

Scattering of massive spin-2 field via graviton exchanges with different spin fields and the long range gravitational potential

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In this work, we compute the graviton mediated scattering amplitude of a massive spin-2 Fierz-Pauli field with various other massive spin fields, and in the nonrelativistic limit, find out the corresponding two-body gravitational potentials. The massive spin-2 field does not represent gravity here. The theory of gravity is taken to be the usual massless general relativity, and the massive spin-2 field is taken as a test quantum field coupled to gravity via the standard minimal prescription. We first compute the tree level 2-2 scattering of a massive spin-2 field with massive scalar, spin-1, and spin-1/2 fields with one graviton exchange. Leading Newton potential, as well as the subleading spin or polarization dependent terms at $\mathcal{O}(G)$ have been computed. We also consider the next to the leading order [$\mathcal{O}(G^2)$] scattering of the massive spin-2 field with a massive scalar, and demonstrate the spin independent, spherically symmetric leading part of the two body gravitational potential. The present paper can be considered as an attempt to compute the gravitational potential in the context of a higher spin field theory.

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I. INTRODUCTION

One of the most challenging unsolved tasks of modern physics is to unify the four fundamental forces of nature. Quantizing gravity is supposed to be an integral part of this program. However, it has been well known for long that gravity is not renormalizable perturbatively when treated as a quantum field [1–12], suggesting there must be some alternative, or more fundamental description of it that remains hitherto unknown. Despite this drawback, there has been a lot of effort given by the community to investigate such a perturbative quantum gravity framework at the first few orders and to produce physical predictions, with the hope that they will be testable in a not too far away future. Most importantly, such predictions, if or when tested, can tell us whether the *graviton*, i.e., a massless spin-2 particle as the quantum of gravity, makes any sense at all. If it indeed does, one may also expect to get insight about extending this perturbative framework into a bigger and complete theory of quantum gravity. We refer our reader to, e.g., [13–16] and references therein for various discussions on and computations of such aspects.

A very interesting and physically well motivated such perturbative computation is to investigate the gravitational potential between two particles, via the nonrelativistic limit of the 2-2 scattering amplitude with graviton exchanges. We refer our reader to [17–31] and references therein for such computations for massive spin-0, spin-1/2, and spin-1 fields. For fields with spin, polarization or spin dependence of the gravitational potential were computed. We further refer our reader to [32–53] and references therein for various computations and predictions on perturbative quantum gravity, including computation of gravitational potential and bending of light using the effective field theory framework at various post-Minkowski orders.

In this paper, we wish to compute the scattering of a massive spin-2 Fierz-Pauli field [54–63] with various other spin fields, and compute the two-body gravitational potential from the nonrelativistic limit of the 2-2 scattering amplitude. We will take the gravity to be Einstein's general relativity, with the usual massless spin-2 graviton as its quanta. In other words, the gravity will *not* be taken to be massive here. The massive spin-2 field will be taken as a test field minimally and covariantly coupled to Einstein's gravity. The present analysis should be interpreted as probing a spin-2 field as a massive particle, analogous to the Proca (massive spin-1) field. In other words, along with the usual graviton, we are considering another spin-2 field here as an independent quantum field. The chief physical motivation behind such a study is to understand the perturbative quantum gravity effect in the presence of a massive higher spin field theory. Indeed, such consideration

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of a massive spin-2 meson field is not new, and can be seen in earlier references, e.g., [55] in the context of black hole no hair theorems. See also [56] for a treatment of a black hole as a massive spin-2 condensate. It was argued in [59] that any massive spin-2 field theory cannot be unique. In [61], the Hamiltonian structure of the Pauli-Fierz theory can be seen in curved spacetimes. Graviton-photon production in the presence of a massive spin-2 field can be seen in [62]. Finally, we also refer our reader to, e.g., [64–69] and references therein for various aspects of massive gravity theory.

Thus, this paper can be taken as an attempt to compute the gravitational potential in the context of a higher spin field theory, which happens to be the Fierz-Pauli field presently. The rest of the paper is organized as follows. In the next Section, we briefly review the basic setup we will be working in. In Sec. III we compute the tree level 2-2 scattering of a massive spin-2 field, respectively with massive spin-0, spin-1, and spin-1/2 fields via one graviton exchanges. In the nonrelativistic limit of the scattering amplitude, we will compute the tree level $\mathcal{O}(G)$

gravitational potential. Various spin and polarization dependent terms will be explicitly demonstrated. We further extend the massive spin-2-spin-0 scattering to $\mathcal{O}(G^2)$ in Sec. IV. We explicitly demonstrate only the polarization independent, leading spherically symmetric parts of the potential in this case. Finally, we conclude in Sec. V. We will work with the mostly positive signature of the metric in $3 + 1$ -dimensions and will set $c = \hbar = 1$. We will also use the notation for symmetrization: $X_{(\mu\nu)} = X_{\mu\nu} + X_{\nu\mu}$.

II. THE BASIC INGREDIENTS

We now wish to outline the basic setup we will be working in. The general, minimally coupled action reads

$$S = \int d^4x \sqrt{-g} \left[\frac{2R}{\kappa^2} + \mathcal{L}_M \right] \quad (1)$$

where $\kappa^2 = 32\pi G$ and $\mathcal{L}_M$ collectively represent the matter fields. Perturbative expansions around the flat or Minkowski background for our purpose read,

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}; \quad g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^\mu{}_\alpha h^{\alpha\nu} + \mathcal{O}(\kappa^3); \quad \sqrt{-g} = 1 + \frac{\kappa h}{2} + \frac{\kappa^2 h^2}{8} - \frac{\kappa^2}{4} h_{\mu\nu} h^{\mu\nu} + \mathcal{O}(\kappa^3),$$

$$\Gamma_{\nu\rho}^\mu = \frac{\kappa}{2} \eta^{\mu\alpha} (\partial_\nu h_{\rho\alpha} + \partial_\rho h_{\nu\alpha} - \partial_\alpha h_{\nu\rho}) - \frac{\kappa^2}{2} h^{\mu\alpha} (\partial_\nu h_{\rho\alpha} + \partial_\rho h_{\nu\alpha} - \partial_\alpha h_{\nu\rho}) + \frac{\kappa^3}{2} h^{\mu\beta} h_\beta{}^\alpha (\partial_\nu h_{\rho\alpha} + \partial_\rho h_{\nu\alpha} - \partial_\alpha h_{\nu\rho}) + \mathcal{O}(\kappa^4). \quad (2)$$

In the de Donder gauge,

$$\partial_\mu \left(h^{\mu\nu} - \frac{1}{2} \eta^{\mu\nu} \kappa h \right) = 0, \quad (3)$$

the free graviton propagator, appropriate for the mostly positive metric signature reads

$$\Delta_{\mu\nu\alpha\beta}(k) = -\frac{i\mathcal{P}_{\mu\nu\alpha\beta}}{k^2} \quad (4)$$

where

$$\mathcal{P}_{\mu\nu\alpha\beta} := \frac{1}{2} (\eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\mu\beta} \eta_{\nu\alpha} - \eta_{\mu\nu} \eta_{\alpha\beta}). \quad (5)$$

A. The massive spin-2 setup

Following [54–56, 58–63, 68, 69], we now wish to review very briefly the basic structure of a massive spin-2 field theory coupled to gravity. The general action reads

$$S_{\text{massive spin-2}} = \int d^4x \sqrt{-g} \left[-\frac{1}{2} (\nabla_\lambda H^{\mu\nu}) (\nabla^\lambda H_{\mu\nu}) + (\nabla^\lambda H_{\mu\lambda}) (\nabla_\alpha H^{\mu\alpha}) - (\nabla_\lambda H) (\nabla_\mu H^{\mu\lambda}) \right. \\ \left. + \frac{1}{2} (\nabla_\lambda H) (\nabla^\lambda H) - \frac{1}{2} M^2 (H_{\mu\nu} H^{\mu\nu} - H^2) \right] \quad (6)$$

where $H_{\mu\nu}$ is the symmetric Fierz-Pauli massive spin-2 field, all the contractions are done with respect to $g_{\mu\nu}$, with $H = g_{\mu\nu} H^{\mu\nu}$. “ ∇ ” is the covariant derivative defined with respect to the metric $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$. The equation of motion corresponding to Eq. (6) in the flat background reads

$$\partial^2 H_{\mu\nu} - \partial_\nu \partial^\alpha H_{\mu\alpha} + \eta_{\mu\nu} \partial_\alpha \partial_\beta H^{\alpha\beta} + (\partial_\mu \partial_\nu - \eta_{\mu\nu} \partial^2) H - M^2 (H_{\mu\nu} - H \eta_{\mu\nu}) = 0 \quad (7)$$

where $H = \eta^{\mu\nu} H_{\mu\nu}$ above. Acting ∂^μ from the left, we find the constraint

$$\partial^\nu (H_{\mu\nu} - \eta_{\mu\nu} H) = 0. \quad (8)$$

Plugging this back into Eq. (7), we have

$$\partial^2 H_{\mu\nu} - \partial_\mu \partial_\nu H - M^2 (H_{\mu\nu} - \eta_{\mu\nu} H) = 0, \quad (9)$$

which yields another constraint $H = H_{\mu\nu} \eta^{\mu\nu} = 0$. Thus, the two constraints, $\partial_\mu H^{\mu\nu} = 0 = H$, imply that $H_{\mu\nu}$ has five independent degrees of freedom or polarizations in four spacetime dimensions. Denoting the corresponding symmetric polarization tensors by $\epsilon_{\mu\nu}(k)$, these constraints imply that $k_\mu \epsilon^{\mu\nu}(k) = k_\nu \epsilon^{\mu\nu}(k) = 0$, and $\epsilon^\mu{}_\mu(k) = 0$. We will come to the matter of explicit representation of these polarization tensors in Sec. III A.

We will take the propagator for the massive spin-2 field as [64],

$$\Delta_{\mu\nu\alpha\beta}^M = -\frac{i\mathcal{P}_{\mu\nu\alpha\beta}^M}{k^2 + M^2}, \quad (10)$$

where

$$\mathcal{P}_{\mu\nu\alpha\beta}^M := \frac{1}{2} \left(\eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\mu\beta} \eta_{\nu\alpha} - \frac{2}{3} \eta_{\mu\nu} \eta_{\alpha\beta} \right). \quad (11)$$

Comparing the above propagator for the massive spin-2 field with that of the graviton [Eq. (4)], we see that there is no smooth massless limit for the first. This leads to the famous van Dam-Veltman-Zakharov (vDVZ) discontinuity in the bending of light [64,65].

We will compute the amplitudes for the 2-2 scattering processes between a massive spin-2 field and chiefly with a massive scalar. The incoming (outgoing) momenta will be denoted by $k_1, k_2 (k'_1, k'_2)$, where k_1, k'_1 are the momenta of the massive spin-2 field. We have

$$k_1 + k_2 = k'_1 + k'_2.$$

We will denote the transfer momentum, $k_1 - k'_1 = k'_2 - k_2$, carried by the internal graviton lines by q . In the non-relativistic limit, we have $q^0 \approx 0$. The gravitational potential, as per the first Born approximation, is defined via the Fourier transform of the nonrelativistic limit of the 2-2 scattering Feynman amplitude [26],

$$V(\vec{r}) = -\frac{1}{2m_1} \frac{1}{2m_2} \int \frac{d^3 \vec{q}}{(2\pi)^3} e^{-i\vec{q} \cdot \vec{r}} \mathcal{M}(\kappa, \vec{q}, m_1, m_2)|_{\text{NR}}. \quad (12)$$

$$\begin{aligned} i\mathcal{M}^{(1)}(\kappa^2, q, m_1, m_2) &= V_{\text{spin-0}}^{(1)\mu\nu}(k_1, k'_1, m_1) \Delta_{\mu\nu\alpha\beta}(q) V_{\text{spin-0}}^{(1)\alpha\beta}(k_2, k'_2, m_2) \\ &= -\frac{i\kappa^2}{8q^2} [k_1^\mu k_1'^\nu + k_1'^\mu k_1^\nu - \eta^{\mu\nu} (k_1 \cdot k'_1 + m_1^2)] (\eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\mu\beta} \eta_{\nu\alpha} - \eta_{\mu\nu} \eta_{\alpha\beta}) [k_2^\alpha k_2'^\beta + k_2'^\alpha k_2^\beta - \eta^{\alpha\beta} (k_2 \cdot k'_2 + m_2^2)] \\ &= -\frac{i\kappa^2}{2q^2} \left[(k_1 \cdot k_2)(k'_1 \cdot k'_2) + (k_1 \cdot k'_2)(k'_1 \cdot k_2) - k_1 \cdot k'_1 (k_2 \cdot k'_2 - m_2^2) + k_2 \cdot k'_2 \right. \\ &\quad \left. ((d/2 - 2)k_1 \cdot k'_1 - (d/2 - 1)m_1^2) - \frac{d}{2} (k_2 \cdot k'_2 - m_2^2) ((d/2 - 2)k_1 \cdot k'_1 - (d/2 - 1)m_1^2) \right]. \end{aligned} \quad (15)$$

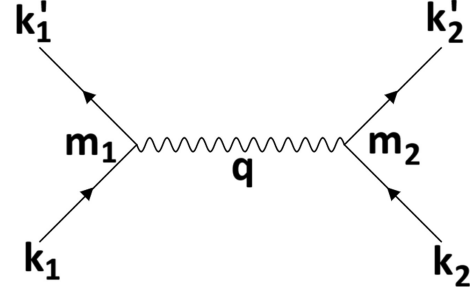


FIG. 1. Tree level diagram of massive spin-0-spin-0 scattering. The straight lines stand for the scalar field, whereas the wavy line represents the graviton. $q = k_1 - k'_1 = k'_2 - k_2$ is the transfer momentum.

We will also require the following Fourier transforms,

$$\begin{aligned} \int \frac{d^3 \vec{q}}{(2\pi)^3} \frac{e^{i\vec{q} \cdot \vec{r}}}{|\vec{q}|} &= \frac{1}{2\pi^2 r^2}; \quad \int \frac{d^3 \vec{q}}{(2\pi)^3} \frac{e^{i\vec{q} \cdot \vec{r}}}{\vec{q}^2} = \frac{1}{4\pi r}; \\ \int \frac{d^3 \vec{q}}{(2\pi)^3} e^{i\vec{q} \cdot \vec{r}} \ln \vec{q}^2 &= -\frac{1}{2\pi r^3}. \end{aligned} \quad (13)$$

The various interaction terms between matter fields and the graviton have been listed in Appendix A. With these ingredients, we are now ready to go into the scattering calculations. We first wish to compute the tree level scattering amplitude of the massive spin-2 field with massive spin-0, spin-1, and spin-1/2 fields.

III. SCATTERING AT TREE LEVEL

In order to clarify notations, let us first review the well-known scattering of two massive scalars, Fig. 1, and derive the Newton potential. Using Eq. (4), and for the two scalar one graviton vertex, e.g., [26],

$$V_{\text{spin-0}}^{(1)\mu\nu}(k, k', m) = -\frac{i\kappa}{2} [k^\mu k'^\nu + k'^\mu k^\nu - \eta^{\mu\nu} (k \cdot k' + m^2)]. \quad (14)$$

The Feynman amplitude for this process is given by

In the nonrelativistic limit, we have $k_1 \cdot k'_1 \approx -m_1^2$, $k_2 \cdot k'_2 \approx -m_2^2$, $k_1 \cdot k_2 \approx -m_1 m_2$, etc. and $q^\mu \approx \vec{q}$, and we obtain

$$i\mathcal{M}^{(1)}(\kappa^2, q, m_1, m_2)|_{\text{NR}} \approx \frac{i\kappa^2 m_1^2 m_2^2}{2\vec{q}^2}. \quad (16)$$

The potential can be found using Eqs. (12) and (13),

$$V(r) = -\frac{\kappa^2 m_1 m_2}{8} \int \frac{d^3 \vec{q}}{(2\pi)^3} \frac{e^{-i\vec{q} \cdot \vec{r}}}{\vec{q}^2} = -\frac{G m_1 m_2}{r}. \quad (17)$$

We next wish to compute the tree level scattering of a massive spin-2 field with massive spin-0, spin-1, and spin-1/2 fields.

A. Massive spin-2 and spin-0 scattering

Let us now compute the lowest order scattering amplitude between a massive scalar and a massive spin-2 field, Fig. 2. This requires cubic vertex functions for the two scalar-one graviton, Eq. (14), and two massive spin-2-one graviton interactions. The first contains two indices, whereas it is clear that the second will contain six indices. A pair of them will correspond to the graviton, whereas the remaining four will pairwise correspond to the massive spin-2 field. However, note that these latter indices must be contracted with the polarization of the massive spin-2 field. Due to this reason, and in order to make things look simple, we would retain those polarization tensors in the three point vertex function (also in the four point vertex later), and *effectively* would represent the same with two indices only, and will denote it as $V_{\mu\nu, \text{eff.}}^{(1)\text{spin-2}}(k, k', M)$. It can be read off from Eq. (A5),

$$\begin{aligned} V_{\mu'\alpha', \text{eff.}}^{(1)\text{spin-2}}(k, k', M) = & -\frac{i\kappa}{2} [2k \cdot k' \epsilon_{(\mu'}^\nu(k) \epsilon_{\alpha')\nu}^*(k') - (k_{(\mu'} k_{\alpha')} - \eta_{\mu'\alpha'} k \cdot k') \epsilon_{\alpha\beta}(k) \epsilon^{\alpha\beta}(k') + M^2 (2\epsilon^\mu_{(\mu'}(k) \\ & \times \epsilon_{\alpha')\mu}^*(k') + \eta_{\mu'\alpha'} \epsilon^{\alpha\beta}(k) \epsilon_{\alpha\beta}^*(k')) + q^2 \epsilon_{\nu(\alpha'}(k) \epsilon^{\star\nu}_{\mu')} (k') + (k_{(\mu'} \eta_{\alpha')\alpha} + k'_{(\mu'} \eta_{\alpha')\alpha}) \\ & (\epsilon^\alpha_{\nu}(k) q_\mu \epsilon^{\star\mu\nu}(k') - q_\mu \epsilon^\mu_{\nu}(k) \epsilon^{\star\alpha\nu}(k')))]. \end{aligned} \quad (18)$$

Using the above expression, Eqs. (4), (14), and also noting that the external momenta are on shell, we write down the Feynman amplitude in the leading order as,

$$\begin{aligned} i\mathcal{M}^{(1)}(\kappa^2, q, M, m) = & V_{\mu'\alpha', \text{eff.}}^{(1)\text{spin-2}}(k_1, k'_1, M) \Delta^{\mu'\alpha'\nu'\beta'}(q) V_{\nu'\beta'}^{(1)\text{spin-0}}(k_2, k'_2, m) \\ = & -\frac{i\kappa^2 (\eta^{\mu'\nu'} \eta^{\alpha'\beta'} + \eta^{\mu'\beta'} \eta^{\nu'\alpha'} - \eta^{\mu'\alpha'} \eta^{\nu'\beta'}) (k_{2\nu'} k'_{2\beta'} + k'_{2\nu'} k_{2\beta'} - \eta_{\nu'\beta'} (k_2 \cdot k'_2 + m^2))}{q^2} \\ & \times [2k_1 \cdot k'_1 \epsilon_{(\mu'}^\nu(k_1) \epsilon_{\alpha')\nu}^*(k'_1) - (k_{1(\mu'} k'_{1\alpha')} - \eta_{\mu'\alpha'} k_1 \cdot k'_1) \epsilon_{\alpha\beta}(k_1) \epsilon^{\alpha\beta}(k'_1) \\ & + M^2 (2\epsilon^\mu_{(\mu'}(k_1) \epsilon_{\alpha')\mu}^*(k'_1) + \eta_{\mu'\alpha'} \epsilon^{\alpha\beta}(k_1) \epsilon_{\alpha\beta}^*(k'_1)) + q^2 \epsilon_{\nu(\alpha'}(k_1) \epsilon^{\star\nu}_{\mu')} (k'_1) \\ & + \{k_{1(\mu'} \eta_{\alpha')\alpha} + k'_{1(\mu'} \eta_{\alpha')\alpha}\} \{ \epsilon^\alpha_{\nu}(k_1) q_\mu \epsilon^{\star\mu\nu}(k'_1) - q_\mu \epsilon^\mu_{\nu}(k_1) \epsilon^{\star\alpha\nu}(k'_1) \}]. \end{aligned} \quad (19)$$

Before we proceed, let us clarify the issue of the polarization tensors $\epsilon_{\mu\nu}(k)$, appearing above. In the *rest frame*, the five independent polarization tensors can be represented by five trace free symmetric matrices as [64],

$$\begin{aligned} \epsilon_{\mu\nu}^{(1)} = & \sqrt{\frac{2}{3}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, & \epsilon_{\mu\nu}^{(2)} = & \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, & \epsilon_{\mu\nu}^{(3)} = & \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \\ \epsilon_{\mu\nu}^{(4)} = & \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, & \epsilon_{\mu\nu}^{(5)} = & \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \end{aligned} \quad (20)$$

The inner product between different polarizations is defined via the trace norm as

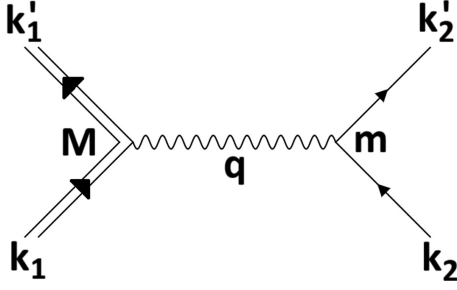


FIG. 2. Tree level scattering of massive spin-2-spin-0 fields. The double lines represent the massive spin-2 field.

$$\epsilon_{\mu\nu}^{(a)} \epsilon_{(a')}^{\star\mu\nu} \equiv \text{Tr}(\epsilon_a^\dagger \epsilon_{a'}) = \delta_{aa'}, \quad (a, a' = 1, 2, 3, 4, 5). \quad (21)$$

For any momentum $\vec{k}$, the corresponding polarization tensors can be found by boosting them through the appropriate Lorentz transformation matrix, $\epsilon_{\mu\nu} \rightarrow \Lambda_\mu^\alpha(\vec{k}) \Lambda_\nu^\beta(\vec{k}) \epsilon_{\alpha\beta}$. Clearly, Eq. (21) remains invariant under this boost transformation,

$$\epsilon_{\mu\nu}^{(a)}(\vec{k}) \epsilon_{(a')}^{\star\mu\nu}(\vec{k}) = \delta_{aa'}.$$

Note also that the rest frame representation Eq. (20) trivially satisfies $k^\mu \epsilon_{\mu\nu} = 0 = k^\nu \epsilon_{\mu\nu}$. These tensor equations will hold in the boosted frame as well.

Let us now try to understand the nonrelativistic limit of the boosted polarization tensors. Writing $\Lambda_\mu^\alpha = \delta_\mu^\alpha + \omega_\mu^\alpha$, for a small Lorentz boost, we have from Eq. (20),

$$\epsilon_{00}(\vec{k}) = 0, \quad \epsilon_{0i}(\vec{k}) = \omega_{0j}(\vec{k}) \epsilon_{ji}.$$

Since $\epsilon_{i0}(\vec{k} = 0) = 0$, in the product of two polarization tensors, we may safely keep the pure spacelike components of $\epsilon_{\mu\nu}(\vec{k})$ in the nonrelativistic limit up to $\mathcal{O}(\omega)$.

We next simplify the scattering amplitude of Eq. (19) as

$$\begin{aligned} i\mathcal{M}_{aa'}^{(1)} = & \frac{16i\pi G}{q^2} [m^2 M^2 \epsilon_{\mu\nu}^{(a)}(k_1) \epsilon_{(a')}^{\star\mu\nu}(k'_1) \\ & + 2m(m + 4M) q_\mu q_\nu \epsilon_\alpha^{(a)\mu}(k_1) \epsilon_{(a')}^{\star\nu\alpha}(k'_1) \\ & + 8mM(k_{2\alpha} q_\mu - k'_{2\mu} q_\alpha) \epsilon_\nu^{(a)\alpha}(k_1) \epsilon_{(a')}^{\star\mu\nu}(k'_1)]. \quad (22) \end{aligned}$$

Note that the representation of Eq. (20) is real. Thus the complex conjugation symbol appearing above is spurious. However, we wish to keep them just to remind us that a particle with momentum k'_1 is created here, whereas the one without complex conjugation indicates annihilation of a particle with momentum k_1 . Note that the leading contribution to the potential comes from the first term on the right-hand side of the above equation. Taking the nonrelativistic as well as *rest frame* limit, we have

$$i\mathcal{M}^{(1)}(\kappa^2, q, M, m)|_{\text{NR}} = \frac{i\kappa^2 m^2 M^2}{2q^2} \delta_{aa'}, \quad (23)$$

which reproduces the Newton potential.

The general form of the long range gravitational potential reads

$$V = -\frac{GmM}{r} \epsilon_{ij}^{(a)}(\vec{k}_1) \epsilon_{(a')}^{\star ij}(\vec{k}'_1) - \frac{2Gm}{Mr^3} \left(\delta_{ij} - \frac{3r^i r^j}{r^2} \right) \epsilon_k^{(a)i}(\vec{k}_1) \epsilon_{(a')}^{\star jk}(\vec{k}'_1) + \mathcal{O}(r^{-4}). \quad (24)$$

B. Massive spin-2 and massive spin-1 scattering

From Eq. (A5), we read off the one graviton two massive spin-1 vertex [41],

$$V_{\mu\nu}^{(1)\text{spin-1}}(k, k', m_v) = \frac{i\kappa}{2} [-k \cdot \epsilon' \epsilon_{(\mu} k'_{\nu)} - k' \cdot \epsilon \epsilon'_{(\mu} k_{\nu)} + \epsilon \cdot \epsilon' k_{(\mu} k'_{\nu)} + \eta_{\mu\nu} k \cdot \epsilon' k' \cdot \epsilon], \quad (25)$$

where we have used the fact that the external momenta are on-shell and nonrelativistic, and $\epsilon_\mu, \epsilon'_\mu$'s are the polarization vectors for the massive spin-1 field. Remembering the constraint $k \cdot \epsilon = 0 = k' \cdot \epsilon'$, the Feynman amplitude for this process is written as,

$$\begin{aligned} i\mathcal{M}^{(1)}(\kappa^2, q, M, m_v) = & V_{\mu\alpha, \text{eff.}}^{(1)\text{spin-2}}(k_1, k'_1, M) \Delta^{\mu\alpha\nu\beta}(q) V_{\nu\beta}^{(1)\text{spin-1}}(k_2, k'_2, m_v) \\ = & -\frac{i\kappa^2 (\eta^{\mu'\nu'} \eta^{\alpha'\beta'} + \eta^{\mu'\beta'} \eta^{\nu'\alpha'} - \eta^{\mu'\alpha'} \eta^{\nu'\beta'})}{8q^2} [-k_{1(\mu'} k'_{1\alpha')} \epsilon_{\mu\nu}(k_1) \epsilon^{\star\mu\nu}(k'_1) + (k_1 + k'_1)_{(\mu'} \eta_{\alpha')} \\ & \times \{ \epsilon^\alpha{}_\nu(k_1) q_\mu \epsilon^{\star\mu\nu}(k'_1) - q_\mu \epsilon^\mu{}_\nu(k_1) \epsilon^{\star\alpha\nu}(k'_1) \}] [-k_2 \cdot \epsilon'_2 \epsilon_{(\nu'}(k_2) k'_{2\beta')} - k'_2 \cdot \epsilon_2 \epsilon_{(\nu'}(k'_2) k_{2\beta')} \\ & + \epsilon_2 \cdot \epsilon'_2 k_{2(\nu'} k'_{2\beta')} + \eta_{\nu'\beta'} k_2 \cdot \epsilon'_2 k'_2 \cdot \epsilon_2] \\ = & \frac{iG\pi}{q^2} [16M^2 m_v^2 \vec{\epsilon}_2 \cdot \vec{\epsilon}'_2 - 8M^2 i(\vec{k} \times \vec{q}) \cdot \vec{S} + 4M^2 \vec{q} \cdot \vec{\epsilon}'_2 \vec{q} \cdot \vec{\epsilon}_2] \epsilon_{ij}^{(a)}(k_2) \epsilon_{(a')}^{\star ij}(k'_2) + \text{sub-leading contributions}. \quad (26) \end{aligned}$$

Since $\vec{k}_1 - \vec{k}'_1 = \vec{k}'_2 - \vec{k}_2 = \vec{q}$, we have written above

$$\vec{k}_1 = \vec{k} + \frac{\vec{q}}{2}, \quad \vec{k}'_1 = \vec{k} - \frac{\vec{q}}{2}; \quad \vec{k}_2 = \vec{k} - \frac{\vec{q}}{2}, \quad \vec{k}'_2 = \vec{k} + \frac{\vec{q}}{2}, \quad (27)$$

where $\vec{k}$ should be interpreted as the momentum of the center of momentum. We also write in the nonrelativistic limit for the polarization vectors of the massive spin-1 field following [41],

$$\epsilon^\mu(k)|_{\text{NR}} \simeq \left(\frac{\vec{k} \cdot \vec{\epsilon}}{m_v}, \vec{\epsilon} \right), \quad (28)$$

which trivially satisfies, $k \cdot \epsilon = 0$ in the nonrelativistic limit, $k^0 \approx -M$. We have

$$\epsilon_\mu(k_1)\epsilon^{\star\mu}(k'_1)|_{\text{NR}} = \vec{\epsilon}(\vec{k}_1) \cdot \vec{\epsilon}^\star(\vec{k}'_1) - \frac{i}{2m_v^2} \vec{S} \cdot (\vec{k} \times \vec{q}) - \frac{1}{m_v^2} \vec{k} \cdot \vec{\epsilon}(\vec{k}_1) \vec{k} \cdot \vec{\epsilon}^\star(\vec{k}'_1) + \frac{1}{4m_v^2} \vec{q} \cdot \vec{\epsilon}(\vec{k}_1) \vec{q} \cdot \vec{\epsilon}^\star(\vec{k}'_1), \quad (29)$$

where we have written

$$-i\vec{S} = \vec{\epsilon}(\vec{k}_1) \times \vec{\epsilon}^\star(\vec{k}'_1), \quad (30)$$

and have used the trivial identity,

$$\vec{q} \cdot \vec{\epsilon} \vec{k} \cdot \vec{\epsilon}^\star = \vec{k} \cdot \vec{\epsilon} \vec{q} \cdot \vec{\epsilon}^\star + i\vec{S} \cdot (\vec{k} \times \vec{q}).$$

Putting everything together now, we have the two body long range gravitational potential,

$$V(r)|_{\text{Tree}} = \left[-\frac{Gm_v M}{r} \vec{\epsilon}_2 \cdot \vec{\epsilon}_2' - \frac{GM}{2m_v r^2} (\vec{k} \times \hat{r}) \cdot \vec{S} + \frac{3GM}{4m_v r^5} \vec{r} \cdot \vec{\epsilon}_2 \vec{r} \cdot \vec{\epsilon}_2' \right] \epsilon_{ij}^{(a)}(\vec{k}_1) \epsilon_{(a')}^{\star ij}(\vec{k}'_1) + \mathcal{O}(r^{-4}). \quad (31)$$

C. Massive spin-2 and spin-1/2 scattering

Let us now consider the massive spin-1/2 and massive spin-2 scattering at the lowest order. We have the one graviton two fermion vertex in the momentum space [41],

$$V_{\text{spin-1/2}}^{(1)\mu\nu}(k, k', m_f) = -\frac{i\kappa}{2} \left[\frac{1}{4} \gamma^\mu(k + k')^\nu - \eta^{\mu\nu} \left(\frac{k + k'}{2} - m_f \right) \right]. \quad (32)$$

The tree level scattering amplitude reads,

$$\begin{aligned} i\mathcal{M}^{(1)}(\kappa^2, q, M, m_f) &= V_{\mu'\alpha', \text{eff}}^{(1)\text{spin-2}}(k_1, k'_1, M) \Delta^{\mu'\alpha'\nu'\beta'}(q) V_{\nu'\beta'}^{(1)\text{spin-1/2}}(k_2, k'_2, m_f) \\ &= -\frac{i\kappa^2 (\eta^{\mu'\nu'} \eta^{\alpha'\beta'} + \eta^{\mu'\beta'} \eta^{\nu'\alpha'} - \eta^{\mu'\alpha'} \eta^{\nu'\beta'})}{8q^2} [2k_1 \cdot k'_1 \epsilon_{(\mu'}^\nu(k_1) \epsilon_{\alpha')\nu}^\star(k'_1) - (k_1 \mu' k'_1 \alpha') \\ &\quad - \eta_{\mu'\alpha'} k_1 \cdot k'_1 \epsilon_{\alpha\beta}(k_1) \epsilon^{\star\alpha\beta}(k'_1) + M^2 (2\epsilon_{(\mu'}^\mu(k_1) \epsilon_{\alpha')\mu}^\star(k'_1) + \eta_{\mu'\alpha'} \epsilon^{\alpha\beta}(k_1) \epsilon_{\alpha\beta}^\star(k'_1)) \\ &\quad + q^2 \epsilon_{\nu(\alpha'}(k_1) \epsilon^{\star\nu}_{\mu')}(k'_1) + \{k_1(\mu' \eta_{\alpha')\alpha} + k'_1(\mu' \eta_{\alpha')\alpha}\} \{ \epsilon_{\nu}^\alpha(k_1) q_\mu \epsilon^{\star\mu\nu}(k'_1) - q_\mu \epsilon_{\nu}^\mu(k_1) \epsilon^{\star\mu\nu}(k'_1) \}] \\ &\quad \times \bar{u}_{s'}(k'_2) \left[\frac{1}{4} \gamma_{(\nu'}(k_2 + k'_2)_{\beta')} - \eta_{\nu'\beta'} \left(\frac{k_2 + k'_2}{2} - m_f \right) \right] u_s(k_2) \\ &= \frac{iG\pi}{q^2} \left[(16m^2 M^2 \epsilon_{\mu\nu}^{(a)} \epsilon_{(a')}^{\star\mu\nu} - 112m^2 k_\mu q_\alpha \epsilon_{\nu}^{(a)} \epsilon_{(a')}^{\star\mu\nu} - 16m^2 k_\mu q_\alpha \epsilon_{\nu}^{(a)\mu} \epsilon_{(a')}^{\star\nu\alpha} + 128m^2 q_\mu \epsilon_{\nu\alpha}^{(a)} k^\nu \epsilon_{(a')}^{\star\mu\alpha} \right. \\ &\quad - 16mM q_\alpha k^\mu \epsilon_{\nu}^{(a)\alpha} \epsilon_{(a')\mu}^{\star\nu} + 16mM q_\alpha k^\mu \epsilon_{\mu\nu}^{(a)} \epsilon_{(a')}^{\star\nu\alpha} \delta_{ss'} + \left(\frac{8M q_\alpha k^\mu \epsilon_{\nu}^{(a)\alpha} \epsilon_{(a')\mu}^{\star\nu}}{m} - \frac{8M q_\alpha k^\mu \epsilon_{\mu\nu}^{(a)} \epsilon_{(a')}^{\star\nu\alpha}}{m} \right. \\ &\quad \left. \left. + 8k_\mu q_\alpha \epsilon_{\nu}^{(a)\alpha} \epsilon_{(a')}^{\star\mu\nu} + 16mM \epsilon_{\mu\nu}^{(a)} \epsilon_{(a')}^{\star\mu\nu} - 12M^2 \epsilon_{\mu\nu}^{(a)} \epsilon_{(a')}^{\star\mu\nu} - 8k_\mu q_\alpha \epsilon_{\nu}^{(a)\mu} \epsilon_{(a')}^{\star\nu\alpha} \right) i\vec{k} \times \vec{q} \cdot \vec{S}_{1/2} \right] \end{aligned} \quad (33)$$

where $s = \pm$, and we used the Gordon identity,

$$\bar{u}_{s'}(k'_2)\gamma^\mu u_s(k_2) = \frac{1}{2m_f}\bar{u}_{s'}(k'_2)\left[(k_2 + k'_2)^\mu - \frac{1}{2}[\gamma^\mu, \gamma^\nu]q_\nu\right]u_s(k_2). \quad (34)$$

We also take

$$u_s(\vec{k}) = \sqrt{E_{\vec{k}} + m_f}\left(\frac{1}{E_{\vec{k}} + m_f}\right)\chi_s \quad (35)$$

where

$$\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (36)$$

We next compute using Eqs. (35) and (36), in the non-relativistic limit,

$$\bar{u}_{s'}(\vec{k}'_2)u_s(\vec{k}_2) = 2m_f\left[\delta_{ss'} - \frac{i}{2m_f^2}(\vec{k} \times \vec{q}) \cdot \vec{S}_{1/2}^{ss'}\right] + \text{subleading terms} \quad (37)$$

where we have defined the matrix element of the spin vector for the spin-1/2 field as ($\hbar = 1$),

$$\vec{S}_{1/2}^{ss'} = \frac{1}{2}\chi_{s'}^\dagger \vec{\sigma} \chi_s.$$

Putting things together now, the gravitational potential reads,

$$V(r)|_{\text{Tree}} = \left[-\frac{Gm_f M}{r}\delta_{ss'} + \left(\frac{G}{r^2} - \frac{3GM}{4m_f r^2}\right)(\vec{k} \times \hat{r}) \cdot \vec{S}\right] \times \epsilon_{ij}^{(a)}(\vec{k}_1)\epsilon_{(a')}^{*ij}(\vec{k}'_1) + \mathcal{O}(r^{-3}). \quad (38)$$

IV. MASSIVE SPIN-2 AND SPIN-0 SCATTERING AT $\mathcal{O}(G^2)$

We wish to compute below the one loop correction [$\mathcal{O}(\kappa^4)$] to the 2-2 scattering for the massive spin-2 and spin-0 fields. The expressions are extremely lengthy compared to the tree level ones we have computed above, and we will present below only the explicitly spin or polarization independent leading parts. The integrals necessary for the following computations are provided in Appendix B.

A. The ladder and the cross-ladder diagrams

We begin by considering the ladder and cross-ladder diagrams, Fig. 3. Note that a massive spin-2 field is created/annihilated with momentum k'_1/k_1 in this process. Accordingly, there will be ϵ^*, ϵ at these two vertices. The remaining two polarizations would sum up to yield the massive spin-2 propagator. Accordingly, the Feynman amplitudes for the ladder and the cross-ladder diagrams are, respectively, given by,

$$i\mathcal{M}_{\text{ladder}} = \int \frac{d^4 l}{(2\pi)^4} \bar{V}_{\mu\nu, \text{eff.}}^{(1)\text{spin-2}}(k_1, l + k_1, M) \epsilon_{\lambda'\eta'}^*(k'_1) \frac{-i\mathcal{P}_M^{\lambda'\eta'\epsilon'\psi'}}{(l + k_1)^2 + M^2} \bar{V}_{\alpha\beta, \text{eff.}}^{(1)\text{spin-2}}(l + k_1, k'_1, M) \epsilon_{\epsilon'\psi'}(k_1) \\ \times V_{\gamma\delta}^{(1)\text{spin-0}}(k_2, k_2 - l, m) V_{\rho\sigma}^{(1)\text{spin-0}}(k_2 - l, k'_2, m) \frac{-i}{(l - k_2)^2 + m^2} \frac{-i\mathcal{P}^{\mu\nu\gamma\delta}}{l^2} \frac{-i\mathcal{P}^{\alpha\beta\rho\sigma}}{(l + q)^2}, \quad (39)$$

and

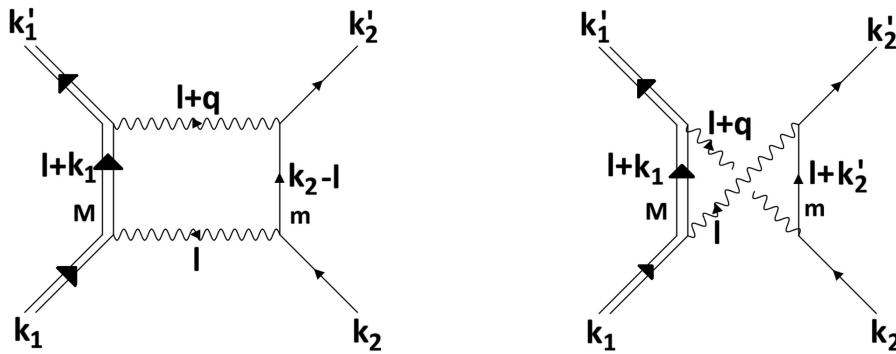


FIG. 3. Ladder (left) and the cross-ladder diagram for massive spin-2-spin-0 fields scattering. As earlier, double and single straight lines, respectively, stand for the massive spin-2 and spin-0 fields, whereas a wavy line represents the graviton. $q = k_1 - k'_1 = k'_2 - k_2$ is the transfer momentum.

$$\begin{aligned}
 i\mathcal{M}_{\text{cross-ladder}} = & \int \frac{d^4 l}{(2\pi)^4} \bar{V}_{\mu\nu}^{(1)\text{spin-2}}(k_1, l + k_1, M) \epsilon_{\lambda'\eta'}^*(k'_1) \frac{-i\mathcal{P}_M^{\lambda'\eta'\epsilon'\psi'}}{(l + k_1)^2 + M^2} \bar{V}_{\alpha\beta}^{(1)\text{spin-2}}(l + k_1, k'_1, M) \epsilon_{\epsilon'\psi'}(k_1) \\
 & \times V_{\rho\sigma}^{(1)\text{spin-0}}(k_2, l + k'_2, m) V_{\gamma\delta}^{(1)\text{spin-0}}(l + k'_2, k'_2, m) \frac{-i}{(l + k'_2)^2 + m^2} \frac{-i\mathcal{P}^{\mu\nu\gamma\delta}}{l^2} \frac{-i\mathcal{P}^{\alpha\beta\rho\sigma}}{(l + q)^2},
 \end{aligned} \quad (40)$$

where $\mathcal{P}_M$ is given by Eq. (11), and $\bar{V}_{\mu\nu}^{(1)\text{spin-2}}$ contains *no* polarization tensor. We have, after some algebra,

$$\begin{aligned}
 \mathcal{M}_{\text{lad.+cr.-lad.tot.}} = & \left[-\frac{16\pi^2 G^2 m^3 M^2}{3q} + \frac{10\pi^2 G^2 m^2 M^3}{3q} - \frac{46}{3} G^2 m^2 M^2 \ln q^2 \right] \epsilon_{\mu\nu}^{(a)}(k_1) \epsilon^{\star\mu\nu(a')}(k'_1) \\
 & + \text{polarisation-dependent subleading terms.}
 \end{aligned} \quad (41)$$

The corresponding correction to the Newton potential becomes,

$$\begin{aligned}
 V_{\text{lad.+cr.-lad.tot.}}(r) = & V_{\text{ladder}}(r) + V_{\text{cross-ladder}}(r) = \left[\frac{2G^2 m^2 M}{3r^2} - \frac{5G^2 m M^2}{12r^2} - \frac{23G^2 m M}{12\pi r^3} \right] \epsilon_{ij}^{(a)}(\vec{k}_1) \epsilon^{\star ij(a')}(\vec{k}'_1) \\
 & + \text{subleading terms.}
 \end{aligned} \quad (42)$$

B. The triangle diagrams

Let us next consider the triangle diagrams, Fig. 4. We need to consider the two-scalar and two-graviton vertex for our purpose [26],

$$\begin{aligned}
 V_{\text{spin-0}}^{(2)\mu\lambda\rho\sigma}(k, k', m) = & i\kappa^2 \left[\left\{ I^{\mu\lambda\alpha\nu} I^{\rho\sigma\beta}{}_{\nu} - \frac{1}{4} (\eta^{\mu\lambda} I^{\rho\sigma\alpha\beta} + \eta^{\rho\sigma} I^{\mu\lambda\alpha\beta}) \right\} (k_{\alpha} k'_{\beta} + k_{\beta} k'_{\alpha}) \right. \\
 & \left. - \frac{1}{2} \left(I^{\mu\lambda\rho\sigma} - \frac{1}{2} \eta^{\mu\lambda} \eta^{\rho\sigma} \right) (k \cdot k' + m^2) \right]
 \end{aligned} \quad (43)$$

where,

$$I_{\mu\nu\lambda\rho} = \frac{1}{2} (\eta_{\mu\lambda} \eta_{\nu\rho} + \eta_{\mu\rho} \eta_{\nu\lambda}).$$

The Feynman amplitudes of triangle-1 and triangle-2 diagrams, respectively, read, Fig. 4,

$$\begin{aligned}
 i\mathcal{M}_{\text{triangle-1}} = & \int \frac{d^4 l}{(2\pi)^4} \bar{V}_{\mu\nu}^{(1)\text{spin-2}}(k_1, l + k_1, M) \epsilon_{\lambda'\eta'}^*(k'_1) \left[-\frac{i\mathcal{P}_M^{\lambda'\eta'\epsilon'\psi'}}{(l + k_1)^2 + M^2} \right] \bar{V}_{\alpha\beta}^{(1)\text{spin-2}}(l + k_1, k'_1, M) \epsilon_{\epsilon'\psi'}(k_1) \\
 & \times V_{\sigma\rho\gamma\delta}^{(2)\text{spin-0}}(k_2, k'_2, m) \frac{-i\mathcal{P}^{\alpha\beta\gamma\delta}}{(l + q)^2} \frac{-i\mathcal{P}^{\mu\nu\sigma\rho}}{l^2}
 \end{aligned} \quad (44)$$

and,

$$\begin{aligned}
 i\mathcal{M}_{\text{triangle-2}} = & \int \frac{d^4 l}{(2\pi)^4} V_{(2)\text{spin-2}}^{\mu\nu\alpha\beta}(k_1, k'_1, M) V_{(1)\text{spin-0}}^{\sigma\rho}(k_2, k_2 - l, m) V_{(1)\text{spin-0}}^{\gamma\delta}(k_2 - l, k'_2, m) \\
 & \times \frac{-i\mathcal{P}^{\mu\nu\sigma\rho}}{l^2} \frac{-i\mathcal{P}^{\alpha\beta\gamma\delta}}{(l + q)^2} \frac{-i}{(l - k_2)^2 + m^2},
 \end{aligned} \quad (45)$$

where $V_{(2)\text{spin-2}}^{\mu\nu\alpha\beta}(k_1, k'_1, M)$ is the *effective* two massive spin-2-two graviton vertex containing two polarizations $\epsilon_{\mu\nu}(k_1)$ and $\epsilon_{\alpha\beta}^*(k'_1)$, and can be read off from Eq. (A6). Note that the terms containing the derivative of the graviton field would yield terms containing the transfer momentum q , and such terms will make a subleading contribution to the potential. Simplifying now the above equations, we obtain,

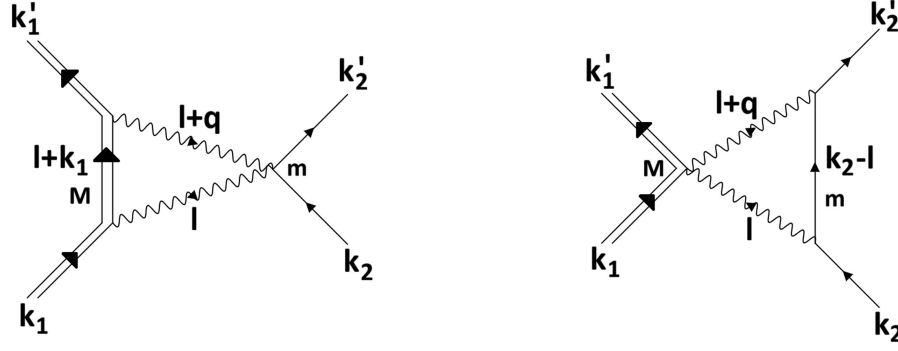


FIG. 4. Triangle diagrams for massive spin-2-spin-0 fields scattering.

$$\mathcal{M}_{\text{triangle-1}} = \left[-\frac{32\pi^2 G^2 m^3 M^2}{3q} + \frac{24\pi^2 G^2 m^2 M^3}{q} + 24G^2 m^2 M^2 \ln q^2 \right] \epsilon_{\mu\nu}^{(a)}(k_1) \epsilon^{\star\mu\nu(a')}(k'_1) + \text{subleading terms}, \quad (46)$$

and

$$\mathcal{M}_{\text{triangle-2}} = \left[\frac{3G^2 m^3 M^2 \pi^2}{q} + \frac{3G^2 m^2 M^3 \pi^2}{q} + 3G^2 m^2 M^2 \ln q^2 \right] \epsilon_{\mu\nu}^{(a)}(k_1) \epsilon^{\star\mu\nu(a')}(k'_1) + \text{subleading terms}. \quad (47)$$

The corresponding correction to the Newton potential in the nonrelativistic limit becomes,

$$V_{\text{triangle tot.}}(r) = V_{\text{triangle-1}}(r) + V_{\text{triangle-2}}(r) = \left[\frac{27G^2 m M}{8\pi r^3} + \frac{23G^2 m^2 M}{24r^2} - \frac{27G^2 m M^2}{8r^2} \right] \epsilon_{ij}^{(a)}(\vec{k}_1) \epsilon^{\star ij(a')}(\vec{k}'_1) + \text{subleading terms}. \quad (48)$$

C. The double seagull diagram

We next consider the double seagull diagram of Fig. 5. We have the Feynman amplitude,

$$i\mathcal{M}_{\text{double-seagull}} = \frac{1}{2!} \int \frac{d^4 l}{(2\pi)^4} V_{(2)\text{spin-2}}^{\alpha\beta\gamma\delta}(k_1, k'_1, M) V_{(2)\text{spin-0}}^{\sigma\rho\mu\nu}(k_2, k'_2, m) \frac{-i\mathcal{P}_{\alpha\beta\mu\nu}}{(l+q)^2} \frac{-i\mathcal{P}_{\gamma\delta\sigma\rho}}{l^2}, \quad (49)$$

where as earlier $V_{(2)\text{spin-2}}^{\alpha\beta\gamma\delta}(k_1, k'_1, M)$ contains contractions with two polarizations of the massive spin-2 field. The above expression yields,

$$\mathcal{M}_{\text{double-seagull}} = -\frac{1}{8} G^2 m^2 M^2 \ln q^2 \epsilon_{\mu\nu}^{(a)}(k_1) \epsilon^{\star\mu\nu(a')}(k'_1) + \text{subleading terms}, \quad (50)$$

which gives the correction to the potential,

$$V_{\text{double-seagull}}(r) = -\frac{G^2 m M}{64\pi r^3} \epsilon_{ij}^{(a)}(\vec{k}_1) \epsilon^{\star ij(a')}(\vec{k}'_1) + \text{subleading terms}. \quad (51)$$

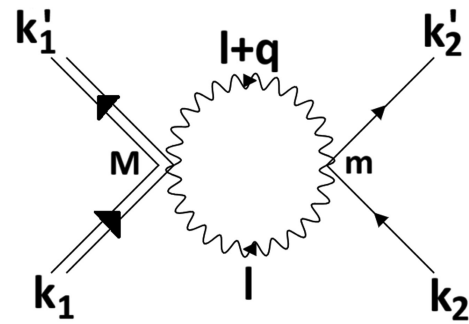


FIG. 5. Double seagull diagram for massive spin-2-spin-0 fields scattering.

D. The vertex correction diagram-1

Let us now come to the contribution from the vertex correction diagrams, the first of which is given by Fig. 6. To evaluate this diagram, we need to consider the three graviton vertex given by [26],

$$\begin{aligned}
 V_{\alpha\beta\gamma\delta}^{(3)\mu\nu}(k, q) = & -\frac{i\kappa}{2} \left[P_{\alpha\beta\gamma\delta} \left(k^\mu k^\nu + (k-q)^\mu (k-q)^\nu + q^\mu q^\nu - \frac{3}{2} \eta^{\mu\nu} q^2 \right) + 2q_\lambda q_\sigma (I_{\alpha\beta}^{\sigma\lambda} I_{\gamma\delta}^{\mu\nu} + I_{\gamma\delta}^{\sigma\lambda} I_{\alpha\beta}^{\mu\nu} \right. \\
 & - I_{\alpha\beta}^{\mu\sigma} I_{\gamma\delta}^{\nu\lambda} - I_{\gamma\delta}^{\mu\sigma} I_{\alpha\beta}^{\nu\lambda}) + \{ q_\lambda q^\mu (\eta_{\alpha\beta} I_{\gamma\delta}^{\nu\lambda} + \eta_{\gamma\delta} I_{\alpha\beta}^{\nu\lambda}) + q_\lambda q^\nu (\eta_{\alpha\beta} I_{\gamma\delta}^{\mu\lambda} + \eta_{\gamma\delta} I_{\alpha\beta}^{\mu\lambda}) \\
 & - q^2 (\eta_{\alpha\beta} I_{\gamma\delta}^{\mu\nu} + \eta_{\gamma\delta} I_{\alpha\beta}^{\mu\nu}) - \eta^{\mu\nu} q_\sigma q_\lambda (\eta_{\alpha\beta} I_{\gamma\delta}^{\sigma\lambda} + \eta_{\gamma\delta} I_{\alpha\beta}^{\sigma\lambda}) \} + \{ -2q_\lambda (I_{\alpha\beta}^{\lambda\sigma} I_{\gamma\delta}^{\nu\sigma} (k-q)^\mu \\
 & + I_{\alpha\beta}^{\lambda\sigma} I_{\gamma\delta}^{\mu\sigma} (k-q)^\nu + I_{\gamma\delta}^{\lambda\sigma} I_{\alpha\beta}^{\nu\sigma} k^\mu + I_{\gamma\delta}^{\lambda\sigma} I_{\alpha\beta}^{\mu\sigma} k^\nu) + q^2 (I_{\alpha\beta}^{\mu\sigma} I_{\gamma\delta}^{\nu\sigma} + I_{\gamma\delta}^{\mu\sigma} I_{\alpha\beta}^{\nu\sigma}) \\
 & + \eta^{\mu\nu} q_\sigma q_\lambda (I_{\alpha\beta}^{\lambda\rho} I_{\gamma\delta}^{\sigma\rho} + I_{\gamma\delta}^{\lambda\rho} I_{\alpha\beta}^{\sigma\rho}) \} + \left\{ (k^2 + (k-q)^2) \left(I_{\alpha\beta}^{\mu\sigma} I_{\gamma\delta}^{\nu\sigma} + I_{\gamma\delta}^{\mu\sigma} I_{\alpha\beta}^{\nu\sigma} \right. \right. \\
 & \left. \left. - \frac{1}{2} \eta^{\mu\nu} \mathcal{P}_{\alpha\beta\gamma\delta} \right) - (I_{\gamma\delta}^{\mu\nu} \eta_{\alpha\beta} k^2 + I_{\alpha\beta}^{\mu\nu} \eta_{\gamma\delta} (k-q)^2) \right\} \Big].
 \end{aligned} \tag{52}$$

The corresponding Feynman amplitude reads,

$$\begin{aligned}
 i\mathcal{M}_{\text{vertex-correction-1}} = & \int \frac{d^4 l}{(2\pi)^4} V_{\alpha\beta, \text{eff}}^{(1)\text{spin-2}}(k_1, k'_1, M) \frac{-i\mathcal{P}_{\gamma\delta}^{\alpha\beta}}{q^2} V_{\lambda\psi\phi\epsilon}^{(3)\gamma\delta}(l, -q) \frac{-i\mathcal{P}^{\phi\epsilon\rho\sigma}}{(l+q)^2} \frac{-i\mathcal{P}^{\lambda\psi\mu\nu}}{l^2} \\
 & \times V_{\mu\nu}^{(1)\text{spin-0}}(k_2, k_2 - l, m) V_{\rho\sigma}^{(1)\text{spin-0}}(k_2 - l, k'_2, m) \frac{-i}{(l-k_2)^2 + m^2},
 \end{aligned} \tag{53}$$

which yields,

$$\begin{aligned}
 \mathcal{M}_{\text{vertex-correction-1}} = & \left[\frac{46\pi^2 G^2 m^3 M^2}{q} + \frac{4\pi^2 G^2 m^2 M^3}{q} + 46G^2 m^2 M^2 \ln q^2 \right] \epsilon_{\mu\nu}^{(a)}(k_1) \epsilon^{\star\mu\nu(a')}(k'_1) \\
 & + \text{subleading terms.}
 \end{aligned} \tag{54}$$

The corresponding correction to the Newton potential reads,

$$\begin{aligned}
 V_{\text{vertex-correction-1}}(r) = & \left[-\frac{23G^2 m^2 M}{4r^2} - \frac{G^2 m M^2}{2r^2} + \frac{23G^2 m M}{4\pi r^3} \right] \epsilon_{ij}^{(a)}(\vec{k}_1) \epsilon^{\star ij(a')}(\vec{k}'_1) \\
 & + \text{subleading terms.}
 \end{aligned} \tag{55}$$

E. The vertex correction diagram-2

The second vertex correction diagram is given by Fig. 7. The Feynman amplitude reads,

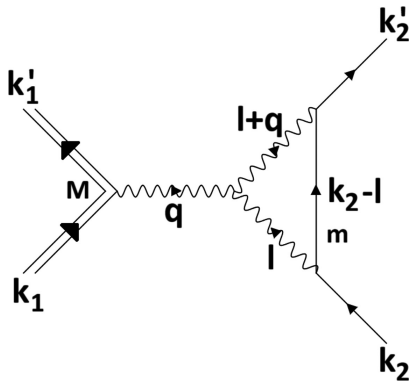


FIG. 6. First vertex correction diagram for massive spin-2-spin-0 fields scattering.

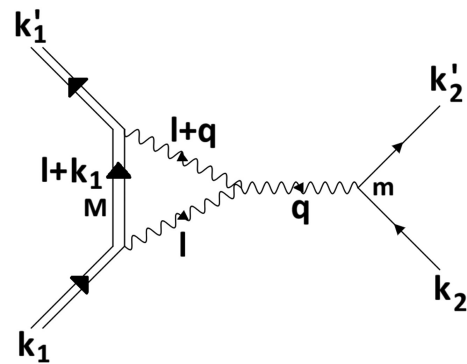


FIG. 7. Second vertex correction diagram for massive spin-2-spin-0 fields scattering.

$$i\mathcal{M}_{\text{vertex-correction-2}} = \int \frac{d^4l}{(2\pi)^4} V_{\mu\nu,\text{eff.}}^{(1)\text{spin-2}}(k_1, l+k_1, M) \epsilon_{\lambda'\eta'}^*(k'_1) \frac{-i\mathcal{P}_M^{\lambda'\eta'\epsilon'\psi'}}{(l+k_1)^2 + M^2} V_{\alpha\beta,\text{eff.}}^{(1)\text{spin-2}}(l+k_1, k'_1, M) \\ \times \epsilon_{\epsilon'\psi'}(k_1) \frac{-i\mathcal{P}^{\mu\nu\rho\sigma}}{l^2} \frac{-i\mathcal{P}^{\alpha\beta\gamma\delta}}{(l+q)^2} V_{\rho\sigma\gamma\delta}^{(3)\phi e}(-l, q) \frac{-iP_{\phi\epsilon\lambda\mu}}{q^2} V_{(1)\text{spin-0}}^{\lambda\mu}(k_2, k'_2, m), \quad (56)$$

which yields,

$$\mathcal{M}_{\text{vertex-correction-2}} = \left[\frac{24\pi^2 G^2 m^3 M^2}{q} - \frac{71\pi^2 G^2 m^2 M^3}{q} - 71G^2 m^2 M^2 \ln q^2 \right] \epsilon_{\mu\nu}^{(a)}(k_1) \epsilon^{\star\mu\nu(a')}(k'_1) \\ + \text{polarisation-dependent subleading terms.} \quad (57)$$

The correction to the gravitational potential reads,

$$V_{\text{vertex-correction-2}}(r) = \left[-\frac{3G^2 m^2 M}{r^2} + \frac{71G^2 mM^2}{8r^2} - \frac{71G^2 mM}{8\pi r^3} \right] \epsilon_{ij}^{(a)}(\vec{k}_1) \epsilon^{\star ij(a')}(\vec{k}'_1) + \text{subleading terms.} \quad (58)$$

F. The vertex correction diagram-3

The third vertex correction diagram is given by Fig. 8. The corresponding Feynman amplitude reads,

$$i\mathcal{M}_{\text{vertex-correction-3}} = \frac{1}{2!} \int \frac{d^4l}{(2\pi)^4} V_{\alpha\beta}^{(1)\text{spin-2,eff}}(k_1, k'_1, M) \frac{-i\mathcal{P}_{\gamma\delta}^{\alpha\beta}}{(l+q)^2} V_{\mu\nu\rho\sigma}^{(3)\gamma\delta}(l, -q) \frac{-i\mathcal{P}^{\rho\sigma\epsilon\phi}}{q^2} \frac{-i\mathcal{P}^{\mu\nu\lambda\mu}}{l^2} V_{\lambda\mu\epsilon\phi}^{(2)\text{spin-0}}(k_2, k'_2, m), \quad (59)$$

which gives,

$$\mathcal{M}_{\text{vertex-correction-3}} = 80G^2 m^2 M^2 \ln q^2 \epsilon_{\mu\nu}^{(a)}(k_1) \epsilon^{\star\mu\nu(a')}(k'_1) + \text{polarization-dependent subleading terms.} \quad (60)$$

The corresponding potential reads,

$$V_{\text{vertex-correction-3}}(r) = \frac{10G^2 mM}{\pi r^3} \epsilon_{ij}^{(a)}(\vec{k}_1) \epsilon^{\star ij(a')}(\vec{k}'_1) + \text{subleading terms.} \quad (61)$$

G. The vertex correction diagram-4

The fourth and the last vertex correction diagram is given by Fig. 9. The Feynman amplitude reads,

$$i\mathcal{M}_{\text{vertex-correction-4}} = \frac{1}{2!} \int \frac{d^4l}{(2\pi)^4} V_{(2)\text{spin-2}}^{\rho\sigma\mu\nu}(k_1, k'_1, M) V_{(1)\text{spin-0}}^{\lambda\kappa}(k_2, k'_2, m) V_{(3)}^{\alpha\beta\gamma\delta\epsilon\phi}(-l, q) \frac{-i\mathcal{P}_{\mu\nu\gamma\delta}}{l^2} \frac{-i\mathcal{P}_{\rho\sigma\alpha\beta}}{(l+q)^2} \frac{-i\mathcal{P}_{\lambda\kappa\epsilon\phi}}{q^2}, \quad (62)$$

which equals,

$$\mathcal{M}_{\text{vertex-correction-4}} = \frac{21}{4} G^2 m^2 M^2 \ln q^2 \epsilon_{\mu\nu}^{(a)}(k_1) \epsilon^{\star\mu\nu(a')}(k'_1) + \text{subleading terms.} \quad (63)$$

The corresponding correction to the gravitational potential reads,

$$V_{\text{vertex-correction-4}}(r) = \frac{21G^2 mM}{64\pi r^3} \epsilon_{ij}^{(a)}(\vec{k}_1) \epsilon^{\star ij(a')}(\vec{k}'_1) + \text{subleading terms.} \quad (64)$$

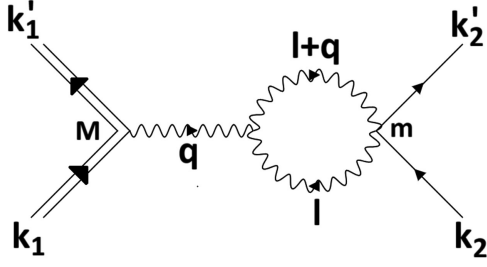


FIG. 8. Third vertex correction diagram for massive spin-2-spin-0 fields scattering.

H. The vacuum polarization and the ghost diagrams

Finally, we come to the contribution from the vacuum polarization plus the ghost loop, Fig. 10. The ghost loop is added to compensate for the gauge noninvariant terms originating from the graviton self energy, Fig. 11. The total

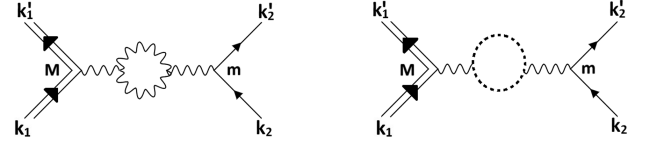


FIG. 10. Vacuum polarization plus ghost diagrams for massive spin-2-spin-0 fields scattering.



FIG. 11. Graviton self-energy from the vacuum polarization diagram along with the ghost contribution.

gauge invariant contribution denoted as $\Pi_{\lambda\xi\mu\nu}(q)$, has been evaluated long ago in [4,64],

$$\begin{aligned} \Pi_{\alpha\beta\gamma\delta} = & -\frac{2G}{\pi} \ln q^2 \left[\frac{21}{120} q^4 I_{\alpha\beta\gamma\delta} + \frac{23}{120} q^4 \eta_{\alpha\beta} \eta_{\gamma\delta} - \frac{23}{120} q^2 (\eta_{\alpha\beta} q_\gamma q_\delta + \eta_{\gamma\delta} q_\alpha q_\beta) - \frac{21}{240} q^2 (q_\alpha q_\delta \eta_{\beta\gamma} + q_\beta q_\delta \eta_{\alpha\gamma} \right. \\ & \left. + q_\alpha q_\gamma \eta_{\beta\delta} + q_\beta q_\gamma \eta_{\alpha\delta}) + \frac{11}{30} q_\alpha q_\beta q_\gamma q_\delta \right]. \end{aligned} \quad (65)$$

We have the Feynman amplitude,

$$i\mathcal{M}_{\text{vac-pol}} = V_{\rho\sigma}^{(1)\text{spin-2,eff.}}(k_1, k'_1, M) \frac{-i\mathcal{P}^{\rho\sigma\lambda\xi}}{q^2} \Pi_{\lambda\xi\mu\nu}(q) \frac{-i\mathcal{P}^{\mu\nu\gamma\delta}}{q^2} V_{\gamma\delta}^{(1)\text{spin-0}}(k_2, k'_2, m), \quad (66)$$

which equals after substituting Eq. (65),

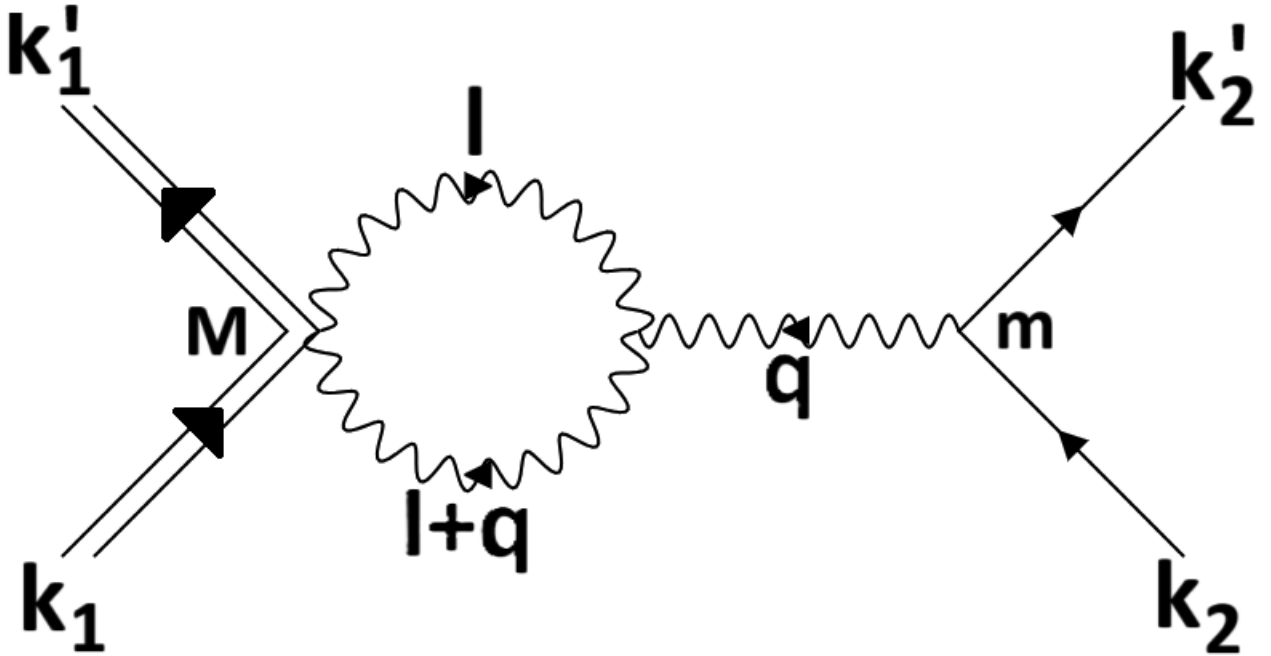


FIG. 9. Fourth vertex correction diagram for massive spin-2-spin-0 fields scattering.

$$\mathcal{M}_{\text{Ghost}} = -\frac{172}{15}G^2m^2M^2 \ln q^2 \epsilon_{\mu\nu}^{(a)}(k_1) \epsilon^{\star\mu\nu(a')}(k'_1) + \text{subleading terms.} \quad (67)$$

The corresponding potential reads,

$$V_{\text{vac-pol}}(r) = -\frac{43MmG^2}{30\pi r^3} \epsilon_{ij}^{(a)}(\vec{k}_1) \epsilon^{\star ij(a')}(\vec{k}'_1) + \text{subleading terms.} \quad (68)$$

I. The final quantum corrected Newtonian potential

Combining everything now, we obtain the leading part of the gravitational potential up to $\mathcal{O}(G^2)$, between a massive spin-2 and spin-0 particle,

$$V(r) = -\frac{GMm}{r} \left[1 + \frac{G(171m - 110M)}{24rc^2} + \frac{577G\hbar}{80\pi r^2 c^3} \right] \times \epsilon_{ij}^{(a)}(\vec{k}_1) \epsilon^{\star ij(a')}(\vec{k}'_1) + \text{subleading terms.} \quad (69)$$

This is one of the main results of this paper. Note that we have put back $\hbar$ in the last term. This is to emphasize that this term has purely quantum origin—i.e., from the $\ln q^2$ term coming from the loops.

V. DISCUSSION

Let us summarize our results now. In this paper we have computed the 2-2 scattering between a massive spin-2 Fierz-Pauli field with various other spin fields, and from the nonrelativistic limit of the scattering amplitudes, we have computed the corresponding two body gravitational potentials at the leading $[\mathcal{O}(G)]$ and next to the leading $[\mathcal{O}(G^2)]$ orders. The massive spin-2 field does *not* stand for any massive gravity here, and gravity is taken to be simply general relativity with the usual massless graviton as its quantum. We have taken the Fierz-Pauli massive spin-2 field as just a field theory with higher spin, minimally coupled to gravity, e.g., [55]. At the leading order $[\mathcal{O}(G)]$, we have computed the gravitational potential for its

scattering with massive scalar, fermion, and Proca fields, Sec. III. The Newton potential, along with the spin and polarization dependent nonspherical terms, have been explicitly computed. For the massive spin-2—massive scalar scattering, we have further extended the above computations to the next to the leading order $[\mathcal{O}(G^2)]$, Sec. IV. For this case, we have explicitly demonstrated the polarization independent spherical parts only. The present computations could be thought of as one of the ongoing attempts to understand the gravitational potential in the context of a higher spin field theory.

As a step forward, perhaps it will be interesting to understand the quantum corrections to Newton's potential in viable massive gravity theories, in which the vDVZ discontinuity can be taken care of via some Vainshtein screening mechanism at scales small compared to the Hubble horizon [67]. We hope to investigate this issue in our future work.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: VARIOUS INTERACTION TERMS

1. Massive spin-0 field

The action for a massive scalar field is given by,

$$S^{\text{spin-0}} = -\frac{1}{2} \int \sqrt{-g} [g^{\mu\nu} (\partial_\mu \phi) (\partial_\nu \phi) + m^2 \phi^2]. \quad (A1)$$

The free as well as the scalar-graviton interaction terms up to $\mathcal{O}(\kappa^2)$ read [using Eq. (2)],

$$\begin{aligned} S^{\text{spin-0}}(\kappa^0) &= -\frac{1}{2} \int d^4x [(\partial_\mu \phi) (\partial^\mu \phi) + m^2 \phi^2], \\ S^{\text{spin-0}}(\kappa^1) &= \frac{\kappa}{2} \int d^4x h^{\mu\nu} \left[(\partial_\mu \phi) (\partial_\nu \phi) - \frac{1}{2} \eta_{\mu\nu} ((\partial\phi)^2 + m^2 \phi^2) \right], \\ S^{\text{spin-0}}(\kappa^2) &= -\frac{\kappa^2}{2} \int d^4x \left[\left(h^\mu{}_\alpha h^{\alpha\nu} - \frac{1}{2} h h^{\mu\nu} + \frac{1}{8} h^2 \eta^{\mu\nu} - \frac{1}{4} h_{\alpha\beta} h^{\alpha\beta} \eta^{\mu\nu} \right) (\partial_\mu \phi) (\partial_\nu \phi) + m^2 \left(\frac{h^2}{8} - \frac{1}{4} h_{\mu\nu} h^{\mu\nu} \right) \phi^2 \right]. \end{aligned} \quad (A2)$$

2. Massive spin-1 field

For a massive spin-1 field we have,

$$S^{\text{spin-1}} = \int \sqrt{-g} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} m_1^2 A_\mu A^\mu \right] \quad (\text{A3})$$

where m_v is the mass of the spin-1 particle and, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The free as well as the two spin-1 one graviton interaction terms read

$$\begin{aligned} S^{\text{spin-1}}(\kappa^0) &= - \int d^4x \left[\frac{1}{4} \eta^{\mu\lambda} \eta^{\nu\rho} F_{\mu\nu} F_{\lambda\rho} + \frac{1}{2} m_v^2 \eta^{\mu\nu} A_\mu A_\nu \right], \\ S^{\text{spin-1}}(\kappa^1) &= \frac{\kappa}{2} \int d^4x \left[\left(\eta^{\mu\lambda} h^{\nu\rho} - \frac{1}{4} \eta_{\alpha\beta} \eta^{\mu\lambda} \eta^{\nu\rho} h^{\alpha\beta} \right) F_{\mu\nu} F_{\lambda\rho} + m_v^2 \left(h^{\mu\nu} - \frac{1}{2} \eta_{\alpha\beta} \eta^{\mu\nu} h^{\alpha\beta} \right) A_\mu A_\nu \right]. \end{aligned}$$

This will be sufficient for our present purpose with the massive spin-1 field.

3. Massive spin-2 field

Let us now consider the Fierz-Pauli massive spin-2 field. The Lagrangian is given in Eq. (6) from where the action can be easily written. We have up to $\mathcal{O}(\kappa^2)$,

$$\begin{aligned} S^{\text{spin-2}}(\kappa^0) &= \int d^4x \left[-\frac{1}{2} (\partial_\lambda H^{\mu\nu}) (\partial^\lambda H_{\mu\nu}) + (\partial^\lambda H_{\mu\lambda}) (\partial_\alpha H^{\mu\alpha}) - (\partial_\lambda H) (\partial_\mu H^{\mu\lambda}) + \frac{1}{2} (\partial_\lambda H) (\partial^\lambda H) \right. \\ &\quad \left. - \frac{1}{2} M^2 (H_{\mu\nu} H^{\mu\nu} - H^2) \right], \end{aligned} \quad (\text{A4})$$

$$\begin{aligned} S^{\text{spin-2}}(\kappa^1) &= -\frac{\kappa}{2} \int d^4x \left[\left(2\eta^{\lambda\rho} \eta_{\nu\beta} h_{\mu\alpha} - \eta_{\mu\alpha} \eta_{\nu\beta} h^{\lambda\rho} + \frac{h}{2} \eta^{\lambda\rho} \eta_{\mu\alpha} \eta_{\nu\beta} \right) (\partial_\lambda H^{\mu\nu}) (\partial_\rho H^{\alpha\beta}) \right. \\ &\quad \left. + 2\eta^{\lambda\rho} \eta_{\nu\beta} (\partial_\rho h_{\mu\alpha} + \partial_\alpha h_{\rho\mu} - \partial_\mu h_{\rho\alpha}) H^{\alpha\beta} (\partial_\lambda H^{\mu\nu}) + M^2 \left(2\eta_{\mu\alpha} h_{\nu\beta} + \frac{h}{2} \eta_{\mu\alpha} \eta_{\nu\beta} \right) H^{\mu\nu} H^{\alpha\beta} \right] \end{aligned} \quad (\text{A5})$$

and,

$$\begin{aligned} S^{\text{spin-2}}(\kappa^2) &= \kappa^2 \int d^4x \left[-\frac{1}{2} \left(\eta^{\lambda\rho} h_{\mu\alpha} h_{\nu\beta} - 2\eta_{\nu\beta} h_{\mu\alpha} h^{\lambda\rho} + h^\lambda{}_{\beta'} h^{\beta'\rho} \eta_{\mu\alpha} \eta_{\nu\beta} + h h_{\mu\alpha} \eta^{\lambda\rho} \eta_{\nu\beta} - \frac{1}{2} h h^{\lambda\rho} \eta_{\mu\alpha} \eta_{\nu\beta} \right) \right. \\ &\quad \left. + \frac{1}{4} \eta^{\lambda\rho} \left(\frac{h^2}{2} - h_{\alpha'\alpha''} h^{\alpha'\alpha''} \right) \right] (\partial_\lambda H^{\mu\nu}) (\partial_\rho H^{\alpha\beta}) - \frac{1}{2} \left(2\eta^{\lambda\rho} \eta_{\nu\beta} h_{\mu\alpha} - \eta_{\mu\alpha} \eta_{\nu\beta} h^{\lambda\rho} + \frac{h}{2} \eta^{\lambda\rho} \eta_{\mu\alpha} \eta_{\nu\beta} \right) \\ &\quad (\partial_\lambda H^{\mu\nu}) (\eta^{\alpha\alpha'} (\partial_\rho h_{\rho'\alpha'} + \partial_{\rho'} h_{\rho\alpha'} - \partial_{\alpha'} h_{\rho\rho'}) H^{\rho'\beta} + \eta^{\beta\alpha'} (\partial_\rho h_{\rho'\beta'} + \partial_{\rho'} h_{\rho\beta'} - \partial_{\beta'} h_{\rho\rho'}) H^{\rho'\alpha}) \\ &\quad + \frac{1}{4} ((2\partial_\lambda h_{\mu\nu} - \partial_\mu h_{\lambda\nu}) H^{\nu\lambda} + \eta_{\rho\mu} \eta^{\lambda\lambda'} \partial_\nu h_{\lambda\lambda'} H^{\nu\rho}) (\eta^{\mu\mu'} (2\partial_\alpha h_{\mu'\beta} - \partial_{\mu'} h_{\alpha\beta}) H^{\alpha\beta} + \eta^{\alpha\alpha'} \partial_\beta h_{\alpha\alpha'} H^{\mu\beta}) \\ &\quad + \frac{1}{2} ((\partial_\mu h_{\alpha\beta}) (\partial^\mu h_{\lambda\rho}) H^{\alpha\beta} H^{\lambda\rho} + 2(\partial_\mu h_{\alpha\beta}) (\partial^\mu H^{\lambda\rho}) h_{\lambda\rho} H^{\alpha\beta} + h_{\alpha\beta} h_{\lambda\rho} (\partial_\mu H^{\alpha\beta}) (\partial^\mu H^{\lambda\rho})) \\ &\quad - \frac{1}{2} ((\partial_\lambda h_{\alpha\beta}) H^{\alpha\beta} + h_{\alpha\beta} (\partial_\lambda H^{\alpha\beta})) (\eta^{\mu\mu'} (\partial_\rho h_{\mu'\mu'}) H^{\rho\lambda} + \eta^{\lambda\lambda'} (2\partial_\mu h_{\rho\lambda'} - \partial_{\lambda'} h_{\mu\rho}) H^{\mu\rho}) \\ &\quad \left. - \frac{1}{2} M^2 \left(h_{\mu\lambda} h_{\nu\rho} - h_{\mu\nu} h_{\lambda\rho} + h \eta_{\mu\lambda} h_{\nu\rho} + \frac{1}{4} \left(\frac{h^2}{2} - h_{\alpha\beta} h^{\alpha\beta} \right) \eta_{\mu\lambda} \eta_{\nu\rho} \right) H^{\mu\nu} H^{\lambda\rho} \right]. \end{aligned} \quad (\text{A6})$$

Note that the terms containing derivatives of the graviton field will carry its momentum explicitly. Such terms will always make a subleading (compared to r^{-3}) or even no contribution to the long range gravitational potential.

APPENDIX B: LIST OF ESSENTIAL INTEGRALS

We list below some integrals useful for the purpose of evaluating the gravitational potential, originating from the parts nonanalytic in the transfer momentum, q^2 [26]. We will display below only those nonanalytic parts.

$$\begin{aligned} \int \frac{d^d l}{(2\pi)^d} \frac{1}{l^2(l+q)^2} &= -\frac{i}{16\pi^2} \ln q^2; \quad \int \frac{d^d l}{(2\pi)^d} \frac{l_\mu}{l^2(l+q)^2} = \frac{i q_\mu}{32\pi^2} \ln q^2, \\ \int \frac{d^d l}{(2\pi)^d} \frac{l_\mu l_\nu}{l^2(l+q)^2} &= -\frac{i \ln q^2}{64\pi^2} \left[\frac{4}{3} q_\mu q_\nu - \frac{1}{3} q^2 \eta_{\mu\nu} \right], \\ \int \frac{d^d l}{(2\pi)^d} \frac{1}{l^2(l+q)^2((l+k)^2 - m^2)} &= -\frac{i}{32\pi^2 m^2} \left[\ln q^2 + \frac{\pi^2 m}{q} \right], \end{aligned} \quad (\text{B1})$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{l_\mu}{l^2(l+q)^2((l+k)^2 - m^2)} = \frac{i}{32\pi^2 m^2} \left[\left\{ \left(-1 - \frac{q^2}{2m^2} \right) \ln q^2 - \frac{1}{4} \frac{q^2}{m^2} \frac{\pi^2 m}{q} \right\} k_\mu + \left\{ \ln q^2 + \frac{\pi^2 m}{2q} \right\} q_\mu \right], \quad (\text{B2})$$

$$\begin{aligned} \int \frac{d^d l}{(2\pi)^d} \frac{l_\mu l_\nu}{l^2(l+q)^2((l+k)^2 - m^2)} &= \frac{i}{32\pi^2 m^2} \left[q_\mu q_\nu \left(-\ln q^2 - \frac{3}{8} \frac{\pi^2 m}{q} \right) + k_\mu k_\nu \left(-\frac{1}{2} \frac{q^2}{m^2} \ln q^2 - \frac{1}{8} \frac{q^2}{m^2} \frac{\pi^2 m}{q} \right) \right. \\ &\quad \left. + (q_\mu k_\nu + q_\nu k_\mu) \left\{ \left(\frac{1}{2} + \frac{1}{2} \frac{q^2}{m^2} \right) \ln q^2 + \frac{3\pi^2 m q}{16} \right\} + q^2 \eta_{\mu\nu} \left(\frac{1}{4} \ln q^2 + \frac{\pi^2 m}{8q} \right) \right], \end{aligned} \quad (\text{B3})$$

$$\begin{aligned} &\int \frac{d^d l}{(2\pi)^d} \frac{1}{l^2(l+q)^2((l+k_1)^2 - m_1^2)((l+k_2)^2 - m_2^2)} \\ &= \frac{i}{16\pi^2 m_1 m_2 q^2} \left(1 - \frac{w}{3m_1 m_2} \right) \ln q^2 \int \frac{d^d l}{(2\pi)^d} \frac{1}{l^2(l+q)^2((l+k_1)^2 - m_1^2)((l+k_2')^2 - m_2^2)} \\ &= \frac{i}{16\pi^2 m_1 m_2 q^2} \left(-1 + \frac{W}{3m_1 m_2} \right) \ln q^2 \end{aligned} \quad (\text{B4})$$

where, $w = -k_1 \cdot k_2 - m_1 m_2$ and $W = -k_1 \cdot k_2' - m_1 m_2$.

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