

Emergence of Unruh prethermalization for uniformly accelerating many-atom system

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A uniformly accelerated atom in an inertial vacuum generally thermalizes to a Gibbs state, which is known as the Unruh effect. In contrast, for a noninteracting many-body system coupled to a common massless scalar field, we find that thermalization proceeds in multiple stages: the system first attains a prethermal generalized Gibbs state, before eventual relaxation to the thermal Gibbs state. The prethermal state is protected by emergent conserved quantities; hence, the system behaves like a nearly integrable one, which shows a sharp distinction from the Unruh effect. We coin the term “Unruh prethermalization” to characterize this phenomenon. The measure of entanglement is a good estimation of the lifetime of the prethermal state and is consistent with previous studies. Finally, we show that in such a regime, the dynamics show a Dicke superradiance-type radiation burst before reaching the prethermal state. In contrast, only a monoexponential decay is observed for Unruh thermalization. In addition, to highlight the significance of our results, we compare them with existing experimental observations.

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I. INTRODUCTION

Thermalization of a many-body quantum system is one of the major areas of research of the last few decades in quantum and statistical physics [1]. A simplified definition of the thermalization phenomenon is given as the complete loss of initial memory in many-body quantum systems where the ergodicity condition holds [2]. Two different approaches are developed throughout the year to explain it, i.e., the first one is for the isolated quantum systems, where such systems thermalize due to their internal dynamics, whereas the second one is strictly for the open quantum systems, where the system is coupled with a comparatively larger subsystem, which is already in thermal equilibrium.

Here, we mostly focus on the latter case, where it is expected that the system will reach the thermal Gibbs state at a late time and the final temperature of the system will be equal to the bath temperature. In the weak-coupling approximation and the Markovian limit, such an irreversible journey of the system is well explained by the Lindblad equation [3]. The dynamics beyond the Markovian regime and the weak-coupling approximation have been explored recently to understand the other demanding aspects of the irreversible dynamics [4].

An exception occurs when a many-body system fails to thermalize. In such cases, the initial memory of the initial states can be kept for an arbitrary long time, e.g., integrable systems [5–7]. It has been shown that the quantum dynamics is constrained by several conserved quantities that help the system to skip thermalization. Such non-thermality has been studied thoroughly in condensed matter physics, atomic physics, and quantum information [8]. A different class of systems that shows striking features of nonthermality is the nearly integrable quantum systems [5,9–12]. There exists a clear timescale separation in the dynamics of such nonintegrable systems. However, the extremely long-time dynamics have no difference from the

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thermalization. But in the intermediate timescale, the system reaches a quasistationary state, which is widely known as the prethermal state. The dynamics are constrained by several quasiconserved quantities in this regime. Recently, it was obtained that an open quantum system consisting of noninteracting spins can show the notion of prethermalization in the presence of an engineered reservoir with a long bath correlation length [11]. Such an environment with a high correlation length can hold the quantum memory for a longer time, which ultimately leads to prethermalization. There also exist several states that act as a decoherence-free subspace in the prethermal regime. Such a state has several applications in making an efficient entanglement storage device, which was explored recently [13]. Motivated by the above-mentioned ideas, we want to study the prethermal behavior of open quantum systems in a different context, i.e., the Unruh effect.

The indistinguishability between the quantum vacuum fluctuation and the thermal fluctuation leads to the discovery of the Unruh effect [14]. Such a connection between the quantum vacuum and the thermal bath is first introduced as the thermalization theorem by Fulling and Davies, which states that the inertial vacuum acts as a thermal bath for a uniformly accelerated observer [15,16]. Moreover, it was shown that the temperature of the bath is proportional to the acceleration of the detector. Later, Unruh put this theorem in the context of particle creation, which, after him, is known as the renowned Unruh effect [17]. It is well known that the Unruh thermalization is also connected to the Hawking radiation in the near-horizon region [18].

From the perspective of a uniformly accelerated atom prepared in an initial pure state, it interacts with the vacuum state of a free scalar field and goes through a nonunitary time evolution. As a result, the system will reach a thermal mixed state at equilibrium. Following the prescription of Benatti *et al.*, we treat the atoms within an open quantum system framework, where the presence of the surrounding environment is modeled as a thermal bath. In this approach, the effective temperature of the bath is proportional to the atomic acceleration α , reflecting the Unruh-like nature of the noise perceived by the accelerated atoms [19]. Hence, the Lindblad equation successfully captures the dynamics of the system. The steady-state solution of the Lindblad equation is the thermal state, which is a mixed state with the maximum von Neumann entropy. The initial memory of the atom is completely lost during this irreversible time evolution, and no conserved quantities survive at thermal equilibrium.

However, more interesting features can be observed when we increase the number of atoms in this system. For example, the emergence of entanglement between two noninteracting accelerated atoms occurs for a particular regime of interatomic distance and acceleration [19]. We note that in such a regime, the thermal bath acts as a

common bath, which creates quantum correlations between the atoms [20]. An interatomic distance-dependent thermal-nonthermal signature of the Unruh effect in resonance Casimir-Polder interaction (RCPI) was reported earlier. At longer atomic separation, the local inertial approximation is not valid, hence the system shows nonthermal behavior. On the other hand, such a response is thermal at lower separation due to the validity of local inertial approximation [21–26]. We argue that in this regime of local inertia, the system may skip the Unruh thermalization, as the dynamics are constrained by several conserved quantities. Here, we explore such a regime, where the time evolution is different from Unruh thermalization.

The significant similarities between the common-environment problem and entanglement generation due to vacuum fluctuation bring our major interest in investigating the existence of an Unruh-prethermal state for a uniformly accelerating two-atom system [11,19,20]. We also extend our analysis to the many-atoms case to observe whether such a system shows the notion of collective behavior or not. A set of noninteracting atoms initially prepared in an excited state shows the notion of Dicke superradiance due to coupling with the common modes of the cavity [27]. One key feature of the Dicke superradiance is that the decay profile of the spontaneous emission is nonmonotonic in nature, and the radiation burst is more intense and short-lived for increasing the number of atoms [28–30]. One of our central results in this paper is the similarities between the intermediate scale dynamics toward the Unruh prethermal state of a many-body quantum system and the Dicke superradiance.

In this manuscript, we confine ourselves to the Markovian regime. We note that the source of non-thermality, which is investigated here, is completely different from the non-Markovian Unruh effect [31]. We also use the exact experimental values estimated by Bell and Leinass in our numerical simulation to provide an estimation of the prethermal regime [32]. Such an estimation will be helpful for future experiments involving the observation of Unruh prethermalization.

Throughout the paper, we follow the given notation. The four-vector is denoted by “ x ,” the time is represented by “ t ,” and the proper time by “ τ .” We denote the space coordinate by “ $\mathbf{x}$,” ($\mathbf{x} = X, Y, Z$). We arrange the manuscript in the following order. In Sec. II, we provide a brief description of the system where we explicitly show the vacuum correlation function and the analytical form of the Lindblad equation. We consider the two-atom problem in Sec. III, where, in terms of the two-atom observable, we calculate the steady state and predict the emergence of a prethermal state, and we also show the entanglement generation as a feature of the prethermal state. In Sec. IV, we extend our approach to the many-atom case, where we particularly focus on two aspects: the von Neumann entropy for the Unruh prethermal state and the Dicke-type short-lived

radiation burst in intermediate-scale dynamics. We also compare our results with the existing experimental works in Sec. V. Finally, in Sec. VI, we will present a general overview and the outlook of Unruh Prethermalization and conclude in Sec. VII.

II. DYNAMICS OF TWO ACCELERATED ATOMS THROUGH A MASSLESS SCALAR FIELD

Here, we consider a system of two atoms (two-level systems) uniformly accelerated in the Minkowski space-time interacting with a free massless scalar field. The atoms are following the hyperbolic trajectory with respect to the inertial observer. The positions of the atoms in terms of proper time are written by,

$$\begin{aligned} t_a &= \frac{1}{\alpha} \sinh \alpha \tau, & X_a &= \frac{1}{\alpha} \cosh \alpha \tau, & Y_a &= 0, & Z_a &= Z_a; \\ t_b &= \frac{1}{\alpha} \sinh \alpha \tau, & X_b &= \frac{1}{\alpha} \cosh \alpha \tau, & Y_b &= 0, & Z_b &= Z_b. \end{aligned} \quad (1)$$

Here, Z_a, Z_b are the constant spatial coordinates. The proper distance (interatomic distance) between the atoms is defined as $L = |Z_a - Z_b|$. We assume that such a distance is constant in time. Here, a and b denote the fixed positions of the two atoms.

The Hamiltonian of the system + scalar field is expressed as,

$$\mathcal{H} = \mathcal{H}_s^\circ + \mathcal{H}_L^\circ + \mathcal{H}_{SL}. \quad (2)$$

Throughout the paper, we use the natural units, $\hbar = c = 1$. Here, we assume the two atoms have the same Zeeman levels, and the free Hamiltonian of the system is $\mathcal{H}_s^\circ = \mathcal{H}_s^{\circ 1} + \mathcal{H}_s^{\circ 2} = \frac{1}{2} \omega_\circ \sigma_3^{(1)} + \frac{1}{2} \omega_\circ \sigma_3^{(2)}$ (The superscript in the Pauli matrices represents the atom number.) σ_i is the Pauli matrix, and $\omega_\circ$ is the frequency of Zeeman levels. For simplicity, we assume that the principal axis of the atoms is in the “Z” direction. The free Hamiltonian of the scalar field $\mathcal{H}_L^\circ$ can be expressed as, $\mathcal{H}_L^\circ = \int \frac{d^3 k}{(2\pi)^3} \omega_k a^\dagger(k) a(k)$. Here, ω_k is the frequency of the free scalar field, and $a^\dagger, a$ are the creation and annihilation operators of the quantized field, respectively. Motivated by the previous work by Benatti *et al.* [19], we define the coupling Hamiltonian between the atom and the scalar field as

$$\mathcal{H}_{SL} = \lambda \sum_{\mu=0}^3 [\sigma_\mu^{(1)} \otimes \phi_\mu(x_1) + \sigma_\mu^{(2)} \otimes \phi_\mu(x_2)], \quad (3)$$

where λ is the coupling constant, ϕ represents the free scalar field, which satisfies the massless Klein-Gordon equation, and x_1, x_2 are the individual trajectories of two atoms. Here, $\phi_\mu(x) = \sum_{a=1}^N [\chi_\mu^a \phi^-(x) + (\chi_\mu^a)^\dagger \phi^+(x)]$. $\phi^\pm(x)$ is the positive and negative field operator of N

independent modes of the free scalar field and χ_μ^a are the corresponding complex coefficients of the field modes. Taking into account the isotropic case, we consider all components $\{X, Y, Z\}$ of the field modes in the system-field coupling. However, it is well known that the “Z” component of the system-environment coupling leads to the dephasing process, and it has no contribution in the decay to the thermal state [3]. Following the same analogy, we expect that the “Z” component has a negligible contribution in this case. It is assumed that the spins and the scalar field are initially uncorrelated, so the total initial density matrix can be written as $\rho(0) = \rho_s(0) \otimes |0\rangle\langle 0|$. $|0\rangle$ is the vacuum state of the scalar field, and $\rho_s(0)$ is the initial density matrix of the system. Such an assumption is consistent with the Born-Markov approximation [3]. We are also working in the weak system-scalar field-coupling limit. Using the von Neumann Liouville equation in the interaction frame, expanding it up to the second-order terms of $\mathcal{H}_{SL}$ and finally taking the trace over the field variable, we get the dynamical equation of the system, which is written below,

$$\frac{d\rho_s(\tau)}{d\tau} = -i[\mathcal{H}_{\text{lamb}}, \rho_s(\tau)] + \mathcal{L}(\rho_s(\tau)). \quad (4)$$

The above Eq. (4) is known as the Lindblad quantum master equation (QME) [3]. Here, $\mathcal{L}(\rho_s(\tau))$ is the dissipator of the master equation and $\mathcal{H}_{\text{lamb}}$ is known as the Lamb-shift Hamiltonian that leads to the renormalization of the Zeeman Hamiltonian. We note that the Lamb shift gives rise to the Casimir-Polder interaction between the spins [24,25]. The analytical forms are given by,

$$\mathcal{L}(\rho_s) = \sum_{a,b=1}^2 \sum_{j,k=1}^3 \gamma_{jk}^{ab} \left(\sigma_b^k \rho_s \sigma_a^j - \frac{1}{2} \{ \sigma_a^j \sigma_b^k, \rho_s \} \right), \quad (5)$$

$$\mathcal{H}_{\text{lamb}} = -\frac{i}{2} \sum_{a,b=1}^2 \sum_{j,k=1}^3 \mathcal{S}_{jk}^{ab} \sigma_a^j \sigma_b^k. \quad (6)$$

The field correlation timescale is taken to be smaller than the relaxation timescale of the system. The dynamics have the following properties of complete positivity and trace preservation. Here, we assume that χ_μ^a satisfies $\sum_{a=1}^N \chi_\mu^a (\chi_\nu^a)^\dagger = \delta_{\mu\nu}$. Hence, the field correlation functions are diagonal, i.e., $G_{ij}^{ab}(x-y) = \delta_{ij} G^{ab}(x-y)$. The expressions of the field-correlation function are obtained by,

$$G^{ab}(\Delta\tau) = \langle 0 | \hat{\Phi}(\tau, \mathbf{x}_a) \hat{\Phi}(\tau', \mathbf{x}_b) | 0 \rangle. \quad (7)$$

Here $\Delta\tau = (\tau - \tau')$. The correlation function is also assumed to be stationary, which is an essential condition for the Markovian time evolution. For a massless scalar field, in the four-dimensional Minkowski spacetime, the expression of this function is written as [17],

$$G(x, x') = -\frac{1}{4\pi^2 R^2}, \quad (8)$$

where $R = \sqrt{(t - t' - i\epsilon)^2 - |\mathbf{x} - \mathbf{x}'|^2}$. It is called the positive-frequency Wightman function. Here $i\epsilon$ is a small constant in the complex plane and acts as a regulator to avoid the divergence and make sure about the function's analytic property over the chosen complex plane. The Fourier transform of the correlation function along the trajectory of the atoms is given below,

$$\mathcal{G}^{ab}(\omega_\circ) = \frac{1}{2\pi} \frac{\omega_\circ}{1 - e^{-2\pi\omega_\circ/\alpha}} f_{ab}(\alpha). \quad (9)$$

Here, $f_{ab}(\alpha) = \frac{\sin(2\frac{\omega_\circ}{\alpha} \sinh^{-1}(\alpha L/2))}{L\omega_\circ \sqrt{1+L^2\alpha^2/4}}$ for $a \neq b$, and $f_{ab}(\alpha) = 1$ for $a = b$. Using the above Eq. (9), the explicit form of the Kossakowski matrix, which is an essential component of the Lindbladian, is given by [19],

$$\gamma_{jk}^{ab} = A^{ab}\delta_{jk} - iB^{ab}\epsilon_{jkl}\delta_{3l} - A^{ab}\delta_{3k}\delta_{3l}. \quad (10)$$

A^{ab} (B^{ab}) are the even (odd) combination of $\mathcal{G}^{ab}$ for the positive and negative Zeeman frequency. Here, γ_{jk}^{ab} has a nonzero contribution for $a \neq b$, which signifies the atom-atom correlation due to the common environment. We also note that the “Z” component has no contribution in the dynamics, $\gamma_{jk}^{ab} = 0$, for $j = 0$, and $k = 0$.

For a massless scalar field, the explicit forms of A^{ab} , B^{ab} are written as,

$$B^{ab} = \frac{\lambda^2 \omega_\circ}{8\pi} f_{ab}(\alpha), \quad A^{ab} = B^{ab} \times \coth(\pi\omega_\circ/\alpha). \quad (11)$$

Here $a, b = \{1, 2\}$. We note that f_{ab} has two extreme limits based on the value of αL . For $L \ll 1/\alpha$, $f_{ab} \approx 1$, and for $L \gg 1/\alpha$, $f_{ab} \approx 0$. In terms of (up)downward transition rate (γ_+) γ_- , the Lindblad equation is written as,

$$\begin{aligned} \mathcal{L}(\rho_s) = & \sum_{a,b=1}^2 \gamma_+ \left(\sigma_+^a \rho_s \sigma_-^a - \frac{1}{2} \{ \sigma_-^a \sigma_+^a, \rho_s \} \right) \\ & + \gamma_- \left(\sigma_-^a \rho_s \sigma_+^a - \frac{1}{2} \{ \sigma_+^a \sigma_-^a, \rho_s \} \right). \end{aligned} \quad (12)$$

Here, $\gamma_+ = \frac{\lambda^2 \omega_\circ}{8\pi} (1 + \tanh(\pi\omega_\circ/\alpha))/2$, and $\gamma_- = \frac{\lambda^2 \omega_\circ}{8\pi} (1 - \tanh(\pi\omega_\circ/\alpha))/2$.

III. DYNAMICAL EQUATION OF THE SYSTEM

The steady-state solution of the QME gives the notion of the equilibrium state of the system. In the Liouville space, the QME can be written as, $\frac{d}{dt} \hat{\rho}_s = \hat{\mathcal{L}} \hat{\rho}_s$. Here, $\hat{\rho}_s$ is a column vector of dimension $N^2 \times 1$ and $\hat{\mathcal{L}}$ is the Lindbladian superoperator of dimension $N^2 \times N^2$ (N is the dimension of the Hilbert space). The eigenvector

corresponding to the zero eigenvalue is the steady-state solution of the QME. In our case, the steady-state solution depends on the behavior of f_{ab} . For a single spin-1/2 system and 2×2 matrix, the observables are $(\sigma_x, \sigma_y, \sigma_z)$ (one constraint, i.e., the trace preservation) and the corresponding equations are known as the Bloch equations. Similarly, for two spin systems, there exist fifteen observables. As the Zeeman levels of the spins are the same, the number of observables is further reduced to nine, which can be written as [10],

$$\begin{aligned} M_i &= \frac{1}{2} \text{Tr}_s([\sigma_i^{(1)} \otimes \mathbb{I} + \mathbb{I} \otimes \sigma_i^{(2)}] \rho_s), \\ M_{ii} &= \frac{1}{4} \text{Tr}_s([\sigma_i^{(1)} \otimes \sigma_i^{(2)}] \rho_s), \\ M_{ij} &= \frac{1}{4} \text{Tr}_s([\sigma_i^{(1)} \otimes \sigma_j^{(2)} + \sigma_j^{(1)} \otimes \sigma_i^{(2)}] \rho_s), \quad (i \neq j). \end{aligned} \quad (13)$$

The Bloch-type equations for our case are written as,

$$\begin{bmatrix} \dot{M}_z \\ \dot{M}_{zz} \\ \dot{M}_c \end{bmatrix} = \begin{bmatrix} -A^{11} & 0 & 2B^{12} \\ B^{11}/2 & -2A^{11} & A^{12} \\ -B^{12}/2 & 2A^{12} & -A^{11} \end{bmatrix} \begin{bmatrix} M_z \\ M_{zz} \\ M_c \end{bmatrix} + \begin{bmatrix} B^{11} \\ 0 \\ 0 \end{bmatrix}. \quad (14)$$

Here $M_c = M_{xx} + M_{yy}$ and f_{ab} varies from $0 \leq f \leq 1$. Here, we are interested in the steady state behavior of the system. Therefore, we focus on the observables that represent the populations of the energy levels (i.e., the diagonal elements), which are described by M_z , M_{zz} , as well as the observable M_c that is dynamically coupled to them through the equations of motion. We find that only these observables remain nonzero in the limit, $t \rightarrow \infty$, due to the presence of inhomogeneous terms in the coupled differential equations for M_z, M_{zz}, M_c . The differential equations for the remaining observables are decoupled from this set and, in the absence of inhomogeneous terms, their solutions decay to zero. Therefore, these observables only characterize transient decoherence and dephasing dynamics and can be neglected when discussing steady-state properties. In the next subsection, we will show the steady state solution for different values of f_{ab} .

A. Steady state solution for $0 \leq f_{ab} < 1$

We note that, in this regime, the Lindbladian has a single zero eigenvalue, which is shown in Fig. 1. We note that the steady-state solution is independent of the initial value dependency. Such a solution is given by,

$$\begin{aligned} M_z^{\text{eq}} &= M_\circ, \\ M_{zz}^{\text{eq}} &= \frac{M_\circ^2}{4}, \\ M_c^{\text{eq}} &= 0. \end{aligned} \quad (15)$$

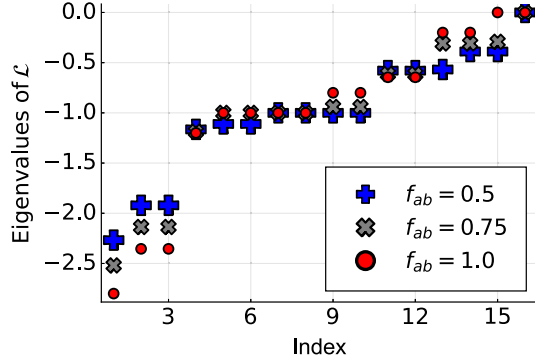


FIG. 1. Plot of the eigenspectrum of the Liouvillian for different values of f_{ab} . The effect of $\mathcal{H}_{\text{lamb}}$ is neglected, hence all the 16 eigenvalues of $\mathcal{L}$ for the two-spin case are real. The x axis denotes the index of eigenvalues arranged in decreasing order of the absolute value of their real parts. Index 1 corresponds to the eigenvalue with the largest absolute real part, while index 16 corresponds to the smallest eigenvalue (which is zero). For $f_{ab} = 1$, we have two zero eigenvalues, whereas for $f_{ab} < 1$, we have a single zero eigenvalue. With decreasing f_{ab} , the 15th eigenvalue also decreases. Hence, at $f_{ab} \rightarrow 1$, it is very close to 1 and hence shows the notion of prethermalization. Here, we choose $\gamma_+ = 0.8$, and $\gamma_- = 0.2$.

Here, $M_{\circ} = B^{11}/A^{11} = \tanh(\pi\omega_{\circ}/\alpha)$, which denotes the equilibrium value of the “Z” magnetization, when the system only interacts with the local environment. Interestingly, $\alpha = 1/\beta$ (β is the typical definition of inverse temperature in statistical mechanics), which ensures that the Unruh temperature is proportional to the acceleration of the atom. Using the above solution, the steady-state density matrix is given by,

$$\begin{aligned} \rho_s^{\text{eq}} &= \frac{e^{-\frac{\pi}{\alpha}\mathcal{H}_s^{\circ}}}{\mathcal{Z}}, \\ &= \mathbb{I}/4 + \frac{M_z^{\text{eq}}}{2}(I_z^1 + I_z^2) + 4M_{zz}^{\text{eq}}I_z^1I_z^2. \end{aligned} \quad (16)$$

Here, $\mathcal{Z}$ is the partition function, and $I_i = \sigma_i/2$. In this regime, the expression for ρ_s^{eq} indicates that the thermalization theorem holds as the final state is a thermal Gibbs state [17].

B. Steady state solution for $f_{ab} = 1$

Before discussing the steady state solution for this case, we give a short review of the symmetries and conserved quantities in open quantum systems [33,34]. Symmetry can be classified into two categories. If an operator $\mathcal{O}$ commutes with both the Hamiltonian and the jump operators of the Liouvillian, then it is defined as the strong symmetry operator. On the other hand, if the symmetry superoperator only commutes with the total Liouvillian, not the individual components (i.e., Hamiltonian and jump operators), then it is called the weak symmetry. Hence, the strong symmetry

conditions already satisfy the weak symmetry condition, but the reverse condition is not always true.

In our case, we find that at $f_{ab} = 1$, the Liouvillian has two zero eigenvalues (shown in Fig. 1), which corresponds to the existence of another symmetry or conserved quantities apart from the trace preservation. Here, $A^{11} = A^{12}$, $B^{11} = B^{12}$. The Lindbladian superoperator $\hat{\mathcal{L}}$ is invariant under the following unitary transformation, $\hat{U}(\kappa)\hat{\mathcal{L}}\hat{U}^{\dagger}(\kappa) = \hat{\mathcal{L}}$ where κ is real. We define, $\hat{U}(\kappa) = \exp[-i\hat{D}\kappa]$ and $\hat{D} = D \otimes \mathbb{I} - \mathbb{I} \otimes D^T$. Here, $D = \vec{\sigma}_i \cdot \vec{\sigma}_j$. $\hat{D}$ is the symmetry superoperator. The analytical form of conserved quantity is written as,

$$\frac{d}{d\tau}(M_{xx} + M_{yy} + M_{zz}) = 0. \quad (17)$$

The steady state solution in terms of observables is given by,

$$\begin{aligned} M_z^{\text{eq}} &= \frac{M_{\circ}(3 + 4(M_2 + M_3))}{3 + M_{\circ}^2}, \\ M_c^{\text{eq}} &= -\frac{M_{\circ}^2 - 4(M_2 + M_3)}{2(3 + M_{\circ}^2)}, \\ M_{zz}^{\text{eq}} &= M_2 + M_3 - M_c^{\text{eq}}. \end{aligned} \quad (18)$$

Here, $M_c(0) = M_2$, $M_{zz}(0) = M_3$ and $M_{\circ} = \tanh(\pi\omega_{\circ}/\alpha)$. We note that the steady state solution has the initial value dependency, which is a sharp contrast from the thermal Gibbs state. The steady state density matrix is given by,

$$\begin{aligned} \rho_s^{\text{eq}} &= e^{-\frac{\pi}{\alpha}\mathcal{H}_s^{\circ}} \frac{l_1 \vec{\sigma}_1 \cdot \vec{\sigma}_2}{\mathcal{Z}_g}, \\ &= \mathbb{I}/4 + \frac{M_z^{\text{eq}}}{2}(I_z^1 + I_z^2) + 4M_{zz}^{\text{eq}}I_z^1I_z^2 \\ &\quad + 2M_c^{\text{eq}}(I_x^1I_x^2 + I_y^1I_y^2). \end{aligned} \quad (19)$$

Here, $l_1 = \ln\left(\frac{1-4(M_2+M_3)}{3+4(M_2+M_3)}(1+2\cosh\frac{\pi\omega_{\circ}}{\alpha})\right)$. $\mathcal{Z}_g$ is the partition function for the generalized Gibbs state.

C. Emergence of prethermalization at $f_{ab} \rightarrow 1$

$f_{ab} = 1$ is an asymptotic limit, as such a value is only possible when $L = 0$, or $1/\alpha = 0$. Such values are not possible experimentally. Here, our main focus is to study the regime of high acceleration and closely spaced atoms. In this regime, for the two-atom case, the Liouvillian has a single zero eigenvalue, but the nearest eigenvalue is very close to zero.

Following our recent work, the prethermalization can be identified by looking at the structure of the eigenspectrum of the Liouvillian, i.e., the eigenspectrum consists of a single zero eigenvalue, and the real part of a few eigenvalues is very close to zero in comparison to the remaining sets of eigenvalues [11]. We note that in this case, the eigen-

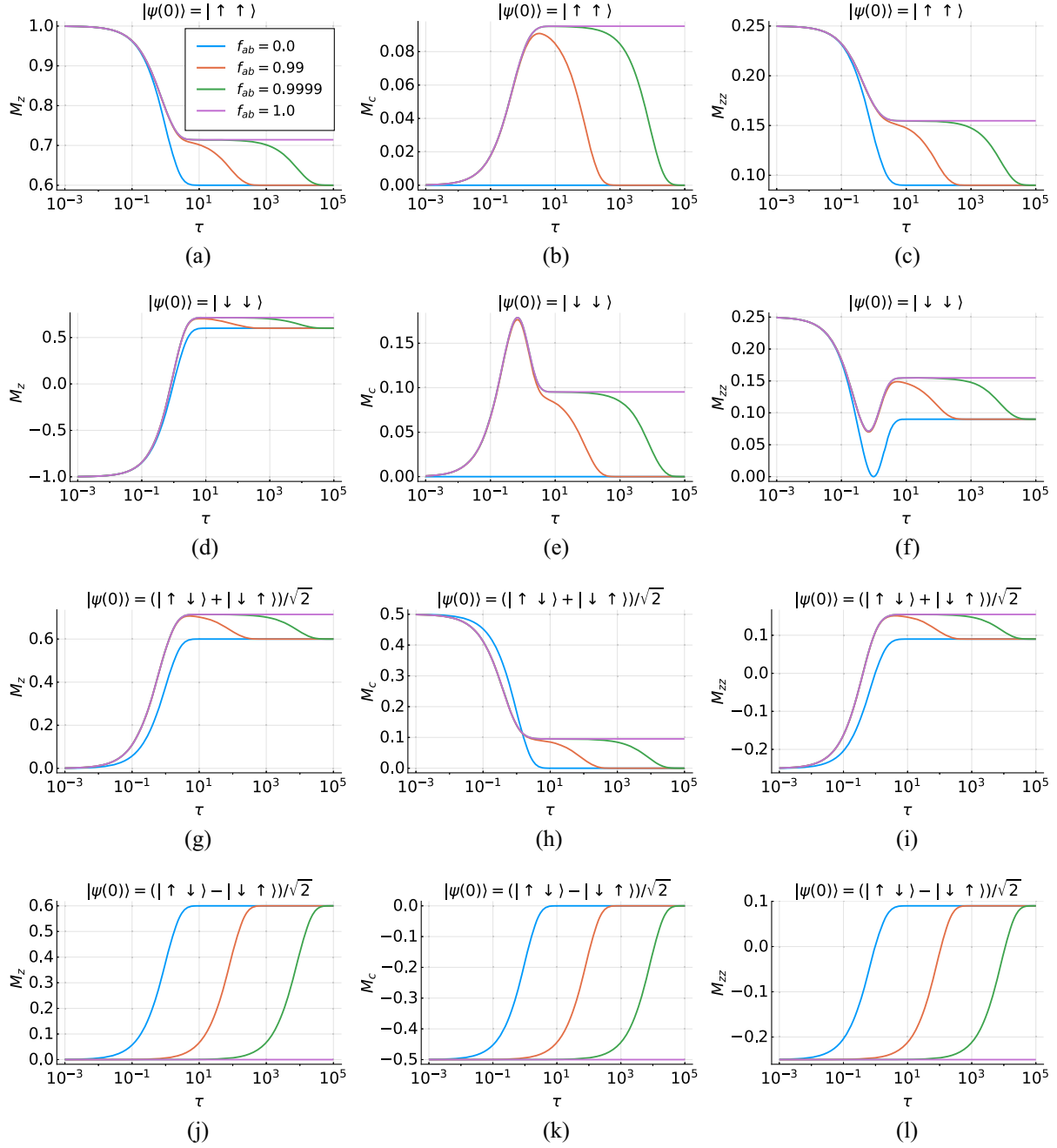


FIG. 2. Plot of the time evolution of the relevant observables $\{M_z, M_{zz}, M_c\}$ for different choices of f_{ab} and initial condition. Here $N = 2$. The initial states are chosen from the dipolar basis as it is the eigenbasis of the symmetry operator $\vec{\sigma}_1 \cdot \vec{\sigma}_2$. We note that the steady state value of the observables is different for $f_{ab} = 1$ and $f_{ab} < 1$. The steady state for $f_{ab} < 1$ is the Gibbs state, and for $f_{ab} = 1$ it is the generalized Gibbs state. For $f_{ab} \rightarrow 1$, the system spends a longer time at a quasisteady state, which is the same as the steady state for $f_{ab} = 1$, which shows the notion of prethermalization. The singlet state $|\psi\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ never evolves for $f_{ab} = 1$, hence acting as a dark state of decoherence-free subspace. Here, we choose $\gamma_+ = 0.8$, and $\gamma_- = 0.2$. As our focus is to explore the Gibbs state, a long-lived prethermal state, and generalized Gibbs state, we choose the following four values of $f_{ab} = \{0.0, 0.99, 0.9999, 1.0\}$.

spectrum follows a similar structure. Hence, we conclude that the system reaches a prethermal quasisteady state before it thermalizes. We call such phenomena the “Unruh prethermalization.” As such, in the intermediate timescale, the system follows the dynamics for $f_{ab} = 1$, so the quasisteady state is a generalized Gibbs state. However, the system

follows the dynamics for $0 \leq f_{ab} < 1$ at a later time. Hence, it reaches the thermal Gibbs state in the long-time limit.

We are interested in the dynamics of the system in the dipolar basis, as it is the eigen-basis of the symmetry operator. The corresponding states are $|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$, $(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$, and $(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$. Among the

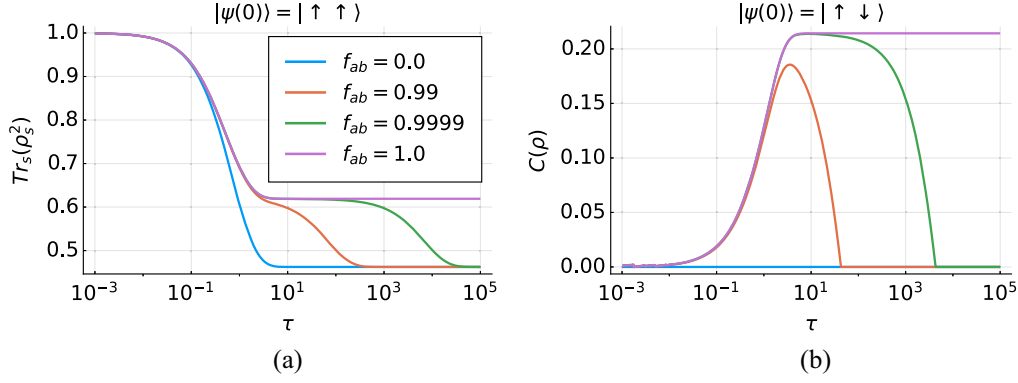


FIG. 3. Plot of the time evolution of the purity operator, $\text{Tr}(\rho_s^2)$ and concurrence $C(\rho_s)$ for different choices of f_{ab} . Here $N = 2$. For $\text{Tr}(\rho_s^2)$, we choose the initial condition as $|\uparrow\uparrow\rangle$. When $f_{ab} < 1$, the steady state purity is minimum, whereas, due to the presence of the symmetry, purity is increased for $f_{ab} = 1$. The system shows the notion of prethermalization for $f_{ab} \rightarrow 1$. The creation and evolution of entanglement is shown for a separable state $|\uparrow\downarrow\rangle$. The maximum value of $C(\rho_s)$ occurs for $f_{ab} = 1$. The entanglement decays as the prethermal state evolves to the thermal state. Hence, the value of $C(\rho_s)$ is a good identifier of the lifetime of the prethermal state. Here, we choose $\gamma_+ = 0.8$, and $\gamma_- = 0.2$.

four states, the first two states are separable, and the next two are maximally entangled states. We show the plots for the relevant observables, $\{M_z, M_{zz}, M_c\}$, using the above four states as an initial configuration. We only consider four values of $f_{ab} = \{0.0, 0.99, 0.9999, 1.0\}$. Apart from the singlet state (i.e., $|\psi(0)\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$), Fig. 2, depicts the emergence of prethermalization for $f_{ab} \rightarrow 1$, as the quasisteady state is similar to $f_{ab} = 1$, whereas, the final steady state is same as $f_{ab} = 0$. We note that $1/(1 - f_{ab})$ is a good identifier of the lifetime of the prethermal state. For the singlet state, there is no evolution for $f_{ab} = 1$, hence it acts as a dark state. A detailed discussion is provided in the next section.

To exhibit the presence of a prethermal state, we also plot the purity of the quantum state as a function of time in Fig. 3(a). The purity is defined as $\text{Tr}(\rho_s^2)$. For a pure quantum state $\text{Tr}(\rho_s^2) = 1$, whereas, for a mixed state $\text{Tr}(\rho_s^2) < 1$. The purity is a minimum for the thermal state. We show that for $f_{ab} \rightarrow 1$, as the system reaches the prethermal state, $\text{Tr}(\rho_s^2)$ becomes constant at that timescale and further it decays to the minimum value.

D. Entanglement as a measure of the lifetime of the prethermal state

In case of multiple zero eigenvalues, the pure eigenvector of the corresponding eigenvalue acts as a dark state or decoherence-free subspace [35]. Hence, if we choose this state as an initial state, it never evolves. Such a state is also represented as an eigenvector of the symmetry operator [11]. In this case for $f_{ab} = 1$, the dark state is $|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$, which is also an eigenstate of D . It is a maximally entangled antisymmetric Bell state. If we choose the value of $M_2 = -1/4$ and $M_3 = -1/2$, (which corresponds to the singlet state), the final steady state solution is the same as the initial state, ($M_z^{\text{eq}} = 0$,

$M_z^{\text{eq}} = M_2$, $M_c^{\text{eq}} = M_3$). The purity of such an initial state is conserved, $[\text{Tr}_s(\rho_s^2) = 1]$. The presence of such entangled dark states indicates that entanglement can be generated between the noninteracting atoms at the $f_{ab} \rightarrow 1$ limit.

1. Measure of concurrence, $C(\rho_s)$

Concurrence is a good measure for the growth of entanglement for a bipartite system. Following the previous works by Wothers *et al.* [36], we define the concurrence of the two-atom system $C(\rho_s)$ is given by,

$$C(\rho_s) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}. \quad (20)$$

Here, λ_i are the eigenvalues (i.e., arranged in decreasing order) of the non-Hermitian matrix, $\rho_s \rho'_s$. We define $\rho'_s = \sigma_y \otimes \sigma_y \rho_s^* \sigma_y \otimes \sigma_y$, where ρ_s^* is the complex conjugate of ρ_s provided it is written in the Zeeman basis. Here, λ_i are the eigenvalues (i.e., arranged in decreasing order) of the non-Hermitian matrix, $\rho_s \rho'_s$. If $C(\rho_s) > 0$, then the system is entangled, whereas for $C(\rho_s) = 0$, the system is in a separable state. In terms of observables, $C(\rho_s)$ is written as [37]

$$C(\rho_s) = \max\left\{0, 2|M_c| - \frac{1}{2}\sqrt{(1 + 4M_{zz})^2 - 4M_z^2}\right\}. \quad (21)$$

For the singlet state, we note that the values of the observables follow the condition of $C(\rho_s) > 0$. Hence, the entanglement is preserved under the evolution. We also show the plot for the creation of entanglement as a distinct feature of Unruh prethermalization. We choose a separable state ($|\psi(0)\rangle = |\uparrow\downarrow\rangle$) for the initial configuration in Fig. 3(b). Our simulation shows that for the prethermal state, the entanglement is created in the intermediate

timescale and preserved for a long time until it decays to the Gibbs state, and the lifetime increases as f_{ab} gets closer to 1. After a very long time, it further decays. Hence, we claim that entanglement is a good measure for the lifetime of the prethermal state, as the decay of the prethermal state in Fig. 3(a) is quite similar to the decay of entanglement in Fig. 3(b). We note that such analysis is in line with the formation of an entangled state in the presence of a common environment [19,38,39]. The value of $C(\rho_s)$ in the prethermal state is less than 1, which ensures that a mixed state emerges due to interaction with the vacuum.

IV. EXTENSION TO MANY-ATOMS CASE

We further extend our analysis to many-atom cases by considering here a bunch of N noninteracting atoms accelerating together. We only focus on the extreme limits of $f_{ab} = 0, 1$. We assume the coupling constant for atom-field interaction is the same for all atoms, which helps us to simulate the dynamics in the collective basis for $f_{ab} = 1$.

A. Measure of von Neumann entropy

Along with purity, the von Neumann entropy is also a good measure for analyzing the mixing of a system due to irreversible dynamics. The expression for the von Neumann entropy is given as, $S = -\text{Tr}(\rho_s \ln \rho_s)$. In the steady state, the von Neumann entropy becomes maximum. In terms of the partition function, such a quantity is also written as $S = -\beta^2 \frac{\partial}{\partial \beta} (\ln \mathcal{Z} / \beta)$. Here $\beta = \pi / \alpha$.

For $f_{ab} = 0$, the atoms are evolved through their individual dynamics, hence the system reaches the Gibbs state at equilibrium. In this case, the steady state configuration is given by,

$$\rho_s^{\text{eq}} = e^{-\sum_{i=1}^N \mathcal{Z} \mathcal{H}_s^{o,N}} / \mathcal{Z}. \quad (22)$$

We note that S is an extensive quantity for the system in a Gibbs state. Hence, for a fixed α , using the above formula for S , one can show that $S \propto N$, which is consistent with the plot shown in Fig. 4(a).

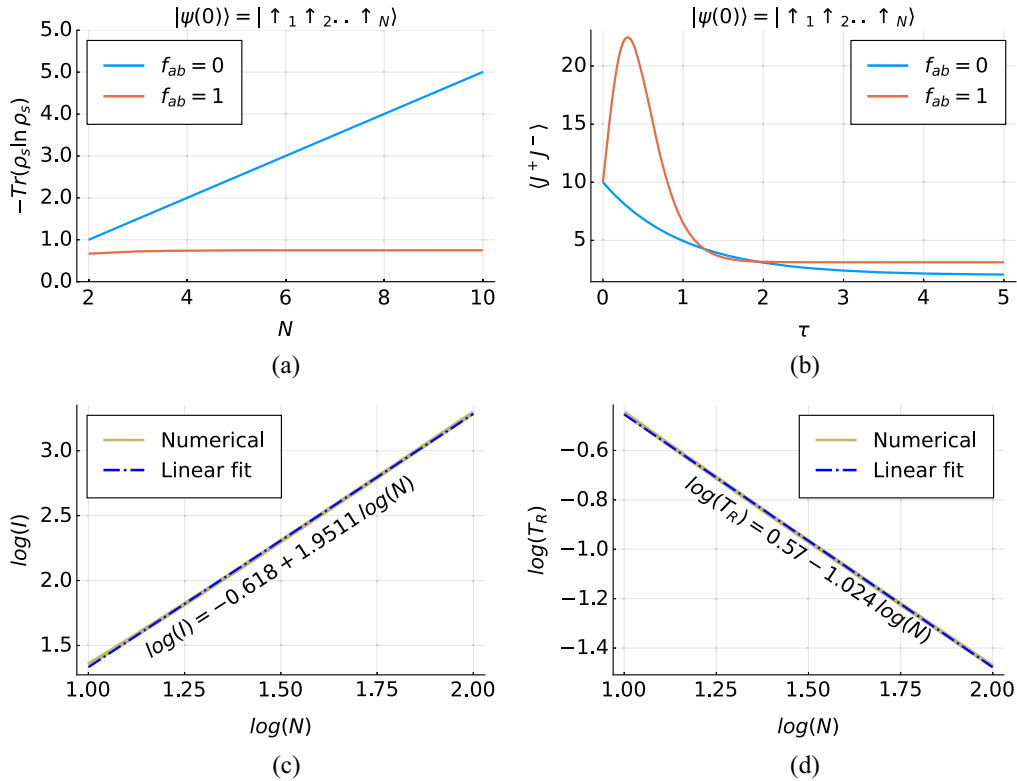


FIG. 4. Plot of von Neumann entropy as a function of N . Here $N = 10$. For $f_{ab} = 0$, as the system thermalizes, entropy behaves like an extensive quantity. On the other hand, for $f_{ab} = 1$, the system skips thermalization, and the emerging conserved quantity leads to constant entropy for large N . We also plot the dynamics of $I = \langle J_+ J_- \rangle$ for $N = 10$ with initial condition $|\psi(0)\rangle = |\uparrow_1 \uparrow_2 \dots \uparrow_N\rangle$. For $f_{ab} = 0$, the dynamics are monoexponential. For $f_{ab} = 1$, a short-lived, intense radiation burst appears in an intermediate timescale before reaching steady state. The maximum intensity plotted as a function of N (from 10 to 100) shows a quadratic scaling: $\log I_{\text{max}} = -0.618 + 1.9511 \log N$. The decay time T_R shows inverse scaling: $\log T_R = -0.57 - 1.024 \log N$. This short-lived, intense radiation is characteristic of Dicke superradiance.

The situation is entirely different for the $f_{ab} = 1$ case. The corresponding dynamical equation for the N spin case is given by,

$$\begin{aligned} \frac{d\rho_s^N}{d\tau} = & -\frac{i}{2} \sum_{a,b=1}^N \sum_{j,k=1}^3 [\mathcal{S}_{jk}^{ab} \sigma_a^j \sigma_b^k, \rho_s^N] \\ & + \sum_{a,b=1}^N \sum_{j,k=1}^3 \gamma_{jk}^{ab} \left(\sigma_b^k \rho_s \sigma_a^j - \frac{1}{2} \{ \sigma_a^j \sigma_b^k, \rho_s^N \} \right). \end{aligned} \quad (23)$$

There exists an exchange symmetry in such a case (i.e., in the exchange of any a th and b th atom, the Liouvillian remains unchanged). Such symmetry leads to $N(N-1)/2$ numbers of conserved quantities for $f_{ab} = 1$ (e.g., the corresponding operator is $\vec{\sigma}_a \cdot \vec{\sigma}_b$), which shows the notion of integrability in the system [11]. The Liouvillian is invariant under the following unitary transformation, $\hat{U} \hat{\mathcal{L}} \hat{U}^\dagger = \hat{\mathcal{L}}$. Here $\hat{U} = e^{-i\hat{O}\phi}$, ϕ is a real parameter, and $\hat{O} = O \otimes \mathbb{I} - \mathbb{I} \otimes O^T$, with $O = \sum_{a,b=1}^N \vec{\sigma}_a \cdot \vec{\sigma}_b$. The number of zero eigenvalues of $\hat{\mathcal{L}}$ increases exponentially with the system size. To explore the dynamics of the system, we use the collective basis for analysis. The collective operator is defined as, $\vec{J} = \sum_{i=1}^N \vec{\sigma}_i$, the corresponding eigenbasis is defined as $|jm\rangle$. Here, $j = 1/2:N/2$, and $m = -j:j$. The number of independent blocks is $N/2 + 1$ if N is even, or $(N+1)/2$. Similar to the two-spin case, the dynamics are confined to a particular block. We also note that the total population of each block depends on the initial configuration of the system $\rho_s(0)$. The separate irreversible dynamics of individual blocks show the notion of the generalized Gibbs ensemble [40]. The dynamical equation of the individual block is given by,

$$\begin{aligned} \frac{d\rho_s^J}{d\tau} = & \gamma_- \left(J_- \rho_s J_+ - \frac{1}{2} \{ J_+ J_-, \rho_s \} \right) \\ & + \gamma_+ \left(J_+ \rho_s J_- - \frac{1}{2} \{ J_- J_+, \rho_s \} \right). \end{aligned} \quad (24)$$

ρ_s^J is the density matrix corresponding to the J th block. We also neglect the effect of the Lamb shift in this case. If the dynamics are initially confined in the principal block (i.e., $J = N/2$), then the corresponding steady state is given by,

$$\begin{aligned} \rho_s^{J,\text{eq}} &= e^{-\frac{\pi}{\alpha} \omega_\circ J_z} / \mathcal{Z}_J, \\ \mathcal{Z}_J &= \text{Tr}(e^{-\frac{\pi}{\alpha} \omega_\circ J_z}) = \frac{\sinh \frac{(N+1)\pi\omega_\circ}{2\alpha}}{\sinh \frac{\pi\omega_\circ}{2\alpha}}. \end{aligned} \quad (25)$$

Using the previous formula of S , we note that for a large N limit, S is independent of N , which is explicitly shown in Fig. 4(a). Next, we focus on the dynamics of the system in the intermediate regime for both values of f_{ab} .

B. Emergence of Dicke superradiance

We find that the main difference between $f_{ab} \rightarrow 1$ and $f_{ab} < 1$ is the presence of collective dissipation at $f_{ab} \rightarrow 1$. Hence, we study the emerging collective features that help to distinguish it from Unruh thermalization. Recently, multiatom superradiance in a cylindrical cavity via vacuum fluctuation was analyzed in the context of detecting the Unruh effect [41]. We have a similar kind of setup. However, the major difference in our work is the presence of linear acceleration instead of circular motion. We study the time evolution of the radiative intensity operator, which is defined as $I = J_+ J_-$. The initial state is chosen as $|\psi(0)\rangle = |\uparrow_1 \uparrow_2 \dots \uparrow_N\rangle$ or $|\psi(0)\rangle = |N/2, N/2\rangle$. For $f_{ab} < 1$, $\langle J_+ J_- \rangle$ shows an exponential decay over time [Fig. 4(b)]. On the other hand, for $f_{ab} \rightarrow 1$, $\langle J_+ J_- \rangle$ shows the characteristic of an intense and short-time radiative burst, which is non-monoexponential and has several similarities with Dicke superradiance [Fig. 4(b)]. We plot the maximum intensity by varying the number of atoms, and we find that $I_{\text{max}} \propto N^2$ in Fig. 4(c). The lifetime of the burst can be calculated from the eigenvalue analysis of the block-diagonalized Liouvillian. The asymptotic decay rate (ADR) Δ is defined as the difference between the real value of the zero eigenvalue and the nearest negative eigenvalue [35]. The inverse of ADR is proportional to the decay time. Here, we plot $1/\Delta$ as a function of N . We show that decay time $\propto 1/N$ [Fig. 4(d)]. Hence, the radiative burst is short-lived and intense and similar to Dicke superradiance. On the other hand, the decay time is constant for the Unruh thermalization case, as the individual atoms reach the steady state separately at the same time. We also show the plot of $\langle J_+ J_- \rangle$ vs τ for a five-atom system at $f_{ab} = 0.999$, to show exact dynamics before it thermalizes in Fig. 5. For Unruh thermalization, the dynamics are always monoexponential toward the Gibbs state. However,

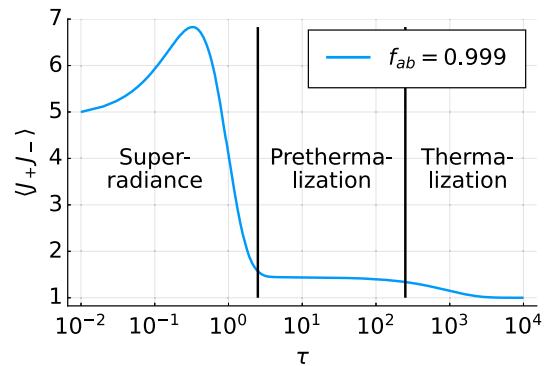


FIG. 5. Plot of $\langle J_+ J_- \rangle$ vs proper time τ for a five atom system with $f_{ab} = 0.999$. The initial state is $|\uparrow_1 \uparrow_2 \dots \uparrow_5\rangle$. The dynamics can be divided into three regions. At a very short timescale, a Dicke superradiance-type radiative burst is observed. In the intermediate time scale, it reaches a prethermal quasisteady state. Finally, the prethermal generalized Gibbs state further decays to a thermal Gibbs state.

Unruh prethermalization at $f_{ab} \rightarrow 1$, the dynamics show an initial radiative burst before reaching the prethermal state. Finally, the prethermal state shows a monoexponential decay to the thermal state. We mostly focus on the collective behavior of the system in this section. Such nonequilibrium phenomena also have acceleration dependence, which we skip in our analysis as it is quite similar to the generalized Gibbs state. As such, if the prethermal magnetization value increases, then the maximum peak will also increase.

V. COMPARISON WITH THE EXPERIMENTAL OBSERVATIONS

The correspondence between the Unruh temperature and the relativistic acceleration is given by, $\kappa T = \hbar\alpha/2\pi c$ [17]. Here κ is the Boltzmann constant, c is the velocity of light, and $\hbar$ is the Planck constant. The above relation shows that for producing 1K temperature, the limiting value of relativistic acceleration $\alpha = 2.4 \times 10^{20}$ m/s². Previous experiments were done in a circular ring using the presence of centrifugal acceleration. Using a 3.1 km accelerator ring of SPEAR at Stanford, the ultrarelativistic electron can be rotated at the value of $\alpha = 2.9 \times 10^{23}$ m/s². Such experiments can be used to produce a range of temperatures from ten Kelvins to a thousand Kelvins [32]. In the case of a circular trajectory, the position of the atoms is given by,

$$\begin{aligned} t_a &= \gamma\tau, & X_a &= R\cos(\Omega\gamma\tau), & Y_a &= R\sin(\Omega\gamma\tau), & Z_a &= Z_a; \\ t_b &= \gamma\tau, & X_b &= R\cos(\Omega\gamma\tau), & Y_b &= R\sin(\Omega\gamma\tau), & Z_b &= Z_b. \end{aligned} \quad (26)$$

Here R is the constant radius, $\gamma = 1/\sqrt{1-v^2}$, and the angular velocity $\Omega = v/R$. In the ultrarelativistic limit ($\gamma \gg 1$), it is known that the system also reaches the thermal state similar to the linear case [32], where the effective Unruh temperature is higher than a factor of $\pi/\sqrt{3}$ of the linear case [42,43]. Similarly, earlier studies have demonstrated that qualitative features remain unchanged for transition probabilities [44]. This correspondence allows us to study the effective regime of prethermalization in the linear case by using the parameter range of the circular case. We conjecture that, in the circular case, the qualitative features of the prethermal state remain the same; however, the quantitative characteristics—such as concurrence, entropy, and emission rate—may differ. For a better understanding, the exact expression for f_{ab} in the circular case should be derived in order to comment on the quantitative features. This constitutes a possible future direction of the present work. We must note that for the circular Unruh effect, we require charged particles experiencing the electromagnetic environment [32]. The dependence of the Unruh prethermalization timescale on

environmental parameters would be an interesting future aspect of the present work.

For synchrotron radiation, the ultrarelativistic electron needs to rotate in a magnetic field at a critical frequency of 10^{15} Hz; hence, we choose $\omega_o = 10^{15}$ Hz for our calculation. We are interested in the values of the minimum interatomic distance for exhibiting such phenomena. Our calculation shows that for varying the values of α between 10^{23} to 10^{26} m/s², we get $f_{ab} \rightarrow 1$ when the average distance between the atoms is between μm to nm range (the yellow region in the figure). Previous experiments show that the minimum distance between the conducting plane for observing the Casimir-Polder interaction is $0.5 - 3.0 \mu\text{m}$ [45]. Hence, using similar atomic separation and the ultrarelativistic centrifugal acceleration used in synchrotron radiation, one can observe that the system will skip the Unruh thermalization and reach a prethermal state. We note that the system generally reaches the thermal state at a timescale proportional to $T_{\text{th}} = 1/\{(\gamma_+ + \gamma_-)(1 - f_{ab})\}$ (shown in Fig. 2) and reaches the prethermal state at a timescale proportional to $T_{\text{pre}} = 1/(\gamma_+ + \gamma_-)$. We define the fractional prethermal lifetime $T_{\text{pre}}^{\text{frac}}$ as,

$$T_{\text{pre}}^{\text{frac}} = \frac{T_{\text{th}} - T_{\text{pre}}}{T_{\text{th}}} \propto f_{ab}. \quad (27)$$

We note that $T_{\text{pre}}^{\text{frac}}$ depends only on f_{ab} , hence such a quantity can be identified using the following experimental values for the circular case shown in Fig. 6. A key advantage of the circular case is that experiments can be carried out within a finite-sized laboratory. However, in circular motion, the proper and coordinate times are related by a time-dependent Lorentz gamma factor, and the circular Unruh temperature depends on the energy gap of the system. These features often hinder the detection of Unruh thermalization in the circular case. In these circumstances, the observation of prethermalization is also challenging and beyond the scope of the present manuscript.

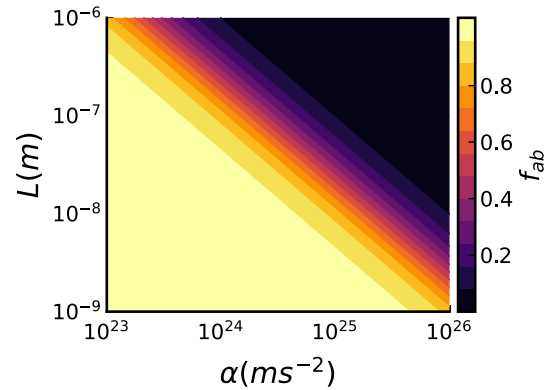


FIG. 6. Contour plot shows the value of f_{ab} for changing L and α . We choose $\omega_o = 10^{15}$ Hz. The yellow region corresponds to $f_{ab} \rightarrow 1$, where the system shows the notion of prethermalization.

VI. DISCUSSIONS AND OUTLOOK

We have revisited the well-known thermalization theorem originally proposed by Unruh and Davies in the context of uniformly accelerating many-body systems. In the case of RCPI, for a small distance $\alpha L \ll 1$, a local inertial frame description is valid, so the response is thermal. However, for the opposite limit $\alpha L \gg 1$, such a description is invalid, and a breakdown of thermal response is reported [21]. An interesting observation has also been made for the steady-state configuration of such systems [19]. The scalar field induced entanglement can be possible in the regime $\alpha L \ll 1$. A common question that arises is how entanglement can be generated in such a regime. We understand that such an answer was still missing in the literature. Hence, in this manuscript, we provide a brief analysis of system dynamics in this regime.

The consequence of the thermalization is the following: the initial memory of the system vanishes, and the system reaches the Gibbs state. Surprisingly, in the case of the Unruh effect, similar things can be observed, where the role of temperature is played by acceleration. In condensed matter physics, for both closed and open quantum systems, the integrability shows a sharp distinction from such thermalization phenomena [11]. It is expected that there should also be a one-to-one correspondence in the case of the Unruh effect. We show that for the $\alpha L \ll 1$ case, the system becomes nearly integrable as there exists an extensive number of quasiconserved quantities. Such phenomena are quite similar to prethermalization in open quantum systems.

Unruh thermalization is related to the dissipation of the system coupled to a local bath. As a result, the atoms individually reach the Gibbs state at equilibrium. The questions remain about the influence of collective dissipation, which has not been investigated previously within the framework of Unruh thermalization. Previous works were confined to the two-atom case, where the entanglement generation is extensively studied. Here we provide the generalized picture of the two-atom case [19]. Our result shows that for the $\alpha L \ll 1$ case, the collective dissipation dominates over the local dissipation. The long-time regime shows the notion of the Unruh effect. However, in the intermediate timescale, the collective dissipation results in the integrability as the dynamics is confined in a symmetric subspace. Similarly, another collective feature, i.e., Dicke superradiance, is also observed before the systems reach the prethermal state. Our recent work on linear acceleration can also be applied to ultrarelativistic particles in circular motion [32], for future experimental verification of the Unruh prethermalization. In this manuscript, we are not claiming any modification of the Unruh thermalization theorem. However, such a theorem does not capture the whole story for noninteracting many-atom systems interacting with a common massless scalar field, as in such cases, a cascaded dynamics is observed: the system reaches

a long-lived prethermal state on an intermediate timescale before thermalizing, which has not been previously explored.

We also comment on the nature of the prethermal state from the perspective of the nonthermal character of the Unruh effect. A number of works have shown that, under certain conditions, the Kubo-Martin-Schwinger condition is violated, and the Unruh effect therefore loses its thermal interpretation [46,47]. In a different work, using the generalized uncertainty principle, the corrections to the Unruh temperature have been calculated [48]. We argue that the quantitative features of the prethermal state would change accordingly. From a statistical mechanical viewpoint, however, the atoms are subject to dissipative noise and ultimately relax to a steady state, which may be thermal or nonthermal. When the interatomic distance is small or the acceleration is large, the evolution toward the steady state occurs in multiple stages. In the intermediate regime, the presence of a symmetry leads to a long-lived quasis-steady state. The symmetry operator is valid for any values of the two-point correlation function, provided that $f_{ab} = 1$. Therefore, under these conditions (high acceleration and/or small interatomic separation), prethermalization can be observed regardless of whether the Unruh effect is thermal or nonthermal.

The only caveat is that at least two atoms are required, since prethermalization involves long-lived many-body correlations. Therefore, this phenomenon cannot be observed in the single-atom case. We recognize several compelling areas where our present work can be explored further. Given that most experimental setups emphasize circular acceleration rather than linear, a critical investigation into the lifetime of the prethermal state under centrifugal acceleration presents a significant opportunity, as our work predicts that using the relativistic particles in a circular ring also shows prethermalization in the ultrarelativistic regime. Additionally, the influence of non-Markovian vacuum fluctuations on Unruh prethermalization is also an interesting area to gain more insight into these complex interactions. We also establish a connection between Dicke superradiance and Unruh prethermalization, thereby enabling new lines of research into the relativistic aspects of several features of the well-known Dicke model [49–54].

VII. CONCLUSION

As a summary, we discuss the correspondence between the open quantum system and uniformly accelerated non-interacting atoms. For a single atom, it is always in the thermal state. On the other hand, for a many-spin case, the system may skip the thermalization, as in $\alpha L \ll 1$, the dynamics is constrained by the extensive number of quasiconserved quantities. Hence, such constrained dynamics lead to Unruh prethermalization. We also show that in that regime, the collective dissipation dominates. As

such, the system shows a Dicke superradiance-type collective emission process before reaching the prethermal state. Finally, we also predict the values of α, L, ω_0 in connection with the experiments using accelerating electrons in a circular ring. We also discuss the relevance of our results in the context of existing experimental proposals and compare our findings with those of previous studies.

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available upon publication because it is not technically feasible and/or the cost of preparing, depositing, and hosting the data would be prohibitive within the terms of this research project. The data are available from the authors upon reasonable request.

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