mathcal{S}_{00}$, $\mathcal{S}_{01}$, $\mathcal{S}_{10}$, and $\mathcal{S}_{11}$ as the LO term, $\mathcal{O}(\alpha_s)$ correction, $\mathcal{O}(v^2)$ correction, and $\mathcal{O}(\alpha_s v^2)$ correction, respectively, and we refer to the complete results including all corrections as the NLO results. Note that, although the right side of Eq. (10) explicitly contains a $1/\epsilon_{\text{IR}}$ term, it cancels against the residue singularities from $\mathcal{M}_{11}$, resulting in a finite $\mathcal{S}_{11}$.

We now clarify the choices of the quarkonium mass in the calculation. For the mass M in the prefactor of Eq. (1), we use the relation $M = 2\sqrt{m^2 + \langle q^2 \rangle}$ [18] and expand it in powers of $\langle q^2 \rangle/m^2$. This treatment enables a partial cancellation between the $\sqrt{2M}$ in Eq. (1) and the $2E_q$ in Eq. (4), thereby reducing the theoretical uncertainty from the charm quark mass m [19]. For the phase-space computation in Eq. (6), we instead use the physical J/ψ mass. This choice minimizes the theoretical uncertainty further, as the J/ψ mass is known very precisely. Note, to

maintain consistency, when determining the input parameters, the physical J/ψ mass M , the charm quark mass m , and the matrix element ratio $\langle q^2 \rangle$ are not treated as independent; they are constrained by the relation $M = 2\sqrt{m^2 + \langle q^2 \rangle}$.

III. DETAILS IN THE PERTURBATIVE CALCULATION

In this section, we elucidate some key points in the calculation of $\mathcal{M}^{\gamma+\gamma \rightarrow c\bar{c}({}^3S_1^{(1)})+\gamma}|_{\text{pert QCD}}$. Considering the partonic process

$$\gamma(p_1) + \gamma(p_2) \rightarrow c\left(\frac{k}{2} + q\right) \bar{c}\left(\frac{k}{2} - q\right) + \gamma(p_3), \quad (11)$$

where the momenta of external particles are indicated in parentheses. The on-shell condition requires that $p_1^2 = p_2^2 = p_3^2 = 0$, $(\frac{k}{2} + q)^2 = (\frac{k}{2} - q)^2 = m^2$. The typical Feynman diagrams involved in the calculation are shown in Fig. 1.

To proceed with the calculation, we work in the $c\bar{c}$ rest frame. Choosing the direction of $\mathbf{p}_3$ as the z axis, the explicit expressions of the external momenta are given by

$$k = 2E_q(1, 0, 0, 0), \quad q = (0, q_x, q_y, q_z), \quad (12)$$

$$p_3 = \frac{2E_q \sin(\theta_1 + \theta_2)}{\sin \theta_1 + \sin \theta_2 - \sin(\theta_1 + \theta_2)}(1, 0, 0, 1), \quad (13)$$

$$p_1 = \frac{2E_q \sin \theta_2}{\sin \theta_1 + \sin \theta_2 - \sin(\theta_1 + \theta_2)}(1, \sin \theta_1, 0, \cos \theta_1), \quad (14)$$

$$p_2 = \frac{2E_q \sin \theta_1}{\sin \theta_1 + \sin \theta_2 - \sin(\theta_1 + \theta_2)}(1, -\sin \theta_2, 0, \cos \theta_2), \quad (15)$$

where θ_1 and θ_2 denote polar angles of $\mathbf{p}_1$ and $\mathbf{p}_2$, respectively. For latter use, we introduce the rescaled momenta $\tilde{p}_1 = p_1/(2E_q)$, $\tilde{p}_2 = p_2/(2E_q)$, $\tilde{p}_3 = p_3/(2E_q)$, $\tilde{k} = k/(2E_q)$, and $\tilde{q} = q/\sqrt{q^2}$ and the Mandelstam variables $\tilde{t} = (\tilde{p}_2 - \tilde{p}_3)^2$ and $\tilde{u} = (\tilde{p}_1 - \tilde{p}_3)^2$.

We employ the covariant projection method to project out the spin-triplet, color-singlet amplitude. The standard projector is [20]

$$\Pi_3 = \frac{1}{4\sqrt{2E_q(E_q+m)}} \left(\frac{k}{2} - \not{q} - m \right) \epsilon^*(k+2E_q) \left(\frac{k}{2} + \not{q} + m \right) \otimes \left(\frac{\mathbf{1}_c}{\sqrt{N_c}} \right), \quad (16)$$

where ϵ is the polarization vector of the spin-triplet state, and $\mathbf{1}_c$ denotes the unit color matrix. When expanding the amplitude in powers of q^2 , note that the rescaled momenta ($\tilde{p}_1$, $\tilde{p}_2$, $\tilde{p}_3$, $\tilde{k}$, and $\tilde{q}$) and polarization vectors [absent in Eqs. (12)–(15)] are q^2 independent. This renders the q^2 expansion straightforward. The amplitude is subsequently averaged over the spatial direction of $\mathbf{q}$, by utilizing the relations

$$\int \frac{d\Omega}{4\pi} \tilde{q}^\mu = 0, \quad \int \frac{d\Omega}{4\pi} \tilde{q}^\mu \tilde{q}^\nu = \frac{-g^{\mu\nu} + \tilde{k}^\mu \tilde{k}^\nu}{D-1}, \quad (17)$$

where D is the spacetime dimensions.

At the LO in α_s , there are six Feynman diagrams, one of which is shown in Fig. 1(a). The calculation at this order is straightforward, yielding relatively compact results,

$$\sum |\mathcal{M}_{00}|^2 = \frac{131072\pi^3\alpha^3}{243m^2(\tilde{t}-1)^2(\tilde{u}-1)^2(\tilde{t}+\tilde{u})^2} (\tilde{t}^4 + 2\tilde{t}^3\tilde{u} - 2\tilde{t}^3 + 3\tilde{t}^2\tilde{u}^2 - 3\tilde{t}^2\tilde{u} + \tilde{t}^2 + 2\tilde{t}\tilde{u}^3 - 3\tilde{t}\tilde{u}^2 + \tilde{t}\tilde{u} + \tilde{u}^4 - 2\tilde{u}^3 + \tilde{u}^2), \quad (18)$$

$$\begin{aligned} \sum 2\text{Re}(\mathcal{M}_{10}\mathcal{M}_{00}^*) = & -\frac{131072\pi^3\alpha^3}{729m^2(\tilde{t}-1)^3(\tilde{u}-1)^3(\tilde{t}+\tilde{u})^3} (4\tilde{t}^6\tilde{u} - 2\tilde{t}^6 + 12\tilde{t}^5\tilde{u}^2 - 18\tilde{t}^5\tilde{u} \\ & + 6\tilde{t}^5 + 20\tilde{t}^4\tilde{u}^3 - 37\tilde{t}^4\tilde{u}^2 + 30\tilde{t}^4\tilde{u} - 3\tilde{t}^4 + 20\tilde{t}^3\tilde{u}^4 - 44\tilde{t}^3\tilde{u}^3 + 44\tilde{t}^3\tilde{u}^2 \\ & - 16\tilde{t}^3\tilde{u} - 4\tilde{t}^3 + 12\tilde{t}^2\tilde{u}^5 - 37\tilde{t}^2\tilde{u}^4 + 44\tilde{t}^2\tilde{u}^3 - 16\tilde{t}^2\tilde{u}^2 - 2\tilde{t}^2\tilde{u} + 3\tilde{t}^2 \\ & + 4\tilde{t}\tilde{u}^6 - 18\tilde{t}\tilde{u}^5 + 30\tilde{t}\tilde{u}^4 - 16\tilde{t}\tilde{u}^3 - 2\tilde{t}\tilde{u}^2 + 2\tilde{t}\tilde{u} - 2\tilde{u}^6 + 6\tilde{u}^5 - 3\tilde{u}^4 - 4\tilde{u}^3 + 3\tilde{u}^2). \end{aligned} \quad (19)$$

Here, $\tilde{t}$ and $\tilde{u}$ are Mandelstam variables, defined previously. Note that we use the physical J/ψ mass in the phase-space calculation, while expanding the $c\bar{c}$ invariant mass $2E_q$ in powers of q^2 when calculating the squared amplitude. This approach, in fact, introduces a projection between the momenta from the phase-space sampling and those entering the squared amplitude calculation. Our projection preserves the spatial directions of all external momenta in the $c\bar{c}$ (or J/ψ) rest frame. Therefore, when performing the phase-space integral, the Mandelstam variables should be evaluated using $\tilde{t} = (\hat{p}_2 - \hat{p}_3)^2/M^2$ and $\tilde{u} = (\hat{p}_1 - \hat{p}_3)^2/M^2$, where $\hat{p}_i$ denotes the momentum from the phase-space sampling.

At the NLO in α_s , there are 48 one-loop diagrams, some of which are shown in Figs. 1(b)–1(f). We encounter both ultraviolet (UV) and infrared (IR) singularities. The conventional dimensional regularization with $D = 4 - 2\epsilon$ is employed to regularize them. The UV singularities, which are contained in self-energy and triangle diagrams, are removed by the renormalization procedure. The relevant renormalization constants include Z_2 and Z_m , corresponding to heavy quark field and heavy quark mass,

respectively. We define them in the on-shell (OS) scheme, with the counterterms given by

$$\delta Z_2^{\text{OS}} = -C_F \frac{\alpha_s}{4\pi} \left[\frac{1}{\epsilon_{\text{UV}}} + \frac{2}{\epsilon_{\text{IR}}} - 3\gamma_E + 3 \ln \frac{4\pi\mu_R^2}{m^2} + 4 \right], \quad (20)$$

$$\delta Z_m^{\text{OS}} = -C_F \frac{\alpha_s}{4\pi} \left[\frac{3}{\epsilon_{\text{UV}}} - 3\gamma_E + 3 \ln \frac{4\pi\mu_R^2}{m^2} + 4 \right]. \quad (21)$$

For the IR singularities, they completely cancel at $\mathcal{O}(\alpha_s v^0)$, while they partially cancel at $\mathcal{O}(\alpha_s v^2)$. The residual IR divergences match the singular term in the NRQCD calculation [Eq. (4)]. Consequently, our final result—the quantity $\mathcal{S}_{11}$ defined in Eq. (10)—is finite, as expected.

As a cross-check to our calculation, we successfully reproduce Eq. (1) of Ref. [21], the $J/\psi \rightarrow 3\gamma$ decay width up to $\mathcal{O}(\alpha_s v^2)$.

TABLE I. Summary of parameters for photon-photon collisions in ultraperipheral Pb-Pb and p-p collisions at the HL-LHC. For each colliding system, we list the nucleon-nucleon center-of-mass energy $\sqrt{s_{\text{NN}}}$, beam energy E_{beam} , photon “maximum” energy E_{γ}^{max} , and effective charge radius R .

System	$\sqrt{s_{\text{NN}}}$ (TeV)	E_{beam} (TeV)	E_{γ}^{max}	R (fm)
Pb-Pb	5.52	2.76	80 GeV	7.1
p-p	14	7.0	2.45 TeV	0.7

IV. NUMERICAL RESULTS

A. Input parameters

We investigate the UPC photon fusion process $A + B \rightarrow A + B + J/\psi + \gamma$ at the HL-LHC. Within the equivalent photon approximation framework [22,23], the final cross section is expressed as a convolution of the $\gamma + \gamma \rightarrow J/\psi + \gamma$ subprocess cross section with the equivalent photon spectral functions,

$$\sigma^{A+B \rightarrow A+B+J/\psi+\gamma} = \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} n_1(x_1) n_2(x_2) \hat{\sigma}^{\gamma+\gamma \rightarrow J/\psi+\gamma}, \quad (22)$$

where $x_i = E_{\gamma,i}/E_{\text{beam}}$ is the energy fraction of each photon, and $n_i(x)$ is the photon spectral function, for which we utilize the formula [24,25]

$$n(x) = \frac{2Z^2\alpha}{\pi} \left\{ \chi K_0(\chi) K_1(\chi) - (1 - \gamma_L^{-2}) \frac{\chi^2}{2} [K_1^2(\chi) - K_0^2(\chi)] \right\}. \quad (23)$$

Here, Z is the charge number of the ion, K_i is the modified Bessel function, and $\gamma_L = E_{\text{beam}}/m_p$ is the Lorentz boost

factor, with m_p denoting the proton mass. The variable x enters through $\chi = xm_p R$, where R is the effective charge radius of the nucleus. Our study focuses on Pb-Pb and p-p collision modes; the relevant parameters [25,26] are summarized in Table I.

Because of the finite coverage of detectors, proper kinematic cuts should be imposed on the final-state particles. Guided by the CMS measurement of $\gamma\gamma \rightarrow \gamma\gamma$ scattering [27] and prior theoretical work [13], we impose transverse momentum and rapidity cuts of $p_T > 2$ GeV and $|y_{\psi,\gamma}| < 2.4$ on both the final-state J/ψ and γ particles. Importantly, the transverse momentum cut serves to suppress the background from the Pomeron exchange process and meanwhile facilitate the exclusive event selection.

Other parameters used in our numerical evaluation are as follows:

$$\begin{aligned} \alpha &= 1/137.065, \quad M = 3.096 \text{ GeV}, \\ m_p &= 0.9315 \text{ GeV}, \\ \langle \mathcal{O}_1 \rangle_{J/\psi} &= 0.440_{-0.055}^{+0.067} \text{ GeV}^3, \quad \langle q^2 \rangle = 0.441_{-0.140}^{+0.140} \text{ GeV}^2. \end{aligned} \quad (24)$$

Here, the LDME and the ratio of matrix elements are taken from Ref. [28], while other parameters are from the Particle Data Group’s Review of Particle Physics [29]. In our calculation, the charm quark mass m is not an independent input, but determined through the relation $M = \sqrt{m^2 + \langle q^2 \rangle}$. This yields $m = 1.40_{-0.05}^{+0.05}$ GeV, where the center value agrees with the value commonly used in the literature.

The renormalization scale μ_R enters only through the strong coupling $\alpha_s(\mu_R^2)$. We set $\mu_R = r_1 \sqrt{4m^2 + p_T^2}$, taking $r_1 = 1$ as the central value. To estimate the renormalization scale uncertainty, we vary r_1 within the range

TABLE II. The total cross sections for the exclusive production of $J/\psi + \gamma$ via photon-photon fusion in Pb-Pb and p-p UPCs. The column labeled “Central” contains the cross sections for central values of parameters. Subsequent columns contain the theoretical uncertainties caused by the variation of corresponding parameters. The kinematic cuts $p_T > 2$ GeV and $|y_{\psi,\gamma}| < 2.4$ are imposed in all cases.

System	Term	Central	$\Delta\langle q^2 \rangle$	$\Delta\langle \mathcal{O}_1 \rangle$	$\Delta\mu_R$	$\Delta\mu_F$
Pb-Pb, $\sqrt{s_{\text{NN}}} = 5.52$ TeV	LO (nb)	87.90	-8.62 +10.38	-10.99 +13.38	...	...
	$\mathcal{O}(\alpha_s)$ (nb)	-43.93	+4.59 -5.49	+5.49 -6.69	-13.72 +8.29	...
	$\mathcal{O}(v^2)$ (nb)	-29.08	+12.38 -17.05	+3.64 -4.43	...	...
	$\mathcal{O}(\alpha_s v^2)$ (nb)	12.98	-5.57 +7.74	-1.62 +1.98	+4.04 -2.44	-1.79 +1.78
	NLO (nb)	27.88	+2.77 -4.42	-3.48 +4.24	-9.68 +5.85	-1.78 +1.78
p-p, $\sqrt{s_{\text{NN}}} = 14$ TeV	LO (fb)	10.970	-1.080 +1.292	-1.371 +1.670	...	...
	$\mathcal{O}(\alpha_s)$ (fb)	-5.462	+0.569 -0.683	+0.683 -0.832	-1.698 +1.025	...
	$\mathcal{O}(v^2)$ (fb)	-3.646	+1.552 -2.136	+0.456 -0.555	...	...
	$\mathcal{O}(\alpha_s v^2)$ (fb)	1.627	-0.698 +0.970	-0.203 +0.248	+0.503 -0.304	-0.222 +0.222
	NLO (fb)	3.489	+0.342 -0.557	-0.436 +0.531	-1.194 +0.720	-0.222 +0.222

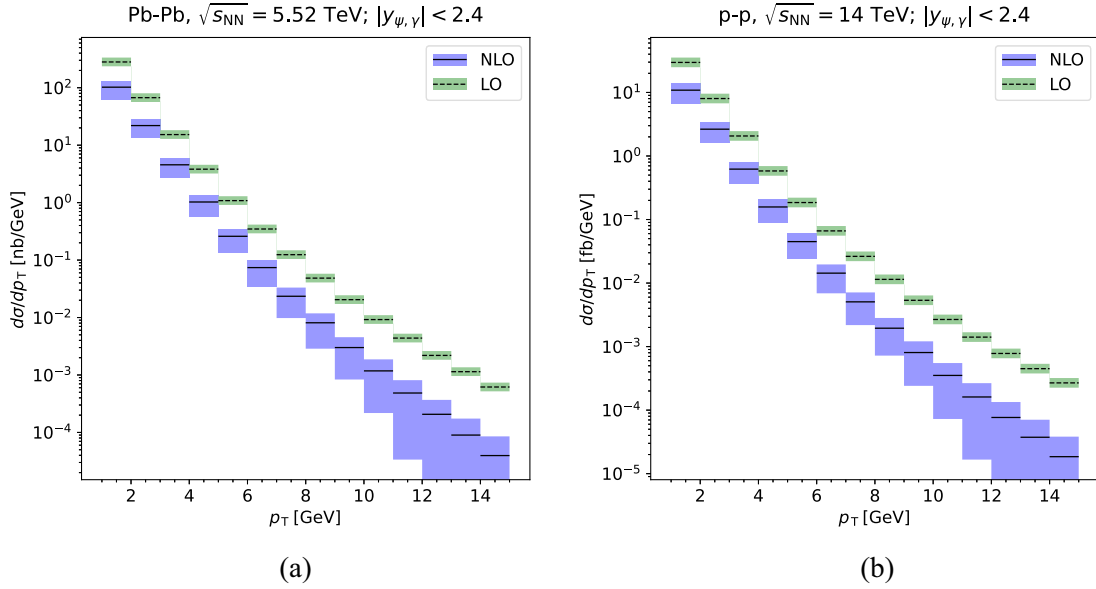


FIG. 2. Differential cross sections with respect to the J/ψ transverse momentum p_T for (a) Pb-Pb collision and (b) p-p collision. Theoretical uncertainties are shown as color bands.

$1/2 < r_1 < 2$. The factorization scale μ_F , however, enters only at $\mathcal{O}(\alpha_s v^2)$, via the $\ln(\mu_F/m)$ term appearing in $\mathcal{S}_{11}$. Therefore, we set $\mu_F = r_2 m$, also with $r_2 = 1$ as the central value, and vary r_2 between $1/2$ and 2 to estimate the factorization scale uncertainty.

For the running of the strong coupling constant, we adopt the two-loop formula [30]

$$\begin{aligned} & \frac{4\pi}{\alpha_s(\mu_R^2)} - \frac{\beta_1}{\beta_0} \ln \left(\frac{4\pi}{\alpha_s(\mu_R^2)} + \frac{\beta_1}{\beta_0} \right) \\ &= \frac{4\pi}{\alpha_s(\mu_0^2)} - \frac{\beta_1}{\beta_0} \ln \left(\frac{4\pi}{\alpha_s(\mu_0^2)} + \frac{\beta_1}{\beta_0} \right) + \beta_0 \ln \frac{\mu_R^2}{\mu_0^2}, \end{aligned} \quad (25)$$

where $\beta_0 = (11/3)C_A - (4/3)T_F n_f$ and $\beta_1 = (34/3)C_A^2 - 4C_F T_F n_f - (20/3)C_A T_F n_f$, with $C_A = 3$, $C_F = 4/3$, and $T_F = 1/2$. We take $n_f = 4$ active flavors and set the initial scale $\mu_0 = m_\tau = 1.777$ GeV, with $\alpha_s(\mu_0^2) = 0.314$ [29].

B. Cross sections

In Table II, we present the cross sections for the exclusive production of $J/\psi + \gamma$ via photon-photon fusion in Pb-Pb and p-p UPCs. The column labeled Central contains the cross sections for central values of parameters. One readily observes that the contributions of $\mathcal{O}(\alpha_s)$, $\mathcal{O}(v^2)$, and $\mathcal{O}(\alpha_s v^2)$ terms are about -50% , -33% , and 15% of the LO term, which suggests reasonable convergence behavior in the NRQCD expansion for both α_s and v^2 . This pattern is in contrast to the situation of $J/\psi \rightarrow 3\gamma$, where the $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s v^2)$ corrections are strikingly significant. In all, these correction terms suppress the LO cross sections by a factor of about $1/3$, which is essential for making reliable phenomenological predictions.

In addition to the central values, the theoretical uncertainties that are associated with the variations of the ratio of the matrix elements $\langle q^2 \rangle$, the LDME $\langle \mathcal{O}_1 \rangle$, the renormalization scale μ_R , and the factorization scale μ_F are separately listed in Table II. Since the J/ψ mass is known very precisely, we omit its variation from our uncertainty analysis. Adding all uncertainties in quadrature, we obtain

$$\begin{aligned} \sigma_{\text{LO}} &= 87.90_{-13.97}^{+16.94} \text{ nb}, & \sigma_{\text{NLO}} &= 27.88_{-11.34}^{+7.94} \text{ nb}, \\ &\text{for Pb-Pb collision;} \end{aligned} \quad (26)$$

$$\begin{aligned} \sigma_{\text{LO}} &= 10.970_{-1.746}^{+2.112} \text{ fb}, & \sigma_{\text{NLO}} &= 3.489_{-1.406}^{+0.984} \text{ fb}, \\ &\text{for p-p collision.} \end{aligned} \quad (27)$$

The results show that the theoretical uncertainties increase after including the QCD and relativistic corrections, which can be explained by the fact that the renormalization and factorization scales are absent in the LO calculation.

The HL-LHC will substantially increase the amount of ion collisions delivered to the LHC experiments. In a typical operation year, the integrated luminosities are about 5 nb^{-1} for Pb-Pb collision [26]¹ and 150 fb^{-1} for p-p collision [25]. Assuming the J/ψ meson is reconstructed through the decay $J/\psi \rightarrow \ell^+ \ell^-$ ($\ell = e, \mu$) with a branching fraction of about 12% [29] and a photon reconstruction and identification efficiency of about 40% [27], the number of reconstructed $J/\psi + \gamma$ candidates per operation year is about 4–8 for Pb-Pb collision and about 15–32 for p-p collision. Although the number of candidates is moderate, the clear event topology makes the experimental

¹This assumes 1 month/yr is allocated to Pb-Pb collision.

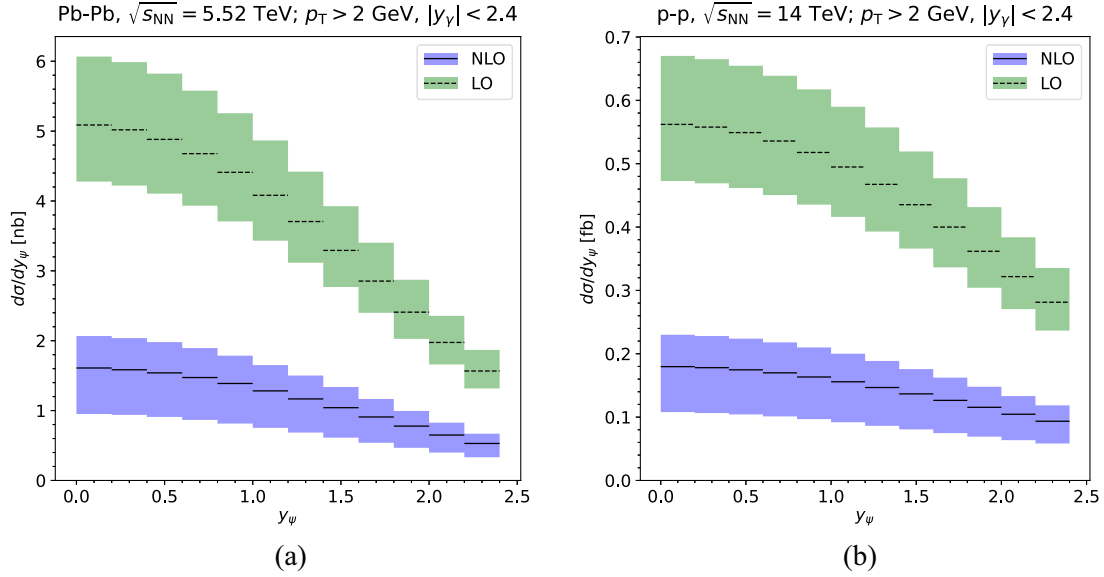


FIG. 3. Differential cross sections with respect to the J/ψ rapidity y_ψ for (a) Pb-Pb collision and (b) p-p collision. Theoretical uncertainties are shown as color bands.

investigations feasible. Moreover, as we will show latter, the number of events could increase substantially by lowering the minimum transverse momentum cut.

Differential cross sections with respect to the J/ψ transverse momentum p_T for Pb-Pb and p-p collisions are shown in Figs. 2(a) and 2(b), respectively. Both the LO and NLO distributions decrease steeply and monotonically as p_T increases from 1 to 15 GeV. Recall that our integrated cross section calculations employ a transverse momentum cut of $p_T > 2$ GeV. As Fig. 2 shows, lowering this cut to $p_T > 1$ GeV would quadruple the integrated cross sections. Another thing to note is that, when setting the renormalization scale to $\mu_R = \sqrt{4m^2 + p_T^2}/2$, the NLO differential cross sections become negative in the last three bins. However, since the contributions from this region are not dominant, and the negative cross section issue does not appear in the central value calculation (where $\mu_R = \sqrt{4m^2 + p_T^2}$), no additional treatment is applied to address this problems.

Differential cross sections with respect to the J/ψ rapidity y_ψ for Pb-Pb and p-p collisions are shown in Figs. 3(a) and 3(b), respectively. Since the distributions are symmetric over $y_\psi = 0$, only the $y_\psi > 0$ region is plotted. It can be seen that, as y_ψ increases from 0 to 2.4, both LO and NLO differential cross sections drop steadily.

V. SUMMARY

In this work, we investigate the exclusive production of $J/\psi + \gamma$ in ultraperipheral Pb-Pb and p-p collisions at the HL-LHC. Working within the NRQCD factorization framework, we calculate the $\mathcal{O}(\alpha_s)$, $\mathcal{O}(v^2)$, and $\mathcal{O}(\alpha_s v^2)$ corrections to the exclusive process $\gamma + \gamma \rightarrow J/\psi + \gamma$. We

evaluate both total cross sections and differential distributions with respect to the J/ψ transverse momentum and rapidity. Numerical results show that the $\mathcal{O}(\alpha_s)$, $\mathcal{O}(v^2)$, and $\mathcal{O}(\alpha_s v^2)$ corrections represent approximately -50% , -33% , and 15% of the LO contribution, exhibiting reasonable convergence behavior both in α_s and v^2 expansion. Collectively, these corrections suppress the LO cross sections by a factor of about $1/3$, which is essential for reliable phenomenological predictions. With appropriate kinematic cuts, our final results including all corrections are $\sigma_{\text{NLO}} = 25.18^{+7.18}_{-10.34}$ nb for Pb-Pb collision and $\sigma_{\text{NLO}} = 3.053^{+0.865}_{-1.238}$ fb for p-p collision. These values suggest that a future experimental analysis is feasible. Finally, we remark that further exploration of the $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(v^4)$ corrections to this process may turn out to be enlightening.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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