

Bounds on SMEFT affecting multigauge and Higgs-gauge couplings using two and three body spin correlations in $e^-e^+ \rightarrow 2je^-e^+\mu\bar{e}$ process

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(Received 20 September 2025; accepted 29 January 2026; published 23 February 2026)

The upcoming electron-positron collider provides an ideal place to probe deviation from the Standard Model predictions with its clean environment, beam polarization, and significant luminosity. We studied anomalous quartic gauge boson couplings (VVW^-W^+ , $V \in \{\gamma, Z\}$), triple gauge couplings ($W^-W^+\gamma/Z$), and Higgs-gauge couplings (HVV , $V \in \{W^\pm, Z, \gamma\}$) induced by $SU(2)_L \times U(1)_Y$ gauge-invariant dimension-6 operators in $2je^-e^+\mu\bar{e}$ final events with initial beam polarization. The phase space of two prominent amplitudes—i.e., triple gauge boson production (WWZ) and vector boson scattering subprocesses—are selected with kinematic cuts on $m_{e^-e^+}$. The full quantum states of the diboson system in vector boson scattering (VBS) and the triboson system in WWZ are given by 80 and 728 independent parameters (polarizations and spin correlations), respectively, which are obtained in the form of angular asymmetries. Further, the reconstruction of vector polarization asymmetries associated with the W boson and their correlations requires flavor tagging of light jets and is thus neglected. We provide one-parameter limits at a 95% confidence level, combining cross section and spin-related observables. The projected one-parameter constraints on Wilson coefficients (WCs) are found to be more stringent than the current limits reported by the LHC experiments. A channel study highlights that WWZ puts stringent bounds on two CP -even WCs, viz. c_W and c_{WWW} , while VBS topology puts stringent bounds on the remaining seven WCs. Finally, marginalized limits on all nine anomalous couplings are obtained with Markov chain Monte Carlo analysis. The limits are found to be insensitive to systematic errors and dominated by statistics.

DOI: [10.1103/ntgz-czmt](https://doi.org/10.1103/ntgz-czmt)

I. INTRODUCTION

Triple and quartic gauge boson couplings are a direct consequence of the non-Abelian structure of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge structure of the Standard Model (SM) of particle physics. The precise measurement of these couplings would test the validity of the SM to a high accuracy. The structure of trilinear gauge couplings (TGCs) are probed via the VV , $V \in \{W^\pm, \gamma, Z\}$ diboson production process at various experiments like L3 [1–5], ALEPH [6–8], DELPHI [9–12], OPAL [13–16], D0 [17–19], CDF [20–22], ATLAS [23–31], and CMS [32–36]. The quartic gauge boson couplings (QGCs) are probed experimentally via vector boson scattering (VBS) in ATLAS [26,27,30,37–39] and CMS [40–44]. At the

leading order in SM, the QGC could also be probed through the production of three massive gauge bosons, ($V_1V_2V_3$, $V_i \in \{W, Z\}$). The CMS Collaboration [45] at the LHC has observed such processes with 137 fb^{-1} datasets. These observations were made in final states involving three, four, five, and six leptons, as well as in channels with two same-sign leptons accompanied by one or two jets. Similar searches for triboson production have been conducted by other experiments. For instance, the ATLAS Collaboration analyzed final states containing WWW , WWZ , and WZZ events [46], while dedicated searches for WWW production are reported in Refs. [47,48]. The experimental results from these studies show a remarkable agreement with the predictions of the SM.

One of the fundamental questions related to the particle is its mass, and the standard consensus is that the masses of all particles are generated via electroweak symmetry breaking or the Higgs mechanism [49–51]. The mechanism suggests the presence of a Higgs field with a nonzero vacuum expectation value (VEV), v , which is responsible for symmetry breaking. The search for the quantum of Higgs field known as the Higgs boson was finally settled with the

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discovery of a scalar particle with mass around 125 GeV at the LHC [52,53]. The properties of this scalar particle (the Higgs boson) have been studied in great detail in various experiments [54–68], and the results are compatible with the prediction of the SM Higgs boson.

With the discovery of the Higgs boson, SM has been the best experimentally validated theory at the fundamental scale, yet there are many experimental and theoretical potholes which cannot be filled within the framework of the SM. The mass of the Higgs boson in the presence of higher quantum corrections being unbounded from above, the enormous gap between the electroweak and the Planck scale, the mass of neutrinos, and the structure of dark matter are a few issues which the SM completely fails to address. There is a large amount of literature which tries to explain the shortcomings of the SM by including new symmetry, particles, and dimensions. The only shortcomings to the majority of these models is the null result from the experiments to date. The absence of any clear evidence of new particles at energies up to several times the Higgs boson mass allows one to parametrize the effects of arbitrary new physics residing at energies $E \gg v$ on physical observables at the electroweak scale in terms of higher-dimension operators. This formalism of expanding SM in a model-independent way is commonly referred to as effective field theory (EFT). In this formalism, SM is expanded by adding a set of higher-dimensional operators to its usual dimension-4 Lagrangian. The framework is constructed with a understanding that if the new physics is heavy, it can be integrated out while keeping its effects in the Wilson coefficients (WCs). The higher-dimensional operators are suppressed by some characteristic energy scale $\Lambda > \sqrt{s}_{\text{LHC}}$. Working with the consideration of linear symmetry breaking (where the Higgs boson is a part of the scalar doublet), such EFT is commonly called SMEFT. Within the SMEFT framework, the effective Lagrangian would be [69]

$$\mathcal{L}_{\text{Eff}} = \mathcal{L}_{\text{SM}} + \sum_{d>4} \sum_i \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^{(d)}, \quad (1)$$

where $c_i^{(d)}$ are the WCs of d -dimensional operators $\mathcal{O}_i^{(d)}$. One can expect Λ to be much larger than the electroweak scale; thus, the higher-order terms of Eq. (1) are largely suppressed, allowing one to truncate the expansion at some order of choice. However, this naive truncation based on dimensionality should be done with the utmost care. In some of the diboson processes, the interference between the SM and dimension-6 operators remains substantially suppressed [70]. However, it has been shown that this interference can be resurrected in the final state with multiparticles or with correlations which we have considered in our current work.

We consider a set of dimension-6 operators that induce changes in triple gauge boson couplings ($W^-W^+\gamma/Z$),

Higgs couplings with gauge bosons ($HVV, V \in \{W^\pm, Z, \gamma\}$), and quartic gauge boson couplings ($VV'W^-W^+, V, V' \in \{\gamma, Z\}$). We investigate these operators in final states with an electron-positron pair, a muon, two jets, and missing energy at a e^-e^+ collider with a center-of-mass energy of 1 TeV, using initially polarized beams.

The process includes a set of amplitudes that allow simultaneous study of the various anomalous couplings described above within a gauge-invariant framework. The desired final states can result from vector boson scattering and triple gauge boson production amplitudes. We select the phase space representing these two dominant amplitudes using a single kinematic variable, viz. the invariant mass of the electron-positron system.

To analyze the polarizations and spin correlations of the weak gauge bosons, we use the angular distribution of their decay products. These parameters are obtained in the form of asymmetries in the angular function of the joint distribution of final fermions. Further, these spin asymmetries have different behavior under parity transformation and the reconstruction of vector polarization, and its correlation asymmetries require the exact identification of final-state objects. This implies that the flavor identity of two jets decayed from the W boson should be known, or such asymmetries average out. One can use multivariate analysis to achieve the flavor tagging [71,72], but in the real collider setup, the reconstruction of the flavor of light jets becomes nontrivial. Hence, in the current work, we neglect the three-vector polarization of the hadronically decayed W boson. For leptonic decays, we assume ideal identification of the final-state leptons. These spin-related observables, along with the cross section, are used to constrain the Wilson coefficients of higher-dimensional operators. We analyze the limits under different luminosities and systematic errors using Markov chain Monte Carlo (MCMC) techniques.

The rest of the paper is structured as follows: In Sec. II, we describe the anomalous $W^-W^+\gamma/Z$, $HVV, V \in \{W^\pm, Z, \gamma\}$, $W^-W^+\gamma Z$, and W^-W^+ZZ couplings in terms of nine independent dimension-6 operators. In Sec. III, we discuss the simulation setup and separation of VBS and WWZ -like phase space using collider variables. In Sec. IV, we describe the construction of spin observables from joint angular distribution. In Sec. V, we describe the probe of anomalous couplings and the limits obtained. We finally conclude in Sec. VI.

II. OPERATORS IN THE ELECTROWEAK SECTOR

Since the discovery of the Higgs boson at the LHC, the electroweak symmetry breaking is considered to be broken linearly, and within this premise, there are a number of higher-dimensional operators that could induce nonstandard effects to TGC, QGC, and HVV couplings. In this

article, we consider the lowest-order ($d = 6$) operators¹ assuming lepton and baryon number conservation. The dimension-6 operators in the bosonic sector affecting TGC, QGC and HVV in the Hagiwara-Ishihara-Szalapski-Zeppenfeld (HISZ) basis are [74–76]

$$\begin{aligned}
\mathcal{O}_{WWW} &= \text{Tr}[W_{\mu\nu} W_{\rho}^{\mu} W^{\nu\rho}], \\
\mathcal{O}_W &= (D_{\mu}\Phi)^{\dagger} W^{\mu\nu} (D_{\nu}\Phi), \\
\mathcal{O}_B &= (D_{\mu}\Phi)^{\dagger} B^{\mu\nu} (D_{\nu}\Phi), \\
\mathcal{O}_{WW} &= \Phi^{\dagger} W_{\mu\nu} W^{\mu\nu} \Phi, \\
\mathcal{O}_{BB} &= \Phi^{\dagger} B_{\mu\nu} B^{\mu\nu} \Phi, \\
\mathcal{O}_{BW} &= \Phi^{\dagger} B_{\mu\nu} W^{\mu\nu} \Phi, \\
\mathcal{O}_{\tilde{W}WW} &= \text{Tr}[\tilde{W}_{\mu\nu} W_{\rho}^{\mu} W^{\nu\rho}], \\
\mathcal{O}_{\tilde{W}} &= (D_{\mu}\Phi)^{\dagger} \tilde{W}^{\mu\nu} (D_{\nu}\Phi), \\
\mathcal{O}_{\tilde{B}} &= (D_{\mu}\Phi)^{\dagger} \tilde{B}^{\mu\nu} (D_{\nu}\Phi), \\
\mathcal{O}_{\tilde{W}W} &= \Phi^{\dagger} \tilde{W}_{\mu\nu} W^{\mu\nu} \Phi, \\
\mathcal{O}_{\tilde{B}B} &= \Phi^{\dagger} \tilde{B}_{\mu\nu} B^{\mu\nu} \Phi, \\
\mathcal{O}_{B\tilde{W}} &= \Phi^{\dagger} B_{\mu\nu} \tilde{W}^{\mu\nu} \Phi.
\end{aligned} \tag{2}$$

Here, the covariant derivative and the field tensors are given as

$$\begin{aligned}
D_{\mu} &= \partial_{\mu} + i \frac{g_W}{2} \sigma^i W_{\mu}^i + i \frac{g_1}{2} B_{\mu}, \\
W_{\mu\nu} &= \frac{\sigma^I}{2} W_{\mu\nu}^I, \\
W_{\mu\nu}^I &= ig_W (\partial_{\mu} W_{\nu}^I - \partial_{\nu} W_{\mu}^I + g_W \epsilon^{IJK} W_{\mu}^J W_{\nu}^K), \\
B_{\mu\nu} &= i \frac{g_1}{2} (\partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu}),
\end{aligned} \tag{3}$$

where g_1 and g_W are the gauge couplings, and σ are the Pauli matrices. The dual of the field tensor is defined as $\tilde{V}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} V^{\rho\sigma}$, where the Levi-Civita tensor follows the standard convention, $\epsilon_{0123} = 1$. In Eq. (2), the 12 operators can be classified in terms of its CP structure as

$$\begin{aligned}
\mathcal{O}_i^E &= \{\mathcal{O}_B, \mathcal{O}_W, \mathcal{O}_{WWW}, \mathcal{O}_{WW}, \mathcal{O}_{BB}, \mathcal{O}_{BW}\}, \\
\mathcal{O}_i^O &= \{\mathcal{O}_{\tilde{W}WW}, \mathcal{O}_{\tilde{W}}, \mathcal{O}_{\tilde{W}W}, \mathcal{O}_{\tilde{B}B}, \mathcal{O}_{B\tilde{W}}, \mathcal{O}_{\tilde{B}}\},
\end{aligned} \tag{4}$$

where $\mathcal{O}_i^E$ are the CP -even operators and $\mathcal{O}_i^O$ denotes the CP -odd operators. Out of the six CP -odd operators, only

¹There is one more operator in the bosonic sector, $\mathcal{O}_{DW} = \text{Tr}([D_{\mu}, W_{\nu\rho}][D^{\mu}, W^{\nu\rho}])$, but it is usually traded using the equation of motion for $\mathcal{O}_{WWW}$ and the fermionic operators [73], which we do not consider in the current work. Thus, it is neglected here.

four of them are linearly independent [75]. For example, $\mathcal{O}_{\tilde{B}}$, and $\mathcal{O}_{B\tilde{W}}$ can be written as

$$\begin{aligned}
\mathcal{O}_{\tilde{B}} &= \mathcal{O}_{\tilde{W}} + \frac{1}{2} \mathcal{O}_{\tilde{W}W} - \frac{1}{2} \mathcal{O}_{\tilde{B}B}, \\
\mathcal{O}_{B\tilde{W}} &= -2\mathcal{O}_{\tilde{W}} - \mathcal{O}_{\tilde{W}W}.
\end{aligned} \tag{5}$$

The operators $\mathcal{O}_{WW}$ and $\mathcal{O}_{BB}$, apart from inducing anomalous contribution to the vertex, would also induce a two-point function of weak states of the form

$$\begin{aligned}
\mathcal{O}_{WW} &= -\frac{c_{WW}}{4\Lambda^2} g_W^2 v^2 \left[W_{\mu\nu}^+ W^{-\mu\nu} + \sin^2 \theta_W (\partial_{\mu} A_{\nu})^2 \right. \\
&\quad \left. + \cos^2 \theta_W (\partial_{\mu} Z_{\nu})^2 - \frac{\sin(2\theta_W)}{2} A_{\mu\nu} Z^{\mu\nu} \right. \\
&\quad \left. - \frac{\sin^2 \theta_W}{2} (\partial_{\nu} A_{\mu})(\partial^{\mu} A^{\nu}) - \frac{\cos^2 \theta_W}{2} (\partial_{\nu} Z_{\mu})(\partial^{\mu} Z^{\nu}) + \dots \right], \\
\mathcal{O}_{BB} &= -\frac{c_{BB}}{4\Lambda^2} g_1^2 v^2 \left[\cos^2 \theta_W (\partial_{\mu} A_{\nu})^2 \right. \\
&\quad \left. + \sin^2 \theta_W (\partial_{\mu} Z_{\nu})^2 + \frac{\sin(2\theta_W)}{2} A_{\mu\nu} Z^{\mu\nu} \right. \\
&\quad \left. - \cos^2 \theta_W (\partial_{\nu} A_{\mu})(\partial^{\mu} A^{\nu}) - \sin^2 \theta_W (\partial_{\nu} Z_{\mu})(\partial^{\mu} Z^{\nu}) + \dots \right],
\end{aligned} \tag{6}$$

where $\dots$ denotes the higher-order Lorentz structures containing a combination of three or a higher number of weak and Higgs states. The parameters c_{WW} and c_{BB} are the associated WCs of the operators $\mathcal{O}_{WW}$ and $\mathcal{O}_{BB}$, respectively. Thus, the operators $\mathcal{O}_{WW}$ and $\mathcal{O}_{BB}$ induce change in the kinetic term of the weak boson after symmetry breaking; thus, these two require the renormalization of the gauge fields and the couplings. However, one can redefine these operators as

$$\begin{aligned}
\mathcal{O}_{WW} &= (\Phi^{\dagger} \Phi - v^2/2) W_{\mu\nu} W^{\mu\nu}, \\
\mathcal{O}_{BB} &= (\Phi^{\dagger} \Phi - v^2/2) B_{\mu\nu} B^{\mu\nu}
\end{aligned} \tag{7}$$

to remove any residual part proportional to v^2 , and this redefinition leads to the absence of any two-point function along with TGC [75] in these operators. After the operator redefinition, only Higgs-gauge couplings exist at the three-point level.

The operator $\mathcal{O}_{BW}$ affects the $Z\gamma$ mixing, which leads to the shift in the mass eigenstates from those in the SM as

$$\begin{aligned}
A_\mu &= \left(1 - \frac{g_1^2 g_W^2 v^2}{4(g_1^2 + g_W^2)} \frac{c_{BW}}{\Lambda^2}\right) A_\mu^{\text{SM}} \\
&\quad + \frac{g_1 g_W v^2}{8} \frac{g_1^2 - g_W^2}{g_1^2 + g_W^2} \frac{c_{BW}}{\Lambda^2} Z_\mu^{\text{SM}}, \\
Z_\mu &= \left(1 + \frac{g_1^2 g_W^2 v^2}{4(g_1^2 + g_W^2)} \frac{c_{BW}}{\Lambda^2}\right) Z_\mu^{\text{SM}} \\
&\quad + \frac{g_1 g_W v^2}{8} \frac{g_1^2 - g_W^2}{g_1^2 + g_W^2} \frac{c_{BW}}{\Lambda^2} A_\mu^{\text{SM}}, \quad (8)
\end{aligned}$$

where c_{BW} is the WC of $\mathcal{O}_{BW}$. The corresponding mass of the Z boson is

$$m_Z^2 = \frac{g_W^2 + g_1^2}{4} v^2 \left[1 - \frac{g_W^2 g_1^2}{g_W^2 + g_1^2} \frac{v^2}{\Lambda^2} c_{BW}\right]. \quad (9)$$

Thus, $\mathcal{O}_{BW}$ induces change in the ρ parameter at the tree level, violating the custodial symmetry. We ignore the operators considering the precision of electroweak precision observables. We describe the relevant gauge and Higgs-gauge couplings below.

A. Triple gauge boson couplings

The anomalous $W^-W^+\gamma/Z$ couplings can be parametrized in terms of 14 parameters as [77]

$$\begin{aligned}
\mathcal{L}_{WWV} &= ig_{WWV} \left(g_1^V (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) V^\nu \right. \\
&\quad + ig_4^V W_\mu^+ W_\nu^- (\partial^\mu V^\nu + \partial^\nu V^\mu) \\
&\quad - ig_5^V \epsilon^{\mu\nu\rho\sigma} (W_\mu^+ \partial_\rho W_\nu^- - \partial_\rho W_\mu^+ W_\nu^-) V_\sigma \\
&\quad + \frac{\lambda^V}{m_W^2} W_\mu^{+\nu} W_\nu^{-\rho} V_\rho^\mu + \frac{\lambda^{\tilde{V}}}{m_W^2} W_\mu^{+\nu} W_\nu^{-\rho} \tilde{V}_\rho^\mu \\
&\quad \left. + \kappa^V W_\mu^+ W_\nu^- V^{\mu\nu} + \kappa^{\tilde{V}} W_\mu^+ W_\nu^- \tilde{V}^{\mu\nu} \right). \quad (10)
\end{aligned}$$

The couplings g_1^V , λ^V , and κ^V are CP -even, while $\lambda^{\tilde{V}}$ and $\kappa^{\tilde{V}}$ are CP -odd. Within the SM, $g_1^V = g_1^Z = 1$, and all other couplings are explicitly zero. The conservation of charge of the W boson implies that any contribution to g_1^V should vanish. The subset of operators listed in Eq. (2) affecting $W^-W^+\gamma/Z$ couplings are

$$\mathcal{O}_i^{WWV} \in \{\mathcal{O}_B, \mathcal{O}_W, \mathcal{O}_{\tilde{W}}, \mathcal{O}_{WWW}, \mathcal{O}_{\tilde{W}WW}\}.$$

The anomalous part of the above WWV Lagrangian can be expressed in terms of the Wilson coefficient (c_i) of dimension-6 operators as [74]

$$\begin{aligned}
\Delta g_1^Z &= c_W \frac{m_Z^2}{2\Lambda^2}, \\
\Delta \kappa_\gamma &= (c_W + c_B) \frac{m_W^2}{2\Lambda^2}, \\
\Delta \kappa_Z &= (c_W - c_B \tan^2 \theta_W) \frac{m_W^2}{2\Lambda^2}, \\
\lambda_\gamma &= \lambda_Z = c_{WWW} \frac{3g_W^2 m_W^2}{2\Lambda^2}, \\
g_4^V &= g_5^V = 0, \\
\tilde{\kappa} &= c_{\tilde{W}} \frac{m_W^2}{2\Lambda^2}, \\
\tilde{\kappa}_Z &= -c_{\tilde{W}} \tan^2 \theta_W \frac{m_W^2}{2\Lambda^2}, \\
\tilde{\lambda}_\gamma &= \tilde{\lambda}_Z = c_{\tilde{W}WW} \frac{3g_W^2 m_W^2}{2\Lambda^2}. \quad (11)
\end{aligned}$$

From the above Eq. (11), one can obtain two more relations:

$$\begin{aligned}
\Delta g_1^Z &= \Delta \kappa_Z + \tan^2 \theta_W \Delta \kappa_\gamma, \\
0 &= \tilde{\kappa}_Z + \tan^2 \theta_W \tilde{\kappa}, \quad (12)
\end{aligned}$$

which reduces the independent anomalous couplings to five. Many studies [26,28,32,72,78–114] have probed anomalous $W^+W^-\gamma/Z$ couplings, employing various kinematic observables like differential rate and angular functions at both hadron and lepton colliders.

B. Higgs couplings to gauge bosons

The operators inducing anomalous couplings of Higgs with gauge bosons are

$$\mathcal{O}_i^{HVV} \in \{\mathcal{O}_B, \mathcal{O}_W, \mathcal{O}_{\tilde{W}}, \mathcal{O}_{WW}, \mathcal{O}_{\tilde{W}W}, \mathcal{O}_{BB}, \mathcal{O}_{\tilde{B}B}\}.$$

These operators give rise to Higgs interactions with gauge boson pairs that take the following form in the unitary gauge [115]:

$$\begin{aligned}
\mathcal{L}_{HVV} &= g_{H\gamma\gamma} H A_{\mu\nu} A^{\mu\nu} + \tilde{g}_{H\gamma\gamma} H \tilde{A}_{\mu\nu} A^{\mu\nu} \\
&\quad + g_{HZ\gamma}^{(1)} A_{\mu\nu} Z^\mu \partial^\nu H + g_{HZ\gamma}^{(2)} H A_{\mu\nu} Z^{\mu\nu} \\
&\quad + \tilde{g}_{HZ\gamma}^{(1)} \tilde{A}_{\mu\nu} Z^\mu \partial^\nu H + \tilde{g}_{HZ\gamma}^{(2)} H \tilde{A}_{\mu\nu} Z^{\mu\nu} \\
&\quad + g_{HZZ}^{(1)} Z_{\mu\nu} Z^\mu \partial^\nu H + g_{HZZ}^{(2)} H Z_{\mu\nu} Z^{\mu\nu} \\
&\quad + \tilde{g}_{HZZ}^{(1)} \tilde{Z}_{\mu\nu} Z^\mu \partial^\nu H + \tilde{g}_{HZZ}^{(2)} H \tilde{Z}_{\mu\nu} Z^{\mu\nu} \\
&\quad + g_{HWW}^{(1)} (W_\mu^+ W_\nu^- \partial^\mu H + H.c) + g_{HWW}^{(2)} H W_\mu^+ W_\nu^- \\
&\quad + \tilde{g}_{HWW}^{(1)} (\tilde{W}_\mu^+ W_\nu^- \partial^\mu H + H.c) + \tilde{g}_{HWW}^{(2)} H \tilde{W}_\mu^+ W_\nu^- \quad (13)
\end{aligned}$$

The effective couplings are related to the Wilson coefficient as

$$\begin{aligned}
g_{H\gamma\gamma} &= -\left(\frac{g_W^2 v \sin^2 \theta_W}{\Lambda^2}\right)(c_{WW} + c_{BB}), \\
g_{HZ\gamma}^{(1)} &= \left(\frac{g_W^2 v \sin \theta_W}{4 \cos \theta_W \Lambda^2}\right)(c_W - c_B), \\
g_{HZ\gamma}^{(2)} &= -\left(\frac{g_W^2 v \sin \theta_W \cos \theta_W}{\Lambda^2}\right)(c_{WW} - \tan^2 \theta_W c_{BB}), \\
\tilde{g}_{H\gamma\gamma} &= \left(\frac{g_W^2 v \sin^2 \theta_W}{2 \Lambda^2}\right)(c_{\tilde{W}W} + c_{\tilde{B}B}), \\
\tilde{g}_{HZ\gamma}^{(1)} &= \left(\frac{g_W^2 v \sin \theta_W}{4 \Lambda^2 \cos \theta_W}\right)c_{\tilde{W}}, \\
\tilde{g}_{HZ\gamma}^{(2)} &= \left(\frac{g_W^2 v \sin \theta_W \cos \theta_W}{2 \Lambda^2}\right)(c_{\tilde{W}W} - \tan^2 \theta_W c_{\tilde{B}B}), \\
g_{HZZ}^{(1)} &= \left(\frac{g_W^2 v}{2 \Lambda^2}\right) \frac{\cos^2 \theta_W c_W + \sin^2 \theta_W c_B}{2 \cos^2 \theta_W}, \\
g_{HZZ}^{(2)} &= -\left(\frac{g_W^2 v}{2 \Lambda^2}\right) \frac{\sin^2 \theta_W c_{BB} + \cos^4 \theta_W c_{WW}}{2 \cos^2 \theta_W}, \\
\tilde{g}_{HZZ}^{(1)} &= \left(\frac{g_W^2 v}{16 \Lambda^2}\right)c_{\tilde{W}}, \\
\tilde{g}_{HZZ}^{(2)} &= -\frac{g_W^2 v \sin^2 \theta_W \tan^2 \theta_W}{4 \Lambda^2} c_{\tilde{W}W} + \frac{g_W^2 v \cos^2 \theta_W}{8 \Lambda^2} c_{\tilde{B}B}, \\
g_{HWW}^{(1)} &= \left(\frac{g_W^2 v}{2 \Lambda^2}\right) \frac{c_W}{2}, \\
g_{HWW}^{(2)} &= -\left(\frac{g_W^2 v}{2 \Lambda^2}\right)c_{WW}, \\
\tilde{g}_{HWW}^{(1)} &= \frac{g_W^2 v}{16 \Lambda^2 \cos \theta_W \sin^2 \theta_W} c_{\tilde{W}}, \\
\tilde{g}_{HWW}^{(2)} &= -\frac{g_W^2 v}{4 \Lambda^2} c_{\tilde{W}W}. \tag{14}
\end{aligned}$$

The probe of Higgs couplings with the gauge boson are of paramount importance to settle the discourse of the CP structure of Higgs, which till the current time is considered to be even. The anomalous HVV couplings have been probed in studies [115–147] and references therein. The Higgs couplings involving Z and photons—i.e., $HZ\gamma, H\gamma\gamma$ —are interesting structures, as they do not exist at SM tree level and arise at the one-loop level. The CMS and ATLAS Collaborations [60] report the first evidence for Higgs decay to Z bosons and photons with a statistical significance of 3.4 standard deviations. Thus, it presents a strong area of interest to probe for new physics. One of the channels that predicts the existence of a scalar with mass around 125 GeV is the two-photon channel [52,53]. This channel would be a clean channel to look for any deviation from SM in upcoming e^-e^+ colliders.

C. Quartic gauge boson couplings

The operators listed in Eq. (2), apart from affecting TGC and HVV , could also directly induce anomalous behavior in quartic gauge boson couplings (QGCs). The operators are

$$\mathcal{O}_i^{WWVV} \in \{\mathcal{O}_W, \mathcal{O}_{WWW}, \mathcal{O}_{WW}, \mathcal{O}_{\tilde{W}WW}\}.$$

The processes like vector boson scattering and triboson production are sensitive to the modifications of quartic couplings. These processes also depend on the trilinear $W^-W^+\gamma/Z$ and HVV couplings. From an operator-focused point of view, the set of operators might affect different Lorentz structures. For example, a set of operators affecting quartic couplings may also induce change in triple gauge couplings. Thus, it becomes important from the sensitivity point of view that such a set of operators be studied in the totality of all affected Lorentz structures. Usually, each set of operators is studied in isolation. For example, diboson processes are mostly probed for $W^-W^+\gamma/Z$ and HZZ couplings; nevertheless, triboson processes become relevant following the helicity selection rules. The helicity rules suppress the linear, tree-level sensitivity of high-energy transverse boson pair production to dimension-6 SMEFT effects [70,148,149]. The study of each set of operators in isolation becomes ambiguous due to each set affecting more than one set of couplings. Thus, the study of a process where all sets of operators can come to life becomes very important to understand the marginalized constraint on them, and also to gauge down the possibility of ultraviolet complete theories. To complement this idea, we probed the $2je^-e^+\mu^-\bar{\nu}_\mu$ production process, which simultaneously provides various amplitudes sensitive to all three sets of operators. A subset of schematic Feynman diagrams for the $2je^-e^+\mu^-\bar{\nu}_\mu$ process at leading order is shown in Fig. 1. The dark blob in each diagram represents the contribution from the dimension-6 operators. The number of points of SMEFT insertion in the amplitudes lies between 0 and 2, where zero insertion is the SM.

We implement the relevant operators in FEYNRULES [150] to obtain the UNIVERSAL FEYNRULES OUTPUT (UFO) [151,152] model file. We used a diagonal CKM matrix and “ α scheme” as an electroweak input scheme. The model files are used in the Monte Carlo generator MADGRAPH5_AMC@NLO (MG5 henceforth) [153] for event generation.

In the next section, we discuss the polarization and spin correlations in VBS and WWZ events and their reconstruction though asymmetries in angular functions.

III. COLLIDER SETUP AND EVENT CLASSIFICATION

The upcoming lepton colliders—like FCC-ee [154], CLIC [155], and ILC [156,157]—would collide initially

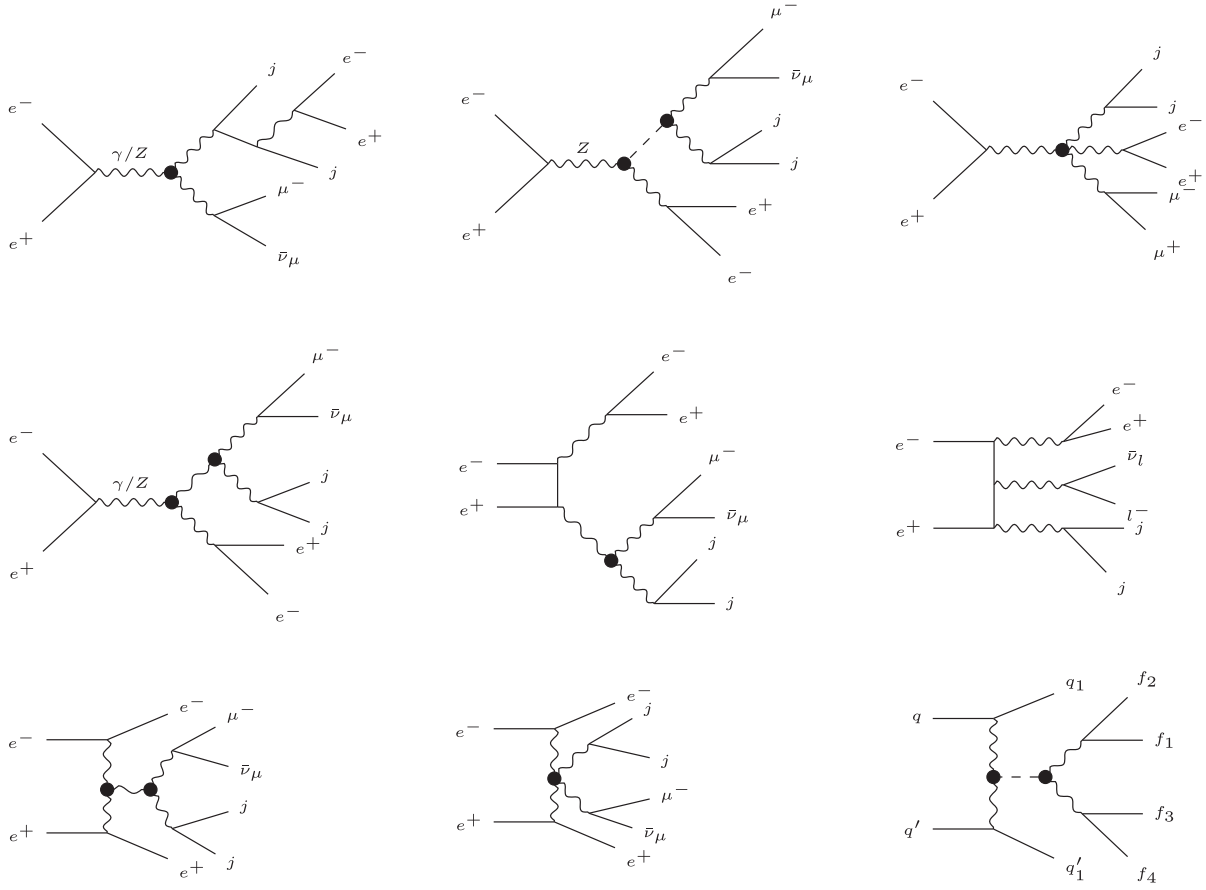


FIG. 1. Representative gauge-invariant set of Feynman diagrams for the electroweak production of three gauge bosons (top and middle rows) and VBS amplitudes (bottom row) that would present the same final states. SMEFT operators modify the triple and quartic couplings.

polarized beams. The use of polarized beams would benefit the search for new physics (NP) signals which are usually drowned in the sea of the SM background. The cross section in the presence of longitudinal beam polarization behaves as [158]

$$\sigma(P_{e^-}, P_{e^+}) = \frac{1}{4} [(\sigma_{\text{RL}} + \sigma_{\text{LR}}) + (P_{e^-} - P_{e^+})(\sigma_{\text{RL}} - \sigma_{\text{LR}}) - P_{e^-}P_{e^+}(\sigma_{\text{RL}} + \sigma_{\text{LR}})],$$

where P_{e^-}, P_{e^+} represent the degree of polarization for the electron and positron beams, respectively. The value σ_{RL} (σ_{LR}) is the cross section when the electron is 100% right-handed (left-handed) polarized and the positron is 100% left-handed (right-handed) polarized. The equation suggests that using the set of two flipped polarizations would provide directional cuts on the differential distribution, which would help to more tightly constrain the parameters of new physics.

We study the process $e^-e^+ \rightarrow 2je^-e^+\mu^-\bar{\nu}_\mu$ at a center-of-mass energy of $\sqrt{s} = 1$ TeV with longitudinally polarized electron and positron beams. At leading order (LO), the

full amplitude receives contributions from several topologies; representative diagrams are shown in Fig. 1. Among these, two gauge-invariant subsets play a dominant role: (i) triple gauge boson production resembling WWZ topologies, and (ii) vector-boson scattering (VBS) topologies characterized by t -channel exchange of electroweak bosons. We aim to identify collider observables that efficiently discriminate between regions of phase space enriched in these two subprocesses.

Matrix-element events are generated with MG5 at LO using the SMEFT UFO model described in the previous section. Parton-shower evolution and hadronization simulations are performed using PYTHIA8(v8.313) [159]. Detector effects, lepton isolation, and jet reconstruction inputs are modeled using DELPHES (v3.5.0) [160] with an ILC-like detector configuration. Jets are reconstructed with the ANTI- k_T [161] algorithm with the radius parameter $R = 0.7$, as implemented in the FASTJET [162] package. Final-state objects are required to satisfy standard acceptance cuts used in e^+e^- analyses.

To obtain samples enriched in the two dominant subprocesses, we generate (i) a “ WWZ -like” sample corresponding to the $e^-e^+ \rightarrow W^-W^+Z$ channel, and (ii) a

“VBS-like” sample from the process $e^-e^+ \rightarrow e^-e^+W^-W^+$ while explicitly removing γ/Z -mediated s -channel diagrams. This isolation enhances the contribution from t -channel electroweak boson exchange characteristic of VBS. Both samples are passed through identical showering and detector simulation to ensure a consistent comparison.

We choose the α scheme $(\alpha_{\text{em}}, m_Z, G_F)$ as the electroweak input parameters, under which the four independent parameters of the electroweak sector are defined as

$$g_1 = \sqrt{\frac{4\pi\alpha_{\text{em}}}{\cos\theta_W^2}}, \quad g_W = \sqrt{\frac{4\pi\alpha_{\text{em}}}{\sin\theta_W^2}}, \quad v = \sqrt{\frac{1}{\sqrt{2}G_F}},$$

$$\lambda = \frac{m_h^2 G_F}{\sqrt{2}}, \quad (15)$$

with the weak mixing angle defined as

$$\sin^2\theta_W = \frac{g_1^2}{g_1^2 + g_W^2} = \frac{1}{2} \left[1 - \sqrt{1 - \frac{2\sqrt{2}\pi\alpha_{\text{em}}}{G_F m_Z^2}} \right]. \quad (16)$$

Here, α_{em} is the electromagnetic coupling strength, m_Z is the mass of the Z boson, and G_F is the Fermi constant. This choice maintains consistency with precision electroweak constraints and provides a stable input basis for high-energy e^-e^+ collisions.

VBS topologies are typically characterized by forward/backward-scattered electrons with large rapidity separation and large invariant mass, originating from the t -channel exchange of weak bosons. To quantify these features, we evaluate various collider observables sensitive to the topology of the final-state leptons and jets. Two variables exhibit particularly strong separation power:

- (1) The pseudorapidity separation between the oppositely charged electrons,

$$\Delta\eta_{e^-e^+} = \eta(e^-) - \eta(e^+).$$

- (2) The invariant mass of the e^-e^+ system,

$$m_{e^-e^+} = \sqrt{(p_{e^-} + p_{e^+})^2}.$$

Figure 2 shows the distributions of $|\Delta\eta_{e^-e^+}|$ and $m_{e^-e^+}$ for the VBS-like and WWZ-like samples along with complete possible LO amplitudes. While $|\Delta\eta_{e^-e^+}|$ provides partial discrimination at large separations, substantial overlap remains in the central region. In contrast, the invariant mass $m_{e^-e^+}$ exhibits a much cleaner separation: applying a threshold around $m_{e^-e^+} \gtrsim 150$ GeV yields nearly disjoint regions of phase space for the two subprocesses. Based on this, we adopt $m_{e^-e^+}$ as the primary observable to select VBS- and WWZ-enriched regions for further analyses.

IV. SPIN OBSERVABLES

For a case where two spin-1 boson are coproduced, the density matrix can be written as

$$\rho_{(11)} = \frac{1}{9} \mathbb{I} \otimes \mathbb{I} + \sum_i p_i^{(1)} \mathbb{I} \otimes J_i + \sum_i p_i^{(2)} J_i \otimes \mathbb{I} + \sum_{i,j} p_{ij}^{(12)} J_i \otimes J_j, \quad (17)$$

where J_i (see Appendix A) are the matrices constructed from fundamental spin-1 operators, and $p_i^{(1/2)}$, $p_{ij}^{(12)}$ are the polarizations and spin correlation parameters. Using the decay density matrix Γ (see Appendix B), we can obtain the joint angular distribution form final decayed fermions of the W boson as

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega^{(1)} d\Omega^{(2)}} = \sum \rho_{(11)} \times (\Gamma_{W^-} \otimes \Gamma_{W^+}). \quad (18)$$

Then, the 80 parameters of the density matrix are obtained by partial integration of the joint angular distribution, which is represented in terms of the asymmetries of various angular functions. In general, the asymmetries are computed numerically as

$$A_{ij}^{12} = \frac{\sigma(C_i^{(1)} C_j^{(2)} > 0) - \sigma(C_i^{(1)} C_j^{(2)} < 0)}{\sigma(C_i^{(1)} C_j^{(2)} > 0) + \sigma(C_i^{(1)} C_j^{(2)} < 0)}, \quad (19)$$

where the angular correlators (C_i) are

$$C_i \in [C_1 = 1, C_2 = \cos\phi, C_3 = \sin\phi, c_4 = \cos\theta, C_5 = C_2 C_3, C_6 = C_2 C_4, C_7 = C_3 C_4, C_8 = C_2^2 - C_3^2, C_9 = \sqrt{C_4^2 - 1}(4C_4^2 - 1)]. \quad (20)$$

The polar (θ) and azimuthal (ϕ) angles are obtained at the rest frame of the W boson; the beam line defines the z axis in the lab, and the production plane of W defines the xz plane ($\phi = 0$). Next, we consider a process where three spin-1 particles are coproduced—i.e., the WWZ production process. Similarly, the density matrix in the case of three spin-1 particles can be expanded as

$$\rho_{(111)} = \frac{1}{27} \mathbb{I}_{27 \times 27} + \sum_i p_i^{(1)} J_i \otimes \mathbb{I} \otimes \mathbb{I} + \sum_i p_i^{(2)} \mathbb{I} \otimes J_i \otimes \mathbb{I} + \sum_i p_i^{(3)} \mathbb{I} \otimes \mathbb{I} \otimes J_i + \sum_{i,j} p_{ij}^{(12)} J_i \otimes J_j \otimes \mathbb{I} + \sum_i p_{ij}^{(13)} J_i \otimes \mathbb{I} \otimes J_j + \sum_{ij} p_{ij}^{(23)} \mathbb{I} \otimes J_i \otimes J_j + \sum_{ijk} p_{ijk}^{(123)} J_i \otimes J_j \otimes J_k, \quad (21)$$

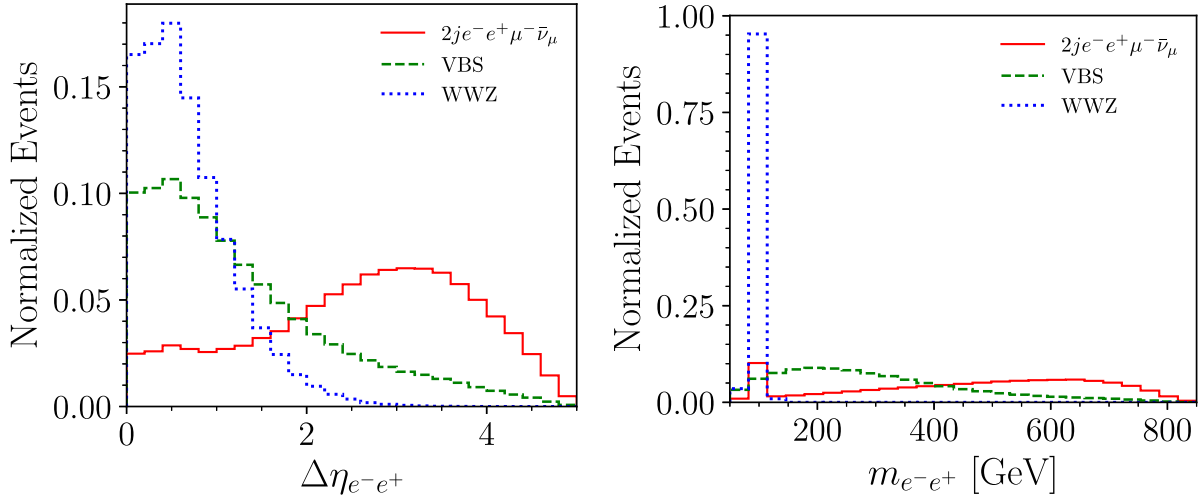


FIG. 2. Detector level distribution of the absolute pseudorapidity difference of the electron and positron (left panel) and the invariant mass of the electron-positron system. The distributions are obtained for three different processes: $e^-e^+ \rightarrow 2je^-e^+\mu^-\bar{\nu}_\mu$ (solid red curve), VBS production (dashed green curve), and the WWZ process (dotted blue curve) at $\sqrt{s} = 1$ TeV.

where $p^{(123)}$ represents the three-particle spin-correlation parameters. Similarly to the two-spin-1-particle case, the parameters for the three-particle spin density are obtained in terms of asymmetries,

$$A_{ijk}^{123} = \frac{\sigma(C_i^{(1)}C_j^{(2)}C_k^{(3)} > 0) - \sigma(C_i^{(1)}C_j^{(2)}C_k^{(3)} < 0)}{\sigma(C_i^{(1)}C_j^{(2)}C_k^{(3)} > 0) + \sigma(C_i^{(1)}C_j^{(2)}C_k^{(3)} < 0)}, \quad (22)$$

where the correlators C_i are given in Eq. (20).

It is important to recall that vector polarization observables (A_x , A_y , A_z) and their correlation asymmetries (e.g., $A_{x,y}^{12}$, $A_{x,z,z}^{123}$, $A_{x,z,y}^{123}$, ...) average out unless the identity of the corresponding fermion is known. While leptonic decay modes permit unambiguous charge and flavor reconstruction, the hadronic sector poses a significant challenge: the two light jets from $W \rightarrow q\bar{q}'$ cannot be reliably distinguished as up- or down-type with standard experimental techniques.

Although jet-level flavor classification can, in principle, be achieved using machine-learning methods exploiting jet substructure (e.g., neural network-based classifiers developed in previous studies [71,72]), such approaches require detailed detector-level information and experimentally calibrated performance to remain applicable in realistic collider conditions. In the absence of robust and validated light-flavor tagging capabilities, we do not perform any explicit quark-jet flavor assignment in this work.

Consequently, all observables whose definitions depend on identifying the flavor of hadronic decay products—in particular, vector polarizations associated with the hadronic W boson and their correlation asymmetries—are excluded from our analysis. The results presented here focus exclusively on those spin observables which remain well defined and experimentally robust without requiring light-jet flavor

tagging. However, by using these collider-safe observables, we would lose significant sensitivity to anomalous couplings, as mentioned in previous studies [72]. We discuss the reconstruction and the impact of the inclusion of flavor-dependent asymmetries in Appendix C.

In the next section, we describe the role of spin-related observables on putting a bound on anomalous couplings for different sets of luminosities and systematic errors.

V. PROBE OF ANOMALOUS COUPLINGS

The angular distribution provides a large set of observables sensitive to the anomalous couplings in the form of polarizations and spin correlations. The general idea is to obtain as many observables as possible in order to better obtain a distinction between the SM and the new physics contribution. As shown in earlier work [163], the asymmetries play a prominent role in parameter extraction given an excess in collider data even if they have a minimal contribution to constraining anomalous couplings. Thus, a large set of polarization and spin correlation asymmetries becomes very important for measurement of the anomalous couplings. To derive a semianalytical expression for observables in terms of anomalous couplings, we fit the data as follows. Considering the amplitudes present in current analysis and assuming the order of SMEFT insertion at the amplitude level to be ≤ 2 , the cross section can be expressed as

$$\begin{aligned} \sigma = & \sigma_0 + \sum_i \sigma_i c_i + \sum_{i \geq j} \sigma_{ij} c_i c_j + \sum_{i \geq j \geq k} \sigma_{ijk} c_i c_j c_k \\ & + \sum_{i \geq j \geq k \geq l} \sigma_{ijkl} c_i c_j c_k c_l. \end{aligned} \quad (23)$$

Here, the coefficients represent different orders of interaction:

- (1) σ_0 is the SM point cross section.
- (2) $\sigma_i [\mathcal{O}(\Lambda^{-2})]$ corresponds to the interference of SMEFT with the SM.
- (3) $\sigma_{ij} [\mathcal{O}(\Lambda^{-4})]$ are the quadratic terms ($i = j$) for one-SMEFT insertion and the interference ($i > j$) of two-SMEFT insertion.
- (4) $\sigma_{ijk} [\mathcal{O}(\Lambda^{-6})]$ represents the three-SMEFT insertion at the cross section level.
- (5) $\sigma_{ijkl} [\mathcal{O}(\Lambda^{-8})]$ signifies the matrix-squared level of amplitude terms with two insertions of SMEFT.

Dimensionally, σ_{ij} correspond to dimension-8 operators. From a naive dimension-counting perspective, a complete analysis requires these higher-dimensional operators to be accounted for. However, due to the suppression of higher-order contributions by larger powers of Λ , one generally proposes a cutoff at $\mathcal{O}(\Lambda^{-4})$. However, cutting off higher dimensions might not be straightforward, as pointed out in Ref. [70], where it was noted that due to the noninterference between the SM and $d = 6$ operators for amplitudes with at least one transversely polarized boson, the relevant effects might actually come from operators with $d > 6$. To substantiate this assumption, we compare different coefficients for a set of (c_{WWW}, c_W) couplings. We obtain that the SM cross section is of the order of 0.1 fb, while the coefficients σ_i and σ_{ii} (for $i = j$) are of the order of 0.001 fb. The coefficients σ_{ij} ($i \neq j$) are found to be of the order of 10^{-6} fb, whereas σ_{ijk} and σ_{ijkl} are suppressed to values below 10^{-9} fb. In fact, the magnitudes of σ_{ijk} and σ_{ijkl} are comparable to or smaller than the intrinsic Monte Carlo statistical uncertainty of the simulations, rendering these contributions effectively indistinguishable from numerical noise. Consequently, they can be safely neglected, and we restrict our analysis to the one-EFT insertion terms—i.e., up to $\mathcal{O}(\Lambda^{-4})$.

As stated in the previous section, there are 80 and 728 asymmetries for the VBS and WWZ processes, respectively, which are further binned in the region of the transverse momenta of muons. Neglecting the vector polarization of hadronically decayed W bosons and their correlation asymmetries, VBS topology offers 53 asymmetries, and WWZ provides a total of 484 flavor-blind asymmetries. We choose seven different bins in a muon momentum width of 50 GeV, thus giving us totals of 371 and 3598 asymmetries for the VBS and WWZ processes, respectively, along with seven binned cross sections. For each bin, we construct cross section weighted spin asymmetries, the observables are classified in terms of their CP structure, and the corresponding fits are performed as

$$\text{Cross section: } \sigma = \sigma_0 + \sum_i \sigma_i c_i + \sum_{i \geq j} \sigma_{ij} c_i c_j, \quad (24)$$

$$\text{CP-even asymmetries: } A_{\text{Even}} = \frac{\Delta\sigma_E}{\sigma}, \quad (25)$$

$$\text{CP-odd asymmetries: } A_{\text{Odd}} = \frac{\Delta\sigma_O}{\sigma}, \quad (26)$$

$$\Delta\sigma_O = \sigma_i c_i + \sum_{i > j} \sigma_{ij} c_i c_j, \quad (27)$$

$$\Delta\sigma_E = \sigma. \quad (28)$$

Here, $\Delta\sigma \equiv \sigma\mathcal{A}$ denotes the product of the cross section and asymmetries in a particular bin of muon transverse momenta. The binned asymmetries, along with the cross section for two sets of beam polarization, are used to perform a chi-squared analysis to obtain the bounds on anomalous couplings c_i . The chi-squared is defined as

$$\begin{aligned} \Delta\chi^2(\mathcal{O}, c_i, \pm\eta_3, \mp\xi_3) \\ = \sum_{l,k} \left[\left(\frac{\mathcal{O}_k^l(c_i, +\eta_3, -\xi_3) - \mathcal{O}_k^l(0, +\eta_3, -\xi_3)}{\delta\mathcal{O}_k^l(0, +\eta_3, -\xi_3)} \right)^2 \right. \\ \left. + \left(\frac{\mathcal{O}_k^l(c_i, -\eta_3, +\xi_3) - \mathcal{O}_k^l(0, -\eta_3, +\xi_3)}{\delta\mathcal{O}_k^l(0, -\eta_3, +\xi_3)} \right)^2 \right]. \end{aligned} \quad (29)$$

Here k, l runs over all bins and observables separately, and $\delta\mathcal{O} = \sqrt{(\delta\mathcal{O}_{\text{stat}})^2 + (\delta\mathcal{O}_{\text{syst}})^2}$ is the estimated error in $\mathcal{O}$. For cross section σ , the estimated error is given by

$$\delta\sigma = \sqrt{\frac{\sigma}{\mathcal{L}} + (\epsilon_\sigma \sigma)^2}, \quad (30)$$

and if the observable is asymmetry, the error is given by

$$\delta\mathcal{A} = \sqrt{\frac{1 - \mathcal{A}^2}{\mathcal{L}\sigma} + \epsilon_{\mathcal{A}}^2}. \quad (31)$$

Here, $\epsilon_{\mathcal{A}}, \epsilon_\sigma$ are the fractional systematic errors in asymmetry and cross section, respectively, and $\mathcal{L}$ is the integrated luminosity. Further, the χ^2 's are multiplied by factors of 8 and 4 for WWZ and VBS topology, respectively, to complement the inclusion of statistics from all leptonic flavors.

We begin by deriving one-parameter constraints on the nine independent anomalous EFT WCs at $\sqrt{s} = 1$ TeV with polarized initial beams with polarization degrees $(\eta_3, \xi_3) = (\mp 0.8, \pm 0.6)$. The analysis is performed in a one-parameter-at-a-time approach, where a single WC is varied while all others are fixed to zero. The limits are extracted using a binned χ^2 analysis that combines information from all seven kinematic bins. The 95% confidence level (CL) bounds are obtained by requiring $\Delta\chi^2 = 3.84$ for one degree of freedom and are summarized in Table I. The χ^2 function includes both statistical uncertainties and systematic uncertainties associated with the total cross section and asymmetry observables. In order to compare

TABLE I. One-parameter 95% CL limits on anomalous WCs c_i obtained from triple gauge boson production (WWZ), vector boson scattering (VBS) amplitudes, and their combination for the process $e^-e^+ \rightarrow 2je^-e^+\ell^-E$ at $\sqrt{s} = 1$ TeV. The limits are derived using a binned χ^2 analysis with beam polarizations ($\eta_3 = \mp 0.8$, $\xi_3 = \pm 0.6$), assuming systematic uncertainties of $\epsilon_\sigma = 20\%$ and $\epsilon_A = 1\%$. WC-dependent integrated luminosities are used to facilitate comparison with existing experimental constraints, which are shown in the last column.

c_i	Limits (TeV ⁻²)			
	VBS	WWZ	VBS + WWZ	Exp
c_W/Λ^2	[-1.25, +0.72]	[-0.96, +0.84]	[-0.91, +0.62]	[-2.50, +0.30] [83]
$c_{\tilde{W}}/\Lambda^2$	[-0.40, +0.40]	[-0.92, +0.92]	[-0.40, +0.40]	[-20.0, +20.0] [85]
c_{WWW}/Λ^2	[-1.73, +1.25]	[-0.95, +0.86]	[-0.93, +0.79]	[-0.90, +0.91] [85]
$c_{\tilde{W}WW}/\Lambda^2$	[-0.21, +0.21]	[-0.89, +0.89]	[-0.21, +0.21]	[-0.45, +0.45] [85]
c_B/Λ^2	[-0.45, +0.14]	[-1.34, +1.24]	[-0.45, +0.14]	[-8.78, +8.54] [93]
c_{WW}/Λ^2	[-0.46, +0.25]	[-1.49, +1.41]	[-0.45, +0.25]	[-4.3, +4.4] [164]
$c_{\tilde{W}W}/\Lambda^2$	[-0.13, +0.13]	[-0.86, +0.86]	[-0.13, +0.13]	
c_{BB}/Λ^2	[-0.41, +0.13]	[-1.49, +1.37]	[-0.41, +0.13]	[-10.0, +7.0] [164]
$c_{\tilde{B}B}/\Lambda^2$	[-0.32, +0.32]	[-1.01, +1.01]	[-0.32, +0.32]	

our results with existing CMS constraints, we evaluate the limits on individual WCs using integrated luminosities chosen to match the corresponding CMS analyses, while adopting a common and conservative treatment of systematic uncertainties. For all WCs considered, we assume a systematic uncertainty of $\epsilon_\sigma = 20\%$ on the total cross section and $\epsilon_A = 1\%$ on the asymmetries, consistent with or more conservative than those employed in Refs. [83,85,93]. Specifically, for the WC c_B , we use an integrated luminosity of $\mathcal{L} = 35.9 \text{ fb}^{-1}$ following Ref. [93]. For the WC c_W , the limits are derived using $\mathcal{L} = 137 \text{ fb}^{-1}$ in accordance with Ref. [83]. Similarly, for the WCs c_{WWW} , $c_{\tilde{W}}$, and $c_{\tilde{W}WW}$, we employ $\mathcal{L} = 137 \text{ fb}^{-1}$ as in Ref. [85]. For c_{WW} and c_{BB} , we refer to [164], where the combined Run-I datasets from ATLAS and CMS at $\mathcal{L} = 25 \text{ fb}^{-1}$ are presented. For the two CP -odd WCs, viz. $c_{\tilde{W}W}$, and $c_{\tilde{B}B}$, we choose $\mathcal{L} = 100 \text{ fb}^{-1}$. No comparison of these WCs to experimental limits is made in the current work.

From Table I, the one-parameter 95% CL constraints derived from the combined VBS + WWZ analysis are competitive with, and in several cases exceed, the current CMS limits. Notable improvements are obtained for the WCs c_W , $c_{\tilde{W}}$, and c_B , with the latter two constrained at a level exceeding the existing experimental bounds by more than an order of magnitude.

For the operators c_{WWW} and $c_{\tilde{W}WW}$, the present sensitivity is comparable to or stronger than that achieved at the LHC. In particular, the constraint on the CP -odd coefficient $c_{\tilde{W}WW}$ improves upon the CMS result by approximately a factor of 2, underscoring the enhanced sensitivity of the VBS topology to CP -violating effects. For WCs c_{WW} and c_{BB} , our limits are tighter by an order of magnitude compared to the experimental ones. A channel-resolved analysis further indicates that, while the dominant contribution to the total χ^2 originates from the VBS topology for

most coefficients, the leading sensitivity to c_W and c_{WWW} is driven primarily by the WWZ channel.

Next, we study the impact of truncating the EFT expansion at linear order in the WCs compared to retaining both linear and quadratic contributions. The linear approximation accounts only for the SM dimension-6 interference terms, $\mathcal{O}(\Lambda^{-2})$, while the quadratic contributions enter at $\mathcal{O}(\Lambda^{-4})$ and are formally of the same order as the interference between dimension-8 operators and the SM amplitude. From the standpoint of EFT power counting, these terms may therefore be regarded as an estimate of higher-dimensional effects and, in principle, introduce an additional source of theoretical uncertainty.

To quantify their numerical impact, we compare the one-parameter $\Delta\chi^2$ distributions obtained using only linear terms with those obtained by including both linear and quadratic contributions for representative CP -even and CP -odd WCs Fig. 3. We find that the inclusion of quadratic terms systematically strengthens the resulting constraints. This improvement arises from the positive-definite nature of the squared EFT contributions, which enhance the sensitivity to deviations from the SM, particularly in kinematic regions where the SMEFT interference is suppressed.

Next, we quantify the relative impact of different observables, viz. the total cross section, polarization, and spin correlation asymmetries arising from the VBS and WWZ topologies on constraining WCs. This is illustrated in Fig. 4 through two-dimensional 95% CL contours obtained from a binned $\Delta\chi^2$ analysis, shown as functions of two WCs at a time, with all remaining coefficients set to zero.

We focus on the WC pairs $(c_W, c_{\tilde{W}})$ and $(c_{BB}, c_{\tilde{B}B})$ to quantify the relative contributions of different observables to the total χ^2 . For all coefficient combinations, the χ^2 decomposition exhibits a clear hierarchy: polarization and

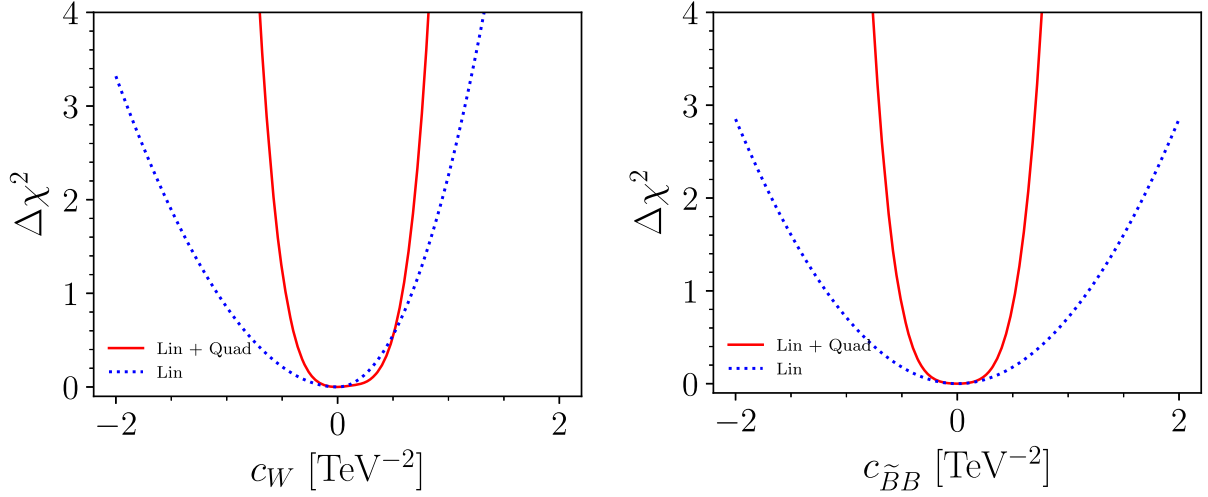


FIG. 3. Comparison of chi-squared distributions for the combination of cross section and asymmetries truncated at the interference (dashed blue) and quadratic (solid red) levels. The distributions are obtained by combining datasets from two different choices of beam polarization at $\mathcal{L} = 100 \text{ fb}^{-1}$. Systematic errors of $\epsilon_\sigma = 20\%$, $\epsilon_A = 1\%$ are considered in this analysis.

spin-correlation asymmetries in the VBS topology provide the dominant contribution, followed by the corresponding asymmetries in the WWZ channel, while the total cross section yields the weakest sensitivity. An exception is observed for c_W , where the WWZ asymmetries contribute more strongly to χ^2 than those from VBS. The tightest constraints are obtained from the simultaneous inclusion of the total cross section and asymmetries from both the VBS and WWZ processes, as illustrated by the dashed blue contours.

This behavior can be understood from the momentum-dependent structure of the effective vertices involving triple gauge, Higgs gauge, and quartic gauge couplings induced by dimension-6 operators after electroweak

symmetry breaking. While such momentum-enhanced terms are more efficiently probed at hadron colliders operating at higher center-of-mass energies, the reduced energy reach at an e^+e^- collider is compensated for by the availability of a large set of clean and well-defined spin observables. Consequently, even at $\sqrt{s} = 1 \text{ TeV}$, polarization and spin correlation measurements play a crucial role in enhancing the overall sensitivity to anomalous gauge interactions.

Finally, we perform multidimensional Markov chain Monte Carlo (MCMC) analysis to obtain the marginalized limits on all nine anomalous couplings for different values of integrated luminosity and systematic errors. The likelihood ratio function for a parameter space, $\vec{x} \equiv (c_i, \eta_3, \xi_3)$,

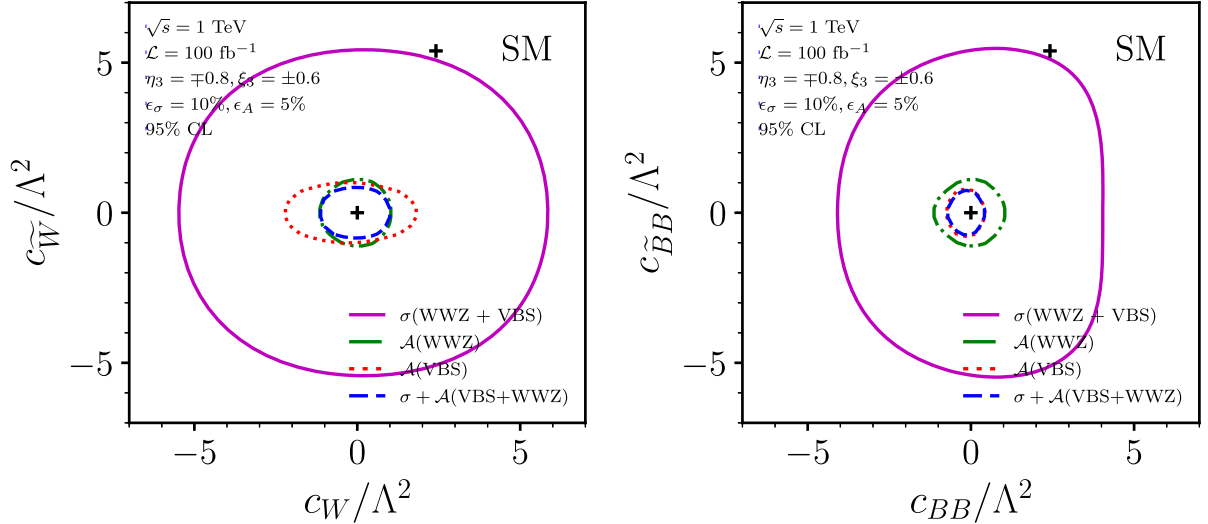


FIG. 4. Two-dimensional 95% CL contours computed with a $\Delta\chi^2$ function for cross section (σ), polarization, and spin correlation asymmetries ($\mathcal{A}$) in all seven bins of $p_T^{\mu^-}$ as a function of two anomalous couplings. The contours are obtained at $\sqrt{s} = 1 \text{ TeV}$, $\mathcal{L} = 1000 \text{ fb}^{-1}$, and $(\epsilon_\sigma = 10\%, \epsilon_A = 5\%)$. The legend details remain the same as those in the top-left panel for all figures.

is considered to be χ^2 , which is given by Eq. (29):

$$-2 \ln(\Delta L(\vec{x})) = \chi^2(\vec{x}).$$

The further interpretation of Monte Carlo samples for Bayesian inference is done using GETDIST [165]. It is a PYTHON-based package that deals with estimating from the MC samples, which include confidence limits and marginalized densities. We perform MCMC analysis based on the METROPOLIS-HASTINGS [166,167] algorithm by combining all observables from seven bins of muon transverse momenta. The different values of integrated luminosities chosen are

$$\mathcal{L} = \{100 \text{ fb}^{-1}, 300 \text{ fb}^{-1}, 1000 \text{ fb}^{-1}, 3000 \text{ fb}^{-1}\},$$

and the systematic errors are

$$(\epsilon_\sigma, \epsilon_A) = \{(0, 0), (1\%, 0.5\%), (5\%, 2\%), (10\%, 5\%)\}.$$

We perform a study to quantify the marginalized limits variation of anomalous couplings with respect to luminosity and systematic errors. In Fig. 5, we show 2D projections at 95% CL for the $(c_{WWW}-c_{\bar{W}W\bar{W}})$ and $(c_{BB}-c_{\bar{B}B})$ planes as a function of luminosity (top row) and systematic error (bottom row). From the figure, it becomes evident that one improves on the precision of probing anomalous effects with increasing datasets. The limits do not show a significant deviation on increasing the systematic error from zero to the more conservative value of 10% for the cross section and 5% for asymmetries. It is due to the low cross section that the errors are dominated by statistical ones. Thus, the future electron-positron collider could be the perfect place to perform a precision measurement on multigauge and Higgs gauge couplings, provided sufficient luminosity is achieved. And finally for completeness, we list the

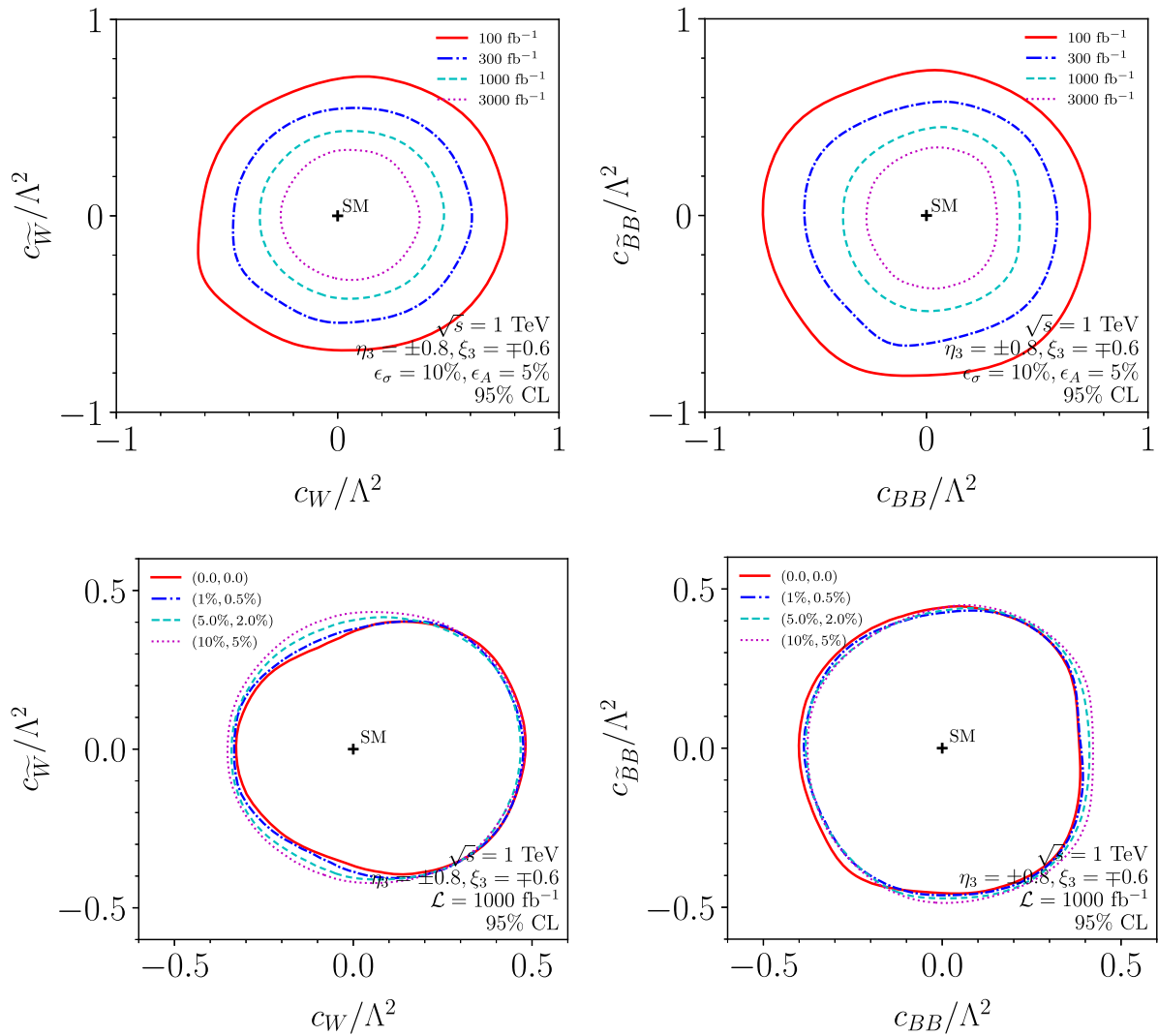


FIG. 5. 95% CL marginalized 2D projections on c_W-c_W and $c_{BB}-c_{\bar{B}B}$ planes from MCMC for $\sqrt{s} = 1$ TeV as a function of integrated luminosity (top row) and systematic errors (bottom row).

TABLE II. Posterior 95% Bayesian confidence intervals for anomalous couplings c_i (TeV^{-2}) obtained from Markov chain Monte Carlo global fits. Limits are shown for the process $e^-e^+ \rightarrow 2je^-e^+l^-E$ at $\sqrt{s} = 1$ TeV with beam polarization $(\eta_3, \xi_3) = (\pm 0.8, \mp 0.6)$, for different integrated luminosities and systematic errors.

c_i (TeV^{-2})	$(\epsilon_\sigma, \epsilon_A)$	100 fb $^{-1}$	300 fb $^{-1}$	1000 fb $^{-1}$	3000 fb $^{-1}$
c_W/Λ^2	(0.0, 0.0)	$[-0.86, +1.06]$	$[-0.66, +0.85]$	$[-0.51, +0.62]$	$[-0.36, +0.49]$
	(1%, 0.5%)	$[-0.85, +1.07]$	$[-0.65, +0.87]$	$[-0.49, +0.64]$	$[-0.35, +0.47]$
	(5%, 2%)	$[-0.88, +1.07]$	$[-0.65, +0.81]$	$[-0.48, +0.64]$	$[-0.35, +0.48]$
	(10%, 5%)	$[-0.86, +1.08]$	$[-1.03, +1.21]$	$[-0.49, +0.65]$	$[-0.35, +0.40]$
$c_{\bar{W}}/\Lambda^2$	(0.0, 0.0)	$[-0.75, +0.82]$	$[-0.63, +0.69]$	$[-0.49, +0.56]$	$[-0.40, +0.44]$
	(1%, 0.5%)	$[-0.82, +0.81]$	$[-0.68, +0.65]$	$[-0.52, +0.55]$	$[-0.39, +0.45]$
	(5%, 2%)	$[-0.81, +0.82]$	$[-0.67, +0.67]$	$[-0.54, +0.58]$	$[-0.43, +0.43]$
	(10%, 5%)	$[-0.86, +0.84]$	$[-0.62, +0.87]$	$[-0.53, +0.59]$	$[-0.43, +0.44]$
c_{WWW}/Λ^2	(0.0, 0.0)	$[-1.05, +0.81]$	$[-0.79, +0.58]$	$[-0.55, +0.43]$	$[-0.43, +0.33]$
	(1%, 0.5%)	$[-1.07, +0.84]$	$[-0.77, +0.61]$	$[-0.54, +0.44]$	$[-0.43, +0.37]$
	(5%, 2%)	$[-1.02, +0.80]$	$[-0.77, +0.65]$	$[-0.55, +0.54]$	$[-0.42, +0.42]$
	(10%, 5%)	$[-0.99, +0.84]$	$[-0.73, +0.72]$	$[-0.54, +0.56]$	$[-0.42, +0.43]$
$c_{\bar{W}WW}/\Lambda^2$	(0.0, 0.0)	$[-0.87, +0.91]$	$[-0.65, +0.75]$	$[-0.51, +0.54]$	$[-0.43, +0.44]$
	(1%, 0.5%)	$[-0.86, +0.89]$	$[-0.66, +0.75]$	$[-0.53, +0.55]$	$[-0.44, +0.44]$
	(5%, 2%)	$[-0.84, +0.94]$	$[-0.71, +0.76]$	$[-0.57, +0.59]$	$[-0.44, +0.45]$
	(10%, 5%)	$[-0.92, +0.90]$	$[-0.75, +0.76]$	$[-0.62, +0.64]$	$[-0.48, +0.49]$
c_B/Λ^2	(0.0, 0.0)	$[-0.89, +0.95]$	$[-0.69, +0.71]$	$[-0.52, +0.55]$	$[-0.38, +0.41]$
	(1%, 0.5%)	$[-0.91, +0.90]$	$[-0.74, +0.68]$	$[-0.55, +0.55]$	$[-0.37, +0.42]$
	(5%, 2%)	$[-0.91, +0.93]$	$[-0.72, +0.74]$	$[-0.54, +0.58]$	$[-0.41, +0.42]$
	(10%, 5%)	$[-1.00, +1.00]$	$[-0.74, +0.79]$	$[-0.54, +0.65]$	$[-0.42, +0.43]$
c_{WW}/Λ^2	(0.0, 0.0)	$[-1.11, +1.02]$	$[-0.85, +0.79]$	$[-0.61, +0.57]$	$[-0.44, +0.43]$
	(1%, 0.5%)	$[-1.05, +0.99]$	$[-0.79, +0.79]$	$[-0.59, +0.54]$	$[-0.46, +0.44]$
	(5%, 2%)	$[-0.88, +0.87]$	$[-0.79, +0.84]$	$[-0.57, +0.58]$	$[-0.45, +0.47]$
	(10%, 5%)	$[-1.09, +1.08]$	$[-0.75, +0.78]$	$[-0.65, +0.62]$	$[-0.45, +0.45]$
$c_{\bar{W}W}/\Lambda^2$	(0.0, 0.0)	$[-0.91, +0.98]$	$[-0.71, +0.73]$	$[-0.58, +0.59]$	$[-0.43, +0.44]$
	(1%, 0.5%)	$[-0.92, +0.98]$	$[-0.73, +0.79]$	$[-0.57, +0.58]$	$[-0.43, +0.47]$
	(5%, 2%)	$[-1.08, +1.12]$	$[-0.82, +0.77]$	$[-0.64, +0.64]$	$[-0.47, +0.48]$
	(10%, 5%)	$[-1.07, +1.05]$	$[-0.79, +0.87]$	$[-0.65, +0.66]$	$[-0.47, +0.51]$
c_{BB}/Λ^2	(0.0, 0.0)	$[-0.96, +0.92]$	$[-0.73, +0.74]$	$[-0.53, +0.51]$	$[-0.38, +0.38]$
	(1%, 0.5%)	$[-0.94, +0.92]$	$[-0.75, +0.74]$	$[-0.52, +0.50]$	$[-0.36, +0.39]$
	(5%, 2%)	$[-0.98, +1.01]$	$[-0.71, +0.75]$	$[-0.50, +0.53]$	$[-0.37, +0.40]$
	(10%, 5%)	$[-0.98, +0.95]$	$[-0.73, +0.76]$	$[-0.53, +0.57]$	$[-0.38, +0.42]$
$c_{\bar{B}B}/\Lambda^2$	(0.0, 0.0)	$[-0.96, +0.95]$	$[-0.81, +0.75]$	$[-0.54, +0.56]$	$[-0.44, +0.44]$
	(1%, 0.5%)	$[-0.97, +0.99]$	$[-0.81, +0.72]$	$[-0.60, +0.57]$	$[-0.47, +0.46]$
	(5%, 2%)	$[-0.99, +1.00]$	$[-0.79, +0.77]$	$[-0.58, +0.56]$	$[-0.46, +0.46]$
	(10%, 5%)	$[-0.97, +0.96]$	$[-0.78, +0.73]$	$[-0.57, +0.57]$	$[-0.46, +0.50]$

marginalized limits on all nine WCs in Table II for different values of luminosity and systematic errors.

VI. CONCLUSION AND DISCUSSIONS

The effects of higher-order operators are usually translated as an anomalous contribution to SM couplings, or they induce a non-SM vertex altogether, as in neutral triple gauge boson couplings. The single SMEFT operator might also effect different Lorentz structure, and in this article we tried to study a set of dimension-6 operators constructed out of Higgs and weak boson states which have an impact on

triple gauge, Higgs gauge, and quartic gauge couplings. We achieved this by probing a $e^-e^+ \rightarrow 2je^-e^+\mu\bar{\nu}_\mu$ process at $\sqrt{s} = 1$ TeV in the presence of polarized beams. We classified the amplitudes in two types as vector boson scattering (VBS) and triple gauge boson (WWZ) production-like amplitudes, and this classification is obtained by using the kinematic cuts in $m_{e^-e^+}$. We exploit the two-body and three-body spin correlations in VBS and WWZ subprocesses, respectively, to constrain the relevant anomalous couplings. The vector polarization of W bosons and their correlation demands the reconstruction of the identity of

jets from W bosons and is thus neglected, while for the case of the lepton final state, we assume the ideal identification of the flavor of those leptons.

The importance of angular asymmetries constructed from final-state fermions is clearly illustrated by the two-dimensional confidence contours in Fig. 4. For all operator combinations considered, the dominant contribution to the total χ^2 arises from polarization and spin correlation asymmetries rather than from inclusive cross sections. This reflects the strong sensitivity of angular observables to interference effects between the SM and dimension-6 operators. The resulting gains in precision are summarized in the one-parameter 95% confidence level limits presented in Table I, which are found to be stronger than current experimental bounds. A comparison between the two production topologies shows a clear complementarity in their sensitivities. The WWZ process provides tighter constraints on the WCs c_W and c_{WWZ} , while VBS dominates the sensitivity for the remaining seven coefficients. We further perform a global Markov chain Monte Carlo analysis combining cross-section and asymmetry observables from both WWZ and VBS channels to obtain simultaneous constraints on all nine Wilson coefficients. The resulting limits show a weak dependence on systematic uncertainties and are predominantly driven by the assumed integrated luminosity.

The LHC enjoys a larger center-of-mass energy and a larger set of amplitudes due to different interacting particles; thus, for a set of operators inducing momentum-dependent anomalous couplings, the sensitivity from the differential rate could be increased. Nevertheless, we have shown in the current analysis that the large set of observables obtained from the angular distribution of final fermions could complement the lower sensitivity rate in a future electron-positron collider.

ACKNOWLEDGMENTS

A. Subba acknowledges the support of ANRF Grant No. CRG/2023/000580. Authors also acknowledge the various comments by an anonymous referee which have improved the article to its current form.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX A: ORTHONORMAL J BASIS

For completeness, the set of orthonormal J_i matrices constructed out of spin-1 operators are

$$\begin{aligned} S_x &= \sqrt{\frac{1}{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, & S_y &= \frac{1}{i\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \\ S_z &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \end{aligned} \tag{A1}$$

The J'_i s are defined as

$$\begin{aligned} J_1 &= \frac{S_x}{2}, & J_2 &= \frac{S_y}{2}, & J_3 &= \frac{S_z}{2} J_4 = \frac{1}{2} (S_x \cdot S_y + S_y \cdot S_x), \\ J_5 &= \frac{1}{2} (S_x \cdot S_z + S_z \cdot S_x), & J_6 &= \frac{1}{2} (S_y \cdot S_z + S_z \cdot S_y), \\ J_7 &= \frac{1}{2} (S_x \cdot S_x - S_y \cdot S_y), & J_8 &= \sqrt{3} \left(\frac{S_z \cdot S_z}{2} - \frac{\mathbb{I}}{3} \right). \end{aligned} \tag{A2}$$

The J'_i s matrices follow $\text{Tr}[J_i \cdot J_j] = \delta_i^j/2$.

APPENDIX B: NORMALIZED DECAY DENSITY MATRIX

The normalized decay density matrices at the helicity rest frame for spin-1 bosons are given by

$$\Gamma(\lambda, \lambda') = \begin{bmatrix} \frac{1+\delta+(1-3\delta)\cos^2\theta+2\alpha\cos\theta}{4} & \frac{\sin\theta(\alpha+(1-3\delta)\cos\theta)}{2\sqrt{2}} e^{i\phi} & (1-3\delta)\frac{1-\cos^2\theta}{4} e^{i2\phi} \\ \frac{\sin\theta(\alpha+(1-3\delta)\cos\theta)}{2\sqrt{2}} e^{-i\phi} & \delta + (1-3\delta)\frac{\sin^2\theta}{2} & \frac{\sin\theta(\alpha-(1-3\delta)\cos\theta)}{2\sqrt{2}} e^{i\phi} \\ (1-3\delta)\frac{(1-\cos^2\theta)}{4} e^{-i2\phi} & \frac{\sin\theta(\alpha-(1-3\delta)\cos\theta)}{2\sqrt{2}} e^{-i\phi} & \frac{1+\delta+(1-3\delta)\cos^2\theta}{4} 2\alpha\cos\theta \end{bmatrix}, \quad (\text{B1})$$

with the polar θ and azimuthal angles ϕ obtained at the rest frame of the mother particle. Considering the decay of the spin-1 boson (W/Z) to two fermions via a vertex of the form $\bar{f}_1\gamma^\mu(C_L P_L + C_R P_R)f_2 V_\mu$ with real $C_{L,R}$, the expression for the spin-analyzing parameters α and δ are given by [168]

$$\alpha = \frac{2(C_R^2 - C_L^2)\sqrt{1 + (x_1^2 - x_2^2)^2 - 2(x_1^2 + x_2^2)}}{12C_L C_R x_1 x_2 + (C_R^2 + C_L^2)[2 - (x_1^2 - x_2^2)^2 + (x_1^2 + x_2^2)]},$$

$$\delta = \frac{4C_L C_R x_1 x_2 + (C_R^2 + C_L^2)[(x_1^2 + x_2^2) - (x_1^2 - x_2^2)^2]}{12C_L C_R x_1 x_2 + (C_R^2 + C_L^2)[2 - (x_1^2 - x_2^2)^2 + (x_1^2 + x_2^2)]}, \quad (\text{B2})$$

where $x_i = m_i/m$, m_i is the mass of the final fermion and m is the mass of the mother boson. At the high-energy limit, the final fermions decayed from the W/Z are taken to be massless, $x_1 \rightarrow 0$, $x_2 \rightarrow 0$, and then one obtains $\alpha \rightarrow (C_R^2 - C_L^2)/(C_R^2 + C_L^2)$ and $\delta \rightarrow 0$. Further, within the SM, in the decay of W , we have $C_R = 0$, hence $\alpha = -1$.

APPENDIX C: RECONSTRUCTION OF VECTOR POLARIZATION USING FLAVOR-DEPENDENT OBSERVABLES

As discussed in the previous section, several polarization and spin-correlation asymmetries constructed from the decay products of electroweak gauge bosons are odd under parity. Representative examples include single-boson polarization observables such as A_y , A_{xy} , as well as multiboson spin-correlation asymmetries like $A_{x,y}^{12}$, $A_{xy,z}^{12}$, and $A_{x,z,y}^{123}$. These observables vanish identically when averaged over charge- or flavor-unidentified hadronic final states. Consequently, accessing the full spin-density matrix of a hadronically decaying W boson requires identification of the flavor of the daughter jets.

For leptonic decays, such an identification is straightforward due to the unambiguous charge measurement of leptons. In contrast, for hadronic W decays, the experimentally observed final states consist of jets, rendering the determination of the parent quark flavor nontrivial. To address this limitation, we develop a dedicated jet-flavor tagger based on artificial neural networks (ANNs), enabling a probabilistic classification of jets originating from *up*-type and *down*-type quarks. This classification allows the reconstruction of parity-odd polarization and associated

spin correlation observables otherwise inaccessible with flavor-blind analyses.

Separate neural network classifiers are trained for VBS-like and WWZ event topologies, reflecting their distinct kinematic and color-flow properties. In each event, we consider the two hardest jets associated with the hadronic decay of a W boson. At the truth level, jets are matched to their parent light quarks using the angular distance

$$\Delta R_{jq} = \sqrt{(\Delta\eta_{jq})^2 + (\Delta\phi_{jq})^2}, \quad (\text{C1})$$

and the jet is assigned the flavor of the closest quark.

The input feature set is constructed from jet constituents and divided into discrete and continuous variables, following Ref. [72]. Discrete variables include charged-particle multiplicities, charge-separated hadron and lepton counts, and displaced-track information binned according to the proper lifetime of the parent hadrons. Continuous variables encode energy and momentum flow carried by different particle species within the jet, including charge-separated hadronic energy deposits and summed four-momenta.

All input features are normalized to the range $[0, 1]$ using a MINMAXSCALER. The ANN architecture consists of three fully connected hidden layers with 120, 60, and 30 neurons, respectively, employing RELU activations. A single output node with a SIGMOID activation performs binary classification between *up*- and *down*-type jets. The network is trained using the binary cross-entropy loss function and optimized with the ADAM algorithm.

The performance of the tagger is evaluated using a randomly selected 40% test sample over 1000 independent training iterations. The resulting accuracy distributions are

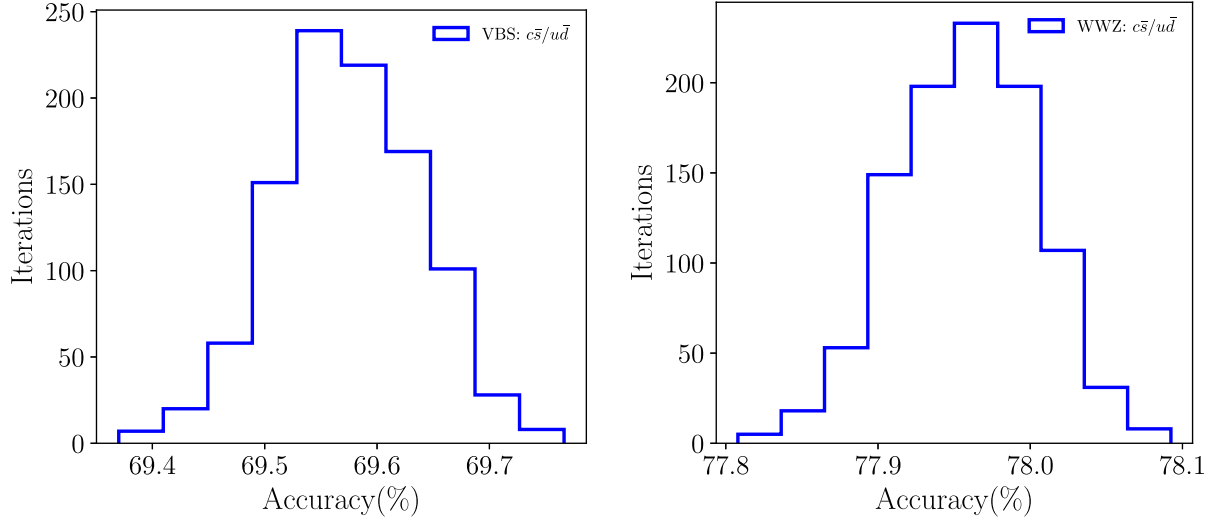


FIG. 6. Distribution of accuracy on classifying the two hardest jets as *up/down*-type jets using neural networks for VBS and WWZ events.

shown in Fig. 6. We obtain an average classification accuracy of approximately 78% for WWZ events and 70% for VBS events. The broader distribution and slightly reduced performance in the VBS case can be attributed to the presence of additional QCD radiation and more complex color connections, which dilute flavor-sensitive jet substructure information.

Despite these limitations, the achieved tagging efficiencies are sufficient to retain significant sensitivity to parity-odd polarization observables and spin correlation effects.

With the probabilistic jet-flavor assignment provided by the ANN, we reconstruct the decay angles of the W boson in its rest frame, enabling the extraction of the vector polarization components (P_x, P_y, P_z) and higher-rank spin correlation observables. These reconstructed observables

are then included in the global likelihood used to derive constraints on WCs.

The effect of including flavor-dependent asymmetries is illustrated in Fig. 7, where we compare the 95% confidence level contours obtained using

- (1) Flavor-blind observables only.
- (2) The full set of observables, including flavor-dependent polarization and spin correlation asymmetries.

For both $(c_W, c_{\bar{W}})$ and $(c_{WW}, c_{\bar{W}W})$ operator pairs, the inclusion of flavor-dependent asymmetries leads to a systematic tightening of the allowed parameter space. Quantitatively, we observe an improvement in the limits at the level of $\mathcal{O}(10\%)$, driven primarily by the additional sensitivity to CP -odd and interference terms that enter linearly in parity-odd observables.

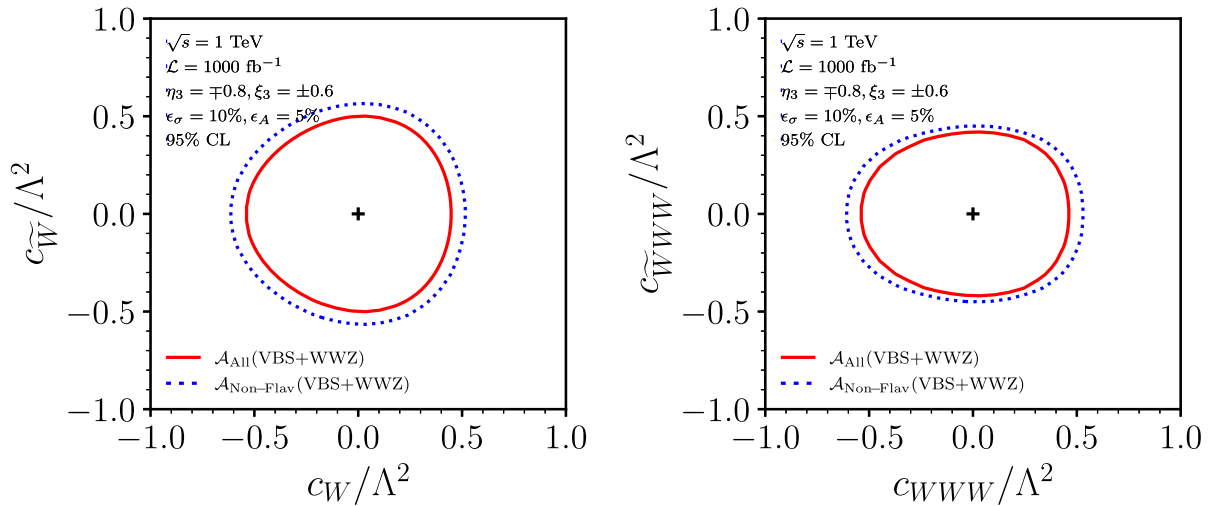


FIG. 7. Two-dimensional 95% CL contours for all asymmetries (red curve) and flavor-blind asymmetries (dashed blue curve) obtained at $\mathcal{L} = 1000 \text{ fb}^{-1}$ with $\epsilon_\sigma = 10\%$ and $\epsilon_A = 5\%$.

These results demonstrate that even imperfect jet-flavor identification significantly enhances the sensitivity to anomalous gauge interactions by restoring access to otherwise suppressed polarization information. The method

therefore provides a viable pathway to exploiting hadronic W decays for precision studies of electroweak symmetry breaking and new physics effects at future lepton colliders.

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