Linearized gravity as edge modes and manifest duality and Lorentz covariance

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We present the first formulation of linearized gravity in four dimensions which is manifestly Lorentz covariant and democratic, i.e., treats the two frames related by electric-magnetic duality on equal footing. It is well known that four-dimensional linearized gravity belongs to a class of singleton representations of the four-dimensional conformal algebra $\mathfrak{so}(2,4)$. Our key insight is viewing this algebra as the isometry of AdS_5 and realising the massless spin-2 field as edge modes of a five-dimensional topological field taking values in a specific finite-dimensional representation of $\mathfrak{so}(2,4)$. The desired four-dimensional action is then found by a covariant boundary reduction procedure.

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I. INTRODUCTION

Electric-magnetic duality is a fascinating symmetry of the eponymous electromagnetism, which rotates the electric and magnetic fields into each other in four spacetime dimensions [1–7]. Linearized gravity is known to enjoy an analogous duality symmetry, which exchanges the linearized Riemann tensor and its dual. The fact that this so-called electric-magnetic duality is an off-shell symmetry of the theory was shown explicitly in [8].

As opposed to electromagnetism and its p-form generalizations, for which Lorentz covariant approaches are available, the only action formulation of linearized gravity with manifest duality is the Hamiltonian formulation by Bunster and Henneaux [8,9]. This formulation does not retain manifest Lorentz covariance—the action is Lorentz invariant, but manifest it is not. To date, no formulation exists which retains both manifest duality and Lorentz covariance—it is widely believed that these are in tension. In this article, we will derive an action formulation of linearized gravity in four spacetime dimensions in which invariance under Lorentz transformations and duality rotations are both manifest. This is made possible by first realising it as a boundary mode for a topological bulk theory.

Let us begin by recalling how electric-magnetic duality is realized in linearized gravity. We consider four-dimensional Minkowski space $\mathbb{R}^{1,3}$, with coordinates denoted by x^μ. In the standard field-theoretic realization, the fundamental degrees of freedom of linearized gravity are described by a rank-2 symmetric tensor $h_{\mu\nu}$ whose field strength is the linearized Riemann tensor

$$R^{\mu\nu}{}_{\rho\sigma} \equiv -2\partial^{[\mu}\partial_{[\rho}h_{\sigma]}{}^{\nu]}. \tag{1}$$

By construction, this field strength satisfies

$$R_{\mu[\nu\rho\sigma]} = 0, \qquad \partial_{[\lambda}R_{\mu\nu]\rho\sigma} = 0, \tag{2}$$

which are known as the Bianchi identities. The dynamics of the helicity-2 field $h_{\mu\nu}$ in vacuum are then encoded in the Ricci-flatness condition (tracelessness of the field strength)

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$$R^{\mu\nu}{}_{\mu\rho} = 0. \tag{3}$$

Now, the totally antisymmetric Levi-Civita tensor ϵ on $\mathbb{R}^{1,3}$ allows us to define the dual Riemann tensor as

$$\star R_{\mu\nu\rho\sigma} \equiv \frac{1}{2}\epsilon_{\mu\nu}{}^{\alpha\beta}R_{\alpha\beta\rho\sigma}. \tag{4}$$

It is straightforward to see this also satisfies the Bianchi identities (2) and the flatness condition (3) on shell. Therefore, SO(2) rotations of the Riemann tensor and its dual

$$\begin{pmatrix} R \\ \star R \end{pmatrix} \mapsto \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} R \\ \star R \end{pmatrix} \tag{5}$$

leave the dynamical content of the theory, i.e., (2) and (3), invariant. In [8], it was in fact shown that there exist off-shell symmetry transformations that reduce to duality rotations of the Riemann tensor and its dual on shell. In other words, not only do duality rotations leave the equations of motion invariant, but they are a genuine symmetry of the theory, which are associated with a nonvanishing conserved charge generating these transformations [10].

Despite the fact that the linearized theory enjoys duality symmetry, the familiar formulation of full nonlinear Einstein gravity clearly treats the fundamental field and its dual asymmetrically. In this case, the choice of one over the other duality frame is justified by the fact that the duality symmetry of the free theory does not extend straightforwardly to the interactions [11–13].

In the linear theory (or at least when it is a good approximation [14]), it is sensible to consider a formulation in which duality symmetry is manifest. This is automatically achieved by any formulation in which the two dual field variables are treated on equal footing—such a formulation is said to be democratic. This is typically realized by doubling the field content and treating the field and its dual independently. For linearized gravity, we relabel

$$R^{(1)}_{\mu\nu\rho\sigma} \equiv R_{\mu\nu\rho\sigma}, \qquad R^{(2)}_{\mu\nu\rho\sigma} \equiv \star R_{\mu\nu\rho\sigma}, \tag{6}$$

and the original equations of motion can then be shown to be fully equivalent to

$$\star R^{(a)} = \varepsilon^{a}{}_{b}R^{(b)}, \tag{7}$$

which is known as a twisted self-duality relation.

The authors of [8,9] (see also [19–22]) derive a democratic action for linearized gravity (in parallel with the formulation for spin-1 [23–26]), by passing to the Hamiltonian formalism and solving the constraints in favor of so-called superpotentials which transform locally under off-shell duality rotations. The price to pay is manifest Lorentz covariance. These are analogous to the noncovariant formulations of chiral degrees of freedom in [23,24,27,28].

However, with an eye toward constructing more general classes of theories, retaining manifest Lorentz invariance is clearly at the very least as useful as making duality invariance manifest. For that reason, in the case of p-forms, several approaches which recover manifest Lorentz covariance have emerged by now [29–42]. More recently, these have been systematically derived by a boundary reduction procedure of topological theories [40,41].

The extension to spin-2 and higher is however nontrivial: The relevant gauge fields take values in a nontrivial representation of the spacetime symmetries, so that dualities mix with the latter. This is an obvious obstruction to formulating such theories in a manifestly Lorentz covariant and democratic manner, and is precisely why no such formulation has been found to date.

In this article, we will present a formulation of linearized gravity in four dimensions which preserves manifest Lorentz covariance and duality. We thereby initiate a program which considerably extends the class of theories which we can describe democratically. At the heart of our construction is the intimate connection of duality with conformal symmetry: Massless fields of spin-two (and higher) in four spacetime dimensions are so-called singleton representations of the conformal algebra $\mathfrak{so}(2,4)$, which are special unitary irreducible representations of $\mathfrak{so}(2,4)$ which stay irreducible when restricted to its subalgebras $\mathfrak{iso}(1,3)$, $\mathfrak{so}(2,3)$, or $\mathfrak{so}(1,4)$. This is despite the fact that the standard description in terms of the field potential does not have conformal symmetry [43–49]. To exploit this property of the singleton representations, we identify a starting point which manifests this $\mathrm{SO}(2,4)$ conformal symmetry in four dimensions as AdS_5 isometry in the bulk. This enables us to construct a bulk topological theory which propagates four-dimensional linearized gravity as an edge mode, a possibility raised in [50]. We are then able to build on the topological boundary reduction procedure developed in [40,41,51]. More precisely, to arrive at the desired massless spin-two field theory on the boundary, we pick the antisymmetric rank-three tensor representation of the conformal group with a specific set of asymptotic boundary conditions for the bulk topological theory. Further generalizations to other singletons in higher dimensions and arbitrary spin will be spelled out in forthcoming work.

II. SETUP

Let us first set the stage by introducing the necessary ingredients for our setup.

Consider $M = \mathrm{AdS}_5$—we are ultimately interested in field theory on the conformal boundary $\partial M = \mathbb{R}^{1,3}$. Let $x^A = (x^\alpha, z)$ and $x^\alpha = (t, x^i)$ be coordinates on AdS_5 and

$\mathbb{R}^{1,3}$ respectively, such that the bulk metric in Poincaré coordinates takes the form

$$\mathrm{d}s^2 = \frac{\eta_{\alpha\beta}\mathrm{d}x^\alpha \mathrm{d}x^\beta + \mathrm{d}z^2}{z^2}. \tag{8}$$

It is also convenient to use embedding space $\mathbb{R}^{2,4} \supset \mathrm{AdS}_5$, on which we will pick coordinates $x^I = (y, x^A)$. In that case, we will trade in the z- and y-coordinates for light cone coordinates with $x^\pm = (z \pm y)/\sqrt{2}$ such that $x^I = (x^-, x^+, x^\alpha)$.

Let us further consider a vector space V of totally antisymmetric 3-tensors of $\mathfrak{so}(2,4)$. To distinguish elements of this vector space (the fiber) from antisymmetric tensors on M (the base) itself, we denote the components of the former using underlined indices, e.g., for $A \in V$,

$$A = \frac{1}{3!} A_{\underline{IJK}} V^{\underline{IJK}}, \tag{9}$$

where $V^{\underline{IJK}}$ are the generators of V. Now, note that V admits the invariant bilinear form

$$\langle A, B\rangle = \frac{1}{3!} A_{\underline{IJK}} B^{\underline{IJK}}. \tag{10}$$

Furthermore, the totally antisymmetric Levi-Civita tensor ε of $\mathfrak{so}(2,4)$ defines an automorphism—the dualization operator $\underline{\star}$, which acts as

$$(\underline{\star} B)_{\underline{IJK}} = \frac{1}{3!} \varepsilon_{\underline{IJK}}{}^{\underline{LMN}} B_{\underline{LMN}}. \tag{11}$$

Since $\underline{\star}^2 = -1$, this automorphism defines a complex structure, which allows us to decompose the vector space V into two orthogonal vector spaces of antisymmetric 2-tensors. This split is not unique, but to make it explicit we can single out a direction by picking an $\mathrm{SO}(2,4)$ vector $w_{\underline{I}}$. Then, we can take these subspaces to be

$$\begin{aligned} W^{(1)} &\equiv \mathrm{Span}_{\mathbb{R}}\{W^{(1)\underline{JK}} \equiv w_{\underline{I}} V^{\underline{IJK}}\}, \\ W^{(2)} &\equiv \mathrm{Span}_{\mathbb{R}}\{W^{(2)\underline{JK}} \equiv w_{\underline{I}} (\underline{\star} V)^{\underline{IJK}}\}. \end{aligned} \tag{12}$$

Finally, we denote the $\mathfrak{so}(2,4)$-covariant derivative by D, which acts on tensors via the action of the commutator as $\mathrm{D}\cdot = \mathrm{d}\cdot + [\Omega, \cdot]$. Here, d is the standard exterior derivative on AdS_5 and Ω is the AdS_5 background connection taking value in the symmetry algebra $\mathfrak{so}(2,4)$ of AdS_5. This connection is flat, so the covariant derivative D is nilpotent. Explicit components of the connection and covariant derivatives acting on tensors are given in Appendix A.

III. DEMOCRATIC LINEARIZED GRAVITY

We are now in a position to present the theory which will provide a manifestly Lorentz covariant and democratic description of linearized gravity in four dimensions.

A. Bulk description

The theory of interest is a topological field theory on M, whose boundary degrees of freedom subject to certain constraints will eventually describe linearized gravity in a democratic manner.

Our starting point for this is the field B, which is a two-form in the base AdS_5 spacetime and takes values in the vector space V—it has indices in both the base M and the fiber V. Our theory of interest is described by the following action

$$S = \int_M \langle \underline{\star} B \wedge \mathrm{D}B\rangle + \frac{1}{2}\int_{\partial M} \langle B \wedge \star B\rangle. \tag{13}$$

The bulk part of this is a Chern-Simons term with the covariant derivative D with respect to the flat $\mathfrak{so}(2,4)$-connection. The boundary term will be essential for what follows.

The classical equations of motion in the bulk and boundary are, respectively,

$$\mathrm{D}B = 0, \qquad \star B + \underline{\star} B = 0. \tag{14}$$

The bulk field is closed and therefore pure gauge, and the degrees of freedom in this theory hence reside purely within the boundary. We further note that the boundary equation of motion already has the suggestive form of the twisted self-duality relation (7) we are after.

On its own, however, (14) propagates too many degrees of freedom, some of which are necessarily not unitary. To this end, we impose a specific set of asymptotic boundary conditions on the components of B which differ for its various base and fiber components. We claim this is precisely the necessary on-shell condition which leaves us with the correct degrees of freedom. This is the higher-dimensional analog of the conditions imposed in [51–54].

B. Boundary reduction

Our goal is now to describe the boundary dynamics directly covariantly. To that end, we perform the Lorentz covariant boundary reduction introduced in [40]. For this, instead of fixing a time direction (along which the usual Hamiltonian reduction is performed), we define a foliation of ∂M (extended into M) by picking a nowhere-null closed one-form v. Thanks to this, we can define the Lorentz covariant decomposition

$$B = C + v \wedge \tilde{R}, \tag{15}$$

for $C \in \Omega^2(M)$ with $\iota_v C = 0$, and $\tilde{R} \in \Omega^1(M)$, both taking values in V. Substituting this into the bulk part of the action (13), we find that this yields

$$S_{\text{bulk}} = \int_M \langle \underline{\star} C \wedge \mathrm{D}C - 2v \wedge \tilde{R} \wedge \underline{\star} \mathrm{D}C \rangle + \int_{\partial M} \langle v \wedge \tilde{R} \wedge \underline{\star} C \rangle. \tag{16}$$

In the bulk, $\tilde{R}$ acts as a Lagrange multiplier enforcing the condition that $v \wedge \mathrm{D}C = 0$. Since D is nilpotent, this is locally solved by $C = \mathrm{D}A + v \wedge P$, where $A, P \in \Omega^1(M)$ are V-valued [55]. This amounts to taking our Ansatz (15) to be

$$B = H \equiv \mathrm{D}A + v \wedge R, \qquad R = \tilde{R} + P. \tag{17}$$

We will frequently make use of the shorthand $F \equiv \mathrm{D}A$. When plugged back into the bulk, this Ansatz reduces it to a total-derivative term, and we are left with the following pure boundary action:

$$S_{\text{bdry}} = \frac{1}{2} \int_{\partial M} \langle H \wedge \star H + 2v \wedge R \wedge \underline{\star} F \rangle. \tag{18}$$

As noted previously, this should be supplemented with a set of asymptotic fall-off conditions. We will use the symbol $\overset{!}{=}$ to indicate that we are manually imposing the equivalence of the left- and right-hand side as a constraint. As we shall see shortly, the following conditions turn out to pick out the correct four-dimensional theory

$$v \wedge F_{\underline{+\alpha\beta}} \overset{!}{=} \mathcal{O}(z^0), \quad v \wedge F_{\underline{-+\alpha}}, v \wedge F_{\underline{\alpha\beta\gamma}} \overset{!}{=} \mathcal{O}(z), \tag{19}$$

where the leading power of z of each field is determined by the conformal dimension of its fiber. These can be imposed directly in the boundary action via Lagrange multipliers

$$S_{\text{constr.}} = \int_{\partial M} v \wedge \chi \wedge F, \tag{20}$$

with $\chi^{\underline{+\alpha\beta}} = 0$. In Appendix B, we show that χ is trivial on shell. Our theory is therefore specified by the total action

$$S = S_{\text{bdry}} + S_{\text{constr.}}. \tag{21}$$

Subtleties that $S_{\text{constr.}}$ introduces to the forthcoming discussions regarding the dynamical content are postponed to Appendix B.

C. Dynamical content

Let us begin by examining the boundary dynamics described by the action (18) itself.

First note that the action (18) has various gauge symmetries. These, separately, comprise of

(i) D-exact shifts in A: $\delta A = \mathrm{D}\alpha$,
(ii) v-exact shifts in R: $\delta R = v \wedge \rho$, and
(iii) Combined shifts which leave B invariant: $\delta A = v\lambda$, $\delta R = \mathrm{D}\lambda$.

with α, ρ, and λ taking values in V. This means that determining the physical content of the theory is not straightforward.

Let us make a few more comments on the boundary equation of motion. From the equation for R, we see

$$v \wedge (\star H + \underline{\star} H) = 0. \tag{22}$$

Since $v^2 \neq 0$, we can follow the steps outlined in [56,57] to show that the expression within square brackets has to vanish

$$\star H + \underline{\star} H = 0. \tag{23}$$

The equation for A is $\mathrm{D}(\star H - \underline{\star} H) = 0$, which, using (23), simplifies to

$$v \wedge \mathrm{D}R = 0. \tag{24}$$

This means we can fix the gauge freedom in R to rewrite (23) as

$$\star \mathrm{D}A + \underline{\star} \mathrm{D}A = 0, \tag{25}$$

as expected from the boundary equation of motion (14). This is precisely the equation of motion we are after, i.e., a twisted self-duality relation of the form (7). In Appendix B, we show the same conclusion holds when the constraints are accounted for in (21).

D. Lorentz covariance

For Lorentz covariance of the action defined by (18) to be manifest, the one-form v must not be fixed. It therefore needs to be dynamical, with its equation of motion $\mathrm{d}v = 0$ being imposed by an additional Lagrange multiplier ξ [41]:

$$S_v = \int_{\partial M} \xi \wedge \mathrm{d}v. \tag{26}$$

However, if this were to cause additional degrees of freedom to be propagated, the description of our theory would be spoiled. Fortunately, the resulting theory has a gauge symmetry given by shifts of v by exact forms, rendering v pure gauge. An equivalent perspective (closer in spirit to [30,39,56]) is that, taking $v = \mathrm{d}a$ to be exact by Poincaré's Lemma (when ξ goes on shell), a possesses a shift symmetry. For the action (18) alone, this follows directly in parallel to the spin-1 case [56]. For the full action (21), this argument requires suitable adjustments—this is described in Appendix B.

We are now ready to state our central result. The full theory, defined by the action (18) subject to the on-shell conditions (19), is a manifestly Lorentz covariant and

democratic formulation of linearized gravity in four dimensions.

IV. NONCOVARIANT REDUCTION

To make the connection to linearized gravity precise, we will now further reduce the action (18) supplied with the asymptotic conditions (19) to the Bunster–Henneaux action derived in [8]. It is known that this is equivalent to the linearized Einstein-Hilbert action, so this effectively demonstrates that the degrees of freedom we are describing are indeed those of the free massless spin-two field.

To do so, we will now pick $v = \mathrm{d}t$ and $w_{\underline{I}} = \delta^0_{\underline{I}}$. This defines a foliation of the conformal boundary as $\mathbb{R}_t \times \Sigma_t$, where Σ_t are spatial slices which are normal to v. We will see that with this choice our theory will reproduce their action after solving the constraints (19).

A. Gauge-fixing

The gauge symmetries described in Sec. III C are associated with zero modes of the action. To proceed with the noncovariant reduction, our first step is therefore to fix the gauge by taking $\iota_{\partial_t} A = \iota_{\partial_t} R = 0$, such that only A and R with just spatial base components are nonzero. With explicit base indices (and implicit fiber indices), the action then takes the form

$$S = \int_{\partial M} \mathrm{d}^4x \left\langle \frac{1}{2}\varepsilon^{ijk} R_i (\underline{\star F})_{jk} + \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} + R_i F^{0i} - \frac{1}{2} R_i R^i \right\rangle, \tag{27}$$

where we have taken $\varepsilon_{0ijk} \equiv \varepsilon_{ijk}$. The field R appears quadratically and is hence auxiliary, so we may integrate it out. Doing so, we are left with

$$S = \frac{1}{2}\int_{\partial M} \mathrm{d}^4x \langle F_{ij} F^{ij} + \varepsilon^{ijk} F^0{}_i (\underline{\star F})_{jk} \rangle. \tag{28}$$

Note the similarity of this with the Schwarz–Sen action [24] (*modulo* the suppressed $\mathrm{SO}(2,4)$ indices).

B. Asymptotic conditions

We will now solve the asymptotic conditions (19) to isolate the correct degrees of freedom. As promised, this will reduce the action (28) to the one in [8]. For this, it is crucial to make the symmetries involving just spatial indices of both the base M and the fiber V explicit. We do so by using the decomposition of the vector space $V = W^{(1)} \oplus W^{(2)}$ with respect to w from (12). For a consistent boundary theory, it is also necessary to perform an asymptotic expansion of the fields in z, as defined in (A6). In the process of this, we also relabel various fiber components of A by $\{Z, S, \omega, \phi\}$.

Now, at a practical level, the constraints (19) set the spatial components of $F_{\underline{\alpha\beta\gamma}}$, $F_{\underline{+\alpha\beta}}$, and $F_{\underline{-+\alpha}}$ (restricted to ∂M) to zero. As an immediate consequence, we find that the only terms in the action that survive are

$$S = -\frac{1}{2}\int_{\partial M} \mathrm{d}^4x \varepsilon_{ab}\varepsilon^{ijk} F^{(a)}{}_{\underline{+m}0i} F^{(b)}{}_{\underline{-}}{}^{\underline{m}}{}_{jk}. \tag{29}$$

Before solving the asymptotic conditions further, let us first make our life easier by making use of the gauge redundancy $\delta A = \mathrm{D}\Lambda$, for a gauge parameter $\Lambda \in \Omega^0(M)$. The background connection Ω relates base and fiber indices, so we will now begin to see expressions which mix the two types of indices—this will eventually allow us to remove the distinction. We denote the gauge parameters for the fields $\{Z, S, \omega, \phi\}$ by $\{\xi, \kappa, \rho, \sigma\}$ respectively (with the appropriate rescalings in z). First note that the gauge transformation on $\phi^{(a)}$ takes the form

$$\delta\phi^{(a)}_j = \partial_j \sigma^{(a)} + 4\kappa^{(a)}_{\underline{j}}. \tag{30}$$

It is clear that we can use the freedom in $\kappa^{(a)}$ to set $\phi^{(a)} = 0$ fully. Similarly, a gauge transformation in $Z^{(a)}$ takes the form

$$\delta Z^{(a)}{}_{\underline{ij}} = \partial_j \xi^{(a)}{}_{\underline{i}} - 2\rho^{(a)}{}_{\underline{ji}} + 2\delta_{ij}\sigma^{(a)}. \tag{31}$$

Here, we can use the freedom in our choice of $\rho^{(a)}$ (which is antisymmetric) to fix $Z^{(a)}$ to be symmetric (it is now no longer necessary to distinguish its base and fiber indices).

Let us now return to the asymptotic conditions (19). In their most explicit form, these can be read off from the components of F given in (A8). We first consider the constraint on $F_{\underline{+\alpha\beta}}$. From (A8a), we see this allows us to solve for $\omega^{(a)}$ in terms of $Z^{(a)}$. Then, since $Z^{(a)}$ is symmetric, the only nonzero projection of $\partial Z^{(a)}$ and hence ω is of [2, 1] Young-type symmetry. By Bianchi's identity

$$\omega^{(a)\underline{jk}}{}_{\underline{i}} = 2\partial^{[j} Z^{(a)}{}_i{}^{k]}. \tag{32}$$

Next, consider the asymptotic condition on $F_{\underline{+-\alpha}}$, given by (A8b). This is an algebraic constraint on $S^{(a)}$, forcing it to be symmetric (it is also now no longer necessary to distinguish its base and fiber indices). Finally, we have the constraint on $F_{\underline{\alpha\beta\gamma}}$, given in (A8c). This equation has two sets of antisymmetric spatial indices. Since there is no rank-4, two-column Young diagram for SO(3), the only content in this equation is therefore in its trace, and solving this algebraically gives

$$S^{(a)}_{ij} = -\frac{1}{4}(\Delta Z^{(a)} - \partial_m\partial_n Z^{(a)mn})\delta_{ij} + \frac{1}{2}(\partial_i\partial_j Z^{(a)} + \Delta Z^{(a)}_{ij}) - \partial^k\partial_{(i}Z^{(a)}_{j)k}, \quad (33)$$

where $\Delta\cdot \equiv \partial_i\partial^i\cdot$ and $Z \equiv Z^i{}_i$. This fully solves the constraints (19) and allows us to write the unconstrained components of $F_{\underline{+\alpha\beta}}$ as

$$\lim_{z\to 0} z F^{(a)}{}_{\underline{+i0j}} = \partial_0 Z^{(a)}{}_{ij} + \frac{1}{2}\varepsilon_i{}^{mn}\varepsilon^a{}_b\omega^{(b)}{}_{\underline{mnj}}. \quad (34)$$

Plugging everything back into the action (29), we arrive at

$$S = -\int_{\partial M} \mathrm{d}^4x \left[\varepsilon^{ijk}\varepsilon_{ab}\partial_0 Z^{(a)}{}_{mi}\partial_j S^{(b)m}{}_k + \frac{1}{2}\delta_{ab}S^{(a)\ell}{}_i\delta^{ijk}_{\ell mn}\partial_j\omega^{(b)\underline{mn}}{}_k\right], \quad (35)$$

after integration by parts. It is straightforward to see that this is precisely the Bunster–Henneaux action presented in [8], as promised. The fields $Z^{(a)}$ precisely correspond to the so-called superpotentials in their formulation. It is a democratic formulation: Manifest invariance under SO(2) rotations between the $Z^{(a)}$'s is implied because ε and δ are invariant tensors of SO(2).

V. CONCLUDING REMARKS

In this article, we presented the first formulation of linearized gravity in four dimensions with manifest Lorentz covariance and democracy. To do so, we leverage the fact the free massless spin-2 fields in four dimensions belong to a class of singleton representations of $\mathfrak{so}(2,4)$. In particular, we first use this insight to realize linearized gravity as the edges modes of a Chern-Simons theory for a two-form in AdS_5 valued in the antisymmetric three-tensor representation of $\mathfrak{so}(2,4)$. The boundary reduction procedure introduced in [40] then leads to a manifestly Lorentz covariant boundary action (18). Our central claim is then that, when supplied with the on-shell conditions (19), the resulting boundary theory is a democratic description of linearized gravity in four dimensions. We demonstrated this by breaking Lorentz covariance appropriately to recover the familiar democratic formulation of linearised gravity in four dimensions given by Bunster and Henneaux [8]. Our work resolves this longstanding open problem and lends further credence to the idea that manifest Lorentz covariance and duality are not incompatible.

In higher dimensions, there are multiform fields with rectangular Young diagram $[s^p]$, which are the natural generalizations of spin-s singleton fields in $D = 4$ to $D = 2p + 2$ spacetime dimensions. This relies on the special property of how the rectangular $(p + 1)$-row Young diagram representations of $\mathrm{SO}(2, 2p)$ split to two isomorphic $\mathrm{SO}(1, 2p)$ representations corresponding to rectangular diagrams with p rows. The case with generic s and p will be addressed in follow-up work. For $p = 2$, this describes massless higher-spin fields in four dimensions, of which a democratic description was given in [58]. For $s = 2$ and $p = 3$, these are conformal biforms featuring in so-called (4, 0)-theory [59–61] (extension to arbitrary p is given in [62]).

The above corresponds to unitary conformal representations (singletons), but the pattern can be further generalized to more generic conformal representations, such as Fradkin–Tseytlin fields [63], Weyl-like fields of [64] (which will be shown to describe conformal representations in four dimensions), and generalizations to higher dimensions.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: COVARIANT DERIVATIVES

The generators $J_{\underline{MN}}$ of $\mathfrak{so}(2,4)$ satisfy

$$[J_{\underline{MN}}, J_{\underline{KL}}] = \eta_{\underline{MK}}J_{\underline{LN}} + \ldots, \quad (A1)$$

where $\eta_{\underline{IJ}}$ is the flat metric on $\mathbb{R}^{2,4}$. More precisely, one can show that the components of an arbitrary element $A \in V$ are eigenvectors of $J_{\underline{-+}}$ with

$$[J_{\underline{+-}}, A_{\underline{\alpha\beta\gamma}}] = 0 \quad (A2a)$$

$$[J_{\underline{+-}}, A_{\underline{\pm\alpha\beta}}] = \pm A_{\underline{\pm\alpha\beta}} \quad (A2b)$$

$$[J_{\underline{+-}}, A_{\underline{+-\alpha}}] = 0, \quad (A2c)$$

and transform under $J_{\underline{\pm\alpha}}$ as

$$[J_{\underline{\pm\lambda}}, A_{\underline{\alpha\beta\gamma}}] = 3\eta_{\underline{\lambda[\alpha|}} A_{\underline{\pm|\beta\gamma]}} \tag{A3a}$$

$$[J_{\underline{\pm\lambda}}, A_{\underline{\mp\alpha\beta}}] = -A_{\underline{\lambda\alpha\beta}} \pm 2\eta_{\underline{\lambda[\alpha|}} A_{\underline{-+|\beta]}} \tag{A3b}$$

$$[J_{\underline{\pm\lambda}}, A_{\underline{\pm\alpha\beta}}] = 0 \tag{A3c}$$

$$[J_{\underline{\pm\lambda}}, A_{\underline{-+\alpha_1\ldots\alpha}}] = A_{\underline{\pm\lambda\alpha}}. \tag{A3d}$$

Next, note the nonzero components of the connection one-form $\Omega = \frac{1}{2}\Omega^{\underline{MN}} J_{\underline{MN}}$ are

$$\Omega^{\underline{+-}} = \frac{\mathrm{d}z}{z}, \qquad \Omega^{\underline{-\alpha}} = \sqrt{2}\frac{\mathrm{d}x^\alpha}{z}. \tag{A4}$$

Putting everything together, the components of a component derivative $F \equiv \mathrm{D}A$ can be shown to be

$$F_{\underline{+\alpha\beta}} = \mathrm{d}A_{\underline{+\alpha\beta}} + z^{-1}\mathrm{d}z \wedge A_{\underline{+\alpha\beta}} - \sqrt{2}z^{-1}\mathrm{d}x^\lambda \wedge A_{\underline{\lambda\alpha\beta}} - \sqrt{2}2z^{-1}\mathrm{d}x_{[\alpha|} \wedge A_{\underline{-+|\beta]}} \tag{A5a}$$

$$F_{\underline{\alpha\beta\gamma}} = \mathrm{d}A_{\underline{\alpha\beta\gamma}} + 3\sqrt{2}z^{-1}\mathrm{d}x_{[\alpha|} \wedge A_{\underline{-|\beta\gamma]}} \tag{A5b}$$

$$F_{\underline{+-\alpha}} = \mathrm{d}A_{\underline{+-\alpha}} - \sqrt{2}z^{-1}\mathrm{d}x^\lambda \wedge A_{\underline{-\lambda\alpha}} \tag{A5c}$$

$$F_{\underline{-\alpha\beta}} = \mathrm{d}A_{\underline{-\alpha\beta}} - z^{-1}\mathrm{d}z \wedge A_{\underline{-\alpha\beta}}. \tag{A5d}$$

1. Asymptotic expansions

Consider the asymptotic expansions

$$A_{\underline{+\alpha\beta}}(z, x^\mu) = z^{-1}[Z_{\underline{\alpha\beta}}(x^\mu) + \mathcal{O}(z)], \tag{A6a}$$

$$A_{\underline{\alpha\beta\gamma}}(z, x^\mu) = \frac{1}{\sqrt{2}}\omega_{\underline{\alpha\beta\gamma}}(x^\mu) + \mathcal{O}(z), \tag{A6b}$$

$$A_{\underline{-+\alpha}}(z, x^\mu) = \frac{1}{\sqrt{2}}\phi_{\underline{\alpha}}(x^\mu) + \mathcal{O}(z), \tag{A6c}$$

$$A_{\underline{-\alpha\beta}}(z, x^\mu) = z^{+1}[S_{\underline{\alpha\beta}}(x^\mu) + \mathcal{O}(z)]. \tag{A6d}$$

The choice of scaling in z reflects the eigenvalues of the components of elements in V under the dilatations given in (A2). With this, the leading-order terms in (A5) become

$$z^{+1}F_{\underline{+\alpha\beta}} = \mathrm{d}Z_{\underline{+\alpha\beta}} - \mathrm{d}x^\lambda \wedge \omega_{\underline{\lambda\alpha\beta}} - 2\mathrm{d}x_{[\alpha} \wedge \phi_{\underline{\beta]}} + \mathcal{O}(z) \tag{A7a}$$

$$F_{\underline{\alpha\beta\gamma}} = \frac{1}{\sqrt{2}}\mathrm{d}\omega_{\underline{\alpha\beta\gamma}} + 3\sqrt{2}\mathrm{d}x_{[\alpha} \wedge S_{\underline{\beta\gamma]}} + \mathcal{O}(z) \tag{A7b}$$

$$F_{\underline{+-\alpha}} = -\frac{1}{\sqrt{2}}\mathrm{d}\phi_{\underline{\alpha}} - \sqrt{2}\mathrm{d}x^\lambda \wedge S_{\underline{\lambda\alpha}} + \mathcal{O}(z) \tag{A7c}$$

$$z^{-1}F_{\underline{-\alpha\beta}} = \mathrm{d}S_{\underline{\alpha\beta}} + \mathcal{O}(z). \tag{A7d}$$

We can further write the fields in terms of the decomposition (12). Now making base indices explicit, (A7) reads

$$\frac{z}{2}F^{(a)}{}_{\underline{+i}}{}^{jk} = \partial^{[j}Z^{(a)}{}_i{}^{k]} - \omega^{(a)[\underline{j}}{}_{\underline{i}}{}^{k]} - \delta_i^{[j}\phi^{(a)\,k]} \tag{A8a}$$

$$\frac{1}{\sqrt{2}}F^{(a)}{}_{\underline{+-}}{}^{ij} = -\partial^{[i}\phi^{(a)j]} - 2S^{(a)[\underline{ij}]} \tag{A8b}$$

$$\frac{1}{\sqrt{2}}F^{(a)}{}_{\underline{ij}}{}^{mn} = \partial^{[m}\omega^{(a)}{}_{\underline{ij}}{}^{n]} + 4\delta^{[m}_{[i}S^{(a)}{}_{\underline{j]}}{}^{n]} \tag{A8c}$$

$$\frac{1}{2z}F^{(a)}{}_{\underline{-i}}{}^{jk} = \partial^{[j}S^{(a)}{}_{\underline{i}}{}^{k]}. \tag{A8d}$$

APPENDIX B: CONSTRAINTS

The theory of interest is described by the action (18) subject to the on-shell conditions (19). Although there is *a priori* no issue with implementing these manually, we may choose to implement these using a Lagrange multiplier χ as in (20). In this Appendix, we will show how the discussions in Secs. III C and III D follow through regardless.

We begin by arguing the boundary equation of motion (25) should continue to hold. First note, we had to pick χ to be projected onto the subspace with $\chi^{\underline{+\alpha\beta}} = 0$. Therefore in the subspace orthogonal to χ, the discussion in the main text applies straightforwardly. The nontrivial part is the subspace parallel to χ.

Let us denote the components of A parallel to χ, i.e., for $A_{\underline{\alpha\beta\gamma}}$, $A_{\underline{-\alpha\beta}}$, and $A_{\underline{-+\alpha}}$, collectively as $A_\parallel$. First note the equations of motion for R are the same, i.e., (23) suitably projected. Next, the equation for χ are precisely the constraints (19) themselves

$$v \wedge H_\parallel = 0. \tag{B1}$$

Because these are closed under fiber dualization, the fiber dual of this should also hold. Together with the equation of R, (23), we then have

$$v \wedge \underline{\star}H_\parallel = 0 \Rightarrow v \wedge \star H_\parallel = 0. \tag{B2}$$

For the same reasons as mentioned below (22), this, combined with the original form of the constraint, implies $H_\parallel = 0$ itself. From this, the projection of (24) onto the subspace parallel to χ follows, and we infer $R_\parallel$ is pure gauge. Now, the equation for $A_\parallel$ is

$$\mathrm{D}(\star H_{\|} - \underline{\star} H_{\|} + v \wedge \chi) = 0. \tag{B3}$$

However, we just showed that $H_{\|} = 0$, so

$$v \wedge \mathrm{D}\chi = 0. \tag{B4}$$

Now, note that the action (21) is also invariant under gauge transformations $\delta\chi = \mathrm{D}\alpha + v \wedge \beta$. Therefore, just as for R, (B4) implies that χ is pure gauge.

The observation that $H_{\|} = 0$ also enables to argue that the one-form $v = \mathrm{d}a$ has a shift-symmetry by exact forms. For dynamical v, the equation of motion for a is

$$\mathrm{d}\langle R \wedge (\star H + \underline{\star} H) - \chi \wedge H_{\|} \rangle = 0. \tag{B5}$$

This vanishes identically as a consequence of the other equations of motion, implying a has a shift symmetry.

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