

Preparing a thermofield double state with feedback quantum algorithms

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The efficient preparation of correlated thermal states, such as the thermofield double state, is a fundamental prerequisite for simulating quantum gravity models and many-body thermodynamics on quantum processors. In this work, we investigate the ground-state preparation of the two coupled Sachdev-Ye-Kitaev model, known as the Maldacena-Qi model, which is dual to a traversable wormhole in anti-de Sitter space in two dimensions, utilizing feedback-based quantum algorithms. We demonstrate that the standard feedback-based quantum algorithm (FALQON) and its time-rescaled variant (TR-FALQON) face severe kinetic limitations in this system, failing to converge to the highly entangled ground state when initialized in trivial product states. To overcome these barriers, we propose the hybrid ITE-TR-FALQON protocol, which integrates the imaginary-time evolution present in imaginary-time-enhanced FALQON with the time-rescaling mechanism. Our ideal state-vector simulations show that imaginary-time filtering overcomes the stagnation observed for the unitary protocols with the chosen driver and initial state, while time rescaling reduces the number of algorithmic layers. The hybrid protocol achieves fidelities close to unity with the exact-diagonalization ground state and reproduces its von Neumann and Rényi entropies with high precision. These results constitute a numerical proof of principle: implementing the nonunitary updates, repeated feedback measurements, and the resulting compiled circuits remains a substantial obstacle, and the protocol in its present form is not a practical state-preparation method for current noisy intermediate-scale quantum devices.

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I. INTRODUCTION

The Sachdev-Ye-Kitaev (SYK) model has garnered significant interest within the theoretical physics community due to its rich connections to quantum chaos, quantum gravity, and nonquasiparticle states of matter. In its original incarnation, it consists of a system of N Majorana fermions with all-to-all random q -body interactions sampled from a Gaussian distribution, exhibiting maximally chaotic dynamics [1,2]. In the low-energy regime, it admits a holographic dual description as Jackiw-Teitelboim gravity

in anti-de Sitter space in two dimensions (AdS_2) [3,4]. In the large- N limit, the SYK spectrum becomes effectively continuous, and the model saturates the Maldacena-Shenker-Stanford bound on Lyapunov exponents [5,6], serving as a theoretical laboratory for investigating quantum thermalization, entanglement entropy, and nonperturbative phenomena in nearly conformal field theories [7,8]. Furthermore, as an exactly solvable example of a strange metal, the SYK model has deepened our understanding of non-Fermi liquids and novel strongly correlated electronic phases [9,10].

Over the years, various extensions and generalizations of the model have been proposed. Among them, a prominent case is the two coupled SYK model, introduced by Maldacena and Qi [11], in which two identical copies of the SYK model (that are often referred to as “left” [L] and “right” [R], respectively) are connected by a coupling term,

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bilinear in the Majorana fermions and having strength parametrized by an extra parameter μ . For a certain range of μ values, this system exhibits a ground state that approximates the thermofield double (TFD) state between the left and right copies with high precision, with an inverse temperature β functionally determined by the coupling strength. The bilinear interaction is given by (we denote the N Majorana fermions belonging to the left system as χ_j^L , and the Majorana fermions belonging to the right system as χ_j^R)

$$i\mu \sum_{j=1}^N \chi_j^L \chi_j^R. \quad (1)$$

When the value of μ is properly tuned, it induces a bilateral correlation that, in the holographic interpretation, corresponds to two AdS_2 geometries connected by a traversable wormhole [11,12]. This arrangement has been explored as a prototype for studying the traversability of quantum wormholes and the emergence of gravity from strongly correlated quantum systems.

Beyond the profound interest in the context of quantum gravity, the construction and manipulation of TFD states is of paramount importance for the development of quantum information and emerging quantum technologies. The TFD state is a valuable resource for quantum teleportation protocols, serves as a basis for quantum error correction codes, and offers a robust benchmark for testing scrambling dynamics and information propagation in quantum processors [13,14]. The ability to efficiently prepare such states is, therefore, a crucial step toward validating scalable quantum computing architectures.

However, the experimental preparation of states such as the TFD on quantum devices remains challenging. Conventional hybrid variational methods, such as the variational quantum eigensolver, rely on the classical optimization of parameters in a parametrized circuit. This approach becomes computationally expensive as the number of qubits increases, and faces severe obstacles, such as the Barren Plateau problem, where the gradient of the cost function vanishes exponentially with system size, rendering optimization unfeasible [15]. Moreover, achieving a faithful TFD state requires deep circuits, which are highly susceptible to noise in noisy intermediate-scale quantum (NISQ) hardware.

In this scenario, feedback-based quantum algorithms, and more specifically the feedback-based algorithm for quantum optimization (FALQON) protocol [16,17], emerge as a promising alternative. From the viewpoint of quantum gravity, this type of protocol is particularly appealing, since the efficient preparation of TFD states is directly connected to the study of traversable wormholes and to the holographic implementation of correlations between two coupled systems. FALQON is a closed-loop control approach that avoids classical variational optimization, updating circuit

parameters deterministically from observables measured during the evolution. This avoids an external variational parameter search, although it does not by itself ensure shallow circuits or low measurement costs on noisy hardware. In these algorithms, circuit parameters are defined deterministically through rules inspired by quantum Lyapunov control (QLC), eliminating the need for an external classical optimizer; this does not establish immunity to small feedback signals or measurement noise. Although standard FALQON and its time-rescaled (TR) variant, TR-FALQON, are efficient for many Hamiltonians [18], previous work has already shown that standard FALQON can be inefficient for certain models, as in the case of the anisotropic next-nearest-neighbor Ising model [17]. In this work, we will show that standard FALQON is inefficient also for the Maldacena-Qi model when the circuit is initialized in trivial quantum computational states, such as product states with no prior correlations. This choice of initial state is natural, since quantum processors typically provide trivial initial states; therefore, to ensure a fair comparison, the algorithm should start from such states.

To overcome this limitation and enhance the method's convergence, we propose and develop in this work the hybrid ITE-TR-FALQON protocol. This new variant integrates imaginary-time evolution (ITE) [19], which introduces an effective nonunitary dynamics capable of filtering out excited states, with the TR mechanism, which dynamically adjusts the magnitude of evolution steps. For the driver, product initial state, and parameter choices studied here, this combination improves convergence to the ground state in ideal numerical simulations where the standard versions stagnate.

In this work, we simulate FALQON, TR-FALQON, ITE-FALQON, and the proposed combined ITE-TR-FALQON on the Maldacena-Qi model. Our main results demonstrate that: (i) among the protocols and parameter choices investigated, ITE-TR-FALQON reaches the ground state in fewer algorithmic layers than ITE-FALQON, whereas the purely unitary variants stagnate within the simulated layer budget; (ii) the prepared state has high fidelity with the exact-diagonalization ground state and reproduces its von Neumann and Rényi entropies. These comparisons concern ideal state-vector evolution. They do not establish a reduction in total hardware resources or a scalable NISQ implementation. The obstacles to digital implementation are discussed explicitly in Sec. IV C.

This paper is organized as follows: in Sec. II, we review the feedback-based quantum algorithms employed in this work, including the TR-FALQON and ITE-FALQON variants, and introduce the hybrid ITE-TR-FALQON protocol proposed here. In Sec. III, we review the Maldacena-Qi model and define the quantities used to characterize the quality of the prepared state, in particular the fidelity and the entanglement entropies. In Sec. IV, we present the numerical results for the preparation of the ground state of

the model and compare the performance of the different algorithms. Finally, in Sec. V, we summarize our conclusions and discuss possible directions for future work.

II. FEEDBACK QUANTUM ALGORITHMS

The FALQON algorithm relies on QLC [16] to design controls that asymptotically guide a quantum system toward a target state. In this context, we consider the time-dependent Hamiltonian

$$H(t) = H_p + \beta(t)H_d, \quad (2)$$

where H_p is the problem Hamiltonian, namely the Hamiltonian whose ground state we aim to prepare, and H_d is the driver Hamiltonian, which induces the dynamics during the evolution. These names follow the standard terminology used in the literature, as in Ref. [20]. The corresponding Schrödinger equation (with $\hbar = 1$) is

$$i \frac{d}{dt} |\psi(t)\rangle = (H_p + \beta(t)H_d) |\psi(t)\rangle. \quad (3)$$

Defining the Lyapunov function $J(t) = \langle \psi(t) | H_p | \psi(t) \rangle$, one aims to design $\beta(t)$ to ensure

$$\frac{dJ(t)}{dt} \leq 0, \quad \forall t, \quad (4)$$

which ensures nonincreasing energy in the continuous-time description, but does not alone guarantee convergence to the ground state. Calculating the derivative yields $\frac{dJ}{dt} = A(t)\beta(t)$, with $A(t) = \langle \psi(t) | i[H_d, H_p] | \psi(t) \rangle$. Therefore, a choice that guarantees the validity of Eq. (4) is

$$\beta(t) = -A(t), \quad (5)$$

provided that $[H_d, H_p] \neq 0$. Instead of relying on an external classical optimizer, FALQON iteratively constructs a Trotterized circuit and updates, via feedback, the parameters of each layer based on measurements of the state generated by previous layers. The formal solution discretized by Suzuki-Trotter in k steps (Δt) is

$$|\psi_k\rangle = U_d(\beta_k)U_p \dots U_d(\beta_1)U_p |\psi_0\rangle, \quad (6)$$

with $U_d(\beta_k) = e^{-i\beta_k H_d \Delta t}$ and $U_p = e^{-iH_p \Delta t}$. The feedback law used in the layer construction is

$$\beta_k = -A_{k-1} = -\langle \psi_{k-1} | i[H_d, H_p] | \psi_{k-1} \rangle, \quad (7)$$

where A_{k-1} is estimated from the state of the previous iteration. The procedure is repeated until $J_k = \langle \psi_k | H_p | \psi_k \rangle$ converges. Figure 1 illustrates the diagram of the method and its variants.

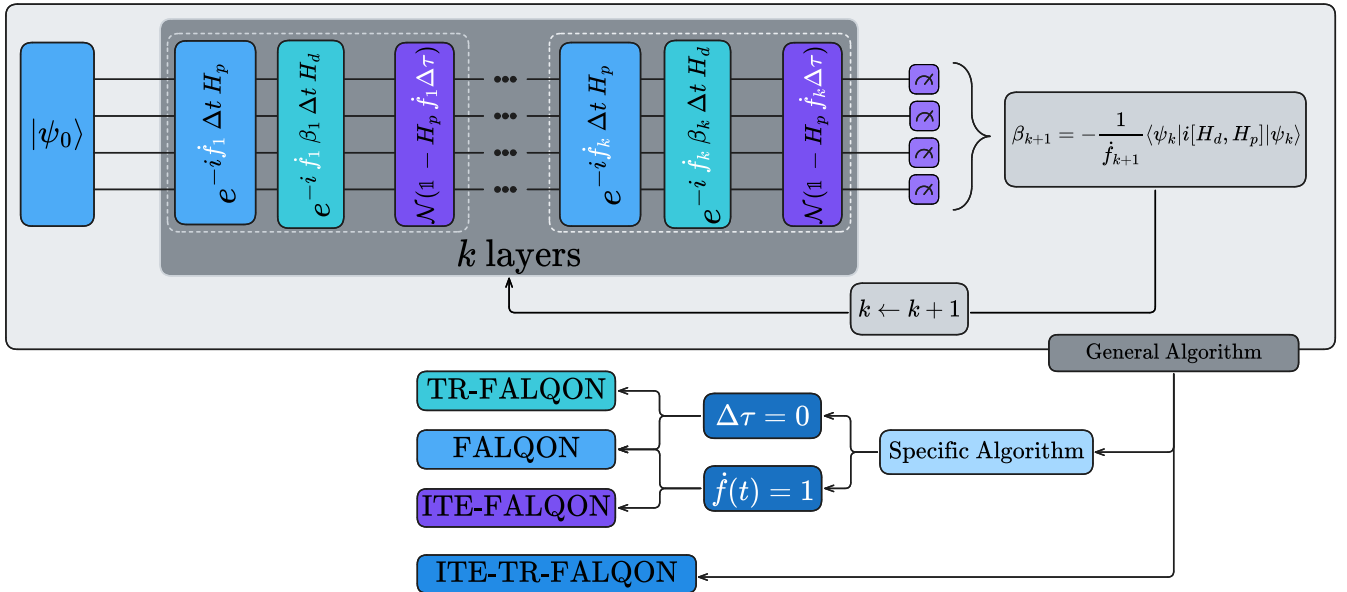


FIG. 1. Unified schematic representation of the FALQON protocols. The upper diagram illustrates the general algorithm, where the initial state $|\psi_0\rangle$ evolves through k layers. Each layer sequentially applies unitary operators generated by H_p and H_d , modulated by the time-rescaling factor $\dot{f}_k$ and the time step Δt , followed by a nonunitary operator $\mathcal{N}(I - H_p \dot{f}_k \Delta \tau)$ associated with imaginary-time evolution. The parameter β_{k+1} is calculated deterministically at each step via the expectation value of the commutator $\langle i[H_d, H_p] \rangle$. The lower flowchart demonstrates how the general algorithm specializes into specific methods: the condition $\Delta \tau = 0$ removes the nonunitary component, recovering FALQON and TR-FALQON (the latter dependent on the modulation of $\dot{f}$); alternatively, fixing $\dot{f}(t) = 1$ preserves the dissipative dynamics, characterizing the ITE-FALQON protocol.

A. TR-FALQON

The TR-FALQON protocol [18,21,22] modifies the standard FALQON algorithm just described by applying an extra time variable change $t = f(t)$, where the evolution satisfies

$$\frac{d}{dt}|\psi(t)\rangle = -iH(f(t))\dot{f}(t)|\psi(t)\rangle \equiv -i\tilde{H}(t)|\psi(t)\rangle. \quad (8)$$

Repeating the calculation of $\frac{d}{dt}\langle H_p \rangle$ yields a term proportional to $\tilde{\beta}(t)\dot{f}(t)A(t)$, with $A(t) = \langle \psi(t) | i[H_d, H_p] | \psi(t) \rangle$ and $\tilde{\beta}(t) = \beta(f(t))$. Choosing appropriate continuous functions, the control law becomes

$$\tilde{\beta}(t) = -A(t) \frac{1}{\dot{f}(t)}, \quad (9)$$

which preserves the monotonicity of $J(t)$. Discretizing into steps Δt and Trotterizing, the rescaled evolution is given by

$$\tilde{U}(t) = \tilde{U}_d(\tilde{\beta}(t))\tilde{U}_p, \quad \tilde{U}_d(\tilde{\beta}) = \exp(-iH_d\tilde{\beta}(\tau)\dot{f}(\tau)\Delta t), \quad (10)$$

with

$$\tilde{U}_p = \exp(-iH_p\dot{f}(\tau)\Delta t), \quad (11)$$

leading to the circuit

$$|\psi_k\rangle = \tilde{U}_d(\tilde{\beta}_k)\tilde{U}_p \dots \tilde{U}_d(\tilde{\beta}_1)\tilde{U}_p|\psi_0\rangle, \quad (12)$$

with the discretized feedback law

$$\tilde{\beta}_k = -A_{k-1} \frac{1}{\dot{f}(k\Delta t)}. \quad (13)$$

TR-FALQON uses this transformation to accelerate convergence in terms of algorithmic layers, with the choice of $f(t)$ influencing performance depending on the problem. A smaller layer count need not imply a smaller compiled gate depth, since the cost of implementing the rescaled propagators also has to be considered.

A common choice for the rescaling function is

$$f(t) = at - \frac{t_f}{2\pi a}(a-1)\sin\left(\frac{2\pi at}{t_f}\right), \quad (14)$$

where a is a parameter controlling time contraction. Its inverse function $f^{-1}(t)$, which cannot be expressed exactly in terms of standard functions, satisfies the properties $f^{-1}(0) = 0$, $f^{-1}(t_f) = \frac{t_f}{a}$, $f'(0) = 1$, and $f'(\frac{t_f}{a}) = 1$, which can be easily verified. These elements qualify this function as an appropriate time-rescaling function for any $a > 1$ [18].

B. ITE-FALQON

The ITE-FALQON protocol, proposed by Van Long *et al.* [19], introduces an effective dissipative dynamics that suppresses excited state amplitudes [23–28]. The imaginary-time Schrödinger equation,

$$\partial_\tau |\psi(\tau)\rangle = -H_p |\psi(\tau)\rangle, \quad (15)$$

possesses a formal solution analogous to diffusion, as follows:

$$|\psi(\tau)\rangle = e^{-H_p\tau}|\psi(0)\rangle = \sum_n c_n e^{-E_n\tau} |n\rangle. \quad (16)$$

After normalization, imaginary-time evolution suppresses excited-state components relative to the ground-state component and approaches the ground-state subspace as $\tau \rightarrow \infty$, provided that the initial state has nonzero overlap with that subspace. Applying the Taylor expansion for finite steps, one obtains

$$|\Phi_{\Delta\tau}(\psi)\rangle \approx \frac{(I - H_p\Delta\tau)|\psi(\tau)\rangle}{\|(I - H_p\Delta\tau)|\psi(\tau)\rangle\|}. \quad (17)$$

ITE-FALQON inserts the operation from Eq. (17) periodically between unitary steps, resulting in

$$|\psi_k\rangle = \mathcal{N}[(I - H_p\Delta\tau)e^{-iH_p\Delta t}e^{-i\beta_{k-1}H_d\Delta t}|\psi_{k-1}\rangle], \quad (18)$$

where $\mathcal{N}$ is the normalization factor. The term $(I - \Delta\tau H_p)$ redistributes the population to lower-energy sectors, reactivating the feedback term. Let H_p be bounded such that $\|H_p\| \leq h$. The hybrid algorithm interleaves ITE updates with the feedback control $\beta(t) = -i\langle [H_d, H_p] \rangle_t$. In this work, the nonunitary update and its normalization are evaluated directly in ideal state-vector simulations. This numerical operation is not an elementary gate available on a quantum processor. One possible implementation uses a unitary dilation with ancilla qubits and postselection; approximate quantum imaginary-time evolution offers a different route, with additional approximation and measurement costs. No such implementation is constructed or benchmarked here. The associated limitations, especially in an iterative feedback protocol, are discussed in Sec. IV C. At each ITE update, the normalized cost function is given by

$$C(\Phi_{\Delta\tau}(\psi)) = \frac{\langle \psi | (I - \Delta\tau H_p) H_p (I - \Delta\tau H_p) | \psi \rangle}{\langle \psi | (I - \Delta\tau H_p)^2 | \psi \rangle}. \quad (19)$$

Furthermore, one has the upper bound

$$C(\Phi_{\Delta\tau}(\psi)) \leq E - \frac{2\Delta\tau(1 - \frac{1}{2}h\Delta\tau)}{(1 + h\Delta\tau)^2} \mathcal{V}, \quad (20)$$

where $E = \langle H_p \rangle_\psi$ and $\mathcal{V} = \langle (H_p - E)^2 \rangle_\psi$. The bound holds for $0 < \Delta\tau < 1/h$. If $\mathcal{V} > 0$, then $C(\Phi_{\Delta\tau}(\psi)) < C(\psi)$, ensuring strict energy decrease at that ITE update [19].

III. MALDACENA-QI MODEL

The Maldacena-Qi model consists of two identical copies of the Sachdev-Ye-Kitaev model, denoted by L and R , coupled through a bilinear interaction term. For Majorana fermions, the total Hamiltonian is expressed as

$$H = H_L + H_R + i\mu \sum_{j=1}^N \chi_j^L \chi_j^R, \quad (21)$$

where N is the number of Majorana fermions in each copy and μ represents the coupling strength. The individual Hamiltonians for each copy, for an interaction order q , are given by

$$H_\alpha = i^{q/2} \sum_{1 \leq j_1 < \dots < j_q \leq N} J_{j_1 \dots j_q}^\alpha \chi_{j_1}^\alpha \dots \chi_{j_q}^\alpha, \quad (\alpha = L, R). \quad (22)$$

The Majorana operators satisfy the Clifford algebra $\{\chi_j^\alpha, \chi_k^\beta\} = 2\delta^{\alpha\beta}\delta_{jk}$. In this work, we focus on the $q = 4$ case. As numerically implemented, the symmetry between the copies correlates the random couplings such that $J_{j_1 \dots j_q}^R = (-1)^{q/2} J_{j_1 \dots j_q}^L$. For $q = 4$, this results in identical couplings for both copies ($J_{j_1 \dots j_q}^L = J_{j_1 \dots j_q}^R \equiv J_{j_1 \dots j_q}$).

The coefficients $J_{j_1 \dots j_q}$ are independent random variables drawn from a Gaussian distribution with zero mean and variance defined by

$$\overline{J_{j_1 \dots j_q}^2} = \frac{2^{q-1}(q-1)!}{qN^{q-1}} \mathcal{J}^2, \quad (23)$$

where $\mathcal{J}$ sets the microscopic energy scale, fixed as $\mathcal{J} = 1$ in our simulations.

In the weak but finite coupling regime, the ground state of this Hamiltonian, $|\psi_0\rangle$, approximates with high fidelity a TFD state [11], as follows:

$$|\text{TFD}(\beta)\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\beta E_n/2} |n\rangle_L \otimes |\bar{n}\rangle_R, \quad (24)$$

where $|n\rangle$ are the energy eigenstates of the isolated SYK system and β is the effective inverse temperature determined by the coupling μ .

A. Fidelity

To quantify the effectiveness of the FALQON in preparing the ground state, we calculate the fidelity between the state obtained via the algorithm, $|\psi_{\text{FALQON}}\rangle$, and the exact ground eigenvector obtained by numerical diagonalization, $|\psi_0\rangle$, as follows:

$$F = |\langle \psi_{\text{FALQON}} | \psi_0 \rangle|^2. \quad (25)$$

Values of $F \approx 1$ confirm the convergence of the algorithm to the vacuum of the coupled system.

B. Entanglement entropy

The characterization of quantum entanglement is fundamental to validating the holographic and thermodynamic nature of the prepared state. In the context of the AdS/CFT duality and the coupled SYK model, the entanglement between copies L and R in the TFD state is not merely a measure of nonlocal correlations, but the geometric manifestation of the connectivity of the dual spacetime (the Einstein-Rosen bridge).

To quantify these correlations, we perform a bipartition of the total system, defining the reduced density matrix of one of the subsystems (e.g., the L side) as $\rho_L = \text{Tr}_R(|\psi\rangle\langle\psi|)$. The primary measure of interest is the von Neumann entropy,

$$S_L^{(\text{vN})} = -\text{Tr}(\rho_L \ln \rho_L). \quad (26)$$

Physically, for an ideal TFD state, $S_L^{(\text{vN})}$ corresponds to the thermodynamic entropy of the SYK system at inverse temperature β . In the holographic context, this quantity is dual to the area of the minimal surface in the bulk (Ryu-Takayanagi formula), serving as a direct diagnostic of the geometry of the formed wormhole.

Complementarily, we calculate the Rényi entropies of order n , as follows:

$$S_L^{(n)} = \frac{1}{1-n} \ln \text{Tr}(\rho_L^n). \quad (27)$$

The inclusion of Rényi entropies is strategically crucial in this work for two main reasons. First, from the perspective of quantum computing implementation, the von Neumann entropy is computationally expensive to estimate experimentally, requiring full state tomography. In contrast, the second-order Rényi entropy $[S_L^{(2)}]$, related to subsystem purity, is directly accessible on NISQ processors via efficient protocols such as the swap test or randomized measurements.

Second, the family of entropies $S_L^{(n)}$ probes the fine structure of the entanglement spectrum (the eigenvalues of ρ_L). For the TFD state, this spectrum must follow a Boltzmann thermal distribution. By verifying the agreement of Rényi entropies for different orders n , we confirm that the algorithm not only reproduces the correct average entanglement but faithfully captures the thermal statistics of the state, which is a prerequisite for gravitational duality.

IV. RESULTS

In this section, we compare the ideal numerical evolution generated by FALQON and its variants for a strongly interacting, long-range fermionic model. The reported results are obtained from state-vector simulations, with exact diagonalization as the reference. First, we compare ground-state energy and fidelity; then, we examine the entanglement entropies. Device noise, finite-shot feedback estimation, ancilla overhead, and hardware compilation are not included in these calculations.

To implement the problem Hamiltonian H_p , we map the fermionic degrees of freedom onto a qubit register using the Jordan-Wigner transformation [29,30]. In the notation adopted in this work, N denotes the number of Majorana fermions in each SYK copy, so that the full Maldacena-Qi system contains $2N$ Majorana modes in total. After the Jordan-Wigner mapping, the model is encoded into a register of $N_q = N$ qubits. In the convention used in this work, the Majorana operators are written as

$$\begin{aligned} \chi_{2j-1} &= \left(\prod_{k=1}^{j-1} \sigma_k^z \right) \sigma_j^x, \\ \chi_{2j} &= \left(\prod_{k=1}^{j-1} \sigma_k^z \right) \sigma_j^y, \quad j = 1, \dots, N_q. \end{aligned} \quad (28)$$

Equivalently, the ordering of the Majorana modes in Eq. (21) is identified with this standard Jordan-Wigner convention. Under this transformation, each Majorana operator is represented by a nonlocal Pauli string that preserves the original Clifford algebra. Consequently, both the four-body interaction terms and the bilinear coupling defined in Eq. (21) are converted into a sum of tensor products of Pauli operators $\{I, X, Y, Z\}^{\otimes N_q}$, allowing for the explicit construction of the Hamiltonian matrix for simulating time evolution and evaluating the cost function.

To employ FALQON, TR-FALQON, and ITE-FALQON as tools for preparing the TFD type ground state of the Maldacena-Qi model, we define the driver Hamiltonian H_d as

$$H_d = \sum_{j=1}^{N_q} \sigma_j^x. \quad (29)$$

Here, σ_j^x is the Pauli-X matrix acting on qubit j . In the computational basis $\{|0\rangle, |1\rangle\}$, the σ^x matrix has the representation

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (30)$$

such that

$$\sigma^x|0\rangle = |1\rangle, \quad \sigma^x|1\rangle = |0\rangle. \quad (31)$$

The choice of the initial state used in our experiments is the product state

$$|\psi_0\rangle = \bigotimes_{j=1}^{N_q} |+\rangle_j \equiv |+\rangle^{\otimes N_q}, \quad (32)$$

that is, each qubit is prepared in the $|+\rangle$ state. We justify this choice for the following practical and conceptual reasons: (i) $|+\rangle^{\otimes N_q}$ is easy to prepare on quantum hardware (by applying Hadamard gates to all qubits starting from $|0\rangle^{\otimes N_q}$); (ii) it is a uniform superposition over the computational basis, thus introducing no initial bias towards specific classical configurations and allowing the algorithm to broadly explore the Hilbert space; (iii) being a product state with no initial entanglement, any entanglement generated during evolution is solely the result of interactions and the preparation protocol, which facilitates the interpretation of entanglement entropy measurements.

Finally, we emphasize that although $H_d = \sum_j \sigma_j^x$ is the standard mixer and offers implementation advantages, there are alternatives that may be preferable in scenarios with conservation constraints or reduced state spaces. We opted for the above definition for consistency with the literature and to facilitate direct comparisons between our results and related works.

A. Ground-state preparation and fidelity

To evaluate the numerical convergence of the proposed protocols in preparing the ground state of the Maldacena-Qi model, we analyze the energy and the fidelity of the prepared state as a function of the number of algorithmic layers k . Simulations were performed for a system with $N = 8$ Majorana fermions per SYK copy ($2N = 16$ Majorana fermions in total, corresponding to $N_q = 8$ qubits after the Jordan-Wigner mapping), considering different coupling regimes μ .

Figure 2 presents the evolution of the energy expectation value $\langle H \rangle_k$ across layers for three distinct coupling strengths. The curves represent the average over disorder realizations, comparing the standard FALQON, ITE-FALQON, TR-FALQON algorithms, and the hybrid ITE-TR-FALQON method.

It is observed that for the uncoupled case ($\mu = 0.0$) and weak coupling ($\mu = 0.3$), the purely unitary methods (FALQON and TR-FALQON with $a = 4$) manage to reduce the energy but exhibit a slow convergence rate and a tendency to saturate above the ground state. TR-FALQON demonstrates a steeper initial descent than standard FALQON due to time rescaling, which effectively amplifies gradients at the beginning of the evolution, yet it still fails to reach the exact energy within the simulated layer budget.

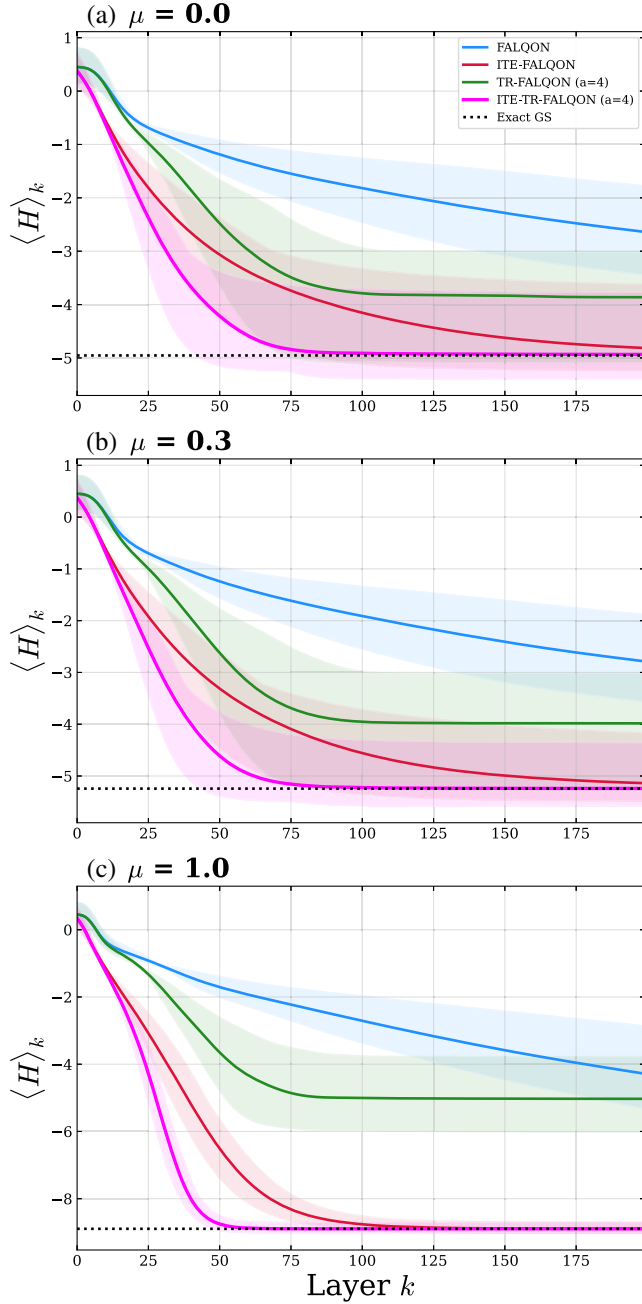


FIG. 2. Convergence of the expected energy value $\langle H \rangle_k$ as a function of the number of layers k for $N = 8$ Majorana fermions per SYK copy. Panels show different coupling strengths: (a) $\mu = 0.0$, (b) $\mu = 0.3$, and (c) $\mu = 1.0$. Curves represent the average of ten realizations, with shaded areas indicating the standard deviation. The black dotted line indicates the exact ground-state energy obtained by diagonalization. Simulations use $\Delta t = 0.01$ and $\Delta \tau = 0.01$.

The introduction of nonunitary dynamics (such as ITE) qualitatively alters the convergence behavior. As evidenced in panels (b) and (c) of Fig. 2, as the coupling μ increases—making the optimization landscape more complex due to correlations between systems L and R —the need to filter

out excited states becomes critical. ITE-FALQON achieves significantly lower energies than the unitary versions. However, it is the hybrid ITE-TR-FALQON protocol that displays superior performance in all scenarios. By combining the exponential filtering of excited states from imaginary-time evolution with the kinetic acceleration provided by time rescaling ($a = 4$), the algorithm reaches the exact ground state ($\langle H \rangle_k \approx E_{GS}$) in fewer layers for $\mu = 1.0$, a regime where standard methods fail or stagnate in local minima.

To quantify the quality of the prepared quantum state, we calculate the fidelity $|\langle \psi_{ED} | \psi_k \rangle|^2$ relative to the ground state obtained by exact diagonalization. Figure 3 illustrates the fidelity on a logarithmic scale of layers (up to $k = 10^3$) for three different coupling strengths: (a) $\mu = 0.0$, (b) $\mu = 0.3$, and (c) $\mu = 1.0$.

Analysis of Fig. 3 shows that the purely unitary algorithms (FALQON and TR-FALQON) remain at low fidelity within the simulated budget of 10^3 layers for the chosen driver, initial state, and parameters. In contrast, the ITE-based methods exhibit a rapid rise toward unit fidelity. This comparison demonstrates the benefit of imaginary-time filtering in the tested setting; it does not imply that unitary state preparation is generally impossible or that a function of H_p can create ground-state overlap from a state with exactly zero overlap.

The impact of the time-rescaling parameter $a = 4$ on the hybrid method is evident in all three panels: while ITE-FALQON requires approximately 10^2 layers to initiate the transition to high fidelity, ITE-TR-FALQON anticipates this convergence, reaching fidelity > 0.99 significantly earlier. This result confirms that time rescaling acts not only on energy minimization but also accelerates the rotation of the state vector toward the ground-state subspace across different values of μ , reducing the number of ideal algorithmic layers needed to approach the target ground state. This metric excludes the gate decomposition of each propagator, nonunitary implementation, and repeated state preparations required for feedback measurements.

B. Entanglement entropy

The entanglement entropy between subsystems L and R offers a direct probe of the emergent geometry and the thermality of the prepared state. In the context of holographic duality, the von Neumann entropy S_{vN} of subsystem L is dual to the area of the minimal surface in the bulk (Ryu-Takayanagi surface). To characterize the states obtained by the feedback algorithms against the exact-diagonalization ground-state reference, we analyze both entropy convergence and the Rényi entropies.

Figure 4(a) presents the convergence dynamics of S_{vN} as a function of the number of algorithmic layers (k) for a system with $N = 8$ Majorana fermions per SYK copy and coupling $\mu = 0.15$. The FALQON and TR-FALQON algorithms do not converge to the exact-diagonalization

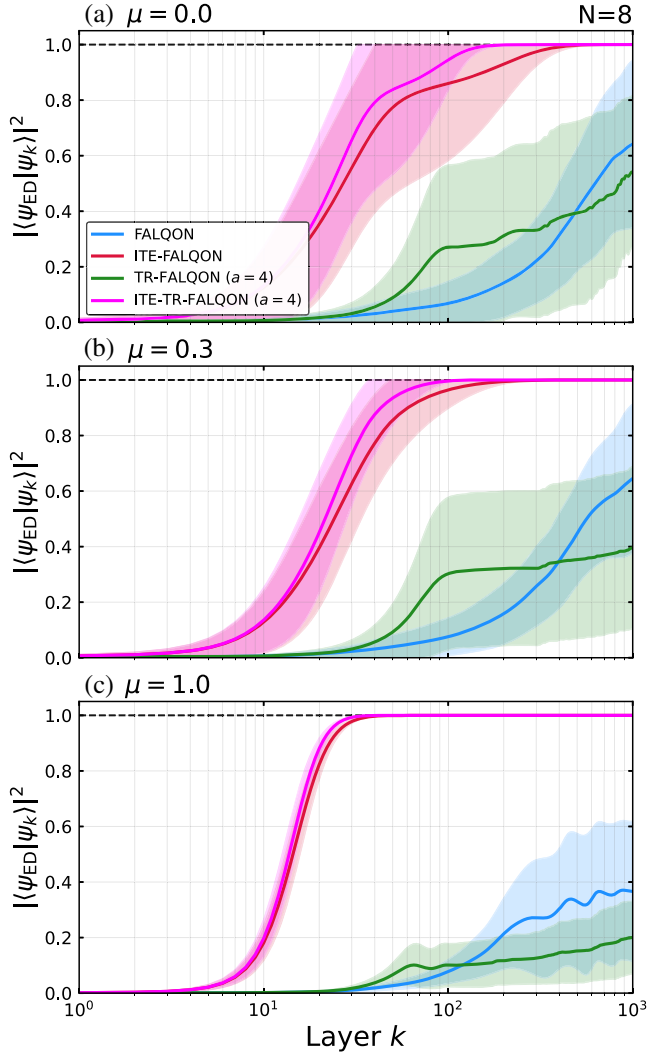


FIG. 3. Fidelity of the prepared state $|\langle\psi_{\text{ED}}|\psi_k\rangle|^2$ relative to the exact ground state as a function of the layer number k on a logarithmic horizontal axis for $N = 8$ Majorana fermions per SYK copy. The panels show three different coupling strengths: (a) $\mu = 0.0$, (b) $\mu = 0.3$, and (c) $\mu = 1.0$. Comparison is made between FALQON, ITE-FALQON, TR-FALQON, and ITE-TR-FALQON methods. The time-rescaling parameter is fixed at $a = 4$. Curves represent the average of ten realizations, with shaded areas indicating the standard deviation. Simulations use $\Delta t = 0.01$ and $\Delta \tau = 0.09$.

entropy within the simulated layer budget. In contrast, the ITE-assisted variants, specifically ITE-TR-FALQON, converge to the exact ground-state entropy. This behavior supports the usefulness of imaginary-time filtering in the tested setup. Since the filter is a function of H_p , it preserves the symmetries of H_p ; its benefit should not be interpreted as breaking those symmetries.

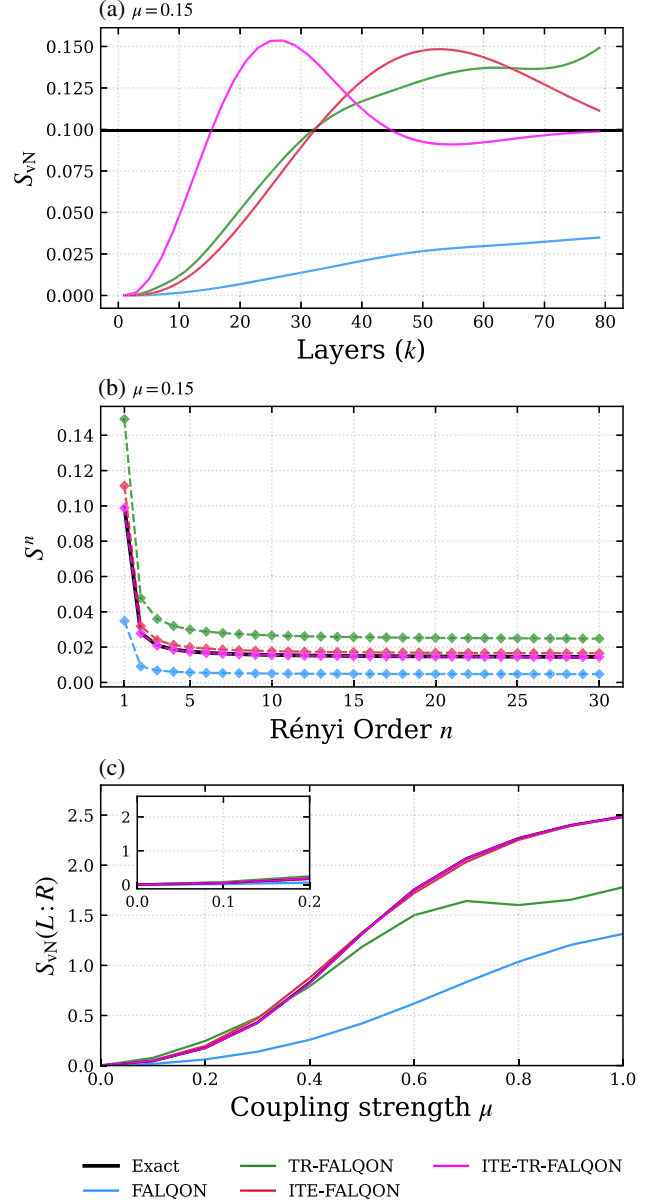


FIG. 4. Entanglement characterization of the ground state prepared for the coupled SYK model with $N = 8$ Majorana fermions per SYK copy. (a) Evolution of von Neumann entropy $S_{\text{vN}}(L:R)$ as a function of algorithmic layers (k) for $\mu = 0.15$. The hybrid ITE-TR-FALQON method (magenta) reaches the exact value, while purely unitary methods saturate prematurely. (b) Spectrum of Rényi entropies S^n as a function of order n for the final state obtained (state at $k = 80$ for $\mu = 0.15$). The agreement with the exact-diagonalization curve (black) confirms that the algorithm reproduces the reference ground-state Rényi entropies for the orders shown. (c) Dependence of the steady-state von Neumann entropy on the coupling strength μ . The proposed algorithm correctly captures the transition from a product state ($\mu = 0$) to a highly entangled state ($\mu = 1$). Parameters: Time step $\Delta t = 0.02$, time-scaling factor $a = 4$ (for TR variants), and ITE rate $\Delta \tau = 0.2$.

A more rigorous verification of the state's thermal nature is provided by the spectrum of Rényi entropies, $S^n = (1 - n)^{-1} \ln \text{Tr}(\rho_L^n)$, shown in Fig. 4(b). While S_{vN} provides a single scalar number, the dependence of S^n on the order n encodes the distribution of the reduced density matrix eigenvalues ρ_L . The agreement between the ITE-TR-FALQON result and the exact-diagonalization curve for the orders n shown demonstrates that the algorithm reproduces the entanglement properties of the target ground state. Its relation to a TFD state is understood within the ground-state approximation described in Sec. III. The discrepancies observed for unitary methods at higher orders ($n > 5$) reveal that, although the energy may be close to the ground state, the fine structure of the entanglement differs significantly from the thermal target.

Finally, we investigate in Fig. 4(c) the robustness of state preparation under variation of the coupling strength $\mu \in [0, 1.0]$. It is crucial to note that the physically relevant regime for the holographic interpretation of a traversable wormhole in AdS_2 occurs at weak coupling. As established in Ref. [12], for large values of the coupling (e.g., $\mu \approx 1.0$), the direct one-body interaction between the two SYK models strongly dominates over the many-body term. In this regime, the system becomes essentially trivial and noninteracting, characterized by an energy gap that scales linearly with the coupling.

Furthermore, the detailed study of the energy gap and the Schwinger-Dyson equations performed in Ref. [12] demonstrated that the nontrivial phase is located below a critical coupling $\mu_c \approx 0.175$. Below this threshold, the system genuinely dualizes to a traversable wormhole and the energy gap exhibits a fractional power-law behavior. Based entirely on the theoretical limits found in that reference, we selected $\mu = 0.15$ (well within the stable wormhole phase) as the strictly physically motivated benchmark for our detailed entanglement entropy calculations.

To properly highlight the performance of our approach in this region of actual holographic interest, we include an inset in Fig. 4(c) magnifying the interval $\mu \in [0, 0.2]$. Our results show that the ITE-TR-FALQON protocol accurately tracks the exact theoretical curve precisely within this gap-stabilized regime, evidencing its applicability for preparing the relevant vacuum states of the model without being restricted to the trivial free-fermion limit.

C. Feasibility on current quantum devices

The simulations above provide a proof of principle for convergence under ideal updates, rather than a hardware demonstration. In its present form, ITE-TR-FALQON should not be regarded as a practical state-preparation protocol for current NISQ devices. The nonunitary filtering, the cost of simulating the coupled SYK Hamiltonian, and the repeated measurements needed for feedback all contribute to this limitation.

The update $(I - \Delta\tau H_p)|\psi\rangle$ followed by normalization cannot, in general, be applied deterministically to an arbitrary input state using only unitary gates on the system register. In a direct ancilla-assisted realization, the filter must be embedded with a suitable normalization into a larger unitary operation, and the desired output is obtained conditional on an ancilla measurement outcome. If p_k denotes the conditional success probability of the k th filter, a sequence of K successful filters without recovery has probability $P_{\text{succ}} = \prod_{k=1}^K p_k$. This probability can become small over many iterations, requiring repeated preparations. The values of p_k depend on the chosen encoding, normalization, and evolving state; they are not evaluated in this work. Ancilla-assisted filtering and approximate quantum imaginary-time evolution therefore represent possible research directions, rather than implementation steps whose efficiency is established here.

Even setting aside the filtering cost, an algorithmic layer is not an elementary gate layer. For $q = 4$, the Hamiltonian contains $2\binom{N}{4}$ quartic interaction terms and N inter-copy bilinear terms before any compilation or grouping. At $N = 8$, this gives up to 148 nonzero Hamiltonian terms on eight system qubits, corresponding to four qubits per copy; the bilinear terms vanish at $\mu = 0$. Their Jordan-Wigner images are Pauli strings whose exponentials generally require multiple elementary gates. The compiled depth further depends on the simulation accuracy, term grouping, gate set, and device connectivity, including any routing overhead. In a direct term-by-term digital implementation, a trajectory with approximately 50 algorithmic layers would thus require substantially more than 50 elementary gate layers, even before the ancilla operations are included. This illustrative layer count is not a universal convergence threshold. Time rescaling also changes the propagators to be implemented, so fewer algorithmic layers do not by themselves establish a lower total gate cost.

Feedback adds a separate sampling cost. Estimating $\langle i[H_d, H_p] \rangle$ at an iteration requires measurements on separately prepared copies of the current state. These state preparations must reproduce the preceding evolution, including any successful conditional filters, and must be repeated sufficiently often to control statistical uncertainty. Direct numerical evaluation of the commutator expectation value and state normalization does not include this cost. Consequently, the convergence curves do not establish an end-to-end resource advantage over other preparation methods. Gate-level compilation, a treatment of filtering success probabilities, and an analysis of noise and finite-shot feedback are required before a practical implementation claim can be made.

V. CONCLUSION

In this work, we investigated, through ideal state-vector simulations, feedback-based preparation of the ground state

of the Maldacena-Qi model, a system of interest for AdS/CFT holography and quantum chaos. We compared FALQON, TR-FALQON, ITE-FALQON, and ITE-TR-FALQON starting from an uncorrelated product state.

For the chosen driver, initial state, and parameter regimes, the purely unitary protocols exhibit slow convergence or stagnation within the simulated layer budget, including at strong coupling ($\mu \approx 1.0$). These findings characterize the tested protocols and do not constitute a general obstruction to unitary ground-state preparation.

Imaginary-time filtering improves convergence in this setting. Among the methods compared, the hybrid ITE-TR-FALQON protocol reaches high ground-state fidelity in fewer algorithmic layers than ITE-FALQON. This is an advantage in ideal numerical convergence; a corresponding reduction in hardware cost has not been established.

Beyond energy and fidelity, the agreement of the von Neumann and Rényi entropies with the exact-diagonalization reference supports the quality of the prepared ground state. In the regime where that ground state approximates a TFD state, this agreement is consistent with the thermal-state motivation of the protocol.

The implementation limitations detailed in Sec. IV C are central to the interpretation of these results. Direct numerical application and normalization of $(I - \Delta\tau H_p)$ do not supply an efficient physical implementation. Ancilla preparation, conditional measurements, possible repetitions, Hamiltonian compilation, and feedback sampling can outweigh a reduction in algorithmic layers. Resolving these costs within the full iterative protocol remains future work, and the present study does not establish feasibility on current NISQ hardware.

The contribution of this study is therefore a numerical proof of principle for combining deterministic feedback with imaginary-time filtering in the coupled SYK model. It

motivates further work on implementable filtering schemes and complete resource estimates, while leaving practical and scalable quantum-device preparation as an open task.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

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