

Effect of a nonextensive parameter on the Page curve

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(Received 8 February 2025; accepted 16 June 2025; published 8 July 2025)

This work employs the quantum extremal surface framework to compute the Page curve for black holes corrected by nonextensive entropy. The entropy of Hawking radiation increases linearly with time, leading to the persistence of the information paradox for nonextensive entropy-corrected black holes. At late time, we extremize the generalized entropy functional; incorporating contributions from both matter and the quantum extremal island, we establish that the entanglement entropy of Hawking radiation saturates to the nonextensive extension of the Bekenstein-Hawking entropy. Finally, we study the dependence of nonextensive parameters on the Page time.

DOI: [10.1103/njpw-wlf3](https://doi.org/10.1103/njpw-wlf3)

I. INTRODUCTION

The study of black hole entropy is crucial for understanding the intricate microscopic structure of spacetime within the framework of quantum gravity. Unlike classical thermodynamic systems, where entropy is proportional to mass and volume, black hole entropy uniquely scales with the horizon area. This deviation from the extensive nature of conventional entropy poses a significant mystery, as the origin of this nonextensivity remains unresolved [1]. Building on the foundation of Bekenstein-Hawking entropy, various extensions incorporating nonextensive statistical frameworks have been proposed. Notable examples include Rényi entropy [2] and Tsallis entropy [3]. Recent advancements have introduced alternative formulations such as Barrow entropy [4], Sharma-Mittal entropy [5], and Kaniadakis entropy [6,7]. These generalized entropies aim to deepen our understanding of black hole thermodynamics and its connection to quantum gravity. The first law of thermodynamics, which relates entropy, temperature, internal energy, and heat transfer, is sensitive to the choice of entropy definition. Altering the entropy framework, as discussed in [8], consequently modifies

other thermodynamic parameters, offering new perspectives on black hole physics and beyond. Lacking a complete quantum gravity theory, effective field theory (EFT) offers a robust framework for exploring quantum effects near the Planck scale [9]. EFT predicts dynamical nonlocality at small distances, modifying black hole metrics to align with quantum-corrected entropy. These corrections, derived via curvature expansions, appear at different orders; third for Schwarzschild [10] and second for Reissner-Nordström [11]. Using the Wald entropy formula [12] and alternative methods [13–18], quantum-corrected geometries and entropies have been established, deepening our understanding of black hole quantum structure and spacetime dynamics.

The information paradox of black holes has been a central puzzle in theoretical physics ever since the discovery of Hawking radiation [19–22]. At the semiclassical level, the unitarity of quantum mechanics seems to break down when a black hole is formed from the collapse of a pure quantum state and eventually evaporates, leaving behind only thermal radiation, which is characterized by a mixed quantum state. The key concept is black hole entropy, specifically the Bekenstein-Hawking entropy, which has garnered significant attention over the past few decades proportional to the horizon area; this entropy serves as a cornerstone in the study of quantum gravity [23,24]. However, within the quantum gravity framework, Bekenstein-Hawking entropy is considered a leading-order, semiclassical approximation, with higher-order corrections offering deeper insights. One particularly important refinement is the logarithmic correction to entropy, widely regarded as universal and foundational. This correction emerges from various approaches, including conical

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singularity and entanglement entropy techniques [24,25], the Euclidean action method [26–29], conformal anomaly considerations [30], the Cardy formula [31], quantum tunneling processes [32,33], and quantum geometric frameworks [34]. The consistent appearance of this logarithmic term across diverse methodologies highlights its critical role in unraveling the microscopic structure of black hole entropy. The Page curve represents the time evolution of this entropy [35,36]. The paradox, therefore, lies in understanding how the Page curve is realized for the entanglement entropy of Hawking radiation. Recently, a potential resolution to this paradox has been proposed through the concept of islands [37–40]. When analyzing the state of Hawking radiation in a region R outside the black hole, the density matrix of R is typically obtained by tracing over the states in its complementary region, R' . However, the quantum extremal surface prescription [41–43] suggests that certain regions within R' , referred to as islands ($I \subset R'$), should not be traced out. As a result, the entanglement entropy of Hawking radiation in R is effectively determined by considering the states in $R \cup I$. The entanglement entropy of the Hawking radiation is therefore expressed as

$$S_{\text{gen}} = \min \left\{ \text{ext} \left[\frac{\text{Area}(\partial I)}{4G_N} + S_{\text{matter}}(R \cup I) \right] \right\}, \quad (1.1)$$

where the minimal quantum extremal surface prescription is applied. We apply this procedure in the corrected spacetime due to nonextensive generalization. The island rule, initially introduced as part of the conjectured quantum extremal surface prescription, was later derived using the replica method within the framework of the gravitational path integral. When applying the replica trick [44–46] to gravitational theories, only the boundary conditions of the replica geometries can be fixed. However, new saddle points must also be considered where bulk wormholes connect multiple replicas of spacetime. These configurations, referred to as replica wormholes, naturally lead to the appearance of islands [47–49]. In the semiclassical limit, the partition function of the replicated geometry is dominated by the saddle point that minimizes the entanglement entropy. This approach ensures that the replica trick in gravitational theories yields the same formula (1.1) as the quantum extremal surface prescription. Since replica wormholes are a direct consequence of the replica trick in gravitational systems, the island conjecture is expected to apply universally to any black hole scenario.

The main motivation of different proposal of entropy comes from the fact that the Bekenstein-Hawking entropy scales with the surface area of a black hole rather than its volume, contrasting with conventional thermodynamic systems where entropy is an extensive quantity typically proportional to mass and, consequently, volume. This area-scaling behavior remains one of the unresolved features of black hole thermodynamics [50]. Recent studies have explored alternative entropy frameworks rooted in

nonextensive statistical mechanics [51–63]. However, adopting such generalized entropic forms introduces significant challenges, as entropy plays a central role in the structure of thermodynamic relations. Altering it necessitates corresponding modifications to other thermodynamic quantities to maintain internal consistency. Although considerable effort has been devoted to redefining black hole entropy within these generalized frameworks, this approach carries inherent risks. This is beyond the scope of this article, and we will get back to these in separate research. Assuming this correction to be consistent,¹ we study the correction to the black holes from the nonextensive entropy. Once we have black hole corrected black holes due to these nonextensive entropy proposals, it is worth studying the information paradox within these black holes and its resolution using the island rule.

The structure of the paper is outlined as follows:

- (i) Section II provides a concise introduction to the nonextensive generalization of black hole entropy and outlines the methodology employed throughout the study;
- (ii) Section III delves into Barrow’s approach, deriving the entanglement entropy to first-order corrections in Barrow’s parameter, and subsequently examines the Page curve with and without the inclusion of the island configuration;
- (iii) Section IV examines the Tsallis-Cirto framework, computing the entanglement entropy up to first-order corrections in Tsallis-Cirto’s parameter, and verifies the Page curve under analogous configurations;
- (iv) Section V discusses Rényi’s approach, determining the entanglement entropy to first-order corrections in Rényi’s parameter, and investigates the Page curve both with and without the island configuration;
- (v) Section VI explores Kaniadakis’s approach, calculating the entanglement entropy up to first-order corrections in Kaniadakis’s parameter, and evaluates the Page curve under similar conditions; and
- (vi) Section VII presents the key findings and conclusions of the study.

II. MOTIVATION FOR NONEXTENSIVE ENTROPY PROPOSAL

In this section, we give a motivation for using the nonextensive entropy proposals. Here we try to argue how these proposals arise in the generalized uncertainty principle (GUP). A common prediction across multiple quantum gravity frameworks is the existence of a minimum measurable length scale [64,65]. In string theory, this minimal length is associated with the fundamental string scale [66,67], while in loop quantum gravity, it is related to

¹In the next section, we explain how nonextensive proposals are related to the GUP.

the nonzero expectation value of the length operator [68]. Additionally, independent arguments from black hole thermodynamics also support the emergence of such a length scale [65]. This recurring feature, typically identified with the Planck length $l_p = \sqrt{G_N \hbar / c^3}$, motivates a modification of the conventional Heisenberg uncertainty principle ($\Delta x_0 \Delta p \geq \hbar$), particularly in regimes where gravitational interactions cannot be neglected.

A heuristic derivation of such a modification can be constructed within Newtonian gravity by considering the gravitational interaction between an electron and a photon of energy E , treated as an effective mass E/c^2 [69]. The gravitational acceleration ($\vec{g}$) of the electron at a distance r from the photon is $-\frac{G_N(E/c^2)}{r^2} \vec{r}$. Assuming the interaction occurs over a characteristic spatial scale $L \sim r$ and timescale $t \sim L/c$, the velocity acquired by the electron is approximately $(G_N E L)/(c^3 r^2)$. The corresponding displacement due to this acceleration is $\frac{l_p^2 \Delta p}{\hbar}$, where $\Delta p \sim E/c$ has been assumed. The total position uncertainty, incorporating both quantum and gravitational contributions, and it becomes

$$\Delta x \sim \frac{\hbar}{\Delta p} + \frac{l_p^2}{\hbar} \Delta p,$$

which implies the GUP,

$$\Delta x \Delta p \gtrsim \hbar \left(1 + \frac{l_p^2}{\hbar^2} (\Delta p)^2 \right). \quad (2.1)$$

The GUP modifies the standard uncertainty relation by introducing Planck-scale corrections, and is consistent with various theoretical indications of a fundamental limit to spatial resolution in quantum gravitational regimes. Among the various formulations proposed, a widely studied form is the quadratic GUP, expressed as

$$\Delta x \Delta p \geq \frac{1}{2} \left(1 + \beta \frac{(\Delta p)^2}{m_p^2} \right),$$

where natural units are used ($\hbar = c = 1$) and m_p denotes the Planck mass and β is GUP deformation parameter.

A distinguishing feature of Hawking radiation, as compared to classical blackbody radiation, is its pronounced sparsity during black hole evaporation [70–79]. This sparsity is quantified by the ratio of the average time interval between successive emissions to the characteristic time scale defined by the energy of the emitted quanta. It has been demonstrated that Hawking radiation remains sparse throughout most of the evaporation process [80]. However, when quantum gravity corrections, such as those introduced via the GUP, are incorporated, this sparsity significantly decreases at the terminal phase of evaporation, resulting in a final burst of radiation that accelerates the

black hole's demise [75]. In contrast, classical thermodynamics, derived from statistical mechanics, assumes the dominance of short-range interactions and the extensivity of thermodynamic variables. Under these conditions, macroscopic quantities are well-defined, and the entropy can be computed via the Boltzmann-Gibbs additive formalism. However, in strongly gravitating systems, such as black holes, the long-range nature of gravity invalidates these assumptions. As a result, conventional extensive variables become ill-defined, and the standard Boltzmann-Gibbs entropy may no longer be applicable [3]. Gibbs thermodynamics assumes entropy to be extensive and additive, meaning it increases proportionally with the size of the system. This assumption relies on neglecting long-range interactions between subsystems, as such forces are considered negligible when the system's size exceeds the range of interactions. Under these conditions, the total entropy is simply the sum of the entropies of its components. However, long-range interactions are significant for systems like black holes and cannot be ignored. As a result, the principles of Gibbs thermodynamics are insufficient for describing black hole thermodynamics. A formulation of nonextensive thermodynamics that is consistent with the zeroth law has remained a long-standing challenge, with a potential solution emerging only recently. To account for the nonextensive behaviour of black hole entropy, several modified frameworks extending Gibbs entropy have been introduced, and in every proposal, we explain how nonextensive entropies are related to the GUP frameworks.

A. Barrow proposal

Barrow explored the effects of quantum gravity on the conventional notion of a smooth and uniform event horizon. He introduced a modified entropy formulation for black holes to account for distortions in the event horizon caused by quantum gravity effects. These distortions were quantified using a fractal parameter Δ . The resulting Barrow entropy [4] is expressed as

$$S_B = (S_{\text{BH}})^{1+\frac{\Delta}{2}}, \quad (2.2)$$

where Δ lies in the range $0 \leq \Delta \leq 1$. For $\Delta = 0$, the standard Bekenstein-Hawking entropy is recovered, signifying the absence of any fractal structure and $\Delta = 1$, signifies the most intricate surface (or maximal deformation) that may account for one higher geometric dimension from an information viewpoint. The above entropy relation (2.2) associates the black hole area entropy with the horizon area of the black hole.

Barrow's proposal of a fractal structure for the black hole horizon can be naturally connected to the GUP [81]. In a recent reformulation by Wang and Ma [81], the minimal increase in the black hole horizon area is identified with the area of the smallest “fractal bubble”, assumed to be of

Planck-scale size. This leads to an entropy expression of the form,

$$S \propto \ln \left(\frac{A}{A_{\text{bubble}}} \right),$$

which mirrors the Boltzmann entropy $S = k_B \ln W$, with the number of microstates $W \sim A/A_{\text{Pl}}$ interpreted as the number of Planck-area elements on the horizon. Thus, by incorporating a GUP-motivated minimal area element into a fractal horizon framework, a Barrow-type logarithmic entropy naturally emerges. Notably, this entropy formulation leads to thermodynamic stability of the black hole, characterized by a monotonic temperature and finite heat capacity, contrasting with the classical thermodynamic instability of Schwarzschild black holes. Hence, the interplay between GUP-induced minimal length and fractal horizon geometry provides a compelling foundation for Barrow entropy.

B. Tsallis-Cirto proposal

The Tsallis-Cirto entropy is a generalization of the standard Bekenstein-Hawking entropy, incorporating the principles of nonextensive statistical mechanics. This formulation is particularly significant for studying gravitational systems, such as black holes and the Universe, where traditional extensive entropy may not be applicable. In cosmology, the Tsallis-Cirto entropy has been utilized to derive modified Friedmann equations that describe the Universe's expansion. These modifications offer valuable insights into dark energy and the observed accelerated expansion of the Universe. A notable application arises in the framework of entropic cosmology, where gravity is viewed as an emergent phenomenon arising from the statistical behavior of microscopic degrees of freedom. This perspective provides fresh insights into the nature of dark energy and the dynamical evolution of the cosmos [82]. The Tsallis-Cirto entropy can be expressed as

$$S_{\text{TC}} = S_{\text{BH}}^{\delta+1}, \quad (2.3)$$

where δ is a nonextensive parameter. In the limit $\delta \rightarrow 0$, the entropy reduces to the standard Bekenstein-Hawking entropy, recovering the classical thermodynamic description.

Luciano [83] established an explicit correspondence between the GUP deformation parameter β and the Tsallis nonextensivity index q . By comparing the thermodynamics derived from GUP-corrected black hole entropy and temperature with those emerging from Tsallis statistics, a nontrivial monotonic relation $q = q(\beta, M)$ was obtained. Specifically, to leading order, this relation takes the form,

$$q \simeq 1 + \frac{9\beta}{512\pi^3} \frac{m_p^4}{M^4} \ln \left(\frac{4\pi M^2}{m_p^2} \right),$$

where S_{BH} is the standard Bekenstein-Hawking entropy. This implies $q > 1$ for $\beta > 0$, and $q \rightarrow 1$ as $M \rightarrow \infty$, thereby recovering the extensive entropy in the classical limit. The result suggests that GUP-induced quantum gravitational corrections can be effectively encoded in a Tsallis-type nonextensive entropy, with the Tsallis parameter q reflecting the impact of a minimal length. Luciano concludes that nonextensive statistical mechanics provides a natural framework for incorporating quantum gravity effects, and further shows that the GUP-modified entropy admits an exact Tsallis form,

$$S_q \propto \ln_q W, \quad \text{with} \quad W \propto e^{S_{\text{BH}}},$$

valid to all orders in β . This formulation strengthens the conceptual bridge between quantum gravity phenomenology and generalized statistical mechanics.

C. Rényi proposal

Rényi entropy generalizes the concept of entropy from information theory and statistical mechanics, offering a robust framework with significant applications in black hole thermodynamics. Unlike the Bekenstein-Hawking entropy, which is extensive and proportional to the event horizon's surface area, Rényi entropy introduces a nonextensive parameter λ , enabling a broader description of entropy in systems exhibiting nonstandard statistical behavior. By treating λ as a thermodynamic variable, the thermodynamic phase space of black holes has been extended, allowing for the derivation of a modified Smarr relation and an extended first law of black hole thermodynamics. This generalization has revealed that black holes governed by Rényi entropy can exhibit stability properties distinct from those described by traditional Gibbs-Boltzmann statistics [60,84]. In the limit $\lambda \rightarrow 0$, Rényi entropy reduces smoothly to the Bekenstein-Hawking entropy, ensuring consistency with classical results. The Rényi entropy is given by

$$S_R = \frac{\log(1 + \lambda S_{\text{BH}})}{\lambda}. \quad (2.4)$$

The Rényi entropy is related to Tsallis entropy via a logarithmic transformation, it arises naturally in contexts involving nonextensive statistical mechanics. If the Tsallis entropy S_T is identified with the Bekenstein-Hawking entropy S_{BH} , the corresponding Rényi entropy takes the form,

$$S_R = \frac{1}{\lambda} \ln(1 + \lambda S_{\text{BH}}),$$

a structure previously employed in the study of black hole thermodynamics [85]. [86] extended this framework by incorporating GUP corrections into S_{BH} , resulting in a GUP-modified Rényi entropy. Their results, corroborated

in [87], demonstrate that both the entropy and the Hawking temperature remain finite in the final stages of black hole evaporation, thereby implying the existence of a black hole remnant. Thus, the Rényi entropy serves as an effective equilibrium description that encapsulates the quantum-gravity-induced nonextensivity, with the GUP providing the minimal length scale underlying the deformation. This supports the broader view that Rényi and Tsallis entropies offer viable thermodynamic formalisms for black holes in quantum gravitational regimes.

D. Kaniadakis proposal

Kaniadakis's proposal [6,88] arises from a relativistic statistical framework that aligns with the core principles of conventional statistical mechanics. When applied to black hole thermodynamics, Kaniadakis entropy offers a powerful tool for investigating deviations from classical thermodynamic behavior, especially in holographic contexts. The Kaniadakis black hole entropy is expressed as

$$S_K = \frac{\sinh(\kappa S_{\text{BH}})}{\kappa}, \quad (2.5)$$

where κ is a deformation parameter that governs the deviation from standard entropy. In the limit $\kappa \rightarrow 0$, the Kaniadakis entropy smoothly recovers the Bekenstein-Hawking entropy, ensuring consistency with classical results. Its nonextensive nature occurs precisely due to deviations from classical Boltzmann-Gibbs statistics, for systems with long-range interactions or fractal-like structures in particular. As it exhibits nonextensive thermodynamic behavior and accounts for corrections to quantum gravity, it can be considered a valuable tool for investigating black hole thermodynamics in modified gravity theories.

Although a direct derivation of Kaniadakis entropy from the GUP is absent in the literature, Kaniadakis statistics arises from a κ -deformation of the Lorentz symmetry, typically within the context of doubly special relativity frameworks. Despite its distinct origin, this formalism has been applied to black hole thermodynamics. Notably, [89] introduced a dual Kaniadakis entropy, related to Rényi entropy, and derived the corresponding Hawking temperature and heat capacity. Their results show that for positive deformation parameter κ , the black hole heat capacity becomes positive, indicating thermodynamic stability, and a new phase transition emerges—features not present in standard Bekenstein-Hawking thermodynamics. While GUP is not explicitly involved, Kaniadakis' statistics can be interpreted as an alternative deformation scheme that captures quantum gravitational corrections, analogous to Tsallis' formalism. Therefore, in our context, Kaniadakis entropy offers a viable route for deforming black hole thermodynamics, resulting in bounded entropy and modified stability behavior, potentially linked to underlying quantum gravity effects.

The combination of the GUP with nonextensive entropy frameworks leads to several robust predictions in black hole thermodynamics. One of the key results is the existence of black hole remnants. Across various generalized entropies—Barrow, Tsallis-Cirto, Rényi, and Kaniadakis—the incorporation of GUP yields finite temperature and entropy as the mass approaches a minimum value M_{min} . [87] confirm that $T(M)$ plateaus and $S \rightarrow \text{const}$, signifying evaporation halts near the Planck scale, independent of the specific entropy model. Nonextensive corrections also impact thermodynamic stability. While classical Schwarzschild black holes possess negative heat capacity ($C < 0$), several modified models yield stable branches. For instance, Rényi black holes exhibit $C > 0$ for sufficiently large deformation parameter λ , and Kaniadakis entropy leads to a stable phase for $\kappa > 0$, as shown by Luciano and Saridakis [90]. Similarly, Barrow entropy with fractal horizons results in monotonic $T(A)$ and positive heat capacity, indicating thermodynamic stability. Another consequence is the bounded nature of entropy. Barrow entropy reaches a maximum for $\Delta = 1$; Kaniadakis entropy is constrained by equipartition bounds, and Tsallis entropy, through its nonadditive rule, effectively limits entropy growth. These bounds arise naturally from the presence of a minimal length scale introduced by GUP, which regularizes divergences at small scales. Moreover, GUP universally modifies the sparsity of Hawking radiation. [87] demonstrated that while different entropy models yield distinct sparsity profiles, GUP consistently reduces sparsity near the endpoint of evaporation. This implies a transition to more burstlike, less sparse emission in the final stages. In sum, GUP-induced minimal length effects, when combined with nonextensive statistics, generically predict black hole remnants, enhanced stability, entropy bounds, and modified emission profiles—regardless of the specific entropy model employed.

1. Connections to quantum gravity frameworks

Nonextensive entropy formalisms closely align with quantum gravity features emerging from various approaches. Modified dispersion relations (MDRs) [67,91], as in doubly special relativity and Gravity's Rainbow, naturally incorporate a minimal length and can reproduce GUP via deformed commutators. These frameworks often employ Tsallis or Rényi entropies to encode Planck-scale corrections in black hole thermodynamics, with consistent predictions of remnants and modified heat capacities. In holographic and emergent gravity scenarios, nonextensive entropies parametrize deviations from the area law. Tsallis and Kaniadakis entropies have been used to model horizon degrees of freedom in holographic dark energy, while Barrow entropy introduces fractal modifications. Such entropic deformations yield modified Friedmann equations [92,93], and MOND-like force laws [94], linking entropy corrections to gravitational dynamics. Loop quantum gravity [57,60,95], with its

discrete area spectrum, offers a natural setting where generalized entropies affect black hole microstate counting. Using Tsallis or κ -deformed statistics modifies the Immirzi parameter and effective area quanta, suggesting a nonextensive underpinning of quantum geometry. Similarly, non-commutative geometry [96] and gravity's rainbow [97] lead to thermodynamic corrections analogous to GUP, often producing stable remnants. Overall, the mapping of GUP corrections to Tsallis/Rényi forms indicates that nonextensive entropy may serve as an effective description of underlying quantum gravitational microstates, highlighting a potentially universal signature of quantum geometry.

2. Methodology

We assume a spherically symmetric shell of dust with mass M and an initially large radius. By the Birkhoff theorem, the spacetime outside the shell is described by the Schwarzschild metric, with mass M . Inside the shell, the spacetime remains empty and flat. As the shell collapses, its radius decreases progressively. Once the shell crosses its Schwarzschild radius, a black hole forms. The geometry remains asymptotically flat throughout the process, and the mass M of the shell, appearing in the Schwarzschild metric, corresponds to the total energy E of the system, such that $E = M$. With the help of these assumptions, we derive the temperature related to all the nonextensive generalizations with the help of

$$\frac{1}{T_{\text{nonextensive}}} = \left(\frac{dS_{\text{nonextensive}}}{dM} \right), \quad (2.6)$$

and derive the metric function.

Equation (1.1) consists of two components the first term represents the gravitational contribution to the generalized entropy, denoted as S_{gravity} . This term is proportional to the area (or volume in higher dimensions) of the boundary of the island, $\partial\mathcal{I}$, and is expressed as

$$S_{\text{gravity}} = \frac{\text{Area}(\partial\mathcal{I})}{4G_{\text{N}}}. \quad (2.7)$$

This term reflects the fact that region $\mathcal{R}$ is separated from the rest of the system, contributing to the entropy of the Hawking radiation. Notably, this term appears even in the case of empty flat spacetime, provided we consider gravitational theories in the bulk. The second term is the matter entanglement entropy S_{matter} associated with the region $\mathcal{R} \cup \mathcal{I}$ in the curved spacetime. If no islands are present, the gravitational entropy of an island vanishes. In higher dimensions, it is well-known that the matter entanglement entropy exhibits arealike divergences that depend on the short-distance cutoff [98,99]. This leads to a divergence for the matter entropy, given by

$$S_{\text{matter}}(\mathcal{R} \cup \mathcal{I}) = \frac{\text{Area}(\partial\mathcal{I})}{\epsilon^2} + S_{\text{matter}}^{(\text{finite})}(\mathcal{R} \cup \mathcal{I}), \quad (2.8)$$

where ϵ represents the short-distance cutoff scale. A similar divergence arises from the boundary of the region $\mathcal{R}$, but this term is typically neglected in our calculations. This divergence can be absorbed by renormalizing the Newton constant as

$$\frac{1}{4G_{\text{N}}^{(r)}} = \frac{1}{4G_{\text{N}}} + \frac{1}{\epsilon^2}, \quad (2.9)$$

where G_{N} is the bare Newton constant and $G_{\text{N}}^{(r)}$ is the renormalized Newton constant. Given this, if we treat G_{N} in Eq. (1.1) as the renormalized Newton constant, the leading divergence of $S_{\text{matter}}(\mathcal{R} \cup \mathcal{I})$ is already accounted for. Thus, $S_{\text{matter}}(\mathcal{R} \cup \mathcal{I})$ contributes only a finite amount, denoted as $S_{\text{matter}}^{(\text{finite})}(\mathcal{R} \cup \mathcal{I})$. Therefore, in higher dimensions, the revised formula becomes

$$S(R) = \min \left\{ \text{ext} \left[\frac{\text{Area}(\partial\mathcal{I})}{4G_{\text{N}}^{(r)}} + S_{\text{matter}}^{(\text{finite})}(\mathcal{R} \cup \mathcal{I}) \right] \right\}. \quad (2.10)$$

By evaluating this formula using the minimal quantum extremal surface prescription in corrected spacetime, we aim to derive the Page curve. For this, we consider the region for the Hawking radiation $\mathcal{R}$ in Schwarzschild spacetime. This region consists of two separate parts, $\mathcal{R}_+$ and $\mathcal{R}_-$, located in the right and left wedges of the Penrose diagram, respectively (as illustrated in Fig. 1). As time progresses, the distance between $\mathcal{R}_+$ and $\mathcal{R}_-$ becomes large, and the entanglement entropy of the Hawking radiation without islands grows significantly. Consequently, at late times, the configuration that includes islands is expected to provide the dominant contribution.

To begin, we examine the case without islands. The matter entanglement entropy, in this case, will be calculated for the two separated regions $\mathcal{R}_+$ and $\mathcal{R}_-$. Now, we examine the configuration involving an island $\mathcal{I}$. At late times, the boundaries of $\mathcal{I}$ are much closer to the boundary of the region $\mathcal{R}$ within the same wedge than to the boundaries of $\mathcal{R}_-$ and $\mathcal{R}_+$ in the opposite wedge. The volume of an extremal surface connecting these boundaries determines the distance between the boundaries in the right and left wedges [100,101]. To determine the distance, we introduce the Kruskal-like coordinates to achieve the maximal extension of the spacetime [102]. These coordinates are defined as

$$\begin{aligned} \text{Right Wedge: } & U = -e^{-\frac{2\pi}{\beta}(t-r_\star)} \quad \text{and} \quad V = e^{\frac{2\pi}{\beta}(t-r_\star)}, \\ \text{Left Wedge: } & U = e^{-\frac{2\pi}{\beta}(t-r_\star)} \quad \text{and} \quad V = -e^{\frac{2\pi}{\beta}(t-r_\star)}. \end{aligned} \quad (2.11)$$

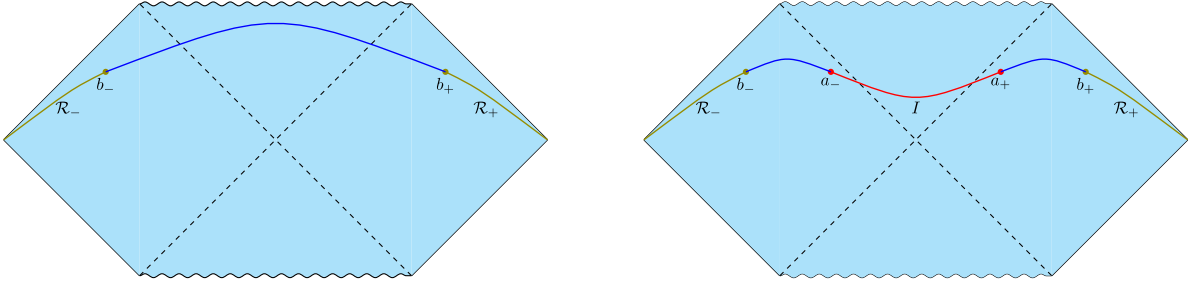


FIG. 1. The Penrose diagram of Schwarzschild black hole. *Left*: The configuration without any islands. *Right*: The configuration with an island $\mathcal{I}$, shown in red, with endpoints a_- and a_+ . The radiation region $\mathcal{R} = \mathcal{R}_+ \cup \mathcal{R}_-$ is highlighted in olive, with its endpoints $b_{\pm}$ located in the baths. Assuming the state on the full Cauchy slice is pure, the entanglement entropy of the matter fields, $S_{\text{matter}}(\mathcal{R} \cup \mathcal{I})$, can be calculated. In the left panel, this is evaluated over the complement interval $[b_-, b_+]$, while in the right panel, it is evaluated over $[b_-, a_-] \cup [a_+, b_+]$, as indicated in blue.

Here, β represents the inverse of nonextensive corrected temperature, and r_* is the tortoise coordinate for the nonextensive corrected spacetime. We can easily write the metric in terms of the Kruskal coordinates, which can be represented as

$$ds^2 = \Omega_{\text{NE}}^{-2} dU dV, \quad (2.12)$$

where Ω_{NE} is the conformal factor.

The general formula we will use to calculate the matter contribution to the entanglement entropy is

$$S_{\text{matter}} = \frac{c}{3} \log \frac{d(a_+, a_-)d(b_+, b_-)d(a_+, b_+)d(a_-, b_-)}{d(a_+, b_-)d(a_-, b_+)}, \quad (2.13)$$

where $d(x, y)$ denotes the proper distance between the points x and y . This formula accounts for the contributions from the boundaries of the entanglement region (b_+, b_-) and the island boundaries (a_+, a_-) , capturing the interplay between these regions. We analyze the entanglement entropy at early times when no island is present. For the two-dimensional s -wave approximation to hold, the radiation region's points must be sufficiently separated from the sphere's radius. In the absence of an island, the entanglement regions are defined by only two boundary points, located in the right wedge $\mathcal{R}_+$ and the left wedge $\mathcal{R}_-$ (see Fig. 1, Left). At late times, the matter contribution to the entanglement entropy can be expressed as

$$S_{\text{matter}}^{\text{No Island}} = \frac{c}{3} \log d(b_+, b_-), \quad (2.14)$$

where b_+ and b_- denote the boundaries of the entanglement regions in the right and left wedges of the nonextensive-corrected geometry. Specifically, b_+ corresponds to $(t, r) = (t_b, b)$, and b_- corresponds to $(t, r) = (-t_b + i\beta/2, b)$. The imaginary component $i\beta/2$ in b_- arises from its location in the left wedge, introducing a sign reversal for U and V in Kruskal coordinates, as

detailed below. Using a conformal transformation, the matter contribution to the entanglement entropy in the nonextensive-corrected geometry is determined as

$$S_{\text{matter}}^{\text{No-Island}} = \frac{c}{6} \log \frac{(U(b_-) - U(b_+))(V(b_+) - V(b_-))}{\Omega_{\text{NE}}(b_+)\Omega_{\text{NE}}(b_-)}. \quad (2.15)$$

It is believed that the entropy increases linearly with time and results in a paradox known as the black hole information paradox.

To resolve the entropy paradox at late times, we introduce the island construction and compute the entanglement entropy in the presence of a single island. Since the primary focus is on the late-time behavior, we employ the late-time approximation. At early times, no saddle point existed for the island's location, implying that the island did not form. For the late-time regime, particularly in the context of the nonextensive-corrected black hole, the radiation entropy becomes significant, as this is the regime where the entropy paradox emerges. We assume that the boundary $r = b$ of the entanglement region $\mathcal{R}$ lies far from the black hole's horizon ($b \gg r_h$) and use the s -wave approximation to simplify the computations. The total entropy is determined using the matter entropy formula. Using the Kruskal-like coordinates, the generalized entropy S_{gen} is computed as the sum of the semiclassical fine-grained entropy and the contribution from the area of the quantum extremal surface. The generalized formula for entanglement entropy is

$$S_{\text{gen}} = \text{Contribution from Island's area} + S_{\text{matter}}. \quad (2.16)$$

With this framework, we proceed to analyze the corrections to the spacetime geometry introduced by various entropy proposals. It is straightforward to verify that, by switching off the correction parameters, the metric reduces to the standard Schwarzschild spacetime. Naturally, one expects these corrections to influence the entanglement

entropy, particularly at early times, reflecting the contributions from each proposal. Following the approximation scheme outlined in [103], we compute the generalized entanglement entropy incorporating the island contribution. Extremizing this entropy functional yields the position of the island. Substituting this extremal surface back into the expression provides the full entanglement entropy, which we find saturates at late times, thereby resolving the entropy paradox. Furthermore, we calculate the Page time in this corrected setup and observe the emergence of correction terms in comparison to the Page time in pure Schwarzschild spacetime.

III. BARROW'S APPROACH

In this section, we systematically explore the Barrow-corrected framework by utilizing (2.2) to compute the Barrow-corrected temperature, derive the corresponding metric function, and further analyze the spacetime structure. We begin by considering (2.2), which governs the Barrow entropy modification. The Barrow-corrected temperature T_B can be expressed as

$$T_B = \frac{2^{-\Delta-2} \pi^{-\frac{\Delta}{2}-1} G_N M (G_N M^2)^{-\frac{\Delta}{2}-1}}{\Delta + 2}. \quad (3.1)$$

This corrected temperature accounts for quantum-gravitational effects encoded by the Barrow parameter [104]. Using the corrected temperature, we construct the Barrow-corrected metric function $f_B(r)$, which incorporates modifications to the Schwarzschild-like geometry. Assuming a static, spherically symmetric spacetime, the metric function is given as

$$f_B(r) = 1 - \frac{2^\Delta (\Delta + 2) \pi^{\Delta/2} G_N M (G_N M^2)^{\Delta/2}}{r}, \quad (3.2)$$

where Δ represents Barrow parameters. The behaviour of metric function $f_B(r)$ for various values of the Barrow exponent is shown in Fig. 2. By using Eq. (3.2), we can find the horizon radius as

$$\begin{aligned} r_h^{\text{Barrow}} &= (4\pi)^{\Delta/2} G_N^{1+\Delta} M^{1+\Delta} (\Delta + 2) \\ &= 2G_N M + \Delta G_N M (\log(4\pi G_N M^2) + 1) + O(\Delta^2). \end{aligned}$$

We can plot this to see how the horizon radius changes with the Barrow parameter.

We can straightforwardly verify that the metric function (3.2) is a valid solution to the Einstein field equations. Furthermore, the Kretschmann scalar can be explicitly determined as

$$\mathcal{K} = R^{\alpha\beta\mu\nu} R_{\alpha\beta\mu\nu} = \frac{3G(\Delta + 2)^2 (4\pi G_N M^2)^{\Delta+1}}{\pi r^6}. \quad (3.3)$$

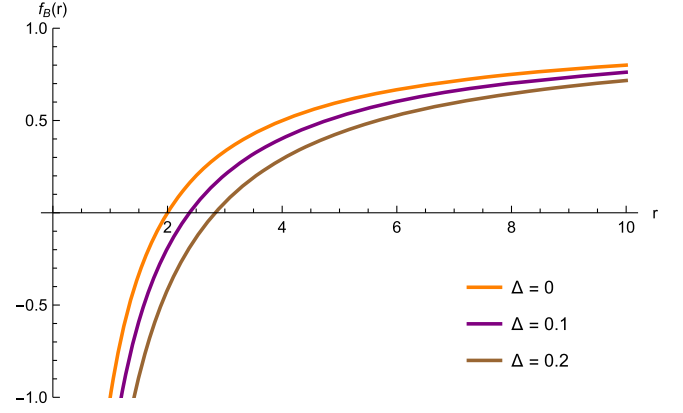


FIG. 2. Plot of Barrow metric function versus r for various values of Barrow parameters.

So, in the Barrow-corrected spacetime, the singularity at $r = r_h^{\text{Barrow}}$ is a coordinate singularity, arising due to the choice of the coordinate and can be resolved through an appropriate transformation. In contrast, the singularity at $r = 0$ corresponds to a true curvature divergence and can be seen through the Eq. (3.3), as the Kretschmann scalar in the limit $r \rightarrow 0$ blows up. To remove the coordinate singularity, we start with the tortoise coordinate from the metric function, which can be derived as

$$\begin{aligned} r_\star(r) &= r + 2^\Delta (\Delta + 2) \pi^{\Delta/2} G_N M (G_N M^2)^{\Delta/2} \\ &\quad \times \log(r - 2^\Delta (\Delta + 2) \pi^{\Delta/2} G_N M (G_N M^2)^{\Delta/2}). \end{aligned} \quad (3.4)$$

This coordinate ensures that the spacetime near the horizon is regular for ingoing and outgoing modes. Now, using Kruskal-like coordinates, the metric can be written similarly to Eq. (2.12) and the conformal factor $\Omega_B(r)$ that encapsulates the Barrow corrections and can be expressed as

$$\Omega_B(r) = \frac{\beta}{2\pi} e^{-\frac{2\pi r_\star(r)}{\beta}} \sqrt{1 - \frac{2^\Delta (\Delta + 2) \pi^{\Delta/2} G_N M (G_N M^2)^{\Delta/2}}{r}}. \quad (3.5)$$

Applying the conformal mapping technique, the matter's contribution to the entanglement entropy within the Barrow-corrected geometry will be discussed.

A. Without island

Now, we compute the entanglement entropy at early times when no island is present. The radiation region points must be sufficiently separated for the two-dimensional s -wave approximation to be valid. Applying a conformal transformation and using Eq. (2.15), we find that the matter contribution to the entanglement entropy in Barrow-corrected geometry is

$$\begin{aligned}
S_{\text{gen}} &= \frac{c}{6} \log \left[\frac{\beta(b - 2^\Delta \pi^{\Delta/2} (\Delta + 2) G_N M (G_N M^2)^{\Delta/2})}{\pi b} \right. \\
&\quad \times \cosh^2 \left(\frac{2\pi t_b}{2^{\Delta+2} (\Delta + 2) \pi^{\Delta/2+1} G_N M (G_N M^2)^{\Delta/2}} \right) \Big] \\
&\approx \frac{c t_b}{12 G_N M} - \frac{\Delta (c t_b (\log(4\pi G_N M^2) + 1))}{24 (G_N M)} + \mathcal{O}(\Delta^2).
\end{aligned} \tag{3.6}$$

Without the island, the information remains trapped within the black hole, causing the entanglement entropy to increase linearly over time. In this scenario, no Page time emerges, and the radiation entropy eventually becomes infinitely larger than the Barrow entropy of the black hole this leads to a paradox, which we will address in the following section, where we show that the island construction resolves this issue and yields the correct Page curve for the Barrow-corrected black holes [105].

B. With island

We will now introduce the concept of island construction and compute the entanglement entropy in the presence of a single island. For the Barrow-corrected black hole, our focus is on the large-time limit of the entanglement entropy—specifically, the radiation at late times, since this is when entropy becomes problematic. Again, using the Kruskal-like coordinates, the generalized entropy is calculated as the semiclassical fine-grained entropy plus the contribution from the area of the quantum extremal surface as in Eq. (2.16). Since we are interested in the late times, we adopt the late-time approximation because, at early times, there is no saddle point that exists for the island's location, which implies that the island does not form and the entropy is the same as we discuss without the island. We have used the approximations [103],

$$\frac{1}{2} \sqrt{\frac{b - 2^\Delta \pi^{\Delta/2} (\Delta + 2) (G_N M^2)^{1+\Delta/2}}{a - 2^\Delta \pi^{\Delta/2} (\Delta + 2) (G_N M^2)^{1+\Delta/2}}} e^{\left(\frac{M(b-a)(G_N M^2)^{-\frac{\Delta}{2}-1}}{(\Delta+2)2^{\Delta+1}\pi^{\frac{\Delta}{2}}} \right)} \approx \cosh \left(\frac{(t_a \pm t_b)(G_N M^2)^{-\frac{\Delta}{2}}}{(\Delta + 2) G_N M \pi^{\frac{\Delta}{2}} 2^{\Delta+1}} \right),$$

where $\approx$ is $\ll$ for $+$ and $\gg$ for $-$ sign, respectively. The generalized entropy S_{gen} is

$$\begin{aligned}
S_{\text{gen}} &= \frac{2\pi a^2}{G_N} + \frac{1}{6} c \log \left(\frac{2^{4\Delta+4} \pi^{2\Delta} (\Delta + 2)^4 (b - 2^\Delta \pi^{\Delta/2} (\Delta + 2) G_N M (G_N M^2)^{\Delta/2})^2}{M^4 (G_N M^2)^{-4-2\Delta} a b} \right) \\
&\quad - \frac{2}{3} c \sqrt{\frac{a - 2^\Delta \pi^{\Delta/2} (\Delta + 2) G_N M (G_N M^2)^{\Delta/2}}{b - 2^\Delta \pi^{\Delta/2} (\Delta + 2) G_N M (G_N M^2)^{\Delta/2}}} \exp \left(\frac{2^{-\Delta-1} \pi^{-\frac{\Delta}{2}} (a-b) (G_N M^2)^{-\frac{\Delta}{2}}}{(\Delta + 2) G_N M} \right) \\
&\quad \times \cosh \left(\frac{2^{-\Delta-1} \pi^{-\frac{\Delta}{2}} (t_a - t_b) (G_N M^2)^{-\frac{\Delta}{2}}}{(\Delta + 2) G_N M} \right) + \frac{1}{6} c \log \left(\frac{\exp \left(\frac{2^{-\Delta} \pi^{-\frac{\Delta}{2}} (b-a) (G_N M^2)^{-\frac{\Delta}{2}}}{(\Delta+2) G_N M} \right)}{a b} \right).
\end{aligned}$$

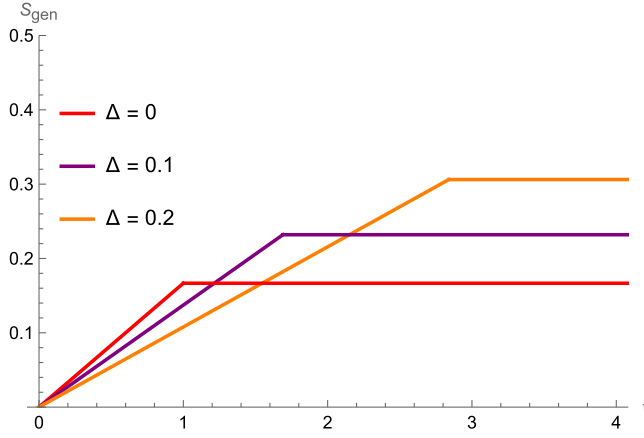
By first extremizing the given expression with respect to t_a , the only solution obtained is $t_a = t_b$. Consequently, the focus shifts to extremizing with respect to a , which determines the location of the island as

$$\begin{aligned}
a &= 2^\Delta \pi^{\Delta/2} (\Delta + 2) G_N M (G_N M^2)^{\Delta/2} \\
&\quad - \frac{c^2 (4\pi)^\Delta (\Delta + 2)^2 G_N (G_N M^2)^{\Delta+1} \exp \left(1 - \frac{b 2^{-\Delta} \pi^{-\frac{\Delta}{2}} M (G_N M^2)^{-\frac{\Delta}{2}-1}}{\Delta+2} \right)}{(2^\Delta \pi^{\Delta/2} (\Delta + 2) G_N M (G_N M^2)^{\Delta/2} - b) (c - 3(4\pi)^{\Delta+1} (\Delta + 2)^2 (G_N M^2)^{\Delta+1})^2}.
\end{aligned} \tag{3.7}$$

By incorporating the island's position, the entanglement entropy can be expressed as

$$\begin{aligned}
S_{\text{gen}} &= \frac{\Delta (\log(4\pi G_N M^2) + 1) (3b^2 - 3b G_N M (c + 32\pi G_N M^2 + 2) + 2G_N^2 M^2 (5c + 96\pi G_N M^2))}{12 G_N M (2G_N M - b)} \\
&\quad + \left(\frac{1}{6} c \log \left(\frac{128 G_N^3 M^3 (b - 2G_N M)^2}{b} \right) + \frac{b}{2G_N M} + 8\pi G_N M^2 - 1 \right) + \mathcal{O}(\Delta^2).
\end{aligned} \tag{3.8}$$

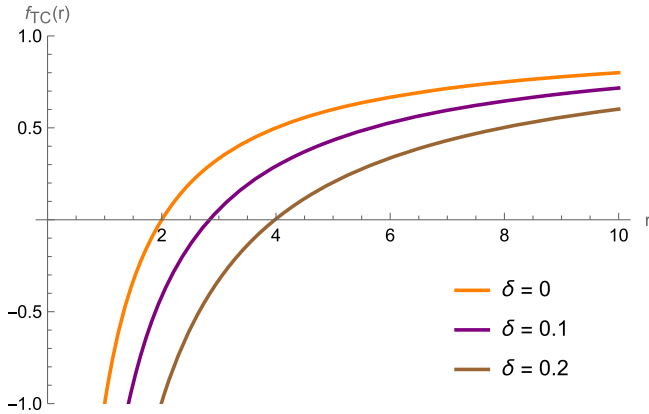
This result demonstrates that the entropy remains constant over time, providing a resolution to the entropy paradox in the late-time regime. In conclusion, the analysis of the result (3.6) and the result (3.8) establishes that the island configuration manifests at late times, effectively terminating the growth of entanglement entropy. The island's boundary is located close to the event horizon [106,107], contributing precisely to the Barrow entropy associated with the Barrow corrected black hole.

FIG. 3. Plot of S_{gen} with and without island.

C. Page time

The Page time refers to the moment when the entropy of radiation reaches its maximum. For an evaporating black hole, the point after which the entropy of the radiation begins to decrease occurs approximately when the black hole has lost half of its initial mass. The entropy remains constant after the Page time for an eternal black hole. The Page time for a Barrow-corrected black hole can be determined by comparing the expressions for the entropy without an island (3.6) and with an island (3.8). The transition from the no-island configuration to the island configuration occurs approximately at the point where these two entropy curves intersect. After this transition, the entanglement entropy remains constant. Using this reasoning, the Page time is

$$t_{\text{Page}} \sim \frac{96\pi^3 G_N^3 M^3}{c} + \frac{144\pi^3 \Delta G_N^3 M^3 (\log(4\pi G_N M^2) + 1)}{c} + \mathcal{O}(\Delta^2). \quad (3.9)$$

FIG. 4. Plot of Tsallis-Cirto metric function versus r for different values of the Tsallis-Cirto parameter.

By combining the results from Eqs. (3.6) and (3.8), the Page curve for the evolution of radiation entropy (or equivalently the entropy of the black hole) can be derived, as shown in Fig. 3. Initially, the entropy of the radiation increases almost linearly with time, as no island is formed. Around the Page time, an island forms near the black hole's horizon, causing the entropy to stabilize at a nearly constant value.

IV. TSALLIS-CIRTO'S APPROACH

Finally, we investigate the Tsallis-Cirto framework by employing Eq. (2.3). The Tsallis-Cirto-corrected temperature and the corresponding metric function is determined as

$$T_{\text{TC}} = \frac{2^{-2\delta-3} \pi^{-\delta-1} M (G_N M^2)^{-\delta-1}}{\delta + 1};$$

$$f_{\text{TC}}(r) = 1 - \frac{2^{2\delta+1} \pi^\delta (\delta + 1) G_N M (G_N M^2)^\delta}{r}, \quad (4.1)$$

where δ denotes the Tsallis-Cirto parameter. By plotting this function, (As seen in Fig. 4 for different values of delta) the dependence of the horizon radius on the Tsallis-Cirto parameter can be visualized. Again, by using Eq. (4.1), the horizon radius in the Tsallis-Cirto-corrected spacetime is expressed as

$$r_h^{\text{Tsallis-Cirto}} = 2G_N M + 2\delta G_N M (\log(G_N M^2) + 1) + 2 \log(2) + \log(\pi) + \mathcal{O}(\delta^2).$$

A direct calculation confirms that the metric function (4.1) satisfies the Einstein field equations. In addition, by computing the Kretschmann scalar, one can confirm that $r = 0$ is a true curvature singularity and can be removed using the tortoise coordinate, which can be derived from the metric function. Using the Eq. (2.11), the metric can be written similar to Eq. (2.12), and it is easy to verify the conformal factor $\Omega_{\text{TC}}(r)$ as

$$\Omega_{\text{TC}}(r) = \frac{\beta}{2\pi} e^{-\frac{2\pi r_{\text{TC}}(r)}{\beta}} \sqrt{1 - \frac{2^{2\delta+1} \pi^\delta (\delta + 1) G_N M (G_N M^2)^\delta}{r}}. \quad (4.2)$$

During the initial stages of evaporation, when no island forms, the entanglement entropy of the radiation grows indefinitely. Using Eq. (2.15) and the entanglement entropy simplifies to

$$S_{\text{gen}} \approx \frac{ct_b}{12G_N M} - \frac{\delta(ct_b(\log(4\pi G_N M^2) + 1))}{12(G_N M)} + \mathcal{O}(\delta^2). \quad (4.3)$$

This result confirms that, in the absence of an island, the entanglement entropy grows linearly with time, creating a

paradox that challenges unitarity and the conservation of information [108]. To resolve this paradox, the island construction is necessary and helps us restore a Page curve consistent with unitarity.

For the computation of the entanglement entropy in the presence of an island and in the late time. We Use Kruskal coordinates and Eq. (2.16), the generalized entropy S_{gen}^2 can be expressed as

$$S_{\text{gen}} = \frac{2\pi a^2}{G_N} - \left\{ \frac{2c}{3} \sqrt{\frac{a - 2^{2\delta+1}\pi^\delta(\delta+1)G_N M(G_N M^2)^\delta}{b - 2^{2\delta+1}\pi^\delta(\delta+1)G_N M(G_N M^2)^\delta}} \exp\left(\frac{2^{-2\delta-2}\pi^{-\delta}(a-b)(G_N M^2)^{-\delta}}{(\delta+1)G_N M}\right) \right. \\ \times \cosh\left(\frac{2^{-2\delta-2}\pi^{-\delta}(t_a - t_b)(G_N M^2)^{-\delta}}{(\delta+1)G_N M}\right) \left. \right\} + \frac{c}{6} \log \left[\frac{2^{8\delta+8}\pi^{4\delta}(\delta+1)^4 G_N^4 M^4 (G_N M^2)^{4\delta}}{ab} \right. \\ \left. \times \frac{(b - 2^{2\delta+1}\pi^\delta(\delta+1)G_N M(G_N M^2)^\delta)^2 \exp\left(\frac{2^{-2\delta-1}\pi^{-\delta}(b-a)(G_N M^2)^{-\delta}}{(\delta+1)G_N M}\right)}{ab} \right]. \quad (4.4)$$

Extremizing (4.4) with respect to t_a and with respect to a determines the island location. Substituting the island location, the entanglement entropy becomes

$$S_{\text{gen}} = \frac{\delta(\log(4\pi G_N M^2) + 1)(3b^2 - 3bG_N M(c + 32\pi G_N M^2 + 2) + 2G_N^2 M^2(5c + 96\pi G_N M^2))}{6G_N M(2G_N M - b)} \\ + \left(\frac{1}{6} c \log\left(\frac{128G_N^3 M^3(b - 2G_N M)^2}{b}\right) + \frac{b}{2G_N M} + 8\pi G_N M^2 - 1 \right) + \mathcal{O}(\delta^2). \quad (4.5)$$

This result demonstrates that the entropy stabilizes at late times and does not depend on time, resolving the entropy paradox and restoring consistency with unitarity. In conclusion, the island configuration emerges at late times, arresting the unbounded growth of entanglement entropy [109,110] and confirms there is no information paradox in this case.

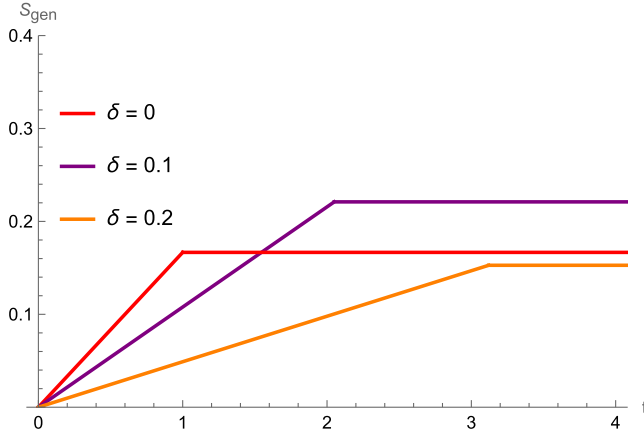
Finally, the Page time can be determined by comparing the entropies, i.e., without an island (4.3) and with an island (4.5). This is the time after which the entanglement entropy stabilizes at a constant value. The Tsallis-Cirto-corrected Page time is given by

$$t_{\text{Page}} \sim \frac{96\pi^2 G_N^3 M^3}{c} + \frac{288\pi^2 G_N^3 M^3 (\log(4\pi G_N M^2) + 1)}{c} \delta \\ + \mathcal{O}(\delta^2). \quad (4.6)$$

Again, the correction comes due to the Tsallis-Cirto correction parameter. The Page curve, as in Fig. 5, which shows the evolution of radiation entropy, can be constructed by combining the expressions for the entropy in the no-island and island configurations.

²The approximations we have used here are

$$\cosh\left(\frac{(t_a + t_b)M^{-2\delta}}{2^{2(\delta+1)}(\delta+1)\pi^\delta(G_N)^{\delta+1}}\right) \gg \frac{1}{2} \sqrt{\frac{b - 2^{2\delta+1}\pi^\delta(\delta+1)G_N M(G_N M^2)^\delta}{a - 2^{2\delta+1}\pi^\delta(\delta+1)G_N M(G_N M^2)^\delta}} \exp\left(\frac{2^{-2\delta-1}M(b-a)(M^2)^{-\delta-1}}{\pi^\delta(\delta+1)G_N^{\delta+1}}\right), \\ \cosh\left(\frac{(t_a - t_b)M^{-2\delta}}{2^{2(\delta+1)}(\delta+1)\pi^\delta(G_N)^{\delta+1}}\right) \ll \frac{1}{2} \sqrt{\frac{b - 2^{2\delta+1}\pi^\delta(\delta+1)G_N M(G_N M^2)^\delta}{a - 2^{2\delta+1}\pi^\delta(\delta+1)G_N M(G_N M^2)^\delta}} \exp\left(\frac{2^{-2\delta-1}M(b-a)(M^2)^{-\delta-1}}{\pi^\delta(\delta+1)G_N^{\delta+1}}\right).$$

FIG. 5. Plot of S_{gen} with and without the island.

V. RÉNYI'S APPROACH

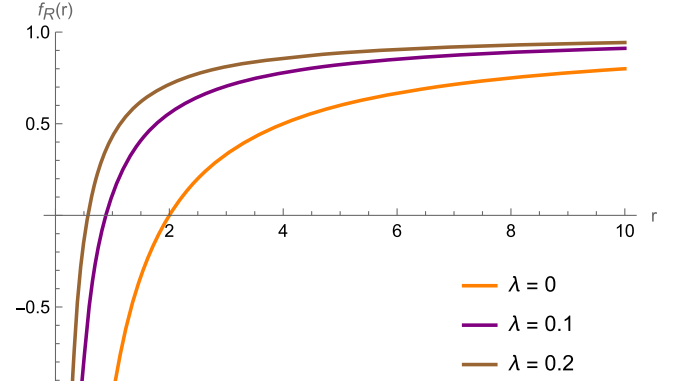
In this section, we systematically investigate the Rényi-corrected framework by utilizing Eq. (2.4) to derive the Rényi-corrected temperature, compute the associated metric function, and analyze the spacetime structure. Beginning with Eq. (2.4), which defines the modification introduced by the Rényi entropy, we derive the corrected temperature T_R , expressed as

$$T_R = \frac{1}{8\pi G_N M} + \frac{\lambda M}{2}. \quad (5.1)$$

This temperature captures quantum-gravitational effects encoded by the Rényi parameter λ . Using this result, we construct the Rényi-corrected metric function $f_R(r)$, which reflects deviations from the standard Schwarzschild geometry. For a static, spherically symmetric spacetime, the metric function is given by

$$f_R(r) = 1 - \frac{2G_N M}{4\pi G_N \lambda M^2 r + r}, \quad (5.2)$$

where λ quantifies the Rényi entropy corrections. A plot of this function as in Fig. 6 reveals the dependence of the horizon radius on the Rényi parameter. By employing

FIG. 6. Plot of the Rényi metric function versus r for various values of the Rényi parameter.

Eq. (5.2), the horizon radius in the Rényi-corrected spacetime is expressed as

$$r_h^{\text{Rényi}} = 2G_N M - 8\lambda\pi G^2 M^3 + \mathcal{O}(\lambda^2).$$

The correction comes due to the correction parameter λ in the horizon radius. A straightforward calculation confirms that the metric function (5.2) is indeed a valid solution to the Einstein field equations. Additionally, the Kretschmann scalar, which serves as an invariant measure of spacetime curvature, is explicitly computed as

$$\mathcal{K} = \frac{48G_N^2 M^2}{r^6(4\pi\lambda G_N M^2 + 1)^2}. \quad (5.3)$$

The singularity at $r = r_h^{\text{Rényi}}$ is a coordinate singularity, which emerges due to the specific choice of coordinates, and the singularity at $r = 0$ is a real curvature singularity, as evidenced by the divergent behavior of the Kretschmann scalar, shown in Eq. (5.3), which diverges as $r \rightarrow 0$, indicating an intrinsic breakdown of spacetime geometry. To resolve the coordinate singularity, we introduce the tortoise coordinate, derived from the metric function as follows:

$$r_\star(r) = \frac{2G_N M(2\pi\lambda M r - 1) + 2G_N M \log(2G_N M(2\pi\lambda M r - 1) + r) + r}{4\pi G_N \lambda M^2 + 1}. \quad (5.4)$$

Using the Kruskal-like coordinates (2.11), the metric takes the form of Eq. (2.12) and the conformal factor $\Omega_R(r)$ incorporating Rényi corrections, given by

$$\Omega_R(r) = \exp\left(-\frac{2\pi(2G_N M \log(r - 2G_N M) - 2G_N M + r)}{\beta}\right) \left[\frac{\beta}{2\pi} \sqrt{1 - \frac{2G_N M}{r}} + \lambda \frac{2G_N^2 M^3 \sqrt{1 - \frac{2G_N M}{r}} (\beta + 4\pi(r - 2G_N M) \log(r - 2G_N M) + 8\pi G_N M - 8\pi r)}{r - 2G_N M} \right] + \mathcal{O}(\lambda^2).$$

When no island forms during the initial stage of the evaporation process, the entanglement entropy grows linearly and can be verified by using Eq. (2.15). Using the approximation before island formation it is easy to verify that the entanglement entropy in the Rényi-corrected geometry is

$$S_{\text{gen}} \approx \frac{ct_b}{12G_N M} + \frac{1}{3}\pi c\lambda M t_b + O(\lambda^2). \quad (5.5)$$

In the absence of an island, the entanglement entropy grows linearly with time and gets a correction factor due to the Rényi modification. This linear growth ultimately leads the

entropy to exceed the Rényi-corrected black hole entropy, creating a paradox. This highlights the necessity of introducing the island construction to resolve the paradox and recover a consistent Page curve.

We now introduce the concept of island construction to compute the entanglement entropy in the presence of a single island. For the Rényi-corrected black hole, we focus on the late-time behavior of the entanglement entropy, particularly the radiation entropy at late times, where the growth of entropy poses significant challenges. Using Eq. (2.13), we can compute the entanglement entropy for conformal matter as

$$S_{\text{matter}} = \frac{c}{6} \log \left(\frac{256G_N^4 M^4 (4\pi b G_N \lambda M^2 + b - 2G_N M)^2 e^{\frac{(b-a)(4\pi G_N \lambda M^2 + 1)}{2G_N M}}}{ab(4\pi G_N \lambda M^2 + 1)^6} \right) - \frac{2c}{3} \left(e^{\frac{(a-b)(4\pi G_N \lambda M^2 + 1)}{4G_N M}} \sqrt{\frac{4\pi a G_N \lambda M^2 + a - 2G_N M}{4\pi b G_N \lambda M^2 + b - 2G_N M}} \cosh \left(\frac{(t_a - t_b)(4\pi G_N \lambda M^2 + 1)}{4G_N M} \right) \right).$$

Now, adding the island contribution $\frac{2\pi a^2}{G_N}$ and using the approximation as above sections and extremizing the expression first with respect to t_a and with respect to a determines the island's position as

$$t_a = t_b; \quad a = 2G_N M \left(2G_N M \left(\frac{e^{-\frac{b}{2G_N M} - 2\pi b \lambda M + 1}}{b} - 2\pi \lambda M \right) + 1 \right). \quad (5.6)$$

Including the island's location, one can confirm that the entanglement entropy becomes stabilized at late times, and is independent of time, resolving the entropy paradox. The expression of generalized entropy at late time is

$$S_{\text{gen}} = \left(\frac{1}{6} c \log \left(\frac{128G_N^3 M^3 (b - 2G_N M)^2}{b} \right) + \frac{b}{2G_N M} + 8\pi G_N M^2 - 1 \right) + \frac{2}{3} \pi \lambda M \left(\frac{cG_N M (10G_N M - 3b)}{b - 2G_N M} + 3b - 96\pi G_N^2 M^3 \right) + O(\lambda^2). \quad (5.7)$$

Again, the final quantity, i.e., the time (the Page time) after which the entropy stabilizes. The Page time can be determined by comparing the entropies for scenarios without an island (5.5) and with an island (5.7). The Rényi-corrected Page time is given by

$$t_{\text{Page}} \sim \frac{96\pi^2 G_N^3 M^3}{c} - \frac{1152\lambda(\pi^3 G_N^4 M^5)}{c} + O(\lambda^2), \quad (5.8)$$

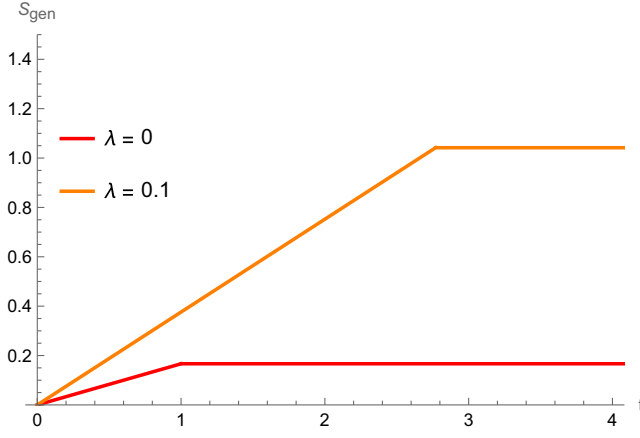
again λ the Rényi correction parameter corrected the Page time. In nutshell, the absence of an island, the radiation entropy increases linearly with time. Around the Page time, an island forms near the black hole horizon, leading to the stabilization of entropy. This curve effectively captures the transition and eventual stabilization of entropy due to the island formation. The Page curve looks like the Fig. 7..

VI. KANIADAKIS'S APPROACH

Finally, we systematically investigate the Kaniadakis-corrected framework by utilizing Eq. (2.5) to derive the Kaniadakis-corrected temperature, construct the corresponding metric function, and analyze the modified spacetime structure. We begin by considering the Kaniadakis entropy modification governed by Eq. (2.5). The Kaniadakis-corrected temperature T_K is given by

$$T_K = \frac{\text{sech}(4\pi G_N \kappa M^2)}{8\pi G_N M}, \quad (6.1)$$

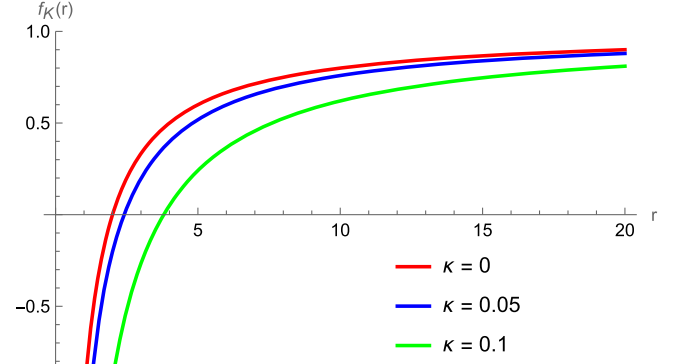
which incorporates quantum gravitational effects as characterized by the Kaniadakis parameter κ . This corrected temperature represents the deviations from the standard Schwarzschild temperature, influenced by the quantum-gravitational modifications to the spacetime geometry [111,112]. Using this corrected temperature, we proceed

FIG. 7. Plot of S_{gen} with and without the island.

to construct the Kaniadakis-corrected metric function $f_K(r)$, which modifies the Schwarzschild geometry to account for the Kaniadakis parameter. Assuming a static, spherically symmetric spacetime, the metric function is expressed as

$$f_K(r) = 1 - \frac{2G_N M \cosh(4\pi\kappa G_N M^2)}{r}. \quad (6.2)$$

Here, κ represents the Kaniadakis parameter, quantifying the deviation from the Schwarzschild black hole. The metric function $f_K(r)$ reflects the changes in the horizon radius due to the quantum-gravitational modifications. A plot of the Kaniadakis metric function as a function of r for various

FIG. 8. Plot of the Kaniadakis metric function versus r for different values of the Kaniadakis parameter.

values of κ is shown in Fig. 8. Utilizing Eq. (6.2), the horizon radius can be determined as

$$r_h^{\text{Kaniadakis}} = 2G_N M + 16\pi^2 G_N^3 \kappa^2 M^5 + \mathcal{O}(\kappa^4).$$

Furthermore, a direct verification confirms that the metric function (6.2) satisfies the Einstein field equations. The Kretschmann scalar inversely proportional to r^6 , which characterizes the curvature properties of spacetime, confirms that $r = r_h^{\text{Kaniadakis}}$ is a coordinate singularity, and $r = 0$ represents a true curvature singularity. To eliminate the coordinate singularity, we introduce the tortoise coordinate, derived from the metric function, as follows:

$$r_\star(r) = r + 2G_N M \cosh(4\pi G_N \kappa M^2) \log(r - 2G_N M \cosh(4\pi G_N \kappa M^2)). \quad (6.3)$$

The metric in these coordinates can be written similarly to Eq. (2.12) using Eq. (2.11). It is easy to verify $\Omega_K(r)$ as the conformal factor that incorporates the Kaniadakis corrections. The conformal factor $\Omega_K(r)$ is given by

$$\begin{aligned} \Omega_K(r) = & \frac{\beta \sqrt{1 - \frac{2G_N M}{r}} e^{-\frac{2\pi(2G_N M \log(r - 2G_N M) + r)}{\beta}}}{2\pi} - 4\kappa^2 \left(\pi G_N^3 M^5 e^{-\frac{2\pi r}{\beta}} \sqrt{1 - \frac{2G_N M}{r}} \right. \\ & \left. \times (r - 2G_N M)^{-\frac{4\pi G_N M}{\beta} - 1} (\beta + 4\pi(r - 2G_N M) \log(r - 2G_N M) - 8\pi G_N M) \right) + \mathcal{O}(\kappa^4). \end{aligned} \quad (6.4)$$

By employing Kruskal-like coordinates and this conformal factor, we obtain a description of the Kaniadakis-corrected spacetime in these coordinates. Again, we apply the conformal mapping technique to investigate the matter's contribution to the entanglement entropy within the Kaniadakis-corrected geometry. By performing a similar line of calculation to the Barrow case as in Sec. III and using the argument of no island during the

early stages of the evaporation process. We use the Eq. (2.15) and compute the matter contribution to the entanglement entropy in Kaniadakis-corrected geometry, which simplifies the approximation

$$S_{\text{gen}} \approx \frac{ct_b}{12G_N M} - \frac{2}{3} \kappa^2 (\pi^2 c G_N M^3 t_b) + \mathcal{O}(\kappa^3). \quad (6.5)$$

In the absence of an island, the entanglement entropy continues to grow linearly with time as the information remains trapped inside the black hole. This linear growth leads to the entanglement entropy eventually surpassing the Kaniadakis-corrected entropy of the black hole, creating a paradox. The radiation entropy becomes infinitely larger than the Kadiakis entropy. To resolve this issue, the introduction of the island construction is necessary.

To address the entropy paradox at late times, we now incorporate the island construction and compute the entanglement entropy in the presence of a single island. We adopt the s-wave approximation to simplify calculations, and using the matter entropy formula, the total entropy is determined by using Kruskal-like coordinates, and the generalized entropy S_{gen} can be computed with the help of Eq. (2.16) as³

$$S_{\text{gen}} = \frac{2\pi a^2}{G_N} - \frac{2}{3}c \sqrt{\frac{a - 2G_N M \cosh(4\pi G_N \kappa M^2)}{b - 2G_N M \cosh(4\pi G_N \kappa M^2)}} e^{\frac{(a-b)\text{sech}(4\pi G_N \kappa M^2)}{4G_N M}} \cosh\left(\frac{(t_a - t_b)\text{sech}(4\pi G_N \kappa M^2)}{4G_N M}\right) + \frac{1}{6}c \log\left(\frac{256G_N^4 M^4 \cosh^4(4\pi G_N \kappa M^2) (b - 2G_N M \cosh(4\pi G_N \kappa M^2))^2 e^{\frac{(b-a)\text{sech}(4\pi G_N \kappa M^2)}{2G_N M}}}{ab}\right). \quad (6.6)$$

Finally, to find the location of the island, we first extremize this expression with respect to t_a , the only solution obtained is $t_a = t_b$. Subsequently, the focus shifts to extremizing with respect to a , which determines the location of the island as

$$a = 2G_N M + \frac{4G_N^2 M^2 e^{4\pi^2 b G_N \kappa^2 M^3 - \frac{b}{2G_N M} + 1}}{b}. \quad (6.7)$$

By substituting the location of the island, we have the entanglement entropy as

$$S_{\text{gen}} = \left(\frac{1}{6}c \log\left(\frac{128G_N^3 M^3 (b - 2G_N M)^2}{b}\right) + \frac{b}{2G_N M} + 8\pi G_N M^2 - 1\right) - \frac{4\kappa^2(\pi^2 G_N M^3 (3b^2 - 3bG_N M(c + 32\pi G_N M^2 + 2) + 2G_N^2 M^2(5c + 96\pi G_N M^2)))}{3(b - 2G_N M)}. \quad (6.8)$$

This result indicates that the entropy does not grow with time, confirming the resolution of the entropy paradox at late times. Combining the results for both early and late times, we obtain the Page curve, which encapsulates the initial linear growth of entropy in Eq. (6.5), followed by its eventual saturation to a constant value in Eq. (6.8). This behavior demonstrates the preservation of information during black hole evaporation and resolves the information paradox. In conclusion, at first, without an island, the radiation entropy grows linearly with time, and then the entropy stabilizes around the Page time (which we will compute next) when an island forms close to the black hole horizon. The Page curve for the Kaniadakis-corrected black hole entropy is shown in Fig. 9.

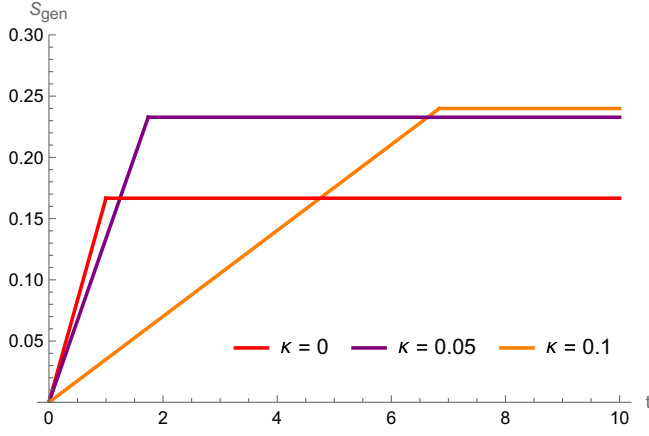
To close this section, we compute the final quantity of interest, i.e., the Page time. The Page time can be determined by comparing the entropy expressions for configurations without an island (6.5) and with an island (6.8). The transition from a no-island to an island arrangement is represented by the intersection of these two entropy curves, which is precisely the Page time. The entanglement entropy stabilizes at an almost constant value following this transition. The Kaniadakis-corrected Page time is given by

$$t_{\text{Page}} \sim \frac{96\pi^3 G_N^3 M^3}{c} + \frac{2304\pi^5 G_N^5 \kappa^2 M^7}{c}. \quad (6.9)$$

This result encapsulates the influence of Kaniadakis's corrections on the Page time. The first term is nothing

³Here we have used the approximations

$$t_a + t_b \gg 4G_N M \cosh(4\pi G_N \kappa M^2) \cosh^{-1}\left(\frac{1}{2} \sqrt{\frac{b - 2G_N M \cosh(4\pi G_N \kappa M^2)}{a - 2G_N M \cosh(4\pi G_N \kappa M^2)}} e^{\frac{(b-a)\text{sech}(4\pi G_N \kappa M^2)}{4G_N M}}\right) \\ t_a - t_b \ll 4G_N M \cosh(4\pi G_N \kappa M^2) \cosh^{-1}\left(\frac{1}{2} \sqrt{\frac{b - 2G_N M \cosh(4\pi G_N \kappa M^2)}{a - 2G_N M \cosh(4\pi G_N \kappa M^2)}} e^{\frac{(b-a)\text{sech}(4\pi G_N \kappa M^2)}{4G_N M}}\right).$$

FIG. 9. Plot of S_{gen} with and without island.

but the Page time for Schwarzschild spacetime, and the second term incorporates the correction due to Kandiakis's Correction.

VII. CONCLUSIONS

This paper investigated the black holes characterized by nonextensive entropy formulations, examining their implications for the information paradox and their resolution using the island formulation. The nonextensive correction to Bekenstein-Hawking entropy provides an alternative framework often linked to modifications from the Generalized Uncertainty Principle. Notably, in the limit where the nonextensive parameter vanishes, the entropy recovers its standard Bekenstein-Hawking form. The horizon radius expands in all cases, and restoring classical parameters recovers the Schwarzschild solution. However, altering entropy inevitably impacts other thermodynamic quantities, posing challenges in maintaining consistency within the broader thermodynamic framework.

We have studied the metric from the nonextensive corrections. It also verified that these solutions satisfied Einstein's equation, and by computing the Krestchmann Scalar, it is shown that $r = 0$ is the true physical singularity of these black holes, too. Furthermore, nonextensive parameter modification and the corrected black holes shift

the event horizon from the standard Schwarzschild radius. Then, we explored the Page curves and information paradox resolutions for corrected black holes. In the early stages of evaporation, no island forms as the entanglement entropy is dominated by radiation. The entanglement entropy before the island in each case is obtained, and it is easy to see from Eqs. (3.6), (4.3), (5.5), and (6.5) that there is a correction in addition to the Schwarzschild black holes. At late times, the entropy is determined by the island's area within the stretched horizon. Using the s-wave approximation and 2D CFT, we computed the entanglement entropy of Hawking radiation and analyzed the effect of nonextensive parameters on the Page curve. Also, the Page time in each case gets corrected with the Page time in Schwarzschild black holes.

Our results generalize the classic Page curve of a Schwarzschild black hole. In the limit of vanishing non-extensivity parameters ($\Delta, \delta, \lambda, \kappa \rightarrow 0$), each model reduces to the usual S_{BH} , and the Page curve reverts to the standard one. For nonzero parameters, the curve shape is qualitatively the same (linear growth then plateau), but the height and slope differ. The key similarity is that with islands, information is recovered (the curves turn over) in all cases. The difference is that the plateau entropy is model-dependent. In effect, the island rule ensures unitarity in every case, but the final entropy is the generalized form. Physically, this means that while the mechanism of island formation is robust, the amount of information encoded (entropy bound) changes with the entropy formula.

A. Comparison of our results in each case and comparison to the existing literature

It is important to see that there are three important quantities that need to be compared. They are the entanglement entropy before and after island formation and the Page time. The entropy after the island formation is complex, so we leave that to read from the individual sections. Here, we compare the entropy before the formation of the island and the Page time.

(1) Generalized entropy without island:

- a. Schwarzschild case: $\approx \frac{ct_b}{12G_N M}$;
- b. Barrow case: $\approx \frac{ct_b}{12G_N M} - \frac{\Delta(ct_b(\log(4\pi G_N M^2) + 1))}{24(G_N M)} + \mathcal{O}(\Delta^2)$;
- c. Tsallis-Cirto case: $\approx \frac{ct_b}{12G_N M} - \frac{\delta(ct_b(\log(4\pi G_N M^2) + 1))}{12(G_N M)} + \mathcal{O}(\delta^2)$;
- d. Rényi case: $\approx \frac{ct_b}{12G_N M} + \frac{1}{3}\pi c\lambda M t_b + \mathcal{O}(\lambda^2)$;
- e. Kaniadakis case: $\approx \frac{ct_b}{12G_N M} - \frac{2}{3}\kappa^2(\pi^2 c G_N M^3 t_b) + \mathcal{O}(\kappa^3)$.

(2) The Page time:

$$\begin{aligned}
 \text{a. Schwarzschild case: } & \sim \frac{96\pi^3 G_N^3 M^3}{c}; \\
 \text{b. Barrow case: } & \sim \frac{96\pi^3 G_N^3 M^3}{c} + \frac{144\pi^3 \Delta G_N^3 M^3 (\log(4\pi G_N M^2) + 1)}{c} + \mathcal{O}(\Delta^2); \\
 \text{c. Tsallis-Cirto case: } & \sim \frac{96\pi^2 G_N^3 M^3}{c} + \frac{288\pi^2 G_N^3 M^3 (\log(4\pi G_N M^2) + 1)}{c} \delta + \mathcal{O}(\delta^2); \\
 \text{d. Rényi case: } & \sim \frac{96\pi^2 G_N^3 M^3}{c} - \frac{1152\lambda(\pi^3 G_N^4 M^5)}{c} + \mathcal{O}(\lambda^2); \\
 \text{e. Kaniadakis case: } & \sim \frac{96\pi^3 G_N^3 M^3}{c} + \frac{2304\pi^5 G_N^5 \kappa^2 M^7}{c}.
 \end{aligned}$$

The Page time in each case gets corrected with the Page time in Schwarzschild black holes and confirms that the Page time depends on the model one chooses. In summary, we have shown that incorporating nonextensive entropy forms such as Barrow, Tsallis-Cirto, Rényi, and Kaniadakis modifies the Page curve by affecting both the saturation behavior and the Page time, while preserving the qualitative structure of information recovery. A comparative analysis reveals that although each entropy model leads to quantitatively distinct Page times and entropy growth profiles, they all share a common feature: the deviation from the standard Bekenstein-Hawking behavior becomes prominent near the black hole's final stages. This suggests a potential universality in how generalized entropic corrections influence black hole evaporation. Our results bear interesting similarities with those obtained via higher-derivative gravity corrections, where the black hole entropy receives geometric modifications (e.g., through Gauss-Bonnet or Weyl terms) [113–115]. Both approaches lead to altered Page times and saturation behavior, suggesting that the influence of entropy corrections—whether statistical or gravitational—plays a central role in quantum information recovery. A sketch of the Page-curve behavior is shown in Figs. 3, 5, 7, and 9. Further work is needed to explore whether these similarities point to deeper connections between microscopic entropy deformations and effective gravitational dynamics.

At present, there exists no empirical evidence suggesting that black hole entropy deviates from the Bekenstein-Hawking formulation in favor of nonextensive statistical frameworks. However, future observational advances, such as precision imaging of black hole shadows, the detection of primordial gravitational waves originating from early Universe black holes, and refined cosmological measurements, may potentially reveal inconsistencies with the predictions of general relativity. Such anomalies could

provide strong motivation for exploring modified theories of gravity and the associated generalized entropy formalisms considered in this work. In anticipation of such developments, it is both timely and necessary to investigate the novel physical phenomena that may arise from the adoption of generalized entropy structures. Simultaneously, it is imperative to examine whether such generalizations can be derived from a more fundamental quantum theory of gravity, such as superstring theory or loop quantum gravity. Furthermore, generalized statistical mechanics may offer new insights into the interplay between black hole thermodynamics, cosmology, and information theory, for instance, through the application of principles such as Landauer's bound [116,117]. Our study is limited to zero or one island cases, though more complex formations may exist. Notably, the sharp transition at the Page time could be smoothed by multiple islands. We have also observed that the Page time changes with the inclusion of nonextensive parameters and plotted for zero as well as nonzero cases. While our results confirm information conservation in the corrected black hole background, the exact dynamics of information transfer into the radiation zone remain an open question, and we will get back to these issues soon.

ACKNOWLEDGMENTS

A. A. is financially supported by the IIT Kanpur Institute's postdoctoral fellowship. A. S. would like to thank CSIR-HRDG for the financial support received as a Postdoctoral Research Associate working under Project No. 03WS(003)/2023-24/EMR-II/ASPIRE.

DATA AVAILABILITY

No data were created or analyzed in this study.

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