

Improved prediction of the mass splitting for P -wave Ω baryons

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Using the QCD sum rule method, we investigate the mass splitting for the spin-orbit partner states of the $\Omega(2012)$ baryon assuming that it is a P -wave excitation with $J^P = 3/2^-$. This study is an extension of the previous work [N. Su *et al.* *Phys. Rev. D* **110**, 034007 (2024)] in which the masses of these states were estimated with uncertainties too large to extract the reliable mass splitting. In the present study, by directly formulating a sum rule for the mass splitting, we obtain an improved prediction, $\delta M = M_{3/2^-} - M_{1/2^-} = -18.0^{+33.6}_{-17.1}$ MeV. This result provides a more quantitative insight into the spectrum of P -wave Ω baryons and serves as a useful reference for future experiments.

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I. INTRODUCTION

Hadrons are composed of quarks and gluons, with their strong interactions governed by quantum chromodynamics (QCD). The quark model systematically classified hadrons into mesons, made of a quark-antiquark pair, and baryons, composed of three quarks [1,2]. This theoretical framework has achieved remarkable success in explaining hadron properties. Experimentally, all ground-state light-flavor mesons and baryons consisting of u , d , and s quarks have been established [3], and a large number of their excited states have also been observed. However, the existence and structure of some highly excited states remain controversial, potentially providing crucial insights into the non-perturbative features of QCD in the low-energy regime [4–17].

In the Ω baryon sector, the Particle Data Group (PDG) currently lists only five states. Besides the ground-state $\Omega(1672)$, $\Omega(2012)$ and $\Omega(2250)$ are rated with three stars, indicating likely existence but requiring further experimental confirmation and theoretical investigation, while $\Omega(2380)$ and $\Omega(2470)$ have two stars, reflecting weaker evidence. The

present moment provides a timely opportunity to study the Ω baryon spectrum. Among these, $\Omega(2012)$ has attracted considerable attention since its discovery by the Belle experiment in 2008 [18–45]. In the past year, both the BESIII and ALICE experiments have reported new supporting evidence [46,47] and the BESIII Collaboration has also observed a potential new state $\Omega(2109)$. In our previous study [48], we analyzed the mass and structure of $\Omega(2012)$ using parity-projected QCD sum rules and found it is likely a P -wave excitation with $J^P = 3/2^-$. We further predicted the mass of its spin-orbit partners with $J^P = 1/2^-$, they are

$$M_{3/2^-} = 2.05^{+0.09}_{-0.10} \text{ GeV}, \quad (1)$$

$$M_{1/2^-} = 2.07^{+0.07}_{-0.07} \text{ GeV}. \quad (2)$$

Although these results imply that mass splitting of the two spin states is not large, it remained poorly constrained due to the inherent uncertainties. To improve the precision of the mass splitting for P -wave Ω excitations and to explore whether $\Omega(2109)$ could correspond to the spin-orbit partner of $\Omega(2012)$, we extend our previous work by formulating a dedicated QCD sum rule for the splitting, thereby providing a more accurate theoretical prediction.

This paper is organized as follows: In Sec. II, we present the interpolating currents of the P -wave Ω baryon used in the calculations. In Sec. III, we perform a detailed description for formulating a QCD sum rule approach to evaluate the mass splittings. Finally, Sec. IV concludes with a brief summary and discussion.

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II. P-WAVE Ω BARYON CURRENTS

The Ω baryon consists of three identical s quarks which obeys Fermi-Dirac statistics. For P -wave Ω baryons with orbital angular momentum $L = 1$, there exist two spin-orbit partner states with quantum number $J^P = 1/2^-$ and $J^P = 3/2^-$. The detailed construction of the currents for these states can be found in our previous work [48], where the currents for the corresponding excited states are constructed as ρ -mode excitations:

$$J = -2\epsilon^{abc}[(D^\mu s_a^T)C\gamma_5 s_b]\gamma_\mu s_c, \quad (3)$$

$$J_\mu = -2\epsilon^{abc}[(D^\nu s_a^T)C\gamma_5 s_b]\left(g_{\mu\nu} - \frac{1}{4}\gamma_\mu\gamma_\nu\right)s_c. \quad (4)$$

Note that these currents can also be expressed as λ -mode currents via Fierz transformation, which is the case for three identical strange quarks. In this study, we employ them to couple to the physical states as

$$\langle 0|J|\Omega; 1/2^-\rangle = f_{1/2^-}\gamma_5 u(q), \quad (5)$$

$$\langle 0|J_\mu|\Omega; 3/2^-\rangle = f_{3/2^-}u_\mu(q), \quad (6)$$

where $f_{1/2^-(3/2^-)}$ are the coupling strength of the currents to physical states, and u and u_μ are the spinors for spin $1/2$ and $3/2$ states.

III. QCD SUM RULE ANALYSES

In this section, we formulate the QCD sum rules to calculate the mass splitting between the P -wave Ω baryons. We firstly study their correlation functions:

$$\Pi_{1/2}(q^2) = i \int d^4x e^{iqx} \langle 0|\mathbf{T}[J(x)J^\dagger(0)]|0\rangle, \quad (7)$$

$$\begin{aligned} \Pi_{\mu\nu,3/2}(q^2) &= i \int d^4x e^{iqx} \langle 0|\mathbf{T}[J_\mu(x)J_\nu^\dagger(0)]|0\rangle \\ &= \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\Pi_{3/2}(q^2) + \dots, \end{aligned} \quad (8)$$

where we have isolated the Lorentz structure corresponding to the spin-3/2 component $\Pi(q^2)_{3/2}$. The function $\Pi_\xi(q^2)$ can then be represented by a dispersion relation:

$$\Pi_\xi(q^2) = \int_{s_<}^\infty \frac{\rho_\xi(s)}{s - q^2 - i\epsilon} ds, \quad (9)$$

where ξ denotes either $1/2$ or $3/2$, $\rho(s) \equiv \text{Im } \Pi(s)/\pi$ is the spectral density, and $s_< = 9m_s^2$ is the physical threshold.

At the hadron level, we insert a complete set of intermediate states into the correlation function and extract the corresponding spectral densities:

$$\begin{aligned} \rho_{1/2}^{\text{phen}}(s) &\equiv \sum_m \delta(s - M_m^2) \langle 0|J|m\rangle \langle m|J^\dagger|0\rangle \\ &= f_{1/2^-}^2 (\not{q} + M_{1/2^-}) \delta(s - M_{1/2^-}^2) \\ &\quad + f_{1/2^+}^2 (\not{q} + M_{1/2^+}) \delta(s - M_{1/2^+}^2) \\ &\quad + \text{continuum}, \end{aligned} \quad (10)$$

$$\begin{aligned} \rho_{3/2}^{\text{phen}}(s) &\equiv \sum_m \delta(s - M_m^2) \langle 0|J_\mu|n\rangle \langle n|J_\nu^\dagger|0\rangle \\ &= f_{3/2^-}^2 (\not{q} + M_{3/2^-}) \delta(s - M_{3/2^-}^2) \\ &\quad + f_{3/2^+}^2 (\not{q} + M_{3/2^+}) \delta(s - M_{3/2^+}^2) \\ &\quad + \text{continuum}. \end{aligned} \quad (11)$$

These can be rewritten into two parts with different structures, denoted as $\rho_{\xi,1(0)}^{\text{phen}}(s)$:

$$\rho_\xi^{\text{phen}}(s) = \rho_{\xi,1}^{\text{phen}}(s)\not{q} + \rho_{\xi,0}^{\text{phen}}(s). \quad (12)$$

Finally, we extract the spectral densities corresponding exclusively to the positive- or negative-parity states as

$$\rho_{\xi,\mp}^{\text{phen}}(s) = \sqrt{s}\rho_{\xi,1}^{\text{phen}}(s) \pm \rho_{\xi,0}^{\text{phen}}(s). \quad (13)$$

At the quark-gluon level, we use the method of the operator product expansion (OPE) to calculate the correlation function and extract the corresponding spectral densities, denoted as $\rho_{\xi,1(0)}^{\text{OPE}}$:

$$\begin{aligned} \rho_{1/2,1}^{\text{OPE}}(s) &= \frac{7s^3}{10240\pi^4} - \frac{45m_s^2 s^2}{2048\pi^4} + \left(-\frac{7\langle g_s^2 GG \rangle}{12288\pi^4} + \frac{9m_s\langle \bar{s}s \rangle}{128\pi^2}\right)s + \left(-\frac{13m_s\langle g_s \bar{s}\sigma Gs \rangle}{128\pi^2} + \frac{7m_s^2\langle g_s^2 GG \rangle}{2048\pi^4}\right) \\ &\quad - \left(\frac{\langle \bar{s}s \rangle \langle g_s \bar{s}\sigma Gs \rangle}{12} - \frac{7m_s\langle g_s^2 GG \rangle \langle \bar{s}s \rangle}{3072\pi^2} - \frac{7m_s^2\langle \bar{s}s \rangle^2}{32}\right)\delta(s) \\ &\quad + \left(-\frac{17\langle g_s \bar{s}\sigma Gs \rangle^2}{384} - \frac{7\langle g_s^2 GG \rangle \langle \bar{s}s \rangle^2}{192} + \frac{m_s\langle g_s^2 GG \rangle \langle g_s \bar{s}\sigma Gs \rangle}{512\pi^2} - \frac{m_s^2\langle \bar{s}s \rangle \langle g_s \bar{s}\sigma Gs \rangle}{384}\right)\delta'(s), \end{aligned} \quad (14)$$

$$\begin{aligned} \rho_{1/2,0}^{\text{OPE}}(s) = & \frac{3m_s s^3}{1024\pi^4} - \frac{3\langle\bar{s}s\rangle s^2}{64\pi^2} - \left(\frac{7m_s\langle g_s^2 GG\rangle}{2048\pi^4} - \frac{17m_s^2\langle\bar{s}s\rangle}{64\pi^2} + \frac{7\langle g_s\bar{s}\sigma Gs\rangle}{128\pi^2} \right) s + \left(\frac{83m_s^2\langle g_s\bar{s}\sigma Gs\rangle}{256\pi^2} + \frac{5\langle\bar{s}s\rangle\langle g_s^2 GG\rangle}{768\pi^2} \right) \\ & - \left(\frac{3m_s^2\langle g_s^2 GG\rangle\langle\bar{s}s\rangle}{128\pi^2} - \frac{\langle g_s^2 GG\rangle\langle g_s\bar{s}\sigma Gs\rangle}{1536\pi^2} + \frac{11m_s\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma Gs\rangle}{32} \right) \delta(s) \\ & + \left(-\frac{25m_s\langle g_s\bar{s}\sigma Gs\rangle^2}{384} + \frac{7m_s\langle g_s^2 GG\rangle\langle\bar{s}s\rangle^2}{96} - \frac{m_s^2\langle g_s^2 GG\rangle\langle g_s\bar{s}\sigma Gs\rangle}{256\pi^2} \right) \delta'(s), \end{aligned} \quad (15)$$

$$\begin{aligned} \rho_{3/2,1}^{\text{OPE}}(s) = & \frac{126}{25} \times \left[\frac{5s^3}{36864\pi^4} - \frac{167m_s^2 s^2}{40960\pi^4} - \left(\frac{5\langle g_s^2 GG\rangle}{49152\pi^4} - \frac{11m_s\langle\bar{s}s\rangle}{1024\pi^2} \right) s + \left(-\frac{263m_s\langle g_s\bar{s}\sigma Gs\rangle}{73728\pi^2} + \frac{155m_s^2\langle g_s^2 GG\rangle}{196608\pi^4} \right) \right. \\ & - \left(\frac{89\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma Gs\rangle}{3072} - \frac{29m_s\langle g_s^2 GG\rangle\langle\bar{s}s\rangle}{147456\pi^2} - \frac{m_s^2\langle\bar{s}s\rangle^2}{128} \right) \delta(s) \\ & \left. + \left(-\frac{361\langle g_s\bar{s}\sigma Gs\rangle^2}{36864} - \frac{\langle g_s^2 GG\rangle\langle\bar{s}s\rangle^2}{1152} + \frac{m_s\langle g_s^2 GG\rangle\langle g_s\bar{s}\sigma Gs\rangle}{3072\pi^2} + \frac{7m_s^2\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma Gs\rangle}{192} \right) \delta'(s) \right], \end{aligned} \quad (16)$$

$$\begin{aligned} \rho_{3/2,0}^{\text{OPE}}(s) = & \frac{126}{25} \times \left[\frac{23m_s s^3}{32768\pi^4} - \frac{\langle\bar{s}s\rangle s^2}{96\pi^2} - \left(\frac{317m_s\langle g_s^2 GG\rangle}{589824\pi^4} - \frac{103m_s^2\langle\bar{s}s\rangle}{1536\pi^2} + \frac{37\langle g_s\bar{s}\sigma Gs\rangle}{3072\pi^2} \right) s \right. \\ & + \left(\frac{441m_s^2\langle g_s\bar{s}\sigma Gs\rangle}{8192\pi^2} + \frac{25\langle\bar{s}s\rangle\langle g_s^2 GG\rangle}{24576\pi^2} \right) - \left(\frac{13m_s^2\langle g_s^2 GG\rangle\langle\bar{s}s\rangle}{4096\pi^2} - \frac{13\langle g_s^2 GG\rangle\langle g_s\bar{s}\sigma Gs\rangle}{294912\pi^2} + \frac{5m_s\langle\bar{s}s\rangle\langle g_s\bar{s}\sigma Gs\rangle}{64} \right) \delta(s) \\ & \left. + \left(-\frac{5m_s\langle g_s\bar{s}\sigma Gs\rangle^2}{2084} - \frac{3m_s\langle g_s^2 GG\rangle\langle\bar{s}s\rangle^2}{1024} + \frac{9m_s^2\langle g_s^2 GG\rangle\langle g_s\bar{s}\sigma Gs\rangle}{8192\pi^2} \right) \delta'(s) \right]. \end{aligned} \quad (17)$$

Note that we multiply an overall factor to the spectral density of the spin-3/2 state. This modification plays a crucial role in the calculation of the mass splitting, as will be discussed in detail later.

By matching the spectral densities at both the hadron and quark-gluon levels, and applying the Borel transformation, we derive the sum rules as follows:

$$\begin{aligned} \Pi_{\xi,\mp}(s_0, M_B) &= 2M_{\xi,\mp} f_{\xi,\mp}^2 e^{-M_{\xi,\mp}^2/M_B^2} \\ &= \int_{s_<}^{s_0} (\sqrt{s} \rho_{\xi,1}^{\text{OPE}}(s) \pm \rho_{\xi,0}^{\text{OPE}}(s)) e^{-s/M_B^2} ds, \end{aligned} \quad (18)$$

and further obtain the formulas for mass and coupling constant:

$$\begin{aligned} M_{\xi,\mp}^2(s_0, M_B) &= \frac{\int_{s_<}^{s_0} (\sqrt{s} \rho_{\xi,1}^{\text{OPE}}(s) \pm \rho_{\xi,0}^{\text{OPE}}(s)) s e^{-s/M_B^2} ds}{\int_{s_<}^{s_0} (\sqrt{s} \rho_{\xi,1}^{\text{OPE}}(s) \pm \rho_{\xi,0}^{\text{OPE}}(s)) e^{-s/M_B^2} ds}, \end{aligned} \quad (19)$$

and

$$\begin{aligned} f_{\xi,\mp}^2(s_0, M_B) &= \frac{\int_{s_<}^{s_0} (\sqrt{s} \rho_{\xi,1}^{\text{OPE}}(s) \pm \rho_{\xi,0}^{\text{OPE}}(s)) e^{-s/M_B^2} ds \times e^{M_{\xi,\mp}^2/M_B^2}}{2M_{\xi,\mp}}. \end{aligned} \quad (20)$$

In this way, we calculate the masses and coupling constants of both positive- and negative-parity states as reported in our previous work [48], with the results summarized in Table I. We find that currents containing the derivative

TABLE I. Masses and coupling constants extracted from the currents J in Eq. (3), and J_μ in Eq. (4).

Current	State	$s_0^{\min}$ (GeV ²)	Working regions		Pole (%)	Mass (GeV)	Couple constant (GeV ³)
			M_B^2 (GeV ²)	s_0 (GeV ²)			
J	$ \Omega; 1/2^+\rangle$	9.7	1.91–2.40	11.0	40–55	$3.05^{+0.21}_{-0.15}$	$0.168^{+0.045}_{-0.040}$
	$ \Omega; 1/2^-\rangle$	5.5	1.58–1.73	6.0	40–47	$2.07^{+0.07}_{-0.07}$	$0.079^{+0.011}_{-0.011}$
J_μ	$ \Omega; 3/2^+\rangle$	10.5	2.09–2.30	11.0	40–46	$3.13^{+0.27}_{-0.18}$	$0.074^{+0.015}_{-0.009}$
	$ \Omega; 3/2^-\rangle$	5.3	1.54–1.76	6.0	40–51	$2.05^{+0.09}_{-0.10}$	$0.037^{+0.007}_{-0.007}$

operator couple better to the negative-parity states, and the calculations for states with different parities are performed independently. Therefore, the contributions from the positive-parity states are neglected in the subsequent calculation of the mass splitting.

Now assuming that the two spin states split by a fine splitting due to spin-orbit dynamics, let us define

$$\Pi_{3/2^-}(q^2) = \Pi_{1/2^-}(q^2) + \Pi_\delta(q^2), \quad (21)$$

both at the hadron and quark-gluon levels. At the hadron level, the correlation functions are expressed as

$$\Pi_{3/2^-}^{\text{phen}}(q^2) = f_{3/2^-}^2 \frac{\not{q} + M_{3/2^-}}{M_{3/2^-}^2 - q^2 - i\epsilon}, \quad (22)$$

$$\Pi_{1/2^-}^{\text{phen}}(q^2) = f_{1/2^-}^2 \frac{\not{q} + M_{1/2^-}}{M_{1/2^-}^2 - q^2 - i\epsilon}. \quad (23)$$

Introducing the mass and coupling differences as $M_{3/2^-} = M_{1/2^-} + \delta M$, $f_{3/2^-} = f_{1/2^-} + \delta f$, we obtain

$$\begin{aligned} & f_{3/2^-}^2 \frac{\not{q} + M_{3/2^-}}{M_{3/2^-}^2 - q^2} \\ &= (f_{1/2^-} + \delta f)^2 \frac{\not{q} + (M_{1/2^-} + \delta M)}{(M_{1/2^-} + \delta M)^2 - q^2} \\ &= f_{1/2^-}^2 \frac{\not{q} + M_{1/2^-}}{M_{1/2^-}^2 - q^2} \\ &\quad + f_{1/2^-}^2 \delta M \frac{(M_{1/2^-}^2 - q^2) - 2M_{1/2^-}(\not{q} + M_{1/2^-})}{(M_{1/2^-}^2 - q^2)^2} \\ &\quad + 2f_{1/2^-} \delta f \frac{\not{q} + M_{1/2^-}}{M_{1/2^-}^2 - q^2} + \dots \end{aligned} \quad (24)$$

Here we only consider the first order corrections with respect to δM and δf . From the above expression, we can extract the difference in the correlation function Π_δ^{phen} as

$$\begin{aligned} \Pi_\delta^{\text{phen}}(q^2) &= f_{1/2^-}^2 \delta M \frac{-2M_{1/2^-} \not{q} - M_{1/2^-}^2 - q^2}{(M_{1/2^-}^2 - q^2)^2} \\ &\quad + 2f_{1/2^-} \delta f \frac{\not{q} + M_{1/2^-}}{M_{1/2^-}^2 - q^2}, \end{aligned} \quad (25)$$

and reorganize it into two parts with different structures:

$$\begin{aligned} \Pi_{\delta,1}^{\text{phen}}(q^2) \not{q} &= \left(2f_{1/2^-} \delta f \frac{1}{M_{1/2^-}^2 - q^2} \right. \\ &\quad \left. - 2f_{1/2^-}^2 M_{1/2^-} \delta M \frac{1}{(M_{1/2^-}^2 - q^2)^2} \right) \not{q}, \end{aligned} \quad (26)$$

$$\begin{aligned} \Pi_{\delta,0}^{\text{phen}}(q^2) &= -f_{1/2^-}^2 \delta M \frac{M_{1/2^-}^2 + q^2}{(M_{1/2^-}^2 - q^2)^2} \\ &\quad + 2f_{1/2^-} M_{1/2^-} \delta f \frac{1}{M_{1/2^-}^2 - q^2}. \end{aligned} \quad (27)$$

At the quark-gluon level, using the results of Eqs. (14–17), $\Pi_{\delta,1(0)}^{\text{OPE}}$ can be derived as

$$\Pi_{\delta,1}^{\text{OPE}}(q^2) = \Pi_{3/2^-,1}^{\text{OPE}}(q^2) - \Pi_{1/2^-,1}^{\text{OPE}}(q^2), \quad (28)$$

$$\Pi_{\delta,0}^{\text{OPE}}(q^2) = \Pi_{3/2^-,0}^{\text{OPE}}(q^2) - \Pi_{1/2^-,0}^{\text{OPE}}(q^2). \quad (29)$$

Equating the two results at the hadron and quark-gluon levels and performing the Borel transformation, we arrive at

$$\begin{aligned} \hat{\mathcal{B}} \prod_{\delta,1}^{\text{OPE}}(s_0, M_B) &= 2f_{1/2^-} \delta f e^{-M_{1/2^-}^2/M_B^2} \\ &\quad - \frac{2f_{1/2^-}^2 M_{1/2^-} \delta M}{M_B^2} e^{-M_{1/2^-}^2/M_B^2}, \end{aligned} \quad (30)$$

$$\begin{aligned} \hat{\mathcal{B}} \prod_{\delta,0}^{\text{OPE}}(s_0, M_B) &= 2f_{1/2^-} M_{1/2^-} \delta f e^{-M_{1/2^-}^2/M_B^2} \\ &\quad + f_{1/2^-}^2 \left(1 - \frac{2M_{1/2^-}^2}{M_B^2} \right) \delta M e^{-M_{1/2^-}^2/M_B^2}. \end{aligned} \quad (31)$$

By solving Eqs. (30) and (31) simultaneously, we extract the quantities δM and δf . Note that in the above calculation of the mass splitting, we modify $\Pi_{3/2^-}^{\text{OPE}}(q^2)$ by multiplying it with an overall factor such that its perturbative term is identical to that of $\Pi_{1/2^-}^{\text{OPE}}(q^2)$ as shown in Eqs. (16) and (17). This technique is equivalent to normalizing the currents. It does not change the extracted mass according to Eq. (19), but affects the coupling constant in Eq. (20). Therefore, the derived δM represents the genuine mass splitting between the two spin-orbit states, whereas the δf does not carry the same physical meaning.

To determine the values of δM and δf , we require the parameters specified in Eqs. (30) and (31), namely the threshold s_0 , the Borel mass M_B^2 , the quantities $f_{1/2}$ and $M_{1/2}$. In our previous work [48], we carried out a detailed analysis of the masses and coupling constants of the P -wave Ω baryons with quantum numbers $J^{PC} = 1/2^-$ and $J^{PC} = 3/2^-$, results are summarized in Table I. For both states, the central values of the input parameters are taken as $s_0 = 6.0 \text{ GeV}^2$ and $M_B^2 = 1.65 \text{ GeV}^2$. The extracted mass and coupling constant for $J^{PC} = 1/2^-$ state are

TABLE II. The uncertainties of the mass splitting arise from the s_0 , M_B^2 and the condensates.

	$s_0(\text{GeV}^2)$		$M_B^2(\text{GeV}^2)$		$\langle\bar{q}q\rangle(\text{GeV}^3)$		$\langle\bar{s}s\rangle(\text{GeV}^3)$	
	5.5	6.5	1.58	1.73	-0.01	+0.01	-0.1	+0.1
$\delta m(\text{GeV})$	+0.0069	-0.0066	+0.0079	-0.0057	-0.0070	+0.0079	-0.0073	+0.0075
$\delta f(\text{GeV}^3)$	-0.0006	+0.0006	-0.0004	+0.0004	+0.0005	-0.0006	+0.0005	-0.0006
	$\langle g_s \bar{s} \sigma G s \rangle(\text{GeV}^3)$		$\langle \alpha_s G G \rangle(\text{GeV}^4)$		$m_s(\text{MeV})$		Total	
	-0.2	+0.2	-0.0035	+0.0035	-0.003	+0.009	-	+
$\delta m(\text{GeV})$	-0.0106	+0.0300	-0.0003	+0.0003	-0.0003	+0.0003	-0.0171	+0.0336
$\delta f(\text{GeV}^3)$	+0.0021	-0.0025	+0.00002	-0.00002	+0.0002	-0.0005	-0.0028	+0.0023

$$M_{1/2^-} = 2.07_{-0.07}^{+0.07} \text{ GeV},$$

$$f_{1/2^-} = 0.079_{-0.011}^{+0.011} \text{ GeV}^3. \quad (32)$$

Utilizing these inputs, we can straightforwardly determine δM and δf as

$$\delta M = -0.018_{-0.0171}^{+0.0336} \text{ GeV}, \quad (33)$$

$$\delta f = 0.00017_{-0.0028}^{+0.0023} \text{ GeV}^3. \quad (34)$$

The uncertainties arise from variations in s_0 , M_B^2 as reported in Ref. [48] where $5.5 \text{ GeV}^2 \leq s_0 \leq 6.5 \text{ GeV}^2$ and $1.58 \text{ GeV}^2 \leq M_B^2 \leq 1.73 \text{ GeV}^2$, together with the condensates listed below [3,49–56]:

$$\begin{aligned} \langle\bar{q}q\rangle &= -(0.240 \pm 0.010)^3 \text{ GeV}^3, \\ \langle\bar{s}s\rangle &= (0.8 \pm 0.1) \times \langle\bar{q}q\rangle, \\ \langle g_s \bar{s} \sigma G s \rangle &= (0.8 \pm 0.2) \times \langle\bar{s}s\rangle, \\ \langle \alpha_s G G \rangle &= (6.35 \pm 0.35) \times 10^{-2} \text{ GeV}^4, \\ m_s &= 93_{-3}^{+9} \text{ MeV}. \end{aligned} \quad (35)$$

As summarized in Table II, the resulting uncertainties are relatively small, with the mixed condensate providing the dominant contribution. This can be attributed to the fact that the spectral densities of the $3/2$ and $1/2$ states share a similar structure: each term appears in the same form with only small differences in their coefficients. The relative ratios of the corresponding terms are close, except for the mixed condensate contribution. Therefore, when evaluating the mass difference, the uncertainties induced by these common parameters largely cancel each other, leaving the mixed condensate as the primary source of uncertainty.

For completeness, we have also examined an alternative treatment in which $\delta f = 0$ is imposed by redefining the current $J'_\mu = f_{1/2}/f_{3/2} J_\mu$. The resulting mass splitting is found to be similar to that obtained in our main analysis,

reinforcing the robustness of our conclusion that the splitting is small.

IV. SUMMARY AND DISCUSSION

In this work, we have formulated the QCD sum rules to investigate the mass splitting for P -wave Ω baryons based on our previous work [48] where $\Omega(2012)$ was identified as the P -wave excitation with $J^P = 3/2^-$. There we constructed the currents for the two spin-orbit partner states of P -wave Ω baryons and applied parity-projected QCD sum rules to have extracted their masses as $M_{3/2^-} = 2.05_{-0.10}^{+0.09} \text{ GeV}$ and $M_{1/2^-} = 2.07_{-0.07}^{+0.07} \text{ GeV}$ as in Eqs. (1) and (2). These values have relatively large uncertainties. To improve them, we have formulated dedicated sum rules directly for the mass splitting. By parametrizing the hadron masses and couplings as $M_{3/2^-} = M_{1/2^-} + \delta M$ and $f_{3/2^-} = f_{1/2^-} + \delta f$, and then expanding the correlation functions in a Taylor series, we obtain expressions for the mass splitting. At the quark-gluon level, the corresponding expressions are derived from the calculated spectral densities. Matching the two representations, we extract the mass splitting as

$$\delta M = -18.0_{-17.1}^{+33.6} \text{ MeV}. \quad (36)$$

Through this procedure, we obtain an improved prediction of small mass splitting between the P -wave Ω baryons. Considering expecting decay patterns of the $J^P = 3/2^-$ and $J^P = 1/2^-$ states, where $J^P = 1/2^-$ state has generally a large width, the resulting two states may be observed experimentally as one peak of $J^P = 3/2^-$ state and a broad bump due to the $J^P = 1/2^-$ state. The results indicate that the $\Omega(2109)$ is unlikely to be the spin-orbit partner of the $\Omega(2012)$, since their experimental gap of about 100 MeV is significantly larger than our calculated splitting.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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