

Possibility of superradiant neutrino emission by atomic condensates

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In a recent work [B. J. P. Jones and J. A. Formaggio, *Phys. Rev. Lett.* **135**, 111801 (2025)], the possibility of superradiant neutrino emission from atomic condensates has been theoretically proposed. Subsequent analysis by Y. K. Lu, H. Lin, and W. Ketterle [arXiv:2510.21705] questioned this scenario, emphasizing the limiting role of the fermionic nature of the decayed atoms. In this study, we revisit the problem and discuss under which conditions collective emission phenomena might still emerge in cold-atom systems.

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I. INTRODUCTION

The prospect of generating coherent radiation outside the electromagnetic sector represents an open direction in quantum many-body physics. In particular, the possibility of realizing collective or even stimulated neutrino emission has attracted recent interest [1–3], as it would open a qualitatively new regime of weak-interaction physics while offering a unique window into quantum coherence in neutrino production processes.

Jones and Formaggio [1] argued that a Bose-Einstein condensate (BEC) of radioactive atoms could, under suitable conditions, exhibit a form of neutrino superradiance analogous to Dicke-enhanced photon emission [4]. Their analysis highlighted a potential amplification mechanism but relied on analogy-based arguments and on an effective single-channel treatment of the decay. Shortly thereafter, a critical assessment by Lu, Lin, and Ketterle [2] emphasized that, for realistic electron-capture isotopes, the fermionic statistics of the daughter atoms leads to severe Pauli blocking, which would suppress the cooperative enhancement presumed in the original scenario. These observations raise the broader question of whether superradiant neutrino emission may survive in alternative settings, or whether the underlying idea is fundamentally incompatible with the statistics of available decay channels.

The aim of this work is to clarify this point by developing and analyzing a set of minimal, yet microscopically motivated, models that isolate the statistical constraints governing cooperative emission. Instead of focusing on a single decay channel, we consider three statistically distinct possibilities: bosonic atoms decaying into bosonic species which could appear for gamma decay of metastable nuclear excited states, bosonic atoms decaying into fermions, and fermionic atoms (in the deep BEC regime) decaying into bosons; these last two options are accessible via electron capture. Each case captures a different statistical pathway and allows us to quantify whether and how bosonic stimulation or Pauli blocking affects the onset of collective emission. Our approach, based on Lindblad dynamics and effective few-mode Hamiltonians derived from a real-space microscopic model, provides a controlled framework within which the competing roles of cooperativity, statistics, and dissipation can be assessed.

We find that while the boson-to-fermion channel is indeed strongly suppressed by Pauli blocking, consistent with Ref. [2], cooperative behavior can persist in scenarios beyond those considered so far—most notably when fermionic atoms in the deep BEC regime decay into bosonic species. This indicates that the conceptual idea underlying Ref. [1], which is realized by a boson-to-boson model, is not ruled out on fundamental grounds, but that its realization requires physical systems with suitable decay pathways.

The paper is organized as follows. In Sec. II we review the essential features of the electron-capture process, with emphasis on how it constrains the statistics of the decay products. Section III introduces a minimal model in which bosonic atoms in a BEC decay into a bosonic daughter species, reproducing the superradiant neutrino-emission mechanism of Ref. [1]. In Sec. IV we examine the

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complementary case in which bosonic atoms decay into fermionic products; here we recover the central conclusion of Ref. [2], namely, the suppression of cooperative emission due to Pauli blocking. Section V then considers a scenario where fermionic atoms in the deep BEC regime decay into bosonic species, a situation in which a pronounced super-radiant behavior emerges. Experimental considerations about the possibility of stabilizing radioactive atomic condensates are reported in Sec. VI. Finally, Sec. VII summarizes our findings and discusses prospects for the possible experimental realization of the scenarios analyzed in this work. Technical details about the theoretical model discussed in Sec. III are reported in the Appendix.

II. ELECTRON CAPTURE

Before discussing the possibility to have an enhanced decay rate for electron capture in BEC of suitable radioactive isotopes, it is important to spend a few words on the electron-capture process. Nuclei with a high proton number could undergo a pure electron-capture process if the energy available for the decay, given by the difference between the mass of the parent atom and the mass of the daughter atom, is smaller than 1022 keV, the threshold for the competing β^+ decay. If this energy constraint is fulfilled, then an electron of the inner shells can be captured in the nucleus where a proton turns to a neutron and an electron neutrino is emitted. The vacancy in the electron shells corresponds to a particular excitation energy of the daughter atom, which is then released within fs.

The first important aspect to notice is that the spin of the electron system for parent and daughter atoms will change from integer to half-integer or vice versa while the nucleus will maintain the spin either integer or half-integer given the fact that the number of nucleons does not change. This aspect imposes limits on the stability of the condensate after electron capture.

A second important fact to be considered is the energy spectrum of neutrinos. The total energy available to the decay is shared among the neutrino, the atomic excitation energy and the nuclear recoil. The atomic excitation energy is, in first approximation, given by the binding energy of the captured electron in the field generated by the daughter nucleus. The capture probability is proportional to the probability to find the electron in the nucleus; therefore it is proportional to square of the electron wave function calculated at the nucleus. Electrons from the s and $p_{1/2}$ shells have the highest probability to be captured. The important implication of this is that the neutrino energy spectrum is not monochromatic, but is characterized by several resonances with intrinsic width in the eV scale and not negligible tails. Nevertheless, for the majority of the decays the capture of $1s$ electrons is dominant and can reach a fraction greater than 80%.

Two prominent examples of electron-capture spectra which have been recently studied with high precision are the one of ^{163}Ho [5–8] and ^7Be [9,10]. In the first case, the

parent atom is a boson while the daughter atom ^{163}Dy is a fermion. The energy available for the decay is just $Q = 2863.2(6)$ eV [11], which allows captures only from the $3s$ shell, while the half-life is about 4570 years [12]. A theoretical description of the electron capture in ^{163}Ho using an *ab initio* approach was presented in [13,14]. Although the intensity of the tails is still not perfect, the discussed model well captures the several structures of the spectrum and, in particular, the multiplet nature of the main resonances. The nuclear recoil of the daughter atom ^{163}Dy , due to the size of the nucleus and the small energy of the emitted neutrinos, is of the order of a few μeV . The ^7Be has, on the other hand, very different properties. First of all, the parent atom is a fermion and, consequently, the daughter atom is a boson. The half-life of this decay is about 52 days and could be suitable for testing decay rate enhancement. At the same time, the energy available to the decay is $Q = 861.963(23)$ keV [15], which leads to a remarkable recoil energy for the daughter nucleus ^7Li of about 50 eV possibly affecting the stability of the condensate. While in the electron capture of ^7Be more than 90% of the decay occurs via capture of $1s$ electrons, it is also important to mention the presence of a small fraction of $2s$ electron capture and a small fraction of decays, of the order of 10%, which populate a short lived excited nuclear state in ^7Li . This excited state has half-life $\tau_{1/2} = 728(20)$ fs and it relaxes to the ground state via the emission of a γ ray with energy 477.603(2) keV.

An analysis of the nuclide chart for electron captures in which the parent atom decays to a stable atom in the ground state with half-life longer than one day and Q value smaller than 1022 keV showed that the candidates are typically nuclei with an odd number of nucleons, therefore with semi-integer nuclear spin. The number of nuclides whose decays belong either to the boson-to-fermion or fermion-to-boson groups is equivalent and covers a wide range of half-lives and decay energies.

The microscopic picture outlined above illustrates that electron capture is a multiscale process involving changes in atomic statistics, a structured neutrino spectrum, and fast electronic and, possibly, also nuclear relaxation channels. A complete dynamical treatment of all these ingredients is beyond present analytical control and would require a fully *ab initio* description of the coupled atomic, nuclear, and neutrino degrees of freedom. For this reason, in the remainder of this work we resort to minimal effective models that retain only the degrees of freedom most relevant for assessing whether cooperative decay can emerge, neglecting the energy scale of the nuclear recoil and the energy released in the atomic deexcitation process. Among the essential features that must be preserved are the quantum statistics of both the parent and daughter atoms, and the manner in which these statistics influence bosonic stimulation or Pauli blocking. In the following sections, we formulate controlled simplified models tailored to isolate

the statistical constraints imposed by the decay process and to quantify their impact on the possibility of realizing a superradiant neutrino-emission regime in ultracold atomic systems.

III. NEUTRINO SUPERRADIANCE IN A BOSONIC-BOSONIC MODEL

In this section, we introduce an effective microscopic model aimed at capturing the seminal idea, originally put forward by Jones and Formaggio, that a radioactive atomic Bose-Einstein condensate (BEC) could serve as a source of neutrinos exhibiting a form of superradiant amplification. It is important to emphasize, however, that the analogy with photon superradiance is necessarily incomplete. Neutrinos obey fermionic statistics and cannot be trapped or confined in a cavity; as a consequence, phase coherence in the emitted field cannot develop in the standard Dicke sense, and the phenomenon effectively reduces to a collective avalanche emission, whenever such a regime is dynamically accessible. This limitation, already noted in Ref. [1], must be kept in mind when discussing the possible cooperative enhancement of the decay rate.

Starting from a real-space microscopic description of the emitting region, i.e., the spatial domain hosting the radioactive condensate, one can derive an effective low-energy Hamiltonian expressed in terms of three collective modes: a bosonic operator a describing the initial condensate, a bosonic operator b associated with the daughter atoms produced via electron capture, and a fermionic operator c representing the neutrino field. A complete derivation of this reduction, including the approximations leading from the full many-body Hamiltonian to the single-mode description of the neutrino field adopted below, is detailed in the Appendix.

Within this effective framework, the coherent part of the dynamics is governed by the Hamiltonian

$$H = \epsilon_a a^\dagger a + \epsilon_b b^\dagger b + E_\nu c^\dagger c + g c^\dagger b^\dagger a + g^* a^\dagger b c, \quad (1)$$

where ϵ_a and ϵ_b denote the effective energies of the initial and final atomic modes, and E_ν is the neutrino energy. The effective vertex term encodes the conversion of one bosonic atom in the condensate into a daughter boson and an emitted neutrino. In order to consider a fermionic leaking channel corresponding to the neutrino emission, we adopt a Lindblad formalism according to which the time evolution of a generic operator O in the Heisenberg picture takes the form [16]:

$$\frac{dO}{dt} = \frac{i}{\hbar} [H, O] + \sum_n \left(L_n^\dagger O L_n - \frac{1}{2} \{ L_n^\dagger L_n, O \} \right), \quad (2)$$

with L_n a set of jump operators.

Taking into account a single dissipation channel with dissipator $L = \sqrt{\gamma} c$ and dissipation rate $\gamma = 2\Gamma/\hbar$, the time

evolution of the atomic populations is described by the following equations:

$$\frac{d}{dt} \langle n_a \rangle = \frac{i}{\hbar} [g \langle c^\dagger b^\dagger a \rangle - g^* \langle a^\dagger b c \rangle], \quad (3)$$

$$\frac{d}{dt} \langle n_b \rangle = \frac{i}{\hbar} [-g \langle c^\dagger b^\dagger a \rangle + g^* \langle a^\dagger b c \rangle], \quad (4)$$

from which one immediately obtains the conservation law $d/dt(\langle n_a \rangle + \langle n_b \rangle) = 0$.

The dynamics of the populations depends on the correlation function $S \equiv \langle a^\dagger b c \rangle$, with $S^* = \langle c^\dagger b^\dagger a \rangle$. Thus the equation for n_a can be written as

$$\langle \dot{n}_a \rangle = \frac{i}{\hbar} [g S^* - g^* S] = \frac{2}{\hbar} \Re[ig S^*] = -\frac{2}{\hbar} \Im[g S^*], \quad (5)$$

which, taking into account the conservation of the total number of atoms, also implies

$$\langle \dot{n}_a \rangle = -\langle \dot{n}_b \rangle = \frac{2}{\hbar} \Im[g^* S]. \quad (6)$$

The average number of neutrinos in the volume of the emission region is obtained by considering the Lindblad-Heisenberg equation of motion for $n_c = c^\dagger c$. Thus, the time evolution of $\langle n_c \rangle$ is regulated by the following equation:

$$\langle \dot{n}_c \rangle = -\frac{2}{\hbar} \Im[g^* S] - \frac{2\Gamma}{\hbar} \langle n_c \rangle, \quad (7)$$

where the second term in the rhs of the equation accounts for the neutrino escaping from the emission region. The irreversible leakage of neutrinos out of the emission region leads to an exponential relaxation of the neutrino population with a characteristic escape time $\tau = \hbar/(2\Gamma)$. Since neutrinos propagate essentially at the speed of light c , the escape time τ is set by the typical linear size ℓ of the condensate cloud. A simple estimate therefore yields the relation $\tau \sim \ell/c$ and $\Gamma \approx \hbar c/\ell$, up to numerical factors of order unity that depend on the detailed geometry of the emitting region. Once Γ is identified as the largest energy scale of the system (or equivalently identifying τ as the shorter time scale of the system), the instantaneous mean number of neutrinos which are present in the emission region is well approximated by the quantity $\langle n_c \rangle^{eq} \approx -\Im[g^* S]/\Gamma$, which is the stationary solution of Eq. (7).

The populations dynamics is evidently affected by $S(t) = \langle a^\dagger b c \rangle$, whose equation of motion is given by:

$$\begin{aligned} \dot{S} = \frac{i}{\hbar} [(\Delta + i\Gamma)S + g \langle n_c n_b (1 + n_a) \\ - n_a (1 + n_b) (1 - n_c) \rangle], \end{aligned} \quad (8)$$

with $\Delta = \epsilon_a - \epsilon_b - E_\nu$.

Hereafter, we neglect high order correlations and adopt the following factorization:

$$\langle n_c n_b (1 + n_a) \rangle \approx \langle n_c \rangle \langle n_b \rangle (1 + \langle n_a \rangle) \quad (9)$$

$$\langle n_a (1 + n_b) (1 - n_c) \rangle \approx \langle n_a \rangle (1 + \langle n_b \rangle) (1 - \langle n_c \rangle), \quad (10)$$

so that we obtain

$$\begin{aligned} \dot{S} = & \frac{(i\Delta - \Gamma)}{\hbar} S + \frac{ig}{\hbar} \langle n_c \rangle \langle n_b \rangle (1 + \langle n_a \rangle) \\ & - \frac{ig}{\hbar} \langle n_a \rangle (1 + \langle n_b \rangle) (1 - \langle n_c \rangle), \end{aligned} \quad (11)$$

which is clearly affected by the Pauli blocking term $(1 - \langle n_c \rangle)$.

An important simplification of the equation of motion of S is obtained observing that $\langle n_c \rangle \approx \langle n_c \rangle^{eq} \ll 1$, in view of the fact that $\Gamma \gg |g|$. Following the above line of reasoning, we obtain the approximate equation:

$$\dot{S} = \frac{i}{\hbar} [(\Delta + i\Gamma)S - g\langle n_a \rangle (1 + \langle n_b \rangle)]. \quad (12)$$

Assuming that the relaxation dynamics of S (controlled by Γ) is much faster than the shorter time scale of the forcing term $-ig\langle n_a \rangle (1 + \langle n_b \rangle)/\hbar$, the solution of Eq. (12) can be accurately approximated by the expression:

$$S \approx \frac{ig\langle n_a \rangle (1 + \langle n_b \rangle)}{i\Delta - \Gamma}. \quad (13)$$

Using the above relation, the dynamics for $\langle n_b \rangle$ takes the following form:

$$\langle \dot{n}_b \rangle = -\frac{2}{\hbar} \Im \left[\frac{|g|^2 \langle n_a \rangle (1 + \langle n_b \rangle)}{\Delta + i\Gamma} \right]. \quad (14)$$

The latter, can be rewritten in the form:

$$\begin{aligned} \langle \dot{n}_b \rangle = & \frac{2|g|^2}{\hbar} \frac{\Gamma}{\Delta^2 + \Gamma^2} \langle n_a \rangle (1 + \langle n_b \rangle) \\ = & \Omega (1 + \langle n_b \rangle) (N - \langle n_b \rangle), \end{aligned} \quad (15)$$

where we used $\langle n_a \rangle + \langle n_b \rangle = N$ and set $\Omega \equiv \frac{2|g|^2}{\hbar} \frac{\Gamma}{\Delta^2 + \Gamma^2} \approx \frac{2|g|^2}{\hbar\Gamma}$. Such an equation exhibits logistic growth of the number $\langle n_b \rangle$, with solution

$$\frac{\langle n_b \rangle}{N} = \frac{e^{\Omega(N+1)t} - 1}{e^{\Omega(N+1)t} + N}, \quad (16)$$

derived with initial condition $\langle n_b \rangle_{t=0} = 0$.

Equation (16), whose behavior is exemplified in Fig. 1, reproduces exactly the superradiant behavior discussed in Eq. (27) of Ref. [1].

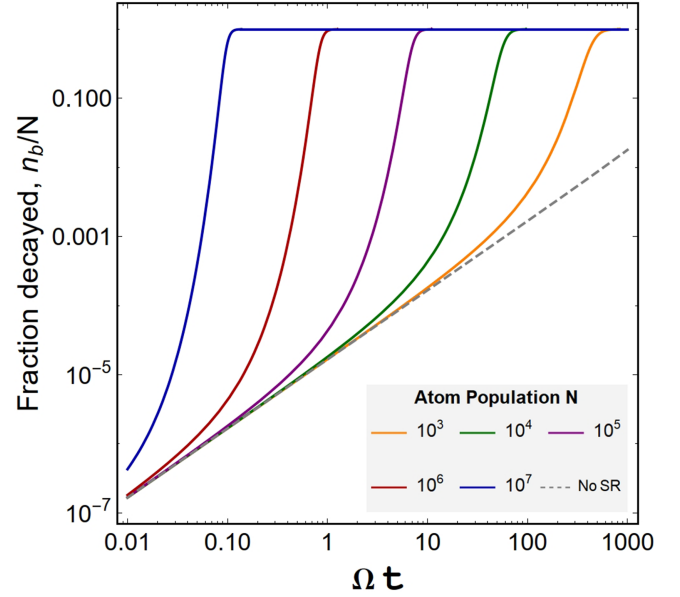


FIG. 1. Plot of the decayed fraction $\frac{\langle n_b \rangle}{N}$, according to Eq. (16), for different atom populations N as specified in the figure legend (see Ref. [1] for comparison). Dashed line corresponds to the case of no superradiance.

A. Effects of atomic interaction

We now include short-range (contact) interactions between atoms through the Bose-Hubbard term

$$H_{\text{coll}} = U_a n_a^2 + U_b n_b^2 + U_{ab} n_a n_b,$$

which supplements the effective Hamiltonian presented in Eq. (1). The contact atomic interaction, written in terms of the intra- and interspecies interaction strengths $U_{a,b}$ and U_{ab} , does not affect the populations dynamics, while it provides modifications to the Heisenberg equation of motion of S (see the Appendix). Indeed, the Heisenberg equation for the mixed correlator $S = \langle a^\dagger b c \rangle$ acquires an additional contribution from H_{coll} , induced by

$$\begin{aligned} [H_{\text{coll}}, a^\dagger b c] = & (2U_a - U_{ab}) a^\dagger b c n_a - (2U_b - U_{ab}) a^\dagger b c n_b \\ & + (U_a + U_b - U_{ab}) a^\dagger b c. \end{aligned} \quad (17)$$

When all coupling constants are taken equal, $U_a = U_b = U_{ab} = U$, the result simplifies to

$$[H_{\text{coll}}, a^\dagger b c] = U a^\dagger b c (n_a - n_b + 1).$$

Inserting this commutator into the Heisenberg equation for S yields

$$\dot{S} = \dots + \frac{iU}{\hbar} \langle a^\dagger b c (n_a - n_b + 1) \rangle,$$

where the dots denote the collisionless terms obtained previously. To close the hierarchy, we apply a mean-field

decoupling,

$$\langle a^\dagger b c (n_a - n_b + 1) \rangle \simeq (\langle n_a \rangle - \langle n_b \rangle + 1) S,$$

neglecting higher-order correlations. With this approximation the equation for S becomes

$$\dot{S} = \frac{i}{\hbar} [\Delta + U(\langle n_a \rangle - \langle n_b \rangle + 1) + i\Gamma] S + \dots,$$

so that the bare detuning Δ in Eq. (12) is replaced by an effective, self-consistent quantity

$$\Delta_{\text{eff}} = \Delta + U(\langle n_a \rangle - \langle n_b \rangle + 1).$$

Hence, atomic collisions do not directly modify the population dynamics but shift the resonance condition through the instantaneous population imbalance between undecayed and decayed atoms. This interaction-induced shift acts as a nonlinear feedback on the cooperative emission, producing a self-consistent renormalization of the effective detuning during the condensate dynamics.

Proceeding under the same assumptions used in the absence of atom-atom interactions and considering the modifications induced by the atom-atom interaction on the dynamics of the mixed correlator S , we are able to derive a modified version of Eq. (15), which takes the following form:

$$\langle \dot{n}_b \rangle = \frac{\Omega}{1 + \eta(1 - 2\frac{\langle n_b \rangle}{N})^2} (1 + \langle n_b \rangle)(N - \langle n_b \rangle), \quad (18)$$

where we have introduced the dimensionless parameter $\eta = (UN/\Gamma)^2$. A close inspection of Eq. (18) shows that, in the relevant regime where $\eta \gg 1$, the decay dynamics is modified compared to the noninteracting case (i.e., $\eta = 0$) in two ways.

At the process beginning, with small $\langle n_b \rangle$ values, the effective decay rate is lowered compared to the noninteracting case and tends to be $\sim \Omega/\eta$, to be compared with the Ω value of the non-interacting case.

Moreover, the collective emission threshold, i.e., the threshold at which the decay process deviates from a pure exponential, shifts from $\langle n_b \rangle = N/2$ of the $\eta = 0$ case to higher values depending on the strength of η . Interestingly, the collective emission threshold can be rigorously characterized by studying the equation $\langle \ddot{n}_b \rangle = 0$ or by direct numerical simulation of Eq. (18).

We implemented the latter strategy and the results of this analysis, shown in Fig. 2, clearly evidence the impact of the atom-atom interaction on the system's dynamics.

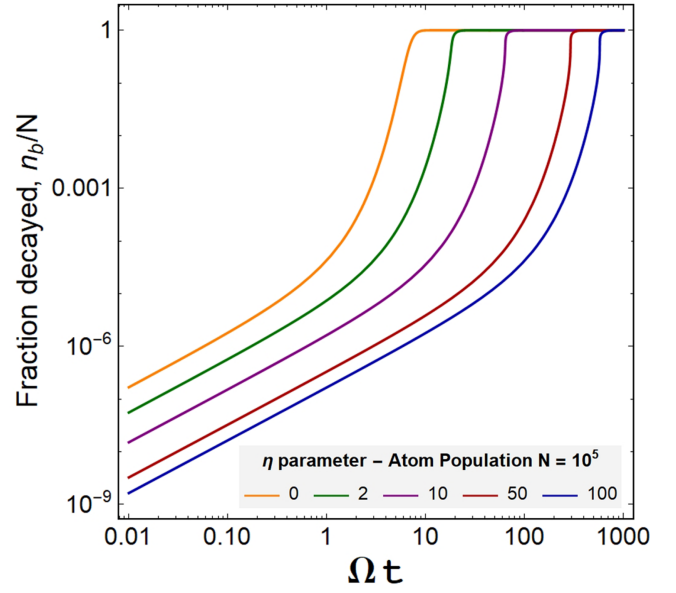


FIG. 2. Plot of the decayed fraction $\frac{\langle n_b \rangle}{N}$, according to Eq. (18), for different values of the η parameter for fixed atom population $N = 10^5$.

IV. NEUTRINO EMISSION IN A BOSONIC-FERMIONIC MODEL

We now turn to the situation in which the initial atomic species is bosonic and forms a Bose-Einstein condensate, while the daughter atoms produced by electron capture are fermions. This case is of particular relevance, as emphasized in Ref. [2], where it was shown that the fermionic statistics of the decay products leads to a severe suppression of superradiant amplification due to Pauli blocking.

In the following, we revisit this problem within the Lindblad formalism, which allows a consistent treatment of the irreversible character of the decay process and provides direct access to the dynamical mechanisms responsible for the inhibition of collective emission.

To this end, we describe the initial condensate by a bosonic annihilation operator a , while the fermionic daughter atoms populate an energy band labeled by single-particle modes b_i with energies E_i . The emitted neutrino is represented by an operator c . Neglecting interactions other than those responsible for the decay, the free Hamiltonian reads

$$H = E_a a^\dagger a + \sum_i E_i b_i^\dagger b_i + E_\nu c^\dagger c, \quad (19)$$

where E_a denotes the energy associated with the bosonic condensate mode, E_i are the fermionic single-particle energies, and E_ν is the neutrino energy.

The decay process is described by the Lindblad jump operators:

$$L_a^{(EC)} = \sqrt{g_a} c^\dagger b_a^\dagger a \quad (20)$$

$$L_\nu^{(NE)} = \sqrt{\Gamma} c \quad (21)$$

$$L_k^{(Th)} = \sqrt{\gamma_k} b_{k-1}^\dagger b_k \quad (22)$$

where the index α denotes a preferred energy level within the energy band in which the atom decays.

A generic observable O evolves according to the Lindblad-Heisenberg equation:

$$\langle \dot{O} \rangle = \frac{i}{\hbar} \langle [H, O] \rangle + \langle D(O) \rangle, \quad (23)$$

where $D(O) = D(O)^{(EC)} + D(O)^{(NE)} + D(O)^{(Th)}$ denotes the Lindblad superoperator accounting for the electron-capture process (EC), the neutrino emission (NE), and the thermalization (Th) of the decayed atoms:

$$\begin{aligned} D(O)^{(EC)} &= L_a^{(EC)\dagger} O L_a^{(EC)} - \frac{1}{2} \{ L_a^{(EC)\dagger} L_a^{(EC)}, O \} \\ D(O)^{(NE)} &= L_\nu^{(NE)\dagger} O L_\nu^{(NE)} - \frac{1}{2} \{ L_\nu^{(NE)\dagger} L_\nu^{(NE)}, O \} \\ D(O)^{(Th)} &= \sum_k \left[L_k^{(Th)\dagger} O L_k^{(Th)} - \frac{1}{2} \{ L_k^{(Th)\dagger} L_k^{(Th)}, O \} \right]. \end{aligned}$$

Using the Lindblad-Heisenberg equation of motion the dynamics of the relevant operators $n_a = a^\dagger a$, $n_c = c^\dagger c$ and $n_k = b_k^\dagger b_k$ can be deduced.

In particular, the equation for n_a can be written as

$$\langle \dot{n}_a \rangle = -g_a \langle a^\dagger a (1 - b_a^\dagger b_a) (1 - c^\dagger c) \rangle, \quad (24)$$

while the equation for n_c takes the form:

$$\langle \dot{n}_c \rangle = g_a \langle a^\dagger a (1 - b_a^\dagger b_a) (1 - c^\dagger c) \rangle - \Gamma \langle n_c \rangle. \quad (25)$$

The thermalization process is described by the equations for n_k , which is written as

$$\begin{aligned} \langle \dot{n}_k \rangle &= \delta_{k\alpha} g_a \langle a^\dagger a (1 - b_a^\dagger b_a) (1 - c^\dagger c) \rangle \\ &\quad - \gamma \langle n_k (1 - n_{k-1}) \rangle (1 - \delta_{k,0}) + \gamma \langle n_{k+1} (1 - n_k) \rangle, \end{aligned} \quad (26)$$

where the label $k = 0$ denotes the lowest single-particle fermionic level and we have assumed $\gamma_k \approx \gamma$.

Equations (24), (25), and (26) do not form a closed set, as they couple to higher-order density-density correlations. In the present treatment, however, we neglect such correlations, thereby truncating the hierarchy and obtaining a self-consistent set of equations that can be solved numerically.

Considering the approximated form of the equations of motion, one can verify the conservation of the total

(decayed and undecayed) number of atoms:

$$\langle \dot{n}_a \rangle + \sum_k \langle \dot{n}_k \rangle = 0. \quad (27)$$

Moreover, close inspection of Eq. (24) reveals that, even under conditions of fast thermalization of the decayed atoms and rapid neutrino emission from the emitting region, the dynamics does not exhibit a superradiant behavior. Instead, it is well described by a simple exponential decay law, $\langle \dot{n}_a \rangle \approx -g_a \langle n_a \rangle$. This result is further corroborated by numerical simulations of the decay dynamics, which confirm the detrimental influence of the Pauli blocking mechanism on the onset of collective emission effects.

The results of the latter numerical analysis are shown in Fig. (3), which illustrates the time evolution of the decayed fraction for different choices of the g_a/γ ratio. When the thermalization process is slow, the occupation of the daughter fermionic level α builds up and produces an intermediate-time Pauli-blocking plateau, temporarily suppressing further decay. This feature disappears when the relaxation dynamics is sufficiently fast to empty the preferential arrival level, thereby allowing decay to proceed unimpeded. At long times, however, Pauli blocking inevitably arrests the dynamics once the fermionic levels become filled up to level α .

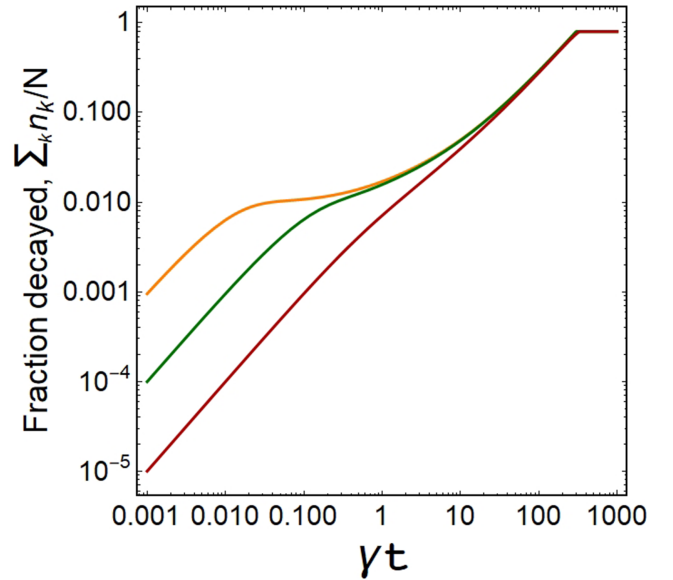


FIG. 3. Plot of the decayed fraction $\sum_k \langle n_k \rangle / N$ as a function of time, obtained from Eqs. (24) and (26) and assuming negligible density-density correlations ($\langle n_k n_{k'} \rangle \approx \langle n_k \rangle \langle n_{k'} \rangle$) and fast neutrino emission ($\langle n_c \rangle \approx 0$). Distinct curves are obtained by fixing the atom population to $N = 100$ and $\alpha = 80$ and considering different values of the parameter g_a/γ . We set $g_a = \gamma$ for upper curve (yellow), $g_a = 0.1\gamma$ for middle curve (green), $g_a = 0.01\gamma$ for lower curve (red).

In the specific cases shown in Fig. (3), this results in a residual population of about 20 undecayed atoms out of 100, explaining the asymptotic saturation observed in the long-time limit (i.e., $\gamma t \approx 10^3$).

These results agree with the general conclusions of Ref. [2], even though we do not observe oscillating behavior in the time evolution of the decaying population, which seems a consequence of the Hamiltonian approach there adopted.

V. NEUTRINO EMISSION IN A FERMIONIC BEC-BOSONIC BEC MODEL

Let us now consider an ultracold gas of fermionic atoms in the deep BEC regime, where tightly bound pairs behave as composite bosons and form a fermionic molecular condensate. In this regime, the condensation energy is typically much larger than any other dynamical scale of the condensate. As a consequence, any unpaired fermion generated by the decay of a molecule rapidly recombines with another unpaired atom, effectively suppressing the occupation of states with broken pairs. Two decay events occurring close in time would transiently produce two unpaired fermions, but these are energetically disfavored and promptly recondense (see Fig. 4). At the level of the effective low-energy theory projected onto the condensate subspace, the dynamical role of unpaired fermions is therefore negligible.

Under these conditions, the decay dynamics can be described in terms of an effective process in which a

fermionic molecule is annihilated and two bosonic daughter atoms are created, with no long-lived intermediate states. The fermionic BEC then behaves as if the system explored virtual states with broken pairs only for a short time, before rapidly returning to the condensate manifold. This leads to a simplified description in which the dynamics is entirely encoded in a pair-destruction operator feeding two particles into the bosonic condensate.

For convenience, we introduce bosonic annihilation operators a and b describing, respectively, the fermionic-pair condensate and the daughter bosonic condensate, with single-mode energies E_a and E_b . Although the operators associated with fermion pairs obey a hardcore boson algebra, the single-mode bosonic approximation provides an accurate description in the dilute deep-BEC regime, where deviations from canonical commutation relations are parametrically suppressed [17].

Within this coarse-grained framework, the irreversible dynamics of the decay can therefore be captured by an effective process in which the composite bosonic mode a is converted into two quanta of the bosonic mode b , consistently with the statistical and energetic structure of the system.

A minimal model for the description of such a process is the following:

$$H = E_a a^\dagger a + E_b b^\dagger b \quad (28)$$

$$L_D = \sqrt{\gamma} (b^\dagger)^2 a \quad (29)$$

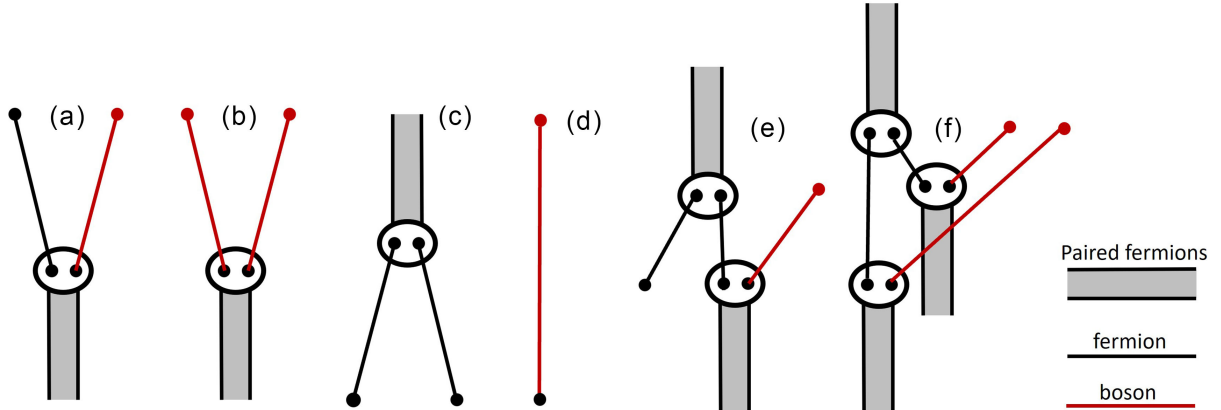


FIG. 4. Schematic representation of the elementary and composite processes occurring in a deeply bound fermionic BEC undergoing electron-capture decay into a bosonic atomic species. Double gray lines denote tightly bound fermion pairs, single black lines indicate unpaired fermions, and single red lines represent bosons produced by decay. Neutrino emission, always accompanying the decay process, is omitted for clarity. The physical regime considered assumes that the recombination time of two unpaired fermions is much shorter than the decay rate, so that the transient population of unpaired fermions is effectively negligible. (a) Decay of a single fermionic constituent within a bound pair, leading to the conversion of one fermion into a boson. (b) Double decay of both fermions forming a pair, resulting in the emission of two bosons. (c) Fast pairing (recombination) of two unpaired fermions into a tightly bound pair. (d) Decay of a single unpaired fermion into a bosonic atom. (e) Decay of one bound fermion within a pair followed by the recombination of the resulting unpaired fermion with another unpaired partner, forming a new pair. (f) A boson-mediated process in which the decay of a bound fermion and the subsequent recombination events effectively lead to the emission of two bosons, with the fermionic condensate providing the intermediate virtual channel.

$$L_d^{(a)} = \sqrt{\gamma_\phi^{(a)}} a^\dagger a \quad (30)$$

$$L_d^{(b)} = \sqrt{\gamma_\phi^{(b)}} b^\dagger b \quad (31)$$

where the Hamiltonian part is diagonal in the two atomic species, while the irreversible part of the dynamics is modeled via jump operators, respectively, describing the decay (L_D) and the decoherence in the fermionic BEC ($L_d^{(a)}$) and in the bosonic BEC ($L_d^{(b)}$).

The degrees of freedom of the emitted neutrinos are neglected, assuming that they escape very rapidly from the emission region, thus ensuring that $\langle n_c \rangle \simeq 0$, as already seen in the previously discussed models.

The quantities of interest are therefore the atomic populations $\langle n_a \rangle$ and $\langle n_b \rangle$, with the constraint of the total atomic number conservation:

$$2\langle n_a \rangle + \langle n_b \rangle = N. \quad (32)$$

The fact that $[H, n_a] = [H, n_b] = 0$, implies that the dynamics of n_a and n_b is entirely regulated by the irreversible dynamics of the decay and by decoherence. In general, for an observable O , we have Lindblad-Heisenberg equation of motion of the form:

$$\langle \dot{O} \rangle = \frac{i}{\hbar} \langle [H, O] \rangle + \langle D(O) \rangle, \quad (33)$$

where $D(O) = D(O)^{(D)} + D(O)^{(d,a)} + D(O)^{(d,b)}$ with

$$D(O)^{(D)} = L_D^\dagger O L_D - \frac{1}{2} \{L_D^\dagger L_D, O\} \quad (34)$$

$$D(O)^{(d,\alpha)} = L_D^{(\alpha)\dagger} O L_D^{(\alpha)} - \frac{1}{2} \{L_D^{(\alpha)\dagger} L_D^{(\alpha)}, O\}, \quad (35)$$

and $\alpha \in \{a, b\}$. Note that, if O commutes both with L and with $L^\dagger$, then $D(O) = 0$.

Within the general framework evoked above, it is possible to derive the equation for $\langle n_a \rangle$ in the form:

$$\langle \dot{n}_a \rangle = \langle D^{(D)}(n_a) \rangle + \langle D^{(d,a)}(n_a) \rangle \quad (36)$$

$$= -\gamma \langle n_a(n_b + 1)(n_b + 2) \rangle. \quad (37)$$

On the other hand, the equation for $\langle n_b \rangle$ gives

$$\langle \dot{n}_b \rangle = \langle D^{(D)}(n_b) \rangle + \langle D^{(d,b)}(n_b) \rangle \quad (38)$$

$$= 2\gamma \langle n_a(n_b + 1)(n_b + 2) \rangle, \quad (39)$$

therefore verifying the above stated conservation law, since $2\langle \dot{n}_a \rangle + \langle \dot{n}_b \rangle = 0$.

Proceeding as done in Secs. III and IV, these equations can be solved by truncating correlations at this level, so that we obtain the following set of equations:

$$\langle \dot{n}_a \rangle = -\gamma \langle n_a \rangle (\langle n_b \rangle + 1)(\langle n_b \rangle + 2), \quad (40)$$

$$\langle \dot{n}_b \rangle = 2\gamma \langle n_a \rangle (\langle n_b \rangle + 1)(\langle n_b \rangle + 2), \quad (41)$$

to be solved taking into account the total number conservation in Eq. (32) and the initial condition $\langle n_b \rangle|_{t=0} = 0$. Interestingly, within the truncation scheme adopted above, the jump operators $L_d^{(a,b)}$ related to the decoherence and proportional to n_a and n_b do not contribute to the closed dynamics of Eqs. (40) and (41).

In a more refined treatment, however, decoherence would enter through the hierarchy of higher-order correlations. Indeed, although the time evolution of the correlator $\langle n_a(n_b + 1)(n_b + 2) \rangle$ is not directly modified by these dissipators, its equation of motion couples to additional mixed correlators whose dynamics would, in principle, be sensitive to the decoherence induced by the jump operators $L_d^{(a,b)}$.

These observations suggest that, although the decoherence channels would, in principle, affect the dynamics through such deeply nested correlators, their overall impact remains negligible at leading order, as already indicated by the truncated equations.

Thus, the resulting equation for $\langle n_b \rangle$ reads

$$\langle \dot{n}_b \rangle = \gamma(N - \langle n_b \rangle)(\langle n_b \rangle + 1)(\langle n_b \rangle + 2). \quad (42)$$

A direct comparison between the numerical integrations of Eq. (42) for different atomic populations reveals the onset of a genuinely *collective* emission process, displaying a markedly superradiant character (see Fig. 5). Once $\langle n_b \rangle$ exceeds a threshold value, the growth of the decayed fraction accelerates abruptly, producing a dynamical “explosion” that is far more pronounced than one would expect from a standard logistic-type equation, as typically arises in mean-field models of boson-to-boson decay (see Sec. III). In the present case, the combination of the bosonic stimulation factor $(\langle n_b \rangle + 2)$ in Eq. (42) and the progressive reduction of the number of fermionic pairs leads to an autocatalytic acceleration of the dynamics, culminating in a rapid collapse toward saturation. The cooperative nature of the process manifests itself through a sharp dynamical transition, which is sensitive to the total number of atoms. Indeed, increasing atomic populations, the onset of the explosive growth shifts dramatically to earlier times, while the initial evolution remains nearly identical across all curves. This behavior indicates that the collective emission emerging in the model cannot be assimilated to a simple logisticlike dynamics, but instead reflects the intrinsic bosonic cooperativity embedded in the decay mechanism.

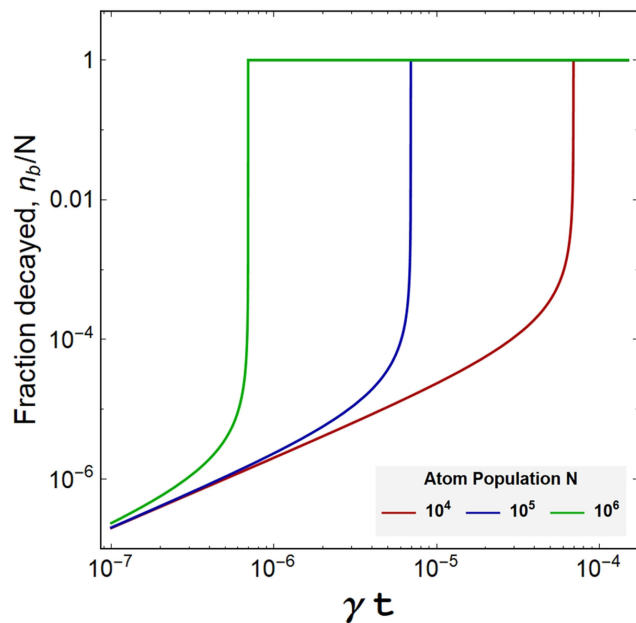


FIG. 5. Plot of the decayed fraction $\frac{\langle n_b \rangle}{N}$, according to Eq. (42), for different atom populations N as specified in the figure legend.

VI. EXPERIMENTAL FEASIBILITY OF RADIOACTIVE ATOMIC CONDENSATES FOR SUPERRADIANT NEUTRINO EMISSION

The mechanisms discussed above rely on the existence of Bose-Einstein condensed systems of radioactive atoms, either bosonic or fermionic, whose experimental realization is clearly demanding. In practice, the atomic half-life must be sufficiently long compared to the time scale required for condensate preparation, while condensate stability may be challenged by the large energy scales involved in the decay process when compared to the intrinsic energy scales of the condensate. Nevertheless, radioactive Bose-Einstein condensates are not excluded on fundamental grounds and therefore represent a legitimate subject of theoretical and experimental investigation.

Radioactive Bose-Einstein condensates of bosonic atoms have been discussed in the literature as potential platforms for collective decay phenomena. In particular, Ref. [1] identified the isotope ^{83}Rb as a promising candidate and suggested that its relatively long half-life of about 86 days could, in principle, be substantially reduced by superradiant effects.

On the other hand, subsequent analyses have clarified that this optimistic scenario is crucially affected by the statistics of the decay products. As shown by Lu, Lin, and Ketterle, when the decay of a bosonic condensate produces fermionic daughter atoms, as is generically the case for electron-capture processes, the buildup of collective emission is suppressed by Pauli blocking. In this sense, the critique of Ref. [2] does not rule out superradiant enhancement on general grounds, but demonstrates that the specific

proposal based on bosonic parent atoms decaying into fermionic products is not viable, precisely because the change in statistics inhibits the cooperative dynamics.

The scenario explored in Sec. V relies instead on the availability of a fermionic condensate in the deep BEC regime. This setting avoids the Pauli-blocking limitations encountered in proposals based on bosonic parent atoms, thereby allowing for a genuinely cooperative decay dynamics. From a many-body perspective, the realization of fermionic superfluids and of the BCS-BEC crossover is by now well established in ultracold atomic gases, most notably for ^6Li and ^{40}K , where pairing and condensation of fermion pairs have been extensively studied [18–20]. Similarly, mixtures of bosonic and fermionic superfluids have also been realized [21]. These systems provide clear experimental evidence that fermionic atoms can be driven into a regime where tightly bound pairs behave as composite bosons and undergo Bose-Einstein condensation.

For the present proposal, however, an additional and more specific requirement must be met: the fermionic atoms forming the condensate must decay exclusively via electron capture, as discussed in Sec. II. This constraint severely restricts the set of viable candidates, but does not render it empty. A particularly natural candidate in this respect is fermionic ^7Be , which decays by electron capture into the bosonic nucleus ^7Li (see Sec. II). While the pairing properties of ^7Be have not been explored experimentally, it is worth noting that its mass is close to that of ^6Li , suggesting that analogous pairing dynamics could, in principle, be achievable under suitable conditions, provided comparable effective interactions and scattering lengths can be engineered.

It should be emphasized that the success of the proposed setup in exhibiting the collective behavior described by the model hinges on the validity of the assumptions underlying the effective description. In particular, the deep BEC regime must ensure that broken pairs recombine on time scales much shorter than the decay rate, so that unpaired fermions do not accumulate and Pauli blocking remains ineffective. A different issue concerns the amount of energy transferred to the condensate during the decay process. Here, a favorable scenario is realized when the recoil associated with the electron-capture process expels the decayed species from the condensate with minimal scattering and the particles produced in the atomic deexcitation fully escape, so that the net energy deposited in the condensate is insufficient to break its coherence. In this scenario, the particle loss associated with the decay would not inhibit the cooperative dynamics and would be well tolerated in the BEC regime, provided that it remains comparable to the loss rates routinely managed in ultracold-atom experiments. Such losses could therefore be incorporated, in a more refined treatment, as an additional source of decoherence.

Finally, it is worth noting that the BEC or superfluid phase should exhibit a lifetime long enough to be operationally relevant as a superradiant neutrino source: present-day ultracold-atom experiments routinely achieve condensate lifetimes ranging from several seconds to tens of seconds, and in favorable conditions even longer, suggesting that further improvements in trapping and loss mitigation could realistically extend this window to regimes compatible with sustained cooperative emission.

Overall, while we fully acknowledge that the experimental realization of the scenario discussed in Sec. V would be technically demanding, we find no fundamental physical principle that forbids it. Rather, the proposal should be viewed as a proof-of-principle identification of a statistical pathway through which cooperative neutrino emissions could arise, provided the relevant microscopic and dynamical conditions are met.

VII. DISCUSSION AND CONCLUSIONS

In this work, motivated by the recent proposal of Ref. [1], we have undertaken a detailed analysis of the mechanism underlying superradiant neutrino emission from radioactive atomic condensates. Our first objective was to clarify the role of particle statistics in such a process.

We have shown that the scenario envisaged in Ref. [1] is, in principle, compatible with both the parent and daughter atoms being bosonic. However, this idealized situation is not realized in practice because the decay process leads to a change in statistics of the daughter species. In particular, starting from a BEC of bosonic atoms, the decay into fermionic atoms strongly suppresses the superradiant amplification, in agreement with the findings of Ref. [2]. Within the Lindblad formalism, we have examined this case in detail and confirmed the dominant role of Pauli blocking in quenching the cooperative emission, thereby reinforcing the conclusion that such a superradiant neutrino burst is effectively forbidden under these conditions.

We have then explored a complementary scenario in which the initial atomic population is fermionic and resides in the deep BEC regime, forming tightly bound pairs that decay into a bosonic condensate. Electron capture destabilizes the pairs, generating unpaired fermions; under the assumption of a rapid recombination of these atoms into new pairs, we have derived an effective model that does exhibit a pronounced superradiant behavior, qualitatively consistent with the mechanism proposed in Ref. [1].

The analysis presented here is based on minimal models designed to isolate and highlight the statistical features that control the cooperative emission process. While simplified, these models capture the essential physics of the phenomenon and delineate the conditions under which superradiance may or may not occur. We expect that this study will help frame a more rigorous understanding of the underlying mechanisms, and more refined investigations are currently underway.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: DERIVATION OF THE EFFECTIVE MODEL USED IN SEC. III

Hereafter, we provide details about the derivation of the effective model used in Sec. III of the main text.

We consider a radioactive Bose-Einstein condensate undergoing electron capture, and construct an effective field-theoretic model retaining only the physically relevant degrees of freedom: two bosonic atomic fields $\psi_a(\mathbf{r})$ and $\psi_b(\mathbf{r})$ associated with the undecayed and decayed internal states, respectively, and a fermionic neutrino field $\nu(\mathbf{r})$ describing the emitted neutrino modes.

The total Hamiltonian of the emission region is thus written as

$$H = H_a + H_b + H_\nu + H_{\text{int}} + H_{\text{coll}}. \quad (\text{A1})$$

The first two terms of H describe the atomic kinetic and potential energies,

$$H_\alpha = \int d^3r \psi_\alpha^\dagger(\mathbf{r}) \left[-\frac{\hbar^2 \nabla^2}{2m_\alpha} + V_\alpha(\mathbf{r}) \right] \psi_\alpha(\mathbf{r}), \quad \alpha = a, b,$$

while the neutrino field contributes

$$H_\nu = \int d^3r \nu^\dagger(\mathbf{r}) h_\nu \nu(\mathbf{r}),$$

where h_ν denotes the single-particle Dirac Hamiltonian generating the free neutrino dispersion. Atom-atom interactions are included through the contact interaction term

$$H_{\text{coll}} = \frac{g_{aa}}{2} \int d^3r \psi_a^\dagger \psi_a^\dagger \psi_a \psi_a + \frac{g_{bb}}{2} \int d^3r \psi_b^\dagger \psi_b^\dagger \psi_b \psi_b + g_{ab} \int d^3r \psi_a^\dagger \psi_b^\dagger \psi_b \psi_a. \quad (\text{A2})$$

Atom decay and subsequent neutrino emission is described in effective manner tracing out the electronic degree of freedom, yielding an effective three-field vertex interaction of the form:

$$H_{\text{int}} = g_v \int d^3r [\chi_\nu^\dagger(r) \psi_b^\dagger(r) \psi_a(r) + \text{H.c.}], \quad (\text{A3})$$

where the quantity

$$\chi_\nu(r) = \sum_{\mathbf{k}s} \mathcal{F}_{\mathbf{k}s}(r) c_{\mathbf{k}s} \quad (\text{A4})$$

can be understood as a dressed field obtained as linear combination, with vertex functions $\mathcal{F}_{\mathbf{k}s}(r)$, of fermionic operators $c_{\mathbf{k}s}$ that destroy a positive energy solution of the single-particle Dirac equation $h_\nu u_{\mathbf{k}s}(r) = E_\nu(\mathbf{k})u_{\mathbf{k}s}(r)$. The same expansion is used to obtain the neutrino field as $\nu(\mathbf{r}) = \sum_{\mathbf{k}s} u_{\mathbf{k}s}(\mathbf{r})c_{\mathbf{k}s}$ with fermionic commutation rule $\{c_{\mathbf{k}s}, c_{\mathbf{k}'s'}^\dagger\} = \delta_{\mathbf{k}\mathbf{k}'}\delta_{ss'}$.

Projecting the atomic fields onto their condensate manifolds, we approximate each bosonic field by its lowest-energy single-particle mode, i.e., $\psi_\alpha(\mathbf{r}) \simeq \phi_\alpha(\mathbf{r})\alpha$, with $\int d^3r |\phi_\alpha|^2 = 1$ and $\alpha \in \{a, b\}$. In the BEC regime this single-mode reduction is accurate, as population of higher orbital modes is energetically suppressed. Together with the scattering-eigenmode expansion of the neutrino field discussed above, the Hamiltonian reduces to the form

$$H = \varepsilon_a a^\dagger a + \varepsilon_b b^\dagger b + \sum_{\mathbf{k},s} E_\nu(\mathbf{k}) c_{\mathbf{k}s}^\dagger c_{\mathbf{k}s} + \sum_{\mathbf{k},s} [g_{\mathbf{k}s} c_{\mathbf{k}s}^\dagger b^\dagger a + g_{\mathbf{k}s}^* a^\dagger b c_{\mathbf{k}s}] + U_a (a^\dagger a)^2 + U_b (b^\dagger b)^2 + U_{ab} (a^\dagger a)(b^\dagger b), \quad (\text{A5})$$

where $\varepsilon_\alpha = \int d^3r \phi_\alpha^* (-\hbar^2 \nabla^2 / 2m_\alpha + V_\alpha) \phi_\alpha$, $E_\nu(\mathbf{k}) = \int d^3r u_{\mathbf{k}s}^\dagger(r) h_\nu u_{\mathbf{k}s}(r)$, $U_a = \frac{g_{aa}}{2} \int d^3r |\phi_a|^4$, $U_b = \frac{g_{bb}}{2} \int d^3r |\phi_b|^4$, $U_{ab} = g_{ab} \int d^3r |\phi_a|^2 |\phi_b|^2$, and the microscopic vertex is

$$g_{\mathbf{k}s} = g_v \int d^3r \mathcal{F}_{\mathbf{k}s}^*(\mathbf{r}) \phi_b^*(\mathbf{r}) \phi_a(\mathbf{r}). \quad (\text{A6})$$

Following Ref. [1], we assume that the continuum of neutrino modes can be effectively projected onto a single dominant channel $(\mathbf{k}_0, s_0)$. Thus, the Hamiltonian in

Eq. (1) is obtained by considering a single neutrino mode described by the second quantization operator $c \equiv c_{\mathbf{k}_0 s_0}$ and energy $E_\nu = E_\nu(\mathbf{k}_0)$.

Using the Lindblad equation of motion for the relevant observables within the effective model framework and neglecting high-order correlations, the complete set of equations describing the system dynamics in the presence of atomic collisions, with $U_a = U_b = U_{ab} = U$, is obtained:

$$\langle \dot{n}_a \rangle = \frac{2}{\hbar} \Im[g^* S] \quad (\text{A7})$$

$$\langle \dot{n}_b \rangle = -\frac{2}{\hbar} \Im[g^* S] \quad (\text{A8})$$

$$\langle \dot{n}_c \rangle = -\frac{2}{\hbar} \Im[g^* S] - \frac{2\Gamma}{\hbar} \langle n_c \rangle \quad (\text{A9})$$

$$\dot{S} = \frac{(i\Delta_{\text{eff}} - \Gamma)}{\hbar} S + \frac{ig}{\hbar} [\langle n_c \rangle \langle n_b \rangle (1 + \langle n_a \rangle) - (1 - \langle n_c \rangle) \langle n_a \rangle (1 + \langle n_b \rangle)] \quad (\text{A10})$$

with $\Delta_{\text{eff}} = \Delta + U(\langle n_a \rangle - \langle n_b \rangle + 1)$ and initial conditions $\langle n_a \rangle_{t=0} = N$ and $\langle n_b \rangle_{t=0} = \langle n_c \rangle_{t=0} = S(t=0) = 0$.

The set of differential equations above can be integrated numerically in their full form, thus retaining the complete dimensionality of the problem. Alternatively, consistent reductions may be implemented by eliminating variables whose evolution occurs on faster time scales or remains only weakly coupled, as discussed in the main text. Both approaches yield mutually consistent evolutions, differing solely in the level of dynamical detail retained.

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