Gauge coupling unification and doublet-triplet splitting via GUT dynamical breaking

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An interesting framework to achieve gauge coupling unification consists in adding to the Standard Model Lagrangian nonrenormalizable operators of $d \geq 5$, which affect the kinetic term of gauge fields. We first review the phenomenology related to this framework in the context of $SU(5)$, identifying which are the most interesting representations for the sake of achieving coupling unification. Second, we point out that in the case of a dynamical breaking pattern, it is possible to relate gauge coupling unification with the doublet-triplet splitting problem. We show that condensates of fermions in the 5 representation do not lead to viable models because of proton decay constraints. At difference, we point out that successful models can be obtained by considering condensates of fermions in the 10, as well as in the 24 representations.

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I. INTRODUCTION

The beta functions of the Standard Model (SM) of particle interactions are such that gauge coupling unification (GCU) is slightly, but unavoidably, missed. In a previous related work, Ref. [1], we discussed a simple general parametrization for the new physics corrections leading to full unification at some scale M_X. We showed that for any new physics model such that the corrections to the non-Abelian couplings are equal (or nearly so), M_X is equal (or close to) the partial unification scale of the SM non-Abelian running gauge couplings, $\mu_{32}^{\rm SM} \approx 2.8 \times 10^{16}$ GeV. The latter scales can be disentangled only if the corrections to the non-Abelian couplings are significantly different.

We already explored in [1] how the parametrization works for some relevant models without a desert up to the scale M_X, as low energy supersymmetry (SUSY), and many others. For models with a desert, GCU relies on corrections at M_X which are instead of ultraviolet (UV) origin. Examples of UV origin corrections include: string inspired corrections [2,3], also explored in [1], and effective corrections induced by an additional nonrenormalizable kinetic term in the Lagrangian [4–7]. In the present work we complete the analysis performed in [1], by focusing on the latter case, whose natural embedding is within a grand unified theory (GUT) [8].

In a GUT framework, gauge couplings near the unification scale can receive corrections, that are mainly of two kinds: (i) corrections related to the mass spectrum of GUT particles, and (ii) the previously mentioned corrections of UV origin.

(i) As for (i), gauge couplings can receive threshold corrections from GUT particles if the latter have masses below the GCU scale, as their presence modifies the beta functions. These corrections are in general subdominant and cannot alone account for GCU. For an example of this approach see, e.g., [9,10].

(ii) As for (ii), gauge couplings can be modified as an effect of nonrenormalizable $d \geq 5$ operators which affect the kinetic term of gauge fields [4–7]. They might arise by embedding the GUT in a larger one, or because of gravitational effects. In any case, the suppression scale of the nonrenormalizable operators has to be associated with the GUT's cutoff scale, Λ (for instance, identified with the Planck scale, M_P, in [4], or with the compactification scale in [5]). These UV origin corrections, which alone might account for GCU, are expected to be small and one would in general expect them to be treated perturbatively.

In this work, we first reconsider the scenario (2) in the framework of $SU(5)$ and in relation with the parametrization proposed in [1]. Second, we discuss its relation with another relevant issue of GUT, that is the doublet-triplet

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splitting (DTS) problem. In particular, we propose that, in the case that the GUT breaking is dynamical, GCU and DTS are related and can be both addressed successfully.

Let us now summarize the status of the art about GCU from nonrenormalizable $d \geq 5$ operators. Since 1984, it has been pointed out, Refs. [4–7], that $d = 5$ operators involving Higgs fields responsible for breaking $SU(5)$ into the SM do affect GCU, and that GCU can be achieved in this way; the related problem of proton decay was also considered. The pioneering studies focused on the 24 representation, including Ref. [11], where the role of an effective Planck scale was emphasized; in Ref. [12] also $d = 6$ operators where studied; Refs. [13,14] extended the calculation to the 75 and 200 representations. These previous numerical analyses have however not been conclusive about the emerging GUT pattern.

In the present work, with a bottom-up attitude, we begin by reconsidering the latter issue, and find that an analytical approach helps in grasping which are the most interesting directions that are supported by phenomenological data, and which are the most relevant ones for model building. In particular, we study and highlight the differences between the scenarios in which the breaking of $SU(5)$ is realized by means of the 24 representation, or rather the 75 and 200 representations. For instance, as a phenomenologically remarkable scenario, we discuss the one in which both the 24 and 75 are present and, as an effect of the relative magnitude of their vacuum expectation values (VEVs), GCU is achieved at the same scale where the SM non-Abelian gauge couplings unify.[1] We provide a short discussion on the issue of proton decay for the scenarios that we find more interesting.

We then investigate the possible relation between GCU and DTS. In the case that the $SU(5)$ breaking is realized via a Higgs mechanism, there is actually no relation. On the contrary, we point out that in the case of a dynamical breaking of $SU(5)$, it is possible to relate the two issues, and even simultaneously address them successfully. We find that the most interesting possibilities are those where the effective representations involved in the $d = 5$ operator arise from a fermion condensate with fermions sitting in the 10 or 24 representations; the case of the 5 representation being ruled out by proton decay constraints.

The paper is organized as follows. In Sec. II we review and update the analysis of GCU from $d = 5$ operators in an $SU(5)$ context. Section III is devoted to the analysis of some relevant scenarios, also in connection with proton decay. In Sec. IV we discuss the relation between GCU and DTS, which follows as a consequence of dynamical breaking. The phenomenology of the latter scenario is studied in detail in Sec. V. Conclusions are drawn in Sec. VI. Appendix A contains technical material about group theory, useful to clarify the calculations presented in the text. Appendix B deals with a useful analytical approximation to study the GCU corrections induced by $d = 5$ kinetic operators. Finally, in Appendix C a toy UV model for nonrenormalizable operators is proposed.

II. GCU FROM $d = 5$ OPERATORS IN $SU(5)$

We consider an $SU(5)$ GUT [8] with fermion matter fields in the 10_F and $\bar{5}_F$ representations. The gauge fields are in the adjoint representation, 24_G. As for scalar fields, the SM Higgs doublet is assumed to be in the 5_H, and its conjugate in the $\bar{5}_H$. For a review of the notation, see Appendix A.

The kinetic term of the gauge bosons is proportional to $\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu})$, where $G_{\mu\nu} \equiv G^A_{\mu\nu}\lambda_A$ is the field strength tensor, λ_A are the generators of $SU(5)$ in the adjoint representation, and the sum over A is understood. From the group theoretical point of view, $G_{\mu\nu}G^{\mu\nu}$ is the symmetric part of the product of two adjoints, that is $(24 \times 24)_s = 1_s + 24_s + 75_s + 200_s$. Taking the trace amounts to considering the singlet 1_s.

As pioneered in [4–6], a correction to the gauge couplings is induced by adding to the standard kinetic term the nonrenormalizable $d = 5$ operator

$$\mathcal{L}_G + \delta\mathcal{L}_G = -\frac{1}{4}\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}) - \frac{1}{4}\sum_r \frac{c_r}{\Lambda}\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu}H_r), \tag{2.1}$$

where the Higgs H_r is a scalar field in the r-dimensional irreducible representation (irrep) of $SU(5)$, for which the allowed values are: $r = 1, 24, 75, 200$. Notice also that H_r might be a propagating field, as well as a nonpropagating one in an effective theory[2]; for the sake of the following analysis, the possibility of having propagating or effective representations makes no difference. Indeed, in any case H_r will acquire a VEV, spontaneously or dynamically breaking the GUT (more on this later).

The $d = 5$ operator in (2.1) leads to the modifications in the kinetic terms of the SM gauge fields

$$-\frac{1}{4}(1+\epsilon_1)(F_{\mu\nu}F^{\mu\nu})_{U(1)} - \frac{1}{2}(1+\epsilon_2)(F_{\mu\nu}F^{\mu\nu})_{SU(2)} - \frac{1}{2}(1+\epsilon_3)(F_{\mu\nu}F^{\mu\nu})_{SU(3)}. \tag{2.2}$$

Upon gauge fields redefinitions leading to canonical kinetic terms, the relations expressing GCU at the scale $\mu = M_X$ become

[1] Since the latter scale is close to the typical unification scale in the framework of low energy supersymmetry, this scenario can be dubbed "mirage SUSY" (mS), as proposed in Ref. [1].

[2] For instance, effective representations are obtained from the product $10 \times \overline{10} = 1 + 24 + 75$.

$$\alpha_G = (1+\epsilon_1)\alpha_1(M_X) = (1+\epsilon_2)\alpha_2(M_X) = (1+\epsilon_3)\alpha_3(M_X), \tag{2.3}$$

where the SM running gauge couplings $\alpha_s(\mu)$, with $s = 1, 2, 3$, are obtained via RGE running from those measured at low energy. For the irreps in Eq. (2.1), the corrections in Eq. (2.3) are given by the sum of four possible contributions [13,14]

$$\epsilon_s = \sum_r \epsilon_s^{(r)}, \tag{2.4}$$

where $s = 1, 2, 3$ and the following relations (see for instance Table 2 of Ref. [14]),

$$\epsilon_1 = \alpha+\beta+\gamma+\delta, \qquad \epsilon_2 = \alpha+3\beta-\frac{3}{5}\gamma+\frac{1}{5}\delta,$$
$$\epsilon_3 = \alpha-2\beta-\frac{1}{5}\gamma+\frac{1}{10}\delta. \tag{2.5}$$

hold for[3]

$$\alpha \equiv \epsilon_1^{(1)} = \epsilon_2^{(1)} = \epsilon_3^{(1)}$$
$$\beta \equiv \epsilon_1^{(24)} = \frac{1}{3}\epsilon_2^{(24)} = -\frac{1}{2}\epsilon_3^{(24)}$$
$$\gamma \equiv \epsilon_1^{(75)} = -\frac{5}{3}\epsilon_2^{(75)} = -5\epsilon_3^{(75)}$$
$$\delta \equiv \epsilon_1^{(200)} = 5\epsilon_2^{(200)} = 10\epsilon_3^{(200)}. \tag{2.6}$$

As for the absolute size of the corrections, we have to define quantities related to $SU(5)$ breaking. For instance, for the VEVs of H_1 and H_{24}, we can exploit a 5×5 matrix representation,

$$\langle H_1\rangle = v_1 \mathbb{1}_5, \qquad \langle H_{24}\rangle = v_{24}\,\mathrm{diag}(\mathbb{1}_3, -3/2\cdot\mathbb{1}_2), \tag{2.7}$$

so that the breaking to the SM is achieved provided $v_{24}\neq 0$. According to our notations, as discussed in Appendix A, the corrections to, e.g., ϵ_3 can be cast in the form

$$\epsilon_3^{(r)} = \frac{c_r v_r}{\Lambda}, \tag{2.8}$$

for any possible value of $r = 1, 24, 75, 200$.

A. Our method

Here we reconsider the phenomenological analysis about achieving GCU from a nonrenormalizable kinetic term. Such an issue can be addressed and solved exactly, as well as in a semianalytical way.

[3]We find it useful this notation, as the increasing alphabetic ordering of the Greek letters corresponds to increasing the dimension of the irreps.

Let us start by putting Eq. (2.3) in the equivalent form

$$1 = \frac{1+\alpha+3\beta-\frac{3}{5}\gamma+\frac{1}{5}\delta}{1+\alpha+\beta+\gamma+\delta}\frac{\alpha_2(M_X)}{\alpha_1(M_X)} = \frac{1+\alpha-2\beta-\frac{1}{5}\gamma+\frac{1}{10}\delta}{1+\alpha+\beta+\gamma+\delta}\frac{\alpha_3(M_X)}{\alpha_1(M_X)}, \tag{2.9}$$

which emphasizes the role of ratios of the SM running gauge couplings,

$$f_{21}(\mu) \equiv \frac{\alpha_2(\mu)}{\alpha_1(\mu)}, \qquad f_{31}(\mu) \equiv \frac{\alpha_3(\mu)}{\alpha_1(\mu)}. \tag{2.10}$$

The running ratios above can be determined with high accuracy by performing the calculation at NNLO (which requires to exploit the RGE and the matching conditions at least at 3-loops and 2-loops respectively), as shown in Fig. 1. Notice that, as already mentioned and discussed in detail in Ref. [1], the non-Abelian gauge couplings unify at the scale $\mu_{32}^{\rm SM}\approx 2.8\times 10^{16}$ GeV, where they take the value $\alpha_{32}^{\rm SM}\approx 0.0217$.

The system in Eq. (2.9) can be solved exactly, as we will do in the following, or within an approximation, as discussed in Appendix B. Let us define

$$\tilde\beta = \beta/(1+\alpha), \quad \tilde\gamma = \gamma/(1+\alpha), \quad \tilde\delta = \delta/(1+\alpha). \tag{2.11}$$

The exact solution of the system in Eq. (2.9) is then given by

$$\tilde\beta = \frac{f_{21}f_{31}(20+\tilde\delta)+15f_{31}(4+\tilde\delta)-40f_{21}(2+\tilde\delta)}{90[f_{31}+f_{21}(2+f_{31})]}$$
$$\tilde\gamma = \frac{f_{21}f_{31}(50+7\tilde\delta)-3f_{31}(10+7\tilde\delta)-4f_{21}(5+7\tilde\delta)}{18[f_{31}+f_{21}(2+f_{31})]}. \tag{2.12}$$

The quantities $\tilde\beta$ and $\tilde\gamma$ are shown in the left plot of Fig. 2, for selected values of $\tilde\delta$. While $\tilde\gamma$ strongly depends on $\tilde\delta$, this

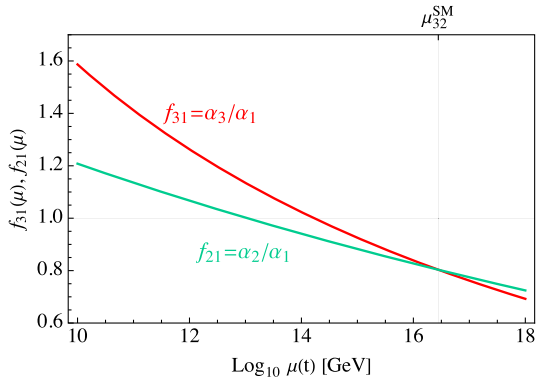


FIG. 1. The quantities $f_{21}(\mu)\equiv\frac{\alpha_2(\mu)}{\alpha_1(\mu)}$ and $f_{31}(\mu)\equiv\frac{\alpha_3(\mu)}{\alpha_1(\mu)}$, from a calculation at NNLO.

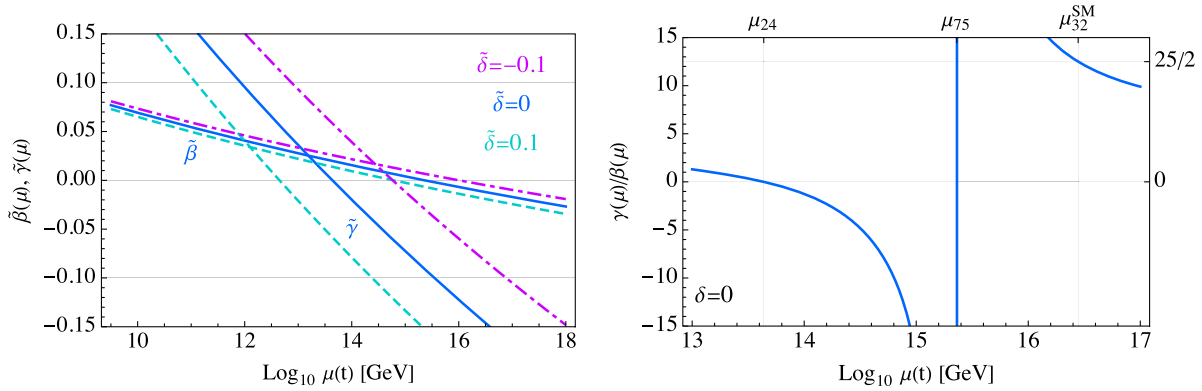


FIG. 2. Left: Solid lines are $\tilde{\beta}(\mu)$ and $\tilde{\gamma}(\mu)$ according to the exact solution, for $\tilde{\delta} = 0, \pm 0.1$. Right: Exact solution for the ratio $\gamma(\mu)/\beta(\mu)$ with $\delta = 0$.

is not the case for $\tilde{\beta}$, for which the dependence is mild. Notice also that one cannot phenomenologically determine α: its variation corresponds to an overall increase or decrease of α_G, and not to a shift of the GCU scale M_X.

In the remaining part of this section, we will first ignore H_{200} (thus taking $\delta = 0$), and explore the contributions from $H_{1,24,75}$. Second, (for reasons that will be clarified later) we will instead ignore H_{24} (thus taking $\beta = 0$), and explore the contributions from $H_{1,75,200}$.

B. Analysis of the contributions from 1, 24, and 75

If we only allow for the presence of the representations 1, 24, and 75, the two unknowns $\tilde{\beta}$ and $\tilde{\gamma}$, can be univocally determined as a function of μ, as shown by the solid lines in Fig. 2, obtained using the exact solution (2.12) with $\tilde{\delta} = 0$. First of all, we can check that for any scale μ in the range $10^{11.5}$–$10^{16.5}$ GeV there are solutions such that both $|\tilde{\beta}|$ and $|\tilde{\gamma}|$ are small (as assumed), say smaller than 0.15. So, GCU can be reasonably be achieved for μ in such a range.

We now focus on two particular cases. One can see that it is possible to realize GCU exploiting solely the 24: this happens for $\tilde{\gamma} = 0$ (no 75) and $\tilde{\beta} \approx 0.02$, in which case $M_X = \mu_{24} \approx 10^{13.6}$ GeV. On the other hand, it is possible to realize GCU also solely with the 75: this happens for $\tilde{\beta} = 0$ (no 24) and $\tilde{\gamma} \approx -0.1$, in which case $M_X = \mu_{75} \approx 10^{15.4}$ GeV.

It is also interesting to inspect the ratio $\tilde{\gamma}(\mu)/\tilde{\beta}(\mu) = \gamma(\mu)/\beta(\mu)$, as shown in the right plot of Fig. 2. Clearly, the ratio is vanishing at μ_{24} and diverges at μ_{75}. An interesting value for the ratio is $25/2$, which corresponds to $\mu = \mu_{32}^{SM} \approx 2.8 \times 10^{16}$ GeV.

The functions $\epsilon_i(\mu)$, obtained by substituting in Eq. (2.4) the exact expressions for $\tilde{\beta}(\mu)$ and $\tilde{\gamma}(\mu)$, Eq. (2.12), are shown in the left plot of Fig. 3, for $\alpha = -0.1$ (dotted), 0 (solid), 0.1 (dashed). Taking in particular $\alpha = 0$, the right plot shows the ratios $\epsilon_{2,3}(\mu)/\epsilon_1(\mu)$.

Exploiting Fig. 3 and taking $\alpha = 0$, we now discuss a few interesting cases for GCU (the extension to $\alpha \neq 0$ will be discussed in the next section).

(i) 24 only ($\alpha = \gamma = \delta = 0$): $M_X = \mu_{24} \approx 10^{13.6}$ GeV. By taking

$$0.020 \approx \beta = \epsilon_1(\mu_{24}) \approx \frac{1}{3}\epsilon_2(\mu_{24}) \approx -\frac{1}{2}\epsilon_3(\mu_{24}), \quad (2.13)$$

GCU happens at μ_{24} with $\alpha_G \approx 0.0247$, as shown in the top left plot of Fig. 4. The solid lines unifying at

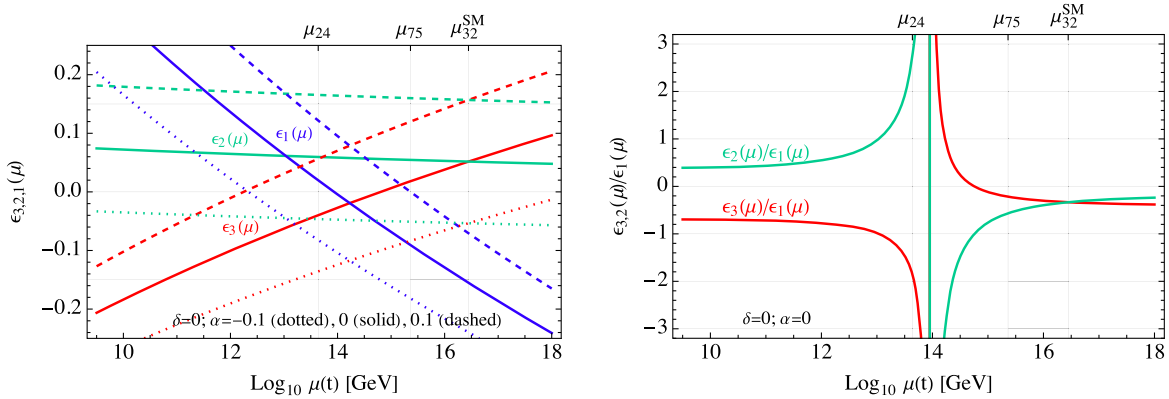


FIG. 3. Left: Dotted, solid, dashed lines are $\epsilon_s(\mu)$ for $\alpha = -0.1$, 0, 0.1 respectively. Right: ratios $\epsilon_{2,3}(\mu)/\epsilon_1(\mu)$, with $\alpha = 0$.

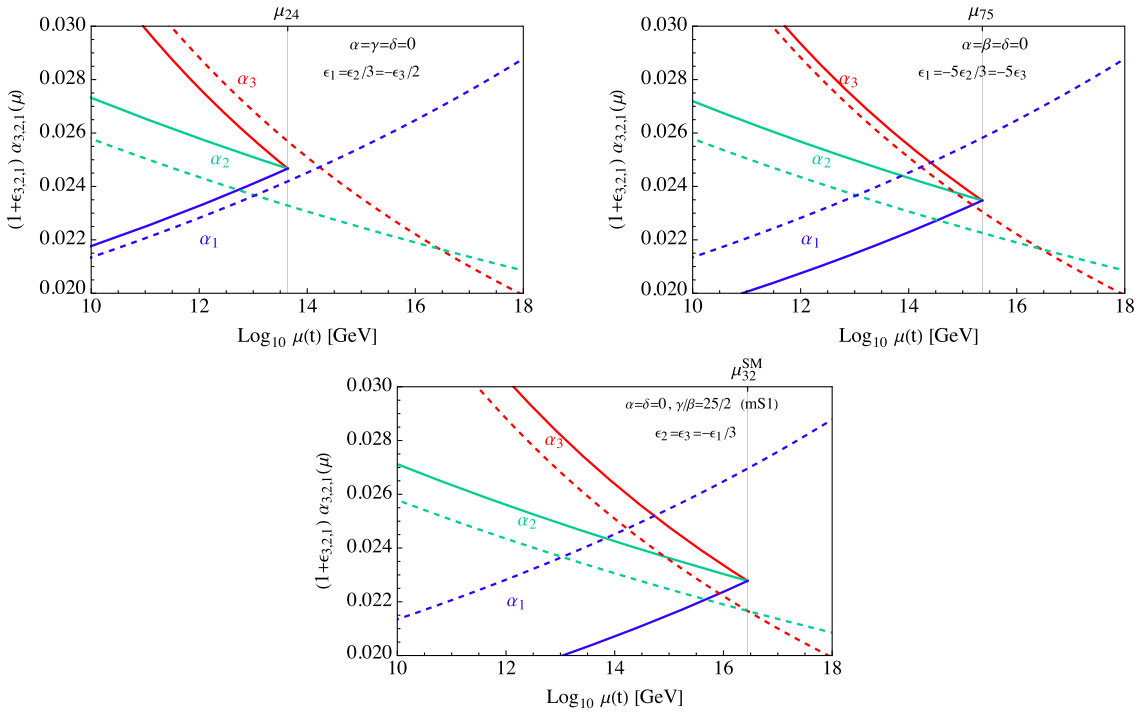


FIG. 4. Top left: GCU happens at μ_{24} by taking $0.020 = \epsilon_1(\mu_{24}) \approx \epsilon_2(\mu_{24})/3 \approx -\epsilon_3(\mu_{24})/2$. Top right: GCU happens at μ_{75} by taking $-0.091 \approx \epsilon_1(\mu_{75}) \approx -5\epsilon_2(\mu_{75})/3 \approx -5\epsilon_3(\mu_{75})$. Bottom: GCU happens at μ_{mS} by taking $0.052 \approx \epsilon_2(\mu_{mS}) \approx \epsilon_3(\mu_{mS}) \approx -\epsilon_1(\mu_{mS})/3$. Dashed lines are the gauge couplings in the SM.

μ_{24} represent the combinations $(1+\epsilon_s)\alpha_s(\mu)$, while dashed lines are the SM running couplings, $\alpha_s(\mu)$.

(ii) 75 only ($\alpha=\beta=\delta=0$): $M_X=\mu_{75}\approx 10^{15.4}$ GeV. By taking

$$-0.091 \approx \gamma = \epsilon_1(\mu_{75}) \approx -\frac{5}{3}\epsilon_2(\mu_{75}) \approx -5\epsilon_3(\mu_{75}), \tag{2.14}$$

GCU happens at μ_{75} with $\alpha_G \approx 0.0235$, as shown in the top right plot of Fig. 4,

(iii) mS_1 ($\alpha=\delta=0$, $\gamma/\beta=25/2$): $M_X=\mu_{32}^{\mathrm{SM}}=\mu_{mS}=10^{16.4}$ GeV $\approx 2.8\times 10^{16}$ GeV. By taking

$$0.052 \approx \epsilon_2(\mu_{mS}) \approx \epsilon_3(\mu_{mS}) \approx -\epsilon_1(\mu_{mS})/3, \tag{2.15}$$

GCU is achieved at the scale μ_{32}^{SM} with $\alpha_G \approx 0.0228$, as shown in the bottom plot of Fig. 4, Since μ_{32}^{SM} is very close to the GCU scale with low energy supersymmetry, this scenario can be denoted as mirage SUSY (mS_1) [1]; hence we define $\mu_{mS}\equiv\mu_{32}^{\mathrm{SM}}$. We find it interesting that the values of the ϵ_s (obtained with just the 24 and 75 irreps) mimicking low energy SUSY correspond to the very simple ratios of Eq. (2.15) which, as already mentioned, derive from the ratio $\gamma/\beta=25/2$. Using $\gamma/\beta=25/2$, from Eq. (2.15) we obtain $0.052\approx-\frac{9}{2}\beta$, namely $\beta\approx-0.011$ and $\gamma\approx-0.144$, in agreement with the left plot of Fig. 2. So, shall low energy SUSY be seen as a mirage induced by those particular ratios? Is it possible to find an $SU(5)$ model where those ratios are obtained from the breaking chains of the 24 and 75 irreps? We will come back to these questions in the following sections.

C. Analysis of the contributions from 1, 75, and 200

If we only allow for the presence of the representations 1, 75, and 200, the two unknowns $\tilde{\gamma}$ and $\tilde{\delta}$, can be univocally determined as a function of μ, by using the exact solution (2.12) with $\tilde{\beta}=0$. The relevance of such curious scenario will be discussed in the following sections. The absence of the 24 (the constraint $\tilde{\beta}=0$), is sufficient to determine that

$$\tilde{\delta} = -20\frac{f_{21}f_{31}+3f_{31}-4f_{21}}{f_{21}f_{31}+15f_{31}-40f_{21}},$$
$$\tilde{\gamma} = -5\frac{f_{21}f_{31}-9f_{31}+8f_{21}}{f_{21}f_{31}+15f_{31}-40f_{21}}, \tag{2.16}$$

which are shown in the left plot of Fig. 5. Requiring that $|\tilde{\delta}|,|\tilde{\gamma}|\lesssim 0.15$, we see from the latter plot that GCU can be achieved for any scale μ in the range $10^{14.5}\lesssim\mu/\mathrm{GeV}\lesssim 10^{16.4}$. The ratio δ/γ is shown in the right plot of Fig. 5.

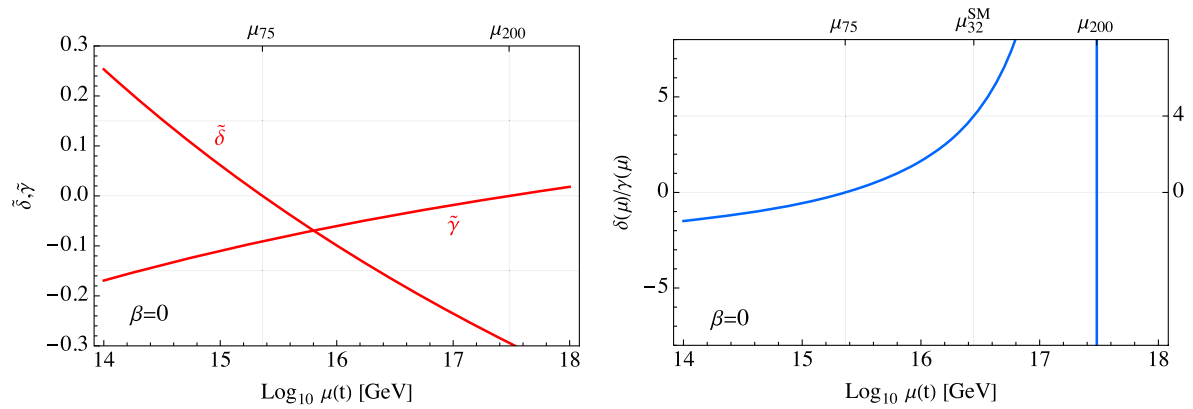


FIG. 5. Left panel: Solid lines are $\tilde{\delta}(\mu)$ and $\tilde{\gamma}(\mu)$ according to the exact solution, for $\tilde{\beta}=0$. Right panel: Ratio $\delta(\mu)/\gamma(\mu)$.

We now discuss a few interesting cases for GCU, taking $\alpha=0$ (the generalization to $\alpha\neq 0$ will be addressed in the following).

(i) 200 only ($\alpha=\beta=\gamma=0$): $M_X=\mu_{200}\approx 10^{17.5}\simeq 3.2\times 10^{17}$ GeV.

From our numerical analysis we found that (see also Ref. [14]), with $\tilde{\delta}\approx -0.29$ (a value which might jeopardize the validity of perturbation theory), GCU can be achieved without 24 and 75, at $\mu_{200}\approx 10^{17.5}$ GeV. In this case, taking $\alpha=0$, we have

$$-0.29\approx\delta=\epsilon_1(\mu_{200})\approx 5\epsilon_2(\mu_{200})\approx 10\epsilon_3(\mu_{200}). \tag{2.17}$$

This scenario is shown in the left plot of Fig. 6.

(ii) mS_2 ($\alpha=\beta=0$, $\delta/\gamma=4$): $M_X=\mu_{32}^{\rm SM}=\mu_{mS}=10^{16.4}$ GeV $\approx 2.8\times 10^{16}$ GeV. Mirage SUSY can be realized in this scenario too, provided $\delta/\gamma=4$ (see the right plot of Fig. 5), so that

$$-0.0081\approx\frac{1}{25}\epsilon_1(\mu_{mS})\approx\epsilon_2(\mu_{mS})\approx\epsilon_3(\mu_{mS})\approx\frac{1}{5}\gamma. \tag{2.18}$$

Within this scenario, that we are going to denote mS_2, the combinations $(1+\epsilon_s)\alpha_s(\mu)$ behave as shown in the right plot of Fig. 6. Notice how it nearly looks like having a suitable normalization for the hypercharge, leading to GCU.

D. Comparison of the previous scenarios

It has to be emphasized that the parametrization in Eq. (2.3) precisely corresponds to the more general one proposed in Ref. [1] for the sake of studying how (partial or full) GCU might be achieved in new physics models; in particular, partial unification of the non-Abelian couplings leads to a useful relation between M_X, ϵ_3, and ϵ_2, as

$$\frac{M_X}{\mu_{32}^{\rm SM}}=\exp\left(\frac{2\pi}{\alpha_{32}^{\rm SM}}\frac{\epsilon_3-\epsilon_2}{(1+\epsilon_3)b_2^{\rm SM}-(1+\epsilon_2)b_3^{\rm SM}}\right), \tag{2.19}$$

where $\mu_{32}^{\rm SM}\approx 2.8\times 10^{16}$ GeV is the non-Abelian gauge couplings partial unification scale, $\alpha_{32}^{\rm SM}\equiv\alpha_2(\mu_{32}^{\rm SM})=\alpha_3(\mu_{32}^{\rm SM})\approx 0.0217$, while $b_2^{\rm SM}=-19/6$ and $b_3^{\rm SM}=-7$ are the SM beta functions at one-loop. Notice that the magnitude of difference $\epsilon_3-\epsilon_2$ is related to the difference between M_X and $\mu_{32}^{\rm SM}$.

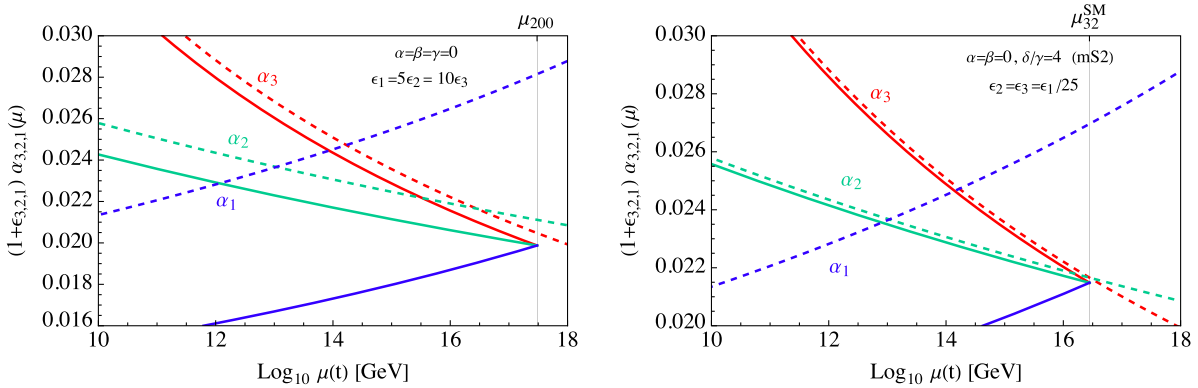


FIG. 6. Left: GCU happens at μ_{200} by taking $-0.029\approx\epsilon_1(\mu_{200})\approx 5\epsilon_2(\mu_{200})\approx 10\epsilon_3(\mu_{200})$. Right: With $\alpha=\beta=0$ and $\delta/\gamma=4$, GCU happens at μ_{mS} by taking $0.052\approx\epsilon_2(\mu_{mS})\approx\epsilon_3(\mu_{mS})\approx\epsilon_1(\mu_{mS})/25$. Dashed lines are the gauge couplings in the SM.

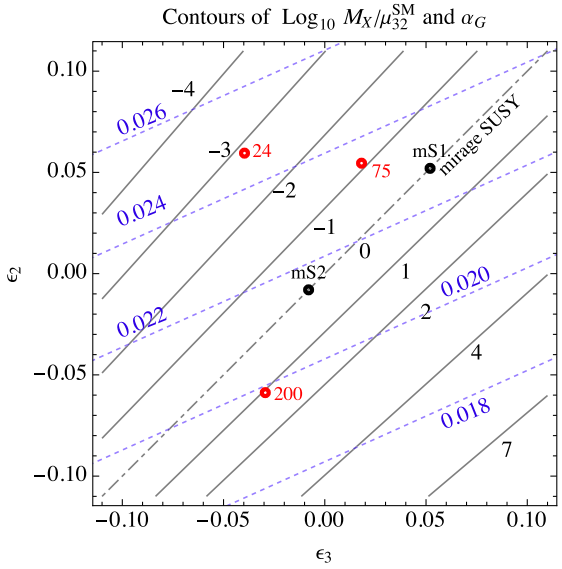


FIG. 7. Contours of $\mathrm{Log}_{10} M_X/\mu_{32}^{\mathrm{SM}}$ (solid black and dot-dashed black) and α_G (dashed blue). The circles emphasize the values corresponding to selected models with $\alpha = 0$ discussed in the text.

It is interesting to localize the previous scenarios in the landscape of ϵ_2 and ϵ_3 (as was done in [1] for other models). Using Eq. (2.19), this can be done by displaying the isolevels of $\log_{10} M_X/\mu_{32}^{\mathrm{SM}}$ (solid lines) and α_G (dashed lines) in the plane (ϵ_2, ϵ_3). Figure 7 shows the location of the previous scenarios, and allows for a direct comparison with other models providing GCU, as those discussed in Ref. [1].

The scenarios denoted by mS_1 and mS_2 lie along the dot-dashed line, characterized by the relation $\epsilon_2 = \epsilon_3$, where all mirage SUSY models are located. Above the dot-dashed line, $\epsilon_3 - \epsilon_2 < 0$, so that $M_X < \mu_{32}^{\mathrm{SM}}$; in this region we find the scenarios denoted as 24 only and 75 only, for which $\epsilon_3 - \epsilon_2 \approx -0.1$ and $\epsilon_3 - \epsilon_2 \approx -0.03$, respectively. Below the dot-dashed line, $\epsilon_3 - \epsilon_2 > 0$, so that $M_X > \mu_{32}^{\mathrm{SM}}$; the 200 only scenario is found here, for which $\epsilon_3 - \epsilon_2 \approx 0.03$.

As already noticed, a nonvanishing value of α does not change the scale where GCU takes place, as α acts as an offset for gauge couplings. Graphically, switching on α is equivalent to moving along the isolevels of M_X shown in Fig. 7: with $\alpha > 0$ ($\alpha < 0$), one moves toward larger (smaller) values of α_G, hence to the top right (bottom left). In the next section we will explore in more details the effect of varying α.

III. DETAILED ANALYSIS OF SOME RELEVANT CASES

Let us fix $\mu = M_X$ as the scale where GCU happens via the corrections that we are studying. We will relate the values of the $\epsilon_s(M_X)$ (hence of the parameters α, β, γ, δ) among themselves, generalizing previous results derived considering $\alpha = 0$. This is equivalent to include in Eq. (2.1) the singlet representation ($r = 1$); even though such representation does not break $SU(5)$, it might in principle be present. The impact of the constraints from nonobservation of proton decay will also be discussed.

A. Generalization with $\alpha \neq 0$

We now focus on the generalization of the three cases of Sec. II B with $\delta = 0$ (in which GCU is achieved at μ_{24}, μ_{75} and μ_{mS}), and of the two cases of Sec. II C with $\beta = 0$ (in which GCU is achieved at μ_{200} and μ_{mS}). The generalization consists in including the singlet irrep.

(i) $1 + 24$ only ($\gamma = \delta = 0$): $M_X = \mu_{24}$

The generalization of Eq. (2.13) to the case $\alpha \neq 0$ is

$$0.020 + \alpha \approx \alpha + \beta = \epsilon_1(\mu_{24}) \approx \frac{1}{3}\epsilon_2(\mu_{24}) + \frac{2}{3}\alpha \approx -\frac{1}{2}\epsilon_3(\mu_{24}) + \frac{3}{2}\alpha. \tag{3.1}$$

As expected, there is no relation between α and β: the latter is fixed to about 0.02, while the former is free and acts as an offset for GCU. The isolevel contour for $M_X = \mu_{24}$ corresponds to the line whose points have coordinates $(\epsilon_2, \epsilon_3) \approx (0.06, -0.04) + \alpha$, as shown in Fig. 8. The three (red) dots emphasize the position associated to some particular values, $\alpha = 0, \pm 0.04$; hence, by increasing α one moves up-right along the line. Notice also that $\alpha_G \approx 0.0247(1 + \alpha)$.

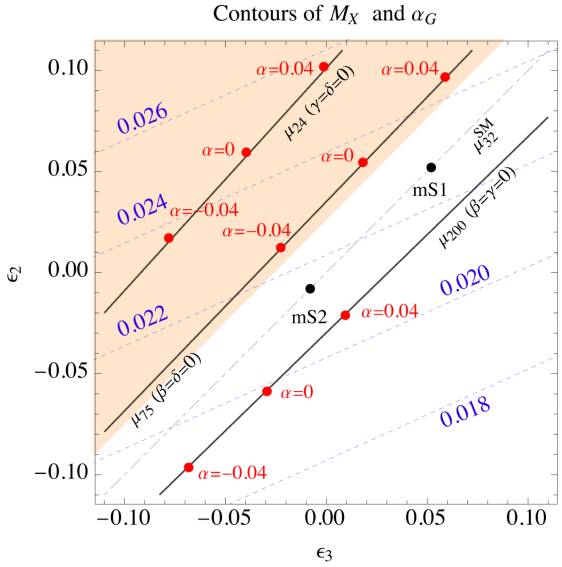


FIG. 8. Right: Contours of M_X (solid black and dot-dashed black) and α_G (dashed blue). The effect of a nonvanishing value for α is shown. The shaded (orange) region is excluded by proton decay contraints.

(ii) $1+75$ only ($\beta=\delta=0$): $M_X=\mu_{75}$
For $\alpha\neq 0$, Eq. (2.14) is generalized to

$$-0.091+\alpha\approx\alpha+\gamma=\epsilon_1(\mu_{75})\approx-\frac{5}{3}\epsilon_2(\mu_{75})+\frac{8}{3}\alpha \approx -5\epsilon_3(\mu_{75})+6\alpha. \tag{3.2}$$

Again, there is no relation between $\gamma\approx-0.091$ and α, which acts as an offset. The isolevel contour corresponding to $M_X=\mu_{75}$ is the line $(\epsilon_2,\epsilon_3)\approx(0.05,0.02)+\alpha$, as shown in Fig. 8. In addition, we have $\alpha_G\approx 0.0235(1+\alpha)$.

(iii) $1+200$ only ($\beta=\gamma=0$): $M_X=\mu_{200}$
Eq. (2.17) is generalized to $\alpha\neq 0$ by

$$-0.3+\alpha\approx\delta+\alpha=\epsilon_1(\mu_{200})\approx 5\epsilon_2(\mu_{200})-5\alpha \approx 10\epsilon_3(\mu_{200})-10\alpha. \tag{3.3}$$

The line corresponding to these scenarios is $(\epsilon_2,\epsilon_3)\approx(-0.06,-0.03)+\alpha$, as shown in Fig. 8. In this case $\alpha_G\approx 0.020(1+\alpha)$.

We now focus on the two scenarios of the mirage SUSY type introduced in the previous section.

(i) mS_1 ($\delta=0$, $\gamma/\beta=25/2$): $M_X=\mu_{mS}$
For $\alpha=0$, we derived Eq. (2.15) and found that $\gamma/\beta=25/2$ implies $\beta\approx-0.011$ and $\gamma\approx-0.144$. For $\alpha\neq 0$, such relation is generalized to

$$0.052+\alpha\approx\epsilon_2(\mu_{mS})\approx\epsilon_3(\mu_{mS})\approx-\frac{1}{3}\epsilon_1(\mu_{mS})+\frac{4}{3}\alpha, \tag{3.4}$$

and the corresponding line in Fig. 8 is simply $(\epsilon_2,\epsilon_3)\approx(0.052,0.052)+\alpha$. Notice also that $\alpha_G\approx 0.0228(1+\alpha)$.

(ii) mS_2 ($\beta=0$, $\delta/\gamma=4$): $M_X=\mu_{mS}$
Eq. (2.18) is generalized to $\alpha\neq 0$ by

$$-0.0081+\alpha\approx\frac{1}{25}\epsilon_1(\mu_{mS})-\frac{1}{25}\alpha\approx\epsilon_2(\mu_{mS}) \approx\epsilon_3(\mu_{mS})\approx\frac{1}{5}\gamma+\alpha. \tag{3.5}$$

The corresponding line in Fig. 8 is simply $(\epsilon_2,\epsilon_3)\approx(-0.008,-0.008)+\alpha$.

B. Relation with proton decay

A high value of $\mu=M_X$ for GCU is welcome to avoid problems with proton decay. As is well known, in $SU(5)$ GUTs, $d=6$ operators are induced by X boson exchange. The main (non-SUSY) decay mode is $p\to e^+\pi^0$, with a lifetime given by [15] (for the lattice coefficients see [16], and for analytic ones see Hisano *et al.* [17])

$$\tau_p=\mathcal{O}(1)\frac{M_X^4}{\alpha_G^2 m_p^5}, \tag{3.6}$$

where m_p is the proton mass and M_X is the X boson mass, to be identified with the GCU scale.

So, to prolong p lifetime, it would be better to have α_G as small as possible and M_X as large as possible. The present experimental bound from Super-Kamiokande is $\tau_p/\mathrm{Br}(p\to e^+\pi^0)>2.4\times 10^{34}$ years at 90% CL, which implies

$$M_X\gtrsim\left(\frac{\alpha_G}{\alpha_{32}^{\mathrm{SM}}}\right)^{1/2}4.5\times 10^{15}\ \mathrm{GeV}, \tag{3.7}$$

where we recall that $\alpha_{32}^{\mathrm{SM}}=0.0217$.

The bound is reported in Fig. 8: models in the shaded region are excluded. This means that the previous cases denoted as $1+24$ only are severely ruled out, while those of the $1+75$ type are disfavored, since $\mu_{75}\approx 2.3\times 10^{15}$ GeV. The scenarios of the type $1+200$ are instead fully acceptable, as well as mirage SUSY models.

Indeed, mirage SUSY turns out to be safe, since $M_X=\mu_{mS}=\mu_{32}^{\mathrm{SM}}=2.8\times 10^{16}$ GeV, and the related value of α_G is quite small: $\alpha_G\approx 0.023(1+\alpha)$ for mS_1 (and even smaller for mS_2). On the other hand, in the case of low energy SUSY, one would have $\alpha_G\approx 0.038$ (and of course $M_X=\mu_{mS}$). Hence, considering mS_1 for definiteness, proton lifetime for the decay $p\to e^+\pi^0$ is longer with respect to the case of low energy SUSY: the enhancement factor is $(0.038/0.023)^2\approx 3$ in the case $\alpha=0$, and larger with α negative.[4]

IV. RELATION WITH THE DOUBLET-TRIPLET SPLITTING PROBLEM

In this section, we investigate possible connections between GCU and the doublet-triplet splitting (DTS) problem. In the following, we will distinguish two scenarios about the nature of the spontaneous $SU(5)$ breaking to the SM: (i) By propagating elementary scalars, as in the Higgs mechanism; (ii) By fermion condensates which dynamically break $SU(5)$ to the SM.

A. Higgs breaking of $SU(5)$

As already discussed in Sec. II [see also the explicit calculation leading to Eq. (A35)], if we allow for the presence of scalar Higgses in the $SU(5)$ representations, $H_r=1,24,75,200$, the nonrenormalizable $d=5$ operator of Eq. (2.1) induces the following corrections

[4]We recall however that in SUSY $SU(5)$ the calculation of the matrix element is slightly different than in the SM, see, e.g., [17]. In addition, the decay mode $p\to e^+\pi^0$ is not even expected to be dominant. The main decay mode is expected to be $p\to K^+\bar{\nu}$, arising from $d=5$ operators with heavy scalar triplet exchange.

$$\epsilon_1 = \sum_r \epsilon_1^{(r)} = \alpha + \beta + \gamma + \delta$$
$$= \frac{1}{\Lambda}\left(c_1 v_1 - \frac{1}{2} c_{24} v_{24} - 5 c_{75} v_{75} + 10 c_{200} v_{200}\right),$$
$$\epsilon_2 = \sum_r \epsilon_2^{(r)} = \alpha + 3\beta - \frac{3}{5}\gamma + \frac{1}{5}\delta$$
$$= \frac{1}{\Lambda}\left(c_1 v_1 - \frac{3}{2} c_{24} v_{24} + 3 c_{75} v_{75} + 2 c_{200} v_{200}\right),$$
$$\epsilon_3 = \sum_r \epsilon_3^{(r)} = \alpha - 2\beta - \frac{1}{5}\gamma + \frac{1}{10}\delta$$
$$= \frac{1}{\Lambda}(c_1 v_1 + c_{24} v_{24} + c_{75} v_{75} + c_{200} v_{200}), \tag{4.1}$$

where we recall the definitions $\langle H_1 \rangle = v_1 \mathbb{1}_5$ and $\langle H_{24} \rangle = v_{24} \mathrm{diag}(\mathbb{1}_3, -3/2 \cdot \mathbb{1}_2)$, while for more details about the contributions from $r = 75, 200$, we refer to Appendix A 2.

As for the DTS problem (in a non-SUSY framework), let us consider the representation in which the SM Higgs doublet is contained, $H_5 = (T, D)^T$. In general, triplet and doublet mass terms, denoted by m_T and m_D respectively, come from the following Lagrangian density terms

$$\mathcal{L} \ni -H_5^\dagger [m_5^2 + m_1 \langle H_1 \rangle + m_{24} \langle H_{24} \rangle + a_1 \langle H_1^2 \rangle + a_{24} \mathrm{Tr}(\langle H_{24} \rangle^2) + b_{24} \langle H_{24} \rangle^2 + c_{124} \langle H_1 H_{24} \rangle] H_5$$
$$= -T^\dagger m_T^2 T - D^\dagger m_D^2 D, \tag{4.2}$$

where the parameters m_5, m_1, m_{24} have the dimension of a mass, while a_1, a_{24}, b_{24}, c_{124} are dimensionless, and we have

$$m_T^2 = m_5^2 + m_1 v_1 + (m_{24} + c_{124} v_1) v_{24} + a_1 v_1^2 + \frac{15}{2} a_{24} v_{24}^2 + b_{24} v_{24}^2,$$
$$m_D^2 = m_5^2 + m_1 v_1 - \frac{3}{2}(m_{24} + c_{124} v_1) v_{24} + a_1 v_1^2 + \frac{15}{2} a_{24} v_{24}^2 + \frac{9}{4} b_{24} v_{24}^2. \tag{4.3}$$

Even if present, the 75 and 200 would provide no contribution to doublet and triplet masses. Notice also that the terms that can make a difference between the doublet and the triplet are those proportional to the combination $(m_{24} + c_{124} v_1)$ and b_{24}. So, ensuring a vanishing mass to the doublet, while keeping the triplet at the GUT scale, can be seen as a problematic tuning. In addition, even accepting the tuning, the doublet mass would be unstable against radiative corrections, unless invoking SUSY. Without SUSY, a reasonable possibility would be to promote the doublet to a pseudo-Nambu-Goldstone boson of some broken symmetry.

So, by comparing Eqs. (4.1) and (4.3), the conclusion is that there is no direct link between GCU and DTS. As we are now going to discuss, a link can however be established in the case that the representations are effective ones, sharing a common origin in a more fundamental theory.

Before doing this, let us discuss another possibility for solving the DTS problem, the so called missing partner mechanism (MDM) [18,19]. It relies on the absence of 1, 24, and on the presence of the 75 to break $SU(5)$. Adding scalars in $50, \overline{50}$ representations, it is possible to give mass to triplets, while keeping doublets (including the SM one) massless. The MDM was originally proposed in the context of SUSY, where GCU is achieved, and was studied in some detail for instance in Refs. [20,21]. The MDM can however be exploited also in a non-SUSY framework, as we are considering here; as for GCU, in this case one has to consider Eq. (4.1) with $\alpha = \beta = \delta = 0$, which corresponds to scenario denoted by 75 only (see the discussion in Sec. II B and Fig. 7). So, also in the case of the non-SUSY MDM, one cannot find any direct link between the parameters involved in DTS and the parameter γ involved in GCU.

B. Dynamical breaking of $SU(5)$

We will now consider a scenario where $SU(5)$ is broken dynamically. Previous studies where the GUT symmetry is broken dynamically by the presence of a strong dynamics have been performed mainly in the context of supersymmetric theories [22–24]. In the case of nonsupersymmetric theories, an option is to start with a supersymmetric theory at high scales, and, e.g., consider a supergravity theory where local supersymmetry is spontaneously broken by a chiral superfield Z; by adopting, e.g., the Polonyi mechanism [25], the scale of supersymmetry breaking is $F_Z \sim M_X M_P$ leading to a gravitino mass $m_{3/2} \sim M_X$. In this way the theory for scales below M_X would behave as a nonsupersymmetric theory. A study in that direction is postponed to further investigations.

Here we will just assume that there is a confining group G which becomes strong at an IR scale $\Lambda_G \simeq M_X$. All conventional GUT fields are then singlets under G, i.e., the matter fermions 10_F and $\bar{5}_F$, the Higgs boson 5_H and the gauge bosons 24_G. Moreover, we will assume that the UV theory for scales $\mu \gtrsim M_X$ contains a set of Dirac (anomaly free) fermions F_R (and $\bar{F}_R$) which transform under the $R = 5, 10, 24$ (and $\bar{R} = \bar{5}, \overline{10}, 24$) representations of $SU(5)$ and under the R_G (and $\bar{R}_G$) representation of G. In this paper we will be agnostic about the group G and the representation R_G and $\bar{R}_G$, but just will consider the most attractive channel X corresponding to the maximal binding strength $\kappa(X) = 2C_2(R_G) - C_2(X)$ to be the singlet representation of G, $X = 1$, as $C_2(1) = 0$, such that the bilinear $\bar{F}_R \times F_R \equiv \mathcal{T}_R$ (in the singlet representation of the confining group G) produces the condensate as

$\langle \mathcal{T}_R \rangle \simeq \Lambda_G^3$ [26]. The theory is then a QCD-like, or better a technicolorlike (techniGUT) theory. For some more recent references on binding strength, see, e.g., Refs. [27,28].

In this way, we recognize that the fermion condensates generate the effective representations of Eq. (2.1)

$$\mathcal{T}_R = \bar{F}_R \times F_R \supset 1 + 24 + a_R \cdot 75 + b_R \cdot 200, \quad (4.4)$$

where the constants a_R and b_R take the following values[5]: $a_5 = b_5 = 0$; $a_{10} = 1$, $b_{10} = 0$; and $a_{24} = b_{24} = 1$. The confining dynamics alone produces a condensate but is blind to alignment. Gauge interactions through the effective potential $V_{\rm eff}(\mathcal{T}_{\mathcal{R}})$ will break the vacua degeneracy leading to the vacuum alignment characteristic of nonsupersymmetric theories. In this paper we will consider the VEVs along the different components of (4.4) as free parameters, and study their role for GCU and the DTS mechanism. In the case of supersymmetric theories the vacuum alignment is much simpler, as it relies on the search for flat directions, and will be done elsewhere.

Taking $R = 5$ is the most economical possibility[6]; the representations 1 and 24 are obtained as effective ones, while the 75 and the 200 are absent. Taking instead $R = 10$, the 200 representation is not obtained. Finally, putting the fermion in the adjoint representation, $R = 24$, one gets all the representations that might contribute to the nonrenormalizable operator of Eq. (2.1).

We will assume that the condensates break $SU(5)$ along the direction of $SU(3) \otimes SU(2) \otimes U(1)$, such that

$$\frac{\langle \mathcal{T}_R \rangle}{\Lambda_G^2} = \frac{\langle \bar{F}_R \times F_R \rangle}{\Lambda_G^2} = \langle H_1^{(R)} \rangle + \langle H_{24}^{(R)} \rangle + a_R \langle H_{75}^{(R)} \rangle + b_R \langle H_{200}^{(R)} \rangle, \quad (4.5)$$

the effective Higgses $H_r^{(R)}$, acquire VEVs as discussed in Appendix A 2.

The branching rules of $SU(5)$ under $SU(3) \otimes SU(2) \otimes U(1)$ are such that $\bar{F}_R \times F_R \supset (3,2)_{5/6} + (\bar{3},2)_{-5/6}$, which are the 12 Goldstone bosons[7] eaten by the X_μ and Y_μ gauge fields to become massive. Moreover in the heavy spectrum, above the condensation scale, there are mesons, from other components of $(\bar{F}_R \times F_R)/\Lambda_G^2$, and baryons, from e.g. $B_R = (\epsilon_{ABCDE} F_R^A F_R^B F_R^C F_R^D F_R^E)/\Lambda_G^6$, which are singlets under $SU(5)$.

[5]The parameters a_R and b_R introduced here should not be confused with those introduced in Eq. (4.3).

[6]This is in a sense a "really minimal $SU(5)$", as there are just 10 fermionic fields, instead of the 24 scalar fields of the "minimal $SU(5)$" model.

[7]For instance for $R = 5$, $\bar{F}_\alpha \gamma_5 F^a$ and $F^\alpha \gamma_5 \bar{F}_a$, where $\alpha = 1, 2, 3$ are $SU(3)$ indices (color), and $a = 1, 2$ $SU(2)$ ones.

1. The gauge coupling unification

Let us first focus on GCU. At the condensation scale $\mu \sim \Lambda_G$, we find (see Appendix C 2) the nonrenormalizable $d = 5$ operator

$$-\frac{1}{4}\frac{c_R}{\Lambda_G} \mathrm{Tr}(G_{\mu\nu}(\langle H_1^{(R)} \rangle + \langle H_{24}^{(R)} \rangle + a_R \langle H_{75}^{(R)} \rangle + b_R \langle H_{200}^{(R)} \rangle) G^{\mu\nu}), \quad (4.6)$$

where R can be chosen among 5, 10, 24. The crucial point is that, unlike Eq. (2.1), the dynamical breaking case of Eq. (4.6) leads to a common coefficient, c_R, for the contributions of the various effective representations. As a result, the relative magnitudes of the contributions to the ϵ's just depend on the effective VEVs.

Defining $\langle H_r^{(R)} \rangle$ as discussed in Appendix A 2, one obtains Eq. (A35), from which it can be seen that the above operator induces the following corrections

$$\begin{aligned}
\epsilon_1 &= \sum_r \epsilon_1^{(r)} = \alpha + \beta + a_R \gamma + b_R \delta \\
&= \frac{c_R}{\Lambda_G}\left(v_1 - \frac{1}{2} v_{24} - 5 a_R v_{75} + 10 b_R v_{200}\right), \\
\epsilon_2 &= \sum_r \epsilon_2^{(r)} = \alpha + 3\beta - \frac{3}{5} a_R \gamma + \frac{1}{5} b_R \delta \\
&= \frac{c_R}{\Lambda_G}\left(v_1 - \frac{3}{2} v_{24} + 3 a_R v_{75} + 2 b_R v_{200}\right), \\
\epsilon_3 &= \sum_r \epsilon_3^{(r)} = \alpha - 2\beta - \frac{1}{5} a_R \gamma + \frac{1}{10} b_R \delta \\
&= \frac{c_R}{\Lambda_G}(v_1 + v_{24} + a_R v_{75} + b_R v_{200}).
\end{aligned} \quad (4.7)$$

One thus obtains relations among α, β, γ, and δ, which depend only on the relations among the various VEVs, v_r.

2. The doublet-triplet splitting

One expects that also the DTS condition will depend on the effective VEVs, and it will thus be possible to relate GCU and DTS. In this section we present the main ideas, and will analyze in detail the phenomenology and the model building, related to the various possible fermion representations, $R = 5$, 10, 24, in the next section.

We will consider an effective Lagrangian as (see Appendix C 1 for a particular simple model)

$$\mathcal{L}_{\rm eff} = -\frac{\lambda_R^2}{\Lambda_G} H_5^\dagger (\bar{F}_R \times F_R) H_5, \quad (4.8)$$

giving rise, upon fermion condensation as in Eq. (4.5), to the mass terms

$$\mathcal{L}_{\rm eff} = -\lambda_R^2 \Lambda_G H_5^\dagger (\langle H_1^{(R)} + H_{24}^{(R)} \rangle) H_5. \tag{4.9}$$

Defining $\langle H_r^{(R)} \rangle$ and the associated VEVs, $v_r^{(R)}$, as discussed in Appendix A 2, it is possible to write

$$\langle H_1^{(R)} \rangle = v_1 \mathbb{1}_5, \quad \langle H_{24}^{(R)} \rangle = v_{24} \mathrm{diag}(\mathbb{1}_3, -3/2 \cdot \mathbb{1}_2) \tag{4.10}$$

for any $R = 5$, 10, 24. The operator above then splits the doublet and triplet masses as

$$\mathcal{L}_{\rm eff} \ni -T^\dagger \underbrace{\lambda_R^2 \Lambda_G (v_1 + v_{24})}_{m_T^2} T - D^\dagger \underbrace{\lambda_R^2 \Lambda_G \left(v_1 - \frac{3}{2} v_{24} \right)}_{m_D^2} D. \tag{4.11}$$

Notice that there is no contribution to DTS from the $H_{75}^{(R)}$ and $H_{200}^{(R)}$ of Eq. (4.5), even in case they are present in the fermion condensate, as they do not couple to $\bar{5} \times 5$.

The doublet is massless if the condition

$$m_D^2 = \lambda_R^2 \Lambda_G \left(v_1 - \frac{3}{2} v_{24} \right) = 0 \tag{4.12}$$

is satisfied, in which case the triplet mass is $m_T^2 = \lambda_R^2 \Lambda_G (v_1 + v_{24}) = \frac{5}{2} \lambda_R^2 \Lambda_G v_{24}$.

3. Doublet-triplet splitting constraints on gauge coupling unification

By comparing Eq. (4.12) with Eq. (4.7), one can recognize that the triplet mass has the structure of the $1 + 24$ contribution to ϵ_3, while the doublet mass has the structure of the $1 + 24$ contribution to ϵ_2. The DTS condition thus implies

$$\alpha = -3\beta. \tag{4.13}$$

If we rewrite Eq. (4.7) using $\alpha = -3\beta$, and the equivalent relation $v_1 = \frac{3}{2} v_{24}$, we obtain

$$\begin{aligned}
\epsilon_1 &= -2\beta + a_R \gamma + b_R \delta = \frac{c_R}{\Lambda_G} (v_{24} - 5 a_R v_{75} + 10 b_R v_{200}), \\
\epsilon_2 &= -\frac{3}{5} a_R \gamma + \frac{1}{5} b_R \delta = \frac{c_R}{\Lambda_G} (3 a_R v_{75} + 2 b_R v_{200}), \\
\epsilon_3 &= -5\beta - \frac{1}{5} a_R \gamma + \frac{1}{10} b_R \delta \\
&= \frac{c_R}{\Lambda_G} \left(\frac{5}{2} v_{24} + a_R v_{75} + b_R v_{200} \right).
\end{aligned} \tag{4.14}$$

Using the above expressions in the system (2.9) accounting for GCU (where we substitute the condition $\alpha = -3\beta$), we obtain the equations

$$\begin{aligned}
\beta(\mu) &= \frac{-4(2 + b_R \delta) f_{21} + (2 + b_R \delta/10) f_{21} f_{31} + (6 + 3 b_R \delta/2) f_{31}}{3(9 f_{31} + f_{21}(-2 + 5 f_{31}))}, \\
a_R \gamma(\mu) &= \frac{-3(5 + 8 b_R \delta) f_{31} + (5 + b_R \delta) f_{21} (-2 + 5 f_{31})}{3(9 f_{31} + f_{21}(-2 + 5 f_{31}))}.
\end{aligned} \tag{4.15}$$

Let us focus on the most general case $R = 24$, so that the parameters a_R, b_R in Eq. (4.15) are equal to one. For fixed values of δ, the dependence of $\beta(\mu)$ and $\gamma(\mu)$ is shown in the left plot of Fig. 9, while the associated ϵ corrections are shown in the right plot; the solid, dashed, and dot-dashed lines refer respectively to $\delta = 0, 0.15, -0.15$. In the case $R = 10$, a_R in Eq. (4.15) is equal to one, while b_R vanishes: this case is thus described by the solid curve ($\delta = 0$) in

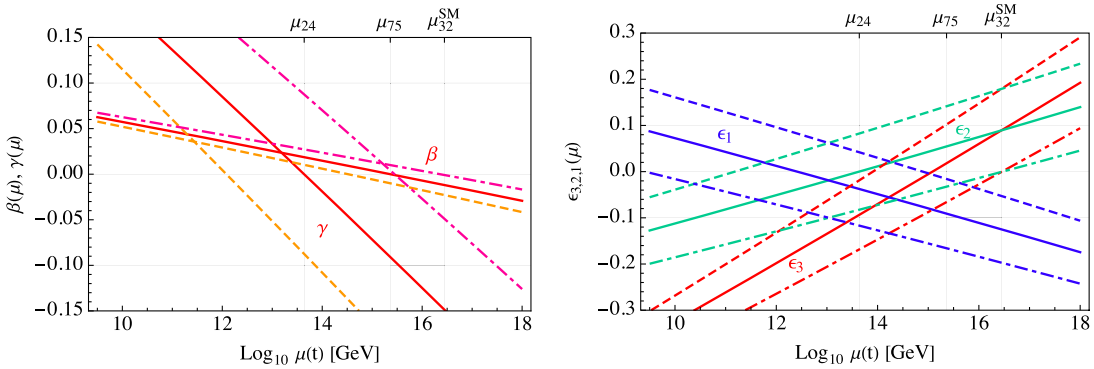


FIG. 9. Left: The solutions for β and γ in Eq. (4.15), with $\delta = 0$ (solid), $\delta = 0.15$ (dashed) and $\delta = -0.15$ (dot-dashed). Right: The associated ϵ corrections, according to Eq. (4.14).

Fig. 9. In the case $R = 5$, both a_R and b_R are vanishing; in particular the second equation in (4.15), gives a constraint on μ, which is satisfied only for $\mu = \mu_{24}$, as can be seen from the dependence of $\gamma(\mu)$ in the left plot of Fig. 9.

V. THE PHENOMENOLOGY OF DYNAMICAL GUT BREAKING

In this section we will consider in turn GCU and DTS for the cases of dynamical breaking such that $\langle \bar{F}_R \times F_R \rangle \neq 0$, for the three different cases where the condensates correspond to the representations $R = 5, 10, 24$; the latter will be also respectively denoted by, F (five), T (ten), A (adjoint), in order to (hopefully) use a more effective notation.

A. Fermion condensates from $R = 5$

We introduce the fermion five-plets $\bar{F} \equiv \bar{F}_5$ and $F \equiv F_5$, so that the tensor

$$\frac{\mathcal{T}_A^B}{\Lambda_G^2} \equiv \frac{\bar{F}_A \times F^B}{\Lambda_G^2} = (H_1^{(5)} + H_{24}^{(5)})_A^B = S_A^B + \Sigma_A^B, \tag{5.1}$$

with indices $A, B = (1, \ldots, 5)$, can be decomposed into the singlet (S) and adjoint (Σ) representations respectively. We recall that $\langle S_A^B \rangle = v_1 \delta_A^B$ and $\langle \Sigma_A^B \rangle = v_{24}(\mathbb{1}_3, -3/2 \cdot \mathbb{1}_2)$.

We denote the Higgs five-plets by $\bar{H}_{\bar{5}} = (\bar{T}, \bar{D})$ and $H_5 = (T, D)^T$; as we have seen in the previous section, the induced effective Lagrangian after fermion condensation is given by Eq. (4.11). The doublet is massless if the condition (4.12) is satisfied, in which case the triplet square mass is $m_T^2 = 5\lambda_F^2 \Lambda_G v_{24}/2$, where we have introduced the notation $\lambda_F \equiv \lambda_5$.

We now discuss in more detail the possible origin of v_1 and v_{24}. Recall that $v_1 \neq 0$ does not break $SU(5)$, while $v_{24} \neq 0$ does. From the analysis in Appendix A 2 a, it turns out that

$$\begin{aligned} v_1 &= \frac{1}{5}(\sqrt{3} s_1^{(5)} + \sqrt{2} s_2^{(5)}), \\ v_{24} &= \frac{\sqrt{2}}{\sqrt{3} \cdot 5}(\sqrt{2} s_1^{(5)} - \sqrt{3} s_2^{(5)}) \end{aligned} \tag{5.2}$$

where

$$\begin{aligned} s_1^{(5)} &= \frac{1}{\sqrt{3}} \frac{\langle \mathcal{T}_1^1 + \mathcal{T}_2^2 + \mathcal{T}_3^3 \rangle}{\Lambda_G^2}, \\ s_2^{(5)} &= \frac{1}{\sqrt{2}} \frac{\langle \mathcal{T}_4^4 + \mathcal{T}_5^5 \rangle}{\Lambda_G^2}, \end{aligned} \tag{5.3}$$

which can be nonvanishing as they are SM singlet VEVs. Notice that

$$m_T^2 = \frac{\lambda_F^2 \Lambda_G s_1^{(5)}}{\sqrt{3}}, \qquad m_D^2 = \frac{\lambda_F^2 \Lambda_G s_2^{(5)}}{\sqrt{2}}. \tag{5.4}$$

The triplet mass squared is thus proportional to $s_1^{(5)}$, while the DTS condition (4.12) is equivalent to $s_2^{(5)} = 0$. The breaking of $SU(5)$ to the SM and the DTS are simultaneously achieved if there is a mechanism such that the SM color singlet $\langle \mathcal{T}_1^1 + \mathcal{T}_2^2 + \mathcal{T}_3^3 \rangle \neq 0$, while the SM weak isospin singlet $\langle \mathcal{T}_4^4 + \mathcal{T}_5^5 \rangle = 0$.

In Ref. [23] it was proven, in the context of supersymmetric theories, that the condition $s_2^{(5)} = 0$ is a flat-direction of the supersymmetric potential and thus a supersymmetric minimum, which triggers automatically the DTS mechanism without any fine-tuning. One could here argue along similar lines, provided that supersymmetry be spontaneously broken at a scale $\sim \Lambda_G$. This is an interesting avenue although outside the scope of the present paper. Still, for the nonsupersymmetric GUT we are here considering it is a very predictive scenario that we will be exploring.

As for the GCU, the correction to the kinetic term Lagrangian, Eq. (4.6), becomes simply

$$\delta \mathcal{L} = -\frac{1}{4} \frac{c_F}{\Lambda_G} \mathrm{Tr}(G_{\mu\nu} \langle S + \Sigma \rangle G^{\mu\nu}), \tag{5.5}$$

where $c_F \equiv c_5$ and, consistently with Eq. (4.7), one obtains

$$\begin{aligned} \epsilon_1 &= \epsilon_1^{(1)} + \epsilon_1^{(24)} = \alpha + \beta = \frac{c_F}{\Lambda_G}\left(v_1 - \frac{1}{2} v_{24}\right) \\ &= \frac{c_F}{\Lambda_G}\left(\frac{2}{5}\frac{s_1^{(5)}}{\sqrt{3}} + \frac{3}{5}\frac{s_2^{(5)}}{\sqrt{2}}\right) \\ \epsilon_2 &= \epsilon_2^{(1)} + \epsilon_2^{(24)} = \alpha + 3\beta = \frac{c_F}{\Lambda_G}\left(v_1 - \frac{3}{2} v_{24}\right) \\ &= \frac{c_F}{\Lambda_G}\frac{s_2^{(5)}}{\sqrt{2}} = c_F \frac{m_D^2}{\lambda_5^2 \Lambda_G^2}, \\ \epsilon_3 &= \epsilon_3^{(1)} + \epsilon_3^{(24)} = \alpha - 2\beta = \frac{c_F}{\Lambda_G}(v_1 + v_{24}) \\ &= \frac{c_F}{\Lambda_G}\frac{s_1^{(5)}}{\sqrt{3}} = c_F \frac{m_T^2}{\lambda_5^2 \Lambda_G^2}. \end{aligned} \tag{5.6}$$

As we can see from Eq. (5.6), the contributions leading to GCU are directly related to the DTS mechanism. In particular, the condition (4.12) applied to (5.6) implies that $\epsilon_2 = 0$, or equivalently $\alpha = -3\beta$. Summarizing, we obtain $\epsilon_1 = -2\beta$, $\epsilon_2 = 0$, $\epsilon_3 = -5\beta$, so that $\epsilon_3/\epsilon_1 = 5/2$.

We now carry out the phenomenological analysis for this scenario, initiated with the discussion leading to Eq. (4.15). We recognize that this case corresponds to a particular case of Eq. (3.1), so that $M_X = \mu_{24}$, $\beta \approx 0.020$ and $\alpha \approx -0.056$, together with $\epsilon_1 \approx -0.04, \epsilon_2 = 0, \epsilon_3 \approx -0.09$. The related quantities $(1 + \epsilon_s)\alpha_s(\mu)$, with $s = 1, 2, 3$, are explicitly shown in Fig. 10.

Notice that this scenario corresponds to a single point in Fig. 11, the one with coordinates $(\epsilon_3, \epsilon_2) = (-0.09, 0)$,

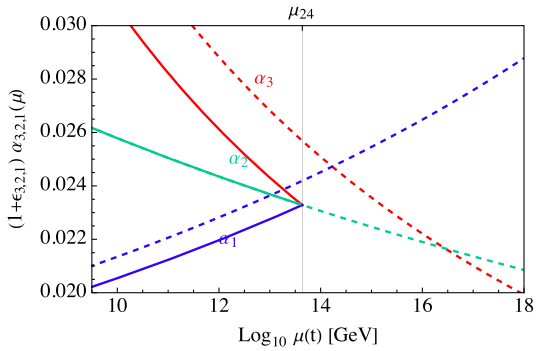


FIG. 10. GCU when the fermion condensate $\bar{5}\times 5$ solves the DTS problem.

explicitly indicated by means of the (green) star lying on the isolevel contour of $M_X = \mu_{24}$. As for proton decay, we have a quite small value for $\alpha_G \approx 0.0247(1+\alpha) \approx 0.023$; despite this, the scale M_X is too low, and the constraint on the proton decay, Eq. (3.7), is not satisfied; the (green) star indeed falls in the excluded shaded region.

This minimalistic and elegant scenario, where DTS and GCU are univocally related, is thus not viable. We envisage two main roads to overcome this impasse: (i) introducing condensates generating also higher dimensional representations, like the 75 and the 200, as we are going to discuss next; (ii) introducing more five-plets, and possibly relating them to flavor; this path will not be followed here.

B. Fermion condensates from $R = 10$

We introduce the fermion ten-plets $\bar{T} \equiv \bar{F}_{10}$ and $T = F_{10}$, in the $\overline{10}$ and 10 representation respectively, so that we can exploit a 10 by 10 matrix notation and decompose the tensor

$$\frac{\mathcal{T}^{AB}_{CD}}{\Lambda_G^2} \equiv \frac{\bar{T}_{CD}\times T^{AB}}{\Lambda_G^2} = (H_1^{(10)} + H_{24}^{(10)} + H_{75}^{(10)})^{AB}_{CD} = S^{AB}_{CD} + \Sigma^{AB}_{CD} + Y^{AB}_{CD}, \tag{5.7}$$

where the latter effective representations are the singlet, adjoint and 75 respectively, whose form is derived in Appendix A 1 b. The ordering of the matrix elements is such that entries from 1 to 3 are related to color, and the last entry is related to the weak hypercharge. The breaking to the SM is realized for

$$\begin{aligned}\langle S\rangle &= v_1 \cdot \mathrm{diag}(\mathbb{1}_3, \mathbb{1}_6, 1),\\ \langle \Sigma\rangle &= 6v_{24}\cdot \mathrm{diag}\left(\mathbb{1}_3, -\frac{1}{4}\cdot\mathbb{1}_6, \frac{3}{2}\right),\\ \langle Y\rangle &= -3v_{75}\cdot \mathrm{diag}(\mathbb{1}_3, -\mathbb{1}_6, 3),\end{aligned} \tag{5.8}$$

where

$$\begin{aligned}v_1 &= \frac{1}{10}(\sqrt{3}s_1^{(10)} + \sqrt{6}s_2^{(10)} + s_3^{(10)}),\\ v_{24} &= \frac{\sqrt{2}}{45\sqrt{3}}(2\sqrt{2}s_1^{(10)} - s_2^{(10)} - \sqrt{6}s_3^{(10)}),\\ v_{75} &= -\frac{1}{18\sqrt{3}}(s_1^{(10)} - \sqrt{2}s_2^{(10)} + \sqrt{3}s_3^{(10)}),\end{aligned} \tag{5.9}$$

and the explicit form of the three SM singlets, $s_i^{(10)}$ with $i = 1, 2, 3$, is given in Appendix A 2 b.

The nonrenormalizable operator relevant to GCU, given in Eq. (4.6), is now

$$\delta\mathcal{L} = -\frac{1}{4}\frac{c_T}{\Lambda_G}\mathrm{Tr}(G_{\mu\nu}\langle S+\Sigma+Y\rangle G^{\mu\nu}) \tag{5.10}$$

where $c_T \equiv c_{10}$ and, consistently with Eq. (4.7), one has

$$\begin{aligned}\epsilon_1 &= \epsilon_1^{(1)} + \epsilon_1^{(24)} + \epsilon_1^{(75)} = \alpha+\beta+\gamma\\ &= \frac{c_T}{\Lambda_G}\left(v_1 - \frac{1}{2}v_{24} - 5v_{75}\right),\\ \epsilon_2 &= \epsilon_2^{(1)} + \epsilon_2^{(24)} + \epsilon_2^{(75)} = \alpha+3\beta-\frac{3}{5}\gamma\\ &= \frac{c_T}{\Lambda_G}\left(v_1 - \frac{3}{2}v_{24} + 3v_{75}\right),\\ \epsilon_3 &= \epsilon_3^{(1)} + \epsilon_3^{(24)} + \epsilon_3^{(75)} = \alpha-2\beta-\frac{1}{5}\gamma\\ &= \frac{c_T}{\Lambda_G}(v_1 + v_{24} + v_{75}).\end{aligned} \tag{5.11}$$

It can be shown that the contribution to ϵ_2 is proportional to

$$v_1 - \frac{3}{2}v_{24} + 3v_{75} = \frac{1}{\sqrt{6}}s_2^{(10)}, \tag{5.12}$$

where $s_2^{(10)}$ turns out to be a SM singlet combination for both color and weak isospin charges.

The DTS for the Higgs $H_5 = (T, D)^T$ works *mutatis mutandis* as in the previous case, as the 75 does not contribute to the induced effective Lagrangian after fermion condensation, Eq. (4.11). Hence, the DTS condition is

$$0 = v_1 - \frac{3}{2}v_{24} = \frac{1}{6\sqrt{3}}(s_1^{(10)} + 2\sqrt{2}s_2^{(10)} + \sqrt{3}s_3^{(10)}). \tag{5.13}$$

The DTS condition above applied to Eq. (5.11) implies that $\alpha = -3\beta$, while applied to Eq. (5.9) allows to write the VEVs as a function of $s_2^{(10)}$ and $s_3^{(10)}$ only,

$$v_1 = -\frac{\sqrt{6}}{10} s_2^{(10)} \left(1 + \frac{\sqrt{2}}{\sqrt{3}} \frac{s_3^{(10)}}{s_2^{(10)}}\right),$$
$$v_{24} = -\frac{\sqrt{6}}{15} s_2^{(10)} \left(1 + \frac{\sqrt{2}}{\sqrt{3}} \frac{s_3^{(10)}}{s_2^{(10)}}\right),$$
$$v_{75} = \frac{\sqrt{6}}{18} s_2^{(10)}. \tag{5.14}$$

We thus recognize that the ratio v_{75}/v_{24} depends on the ratio $s_3^{(10)}/s_2^{(10)}$,

$$\frac{v_{75}}{v_{24}} = -\frac{5}{6} \frac{1}{1 + \frac{\sqrt{2}}{\sqrt{3}} \frac{s_3^{(10)}}{s_2^{(10)}}}. \tag{5.15}$$

We now carry out the phenomenological analysis for this scenario. As anticipated, we obtain Eq. (4.15) with the constraint that $\delta = 0$. As shown by the solid lines in Fig. 9, unification can be achieved for any desired scale μ, hence even larger than μ_{24} (which instead was the only scale allowed in the case $R = 5$ previously studied).

The solid green line of Fig. 11 explicitly shows the location of these models solving simultaneously the DTS and GCU; such line corresponds to the function $\epsilon_2 = \epsilon_3 + 0.45$. Notice that this line includes the (green) star on top of the isolevel for $M_X = \mu_{24}$ (where $\epsilon_2 = \gamma = 0$), as well as the $\alpha = 0$ (red) dot of the isolevel for $M_X = \mu_{75}$ (in the case $\alpha = 0$, due to the DTS condition, $\beta = 0$ too, also implying that the triplet should acquire mass from another mechanism, as for instance the MDM). Both these scenarios are however in trouble with proton decay constraints. On the other hand, all points along the (green solid) branch which do not fall into the shaded (orange) region are safe with respect to proton decay constraints. So, there are many possible viable scenarios with the effective $10 \times \overline{10}$, where the conditions on DTS and GCU are enforced.

In the following, we focus our attention on a particular case, that is the point on top of the isolevel with $M_X = \mu_{32}^{\rm SM}$, emphasized by the open green circle in Fig. 11. It corresponds to the mirage SUSY case denoted by mS_1 (for which $\gamma/\beta = 25/2$), and described by Eq. (3.4), with the additional constraint $\alpha = -3\beta \approx 0.03$ (see also the solid lines of Fig. 9), so that $\epsilon_2 = \epsilon_3 \approx 0.09$. As shown in Fig. 12, unification is achieved with $\alpha_G \approx 0.0236$ (which is significantly smaller than in the low energy SUSY case). As a consequence, this scenario satisfies the constraints on proton lifetime, Eq. (3.7). Notice also that, using the DTS condition ($v_1 = \frac{3}{2} v_{24}$) and the condition $\gamma/\beta = 25/2$, from Eq. (5.11) we get the corresponding values of the parameters

$$\alpha = \frac{3}{2} \frac{c_T}{\Lambda_G} v_{24}, \quad \beta = -\frac{1}{2} \frac{c_T}{\Lambda_G} v_{24}, \quad \gamma = -\frac{25}{4} \frac{c_T}{\Lambda_G} v_{24}. \tag{5.16}$$

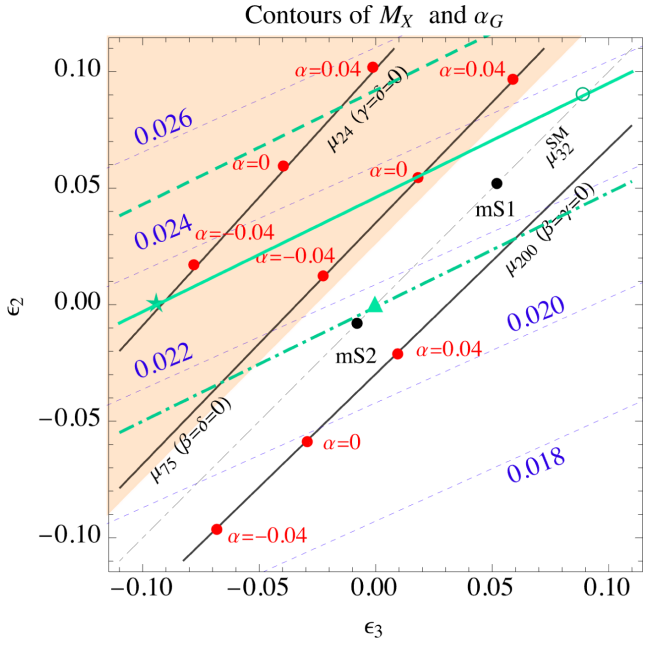


FIG. 11. Contours of M_X (solid black and dot-dashed black) and α_G (dashed blue). Configurations fulfilling the DTS condition $\alpha = -3\beta$ are shown in green. For $R = 5$, the DTS condition is satisfied by a single point, corresponding to the green star on top of the isolevel $M_X = \mu_{24}$. For $R = 10$, the DTS condition is satisfied within the green solid line; the open circle on top of $\mu_{32}^{\rm SM}$ represents the case mS_1, for which $\gamma/\beta = 25/2$. For $R = 24$, the DTS condition is satisfied in the whole plane: the dashed and dot-dashed green lines represent the models with $\delta = +0.15$ and $\delta = -0.15$; the latter crosses the isolevel $\mu_{32}^{\rm SM}$ at the point represented by the green triangle, for which $\epsilon_2 = \epsilon_3 = 0$ and $\alpha_G = \alpha_{32}^{\rm SM}$. The shaded (orange) region is excluded by proton decay constraints.

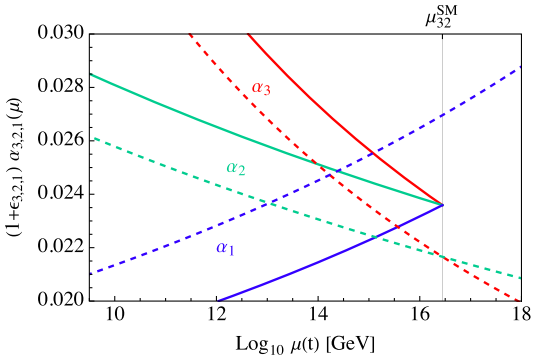


FIG. 12. GCU for the particular case mS_1, when the fermion condensate $\overline{10} \times 10$ solves the DTS problem.

and the relation between the VEVs

$$v_{75} = \frac{5}{4} v_{24}, \tag{5.17}$$

which, according to Eq. (5.15), implies that mS_1 is associated to the following specific direction implementing the

dynamical breaking

$$\frac{3}{\sqrt{6}}\frac{\langle \mathcal{T}^{ab}_{ab}\rangle}{\langle \mathcal{T}^{ab}_{ab}\rangle} = \frac{s_3^{(10)}}{s_2^{(10)}} = -\frac{5}{\sqrt{6}}. \tag{5.18}$$

C. Fermion condensates for $R = 24$

We finally introduce the fermion 24-plet fields $\bar{A} \equiv \bar{F}_{24}$ and $A \equiv F_{24}$, so that the tensor has the decomposition

$$\frac{\mathcal{T}^{AB}_{CD}}{\Lambda_G^2} \equiv \frac{\bar{A}_{CD} \times A^{AB}}{\Lambda_G^2} = (H_1^{(24)} + H_{24}^{(24)} + H_{75}^{(24)} + H_{200}^{(24)})^{AB}_{CD}$$
$$= S^{AB}_{CD} + \Sigma^{AB}_{CD} + Y^{AB}_{CD} + Z^{AB}_{CD}, \tag{5.19}$$

where the latter effective representations are the singlet, adjoint, 75 and 200 respectively. Their form has been derived explicitly in Ref. [14], by using 24×24 matrices. The ordering of the elements is such that entries from 1 to 8 are related to color, those from 9 to 11 to weak isospin, the 12th entry is related to the hypercharge, and those from 13 to 24 are related to the heavy leptoquarks. The breaking to the SM is realized when

$$\begin{aligned}
\langle S\rangle &= v_1 \mathbb{1}_{24},\\
\langle \Sigma\rangle &= v_{24}\mathrm{diag}\left(\mathbb{1}_8, -\frac{3}{2}\cdot\mathbb{1}_3, -\frac{1}{2}, -\frac{1}{4}\cdot\mathbb{1}_{12}\right),\\
\langle Y\rangle &= v_{75}\mathrm{diag}(\mathbb{1}_8, 3\cdot\mathbb{1}_3, -5, -\mathbb{1}_{12}),\\
\langle Z\rangle &= v_{200}\mathrm{diag}(\mathbb{1}_8, 2\cdot\mathbb{1}_3, 10, -2\cdot\mathbb{1}_{12}).
\end{aligned} \tag{5.20}$$

The nonrenormalizable operator of Eq. (4.6) now includes all the contributions which, according to the previous notation, read

$$\delta\mathcal{L} = -\frac{1}{4}\frac{c_A}{\Lambda_G}\mathrm{Tr}(G_{\mu\nu}\langle S + \Sigma + Y + Z\rangle G^{\mu\nu}) \tag{5.21}$$

where $c_A \equiv c_{24}$ and, consistently with Eq. (4.7), one has

$$\begin{aligned}
\epsilon_1 &= \epsilon_1^{(1)} + \epsilon_1^{(24)} + \epsilon_1^{(75)} + \epsilon_1^{(200)} = \alpha + \beta + \gamma + \delta\\
&= \frac{c_A}{\Lambda_G}\left(v_1 - \frac{1}{2}v_{24} - 5v_{75} + 10v_{200}\right),\\
\epsilon_2 &= \epsilon_2^{(1)} + \epsilon_2^{(24)} + \epsilon_2^{(75)} + \epsilon_2^{(200)} = \alpha + 3\beta - \frac{3}{5}\gamma + \frac{1}{5}\delta\\
&= \frac{c_A}{\Lambda_G}\left(v_1 - \frac{3}{2}v_{24} + 3v_{75} + 2v_{200}\right),\\
\epsilon_3 &= \epsilon_3^{(1)} + \epsilon_3^{(24)} + \epsilon_3^{(75)} + \epsilon_3^{(200)} = \alpha - 2\beta - \frac{1}{5}\gamma + \frac{1}{10}\delta\\
&= \frac{c_A}{\Lambda_G}(v_1 + v_{24} + v_{75} + v_{200}).
\end{aligned} \tag{5.22}$$

The DTS works as in the previous cases, since the 75 and 200 do not contribute to the effective Lagrangian after fermion condensation, Eq. (4.11). In particular, the DTS condition, $v_1 - \frac{3}{2}v_{24} = 0$, always implies that $\alpha = -3\beta$. Also in this scenario, where we have one more parameter with respect to the case with $R = 10$, GCU and DTS can be achieved at any scale, μ.

In order to carry out the phenomenological analysis for this scenario, we come back to Eq. (4.15) and Fig. 9. As for the location of these models in Fig. 11, we obtain an area, rather than a line; as an example, the dashed and dot-dashed green lines correspond to selected values of δ, respectively $\delta = +0.15$ and $\delta = -0.15$. As a result, all the mirage SUSY models can be reproduced within a condensate with $R = 24$.

Let us focus in particular on the scenario in which DTS is realized, and GCU is achieved at $M_X = \mu_{32}^{\mathrm{SM}}$, with the additional condition that $\alpha_G = \alpha_{32}^{\mathrm{SM}}$. The latter condition implies $\epsilon_2 = \epsilon_3 = 0$, hence $\delta = 3\gamma$ and $\beta = \gamma/50$, which leads to

$$-\alpha = 3\beta = \frac{3}{50}\gamma = \frac{1}{50}\delta. \tag{5.23}$$

All the job of unification is thus carried out by ϵ_1,

$$-0.197 \approx \epsilon_1 = \alpha + \beta + \gamma + \delta = 198\beta, \tag{5.24}$$

as shown in Fig. 13. Numerically, it turns out that $\beta \simeq 0.001$; it is interesting that such a small value leads to successful GCU and DTS within this scenario. On the other hand, notice that we have $\delta \approx -0.15$. The green triangle of Fig. 11 displays the location of this particular scenario.

Notice that implementing the DTS condition ($\alpha = -3\beta$) within the scenario mS_2 (for which $\delta/\gamma = 4$ and $\beta = 0$), leads to $\alpha = \beta = 0$, that is to a vanishing mass for the Higgs triplet. This scenario, corresponding to the black circle labeled by mS_2 in Fig. 11, would thus require an alternative mechanism for the triplet to acquire a mass, as for instance the MDM.

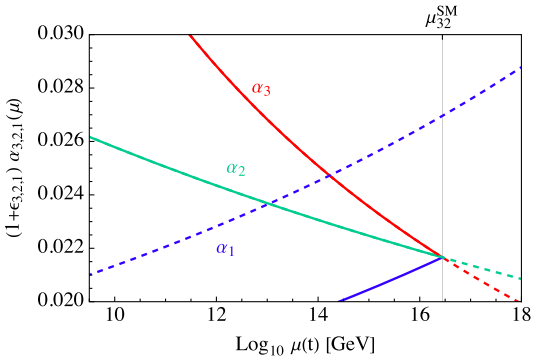


FIG. 13. GCU for the particular mirage SUSY case in which $\alpha_G = \alpha_{32}^{\mathrm{SM}}$, when the fermion condensate 24×24 solves the DTS.

VI. CONCLUSIONS

We considered the phenomenology of a nonsupersymmetric $SU(5)$ framework [8] in which GCU is achieved because the SM gauge couplings receive UV origin corrections, denoted by ϵ_i $(i = 1, 2, 3)$, which are induced by a nonrenormalizable $d = 5$ kinetic operator [4–7,11–14]. The representations that can in principle be involved are denoted by H_r, with $r = 1$, 24, 75, 200, and $SU(5)$ is broken to the SM by the VEVs of the representations $r = 24$, 75, 200.

We first studied the impact on GCU, especially on the unification scale M_X and the unified coupling α_G, for the various representations, separately and in some relevant combinations. According to the proposal in Ref. [1], we compared the most representative scenarios by mapping them in the plane of the non-Abelian corrections, ϵ_2 and ϵ_3, as done in Figs. 7 and 8.

We then questioned the nature of the representations H_r, in relation with the doublet-triplet splitting problem. The electroweak scale Higgs doublet and its conjugate are indeed part of the 5 and $\bar{5}$ scalar Higgs representations, together with the GUT scale triplet and antitriplet. In the former literature, H_r have been considered to be elementary scalars, spontaneously breaking $SU(5)$ (for $r > 1$) as in the Higgs mechanism; in this case there is a huge number of parameters in the Lagrangian, and there is no direct relation between the parameters providing GCU and those involved in the DTS.

A reduction in the number of parameters is achieved by postulating that the representations H_r are effective ones, coming from some UV completion of $SU(5)$. In particular, we postulate that they are originated from the condensation of extra Dirac fermions, F_R and $\bar{F}_R$, sitting in the representation R and $\bar{R}$ of $SU(5)$, and in the representation R_G and $\bar{R}_G$ of some confining group G. Interestingly, the phenomenology of this framework can be studied without specifying the details of the group G and its matter content. Indeed, in the case that $R = 5$, 10, 24, the breaking of $SU(5)$ is achieved dynamically by meson in the effective theory having the form $\langle \bar{F}_R \times F_R \rangle$, which are singlets under G and whose VEV is given by a specific combination of the $\langle H_r^{(R)} \rangle$, which differs upon the choice of R, according to Eq. (4.5). We showed that, due to the reduction in the number of parameters, in this framework it is possible to relate the parameters involved in the DTS with those accounting for GCU; the relation is univocally determined for $R = 5$, and relaxes increasingly for $R = 10$ and $R = 24$.

Specifically, solving simultaneously the DTS and GCU in the case $R = 5$, leads to a sharp prediction for M_X, which turns out to be too low and in conflict with proton decay constraints; this scenario is represented by the green star in Fig. 11. In the case $R = 10$, the parameter space solving simultaneously DTS and GCU becomes a line, as shown via the solid green one in Fig. 11; now M_X can be made large enough to escape the proton decay bounds. With $R = 24$, the whole area of Fig. 11 is at disposal to solve the DTS and GCU, with even larger possibilities to meet proton decay constraints.

Notice that, in the above framework, we have considered the Higgs doublet as an elementary scalar which is contained into the Higgs five-plet H_5 of $SU(5)$. Of course the possibility of considering the Higgs five-plet as a fermion condensate does exist in case there exist extra fermions in the 5 and 10 irreps F and T (and $\bar{F}$, $\bar{T}$) and condensates as $\langle T^{AB} \otimes \bar{F}_B \rangle = H_5^A$ and $\langle \bar{T}_{AB} \otimes F^B \rangle = H_{5A}^\dagger$. The DTS coupling can be generated as in the case of elementary scalar Higgs five-plets, as described in the Appendix C 1.

Summarizing, we proved that in the case of a dynamical breaking of the GUT group—here in particular we considered $SU(5)$—it is possible to achieve gauge coupling unification and simultaneously solve the doublet-triplet splitting problem. We find it remarkable that the phenomenological implications of such a scenario can be grasped even without entering into the details of the confining group. In particular, constraints from proton decay imply that the extra fermions which are going to condensate, dynamically breaking $SU(5)$, cannot belong to the 5 representation (the fundamental representation) of $SU(5)$; interestingly, viable models require such fermions to belong to larger representations, such as the 10 or 24.

To conclude, let us provide some comments about possible extensions of the present work, to be explored elsewhere. An interesting possibility to build viable models is to consider more than a single 5 representation, thus "flavoring" the fermion condensates; in this way it might be possible to achieve GCU and DTS simultaneously, avoiding constraints on proton decay. Notice also that in the present work the Higgs was put in the 5 representation of $SU(5)$, as usually done; another interesting possibility would be to put it in the 45 representation, and to assess the phenomenological impact of such choice.

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DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: GROUP THEORY FOR $SU(5)$

In this Appendix we summarize the methods adopted to deal with tensors in $SU(5)$. Large representations (like the 75) can be conveniently written as products of smaller representations (like 10 and $\overline{10}$). In this way, it is not difficult to recognize how the SM multiplets are embedded in the large representations.

1. Basic conventions for the tensors in $SU(5)$

In $SU(5)$ the matter fields of a single family are contained in the 10 and $\bar{5}$ representations. We write them following the conventions of [21] (see also the Particle Data Group review on *grand unified theories* [15]), so that

$$\psi_{10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & u^1 & d^1 \\ -u_3^c & 0 & u_1^c & u^2 & d^2 \\ u_2^c & -u_1^c & 0 & u^3 & d^3 \\ -u^1 & -u^2 & -u^3 & 0 & e^c \\ -d^1 & -d^2 & -d^3 & -e^c & 0 \end{pmatrix},$$

$$\psi_{\bar{5}} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e \\ -\nu_e \end{pmatrix}. \tag{A1}$$

Our conventions are such that the fundamental representation corresponds to a superscript in tensorial notation and the antifundamental (conjugated fundamental) to a subscript in tensorial notation. The $\bar{5}$ representation, $\psi_{\bar{5}A}$, is written as a tensor with a single lower index, $A = 1, \ldots, 5$; the 10 representation, ψ_{10}^{AB}, is written as a tensor with two antisymmetric upper indices, $A, B = 1, \ldots, 5$. Note that the $SU(3)$ indices correspond to $A, B = 1, 2, 3$, while the $SU(2)$ ones are selected for $A, B = 4, 5$. We will sometimes write α (a) instead of A if $A = 1, 2, 3$ (4, 5).

As for scalars, the SM Higgs doublet fits in the fundamental representation, H_5,

$$H_5 = \begin{pmatrix} T^1 \\ T^2 \\ T^3 \\ D^+ \\ D^0 \end{pmatrix}, \tag{A2}$$

breaking $SU(2)$ when $\langle D^0 \rangle = v/\sqrt{2}$. In a nonsupersymmetric context,[8] one can introduce just H_5^A and its conjugate, $(H_5)_A^\dagger$, which transforms as a $\bar{5}$.

We now discuss how to write large representations in terms of smaller ones.

a. Effective representations from $\bar{5} \times 5$

We define $\bar{5}_A \times 5^B \equiv T_A^B$, which can be split according to

$$T_A^B = \underbrace{S_A^B}_{1} + \underbrace{\Sigma_A^B}_{24} \tag{A4}$$

where we identify the 1 with S, the 24 with Σ. Here and in the following summation over repeated indices has to be understood. The traceless condition for the 24 is thus $\Sigma_A^A = 0$.

Defining

$$s \equiv T_A^A, \tag{A5}$$

one has

$$S_A^B = \frac{1}{5}\delta_A^B s, \qquad \Sigma_A^B = T_A^B - \frac{1}{5}\delta_A^B s. \tag{A6}$$

b. Effective representations from $\overline{10} \times 10$

We define $10^{AB} \times \overline{10}_{CD} \equiv T_{CD}^{AB}$, so that T_{CD}^{AB} is antisymmetric in the upper and lower pair of indices and thus corresponds to 100 independent fields. It can be split according to

$$T_{CD}^{AB} = \underbrace{S_{CD}^{AB}}_{1} + \underbrace{\Sigma_{CD}^{AB}}_{24} + \underbrace{Y_{CD}^{AB}}_{75} \tag{A7}$$

where we identify the 1 with S, the 24 with Σ and the 75 with Y.

Defining

$$s \equiv T_{MN}^{MN}, \qquad \sigma_C^A \equiv T_{MC}^{MA} - \frac{1}{5}\delta_C^A T_{MN}^{MN} \tag{A8}$$

[8] In a supersymmetric framework instead, one has to introduce two Higgs fields, $\bar{H}_{\bar{5}A}$ and H_5^A, belonging to the $\bar{5}$ and 5 representations, respectively,

$$\bar{H}_{\bar{5}} = \begin{pmatrix} T_{d1} \\ T_{d2} \\ T_{d3} \\ D_d^- \\ -D_d^0 \end{pmatrix}, \qquad H_5 = \begin{pmatrix} T_u^1 \\ T_u^2 \\ T_u^3 \\ D_u^+ \\ D_u^0 \end{pmatrix}, \tag{A3}$$

containing the SM Higgs doublet and its conjugate. When $SU(2)$ is spontaneously broken, $\langle D_d^0 \rangle = v_d/\sqrt{2}$, $\langle D_u^0 \rangle = v_u/\sqrt{2}$, giving rise to down and up quark masses respectively.

one has

$$S^{AB}_{CD} = \frac{1}{20}(\delta^A_C\delta^B_D - \delta^A_D\delta^B_C)T^{MN}_{MN}, \tag{A9}$$

$$\Sigma^{AB}_{CD} = \frac{1}{3}\left[\delta^A_C\left(T^{MB}_{MD} - \frac{1}{5}\delta^B_D T^{MN}_{MN}\right) - \delta^A_D\left(T^{MB}_{MC} - \frac{1}{5}\delta^B_C T^{MN}_{MN}\right) - \delta^B_C\left(T^{MA}_{MD} - \frac{1}{5}\delta^A_D T^{MN}_{MN}\right) + \delta^B_D\left(T^{MA}_{MC} - \frac{1}{5}\delta^A_C T^{MN}_{MN}\right)\right], \tag{A10}$$

and

$$Y^{AB}_{CD} = T^{AB}_{CD} - (S^{AB}_{CD} + \Sigma^{AB}_{CD}). \tag{A11}$$

2. The method for catching SM multiplets

One can apply a simple recipe to recognize how the SM multiplets are included in the large effective representations. We will use Table I.

a. SM singlets and VEVs for $\bar{5}\times 5$

The starting point is the identification of the SM multiplets in the $\bar{5}$ and 5 representations

$$\bar{F}_A = \underbrace{\bar{5}_\alpha}_{(\bar{3},1,-1)} + \underbrace{\bar{5}_a}_{(1,2,1/2)}, \quad F^A = \underbrace{5^\alpha}_{(3,1,1)} + \underbrace{5^b}_{(1,2,-1/2)}. \tag{A12}$$

There are two SM singlets (1, 1, 0) arising from the product $\bar{F}\times F$

$$T^\beta_\alpha \equiv \underbrace{\bar{F}_\alpha}_{(\bar{3},1,-1)}\underbrace{F^\beta}_{(3,1,1)} = \underbrace{\frac{1}{3}\delta^\beta_\alpha T^\alpha_\alpha}_{(1,1,0)} + \underbrace{T^\beta_\alpha - \frac{1}{3}\delta^\beta_\alpha T^\alpha_\alpha}_{(8,1,0)}, \tag{A13}$$

and

$$T^b_a \equiv \underbrace{\bar{F}_a}_{(1,2,-1/2)}\underbrace{F^b}_{(1,2,1/2)} = \underbrace{\frac{1}{2}\delta^b_a T^a_a}_{(1,1,0)} + \underbrace{T^b_a - \frac{1}{2}\delta^b_a T^a_a}_{(1,3,0)}. \tag{A14}$$

We call $s^{(5)}_1$ the SM singlet related to color, and call $s^{(5)}_2$ the one related to weak isospin. With a normalization choice, they are

$$s^{(5)}_1 = \frac{1}{\sqrt{3}}(T^1_1 + T^2_2 + T^3_3), \quad s^{(5)}_2 = \frac{1}{\sqrt{2}}(T^4_4 + T^5_5). \tag{A15}$$

From the previous section, Eq. (A5), it turns out that

$$s = \sqrt{3}s^{(5)}_1 + \sqrt{2}s^{(5)}_2. \tag{A16}$$

Now, comparing with the previous expressions for S^B_A and Σ^B_A in Eq. (A6), and adopting a 5×5 matrix notation, it turns out that

$$\langle H^{(5)}_1\rangle = \langle S\rangle = v^{(5)}_1 \mathrm{diag}(1,1,1,1,1),$$
$$\langle H^{(5)}_{24}\rangle = \langle \Sigma\rangle = v^{(5)}_{24}\mathrm{diag}(1,1,1,-3/2,-3/2), \tag{A17}$$

where

$$v^{(5)}_1 = \frac{1}{5}(\sqrt{3}s^{(5)}_1 + \sqrt{2}s^{(5)}_2),$$
$$v^{(5)}_{24} = \frac{\sqrt{2}}{\sqrt{3}\cdot 5}(\sqrt{2}s^{(5)}_1 - \sqrt{3}s^{(5)}_2), \tag{A18}$$

and we have introduced the VEV of $H^{(5)}_1, H^{(5)}_{24}$ to match the notation of Sec. IV B.

With these findings, one can calculate

$$\frac{\mathrm{Tr}(G_{\mu\nu}(\langle H^{(5)}_1\rangle + \langle H^{(5)}_{24}\rangle)G^{\mu\nu})_{G^{\mathrm{SM}}_i}}{\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu})} = v^{(5)}_1 + X_{24,i}v^{(5)}_{24}, \tag{A19}$$

where $i = 1, 2, 3$ corresponds to $G^{\mathrm{SM}} = \{U(1), SU(2), SU(3)\}$ and $X_{24} = \{-1/2, -3/2, 1\}$. The latter results for X_{24} are in agreement with Eq. (2.6).

b. SM singlets and VEVs for $\overline{10}\times 10$

The starting point is the identification of the SM multiplets in the 10 and $\overline{10}$ representations.

$$T^{AB} = \underbrace{10^{\alpha\beta}}_{(\bar{3},1,-2/3)} + \underbrace{10^{ab}}_{(3,2,1/6)} + \underbrace{10^{ab}}_{(1,1,1)},$$
$$\bar{T}_{AB} = \underbrace{10_{\alpha\beta}}_{(3,1,2/3)} + \underbrace{10_{\alpha b}}_{(\bar{3},2,-1/6)} + \underbrace{10_{ab}}_{(1,1,-1)}. \tag{A20}$$

There are three singlets (1, 1, 0) arising from the product $T\times\bar{T}$. The first singlet, $s^{(10)}_1$, is the one related to color only, and is obtained from

$$T^{\alpha\beta}_{\gamma\delta} \equiv \underbrace{T^{\alpha\beta}}_{(\bar{3},1,-2/3)}\underbrace{\bar{T}_{\gamma\delta}}_{(3,1,2/3)} = \underbrace{\frac{1}{6}(\delta^\alpha_\gamma\delta^\beta_\delta - \delta^\alpha_\delta\delta^\beta_\gamma)T^{\mu\nu}_{\mu\nu}}_{(1,1,0)} + \underbrace{T^{\alpha\beta}_{\gamma\delta} - \frac{1}{6}(\delta^\alpha_\gamma\delta^\beta_\delta - \delta^\alpha_\delta\delta^\beta_\gamma)T^{\mu\nu}_{\mu\nu}}_{(8,1,0)}. \tag{A21}$$

TABLE I. The SM multiplets contained in some of the $SU(5)$ representations.

$SU(5) \supset SU(3)\times SU(2)\times U(1)$
$1 = (1,1,0)$
$5 = (1,2,1/2) + (3,1,-1)$
$10 = (1,1,1) + (\bar{3},1,-2/3) + (3,2,1/6)$
$24 = (1,1,0) + (1,3,0) + (3,2,-5/6) + (\bar{3},2,5/6) + (8,1,0)$
$75 = (1,1,0) + (3,1,5/3) + (\bar{3},1,-5/3) + (3,2,-5/6) +$ $(\bar{3},2,5/6) + (\bar{6},2,-5/6) + (6,2,5/6) + (8,1,0) + (8,3,0)$

Choosing a convenient normalization, it is

$$s_1^{(10)} = \frac{1}{\sqrt{3}}(T_{12}^{12} + T_{13}^{13} + T_{23}^{23}) = \frac{1}{\sqrt{3}} T^{\mu\nu}_{\mu\nu}|_{\mu<\nu}. \quad \text{(A22)}$$

The second singlet, $s_2^{(10)}$, related to both color and weak isospin, comes from the product

$$T^{ab}_{\gamma d} \equiv \underbrace{T^{ab}}_{(3,2,1/6)} \underbrace{\bar{T}_{\gamma d}}_{(\bar{3},2,-1/6)} = \underbrace{\frac{1}{6}\delta^a_\gamma \delta^b_d T^{\mu n}_{\mu n}}_{(1,1,0)} + \underbrace{\frac{1}{3}\delta^a_\gamma T^{\mu b}_{\mu d} - \frac{1}{6}\delta^a_\gamma \delta^b_d T^{\mu n}_{\mu n}}_{(1,3,0)} + \cdots, \quad \text{(A23)}$$

and turns out to be given by

$$s_2^{(10)} = \frac{1}{\sqrt{6}}(T_{14}^{14} + T_{24}^{24} + T_{34}^{34} + T_{15}^{15} + T_{25}^{25} + T_{35}^{35}) = \frac{1}{\sqrt{6}} T^{\mu n}_{\mu n}. \quad \text{(A24)}$$

Finally, the last singlet, $s_3^{(10)}$, related to weak isospin only, is obtained from

$$T^{ab}_{cd} \equiv \underbrace{T^{ab}}_{(1,1,1)} \underbrace{\bar{T}_{cd}}_{(1,1,-1)} = \underbrace{\frac{1}{2}(\delta^a_c \delta^b_d - \delta^a_d \delta^b_c) T^{mn}_{mn}}_{(1,1,0)}, \quad \text{(A25)}$$

and is simply

$$s_3^{(10)} = T_{45}^{45} = T^{mn}_{mn}|_{m<n}. \quad \text{(A26)}$$

Now, comparing with the expressions for S, Σ, and Y in Eqs. (A9), (A10), and (A11), and adopting a 10×10 matrix notation, it turns out that

$$\langle H_1^{(10)} \rangle = \langle S \rangle = v_1^{(10)} \cdot \mathrm{diag}(\mathbb{1}_3, \mathbb{1}_6, 1),$$
$$\langle H_{24}^{(10)} \rangle = \langle \Sigma \rangle = 6 v_{24}^{(10)} \cdot \mathrm{diag}\left(\mathbb{1}_3, -\frac{1}{4}\cdot \mathbb{1}_6, \frac{3}{2}\right),$$
$$\langle H_{75}^{(10)} \rangle = \langle Y \rangle = -3 v_{75}^{(10)} \cdot \mathrm{diag}(\mathbb{1}_3, -\mathbb{1}_6, 3), \quad \text{(A27)}$$

where

$$v_1^{(10)} = \frac{1}{10}(\sqrt{3} s_1^{(10)} + \sqrt{6} s_2^{(10)} + s_3^{(10)}) \quad \text{(A28)}$$

$$v_{24}^{(10)} = \frac{\sqrt{2}}{45\sqrt{3}}(2\sqrt{2} s_1^{(10)} - s_2^{(10)} - \sqrt{6} s_3^{(10)}) \quad \text{(A29)}$$

$$v_{75}^{(10)} = -\frac{1}{18\sqrt{3}}(s_1^{(10)} - \sqrt{2} s_2^{(10)} + \sqrt{3} s_3^{(10)}). \quad \text{(A30)}$$

and we have introduced the VEVs of $H_1^{(10)}$, $H_{24}^{(10)}$ and $H_{75}^{(10)}$ to match and clarify the notation of Sec. IV B.

Hence, one finds that, for $i = 1, 2, 3$,

$$\frac{\mathrm{Tr}(G_{\mu\nu}(\langle H_1^{(10)} \rangle + \langle H_{24}^{(10)} \rangle + \langle H_{75}^{(10)} \rangle) G^{\mu\nu})_{G_i^{\mathrm{SM}}}}{\mathrm{Tr}(G_{\mu\nu} G^{\mu\nu})} = v_1^{(10)} + X_{24,i} v_{24}^{(10)} + X_{75,i} v_{75}^{(10)}, \quad \text{(A31)}$$

where $G^{\mathrm{SM}} = \{U(1), SU(2), SU(3)\}$, $X_{24} = \{-1/2, -3/2, 1\}$ and $X_{75} = \{-5, 3, 1\}$. These results are in agreement with Eq. (2.6).

c. SM singlets and VEVs for 24×24

Adopting now a 24×24 matrix notation (see the Appendix of Ref. [14]) the VEVs can be written as

$$\langle H_1^{(24)} \rangle = \langle S \rangle = v_1^{(24)} \mathbb{1}_{24},$$
$$\langle H_{24}^{(24)} \rangle = \langle \Sigma \rangle = v_{24}^{(24)} \mathrm{diag}\left(\mathbb{1}_8, -\frac{3}{2}\cdot \mathbb{1}_3, -\frac{1}{2}, -\frac{1}{4}\cdot \mathbb{1}_{12}\right),$$
$$\langle H_{75}^{(24)} \rangle = \langle Y \rangle = v_{75}^{(24)} \mathrm{diag}(\mathbb{1}_8, 3 \cdot \mathbb{1}_3, -5, -\mathbb{1}_{12}),$$
$$\langle H_{200}^{(24)} \rangle = \langle Z \rangle = v_{200}^{(24)} \mathrm{diag}(\mathbb{1}_8, 2 \cdot \mathbb{1}_3, 10, -2 \cdot \mathbb{1}_{12}). \quad \text{(A32)}$$

Hence, one finds that, for $i = 1, 2, 3$,

$$\frac{\mathrm{Tr}(G_{\mu\nu}(\sum_{r=1,24,75,200} \langle H_r^{(24)} \rangle) G^{\mu\nu})_{G_i^{\mathrm{SM}}}}{\mathrm{Tr}(G_{\mu\nu} G^{\mu\nu})} = v_1^{(24)} + X_{24,i} v_{24}^{(24)} + X_{75,i} v_{75}^{(24)} + X_{200,i} v_{200}^{(24)}, \quad \text{(A33)}$$

where $G^{\mathrm{SM}} = \{U(1), SU(2), SU(3)\}$, $X_{24} = \{-1/2, -3/2, 1\}$, $X_{75} = \{-5, 3, 1\}$ and $X_{200} = \{10, 2, 1\}$. These results are in agreement with Eq. (2.6).

d. Summary and notation

Notice that we defined the VEVs for $r = 10$ in Eq. (A27) in such a way that, for the effective representations 1 and 24, they precisely correspond to those of the case with $r = 5$, see Eq. (A19). Similarly, we defined the VEVs for $r = 24$ in Eq. (A32) in such a way that, for the effective representations 1,24 and 75, they precisely correspond to those of the case with $r = 10$, see Eq. (A31). Hence, in order to simplify the notation, we can define

$$v_1 \equiv v_1^{(24)} = v_1^{(10)} = v_1^{(5)}, \qquad v_{24} \equiv v_{24}^{(24)} = v_{24}^{(10)} = v_{24}^{(5)},$$
$$v_{75} \equiv v_{75}^{(24)} = v_{75}^{(10)}, \qquad v_{200} \equiv v_{200}^{(24)}. \quad \text{(A34)}$$

and summarize our results by means of the following expression

$$\frac{\mathrm{Tr}(G_{\mu\nu}(\langle H_1^{(R)} + H_{24}^{(R)} + a_R H_{75}^{(R)} + b_R H_{200}^{(R)}\rangle)G^{\mu\nu})_{G_i^{\rm SM}}}{\mathrm{Tr}(G_{\mu\nu}G^{\mu\nu})} = v_1 + X_{24,i}v_{24} + a_R X_{75,i}v_{75} + b_R X_{200,i}v_{200}, \quad \text{(A35)}$$

where $a_5 = b_5 = b_{10} = 0$, while $a_{10} = a_{24} = b_{24} = 1$.

In addition, thanks to Eq. (A34), for any $R = 5, 10, 24$ it is possible to write the VEVs of the effective representations $r = 1, 24$ using an equivalent 5×5 matrix form

$$\langle H_1^{(R)}\rangle \doteq v_1 \mathbb{1}_5, \quad \langle H_{24}^{(R)}\rangle \doteq v_{24}\mathrm{diag}(\mathbb{1}_3, -3/2 \cdot \mathbb{1}_2). \quad \text{(A36)}$$

APPENDIX B: APPROXIMATE SOLUTION FOR GCU

As for the approximate solution, from Eq. (2.9) and expanding to first order in $\tilde\beta$, $\tilde\gamma$, and $\tilde\delta$, one obtains

$$f_{21}(\mu) = \frac{1+\alpha+\beta+\gamma+\delta}{1+\alpha+3\beta-\frac{3}{5}\gamma+\frac{1}{5}\delta} \approx 1 - 2\tilde\beta + \frac{8}{5}\tilde\gamma + \frac{4}{5}\tilde\delta$$
$$f_{31}(\mu) = \frac{1+\alpha+\beta+\gamma+\delta}{1+\alpha-2\beta-\frac{1}{5}\gamma+\frac{1}{10}\delta} \approx 1 + 3\tilde\beta + \frac{6}{5}\tilde\gamma + \frac{9}{10}\tilde\delta. \quad \text{(B1)}$$

The system of two equations above can now be solved for any value of $\mu = M_X$, with redundancy of solutions, as there are three unknowns ($\tilde\beta$, $\tilde\gamma$, and $\tilde\delta$),

$$\tilde\beta = -\frac{2}{3}\left(\frac{f_{21}-1}{4} - \frac{f_{31}-1}{3} + \frac{1}{10}\tilde\delta\right),$$
$$\tilde\gamma = \frac{5}{36}\left(3f_{21} + 2f_{31} - 5 - \frac{21}{5}\tilde\delta\right). \quad \text{(B2)}$$

For fixed values of $\tilde\delta$, we compare the exact and approximate solutions displayed in Figs. 2 and 14 respectively: they agree pretty well. The comparison can be used to check the consistency of the perturbative approach.

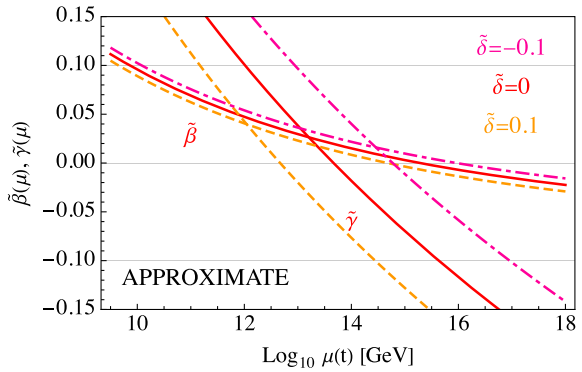


FIG. 14. Solid lines are $\tilde\beta(\mu)$ and $\tilde\gamma(\mu)$ according to the approximate solution, for $\tilde\delta = 0, \pm 0.1$.

APPENDIX C: A (TOY) UV COMPLETION FOR NONRENORMALIZABLE OPERATORS

In this paper we have used the dynamical breaking of $SU(5)$ to impose conditions on GCU via the $d = 5$ operators given by Eq. (2.1). To this end we are using the higher dimensional operator in Eq. (4.9) from the $\langle \bar F_R F_R\rangle$ condensate (the DTS operator), along with the nonrenormalizable operators (5.5), (5.10), and (5.21) from the respective condensates $\langle \bar F F\rangle$, $\langle \bar T T\rangle$ and $\langle \bar A A\rangle$ (kinetic operators). In this section we provide some toy UV completions that can give rise to such operators at the condensation scale.

1. The DTS operator

Here we are considering the $d = 5$ operator

$$\mathcal{L}_{\rm eff} = -\frac{c_R}{\Lambda} H_5^\dagger (F_R \bar F_R) H_5, \quad \text{(C1)}$$

where Λ is the cutoff of the theory, giving rise to the DTS. In our theory we assume that the $SU(5)$ GUT symmetry is dynamically broken when a condensate is formed at some scale Λ_G, when the confining group G is strongly coupled.

Let us start by considering the case $R = 5$. The relevant fields are the fermions, $F(5, R_G)$, $\bar F(\bar 5, \bar R_G)$ (notice that we need both F and $\bar F$ to cancel anomalies), the Higgs field $H_5(5, 1)$, and the $SU(5)$ gauge bosons. As the breaking is dynamical we do not have any scale in the theory, while the only scale Λ_G is dynamically created by the strong force in G. In order to generate a mass term for the Higgses we introduce heavy fermions, transforming as a representation R_G of G and singlets under $SU(5)$, $\psi(1, R_G)$ and $\bar\psi(1, \bar R_G)$, with a mass M, a bit larger than Λ_G to not change the G dynamics at the condensation scale, so that the field ψ can be integrated out. The renormalizable Yukawa Lagrangian is then

$$\mathcal{L} = h_F \bar F_A H_5^A \psi + h_F^* \bar\psi (H_5^\dagger)_A F^A + M\bar\psi\psi, \quad \text{(C2)}$$

where h_F are the Yukawa couplings and $A = 1, \ldots, 5$ refers to $SU(5)$.

After integrating out ψ the Lagrangian (at scales $\mu < M$) is written as

$$\mathcal{L}_{\rm eff} = -\frac{|h_F|^2}{M} H_5^A (\bar F F)_A^B (H_5^\dagger)_B, \quad \text{(C3)}$$

which reduces to (C1) by the identification $c_F = |h_F|^2$ and $\Lambda \sim M$. For scales below the condensation scale the effective Lagrangian can be written as

$$\mathcal{L}_{\rm eff} = -\lambda_F^2 \Lambda_G H_5^\dagger (\langle H_1^{(5)} + H_{24}^{(5)}\rangle) H_5, \text{ with } \lambda_F^2 \equiv |h_F|^2 \Lambda_G/\Lambda. \quad \text{(C4)}$$

Notice that this mechanism is similar to that giving rise to the Weinberg operator in the leptonic sector, after integrating out the heavy right-handed neutrinos. After condensation $\langle \bar{F}_R F_R \rangle$, the previous Lagrangian gives a mass to the Higgs five-plet.

In the case $R = 10$, the condensation fermions are $T(10, R_G)$ and $\bar{T}(\overline{10}, \bar{R}_G)$. The argument is similar, with the only difference being that the heavy fermions ψ and $\bar{\psi}$ cannot be singlets under $SU(5)$, but should rather transform as $\psi(5, R_G)$ and $\bar{\psi}(\bar{5}, \bar{R}_G)$. The Lagrangian becomes

$$\mathcal{L} = h_T \bar{T}_{AB} H_5^A \psi^B + h_T^* \bar{\psi}_A (H_5^\dagger)_B T^{AB} + M \bar{\psi}_A \psi^A, \quad \text{(C5)}$$

giving, upon condensation of $\langle \bar{T} T \rangle = H_1^{(10)} + H_{24}^{(10)} + H_{75}^{(10)}$,

$$\mathcal{L}_{\text{eff}} = -\frac{|h_T|^2}{M} H_5^A \langle H_1^{(10)} + H_{24}^{(10)} \rangle_A^B (H_5^\dagger)_B. \quad \text{(C6)}$$

As $\bar{5} \times 5 = 1 + 24$, the term with the 75 provides indeed no contribution.

Finally, in the case $R = 24$, where the condensation fermions are $A(24, R_G)$ and $\bar{A}(24, \bar{R}_G)$. the above arguments are reproduced by taking for instance $\psi(\bar{5}, R_G)$ and $\bar{\psi}(5, \bar{R}_G)$.

Figure 15 provides a graphical representation for the condensation mechanism within the toy model.

2. The higher dimensional kinetic operators

The modification of GCU in models with $SU(5)$ dynamical breaking is induced via the $d = 7$ operators

$$-\frac{1}{4}\frac{\bar{c}_R}{\Lambda^3} \text{Tr}(G_{\mu\nu} \bar{F}_R F_R G^{\mu\nu}). \quad \text{(C7)}$$

We will now see how can we generate such term in the Lagrangian.

We introduce heavy fermions, singlets under G (so they cannot condense at the scale Λ_G), and transforming under the representation R of $SU(5)$, $f(R, 1)$, and $\bar{f}(\bar{R}, 1)$, with mass M_f, and a heavy real scalar φ with a mass M_s and zero vacuum expectation value, with a Yukawa coupling and Lagrangian

$$\mathcal{L} = \varphi \bar{f} f + M_f \bar{f} f - \frac{1}{2} M_s^2 \varphi^2 \Rightarrow m_f(\varphi) = M_f + \varphi. \quad \text{(C8)}$$

We are assuming the mass M_s to be larger than Λ_G for the scalar field φ to be integrated out before the condensation dynamics.

The fermion f contributes to the renormalization of the $SU(5)$ kinetic term. At one-loop the kinetic Lagrangian is [29]

$$\mathcal{L}_{\text{kin}} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} \frac{b_f g_5^2}{16\pi^2} \log \frac{\Lambda^2}{m_f^2(\varphi)}, \quad \text{(C9)}$$

where b_f is the coefficient of the beta-function contribution from the fermion f and g_5 the $SU(5)$ gauge coupling. The previous Lagrangian can be expanded in a power series in φ and yields

$$\mathcal{L}_{\text{eff}} = \frac{1}{4} \frac{\alpha_5 b_f}{2\pi} \frac{\varphi}{M_f} G_{\mu\nu} G^{\mu\nu}. \quad \text{(C10)}$$

We now introduce the Yukawa coupling of φ with the confining fermions as

$$\Delta\mathcal{L} = \lambda_R \varphi \bar{F}_R F_R, \quad \text{(C11)}$$

and integrate out the scalar φ as $\varphi = -\lambda_R (\bar{F}_R F_R + \bar{f} f)/M_s^2$, which yields

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} \frac{\alpha_5 \lambda_R b_f}{2\pi} \frac{\bar{F}_R F_R}{M_f M_s^2} G_{\mu\nu} G^{\mu\nu}, \quad \text{(C12)}$$

which can then be identified with (C7) assuming $M_f \sim M_s$, $M_f M_s^2 \equiv \Lambda^3$ and

$$\bar{c}_R \equiv \frac{\alpha_5 \lambda_R b_f}{2\pi}. \quad \text{(C13)}$$

Finally, at the condensation scale $\mu \sim \Lambda_G$ the effective Lagrangian can be written as in Eq. (4.6) with

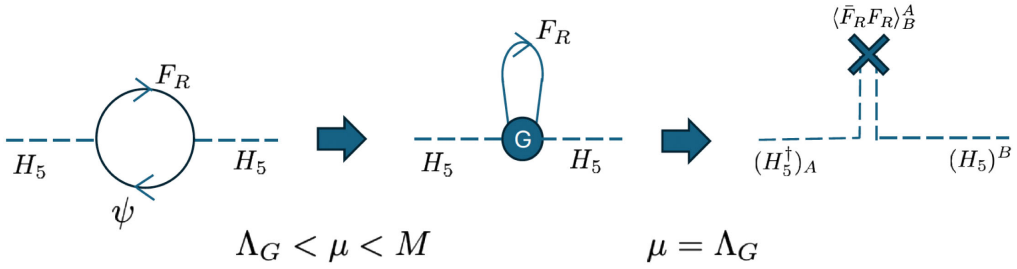


FIG. 15. Toy model's graphical representation of the condensation and the generation of the nonrenormalizable operator providing the DTS.

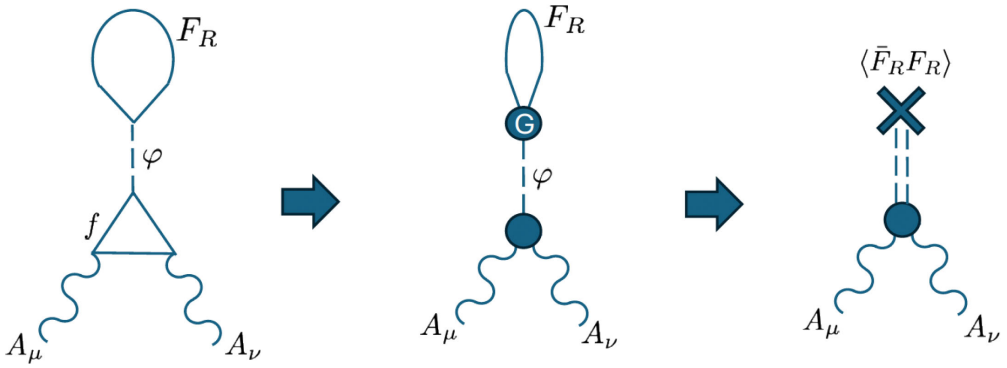


FIG. 16. Toy model's graphical representation of the condensation and the generation of the nonrenormalizable kinetic operator in (C7).

$$c_R \equiv \bar{c}_R(\Lambda_G/\Lambda)^3. \tag{C14}$$

Notice that this effective Lagrangian is similar to that giving rise to the decay $h \to \gamma\gamma$ in the SM, where the scalar field φ plays the role of the Higgs h, and the heavy fermion f that of the top-quark t. Figure 16 provides a graphical representation for this mechanism within the toy model.

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