

Absence of $\eta_1(1855)$ and its exotic partner candidate in a chiral quark model

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Inspired by the experimental discoveries of $X(3872)$ ($c\bar{q}q\bar{c}$) and T_{cc} ($c\bar{q}c\bar{q}$), we systematically study their orbitally excited strange partners, i.e., the $K\bar{K}_1$ ($q\bar{s}s\bar{q}$) system, which may be related to the recently observed $\eta_1(1855)$, and the KK_1 ($q\bar{s}q\bar{s}$) system. Within the framework of an accurate few-body calculation method, we employ the chiral quark model to investigate both the molecular and diquark structures of these two multiquark systems in addition to channel coupling effects. Our results show that the $K\bar{K}_1$ system remains a scattering state, although we have considered two different sets of parameters in our calculations to avoid the influence of parameters. On the other hand, due to the presence of a good-diquark structure in the KK_1 system, we obtain a very shallow bound state in the coupled-channel calculation, where the one-gluon, the η -meson, f_0 -meson, and σ -meson exchanges play crucial roles. The interquark distance and component analysis also suggest that it is a loosely bound molecular state.

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I. INTRODUCTION

To give proper explanations on the newly observed exotic states in experiments, decoding the inner hadronic structures of them is one of the most intriguing and important topics in hadron physics. In the quark model, exotic states can be classified into two categories. One is that their properties cannot be adequately described by the traditional quark model. For example, $X(3872)$ [1,2] is interpreted as a $c\bar{c}$ state with 1^{++} in the traditional quark model, but its mass exceeds the experimentally observed value by several tens of MeV, or the charged $Z_c(3900)^\pm$ cannot be classified as a $c\bar{c}$ state [3–6]. The other is that their quantum numbers cannot be achieved as a meson or baryon, such as the $\pi_1(1600)$ with a quantum number as 1^{-+} . Studying these exotic hadrons can deepen our understanding in hadron spectroscopy and, in turn, lead to a better comprehension on QCD.

Recently, the BESIII Collaboration observed a new structure $\eta_1(1855)$ in the $J/\psi \rightarrow \gamma\eta\eta'$ process with a significance of 19σ [7,8], whose mass and width are measured to be $1855 \pm 9_{-1}^{+6}$ and $188 \pm 18_{-8}^{+3}$ MeV, respectively. Because of its unusual quantum numbers as 1^{-+} just like $\pi_1(1600)$, it absolutely cannot be described by a conventional quark-antiquark structure. Moreover, since its mass is very close to the threshold for $K\bar{K}_1(1400)$, a possible assignment is that it may be a molecular state that mainly composed by $K\bar{K}_1(1400)$.

Currently, theoretical studies on the $\eta_1(1855)$ can be broadly classified into two categories: One interprets it as a hybrid state such as $q\bar{q}g$ [9–20], while the other proposes it as a molecular $K\bar{K}_1$ ($q\bar{s}s\bar{q}$) state [21–25]. For example, the authors in [10] studied the decays of the $J^{PC} = 1^{-+}$ hybrid nonet using a Lagrangian invariant under flavor symmetry and found that the light isoscalar must be significantly narrow, while the width of the heavy isoscalar can match the recently observed $\eta_1(1855)$. Through the study of the corresponding decay modes of $\eta_1(1855)$, the authors in [18] explained it as an $s\bar{s}g$ hybrid meson and found that the QCD axial anomaly enhances the decay width of the $\eta\eta'$ channel, although this mode is strongly suppressed by the small P-wave phase space. This conclusion is confirmed in [17], which is based on the flux tube model. Recently, Ma *et al.* [20] included effective gluons in the constituent quark model and recalculated the 1^{-+} light hybrids, including

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$q\bar{q}g$, $s\bar{s}g$, and $q\bar{s}g$. Their results provide a good explanation of the energy spectrum for $\pi(1600)$ and $\eta_1(1855)$, but the candidate for $\eta_1(1855)$, mainly $s\bar{s}g$, has a nearly zero decay width to $\eta\eta'$, which cannot explain the experimental observation of $\eta_1(1855)$ on $J/\psi \rightarrow \gamma\eta\eta'$ process. On the other hand, some works suggest that $\eta_1(1855)$ could be a molecular state of $K\bar{K}_1$. For instance, in Ref. [22], the authors studied the radiative and strong decays of the S-wave $K\bar{K}_1(1400)$ molecular state within the effective Lagrangian approach and found that its decay width is consistent with the experimentally observed $\eta_1(1855)$, while the study in Ref. [23] obtained the bound state of $K\bar{K}_1(1400)$ based on the one-boson-exchange model. Similarly, Ref. [24] conducted a similar study by using the Bethe-Salpeter equation. In Ref. [25], the tetraquark interpretation for the structure of $\eta_1(1855)$ is examined, where the observed $\eta_1(1855)$ could be embedded in the $[1_c]\bar{s}s \otimes [1_c]\bar{q}q$ configuration. However, within the framework of QCD sum rules, a recent study by the authors of Ref. [26] conducted detailed calculations of the correlation functions for the $K\bar{K}_1$ system, and they find no evidence for a bound state formation.

Apparently, the tetraquark explanation for $\eta_1(1855)$ suggests that it can be seen as an orbital-excited strange partner of $X(3872)$. Since the observation of $X(3872)$, with its quark configuration of $c\bar{q}q\bar{c}$, it has further inspired experimental physicists to discover a similar multi-quark state, T_{cc} , with a quark composition of $c\bar{q}c\bar{q}$. In this article, we systematically study the $K\bar{K}_1$ ($q\bar{s}s\bar{q}$) system, which includes three quark configurations: $q\bar{s}-s\bar{q}$, $s\bar{s}-q\bar{q}$, and $qs-\bar{s}\bar{q}$. Since these tetraquark systems are composed of light and strange quarks, we adopt a potential model based on SU(3) symmetry called the chiral quark model (ChQM) to perform the calculation. In addition, to better explore the influence of the Goldstone-boson-exchange potential in the chiral quark model, we introduce the exchange potential of scalar mesons into our model. Subsequently, we extend this calculation to the KK_1 ($q\bar{s}q\bar{s}$) system. To calculate the properties of these tetraquark systems, we employ the Rayleigh-Ritz variational method combined with the Gaussian expansion method (GEM), which allows us to expand each relative motion in the system in terms of Gaussian basis functions. In addition, we account for the mixing effects between the molecular and diquark-antidiquark configurations.

The paper is organized as follows: After the Introduction, Sec. II presents the details of our model, Sec. III presents the numerical results, and the final Sec. IV is devoted to the summary.

II. MODEL SETUP

A. Chiral quark model

In the chiral quark model [27–29], the potential part of the Hamiltonian consists of the color confinement potential $V_{\text{CON}}(r_{ij})$, the one-gluon-exchange potential $V_{\text{OGE}}(r_{ij})$,

the pseudoscalar-meson-exchange potential $V_{\text{PSC}}(r_{ij})$, which includes pseudoscalar mesons (π , K , and η), and the scalar-meson-exchange potential $V_{\text{SC}}(r_{ij})$, which includes scalar mesons (a_0 , f_0 , κ , and σ). The Hamiltonian of the ChQM used here is given by

$$H = \sum_{i=1}^4 \left(m_i + \frac{\vec{p}_i^2}{2m_i} \right) - T_{\text{c.m.}} + \sum_{i<j=1}^4 (V_{\text{CON}}(r_{ij}) + V_{\text{OGE}}(r_{ij}) + V_{\text{PSC}}(r_{ij}) + V_{\text{SC}}(r_{ij})), \quad (1)$$

where $T_{\text{c.m.}}$ is the kinetic energy operator for the center-of-mass motion of the entire system and r_{ij} refers to the distances between the quarks. Because of the spin-orbit coupling effects of K'_1 and K_1^* in our system, some components of the Hamiltonian, such as $V_{\text{SC}}(r_{ij})$, $V_{\text{CON}}(r_{ij})$, and $V_{\text{OGE}}(r_{ij})$, include both central potentials and spin-orbit couplings, whereas $V_{\text{PSC}}(r_{ij})$ includes only central potentials. The pseudoscalar-meson-exchange potential $V_{\text{PSC}}(r_{ij})$ is expressed as follows:

$$V_{\text{PSC}} = v_\pi(r_{ij}) \sum_{a=1}^3 \lambda_i^a \lambda_j^a + v_K(r_{ij}) \sum_{a=4}^7 \lambda_i^a \lambda_j^a + v_\eta(r_{ij}) [\cos \theta_P (\lambda_i^8 \lambda_j^8) - \sin \theta_P], \quad (2)$$

$$v_{\chi=\pi,K,\eta} = \frac{g_{ch}^2}{4\pi} \frac{m_\chi^2}{12m_i m_j} \frac{\Lambda_\chi^2}{\Lambda_\chi^2 - m_\chi^2} \times m_\chi \left[Y(m_\chi r_{ij}) - \frac{\Lambda_\chi^3}{m_\chi^3} Y(\Lambda_\chi r_{ij}) \right] (\vec{\sigma}_i \cdot \vec{\sigma}_j). \quad (3)$$

The central potential expressions for the scalar-meson-exchange potential V_{SC}^C are given by

$$V_{a_0}^C = -\frac{g_{ch}^2}{4\pi} \frac{\Lambda_{a_0}^2 m_{a_0}}{\Lambda_{a_0}^2 - m_{a_0}^2} \left[Y(m_{a_0} r_{ij}) - \frac{\Lambda_{a_0}}{m_{a_0}} Y(\Lambda_{a_0} r_{ij}) \right] \times \sum_{a=1}^3 \lambda_i^a \lambda_j^a, \quad (4)$$

$$V_\kappa^C = -\frac{g_{ch}^2}{4\pi} \frac{\Lambda_\kappa^2 m_\kappa}{\Lambda_\kappa^2 - m_\kappa^2} \left[Y(m_\kappa r_{ij}) - \frac{\Lambda_\kappa}{m_\kappa} Y(\Lambda_\kappa r_{ij}) \right] \sum_{a=4}^7 \lambda_i^a \lambda_j^a, \quad (5)$$

$$V_{f_0}^C = -\frac{g_{ch}^2}{4\pi} \frac{\Lambda_{f_0}^2 m_{f_0}}{\Lambda_{f_0}^2 - m_{f_0}^2} \left[Y(m_{f_0} r_{ij}) - \frac{\Lambda_{f_0}}{m_{f_0}} Y(\Lambda_{f_0} r_{ij}) \right] \lambda_i^8 \lambda_j^8, \quad (6)$$

$$V_\sigma^C = -\frac{g_{ch}^2}{4\pi} \frac{\Lambda_\sigma^2 m_\sigma}{\Lambda_\sigma^2 - m_\sigma^2} \left[Y(m_\sigma r_{ij}) - \frac{\Lambda_\sigma}{m_\sigma} Y(\Lambda_\sigma r_{ij}) \right]. \quad (7)$$

The spin-orbit coupling potential expressions for the scalar-meson-exchange potential $V_{\text{SC}}^{\text{SO}}$ in the Goldstone-boson-exchange potential and V_{σ}^{SO} are given by

$$V_{a_0}^{\text{SO}} = -\frac{g_{ch}^2}{4\pi} \frac{\Lambda_{a_0}^2}{\Lambda_{a_0}^2 - m_{a_0}^2} \frac{m_{a_0}^3}{2m_i m_j} \left[G(m_{a_0} r_{ij}) - \frac{\Lambda_{a_0}^3}{m_{a_0}^3} \times G(\Lambda_{a_0} r_{ij}) \right] \vec{L} \cdot \vec{S} \sum_{a=1}^3 \lambda_i^a \lambda_j^a, \quad (8)$$

$$V_{\kappa}^{\text{SO}} = -\frac{g_{ch}^2}{4\pi} \frac{\Lambda_{\kappa}^2}{\Lambda_{\kappa}^2 - m_{\kappa}^2} \frac{m_{\kappa}^3}{2m_i m_j} \left[G(m_{\kappa} r_{ij}) - \frac{\Lambda_{\kappa}^3}{m_{\kappa}^3} \times G(\Lambda_{\kappa} r_{ij}) \right] \vec{L} \cdot \vec{S} \sum_{a=4}^7 \lambda_i^a \lambda_j^a, \quad (9)$$

$$V_{f_0}^{\text{SO}} = -\frac{g_{ch}^2}{4\pi} \frac{\Lambda_{f_0}^2}{\Lambda_{f_0}^2 - m_{f_0}^2} \frac{m_{f_0}^3}{2m_i m_j} \left[G(m_{f_0} r_{ij}) - \frac{\Lambda_{f_0}^3}{m_{f_0}^3} \times G(\Lambda_{f_0} r_{ij}) \right] \vec{L} \cdot \vec{S} \lambda_i^8 \lambda_j^8, \quad (10)$$

$$V_{\sigma}^{\text{SO}} = -\frac{g_{ch}^2}{4\pi} \frac{\Lambda_{\sigma}^2}{\Lambda_{\sigma}^2 - m_{\sigma}^2} \frac{m_{\sigma}^3}{2m_i m_j} \left[G(m_{\sigma} r_{ij}) - \frac{\Lambda_{\sigma}^3}{m_{\sigma}^3} \times G(\Lambda_{\sigma} r_{ij}) \right] \vec{L} \cdot \vec{S}, \quad (11)$$

where λ^a are the SU(3) flavor Gell-Mann matrices, $m_{\chi=\pi,K,\eta,a_0,\kappa,f_0}$ is the mass of the Goldstone bosons, and $\Lambda_{\chi=\pi,K,\eta,a_0,\kappa,f_0}$ are the cutoffs. The $Y(x)$ is the standard Yukawa function, defined as $Y(x) = e^{-x}/x$, and $G(x)$ is given by $G(x) = (1 + 1/x)Y(x)/x$. Finally, the chiral coupling constant g_{ch} is determined from the πNN coupling constant via the relation

$$\frac{g_{ch}^2}{4\pi} = \left(\frac{3}{5}\right)^2 \frac{g_{\pi NN}^2}{4\pi} \frac{m_{u,d}^2}{m_N^2}. \quad (12)$$

In the quark model, there are typically three forms of the confinement potential: the linear confinement [30], the quadratic confinement [29], and the color-screened confinement [27]. All of them are effective in describing the energies of ground-state hadrons. Among these, the color-screened confinement potential takes into account the effects of sea quarks by introducing a screening factor to simulate the influence of quark-antiquark pairs excited from the vacuum on the confinement interaction in excited hadrons. Therefore, it is considered more suitable for describing excited states. In our study, which focuses on excited states of the K -meson family, the color-screened confinement potential is adopted. Its explicit form is given as follows:

$$V_{\text{CON}}^C = (-a_c(1 - e^{-\mu_c r_{ij}}) + \Delta)\lambda_i^c \cdot \lambda_j^c, \quad (13)$$

$$V_{\text{CON}}^{\text{SO}} = -\lambda_i^c \cdot \lambda_j^c \frac{a_c \mu_c e^{-\mu_c r_{ij}}}{4m_i^2 m_j^2 r_{ij}} [(m_i^2 + m_j^2)(1 - 2a_s) + 4m_i m_j(1 - a_s)(\vec{S}_+ \cdot \vec{L}) + ((m_j^2 - m_i^2) \times (1 - 2a_s))(\vec{S}_- \cdot \vec{L})], \quad (14)$$

where λ^c are SU(3) color Gell-Mann matrices, a_c and μ_c are model parameters, and $\vec{S}_{\pm} = \vec{S}_i \pm \vec{S}_j$. For the one-gluon exchange, it provides both intermediate and long-range attraction for the multi-quark systems here, and its form is given as

$$V_{\text{OGE}}^C = \frac{\alpha_s}{4} \lambda_i^c \cdot \lambda_j^c \left[\frac{1}{r_{ij}} - \frac{1}{6m_i m_j} \sigma_i \cdot \sigma_j \frac{e^{-r_{ij}/r_0(\mu_{ij})}}{r_{ij}^2(\mu_{ij})} \right], \quad (15)$$

$$V_{\text{OGE}}^{\text{SO}} = -\frac{1}{16} \frac{\alpha_s}{m_i^2 m_j^2} \lambda_i^c \cdot \lambda_j^c \left[\frac{1}{r_{ij}^3} - \frac{e^{-r_{ij}/r_g(\mu)}}{r_{ij}^3} \times \left(1 + \frac{r_{ij}}{r_g(\mu)} \right) \right] [((m_i + m_j)^2 + 2m_i m_j)(\vec{L} \cdot \vec{S}) + (m_j^2 - m_i^2)(\vec{L} \cdot \vec{S}_-)], \quad (16)$$

where α_s is the QCD-inspired strong coupling constant, which is determined by fitting experimental meson data. All model parameters are presented in Table I, and the masses of the K -meson family are shown in Table II. Since the K meson does not possess charge conjugation (C) symmetry, the $K_1(1271)$ and $K_1(1400)$ mesons, which share the same quantum numbers $J^P = 1^+$, cannot be distinguished directly in experiments. As a result, a coupled-channel calculation is

TABLE I. Quark model parameters ($m_{\pi} = 0.7 \text{ fm}^{-1}$, $m_{\sigma} = 3.42 \text{ fm}^{-1}$, $m_{\eta} = 2.77 \text{ fm}^{-1}$, $m_K = 2.51 \text{ fm}^{-1}$, $m_{a_0} = 4.96 \text{ fm}^{-1}$, $m_{f_0} = 2.51 \text{ fm}^{-1}$, and $m_{\kappa} = 4.1 \text{ fm}^{-1}$). There are two sets of chiral quark model parameters, the different parts of which are separated by “/”.

Quark masses	$m_u = m_d$ (MeV) m_s (MeV)	370/310 560/500
Goldstone bosons	$\Lambda_{\pi} = \Lambda_{\sigma} = \Lambda_{a_0}$ (fm $^{-1}$) $\Lambda_{\eta} = \Lambda_K = \Lambda_{f_0} = \Lambda_{\kappa}$ (fm $^{-1}$) $g_{ch}^2/(4\pi)$ θ_p ($^{\circ}$)	4.2 5.2 0.54 -15
Confinement	a_c (MeV) a_s (MeV) μ_c (fm $^{-1}$) Δ (MeV)	430 0.777 0.7 181.1/170.5
OGE	α_{qq} α_{qs} α_{ss} $\hat{r}_0$ (MeV) $\hat{r}_g$ (MeV)	0.56/0.66 0.49/0.48 0.41/0.38 28.17 34.5

TABLE II. Mass of light mesons. QM.1 and QM.2 represent the calculation results under two different sets of parameters, while Mixed.1 and Mixed.2 represent the mass of the states after a coupled-channel calculation between $K_1(1271)$ and $K_1(1400)$ under the two different sets of parameters.

J^P	Spin	State	QM.1	mixed.1	QM.2	mixed.2
0^-	0	$K(495)$	510		518	
1^-	1	$K^*(892)$	943		948	
0^+	1	$K_0(800)$	1270		1313	
1^+	0	$K_1(1400)$	1410	1483	1423	1522
1^+	1	$K_1(1271)$	1406	1328	1414	1301
2^+	1	$K_2(1430)$	1447		1444	
0^-	0	$\eta(547)$	583		537	
1^-	1	$\omega(782)$	663		649	
1^+	1	$f_1(1285)$	1302		1303	
1^+	0	$h_1(1170)$	1277		1267	
0^-	0	$\eta'(958)$	817		847	
1^-	1	$\phi(1020)$	998		998	
1^+	1	$f_1(1420)$	1519		1520	
1^+	0	$h_1(1380)$	1514		1509	

necessary to account for their mixing. The results of this mixing are presented in the fifth and seventh columns in Table II, corresponding to the first and second parameter sets, respectively. In the quark model employed here, both $K_1(1271)$ and $K_1(1400)$ are composed of $L = 1$ states with $q\bar{s}$, but they differ in spin: $K_1(1271)$ has spin 1, while $K_1(1400)$ has spin 0. The coupling between these channels arises primarily from the spin-orbit interaction in the one-gluon-exchange potential, as well as the scalar-meson-exchange potential. These interactions lead to the mixing of the $K_1(1271)$ and $K_1(1400)$ states. For convenience, we denote the $K(495)$ as K , the $K^*(892)$ as K^* , the $K_0(800)$ as K_0^* , the $K_1(1271)$ as K_1' , the $K_1(1400)$ as K_1^* , and the $K_2(1430)$ as K_2^* , $\eta(547)$ as η , $\omega(782)$ as ω , $f_1(1285)$ as f_1 , $h_1(1170)$ as h_1 , $\eta'(958)$ as η' , $\phi(1020)$ as ϕ , $f_1(1420)$ as f_1' , and $h_1(1170)$ as h_1' . Therefore, in our study, the K_1 is treated as a double-pole structure, which includes both K_1' and K_1^* .

B. The wave function of $K\bar{K}_1$ and KK_1 systems

In the quark model, the total tetraquark wave function $\Psi(r_{ij})^{i,j,k,l}$ is constructed by the direct product of the spatial wave function ϕ_L (where L denotes the orbital quantum number), the spin wave function σ_S (where S denotes the spin quantum number), the flavor wave function ζ_I (where I denotes the isospin quantum number), and the color wave function χ_c (where c denotes the color singlet state). Each index i, j, k , and l represents the i th spatial wave function, the j th spin wave function, the k th flavor wave function, and the l th color wave function, respectively. The total tetraquark wave function is then multiplied by the antisymmetrization operator $\mathcal{A}$ to ensure proper symmetry as

$$\Psi^{i,j,k,l} = \mathcal{A}[\phi_L^i \otimes \sigma_s^j \otimes \zeta_I^k \otimes \chi_c^l]. \quad (17)$$

Since the quark components of the $K\bar{K}_1$ system and KK_1 system are different, the antisymmetrization operators for their wave functions are also different. For the $K\bar{K}_1$ system, the antisymmetrization operator is simply $\mathcal{A} = 1$, while for the KK_1 system, it is given by

$$\mathcal{A} = 1 - (13) - (24) + (13)(24),$$

where (13) and (24) denote the exchanges of quarks 1 and 3 and quarks 2 and 4, respectively.

In addition to the antisymmetrization operator, the main distinction between the $K\bar{K}_1$ system and KK_1 system lies in their flavor wave functions ζ_I^k . The quark components of the $K\bar{K}_1$ system are $q\bar{s}s\bar{q}$, and it possesses three possible structures: the molecular structures $q\bar{s}-s\bar{q}$ and $q\bar{q}-s\bar{s}$ as well as the diquark structure $qs-\bar{s}\bar{q}$. To match the isospin $I = 0$ of the $\eta_1(1855)$ state, the three flavor wave functions are chosen as

$$\zeta_0^1 = \frac{1}{\sqrt{2}}(u\bar{s}s\bar{u} + d\bar{s}s\bar{d}), \quad (18)$$

$$\zeta_0^2 = \frac{1}{\sqrt{2}}(u\bar{u}s\bar{s} + d\bar{d}s\bar{s}), \quad (19)$$

$$\zeta_0^3 = \frac{1}{\sqrt{2}}(us\bar{s}\bar{u} + ds\bar{s}\bar{d}). \quad (20)$$

Similarly, the quark components of the KK_1 system are $q\bar{s}q\bar{s}$, and it also has two structures: the molecular structure $q\bar{s}-q\bar{s}$ and the diquark structure $qq - \bar{s}\bar{s}$. The corresponding flavor wave functions are as follows:

$$\zeta_0^4 = \frac{1}{\sqrt{2}}(u\bar{s}d\bar{s} - d\bar{s}u\bar{s}), \quad (21)$$

$$\zeta_0^5 = \frac{1}{\sqrt{2}}(ud\bar{s}\bar{s} - du\bar{s}\bar{s}). \quad (22)$$

For the spins of the quark and antiquark, they are indistinguishable regardless of whether it is a diquark structure or a molecular structure; thus, the spin wave functions of the $K\bar{K}_1$ system and the KK_1 system are the same. For the subclusters, the spin wave functions are shown below as

$$\begin{aligned} \chi_{11}^\sigma &= \alpha\alpha, & \chi_{10}^\sigma &= \frac{1}{\sqrt{2}}(\alpha\beta + \beta\alpha), \\ \chi_{1-1}^\sigma &= \beta\beta, & \chi_{00}^\sigma &= \frac{1}{\sqrt{2}}(\alpha\beta - \beta\alpha), \end{aligned}$$

where the first subscript of χ represents the spin quantum number of the subcluster and the second subscript represents the third component of the spin quantum number. α and β represent the third component of quark spin, taking

values of $\frac{1}{2}$ and $-\frac{1}{2}$, respectively. By coupling the spin wave functions of the two subclusters with Clebsch-Gordan coefficients, the total spin wave function can be written as

$$\begin{aligned}\sigma_0^1 &= \chi_{00}^\sigma \chi_{00}^\sigma, & \sigma_0^2 &= \sqrt{\frac{1}{3}}(\chi_{11}^\sigma \chi_{-1-1}^\sigma - \chi_{10}^\sigma \chi_{10}^\sigma + \chi_{1-1}^\sigma \chi_{11}^\sigma), \\ \sigma_1^3 &= \chi_{00}^\sigma \chi_{11}^\sigma, & \sigma_1^4 &= \chi_{11}^\sigma \chi_{00}^\sigma, & \sigma_1^5 &= \frac{1}{\sqrt{2}}(\chi_{11}^\sigma \chi_{10}^\sigma - \chi_{10}^\sigma \chi_{11}^\sigma), \\ \sigma_2^6 &= \chi_{11}^\sigma \chi_{11}^\sigma.\end{aligned}$$

For the color wave functions of $K\bar{K}_1$ system and KK_1 system, they need to satisfy the color-neutral principle. Therefore, for their molecular state structures, there are two possible configurations, i.e., $1 \otimes 1 \rightarrow 1$ and $8 \otimes 8 \rightarrow 1$. For the diquark-antidiquark structure, the two possible configurations are $\bar{3} \otimes 3 \rightarrow 1$ and $6 \otimes \bar{6} \rightarrow 1$:

$$\begin{aligned}\chi_c^1 &= \sqrt{\frac{1}{9}}(\bar{r}r\bar{r}r + \bar{r}r\bar{g}g + \bar{r}r\bar{b}b + \bar{g}g\bar{r}r + \bar{g}g\bar{g}g \\ &\quad + \bar{g}g\bar{b}b + \bar{b}b\bar{r}r + \bar{b}b\bar{g}g + \bar{b}b\bar{b}b), \\ \chi_c^2 &= \sqrt{\frac{1}{72}}(3\bar{b}r\bar{r}b + 3\bar{g}r\bar{r}g + 3\bar{b}g\bar{g}b + 3\bar{g}b\bar{b}g + 3\bar{r}g\bar{g}r \\ &\quad + 3\bar{r}b\bar{b}r + 2\bar{r}r\bar{r}r + 2\bar{g}g\bar{g}g + 2\bar{b}b\bar{b}b - \bar{r}r\bar{g}g \\ &\quad - \bar{g}g\bar{r}r - \bar{b}b\bar{g}g - \bar{b}b\bar{r}r - \bar{g}g\bar{b}b - \bar{r}r\bar{b}b), \\ \chi_c^3 &= \sqrt{\frac{1}{12}}(r\bar{g}\bar{r}\bar{g} - r\bar{g}\bar{g}\bar{r} + g\bar{r}\bar{g}\bar{r} - g\bar{r}\bar{r}\bar{g} + r\bar{b}\bar{r}\bar{b} \\ &\quad - r\bar{b}\bar{b}\bar{r} + b\bar{r}\bar{b}\bar{r} - b\bar{r}\bar{r}\bar{b} + g\bar{b}\bar{g}\bar{b} - g\bar{b}\bar{b}\bar{g} \\ &\quad + b\bar{g}\bar{b}\bar{g} - b\bar{g}\bar{g}\bar{b}), \\ \chi_c^4 &= \sqrt{\frac{1}{24}}(2\bar{r}r\bar{r}\bar{r} + 2\bar{g}g\bar{g}\bar{g} + 2\bar{b}b\bar{b}\bar{b} + r\bar{g}\bar{r}\bar{g} + r\bar{g}\bar{g}\bar{r} \\ &\quad + g\bar{r}\bar{g}\bar{r} + g\bar{r}\bar{r}\bar{g} + r\bar{b}\bar{r}\bar{b} + r\bar{b}\bar{b}\bar{r} + b\bar{r}\bar{b}\bar{r} \\ &\quad + b\bar{r}\bar{r}\bar{b} + g\bar{b}\bar{g}\bar{b} + g\bar{b}\bar{b}\bar{g} + b\bar{g}\bar{b}\bar{g} + b\bar{g}\bar{g}\bar{b}).\end{aligned}$$

Finally, for the spatial wave function of $K\bar{K}_1$ system and KK_1 system, it consists of the motion of two subclusters and the relative motion between them. Since we consider two spatial structures, i.e., the diquark structure and the molecular structure, the difference in their spatial wave functions lies in the encoding of the particles. For example, in the molecular structure, the encoding of the four-quark spatial wave function is $q_1\bar{s}_2\bar{s}_3\bar{q}_4$, while in the corresponding diquark structure, the encoding of the four-quark spatial wave function is $q_1s_3\bar{s}_2\bar{q}_4$. Here, we set the orbital angular momentum of the second subcluster to 1 and that of the first subcluster to 0. First, we couple them to obtain the orbital quantum number $l_{12} = 1$; then, we couple this spatial wave function with the relative motion wave function to obtain the total orbital wave function $\phi_{L=1}^i$. The expression is as

$$\begin{aligned}\phi_L^1 &= [[\psi_{l_1=0}(\mathbf{r}_{12})\psi_{l_2=1}(\mathbf{r}_{34})]_{l_{12}=1}\psi_{L_r}(\mathbf{r}_{1234})]_{L=1}, \\ \phi_L^2 &= [[\psi_{l_1=0}(\mathbf{r}_{14})\psi_{l_2=1}(\mathbf{r}_{23})]_{l_{12}=1}\psi_{L_r}(\mathbf{r}_{1234})]_{L=1}, \\ \phi_L^3 &= [[\psi_{l_1=1}(\mathbf{r}_{14})\psi_{l_2=0}(\mathbf{r}_{23})]_{l_{12}=1}\psi_{L_r}(\mathbf{r}_{1234})]_{L=1}, \\ \phi_L^4 &= [[\psi_{l_1=0}(\mathbf{r}_{13})\psi_{l_2=1}(\mathbf{r}_{24})]_{l_{12}=1}\psi_{L_r}(\mathbf{r}_{1324})]_{L=1}, \\ \phi_L^5 &= [[\psi_{l_1=1}(\mathbf{r}_{13})\psi_{l_2=0}(\mathbf{r}_{24})]_{l_{12}=1}\psi_{L_r}(\mathbf{r}_{1324})]_{L=1}.\end{aligned}\quad (23)$$

For each relative motion wave function, we use the GEM for the description, and its form is

$$\psi(\mathbf{r}) = \sum_{n=1}^{n_{\max}} c_n N_{nl} r^l e^{-\nu_n r^2} Y_{lm}(\hat{\mathbf{r}}), \quad (24)$$

where N_{nl} are normalization constants:

$$N_{nl} = \left[\frac{2^{l+2} (2\nu_n)^{l+\frac{3}{2}}}{\sqrt{\pi} (2l+1)} \right]^{\frac{1}{2}}. \quad (25)$$

The coefficients c_n are variational parameters, which are determined dynamically. The Gaussian size parameters are chosen according to the geometric progression

$$\nu_n = \frac{1}{r_n^2}, \quad r_n = r_1 a^{n-1}, \quad a = \left(\frac{r_{n_{\max}}}{r_1} \right)^{\frac{1}{n_{\max}-1}}. \quad (26)$$

Finally, by applying the Rayleigh-Ritz variational principle to solve the Schrödinger equation, we obtain the eigenvalues and eigenfunctions:

$$\left((H) - E_n(N) \right) (C_n) = 0, \quad (27)$$

where the Hamiltonian and overlap matrix elements are defined as

$$\langle H \rangle_{ij} = \langle \psi_i | H | \psi_j \rangle, \quad (28)$$

$$\langle N \rangle_{ij} = \langle \psi_i | \psi_j \rangle, \quad (29)$$

respectively. The total wave function Ψ incorporates all individual channel wave functions ψ_i , where the subscript i indicates the tetraquark channel index. Explicitly, the matrices (H) and (N) can be written as

$$(H) = \begin{pmatrix} \langle H \rangle_{11} & \langle H \rangle_{12} & \cdots & \langle H \rangle_{1n} \\ \langle H \rangle_{21} & \langle H \rangle_{22} & \cdots & \langle H \rangle_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \langle H \rangle_{n1} & \langle H \rangle_{n2} & \cdots & \langle H \rangle_{nn} \end{pmatrix}, \quad (30)$$

$$(N) = \begin{pmatrix} \langle N \rangle_{11} & \langle N \rangle_{12} & \cdots & \langle N \rangle_{1n} \\ \langle N \rangle_{21} & \langle N \rangle_{22} & \cdots & \langle N \rangle_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \langle N \rangle_{n1} & \langle N \rangle_{n2} & \cdots & \langle N \rangle_{nn} \end{pmatrix}. \quad (31)$$

C. The root-mean-square distances calculation

In order to study the internal structure of the $KK_1(K\bar{K}_1)$ system, we estimate the distance between quarks using the root-mean-square (rms) distance. It is generally assumed that when the distance between the first cluster of quarks ($q_1\bar{q}_2$) and the second cluster of quarks ($q_3\bar{q}_4$) is less than 1 fm, and the four-quark system is considered a compact four-quark structure. On the other hand, when this distance is significantly greater than 1 fm, the four-quark structure is regarded as a molecular configuration. However, in the presence of identical particles, the conventional rms distance calculated using the full wave function can become ambiguous [31]. The formula for the rms distance is given by

$$r_{q_i q_j} = \sqrt{\frac{\langle \Psi' | r_{ij}^2 | \Psi' \rangle}{\langle \Psi' | \Psi' \rangle}}, \quad (32)$$

where the indices i and j represent the quarks and Ψ' is the wave function without the antisymmetrization operator A . The quantities $r_{q_1\bar{q}_2}$ and $r_{q_3\bar{q}_4}$ refer to the rms distances between the first and second clusters of mesons, while $r_{q_1 q_3}$, $r_{\bar{q}_2\bar{q}_4}$, $r_{q_1\bar{q}_4}$, and $r_{\bar{q}_2 q_3}$ reflect the intermeson distances.

III. RESULT

In this work, we primarily perform the bound state calculation for the four-quark systems of $K\bar{K}_1(q\bar{s}-s\bar{q})$ and $KK_1(q\bar{s}-q\bar{s})$. We will then analyze the contributions from each part of the potential energy for these bound states, as well as the quark distance. Finally, we compare the differences between these two systems.

A. Bound-state calculation of $K\bar{K}_1$ system

In the $K\bar{K}_1$ tetraquark system, considering that $J^P = 1^-$ and the intercluster orbital wave function is in the S wave, there are three possible coupling scenarios for the two subclusters (J_1 and J_2): $J = J_1 \otimes J_2 = 0 \otimes 1, 1 \otimes 0$, or $1 \otimes 1$. Furthermore, for the $K\bar{K}_1$ tetraquark system, it possesses two possible molecular configurations: $q\bar{s}-s\bar{q}$ and $q\bar{q}-s\bar{s}$. Therefore, the possible molecular states include $K\bar{K}_1^*$, $K^*\bar{K}_0^*$, $K^*\bar{K}_1'$, $K\bar{K}_1'$, $K^*\bar{K}_1'$, $\eta'f_1$, $\eta'h_1$, ϕf_1 , ϕh_1 , $f_1'\eta$, $h_1'\eta$, $f_1'\omega$, and $h_1'\omega$, as well as their corresponding color-octet states. For the $K\bar{K}_1$ tetraquark system, it possesses only one diquark structure (where we use $^{\text{spin}}[qq]_{\text{color}}^{\text{L-spin}}[\bar{q}\bar{q}]_{\text{color}}^{\text{L}}$ to represent the diquark structures) and a total of eight diquark physical states, which are all

TABLE III. Results of the bound-state calculations in the $K\bar{K}_1$ system, where E_1 and E_2 represent the calculation results under two different sets of parameters (unit, MeV).

Channel	$\Psi^{i,j,k,l}$	E_1	E_2	Percent
$K\bar{K}_1^*$	$\Psi^{1,3,1,1}$	1918	1934	53.4%
$[K\bar{K}_1^*]_8$	$\Psi^{1,3,1,2}$	2350	2357	0.0%
$K\bar{K}_1'$	$\Psi^{1,1,1,1}$	1921	1943	48.6%
$[K\bar{K}_1']_8$	$\Psi^{1,1,1,2}$	2364	2340	0.0%
$K^*\bar{K}_0^*$	$\Psi^{1,5,1,1}$	2230	2265	0.0%
$[K^*\bar{K}_0^*]_8$	$\Psi^{1,5,1,2}$	2312	2342	0.0%
$K^*\bar{K}_1'$	$\Psi^{1,5,1,1}$	2351	2365	0.0%
$[K^*\bar{K}_1']_8$	$\Psi^{1,5,1,2}$	2312	2343	0.0%
$K^*\bar{K}_1'$	$\Psi^{1,4,1,1}$	2355	2374	0.0%
$[K^*\bar{K}_1']_8$	$\Psi^{1,4,1,2}$	2350	2298	0.0%
$\eta'f_1$	$\Psi^{2,3,2,1}$	2122	2154	0.0%
$[\eta'f_1]_8$	$\Psi^{2,3,2,2}$	2448	2469	0.0%
$\eta'h_1$	$\Psi^{2,1,2,1}$	2097	2118	0.0%
$[\eta'h_1]_8$	$\Psi^{2,1,2,2}$	2438	2456	0.0%
ϕf_1	$\Psi^{2,5,2,1}$	2303	2304	0.0%
$[\phi f_1]_8$	$\Psi^{2,5,2,2}$	2405	2432	0.0%
ϕh_1	$\Psi^{2,4,2,1}$	2278	2269	0.0%
$[\phi h_1]_8$	$\Psi^{2,4,2,2}$	2428	2458	0.0%
$f_1'\eta$	$\Psi^{3,3,2,1}$	2105	2062	0.0%
$[f_1'\eta]_8$	$\Psi^{3,3,2,2}$	2422	2444	0.0%
$h_1'\eta$	$\Psi^{3,1,2,1}$	2101	2050	0.0%
$[h_1'\eta]_8$	$\Psi^{3,1,2,2}$	2402	2416	0.0%
$f_1'\omega$	$\Psi^{3,5,2,1}$	2185	2173	0.0%
$[f_1'\omega]_8$	$\Psi^{3,5,2,2}$	2272	2285	0.0%
$h_1'\omega$	$\Psi^{3,4,2,1}$	2180	2162	0.0%
$[h_1'\omega]_8$	$\Psi^{3,4,2,2}$	2271	2416	0.0%
$0[qs]_3^0-0[\bar{s}\bar{q}]_3^1$	$\Psi^{4,1,3,3}$	2382	2387	0.0%
$0[qs]_6^0-0[\bar{s}\bar{q}]_6^1$	$\Psi^{4,1,3,4}$	2480	2488	0.0%
$0[qs]_3^0-1[\bar{s}\bar{q}]_3^1$	$\Psi^{4,3,3,3}$	2436	2450	0.0%
$0[qs]_6^0-1[\bar{s}\bar{q}]_6^1$	$\Psi^{4,3,3,4}$	2470	2484	0.0%
$1[qs]_3^0-0[\bar{s}\bar{q}]_3^1$	$\Psi^{4,4,3,3}$	2409	2416	0.0%
$1[qs]_6^0-0[\bar{s}\bar{q}]_6^1$	$\Psi^{4,4,3,4}$	2441	2443	0.0%
$1[qs]_3^0-1[\bar{s}\bar{q}]_3^1$	$\Psi^{4,5,3,3}$	2456	2469	0.0%
$1[qs]_6^0-1[\bar{s}\bar{q}]_6^1$	$\Psi^{4,5,3,4}$	2415	2424	0.0%
Complete coupled channels:		1839	1821	
Threshold ($K + \bar{K}_1^*$):		1838	1820	

listed in Table III. To minimize the impact of parameter uncertainties, we considered two sets of parameters, with their corresponding calculation results listed in the third and fourth columns in Table III. Our calculations show that the two lowest-energy channels are $K\bar{K}_1^*$ and $K\bar{K}_1'$, with energies slightly below 1.9 GeV. The energies of other color-singlet states are concentrated around 2.3 GeV and correspond to scattering states. Similarly, for the color structures, the energies of the color-octet and diquark structures are concentrated in the range of 2.2–2.4 GeV, which is significantly higher than the lowest threshold of the $K\bar{K}_1$ tetraquark system. Because of this, their channel

TABLE IV. Results of the bound-state calculations in the KK_1 system, where E_1 and E_2 represent the calculation results under two different sets of parameters (unit, MeV).

Channel	$\Psi^{i,j,k,l}$	E_1	E_2	Percent
KK_1^*	$\Psi^{1,3,4,1}$	1919	1935	52.0%
$[KK_1^*]_8$	$\Psi^{1,3,4,2}$	2374	2327	0.0%
KK_1'	$\Psi^{1,1,4,1}$	1923	1944	47.3%
$[KK_1']_8$	$\Psi^{1,1,4,2}$	2316	2374	0.0%
$K^*K_0^*$	$\Psi^{1,5,4,1}$	2216	2266	0.1%
$[K^*K_0^*]_8$	$\Psi^{1,5,4,2}$	2379	2375	0.1%
$K^*K_1^*$	$\Psi^{1,5,4,1}$	2352	2366	0.0%
$[K^*K_1^*]_8$	$\Psi^{1,5,4,2}$	2329	2387	0.0%
K^*K_1'	$\Psi^{1,4,4,1}$	2356	2374	0.0%
$[K^*K_1']_8$	$\Psi^{1,4,4,2}$	2380	2329	0.0%
$^0[qq]_3^0-^0[\bar{s}\bar{s}]_3^1$	$\Psi^{4,1,5,3}$	2437	2430	0.2%
$^1[qq]_6^0-^1[\bar{s}\bar{s}]_6^1$	$\Psi^{4,5,5,4}$	2464	2480	0.1%
$^1[qq]_3^1-^1[\bar{s}\bar{s}]_3^0$	$\Psi^{5,5,5,3}$	2490	2496	0.1%
$^0[qq]_6^1-^0[\bar{s}\bar{s}]_6^0$	$\Psi^{5,1,5,4}$	2492	2504	0.1%
Complete coupled channels:		1837	1818	
Threshold ($K + K_1^*$):		1838	1820	

coupling effects are very small, and the final complete coupled-channel calculation shows that the $K\bar{K}_1$ tetraquark system does not form a bound state. It is worth noting that the energy of the second molecular configuration, $q\bar{q}-s\bar{s}$, is generally higher, exceeding 2.1 GeV, causing the energy difference between states to reach 200 MeV. Therefore, even though we considered a sufficient number of physical states (34 channels), the channel coupling effects are still insufficient to form a bound state for the $K\bar{K}_1$ tetraquark system. Consequently, our current calculations do not support the $\eta_1(1855)$ as a $K\bar{K}_1$ molecular state, which is consistent with the conclusion in Ref. [26] but contradicts the conclusion in Ref. [24]. It may indicate that a higher partial wave, such as D-wave contribution, is needed, or it might behave similarly as the X(3872) in the unquenched quark model [2], which requires the mixing effect of a $s\bar{s}$ component.

B. Bound-state calculation of KK_1 system

In contrast to the $K\bar{K}_1$ tetraquark system, the KK_1 tetraquark system has only one molecular configuration, $q\bar{s}-q\bar{s}$, which includes five color-singlet channels: KK_1^* ,

TABLE VI. The root-mean-square distances (unit, fm) in KK_1 four-quark systems.

$r_{q_1\bar{s}_2}$	$r_{q_1q_3}$	$r_{q_1\bar{s}_4}$	$r_{\bar{s}_2q_3}$	$r_{\bar{s}_2\bar{s}_4}$	$r_{q_3\bar{s}_4}$
0.4	2.9	2.9	2.9	2.9	0.8

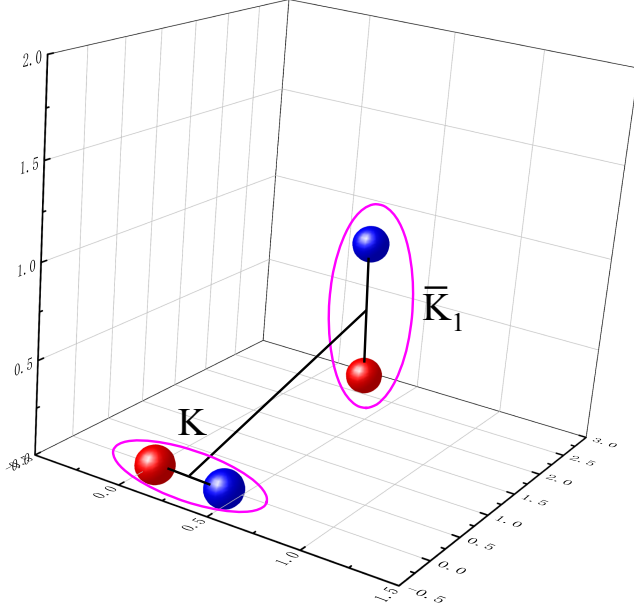
$K^*K_0^*$, $K^*K_1^*$, KK_1' , and K^*K_1' , as well as the corresponding five color-octet states. The difference from the $K\bar{K}_1$ tetraquark system is that, due to symmetry constraints, the KK_1 tetraquark system has only four diquark structures: $\Psi^{2,1,4,3}$, $\Psi^{2,5,4,4}$, $\Psi^{3,5,4,3}$, and $\Psi^{3,1,4,4}$, all of which are listed in Table IV. Our single-channel calculation results show that the lowest-energy channels are still KK_1^* and KK_1' , with the color-octet states concentrated between 2.2 and 2.3 GeV. In the diquark-antidiquark structures, the energy of $\Psi^{4,1,5,3}$ ($^0[qq]_3^0-^0[\bar{s}\bar{s}]_3^1$) is relatively low at 1.98 GeV, while the energies of the other states are between 2.2 and 2.3 GeV. This occurs because, in the diquark structure $\Psi^{4,1,5,3}$, the qq cluster of ($^0[qq]_3^0-^0[\bar{s}\bar{s}]_3^1$) forms a good-diquark structure, meaning the quantum numbers of qq are 00^+ and the color wave function is antisymmetric. Because of the presence of such a low-energy diquark channel close to the lowest threshold, the final complete coupled-channel calculation shows that we obtain a relatively shallow bound state with an energy of 1837 MeV (for the first set of parameters) and 1818 MeV (for the second set of parameters). The binding energy is approximately 1–2 MeV. Both sets of parameters show the existence of a bound state, which indicates that the KK_1 bound state is quite stable. The main components of this bound state are KK_1^* (about 52.0%), KK_1' (about 47.3%), and the other color components (including color-octet states and diquark structures) together account for less than 1%, which indicates that the resulting bound state is predominantly molecular in nature.

C. Comparison between the KK_1 system and the $K\bar{K}_1$ system

To compare the differences between the KK_1 system and the $K\bar{K}_1$ system, we first calculate the contribution of the potential energy for each part of the lowest energy of these two systems (by subtracting the potential energy contributions of the threshold state from the results of the lowest-energy state) and the rms distances, which are listed in

TABLE V. The contributions of all potentials to the binding energy (unit, MeV) in $K\bar{K}_1$ and KK_1 four-quark systems, where Q.M.1 and Q.M.2 represent the calculation results under two different sets of parameters.

	Kinetic	Con	OGE	π	K	η	σ	a_0	κ	f_0
$K\bar{K}_1^*$ (Q.M.1)	10.3	-0.6	-0.4	-0.1	0.0	-0.1	-4.2	-1.6	-0.1	-2.2
$K\bar{K}_1^*$ (Q.M.2)	13.0	-0.5	-0.8	-0.2	0.0	0.0	-4.7	-2.2	-0.2	-3.1
KK_1^* (Q.M.1)	14.8	-0.5	-12.3	-0.1	0.0	-0.9	-1.1	-0.1	0.0	-1.1
KK_1^* (Q.M.2)	16.9	-0.7	-14.3	-0.1	0.0	-1.1	-1.3	-0.1	0.0	-1.3

FIG. 1. The spatial configuration of KK_1 .

Tables V and VI. Table V rows 3 and 4 represent the contributions of the potential energy for the KK_1 system under two sets of parameters, while rows 1 and 2 represent the contributions for the $K\bar{K}_1$ system. Since KK_1 is a shallow bound state and $K\bar{K}_1$ is a scattering state, this is intuitively reflected in the difference in kinetic energy. The kinetic energy repulsion in the KK_1 system is slightly greater than that in the $K\bar{K}_1$ system, while in the long-range attraction (meson exchange potential), the KK_1 system is smaller than the $K\bar{K}_1$ system. In the KK_1 system, the contribution from the one-gluon-exchange potential (oge) is relatively significant, and in addition, the exchanges of the η , σ , and f_0 mesons are quite important. On the other hand, in terms of the rms distances, we find that the intercluster distance between K and K_1 in the KK_1 system is approximately 2.9 fm, which shows a clear loose molecular structure, as shown in Fig. 1. We can see that, since K_1 is an excited state, q_3 and $\bar{s}_4$ of K_1 are relatively farther apart.

IV. SUMMARY

Within the framework of the chiral quark model, we perform a systematical study on the KK_1 and $K\bar{K}_1$ systems. We consider not only the molecular structures, but also the diquark structures and their mixing effects.

Our results show that, under both sets of parameters, for the $K\bar{K}_1$ system, all the color-singlet channels are scattering states, and the energies of the colored structures, including the color octet and diquark structures, have an energy difference of 200 MeV above the threshold, which means their coupling channel effects are minimal. Therefore, despite the involvement of up to 34 physical channels in the coupled-channel calculation, no bound state was obtained. In contrast, due to symmetry constraints, the KK_1 system has only four diquark channels, and one of the diquark structures forms a good-diquark structure, which significantly lowers its energy compared to other colored structures. This good-diquark structure may mix with KK_1^* and KK_1' , which leads to the formation of a relatively shallow bound state. The rms distance calculations show that this bound state is a clear molecular structure. According to our calculations, for the formation of a bound state in the KK_1 system, the dominant contributions come from one-gluon exchange, assisted by meson exchanges including σ , η , and f_0 .

In summary, our current calculations do not support the interpretation of $\eta_1(1855)$ as a $K\bar{K}_1$ molecule, and it is perhaps similar to the $X(3872)$ in the unquenched quark model [2] that requires the mixing effect of a $c\bar{c}$ component. In the $K\bar{K}_1$ tetraquark system, similar mixing effects may be needed, such as the contribution from the D-wave component, or the mixing effect between $(q\bar{s})-(s\bar{q})$ and $s\bar{s}g$. However, our results strongly support the existence of a new bound state in the KK_1 system, and we suggest future experiments to search for it.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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