

Small- x gluon GPD constrained from deeply virtual J/ψ production and gluon PDF through universal-moment parametrization

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We phenomenologically constrain the small- x and small- ξ gluon generalized parton distributions (GPDs) with the deeply virtual J/ψ production (DVJ/ ψ P) in the framework of GPDs through universal moment parameterization (GUMP). We use a hybrid cross-section formula combining collinear factorization to the next-to-leading order (NLO) accuracy of the strong coupling α_s , with corrections from nonrelativistic QCD to account for the power corrections due to the heavy J/ψ mass. We reach reasonable fit to the measured differential cross sections of DVJ/ ψ P by H1 at the Hadron-Electron Ring Accelerator as well as forward gluon parton distribution functions (PDFs) from JAM22 global analysis. We find that both NLO and nonrelativistic corrections are significant for heavy vector meson productions. Of course, the gluon GPD we obtain still contains considerable freedom in need of inputs from other constraints, particularly in the distribution-amplitude-like region.

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I. INTRODUCTION

The multidimensional structure of hadrons, as encoded by their quark and gluon degrees of freedom in quantum chromodynamics (QCD), remains a major open problem in nuclear physics. Several powerful tools for addressing this very problem have been developed in the past few decades, one of which is the generalized parton distributions (GPDs) [1–3]. GPDs leverage nonzero momentum transfer to a hadron to go beyond the forward parton distribution functions (PDFs) that carry the one-dimensional, longitudinal structure, and instead probe the three-dimensional structure in the impact parameter space [4,5]. Thus, GPDs are investigated for the tomography of hadrons [4–7] as well as various properties of the bulk hadron state, such as

the distributions of spin and mass in the quark and gluon fields comprising the bound state [2,8,9]. These functions are related to exclusive production processes like deeply virtual Compton scattering (DVCS) and deeply virtual meson production (DVMP) through factorization theorems [10–12]. However, the amplitudes in these processes are given by convolutions involving GPDs together with other quantities, which essentially eliminates one of their three variables from being directly probed. This poses an inverse problem for the deconvolution of these amplitudes to extract GPDs, as illustrated in the context of shadow GPDs recently [13].

Accordingly, massive efforts have been devoted to searching for complimentary inputs. Recent measurements of time-like Compton scattering [14] and near-threshold J/ψ productions [15–17] provide some new insights to access the GPDs through exclusive photoproductions [18–22]. Moreover, exclusive productions of multiple particles that typically have better sensitivity to the missing variable have been proposed [23–26], and a new factorization theorem for single diffractive hard exclusive processes has been developed recently [27–31]. Probing the nucleon structures with these exclusive processes will be one of the crucial tasks of the future electron-ion collider (EIC) [32]. On the other hand, the lattice QCD offers a different perspective to approach the nuclear structure in

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Euclidean space from the first principle. The simulations of the nucleon form factors (FFs) on lattice have been significantly improved just in the past few years and show exciting results [33–40]. Moreover, developments like the large momentum effective theory have enabled the lattice QCD to directly calculate the parton distributions including GPDs [41,42], with which many pioneering works have been done for the GPDs [43–45]. Therefore, we will need a global analysis program of GPDs that combines the comprehensive present and future inputs on GPDs including both experiment measurements and lattice simulations and perhaps overcomes the inverse problem once enough constraints are gathered.

In the previous works of the authors [46,47], the GPDs through universal moment parametrization (GUMP) framework was put forward for performing such a global analysis by making use of the conformal moment expansion of GPDs [48,49]. A preliminary global analysis was performed by combining input from globally extracted PDFs [50] and FFs [51], lattice calculations [33,43], and DVCS data from Jefferson Lab (JLab) and the Hadron-Electron Ring Accelerator (HERA) [52–56] to constrain the up and down quark GPDs, both in the valence and sea quark regimes at leading order (LO) in the perturbative expansion in α_s [47]. The gluon GPDs were only constrained in the forward (PDF) limit, as the DVCS cross sections are not sensitive to them until at next-to-leading order (NLO) or through the evolution of the sea quarks. In this paper, we extend the analysis by adding the off-forward gluon GPDs together with the deeply virtual J/ψ production (DVJ/ ψ P) measurements at HERA by the H1 Collaboration [57]. Assuming the intrinsic quark content of the target is negligible for heavy quarks like the charm, the heavy quarkonium productions will be mostly sensitive to the gluon GPDs, especially in the gluon-dominant high Q^2 and small- x_B kinematics for the HERA measurements. Due to this unique sensitivity to the gluonic structure in the nucleon, there have been extensive studies on the exclusive productions of the J/ψ in various frameworks [19,58–71].

The exclusive photoproduction of the J/ψ , corresponding to the $Q \rightarrow 0$ limit of leptonproduction has been calculated to NLO in [19] in the framework of non-relativistic QCD (NRQCD), where the heavy meson mass serves as the hard scale for the factorization. It has been later extended to electroproduction [63,66], where the NLO effects are shown to be significant and could strongly cancel the LO contributions with some simple GPD model [63]. However, an analysis strictly within the GPD framework with corresponding NLO evolution has not yet been established. On the other hand, there have been many developments in the analysis of light meson productions in the collinear factorization framework of DVMP, for which the DVMP of light mesons as well as DVCS have been studied up to NLO including GPD evolution [72–77]. This process has also been studied in the small- x color-glass-condensate (CGC) framework [62,78–83] and recently

even at NLO [67,68]. However, an approximate matching between the CGC and the collinear framework has been established for the DVCS [84], but not for the DVJ/ ψ P yet. These developments provide us the opportunity to consider the heavy vector meson production in a strict GPD framework to NLO including evolution, of which the theoretical setup will be introduced in the next section.

The structure of the rest of this paper is as follows, in Sec. II, we introduce the theoretical setup used to study the gluon GPDs with the DVJ/ ψ P cross section, paying special attention to the various difficulties that have arisen in past efforts to extract the gluon GPD or PDF from this process. Next, in Sec. III we specify how we implement constraints from DVJ/ ψ P in the GUMP framework and present the results of our new fit. First, we review the results of our previous global analysis of up and down quarks [47], then we clarify the parameters and formulas used for the modeling of the GPDs and the calculations of the cross section when performing our fit to the data. We then present the results of our fits and compare them to the input DVJ/ ψ P data and PDFs. Finally, in Sec. IV we summarize our results and discuss directions for further work.

II. EXCLUSIVE J/ψ ELECTROPRODUCTION FOR THE GLUONIC STRUCTURE

In this section, we introduce a factorization formula for the electroproduction of heavy vector meson, which will be used to constrain the gluon GPD in this paper. In the deeply virtual, i.e., $Q \rightarrow \infty$ limit where $Q^2 \equiv -q^2$ is the virtuality of the photon with momentum q , a collinear factorization theorem has been proven at leading twist for longitudinally polarized virtual photons [11,12]. Meanwhile, for exclusive heavy meson productions, the NRQCD factorization has been established for both the photo- and electroproduction to NLO in the limit that the meson mass $M_V \rightarrow \infty$ [19,63,66]. A diagrammatic illustration of the two frameworks is shown in Fig. 1. Nevertheless, the two frameworks have been explored in rather different context, focusing on light-meson productions [75,77] and heavy-meson productions [19,63,66], respectively, whereas their connection beyond the LO has not yet been established.

While each framework has been shown to be successful for its own application, the lack of proper matching between them would cause issues for a global analysis of GPDs. On the one hand, a combined analysis of the light and heavy meson productions would allow enhanced sensitivities to GPDs, especially to disentangle their flavor structures. On the other hand, each framework has its limitations, as summarized in Table I, which can be compensated by a hybrid framework. Ideally, a hybrid factorization framework that perturbatively matches the collinear and NRQCD factorization frameworks when Q^2 and M_V^2 are both large, and reduces to them under different limits, respectively, could overcome these issues. However, as we will show below while the matching is trivial at LO,

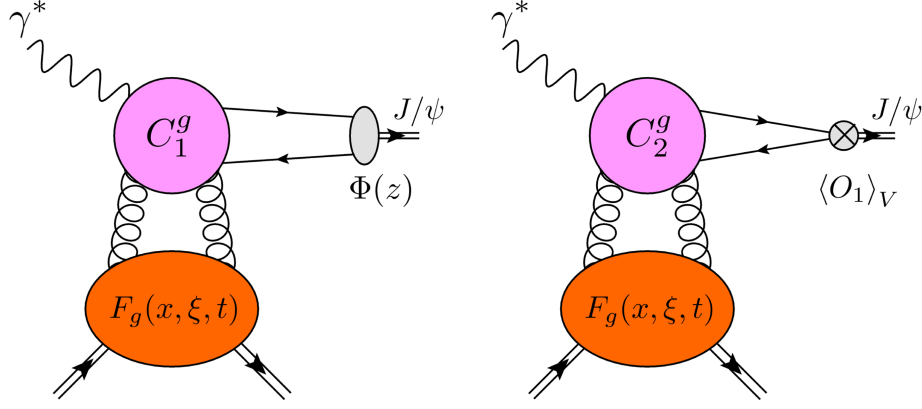


FIG. 1. Illustrations of the factorized amplitudes in the collinear factorization framework (left) and the NRQCD framework (right). The DA $\Phi(z)$ in the collinear framework will be replaced by a local NRQCD matrix element $\langle O_1 \rangle_V$ in the NRQCD framework, with different hard scattering coefficients $C_1^g(x, \xi, z)$ and $C_2^g(x, \xi, M_V, Q)$ to be matched.

it gets much more complicated at NLO, which requires more detailed theoretical studies. In this work, we will consider an approximate hybrid framework based on the LO matching while including the NLO corrections from the collinear framework, whereas refined studies on the NLO matching will be left to future work. This is the best approximation we can come up with based on the state-of-art theoretical work [19,63,66,75,77].

In the following section, we will briefly review the two factorization frameworks and introduce the hybrid framework of this work.

In the collinear factorization of DVMP, the differential cross sections of DVMP for an unpolarized proton target can be written in the small- x_B limit as [77],

$$\frac{d\sigma_{\gamma_L^* p \rightarrow J/\psi p}^{\text{DVMP}}}{dt} = 4\pi^2 \alpha_{\text{EM}} \frac{x_B^2}{Q^4} \left(|\mathcal{H}_{\text{DVMP}}|^2 - \frac{t}{4M_N^2} |\mathcal{E}_{\text{DVMP}}|^2 \right), \quad (1)$$

where α_{EM} is the fine structure constant and $\mathcal{H}_{\text{DVMP}}$ and $\mathcal{E}_{\text{DVMP}}$ are the so-called transition form factors (TFFs) that correspond to the H and E GPDs defined below. Note that the lepton production cross section has been rewritten into a virtual photoproduction cross section by removing the virtual photon flux of the lepton. Thus, the total virtual

photoproduction cross section can be written as the sum of the longitudinal- and the transverse-photon contributions,

$$\frac{d\sigma_{\gamma^* p \rightarrow J/\psi p}}{dt} = \frac{d\sigma_{\gamma_L^* p \rightarrow J/\psi p}}{dt} + \varepsilon \frac{d\sigma_{\gamma_T^* p \rightarrow J/\psi p}}{dt}, \quad (2)$$

where ε is the ratio of longitudinal and transverse photon flux that can be determined with the kinematics of the lepton beam. Moreover, since the factorization theorem is proven at leading twist with the longitudinally polarized photon, only the longitudinal $d\sigma_{\gamma_L^* p \rightarrow J/\psi p}$ can be calculated in this framework.

The TFFs $\mathcal{H}_{\text{DVMP}}$ (and $\mathcal{E}_{\text{DVMP}}$ similarly) can then be written in the factorized form as [75,77]

$$\mathcal{H}_{\text{DVMP}}^g = \frac{e_q f_V}{N_c Q} \int_0^1 dz \int_{-1}^1 dx C_1^g(x, \xi, z) H^g(x, \xi, t) \Phi^V(z), \quad (3)$$

as illustrated in Fig. 1. In the above expression, f_V is the meson decay constant, V denotes the vector meson: $V = \rho, \omega, \phi, J/\psi, \dots$,¹ $N_c = 3$ is the number of color, and e_q is the quark charge in the unit of proton charge e , i.e., $2/3$ for charm and $-1/3$ for strange quark. All the scale dependence will be suppressed in this section for simplicity. The constant f_V can be determined according to physical observables such as the leptonic decay width as [87],

$$\Gamma[V \rightarrow e^+ e^-] = \frac{4\pi e_q^2 \alpha_{\text{EM}}^2 f_V^2}{3M_V}, \quad (4)$$

¹In this work we consider the J/ψ production, which is vector meson. The factorization formula applies to other mesons as well, with slight changes in the formula and GPDs involved in the TFFs [75,77,85,86], which will not be discussed here.

TABLE I. Comparison of the different features of the collinear factorization and NRQCD factorization framework.

Features	DVMP	NRQCD
Light meson production	✓	...
Simple moment space expressions	✓	?
Meson mass correction	...	✓
Transverse photon cross section	...	✓
Fixed-order results up to NLO	✓	✓

where M_V is the vector meson mass. Taking from the particle data group [88] that $\Gamma[J/\psi \rightarrow e^+e^-] = 5.53 \pm 0.10$ keV, one gets $f_{J/\psi} \approx 0.41$ GeV.

Inside the integral for the TFFs, there are three components: the meson distribution amplitude (DA) $\Phi^V(z)$ is defined by [89,90],

$$\begin{aligned} M_V f_V \frac{n \cdot \varepsilon^V}{n \cdot P_V} \Phi_q^V(z) \\ = \frac{1}{P_V^+} \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{i(2z-1)\lambda/2} \langle 0 | \bar{\psi}_q \left(-\frac{\lambda n}{2} \right) \gamma^+ \psi_q \left(\frac{\lambda n}{2} \right) | V_L(P_V) \rangle, \end{aligned} \quad (5)$$

normalized according to $\int_0^1 dz \Phi(z) = 1$, where n is the light-cone vector, conjugating to the meson momentum P_V . At leading twist, the vector meson is longitudinally polarized and so is the corresponding DA. The leading-twist gluon GPDs $F_g(x, \xi, t)$ are defined by [2,91],

$$F_g(x, \xi, t) \equiv \frac{1}{(\bar{P}^+)^2} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | F^{+\mu} \left(-\frac{\lambda n}{2} \right) F_\mu^+ \left(\frac{\lambda n}{2} \right) | P \rangle, \quad (6)$$

where P and P' are the momenta of the initial and final nucleon with $\bar{P} \equiv (P + P')/2$ and $\Delta \equiv P' - P$, x is the average parton longitudinal momentum, $\xi \equiv -n \cdot \Delta / (2n \cdot \bar{P})$ is the skewness parameter, and $t \equiv \Delta^2$ is the squared momentum transfer. They can be further parametrized in terms of scalar functions as [2]

$$\begin{aligned} F_g(x, \xi, t) = \frac{1}{2\bar{P}^+} \bar{u}(P') \left[H_g(x, \xi, t) \gamma^+ \right. \\ \left. + E_g(x, \xi, t) \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M_N} \right] u(P), \end{aligned} \quad (7)$$

where $H_g(x, \xi, t)$ and $E_g(x, \xi, t)$ are the well-known leading-twist gluon GPDs [2]. The hard scattering coefficient $C_1^g(x, \xi, z)$ can be perturbatively calculated. There will also be contributions from quark GPDs $F_q(x, \xi, t)$ with their corresponding Wilson coefficient $C_1^q(x, \xi, z)$ starting at NLO, whose expressions are not explicitly presented here.

This framework has been applied to analyze the measurement of DVMP production of light mesons at HERA at NLO, which shows great success [75,77]. Nevertheless, it is noteworthy that the vector meson mass neglected in the DVMP formula can introduce sizable corrections in the case of J/ψ . Even for the highest Q^2 bin in HERA data [57,92], one has $M_{J/\psi}^2/Q^2 \approx 1/2$. These corrections in the form of $M_{J/\psi}^2/Q^2$ will be higher-twist effects, which lie outside the approximations of the collinear factorization theorem [12].

On the other hand, these higher-twist terms, including the contribution from the transversely polarized virtual

photon, can be calculated within the NRQCD framework, where the heavy meson mass $M_{J/\psi}$ is treated as the hard scale and the formation of a nonrelativistic meson bound state will be factorized into a local NRQCD matrix element. This has been calculated up to the NLO for the photoproduction [19] as well as the leptonproduction [63,66]. In this framework, the corresponding differential cross section in the small x_B limit reads [19,64]

$$\begin{aligned} \frac{d\sigma_{\gamma^* p \rightarrow J/\psi p}^{\text{NRQCD}}}{dt} = 4\pi^2 \alpha_{\text{EM}} \frac{x_B^2}{(Q^2 + M_{J/\psi}^2)^2} \\ \times \left(|\mathcal{H}_{\text{NRQCD}}|^2 - \frac{t}{4M_N^2} |\mathcal{E}_{\text{NRQCD}}|^2 \right), \end{aligned} \quad (8)$$

which formally reduces to the DVMP crosssection in the limit $Q \gg M_{J/\psi}$, though the amplitudes will be different.² In the NRQCD framework, the corresponding amplitudes can be written as [19],

$$\begin{aligned} \mathcal{H}_{\text{NRQCD}}^g = \frac{e_q}{N_C} \left(\frac{\langle O_1 \rangle_V}{m_q} \right)^{1/2} \frac{Q}{Q^2 + M_V^2} \\ \times \int_{-1}^1 dx C_2^g(x, \xi, M_V, Q) H^g(x, \xi, t), \end{aligned} \quad (9)$$

where m_q is the corresponding quark mass, which can be taken to be $M_V/2$ for heavy quarkonia like J/ψ or Υ , omitting the higher-order nonrelativistic corrections suppressed by the heavy quark velocity $v \sim \alpha_s(M_V)$.

The NRQCD matrix element $\langle O_1 \rangle_V$ is defined as $\langle O_1 \rangle_V \equiv \langle J/\psi | \hat{O}_1(^3S_1) | J/\psi \rangle$, where the local NRQCD operator $\hat{O}_1(^3S_1)$ can be written as $\psi^\dagger \sigma_\chi \cdot \chi^\dagger \sigma_\psi \psi$ in terms of the heavy quark field ψ and antiquark field χ . On the other hand, this NRQCD matrix element $\langle O_1 \rangle_V$ can also be related to the meson decay width [93],

$$\Gamma[V \rightarrow e^+e^-] = \frac{2\pi e_q^2 \alpha_{\text{EM}}^2 \langle O_1 \rangle_V}{3m_q^2}. \quad (10)$$

Comparing to Eq. (4), one has the simple relation $\langle O_1 \rangle_V = m_q f_V^2$, with which we have

$$\mathcal{H}_{\text{NRQCD}}^g = \frac{e_q f_V}{N_C} \frac{Q}{Q^2 + M_V^2} \int_{-1}^1 dx C_2^g(x, \xi, M_V, Q) H^g(x, \xi, t), \quad (11)$$

which resembles the DVMP one in Eq. (3). However, in the case of NRQCD, there exist additional nonrelativistic

²Meanwhile, we also note that the application of NRQCD factorization is not limited to just the high-energy productions of heavy mesons, but also lower-energy near-threshold photoproductions [20], since the heavy meson mass serves as the hard scale.

expansion in powers of $v \sim \alpha_s(M_V)$, of which the corresponding corrections can be calculated as well [94].

The matching between the two amplitudes or TFFs in the two frameworks is rather obvious at LO, for which the Wilson coefficients at LO read, respectively,

$$C_1^{g,(\text{LO})}(x, \xi, z) = \frac{1}{1-z} \frac{\alpha_s}{\xi(\xi - x - i\epsilon)}, \quad (12)$$

and

$$C_2^{g,(\text{LO})}(x, \xi) = \frac{2\alpha_s}{\xi(\xi - x - i\epsilon)}, \quad (13)$$

which does not depend on the meson mass M_V or Q . They are almost identical except for the different prefactors, which can be matched after the dz integral if a non-relativistic approximation of the DA $\Phi(z)$ is made—in NRQCD, the relative velocity v of the two quark in the meson is suppressed by the strong coupling constant $v \sim \alpha_s$. Thus, in the LO picture, each of them shares half of the total meson momentum with $z = 1/2$ and we have: $\Phi_{\text{NR}}(z) = \delta(z - 1/2)$ and thus $\int dz \Phi_{\text{NR}}(z)/(1-z) = 2$. This corresponds to the extra factor of 2 in the $C_2^{g,(\text{LO})}(x, \xi)$. Thus, the amplitudes in the two frameworks become identical in the limit $Q \gg M_V$.

While this can be trivially done at LO as shown above, further complexity arises at higher order, which is found necessary in our analysis of the DVJ/ ψ P data—the attempt to fit the data fails with only the LO hard scattering amplitude and QCD scale evolution, indicating significant NLO corrections. While the NLO corrections have been calculated in the NRQCD framework [63,66] as well as the DVMP framework [76,77], there does not exist a smooth transition between them. In NRQCD, the loop integral will be naturally regulated by the mass quark propagators, leading to log terms in the form of $\log(m_c^2/\mu_F^2)$ with μ_F the factorization scale [63,66], of which the limit $m_c \rightarrow 0$ does not exist, whereas in the collinear framework, there are no such dimensional quantities and thus the logs will be in the form of $\log(1/\xi)$ [75], besides the ones that can be factorized into the meson DA. These single-log terms can also combine to form double logs. Consequently, the matching of the two hard scattering coefficients becomes extremely nontrivial. Therefore, we consider a hybrid framework that utilizes the DVMP framework at NLO but only matches the nonrelativistic corrections at LO, whereas a refined study comparing and matching the two frameworks beyond LO will be left to future work. We note that pioneering work in refactorizing NRQCD short-distance coefficients and matching to perturbative QCD has been previously demonstrated in [95] for exclusive heavy quarkonium production, indicating that a similar approach could apply to the GPD framework here.

To do so, we consider the DVMP amplitude with the replacement $1/Q \rightarrow Q/(Q^2 + M_{J/\psi}^2)$,

$$\mathcal{H}_{\text{Hyb}}^g = \frac{e_g f_{J/\psi}}{N_C} \frac{Q}{Q^2 + M_{J/\psi}^2} \int_0^1 dz \times \int_{-1}^1 dx C_1^g(x, \xi, z) H^g(x, \xi, t) \Phi(z), \quad (14)$$

and also replace the kinematic factor in the cross section,

$$\frac{d\sigma_L^{\text{Hyb}}}{dt} = 4\pi^2 \alpha_{\text{EM}} \frac{x_B^2}{(Q^2 + M_{J/\psi}^2)^2} \left(|\mathcal{H}_{\text{Hyb}}|^2 - \frac{t}{4M_N^2} |\mathcal{E}_{\text{Hyb}}|^2 \right), \quad (15)$$

whereas the Wilson coefficients and J/ψ DA $\Phi(z)$ from the DVMP framework will still be used. This hybrid framework allows us not only to take the leading mass corrections from the heavy J/ψ mass into account in the NRQCD framework as well as the large logarithms $\log(1/\xi)$ at NLO for ξ as low as 10^{-4} , but also to extend to light vector meson production for future analyses, which cannot be achieved otherwise with the collinear or NRQCD framework alone. Still, while the replacement gives the correct LO matching between the two frameworks as shown above, the major systematics are from the mismatch of the NLO amplitudes that include both the quark and gluon contributions, which will be studied in a separate future work.

Another advantage of such a hybrid framework lies in the photon polarizations that enter the amplitude. For the collinear case, the leading-twist contribution comes from the longitudinal photon polarization, and this is the only contribution for which the factorization theorem is proven for light vector meson production. Thus, one needs to measure the ratio of the cross section for longitudinally polarized photons to that for transversely polarized photons $R \equiv d\sigma_L/d\sigma_T$ and use this to relate the factorized portion of the cross-section $d\sigma_L$ to the total cross section through

$$\frac{d\sigma_{\text{tot}}^{\text{Hyb}}}{dt} = \frac{d\sigma_L^{\text{Hyb}}}{dt} \left(\epsilon + \frac{1}{R} \right), \quad (16)$$

with the longitudinal cross section given by Eq. (15). The need to separate the longitudinal and transverse contributions requires large experimental efforts and also gives rise to extra experimental uncertainties in the measured longitudinal-transverse ratio R . This can be avoided in the NRQCD treatment, where the hard part of the amplitudes, including the one for transverse photon polarization, can be constructed perturbatively, which also predicts a longitudinal-transverse ratio $R = Q^2/M_{J/\psi}^2$. This eliminates the need for experimental extraction of R , and thus removes one of the largest sources of uncertainty from the

experimental side [57,92], noting that the NRQCD prediction of R will be susceptible to perturbative and relativistic corrections that need further investigation. This prediction for R also connects the amplitude smoothly to the photoproduction limit $Q^2 \rightarrow 0$, which could allow for the inclusion of much more J/ψ photoproduction data in future work. Indeed, theoretical control over the photoproduction process would then allow for the inclusion of ultraperipheral collision data, which in principle provides a probe of both nucleon and nuclear GPDs. The NRQCD framework has also been used to study the near-threshold J/ψ production [20–22,96]. These processes can provide constraints on GPDs in very different regions of (ξ, t) phase space and thus enhance the scope of a global analysis.

III. DVJ/ ψ P WITH GPDs IN THE GUMP FRAMEWORK

In this section, we discuss the implementation of J/ψ electroproduction data as a constraint on gluon GPDs within the GUMP framework. We start by reviewing the ingredients of the GUMP framework and the previous forward [46] and off-forward analyses [47], where the u and d quark valence and sea GPDs were constrained by a combination of experimental data and lattice computations. Then we will discuss how we include J/ψ data from H1 [57] as a constraint on the off-forward parameters entering the gluon GPDs. Finally, we will discuss the results of the fit and the extracted gluon GPDs.

A. Constraints from previous work

The analysis in [47] included conformal moment space models for the valence and sea quark GPDs for the u and d quarks, as well a model for the semiforward limit of the gluon GPDs (t -dependent PDFs). In these models, the GPD conformal moments are directly parametrized, similar to the KM models used in [49,73,74,77,97,98]. The details of the conformal moment parametrization as used in the GUMP framework thus far are given in [46,47], we will briefly review them here. A GPD $F^i(x, \xi, t)$ for parton species i can be expressed as a series expansion in conformal moments $\mathcal{F}_j^i(\xi, t)$ as

$$F^i(x, \xi, t) = \sum_{j=0}^{\infty} (-1)^j p_j^i(x, \xi) \mathcal{F}_j^i(\xi, t), \quad (17)$$

with $p_j^i(x, \xi)$ the conformal wave functions, which can be expressed in terms of Gegenbauer polynomials [48,49]. Each individual moment is ensured to be written in terms of polynomials in ξ by the polynomiality condition for GPDs [3],

$$\mathcal{F}_j^i(\xi, t) = \sum_{k=0, \text{even}}^{j+1} \xi^k \mathcal{F}_{j,k}^i(t), \quad (18)$$

and for the GUMP framework we take the generalized form factors $\mathcal{F}_{j,k}^i(t)$ to be given by

$$\begin{aligned} \mathcal{F}_{j,k}^i(t) &= \sum_{l=1}^{l_{\max}} N_{l,k}^i B(j+1-\alpha_{l,k}, 1+\beta_{l,k}) \\ &\times \frac{j+1-k-\alpha_{l,k}}{j+1-k-\alpha_{l,k}(t)} f(t). \end{aligned} \quad (19)$$

Here we have overall normalization constants $N_{l,k}^i$, Euler beta functions coming from the forward PDF x dependence as $x^{-\alpha_{l,k}}(1-x)^{\beta_{l,k}}$, the Regge trajectory $\alpha_{l,k}(t) = \alpha_{l,k} + \alpha'_{l,k}t$, and the “residual” term $f(t) = e^{b_{l,k}^i t}$, which gives the exponential falloff in t , which is observed at small x . For simplicity, as well to keep the number of free parameters from overfitting the data we use for the analysis, we take a single term in the ansatz (19), setting $l_{\max} = 1$.

Furthermore, we impose several extra constraints to reduce the number of free parameters: first the Regge intercepts of the sea quarks are taken to be equal, $\alpha'^{\bar{u}} = \alpha'^d$, as well as their “residual” terms, $b^{\bar{u}} = b^d$. These quark parameters will be fixed from the previous DVCS analysis [47], whereas the corresponding gluon parameters, α'^g and b^g , will be determined from experimental data. Second, we take the higher-order terms in the ξ polynomial for the moments (18) to be proportional to the leading $k=0$ term so that

$$\mathcal{F}_j^i(\xi, t) = \sum_{k=0, \text{even}}^{j+1} \xi^k R_{\xi^k}^i \mathcal{F}_{j,0}^i(t), \quad (20)$$

with the sum being truncated at $k=2$ in [47], taking advantage of the high-energy limit where ξ is relatively small. Here, we extend to include a $k=4$ term to allow extra flexibility in modeling the ξ dependence for the gluon. Finally, some GPD species are not as well constrained by currently available data, and so we took them to be proportional to more well-constrained GPD species, for example setting the sea quark E GPDs to be proportional to the sea quark H GPDs, $E^{\bar{q}} = R_{\text{sea}}^E H^{\bar{q}}$. The same applies to the gluon GPD: $E^g = R_{\text{sea}}^E H^g$ in the previous fit. As the sensitivity to the gluon E_g GPDs is suppressed in the small- t region as shown in Eq. (15), the R_{sea}^E will be fixed from previous results too. Modifying this constraint on E_g appears to have a small effect on the DVJ/ ψ P cross sections.

With these extra constraints, the ansatz for the four GPD species $\{H, \tilde{H}, E, \tilde{E}\}$ of the five different parton species $\{u, \bar{u}, d, \bar{d}, g\}$ were given in total by around 60 parameters, as summarized in Table II, the majorities of which have been fitted in the previous work [47]. In this work, we will fix those quark parameters as the background and focus on determining the gluon GPDs from experimental data of

TABLE II. Summary of GUMP GPD parameters. The six parameters in bold are free parameters to be fitted in this work, whereas the others are fixed from previous work [47]. The E and $\tilde{E}$ GPDs of $\bar{u}, \bar{d}$, and g are set to be proportional to the corresponding H and $\tilde{H}$ GPDs with the ratio R_{sea}^E and $R_{\text{sea}}^{\tilde{E}}$ due to the lack of constraints on them. More about their definitions can be found in [47]. We note that an extra parameter N_{amp} for the normalization of the amplitude is used in the fit, besides these GPD parameters. More details will be discussed in the next subsection.

Parameters	H	E	$\tilde{H}$	$\tilde{E}$
u_V	$N_{u_V}^H, \alpha_{u_V}^H, \beta_{u_V}^H,$ $\alpha'_{u_V}, R_{2,u}^H$	$N_{u_V}^E, \alpha_{u_V}^E, \beta_{u_V}^E,$ $\alpha'_{u_V}, R_{2,u}^E$	$N_{u_V}^{\tilde{H}}, \alpha_{u_V}^{\tilde{H}}, \beta_{u_V}^{\tilde{H}},$ $\alpha'_{u_V}, R_{2,u}^{\tilde{H}}$	$N_{u_V}^{\tilde{E}}, \alpha_{u_V}^{\tilde{E}}, \beta_{u_V}^{\tilde{E}},$ $\alpha'_{u_V}, R_{2,u}^{\tilde{E}}$
$\bar{u}$	$N_{\bar{u}}^H, \alpha_{\bar{u}}^H, \beta_{\bar{u}}^H,$ $\alpha'_{\text{sea}}^H, b_{\text{sea}}^H, R_{2,u}^H$	R_{sea}^E	$N_{\bar{u}}^{\tilde{H}}, \alpha_{\bar{u}}^{\tilde{H}}, \beta_{\bar{u}}^{\tilde{H}},$ $\alpha'_{\text{sea}}^{\tilde{H}}, b_{\text{sea}}^{\tilde{H}}, R_{2,u}^{\tilde{H}}$	$R_{\text{sea}}^{\tilde{E}}$
d_V	$N_{d_V}^H, \alpha_{d_V}^H, \beta_{d_V}^H,$ $\alpha'_{d_V}, R_{2,d}^H$	$N_{d_V}^E, \alpha_{d_V}^E, \beta_{d_V}^E,$ $\alpha'_{d_V}, R_{2,d}^E$	$N_{d_V}^{\tilde{H}}, \alpha_{d_V}^{\tilde{H}}, \beta_{d_V}^{\tilde{H}},$ $\alpha'_{d_V}, R_{2,d}^{\tilde{H}}$	$N_{d_V}^{\tilde{E}}, \alpha_{d_V}^{\tilde{E}}, \beta_{d_V}^{\tilde{E}},$ $\alpha'_{d_V}, R_{2,d}^{\tilde{E}}$
$\bar{d}$	$N_{\bar{d}}^H, \alpha_{\bar{d}}^H, \beta_{\bar{d}}^H,$ $\alpha'_{\text{sea}}^H, b_{\text{sea}}^H, R_{2,d}^H$	R_{sea}^E	$N_{\bar{d}}^{\tilde{H}}, \alpha_{\bar{d}}^{\tilde{H}}, \beta_{\bar{d}}^{\tilde{H}},$ $\alpha'_{\text{sea}}^{\tilde{H}}, b_{\text{sea}}^{\tilde{H}}, R_{2,d}^{\tilde{H}}$	$R_{\text{sea}}^{\tilde{E}}$
g	$N_g^H, \alpha_g^H, \beta_g^H, \alpha_g^H,$ $b_g^H, R_{g,2}^H, R_{g,4}^H$	R_{sea}^E	$N_g^{\tilde{H}}, \alpha_g^{\tilde{H}}, \beta_g^{\tilde{H}}, \alpha_g^{\tilde{H}},$ $b_g^{\tilde{H}}, R_{g,2}^{\tilde{H}}$	$R_{\text{sea}}^{\tilde{E}}$

DVJ/ ψ P. Besides, we add extra gluon PDF constraints in the small- x region from a recent Jefferson Lab Angular Momentum Collaboration (JAM) global analysis [50] to fix the forward limit of the gluon GPDs to match the kinematics of the J/ψ electroproduction measurement.

B. Fitting to J/ψ electroproduction with GUMP

We now turn to the implementation of deeply virtual J/ψ production in the GUMP framework. The parameters we need to determine for the gluon GPDs are the ones given in the Eq. (19), namely, $N_g, \alpha_g, \beta_g, \alpha'_g$, and b_g . Besides, we have $R_{\xi^2}^g$ and $R_{\xi^4}^g$ for the ξ dependence. However, since the J/ψ electroproduction measurements are mostly in the small- x_B region, which constrain the large- x behavior of the gluon GPD very poorly, we fixed $\beta_g = 7$ from the previous analysis, where β_g accounts for the large- x behavior of gluon PDF. Thus, we are left with six parameters to be determined from the small- x gluon PDFs and the J/ψ electroproduction measurement.

However, an extra subtlety appears in the meson DA of J/ψ . In the last section, we showed that a matching between the NRQCD and DVMP framework can be achieved when choosing the DA $\Phi_{\text{NR}}(z) = \delta(z - \frac{1}{2})$. This approximation shall receive further nonrelativistic corrections, whereas the knowledge of the actual meson DA is rather limited, especially for heavy mesons like J/ψ . There have been recent developments on the calculation of heavy meson DA with lattice simulation from first principle, which was applied to the D meson with heavy-light flavor [99]. However, the meson DA for heavy-heavy flavor remains a question. For simplicity, we will use the asymptotic DA $\Phi_{\text{asym}}(z) = 6z(1-z)$ in the analysis and this will cause a potential mismatch factor, which will be $2/3$ at leading order since $\int dz \Phi_{\text{asym}}(z)/(1-z) = 3$ instead of 2 for the nonrelativistic DA $\Phi_{\text{NR}}(z)$. At NLO and beyond, this mismatch factor is not known, and thus we will parametrize it and determine it from the experiment. Consequently, the amplitude can be written as,

$$\mathcal{A}_{\text{Hyb}} = \sum_{i=q,\bar{q},g,\dots} \frac{e_q f_{J/\psi}}{N_C} \frac{Q}{Q^2 + M_{J/\psi}^2} \int_0^1 dz \int_{-1}^1 dx C^i(x, \xi, z, Q, \mu_R, \mu_f, \text{GPD}, \mu_f, \text{DA}) F^i(x, \xi, t, \mu_f, \text{GPD}) N_{\text{amp}} \Phi_{\text{asym}}(z, \mu_f, \text{DA}), \quad (21)$$

where N_{amp} stands for the mismatch factor after the substitution of asymptotic DA Φ_{asym} .

These seven parameters can then be fit to the small- x gluon PDFs taken from [50] and the DVJ/ ψ P cross section [57]. Before moving on to the actual fit, we note that the above amplitude can be extended to NLO by virtue of the previous development of the NLO Wilson coefficient and GPD evolution approaches in the literature [75–77, 97, 100, 101], which we will not repeat here. Eventually, one can write the amplitude in the form of³

³Note that a factor of C_F has been pulled out of the Wilson coefficient C_k in the conformal moment space to match with the convention in the literature, see for instance [77].

$$\mathcal{A}_{\text{Hyb}} = \frac{e_q f_{J/\psi} C_F}{N_C} \frac{Q}{Q^2 + M_{J/\psi}^2} \sum_{i=u,\bar{u},\dots} \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \xi^{-j-1} \left[i + \tan\left(\frac{\pi j}{2}\right) \right] \\ \times [C_k^{i,LO} E_{kj}^{LO} F_j^i(\xi, t) + C_k^{i,NLO} E_{kj}^{LO} F_j^i(\xi, t) + C_k^{i,LO} E_{jk}^{NLO} F_j^i(\xi, t)], \quad (22)$$

where we have only kept terms up to NLO accuracy. The scale-dependence is suppressed in the above expression for clarity. More details regarding the scale-dependence and the implementation of the GPD evolution can be found in the Appendix.

We also note that the skewness variable ξ will be modified in the presence of vector meson mass $M_{J/\psi}$, which reads

$$\xi = \frac{\tilde{x}_B}{2 - \tilde{x}_B}, \quad (23)$$

in terms of the meson mass corrected Bjorken variable

$$\tilde{x}_B = \frac{M_{J/\psi}^2 + Q^2}{W^2 + Q^2}, \quad (24)$$

with Q^2 the photon virtuality and W^2 the center-of-mass energy squared for the scattering of the virtual photon on the hadron. NLO corrections in the valence quark sector will not be considered in this work because in J/ψ production the valence contribution would require intrinsic charm content in the proton, which we do not consider for now.

At the end of the subsection, we note again that a complete matching of the DVMP and the NRQCD frameworks at NLO will be crucial for the proper understanding of the systematical uncertainties in the analysis and will be left to a separate work.

C. Input PDFs and differential cross sections and fit results

Having established the inputs and framework, we now turn to the results. We sample nine points of gluon PDF from the globally extracted set [50] at $\mu = 2$ GeV in the region $x \in [10^{-4}, 10^{-3}]$, and take the 17 points of differential cross sections from H1 [57] with $\langle Q^2 \rangle \in [7.0, 22.4]$ GeV², $-t \in [0.04, 0.64]$ GeV², and $\tilde{x}_B \in [9 \times 10^{-4}, 6 \times 10^{-3}]$ to fit the seven-parameter set $\{N^g, \alpha^g, \beta^g, R_{\xi^2}^g, R_{\xi^4}^g, N_{\text{amp}}\}$. We perform a sequential fit of N^g and α^g to the forward PDFs first, and then fit the remaining five parameters to the DVJ/ ψ P cross sections. The best-fit parameters are obtained with the MINUIT interface of Minuit2 [102,103] as the minimizer, which are listed with their statistical uncertainties from Minuit2 in Table III, with $\chi_{\text{off-forward}}^2/\text{d.o.f.} \approx 2.41$ for the off-forward fit and $\chi_{\text{tot}}^2/\text{d.o.f.} \approx 1.53$.

In Fig. 2, we compare the fitted PDFs to the samples of PDFs taken from the JAM22 global analysis [50], where the two forward parameters N^g and α^g appear to describe the small- x gluon PDFs well. Extending to the region with larger x would require a more flexible parametrization of the gluon PDFs, which will be left to future work with more inputs. In this work, we focus on the small- x gluon PDF and its impact on the DVJ/ ψ P cross-section data. Thus, the two parameters are adequate for this purpose.

In Figs. 3 and 4, we show the differential cross section for DVJ/ ψ P as a function of t and x_B , respectively, from the H1 data [57], comparing it to the predictions from the best-fit parameter values. We observe large bands obtained by scale variation of μ^2 with a factor of two, indicating potentially large corrections that are higher order of α_s , i.e., next-to-next-to-leading order (NNLO) and beyond. We note that there are measurements of the differential cross section of DVJ/ ψ P by ZEUS [92] as well. However, some slight tensions are noticed when fitting the two measurements together, and thus we pick the H1 data that has more coverage in the high- Q^2 region. Additionally, there are a few more points for discussion. First, we found the NLO corrections are indeed required to describe the data—the LO calculation will not reproduce the correct x_B and Q scaling of the data, no matter the input GPD. The same applies to the leading mass corrections of $M_{J/\psi}$ as well, where the data are found to scale according to $1/(Q^2 + M_{J/\psi}^2)^n$, which simply cannot be described by the $1/Q^n$ scaling with the DVMP framework.

TABLE III. Best-fit values for the seven parameters are listed along with their statistical uncertainties from Minuit2 with $\chi_{\text{tot}}^2/\text{d.o.f.} \approx 1.53$, where $\chi_{\text{tot}}^2 = \chi_{\text{forward}}^2 + \chi_{\text{off-forward}}^2$. We have $\chi_{\text{forward}}^2 = 0.24$ for the forward fit and $\chi_{\text{off-forward}}^2 = 28.94$ for the off-forward fit separately, where the off-forward fit yields $\chi_{\text{off-forward}}^2/\text{d.o.f.} \approx 2.41$.

Parameter	Best-fit value	Statistical uncertainty
<i>Forward fit with nine samples of gluon PDF</i>		
N^g	1.85	0.22
α^g	1.095	0.016
<i>Off-forward fit with 17DVJ/ψP cross-section data</i>		
α'^g	0.02	0.04
$R_{\xi^2}^g$	-0.59	0.27
$R_{\xi^4}^g$	0.25	0.10
b^g	1.84	0.12
N_{amp}	1.38	0.06

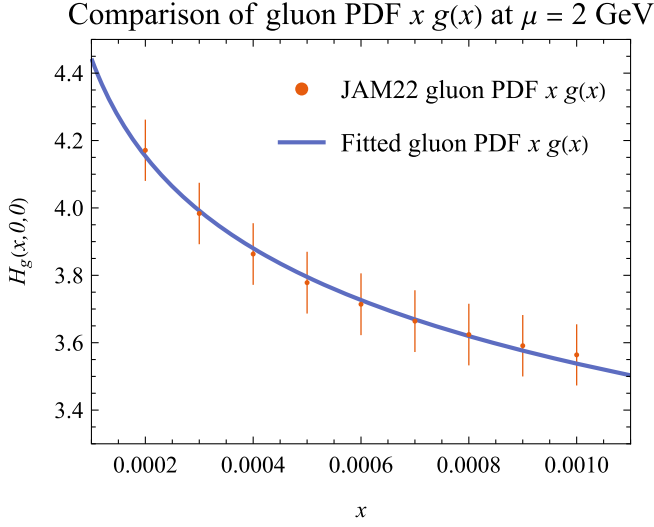


FIG. 2. Comparison of the fitted forward limit of the GPDs to the PDF data points taken from the JAM22 global analysis [50] in the region of small $x \sim 10^{-4}$ – 10^{-3} at the reference scale $\mu_0 = 2$ GeV.

In fact, strong cancelation between the LO gluon and NLO gluon contributions is found in the TFFs and final cross sections. In Fig. 5, we present the relative contributions of the LO gluon $\mathcal{H}_g^{\text{LO}}$, NLO gluon $\mathcal{H}_g^{\text{NLO}}$, and NLO quark $\mathcal{H}_q^{\text{NLO}}$ to the TFF $\mathcal{H}$, respectively, at one reference kinematic point and their evolution with the reference scale μ . Note that the NLO here excludes the LO contributions so that we write,

$$\mathcal{H} = \mathcal{H}_g^{\text{LO}} + \mathcal{H}_g^{\text{NLO}} + \mathcal{H}_q^{\text{NLO}}. \quad (25)$$

It appears that the quark fields always have relatively small contributions, whereas the gluonic contributions from NLO are rather comparable to the LO ones, as shown in the

figure on the right. Although the NLO corrections seem to be suppressed for increasing μ , as expected for a perturbative expansion.

These observations, together with the strong scale dependence shown in Figs. 3 and 4, imply that the perturbative expansion in α_s is not as well controlled in the case of meson production as in, for example, DVCS, where the LO calculation is sufficient to produce a reasonably good fit even when including both low- Q^2 JLab data and small- x_B HERA data. Such observations are consistent with what has been found in the literature [63,66,69] in various frameworks, even including the light-meson production [75,77]. Therefore, further studies on the perturbative corrections as well as nonrelativistic corrections for heavy meson productions are crucial for reducing the systematic uncertainties. We also notice that it has recently been found that the scale dependence in the exclusive photoproduction of J/ψ can be reduced by resumming higher-order QCD corrections [71]. In the case of lepton production here, we find that the best-fit predictions for DVJ/ ψ P are closer to the data as Q^2 increases, which could be an indication of better convergence at large Q^2 . However, there are not many existing high-precision data at large Q^2 , which should be one vital task of the future EIC [32].

As a cross-check, we comment that the results obtained in this work are rather consistent with [63] in the NRQCD framework at NLO, even though only simple GPD models are used there. Comparing to the results therein, we found similar large scale dependence as well as strong cancelation between LO and NLO contributions. Note that the theoretical values agree better with the data in this work, likely by virtue of the extra flexibility of ξ dependence in the GPD parametrization that has been fitted to the data, as well as the full NLO evolution of GPD that potentially helps. This agreement indicates that the hybrid framework in this work

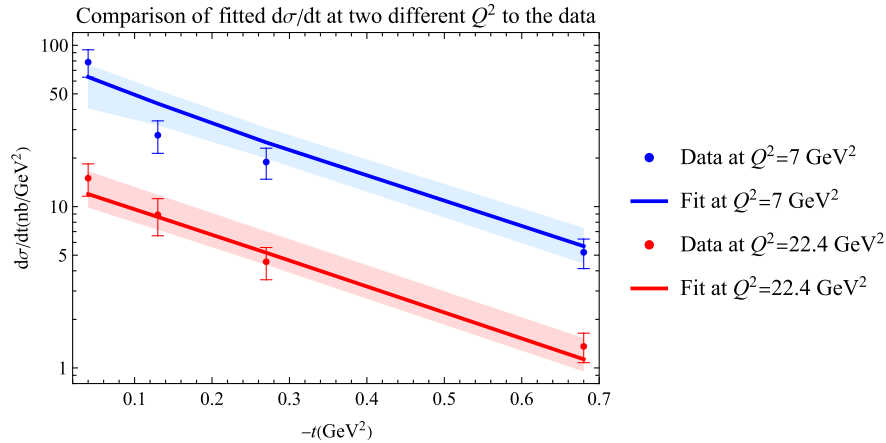


FIG. 3. Comparison of the fitted differential cross section calculated with Eq. (16) and the best-fit parameters in Table III, as a function of t against the H1 data [57] for various Q^2 and x_B ranging from 1.3×10^{-3} up to 3.2×10^{-3} . The bands are calculated by variation of scale with $\mu^2 \in [1/2, 2] \times \mu_F^2$.

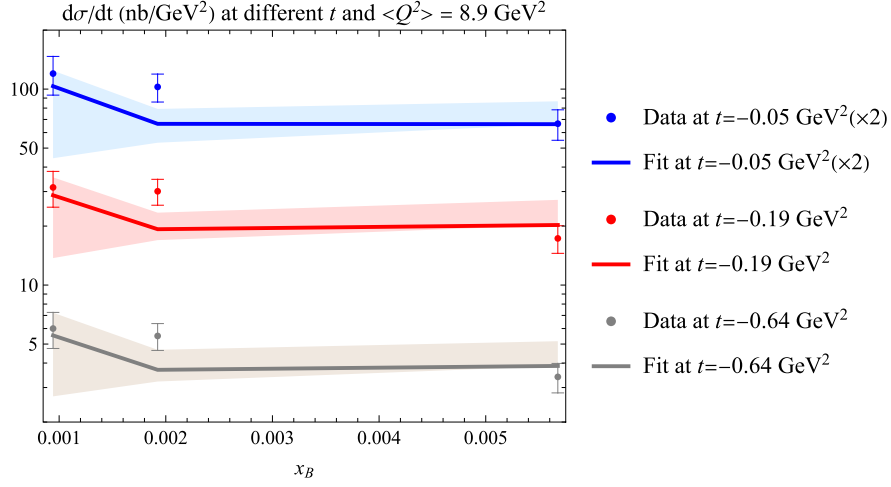


FIG. 4. Similar to the previous plot, we present the comparison of the fitted differential cross section calculated with Eq. (16) and the best-fit parameters in Table III as a function of x_B against the H1 data [57] for various values of t , with $\langle Q^2 \rangle = 8.9 \text{ GeV}^2$. Again, the bands are calculated by variation of scale with $\mu^2 \in [1/2, 2] \times \mu_F^2$. The multipliers in the parentheses have been multiplied to the cross sections to avoid overlaps.

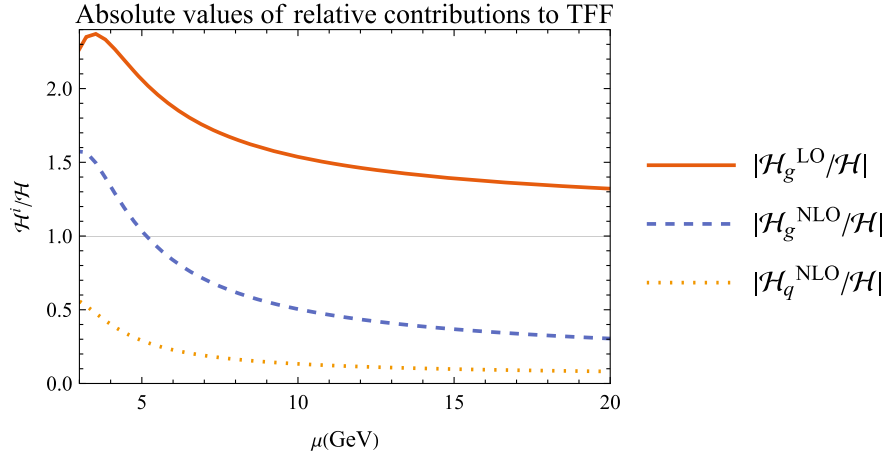


FIG. 5. Relative contributions of LO gluon $\mathcal{H}_g^{\text{LO}}$, NLO gluon $\mathcal{H}_g^{\text{NLO}}$, and NLO quark $\mathcal{H}_q^{\text{NLO}}$ contributions to the full NLO TFF $\mathcal{H}$ decomposed according to Eq. (25) at $x_B = 0.005$ and $t = -0.05 \text{ GeV}^2$. Note that at LO there is no quark contribution. The ratios are complex numbers in general, and their absolute values are presented here.

captures the main feature of NLO corrections. However, we note that even with meson mass and NLO corrections included, this hybrid framework produces a fit for the off-forward data with $\chi^2/\text{d.o.f.} \approx 2.41$, reasonably following the data within known uncertainties but not a very high quality fit. This further suggests that there are significant corrections to be discovered by properly studying the connection between the NRQCD and collinear frameworks in the kinematic regime of large Q^2 and large meson mass. We leave the study of these corrections to future work.

D. Extracted gluon GPDs and other relevant observables

Finally, we also discuss the extracted gluon GPDs themselves. It is well known that processes like DVCS

and DVMP do not provide enough constraints on the shape of the GPDs themselves, as discussed in the context of shadow GPDs [13]. Therefore, all extractions of the GPDs from those process alone without extra off-forward input cannot fully determine the shape of GPDs. Despite that, we present the shape of the gluon GPDs obtained from this extraction for reference in Fig. 6. In the right figure, it is shown that the gluon GPDs approaches the gluon PDFs in the limit $x \gg \xi$ as expected. On the other hand, strong oscillation is shown in the DA-like region of the GPD when $-\xi < x < \xi$, as shown in the left figure. This behavior has been extensively discussed in the previous work for the quark GPDs [47], which can be considered the artifacts of the conformal moment parametrization. A more careful treatment can be used to tune the GPDs in the DA-like region. However, in the small- ξ limit, the DA-like region

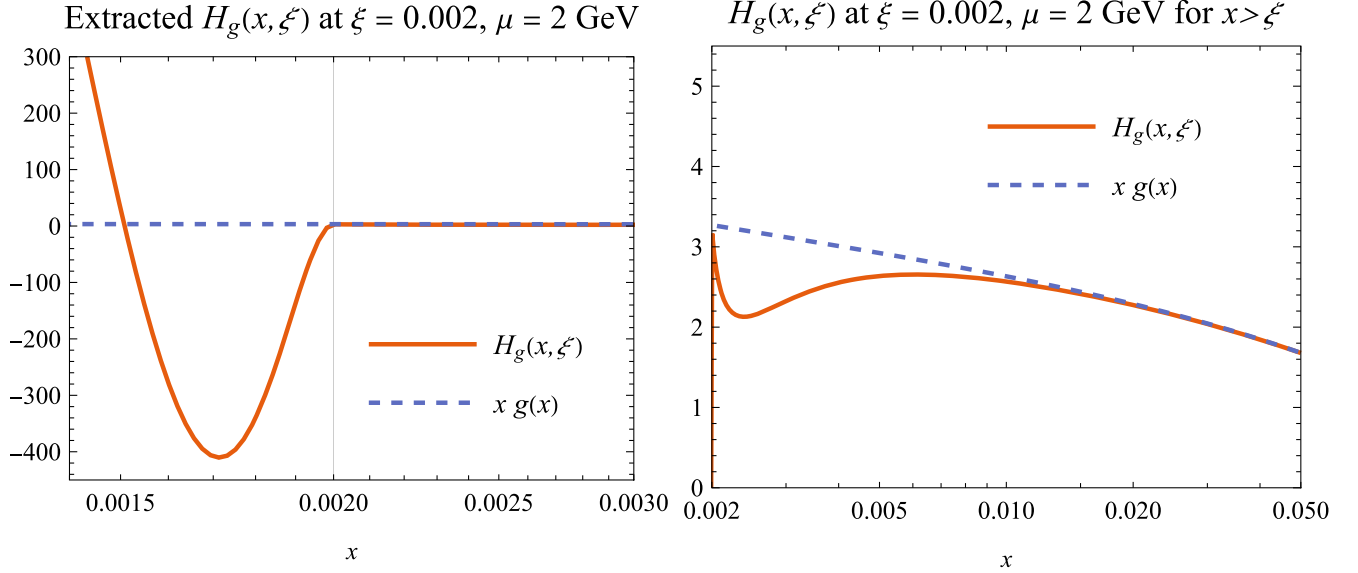


FIG. 6. Extracted gluon GPD $H_g(x, \xi, t=0)$ at $\xi = 0.002$ and reference scale $\mu = 2$ GeV. The left plot shows the oscillating behavior in the DA-like region, and the right plot shows that GPDs reduce to PDFs when $x \gg \xi$ in the PDF-like region with $t = 0$.

becomes singular itself and therefore, those behaviors might not have too much physical impact anyway.

Another interesting feature worth discussing is the ξ dependence of gluon GPDs obtained in this framework. Due to the lack of input of the ξ dependence, it is commonly assumed that the amplitudes can be approximated by the corresponding PDFs $H_g(x, \xi = 0)$ up to some factors for skewness corrections. However, the skewness dependence is likely to be more complicated and more importantly, it will evolve with the renormalization scale μ . Thus, our extraction here provides an opportunity to study such effects, even though we note that it will be model dependent. In Fig. 7, we present the x and μ dependence of the skewness ratio or the so-called R ratio defined by,

$$R_g(x, \mu) \equiv \frac{H_g(x, \xi = x, 0, \mu)}{H_g(x, 0, 0, \mu)}. \quad (26)$$

As clearly shown in Fig. 7, as $x \rightarrow 0$ and $\mu \rightarrow \infty$, the skewness ratio somewhat stabilizes around the value $R_g(x, \mu) \sim 1$ that approaches unity. However, for the majority values of x and μ , the $R_g(x, \mu)$ deviates from unity. This indicates the skewness effect even for the kinematics of the H1 data, which has $\xi \sim 10^{-4}$. We note that in another recent work [77], the DVCS and DVMP measurements at HERA including only light meson productions have been studied at NLO in the GPD framework, based on a similar conformal moment framework [75]. The skewness ratio is found to be around 1.1 therein, which is larger than 1. Nevertheless, we note that the extraction of the ratio $R_g(x, \mu)$ is rather model dependent—the ξ dependence originates from both the ξ dependence of the moments and the scale evolution of GPDs. While the latter does not depend on the GPD parametrization, the former does. Therefore, only in the large- Q and small- x limit, the evolution effect dominates, and universal skewness ratios shall be obtained. This is consistent with both the observation here and that of the previous work where the gluonic skewness ratio approaches around unity in such a limit, which has also been suggested based on the Shuvaev transform method [104,105]. Similar skewness effects have also been seen in small- x frameworks [106,107] and it appears that the off-forward physics separating GPDs from their PDF limits is nontrivial even for very near-forward scattering.

In the end, we comment on the fitting parameter and the model/extraction-dependence of the results. In this work, we aim to bring together the forward small- x gluon PDFs

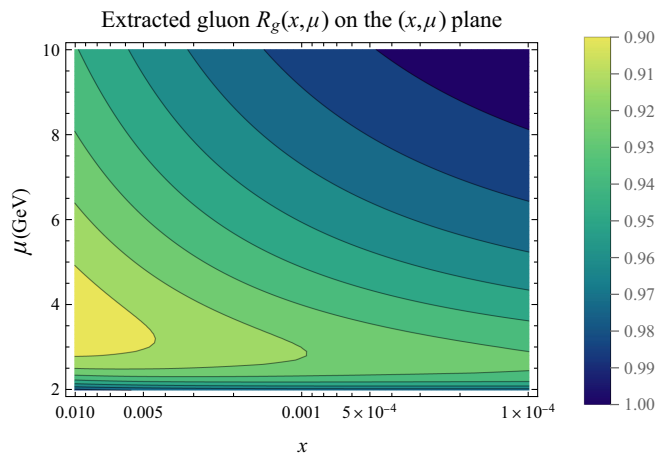


FIG. 7. Skewness ratio $R_g(x, \mu)$ from the extracted gluon GPD shown on the (x, μ) plane in the region covered by the data and extrapolated to large μ .

and the $DVJ/\psi P$ cross-section data in the GPD framework at the NLO. In the forward region, the two parameters N_g and α_g are flexible enough for the small- x gluon PDF. More flexibility is definitely needed to extend the analysis to a larger region to include the larger- x sector of gluon PDFs, as well as lattice calculations of the gluon gravitational form factors [39,40]. However, at this point, we still have to carefully balance the sophistication of our ansatz in the forward limit against the availability of off-forward data inputs.

As for the off-forward parameters, two parameters for the t dependence, b_g and α'_g and another two parameters for the ξ dependence, $R_{\xi^2}^g$ and $R_{\xi^4}^g$ have been used in the ansatz. Although the actual t dependence and ξ dependence can be much more complicated than this, we do see the four parameters reproduce the t and x_B dependence of the data well. One thing worth noting is the fixed quark parameters throughout this analysis, as we assume that the quark contributions are suppressed, at least in the limit $\mu \rightarrow \infty$. While our results in Fig. 5 seem to confirm such behaviors, it should be worthy to consider a simultaneous fit of quark and gluon distributions from combined input to further justify this assumption, which will be left to a future work. The last parameter is the normalization of the amplitude N_{amp} , which depends on the choice of the meson DA $\Phi(z)$. From the fit, we obtain 1.42 ± 0.14 , which is reasonably close to the LO values $2/3$, noting that it strongly depends on the choice of the factorization scale. Since the results receive potentially large perturbative corrections as discussed before, this factor should be more carefully studied by a complete matching of the two frameworks at NLO.

IV. CONCLUSIONS AND OUTLOOK

To conclude, we study the gluon GPDs with $DVJ/\psi P$ process, employing a hybrid framework that combines NLO collinear factorization with LO heavy J/ψ mass corrections from NRQCD. We find that the differential cross section of the $DVJ/\psi P$ measurement by H1 [57] can be described with the gluon GPDs after including the NLO perturbative corrections and LO heavy mass corrections, when simultaneously fitting to the forward gluon PDFs obtained from global analysis [50]. We study the scale dependence as well as the contributions of different order to the amplitudes, and find significant contributions from the gluon at NLO that cancel the LO contributions, as well as strong scale dependence even at NLO, indicating large perturbative corrections that should be further studied. These observations agree with the previous work that studies the exclusive J/ψ leptonproduction in the NRQCD framework at NLO with simple GPD models [63].

Additionally, we present the gluon GPDs as well as the skewness ratio based on the extraction of this work, though the extraction of GPDs based on such exclusive meson productions alone suffers from the inverse problem and

therefore would depend on the specific model. Our extracted skewness ratio appears to be consistent with another extraction from combined DVCS and DVMP analysis with light meson productions at NLO [77], whereas similar skewness effects have been seen in other frameworks as well [107].

For future development, more careful studies to match the collinear DVMP and NRQCD framework are crucial to systematically examine the NLO corrections as well as to better understand the effect of the mismatch factor N_{amp} . Besides, including other light-meson production as well as DVCS measurements altogether at NLO would allow us to better determine the quark and gluon GPDs simultaneously, so that their contributions, especially at NLO, can be better understood. In addition, the GUMP framework also enables us to extend to the larger- x region, so that a global analysis that includes gluon PDFs in larger- x region and gluonic form factors from lattice [39,40] can be achieved.

This research benefits from the open-source Gepard package [108].

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DATA AVAILABILITY

The data that support the findings of this article are openly available [109].

APPENDIX: SETUP FOR THE NLO HARD COEFFICIENTS AND EVOLUTION

In this appendix, we present the details of the implementation of the NLO hard coefficients and evolutions. In Eq. (17), we show that the GPDs are expanded in terms of their conformal moments. As shown in Eq. (21), the DVMP amplitudes are given by a double convolution of GPDs with the Wilson coefficients $C^g(x, \xi, z, Q, \mu)$ as well as the meson distribution amplitude $\Phi_{\text{asym}}(z, \mu)$. In moment space, the double convolution will be turned into a double summation in the form of,

$$\mathcal{A}_{\text{Hyb}} = \frac{e_q f_{J/\psi} C_F}{N_C} \frac{Q}{Q^2 + M_{J/\psi}^2} \sum_{i=q,\bar{q},g} \sum_{j=0}^{\infty} \sum_{l=0}^{\infty} [1 \mp (-1)^j] \xi^{-j-1} C_{jl}^i(Q, \mu_R, \mu_{f,\text{GPD}}, \mu_{f,\text{DA}}) F_j^i(\xi, t, \mu_{f,\text{GPD}}) \Phi_l(\mu_{f,\text{DA}}), \quad (\text{A1})$$

where $F_j^i(\xi, t, \mu_{f,\text{GPD}})$ and $\Phi_l(\mu_{f,\text{DA}})$ are the GPD and DA in conformal moment space. The sign $\mp$ is $-$ for vector GPDs and $+$ for axial-vector GPDs. The asymptotic DA $\Phi^{\text{asym}}(z, \mu)$ contains the leading conformal moment only: $\Phi_l^{\text{asym}}(\mu) = \delta_{l0} \Phi_0^{\text{asym}}(\mu)$.

For simplicity, we set all the factorization and renormalization scales to be the same: $\mu = \mu_{f,\text{DA}} = \mu_{f,\text{GPD}} = \mu_R$, though the choice of μ appears to be a bit subtle. It has been suggested in [63,110] that the scale of a single quark rather than that of the quark pair can be chosen to suppress the higher-order perturbative corrections in the high-energy limit. Namely, the leading double logs are observed to be in the form of $\log(\frac{Q^2 + M_{J/\psi}^2}{4\mu_f^2}) \log(\xi)$. In the DVMP framework, the large logs are in the form of $\log(Q^2/\mu^2)$, and thus we set $\mu = Q$ accordingly to suppress the higher-order perturbative corrections. We note that the results will strongly depend on the choice of scale, which shall be further studied.

$$\mathcal{A}_{\text{Hyb}} = \frac{e_q f_{J/\psi} C_F}{N_C} \frac{Q}{Q^2 + M_{J/\psi}^2} \sum_{i=q,\bar{q},g} \sum_{j=0}^{\infty} [1 \mp (-1)^j] \xi^{-j-1} C_{j0}^i F_j^i(\xi, t, \mu) \Phi_0^{\text{asym}}(\mu), \quad (\text{A2})$$

$$\mathcal{A}_{\text{Hyb}} = \frac{e_q f_{J/\psi} C_F}{N_C} \frac{Q}{Q^2 + M_{J/\psi}^2} \sum_{i,i'=q,\bar{q},g} \sum_{j=0}^{\infty} \sum_{k=0}^j [1 \mp (-1)^j] \xi^{-j-1} C_{j0}^i \left[\frac{1 \mp (-1)^k}{2} \right] E_{jk}^{ii'}(\xi, \mu, \mu_0) F_k^{i'}(\xi, t, \mu_0) \Phi_0^{\text{asym}}. \quad (\text{A4})$$

Again, the signs $\mp$ are $-$ for vector GPDs and $+$ for axial-vector GPDs. The Wilson coefficients C_{j0}^i and the evolution operators $E_{jk}^{ii'}(\xi, \mu, \mu_0)$ and $E_{\Phi_0}(\mu, \mu_0)$ are known to the NLO [97,100,101], whereas the GPD $F_k^i(\xi, t, \mu_0)$ and the DA $\Phi_0^{\text{asym}}(\mu_0)$ are parametrized at the reference scale μ_0 . Thus, the amplitude can in principle be evaluated by performing the double summation, which, however, is more involved than it appears.

1. Resummation of moments with Mellin-Barnes integral

As discussed in the main text as well as many previous developments on conformal moment construction of GPDs, the resummation in moment space is typically divergent and requires proper analytic treatment. The common one in the literature is the Mellin-Barnes integral, which basically states,

$$\sum_{j=j_0}^{\infty} (-1)^j f_j = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} \frac{1}{\sin(\pi j)} f_j, \quad (\text{A5})$$

where the GPD and DA are evolved to the scale μ accordingly. Note that the asymptotic DA corresponds to the asymptotic behavior of DA in the $\mu_f \rightarrow \infty$ limit, and thus its scale dependence is trivial such that we can write it as Φ_0^{asym} for simplicity, which could be recovered if necessary.⁴ On the other hand, the evolution of GPD is more complicated, which can be written in terms of the evolution operator $E_{jk}^{ii'}(\xi, \mu, \mu_0)$ as,

$$F_j^i(\xi, t, \mu) = \sum_{i'=q,\bar{q},g} \sum_{k=0}^j \left[\frac{1 \mp (-1)^k}{2} \right] E_{jk}^{ii'}(\xi, \mu, \mu_0) F_k^{i'}(\xi, t, \mu_0). \quad (\text{A3})$$

Note that the evolution operator includes nontrivial mixing in both the flavor ($i = q, \bar{q}, g$) space and the conformal moment (j) space. Therefore, we can write the amplitude as,

where c is a real number that satisfies $j_0 - 1 < c < j_0$. Requiring that f_j is analytic when $\text{Re } j > c$ and falls fast enough at infinite, the identity can be proven simply with the residue theorem after analytical continuation. This also indicates similar variants,

$$\sum_{j=j_0}^{\infty} [1 - (-1)^j] f_j = -\frac{1}{2i} \int_{c-i\infty}^{c+i\infty} \tan(\pi j/2) f_j, \quad (\text{A6})$$

$$\sum_{j=j_0}^{\infty} [1 + (-1)^j] f_j = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} \cot(\pi j/2) f_j, \quad (\text{A7})$$

which can be used to resum the double summation in Eq. (A4). Also note that since the second summation in

⁴The asymptotic DA is invariant under LO evolution, whereas the NLO evolution effects are negligible. Here we ignore the NLO evolution of DA, which is also partially accounted in the N_{amp} , so the scale dependence of DA becomes trivial.

k only sums over $0 \leq k \leq j$, not to infinity, one can reshuffle the double summation to show that

$$\sum_{j=0}^{\infty} \sum_{k=0}^j F_{jk} = \sum_{k=0}^{\infty} \sum_{j=k}^{\infty} F_{jk} = \sum_{j=0}^{\infty} \sum_{k=j}^{\infty} F_{kj}. \quad (\text{A8})$$

Therefore, we have alternatively for Eq. (A4) [97],

$$\mathcal{A}_{\text{Hyb}} = \frac{e_q f_{J/\psi} C_F}{N_C} \frac{Q}{Q^2 + M_{J/\psi}^2} \sum_{i,i'=q,\bar{q},g} \sum_{j=0}^{\infty} \sum_{k=j}^{\infty} [1 \mp (-1)^k] \xi^{-k-1} C_{k0}^i \left[\frac{1 \mp (-1)^j}{2} \right] E_{kj}^{ii'}(\xi, \mu, \mu_0) F_j^{i'}(\xi, t, \mu_0) \Phi_0^{\text{asym}}. \quad (\text{A9})$$

In both Eqs. (A4) and (A9), the second summation of k should be performed before the j summation. However, the meaning of k has been switched due to the change of variable. In the former one in Eq. (A4), k refers to the k th moment of the unevaluated GPD, whereas in the latter one in Eq. (A9), k refers to the k th moment of the Wilson coefficients. Accordingly, the two different orders of performing the double summation will be referred to as the moment-evolution method and the coefficient-evolution method.

Although the two methods should produce the same results, both setups are needed in the actual implementation of NLO evolution. During the fit, one needs to change the parameters of GPDs and thus the GPD moments frequently. Thus, it will be much more efficient to precompute the evolved Wilson coefficients and sum them with the moments afterward. On the other hand, for the calculation of, e.g., best-fit GPDs, after the fit, the moments of GPDs will be fixed whereas different x and ξ will be put in. Then the moment-evolution method will be the more reasonable choice. Actually, we found it extremely difficult to perform the analysis otherwise—both fitting with the moment-evolution method and computing GPDs with the coefficient-evolution method seem too slow for any practical purpose. More details will be discussed in the following subsection.

2. Implementation and comparison of the two methods

Finally, we discuss how the two methods are numerically implemented. We first note that the NLO evolution has

been implemented in the Gepard package [108]. Our work is based on the existing Gepard development, and aims to provide a complete implementation of the NLO evolution including both methods. Before getting to the details, we first note that there are two useful properties of the evolution kernel for the implementation. First, the ξ dependence of the evolution kernel can be factorized at least to the NLO as,

$$E_{jk}^{ii'}(\xi, \mu, \mu_0) = \xi^{j-k} E_{jk}^{ii'}(1, \mu, \mu_0). \quad (\text{A10})$$

Therefore, we define $\bar{E}_{jk}^{ii'}(\mu, \mu_0) = E_{jk}^{ii'}(1, \mu, \mu_0)$ that is independent of ξ . Second, the evolution kernel can be split into diagonal and off-diagonal parts,

$$\bar{E}_{jk}^{ii'}(\mu, \mu_0) = \delta_{kj} A_j^{ii'}(\mu, \mu_0) + B_{jk}^{ii'}(\mu, \mu_0), \quad (\text{A11})$$

where the diagonal term $A_j^{ii'}(\mu, \mu_0)$ shows up at LO, while the $B_{jk}^{ii'}(\mu, \mu_0)$ is NLO and beyond. The double summation involving the diagonal term reduces to a simple single summation, which can be treated in the same way as in the LO case. Therefore, we focus on the off-diagonal piece associated with the $B_{jk}^{ii'}(\mu, \mu_0)$.

In Eq. (A4) for the moment-evolution method, we have the double summation in the form of,

$$\begin{aligned} & \sum_{j=0}^{\infty} \sum_{k=0}^j [1 - (-1)^j] \xi^{-j-1} C_{j0}^i \left[\frac{1 - (-1)^k}{2} \right] E_{jk}^{ii'}(\xi, \mu, \mu_0) F_k^{i'}(\xi, t, \mu_0) \\ &= \sum_{j=0}^{\infty} \sum_{k=0}^j [1 - (-1)^j] \xi^{-k-1} C_{j0}^i \left[\frac{1 - (-1)^k}{2} \right] [\delta_{jk} A_j^{ii'}(\mu, \mu_0) + B_{jk}^{ii'}(\mu, \mu_0)] F_k^{i'}(\xi, t, \mu_0), \end{aligned} \quad (\text{A12})$$

where we ignore the prefactors that are independent of j and k for clarity. Then the double summation of the off-diagonal term can be converted into the following double integral [111]:

$$\begin{aligned} & \sum_{j=1}^{\infty} \sum_{k=1}^j [1 - (-1)^j] \xi^{-k-1} C_{j0}^i \left[\frac{1 - (-1)^k}{2} \right] B_{jk}^{ii'}(\mu, \mu_0) F_k^{i'}(\xi, t, \mu_0) \\ & \rightarrow \frac{1}{2i} \int_{c_j-i\infty}^{c_j+i\infty} dj \left[i + \tan\left(\frac{\pi j}{2}\right) \right] \xi^{-j-1} C_j^i \int_{c_k-i\infty}^{c_k+i\infty} \frac{dk}{4i} \cot\left(\frac{\pi k}{2}\right) (\xi^{-k} B_{j,k+j}^{ii'} \mathcal{F}_{k+j}^{i'} - \xi^{j-k-1} B_{j,k+1}^{ii'} \mathcal{F}_{k+1}^{i'}), \end{aligned} \quad (\text{A13})$$

where $c_j < 2$ and $c_k < 0$. In the above expression, we explicitly exclude the contributions from the $k = 0$ terms which can be added to the results later. The benefit of doing so is to avoid the pole in the GPD moment F_k as well as the poles in the anomalous dimensions in the evolution kernel. Those poles must be avoided so that the contour integral is properly defined. Similarly, for the coefficient-evolution method, the following double integral representation can be derived:

$$\begin{aligned} & \sum_{j=1}^{\infty} \sum_{k=j}^{\infty} [1 - (-1)^k] \xi^{-j-1} C_{k0}^i \left[\frac{1 - (-1)^j}{2} \right] B_{kj}^{ii'}(\mu, \mu_0) F_j^{i'}(\xi, t, \mu_0) \\ & \rightarrow \frac{1}{2i} \int_{c_j-i\infty}^{c_j+i\infty} dj \left[i + \tan\left(\frac{\pi j}{2}\right) \right] \xi^{-j-1} \mathcal{F}_j^{i'} \int_{c_k-i\infty}^{c_k+i\infty} \frac{dk}{4i} \tan\left(\frac{\pi k}{2}\right) C_{k+j+1}^i B_{k+j+1,j}^{ii'}, \end{aligned} \quad (\text{A14})$$

with $c_j < 1$ and $c_k < 0$. Again, the contributions from $j = 0$ terms are removed in the above expression and will be added back later to avoid the poles for the same reason.

Such techniques also apply to the case of axial-vector GPDs, which replaces $[1 - (-1)^{j,k}]$ with $[1 + (-1)^{j,k}]$, and we have,

$$\begin{aligned} & \sum_{j=1}^{\infty} \sum_{k=1}^j [1 + (-1)^j] \xi^{-k-1} C_{j0}^i \left[\frac{1 + (-1)^k}{2} \right] B_{jk}^{ii'}(\mu, \mu_0) F_k^{i'}(\xi, t, \mu_0) \\ & \rightarrow \frac{1}{2i} \int_{c_j-i\infty}^{c_j+i\infty} dj \left[i - \cot\left(\frac{\pi j}{2}\right) \right] \xi^{-j-1} C_j^i \int_{c_k-i\infty}^{c_k+i\infty} \frac{dk}{4i} \left\{ \tan\left(\frac{\pi k}{2}\right) \xi^{j-k-1} B_{j,k+1}^{ii'} \mathcal{F}_{k+1}^{i'} + \cot\left(\frac{\pi k}{2}\right) \xi^{-k} B_{j,k+j}^{ii'} \mathcal{F}_{k+j}^{i'} \right\}, \end{aligned} \quad (\text{A15})$$

with $c_j < 2$ and $c_k < 0$. And for the coefficient-evolution method, we have

$$\begin{aligned} & \sum_{j=1}^{\infty} \sum_{k=j}^{\infty} [1 + (-1)^k] \xi^{-j-1} C_{k0}^i \left[\frac{1 + (-1)^j}{2} \right] B_{kj}^{ii'}(\mu, \mu_0) F_j^{i'}(\xi, t, \mu_0) \\ & \rightarrow \frac{1}{2i} \int_{c_j-i\infty}^{c_j+i\infty} dj \left[i - \cot\left(\frac{\pi j}{2}\right) \right] \xi^{-j-1} \mathcal{F}_j^{i'} \int_{c_k-i\infty}^{c_k+i\infty} \frac{dk}{4i} \tan\left(\frac{\pi k}{2}\right) C_{k+j+1}^i B_{k+j+1,j}^{ii'}, \end{aligned} \quad (\text{A16})$$

with $c_j < 1$ and $c_k < 0$. Note that all $\tan(\pi j/2)$ are replaced by $-\cot(\pi j/2)$ just like the diagonal case, whereas the replacement of $\tan(\pi k/2)$ and $\cot(\pi k/2)$ is highly nontrivial due to the redefinition of variable in the second integral. These two methods have been numerically checked to produce the same amplitudes/GPDs with NLO evolution. The code has been updated online [109].

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