

Multicentered Myers-Perry black holes in five dimensions

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We present a new family of multicentered rotating black hole solutions in 5D vacuum Einstein gravity, providing explicit examples of cohomogeneity-three spacetimes. It is well known that, in the presence of two commuting Killing vector fields, the theory reduces to 3D gravity coupled to an $SL(3, \mathbb{R})$ nonlinear sigma model with five scalar fields. We show that the scalar fields of the extremal Myers-Perry solution can be expressed in terms of two harmonic functions on 3D flat space, and that promoting these functions to include multiple sources yields explicit multicentered extremal Myers-Perry black holes located at arbitrary positions. Each center forms a smooth S^3 Killing horizon, provided that the rotation parameters satisfy $|j_i| < 1/2$. We further demonstrate that all curvature singularities are hidden behind the horizons and that no closed timelike curves arise on or outside the horizons. The solutions are asymptotically locally Minkowski in the sense that constant-time hypersurfaces are asymptotically locally Euclidean. As a concrete example, we consider a binary configuration, examine its rod structure, and demonstrate the absence of conical singularities between the two black holes, indicating that they are supported by an intermediate bubble region separating them.

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I. INTRODUCTION

Exact solutions describing systems of multiple black holes have long been of interest in both astrophysics and gravitational theory, since they provide analytic control over configurations that model black hole binaries—prime sources of gravitational waves. Constructing such solutions is, however, notoriously difficult: generic multibody dynamics are intrinsically time dependent and lack enough symmetry to reduce the field equations to a tractable form. Nevertheless, by imposing additional structure (such as stationarity, axisymmetry, or integrability in a reduced sigma-model form), several notable families of static or stationary multiblack hole geometries have been found, together with a variety of multiblack object solutions in higher dimensions. A classical four-dimensional example is the Israel-Khan solution [1],

which represents a static, axisymmetric assemblage of Schwarzschild black holes aligned along a common symmetry axis. In this configuration, the individual black holes attract each other, and equilibrium can be maintained only at the cost of introducing conical defects (“struts”) between the horizons. A rotating analog was later obtained by Kramer and Neugebauer [2], who derived the double-Kerr solution describing the mutual interaction of two spinning black holes. Although spin-spin repulsion partly counteracts the gravitational attraction, conical singularities persist. Subsequent analyses of the double-Kerr family clarified that achieving regular, balanced configurations in vacuum is highly nontrivial [3,4]. These results illustrate a general theme: within four-dimensional vacuum gravity, exact static or stationary multiblack hole configurations typically require distributional sources (struts/strings) and are generically singular.

The situation changes when electric charge is included. The Majumdar-Papapetrou solution [5,6] provides an exact static multiblack hole geometry in the Einstein-Maxwell theory, where the gravitational attraction is precisely cancelled by the electrostatic repulsion, yielding completely regular equilibrium configurations without struts. Israel and Wilson [7], and independently Perjés [8], extended this to stationary settings, producing rotating solutions in Einstein-Maxwell theory. It was later shown,

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however, that these spacetimes contain naked singularities and therefore do not represent genuine black holes. This establishes that equilibrium configurations of rotating charged black holes cannot be achieved within Einstein-Maxwell theory alone. More recently, Teo and Wan [9] constructed a new family of exact, fully regular multicentered spinning black holes in 5D Kaluza-Klein theory. Upon dimensional reduction, these yield balanced configurations of arbitrarily many dyonic, rotating black holes in four-dimensional Einstein-Maxwell dilaton theory. Each constituent black hole is described by its own mass, angular momentum, and equal electric and magnetic charges, as well as its position. Moreover, when all angular momenta are set to zero, the dilaton vanishes and the solution smoothly reduces to the Majumdar-Papapetrou solution. More recently, we constructed an exact solution describing multicentered rotating black holes in 5D Kaluza-Klein theory [10]; upon dimensional reduction, it yields multicentered rotating black holes carrying both electric and magnetic charges in four-dimensional Einstein-Maxwell dilaton theory. This solution provides a rotating generalization of the Majumdar-Papapetrou family and extends the multicentered rotating black holes of Teo and Wan [9] to the case of unequal electric and magnetic charges.

In five dimensions, the landscape of multiblack hole and multiblack object solutions is substantially richer than in four dimensions, owing to the existence of nonspherical horizon topologies. A striking demonstration of the richness of 5D gravity is provided by the discovery of black rings [11,12]. The first example of multiblack object solutions is the “black saturn” solution of Elvang and Figueras [13], which describes a spherical Myers-Perry black hole [14] surrounded by a black ring. This composite object is asymptotically flat and exhibits the remarkable possibility of “balanced” configurations without conical singularities, provided the black ring and the central black hole rotate. This black saturn, for the first time, showed that regular multicomponent stationary solutions can exist in vacuum 5D gravity without supersymmetry. Furthermore, Iguchi and Mishima constructed the first exact “black diling” solution [15], consisting of two concentric black rings in stationary equilibrium. By contrast, it remains difficult to construct regular, asymptotically flat vacuum solutions describing multiple black holes with spherical horizons (S^3). A class of 5D, static vacuum solutions describing multiple spherical black holes was obtained by Tan and Teo [16]. Within the generalized Weyl class, they constructed multicentered configurations of Schwarzschild black holes in five dimensions. As in the four-dimensional Israel-Khan solution, these spacetimes inevitably contain conical singularities. Another example is the double Myers-Perry solution found by Herdeiro [17], which may be viewed as a 5D analog of the four-dimensional double-Kerr spacetime. It describes a pair of rotating black holes, each with an S^3 horizon. As in the double-Kerr case, conical

singularities persist and can be interpreted as struts supporting the system, indicating that balanced multicentered rotating black holes remain difficult to realize even in 5D vacuum gravity. These works motivate a closer examination of whether fully regular, asymptotically flat multiblack hole solutions with rotation can exist in the vacuum case, and in particular whether the Myers-Perry black hole admits genuine multicentered generalizations.

In this paper, we construct a new class of multicentered rotating black hole solutions in 5D vacuum Einstein gravity. The solutions describe multicentered extremal Myers-Perry black holes, with independent spin parameters at each center and with the centers located at arbitrary points in 3D Euclidean space. Our construction assumes only two commuting Killing vectors: an asymptotically timelike Killing vector and a single rotational Killing vector. Consequently, the solutions are not, in general, of Weyl class (which requires additional axial symmetries), but instead form a cohomogeneity-three family. Assuming the existence of two commuting Killing vector fields (one timelike and one rotational), the field equations reduce to 3D Euclidean gravity coupled to an $SL(3, \mathbb{R})$ nonlinear sigma model whose scalar sector consists of five fields: three inner products of the Killing vectors and two twist potentials. Using the Maison formulation [18] together with Clément’s construction [19,20], we show that the scalar fields of the extremal Myers-Perry black hole admit a representation in terms of two harmonic functions on flat $\mathbb{E}^3$. By promoting the single-source harmonic functions to multisource ones, we obtain an explicit class of multicentered extremal Myers-Perry configurations with centers located at arbitrary points in $\mathbb{E}^3$. We then analyze the geometry in detail. Each center forms a smooth S^3 Killing horizon provided the spin parameters satisfy $|j_i| < 1/2$, and we derive the corresponding near-horizon geometry and horizon area. We show regularity in the domain of outer communication: all curvature singularities are hidden behind the horizons and closed timelike curves (CTCs) are rigorously excluded. The solutions are asymptotically locally flat, being asymptotically flat modulo $\mathbb{Z}_N$ (where N is the number of black holes) identification, which leads to lens-space asymptotics. As a concrete example, we study a binary configuration, determine its rod structure, and identify the appearance of an intermediate bubble region, highlighting geometric features distinctive to 5D gravity.

We briefly outline the organization of this paper. Section II reviews the Maison formalism and explains how the 5D Einstein equations with two commuting Killing fields can be recast as a 3D gravity-coupled $SL(3, \mathbb{R})$ sigma model. The essential field equations required for the construction of our solution are also collected there, following the formulation of Clément. Section III shows that the extremal Myers-Perry geometry arises from a pair of harmonic functions sourced at a single point, and that promoting these functions to include multiple centers naturally yields a family of multi-Myers-Perry

configurations. In Sec. IV, we investigate the physical and geometric features of these solutions—such as their horizon structure, asymptotics, conditions for regularity, and the exclusion of CTCs. Section V presents a concrete example of a double Myers-Perry black hole configuration. Section VI offers concluding remarks and a summary of our results.

II. NONLINEAR SIGMA MODEL IN 5D EINSTEIN GRAVITY WITH TWO COMMUTING KILLING VECTORS

To study stationary solutions with a single $U(1)$ symmetry, we assume that the spacetime admits two commuting Killing vector fields: a timelike Killing vector and a spacelike rotational Killing vector (at least asymptotically). Under this symmetry assumption, the 5D Einstein equations can be dimensionally reduced to 3D Euclidean gravity coupled to five scalar fields [18]. We briefly recall how these scalars are organized as an $SL(3, \mathbb{R})$ -invariant nonlinear sigma model. In general, the reduced equations remain difficult to solve because the sigma-model fields are coupled to the 3D Euclidean metric. Clément [19,20] showed, however, that if the 3D base is taken to be flat, one can construct a distinguished class of solutions governed by two harmonic functions. While Clément considered the asymptotically Kaluza-Klein case, we will focus on asymptotically flat solutions, or more generally on solutions whose spatial slices are asymptotically locally Euclidean such as the Eguchi-Hanson space. In what follows, we summarize the equations needed for our analysis, following Clément's formulation.

A. 5D Einstein equation with two commuting Killing vectors

Let ξ_a ($a = 0, 1$) be two mutually commuting Killing vector fields, so that $[\xi_a, \xi_b] = 0$ and $\mathcal{L}_{\xi_a} g = 0$. Introducing coordinates x^a adapted to ξ_a (i.e., $\xi_a = \partial/\partial x^a$), the metric g can be written in the form

$$ds^2 = \lambda_{ab}(dx^a + \omega^a_i dx^i)(dx^b + \omega^b_j dx^j) + |\tau|^{-1} h_{ij} dx^i dx^j, \quad (1)$$

where the scalar fields λ_{ab} , $\tau := -\det(\lambda_{ab})$, the functions ω^a_i , and the 3D metric h_{ij} ($i = 2, 3, 4$) are independent of the Killing coordinates x^a . In the vacuum case, there exist locally twist potentials V_a such that

$$\partial_k V_a = \tau \sqrt{|h|} \lambda_{ab} \varepsilon_{kij} h^{im} h^{jn} \partial_m \omega^b_n. \quad (2)$$

With these definitions, the vacuum Einstein equations reduce to the field equations for the five scalar fields $\{\lambda_{ab}, V_a\}$,

$$\Delta_h \lambda_{ab} = \lambda^{cd} h^{ij} \frac{\partial \lambda_{ac}}{\partial x^i} \frac{\partial \lambda_{bd}}{\partial x^j} + \tau^{-1} h^{ij} \frac{\partial V_a}{\partial x^i} \frac{\partial V_b}{\partial x^j}, \quad (3)$$

$$\Delta_h V_a = \tau^{-1} h^{ij} \frac{\partial \tau}{\partial x^i} \frac{\partial V_a}{\partial x^j} + \lambda^{bc} h^{ij} \frac{\partial \lambda_{ab}}{\partial x^i} \frac{\partial V_c}{\partial x^j}, \quad (4)$$

together with the Einstein equations for the 3D metric h_{ij} ,

$$R^h_{ij} = \frac{1}{4} \lambda^{ab} \lambda^{cd} \frac{\partial \lambda_{ac}}{\partial x^i} \frac{\partial \lambda_{bd}}{\partial x^j} + \frac{1}{4} \tau^{-2} \frac{\partial \tau}{\partial x^i} \frac{\partial \tau}{\partial x^j} - \frac{1}{2} \tau^{-1} \lambda^{ab} \frac{\partial V_a}{\partial x^i} \frac{\partial V_b}{\partial x^j}, \quad (5)$$

where Δ_h is the Laplacian and R^h_{ij} is the Ricci tensor associated with h_{ij} .

B. Coset matrix

Maison [18] showed that the action for the scalar fields $\{\lambda_{ab}, V_a\}$ can be described as a nonlinear sigma model with a global $SL(3, \mathbb{R})$ symmetry. This symmetry becomes manifest upon introducing the symmetric 3×3 matrix

$$\chi = \begin{pmatrix} \lambda_{ab} - \frac{V_a V_b}{\tau} & \frac{V_a}{\tau} \\ \frac{V_b}{\tau} & -\frac{1}{\tau} \end{pmatrix}, \quad (6)$$

where this is symmetric, $\chi^T = \chi$, and unimodular, $\det(\chi) = 1$. In terms of χ , the field equations (3), (4), and (5) can be derived from the action

$$S = \int \left(R^h - \frac{1}{4} h^{ij} \text{tr}(\chi^{-1} \partial_i \chi \chi^{-1} \partial_j \chi) \right) \sqrt{|h|} d^3 x, \quad (7)$$

which is invariant under the global transformation

$$\chi \rightarrow \chi' = g \chi g^T, \quad h \rightarrow h, \quad (8)$$

with $g \in SL(3, \mathbb{R})$. Equations (3), (4), and (5) take the compact form

$$d \star_h (\chi^{-1} d\chi) = 0, \quad (9)$$

$$R^h_{ij} = \frac{1}{4} \text{tr}(\chi^{-1} \partial_i \chi \chi^{-1} \partial_j \chi). \quad (10)$$

Thus, the presence of two commuting Killing vector fields reduces 5D vacuum gravity to 3D gravity coupled to an $SL(3, \mathbb{R})$ nonlinear sigma model. If both Killing vectors are spacelike, the target space is the Riemannian coset $SL(3, \mathbb{R})/SO(3)$, whereas if one of them is timelike, it is replaced by the Lorentzian coset $SL(3, \mathbb{R})/SO(2, 1)$.

C. 5D asymptotically flat solutions

Since Eqs. (9) and (10) are coupled, solving them in general is nontrivial. A considerable simplification occurs, however, if the 3D metric h_{ij} is taken to be flat, namely the Euclidean metric on $\mathbb{E}^3$,

$$h_{ij}dx^i dx^j = d\mathbf{x} \cdot d\mathbf{x}, \quad (11)$$

where $\mathbf{x} = (x, y, z)$ are Cartesian coordinates on $\mathbb{E}^3$. In this case the field equations reduce to

$$\partial_i(\chi^{-1}\partial^i\chi) = 0, \quad (12)$$

$$\text{tr}(\chi^{-1}\partial_i\chi\chi^{-1}\partial_j\chi) = 0. \quad (13)$$

Following Clément [19,20], one can represent asymptotically Kaluza-Klein solutions by a coset matrix of the form

$$\chi = \eta e^{fA} e^{gA^2}, \quad (14)$$

where f and g are harmonic functions on $\mathbb{E}^3$, and η and A are a constant 3×3 matrices. Assuming that f and g vanish at infinity, asymptotic flatness of the corresponding 5D metric requires

$$\eta = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}, \quad (15)$$

which is different from that of asymptotically Kaluza-Klein solutions [19,20]. Moreover, if A satisfies

$$A^T = \eta A \eta, \quad \text{tr}(A) = 0, \quad \text{tr}(A^2) = 0, \quad (16)$$

then χ is symmetric ($\chi^T = \chi$), unimodular ($\det \chi = 1$), and automatically obeys the constraint (13).

III. FROM AN EXTREMAL MYERS-PERRY BLACK HOLE TO MULTIBLACK HOLES

The 5D Myers-Perry solution [14] describes a rotating black hole, whose horizon cross section has the topology of a three-sphere S^3 . It is an asymptotically flat, stationary, and biaxisymmetric solution of the vacuum Einstein equations in five dimensions. In this section, we show that the extremal limit can be expressed in terms of the solution with two harmonic functions, Eq. (14), each having a single point source. By extending these harmonic functions to include multiple point sources, we then construct an exact solution describing multicentered rotating black holes of the 5D vacuum Einstein equations.

A. Extremal Myers-Perry black hole

The metric for the 5D Myers-Perry black hole is written as [14]

$$ds^2 = -dt^2 + \frac{m}{\Sigma} (dt - a_1 \sin^2 \bar{\theta} d\phi_1 - a_2 \cos^2 \bar{\theta} d\phi_2)^2 \\ + (\bar{r}^2 + a_1^2) \sin^2 \bar{\theta} d\phi_1^2 + (\bar{r}^2 + a_2^2) \cos^2 \bar{\theta} d\phi_2^2 + \frac{\Sigma}{\Delta} d\bar{r}^2 \\ + \Sigma d\bar{\theta}^2, \quad (17)$$

with the metric functions

$$\Delta = \bar{r}^2 \left(1 + \frac{a_1^2}{\bar{r}^2} \right) \left(1 + \frac{a_2^2}{\bar{r}^2} \right) - m, \quad (18)$$

$$\Sigma = \bar{r}^2 + a_1^2 \cos^2 \bar{\theta} + a_2^2 \sin^2 \bar{\theta}, \quad (19)$$

where m and a_1, a_2 are the mass and rotational parameters. The angular coordinates $(\bar{\theta}, \phi_1, \phi_2)$ take values in the ranges

$$0 \leq \bar{\theta} \leq \frac{\pi}{2}, \quad 0 \leq \phi_1 < 2\pi, \quad 0 \leq \phi_2 < 2\pi. \quad (20)$$

The spacetime admits three mutually commuting Killing vector fields: a timelike Killing vector ∂_t (at least asymptotically) and two rotational Killing vectors ∂_{ϕ_1} and ∂_{ϕ_2} . At spatial infinity $\bar{r} \rightarrow \infty$, the metric approaches the 5D Minkowski form,

$$ds^2 \simeq -dt^2 + d\bar{r}^2 + \bar{r}^2 (d\bar{\theta}^2 + \sin^2 \bar{\theta} d\phi_1^2 + \cos^2 \bar{\theta} d\phi_2^2). \quad (21)$$

Introducing new angular coordinates (θ, ψ, ϕ) via

$$\bar{\theta} = \frac{\theta}{2}, \quad \phi_1 = \frac{\psi + \phi}{2}, \quad \phi_2 = \frac{-\psi + \phi}{2}, \quad (22)$$

we see that ∂_ψ and ∂_ϕ are also Killing vectors. In these coordinates, the S^3 metric at spatial infinity takes the standard Hopf fibration form, exhibiting S^3 as an S^1 bundle over S^2 :

$$ds^2 \simeq -dt^2 + d\bar{r}^2 + \frac{\bar{r}^2}{4} [(d\psi - \cos \theta d\phi)^2 + d\theta^2 + \sin^2 \theta d\phi^2], \quad (23)$$

together with the identifications

$$(\psi, \phi) \sim (\psi + 2\pi, \phi + 2\pi), \quad (\psi, \phi) \sim (\psi + 4\pi, \phi). \quad (24)$$

The horizons are located at the values $\bar{r}$ satisfying $\Delta = 0$. The solution is extremal when Δ has a double zero in $\bar{r}^2$, which occurs for

$$m = (|a_1| + |a_2|)^2. \quad (25)$$

Taking the extremal limit

$$m = (a_1 - a_2)^2, \quad (26)$$

and introducing the new radial coordinate r ,

$$\bar{r} = \sqrt{r - 4a_1 a_2}, \quad (27)$$

together with the new parameters

$$a_{\pm} = \frac{a_1 \pm a_2}{2}, \quad (28)$$

the metric can be rewritten in the form

$$\begin{aligned}
 ds^2 = & \frac{2r^2 + 2a_-^2 r - a_-^3 a_+ \cos \theta + a_-^4}{2r + a_- a_+ \cos \theta + a_-^2} \left[d\psi - \frac{a_-^2 (a_- - a_+ \cos \theta)}{2r^2 + 2a_-^2 r - a_-^3 a_+ \cos \theta + a_-^4} \left(dt + \frac{a_-^2 a_+ \sin^2 \theta}{2r} d\phi \right) \right. \\
 & \left. + \left(-\cos \theta + \frac{a_- a_+ \sin^2 \theta}{2r} \right) d\phi \right]^2 - \frac{2r^2}{2r^2 + 2a_-^2 r - a_-^3 a_+ \cos \theta + a_-^4} \left(dt + \frac{a_-^2 a_+ \sin^2 \theta}{2r} d\phi \right)^2 \\
 & + \frac{2r + a_-^2 + a_- a_+ \cos \theta}{2r^2} [dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)]. \quad (29)
 \end{aligned}$$

For this extremal Myers-Perry solution, the metric h_{ij} takes the flat form:

$$h_{ij} dx^i dx^j = dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (30)$$

and the corresponding coset matrix χ (14) can be written as the form depending on two harmonic functions f and g on the flat space:

$$\chi = \eta e^{fA} e^{gA^2}, \quad (31)$$

with

$$f = \frac{1}{r}, \quad g = \frac{a_+ \cos \theta}{2a_- r^2}, \quad (32)$$

where the matrices η, A are given by

$$\eta = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}, \quad (33)$$

$$A = \begin{pmatrix} -a_-^2 & a_-^3 & 0 \\ 0 & \frac{1}{2} a_-^2 & 1 \\ a_-^3 & -\frac{3}{4} a_-^4 & \frac{1}{2} a_-^2 \end{pmatrix}, \quad (34)$$

where it should be emphasized that, in constructing the scalar fields $\{\lambda_{ab}, V_a\}$, the pair of Killing vectors ξ_a must be chosen as $\xi_0 = \partial_t$ and $\xi_1 = \partial_\psi$, rather than $\{\xi_0, \xi_1\} = \{\partial_t, \partial_{\phi_1}\}$ or $\{\xi_0, \xi_1\} = \{\partial_t, \partial_{\phi_2}\}$.

In what follows, we demonstrate this explicitly. From (32)–(34), the coset matrix χ takes the form

$$\chi = \begin{pmatrix} -\frac{1}{2} a_-^4 (f^2 + 2g) + a_-^2 f - 1 & \frac{1}{4} [a_-^5 (f^2 + 2g) - 4a_-^3 f] & -\frac{1}{2} a_-^3 (f^2 + 2g) \\ \frac{1}{4} [a_-^5 (f^2 + 2g) - 4a_-^3 f] & \frac{1}{8} [6a_-^4 f - a_-^6 (f^2 + 2g)] & \frac{1}{4} [a_-^4 (f^2 + 2g) - 2a_-^2 f - 4] \\ -\frac{1}{2} a_-^3 (f^2 + 2g) & \frac{1}{4} [a_-^4 (f^2 + 2g) - 2a_-^2 f - 4] & \frac{1}{2} [-a_-^2 (f^2 + 2g) - 2f] \end{pmatrix}. \quad (35)$$

Hence, from Eq. (6), we can read off the conformal factor τ and the scalar fields (λ_{ab}, V_a) as

$$\tau = \frac{2}{a_-^2 (f^2 + 2g) + 2f}, \quad (36)$$

$$\lambda_{00} = \frac{\tau}{2} [a_-^2 (f^2 - 2g) - 2f], \quad (37)$$

$$\lambda_{01} = -\frac{\tau}{2} [a_-^3 (f^2 - 2g)], \quad (38)$$

$$\lambda_{11} = \frac{\tau}{2} [a_-^4 (f^2 - 2g) + 2a_-^2 f + 2], \quad (39)$$

$$V_0 = -\frac{\tau}{2} [a_-^3 (f^2 + 2g)], \quad (40)$$

$$V_1 = \frac{\tau}{4} [a_-^4 (f^2 + 2g) - 2a_-^2 f - 4]. \quad (41)$$

From Eqs. (2), (40), and (41), the one-forms $\omega^0 = \omega^0_i dx^i$ and $\omega^1 = \omega^1_i dx^i$ can be expressed as

$$\nabla \times \omega^0 = -a_-^3 \nabla g, \quad (42)$$

$$\nabla \times \omega^1 = -\nabla (f + a_-^2 g). \quad (43)$$

If we define $\tilde{\omega}^1 := \omega^1 - \frac{1}{a_-} \omega^0$, then

$$\nabla \times \tilde{\omega}^1 = -\nabla f. \quad (44)$$

From Eq. (32), these can be solved as

$$\omega^0 = \frac{a_-^2 a_+}{2r} \sin^2 \theta d\phi = -\frac{a_-^2 a_+}{2r^3} [ydx - xdy], \quad (45)$$

$$\tilde{\omega}^1 = -\cos \theta d\phi = \frac{z y dx - x dy}{r^2 x^2 + y^2}, \quad (46)$$

with $(x, y, z) = (r \sin \theta \cos \phi, r \sin \theta \sin \phi, r \cos \theta)$. Thus the 5D metric (1) can be obtained as

$$ds^2 = \frac{H_-}{H_+} \left[d\psi - \frac{a_-^3(f^2 - 2g)}{2H_-} (dt + \omega_\phi^0 d\phi) + \omega_\phi^1 d\phi \right]^2 - \frac{1}{H_-} (dt + \omega_\phi^0 d\phi)^2 + H_+ [dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)], \quad (47)$$

where the functions $H_\pm$ are given by

$$H_- = \frac{1}{2} [a_-^4 (f^2 - 2g) + 2a_-^2 f + 2], \\ H_+ = \frac{1}{2} [a_-^2 (f^2 + 2g) + 2f]. \quad (48)$$

This coincides with the metric (29) corresponding to the extremal Myers-Perry black hole.

B. Multicentered rotating black hole solutions

We now generalize the harmonic functions f and g in Eq. (32) to multicenter configurations by taking

$$f = \sum_{i=1}^N \frac{1}{|\mathbf{x} - \mathbf{x}_i|}, \quad (49)$$

$$g = \sum_{i=1}^N \frac{j_i (z - z_i)}{|\mathbf{x} - \mathbf{x}_i|^3}, \quad (50)$$

where j_i are new parameters. Using Eqs. (42) and (44), we can show that the one-forms ω^0 and $\tilde{\omega}^1$ take the following explicit forms:

$$\omega^0 = - \sum_{i=1}^N a_-^3 j_i \frac{(y - y_i) dx - (x - x_i) dy}{|\mathbf{x} - \mathbf{x}_i|^3}, \quad (51)$$

$$\tilde{\omega}^1 = \sum_{i=1}^N \frac{(z - z_i)}{|\mathbf{x} - \mathbf{x}_i|} \frac{(y - y_i) dx - (x - x_i) dy}{(x - x_i)^2 + (y - y_i)^2}. \quad (52)$$

Consequently, the 5D metric of the resulting multicentered black hole solution can be written as

$$ds^2 = \frac{H_-}{H_+} \left[d\psi - \frac{a_-^3(f^2 - 2g)}{2H_-} (dt + \omega^0) + \omega^1 \right]^2 - \frac{1}{H_-} (dt + \omega^0)^2 + H_+ d\mathbf{x} \cdot d\mathbf{x}, \quad \mathbf{x} = (x, y, z) \quad (53)$$

together with Eqs. (48), (49), (50), (51), and (52). As we shall see in the next section, this metric describes a family of multicentered rotating black holes.

IV. PROPERTIES OF THE MULTIROTATING BLACK HOLE SOLUTION

In this section, we see that this solution is regular and describes asymptotically locally Euclidean, multirotating black holes, each possessing an extremal horizon with the topology of S^3 . We demonstrate that curvature singularities are confined inside the horizons and do not occur on or outside them. Furthermore, we prove the absence of closed timelike curves in the exterior region as well as on the horizons.

A. Near-horizon geometry

The metric apparently diverges at the point sources $\mathbf{x} = \mathbf{x}_i$ in the harmonic functions f and g but we show that they correspond to smooth Killing horizons, provided for all i , the parameters j_i satisfy

$$|j_i| < \frac{1}{2}. \quad (54)$$

Using the new radial coordinate $r := |\mathbf{x} - \mathbf{x}_i|$ and the spherical coordinates $(x, y, z) = (r \sin \theta \cos \phi, r \sin \theta \sin \phi, r \cos \theta)$ so that the i th point source becomes an origin, we examine the behavior of the metric near $r = 0$. First, we introduce new coordinates (t', r', ψ', ϕ') defined by

$$d\psi = d\psi' + (A_0 - B_0) dt, \quad d\phi = d\phi' + (A_0 + B_0) dt, \\ dt = \frac{E}{\varepsilon} dt', \quad dr = \varepsilon D dr', \quad (55)$$

with the constants

$$A_0 = -B_0 = \frac{1}{2a_-}, \quad D = 4a_-^2, \quad E = \frac{\sqrt{a_-^2 - a_+^2}}{2}. \quad (56)$$

Next, we introduce the coordinate (v, ψ'', ϕ'') by

$$dt' = dv + \left(\frac{A_2}{r'^2} + \frac{A_1}{r'} \right) dr', \quad d\psi' = d\psi'' + \frac{B_1}{r'} dr', \\ d\phi' = d\phi'' + \frac{C_1}{r'} dr' \quad (57)$$

with

$$A_1 = \pm \frac{a_- (1 + a_-^2 f_s) \varepsilon}{E \sqrt{1 - 4j_i^2}} = \pm \frac{2a_- (1 + a_-^2 f_s) \varepsilon}{\sqrt{a_-^2 - a_+^2} \sqrt{1 - 4j_i^2}}, \quad (58)$$

$$A_2 = \pm \frac{a_-^3 \sqrt{1 - 4j_i^2}}{2DE} = \pm \frac{a_- \sqrt{1 - 4j_i^2}}{4\sqrt{a_-^2 - a_+^2}}, \quad (59)$$

$$B_1 = \mp \frac{1}{\sqrt{1 - 4j_i^2}}, \quad (60)$$

$$C_1 = \pm \frac{2j_i}{\sqrt{1 - 4j_i^2}}, \quad (61)$$

then, the apparent divergence can be eliminated. Finally, taking the near-horizon limit $\varepsilon \rightarrow 0$, we obtain the metric

$$ds^2 \simeq \frac{2D^2 E^2}{a_-^4 f_+} r'^2 dv^2 \mp \frac{2DEf_+}{a_- \sqrt{1-4j_i^2}} dv dr' + \frac{4DE}{a_- f_+} r' dv [d\psi'' - (\cos \theta - j_i \sin^2 \theta) d\phi''] \\ + \frac{a_-^2 f_-}{f_+} \left[d\psi'' + \frac{2j_i - \cos \theta}{f_-} d\phi'' \right]^2 + \frac{a_-^2 (1-4j_i^2)}{2f_-} \sin^2 \theta d\phi''^2 + \frac{a_-^2 f_+}{2} d\theta^2, \quad (62)$$

where $f_{\pm} := 1 \pm 2j_i \cos \theta$. The horizon area is

$$A = 8\pi a_-^3 \sqrt{1-4j_i^2}. \quad (63)$$

The nonvanishing horizon area together with the absence of CTCs near the horizon requires the conditions

$$a_- > 0, \quad |j_i| < \frac{1}{2}, \quad i = 1, \dots, N. \quad (64)$$

This is the same near-horizon geometry as that of an extremal Myers-Perry black hole whose horizon cross section is S^3 , provided the angular coordinates satisfy the periodicities $\Delta\psi = 4\pi$ and $\Delta\phi = 2\pi$, equivalently $\Delta\psi'' = 4\pi$ and $\Delta\phi'' = 2\pi$.

B. Asymptotic structure

In terms of the standard spherical coordinates $(x, y, z) = (r \sin \theta \cos \phi, r \sin \theta \sin \phi, r \cos \theta)$, the functions, $f, g, H_{\pm}$ and the one-forms $\omega^0, \tilde{\omega}^1$ at $r \rightarrow \infty$ behave asymptotically as

$$f \simeq \frac{N}{r} + \mathcal{O}(r^{-2}), \quad (65)$$

$$g \simeq \frac{\sum_i j_i \cos \theta}{r^2} + \mathcal{O}(r^{-3}), \quad (66)$$

$$H_+ \simeq \frac{N}{r} + \mathcal{O}(r^{-2}), \quad (67)$$

$$H_- \simeq 1 + \frac{Na_-^2}{r} + \mathcal{O}(r^{-2}), \quad (68)$$

and

$$\omega^0 \simeq \left(\frac{a_-^3 \sum_i j_i}{r} \sin^2 \theta + \mathcal{O}(r^{-2}) \right) d\phi, \quad (69)$$

$$\tilde{\omega}^1 \simeq (-N \cos \theta + \mathcal{O}(r^{-1})) d\phi. \quad (70)$$

In terms of the new radial coordinate $\tilde{r} := 2\sqrt{N}r$, the metric at $\tilde{r} \rightarrow \infty$ behaves as

$$ds^2 \simeq -dt^2 + d\tilde{r}^2 + \frac{\tilde{r}^2}{4} \left[\left(\frac{d\psi}{N} - \cos \theta d\phi \right)^2 + d\theta^2 + \sin^2 \theta d\phi^2 \right], \quad (71)$$

which is locally isometric to 5D Minkowski spacetime, with the spatial infinity S^3 replaced with the lens space $L(N; 1) = S^3/\mathbb{Z}_N$.

C. Regularity

If curvature singularities exist outside the horizons, they appear at points where the metric or its inverse diverges, which happens only on the surfaces $H_+(x, y, z) = 0$ or $H_-(x, y, z) = 0$. We can show that such singularities do not exist on and outside the event horizons at $\mathbf{x} = \mathbf{x}_i$ provided that $|j_i| < 1/2$. To demonstrate this, it is sufficient to verify that $H_{\pm} > 0$ on and outside the horizons. First, we note that under the assumptions, we have $f > 0$, while g can take both signs.

We normalized all quantities Q by the mass m , and we denote the corresponding dimensionless quantities by $\hat{Q}$. For example,

$$\hat{\mathbf{x}} = \frac{\mathbf{x}}{m}, \quad \hat{\mathbf{x}}_i = \frac{\mathbf{x}_i}{m}, \quad \hat{f} = mf = \sum_i \frac{1}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|}, \\ \hat{g} = m^2 g = \sum_i \frac{j_i (\hat{z} - \hat{z}_i)}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^2} \quad (72)$$

and, it follows from Eqs. (26) and (28) that

$$\hat{a}_- = \frac{a_-}{\sqrt{m}} = \frac{1}{2}, \quad \hat{j}_i = j_i. \quad (73)$$

Using this, we can show the positivity of the function $\hat{H}_- = H_-$ as

$$2\hat{H}_- = \hat{a}_-^4 (\hat{f}^2 - 2\hat{g}) + 2\hat{a}_-^2 \hat{f} + 2 \\ > \hat{a}_-^4 [(\hat{f}^2 - 2\hat{g}) + 2\hat{f} + 2] \\ = \hat{a}_-^4 [(1 + \hat{f})^2 - 2\hat{g} + 1] \\ \geq \hat{a}_-^4 [(1 + \hat{f})^2 - 2|\hat{g}| + 1] \\ > 0 \quad (74)$$

where we have used the inequality

$$\begin{aligned}
(1 + \hat{f})^2 - 2|\hat{g}| &= \left(1 + \sum_i \hat{f}_i\right)^2 - 2\left|\sum_i \hat{g}_i\right| \\
&> 1 + \sum_i \hat{f}_i^2 - 2\sum_i |\hat{g}_i| \\
&\geq 1 + \sum_i \frac{1 - 2|j_i|}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^2} \\
&> 1
\end{aligned} \tag{75}$$

with

$$\hat{f}_i := \frac{1}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|}, \quad \hat{g}_i := \frac{j_i(\hat{z} - \hat{z}_i)}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^3}. \tag{76}$$

Furthermore, we can show the positivity of the function $\hat{H}_+ := mH_+$ as follows:

$$\begin{aligned}
2\hat{H}_+ &= \hat{a}_-^2(\hat{f}^2 + 2\hat{g}) + 2\hat{f} \\
&\geq \hat{a}_-^2[(\hat{f}^2 + 2\hat{g}) + 2\hat{f}] \\
&= \hat{a}_-^2[(1 + \hat{f})^2 + 2\hat{g} - 1] \\
&\geq \hat{a}_-^2[(1 + \hat{f})^2 - 2|\hat{g}| - 1] \\
&> 0,
\end{aligned} \tag{77}$$

where we have used the inequality (75).

D. Absence of CTCs

Here, we show the nonexistence of CTCs everywhere on and outside the horizons, provided that the inequality

$|j_i| < 1/2$ holds for each $i = 1, \dots, N$. The condition for the absence of CTCs is equivalent to requiring that the 3D matrix g_{IJ} ($I, J = \psi, x, y$) is positive definite everywhere on and outside the horizons. From Sylvester's criterion, the 3D matrix (g_{IJ}) is positive-definite if and only if all the leading principal minors of the matrix (g_{IJ}) are positive as follows:

$$g_{\psi\psi} = \frac{H_-}{H_+} > 0, \tag{78}$$

$$\det \begin{pmatrix} g_{\psi\psi} & g_{\psi x} \\ g_{\psi x} & g_{xx} \end{pmatrix} = \frac{H_+ H_- - (\omega_x^0)^2}{H_+} > 0, \tag{79}$$

$$\det \begin{pmatrix} g_{\psi\psi} & g_{\psi x} & g_{\psi y} \\ g_{\psi x} & g_{xx} & g_{xy} \\ g_{\psi y} & g_{xy} & g_{yy} \end{pmatrix} = H_+ H_- - (\omega_x^0)^2 - (\omega_y^0)^2 > 0. \tag{80}$$

From the inequalities (74) and (77), the condition (78) is satisfied. It is straightforward to verify that if the condition (80) is satisfied, then the condition (79) is also automatically satisfied. Therefore, it is sufficient to consider only the inequality (80).

Using the inequalities (74) and (77)

$$2\hat{H}_+ \geq \hat{a}_-^2[(1 + \hat{f})^2 + 2\hat{g} - 1] > 0, \tag{81}$$

$$2\hat{H}_- \geq \hat{a}_-^4[(1 + \hat{f})^2 - 2\hat{g} + 1] > 0, \tag{82}$$

we have

$$\begin{aligned}
4[\hat{H}_+ \hat{H}_- - (\hat{\omega}_x^0)^2 - (\hat{\omega}_y^0)^2] &\geq \hat{a}_-^6[\{(1 + \hat{f})^2 + 2\hat{g} - 1\}\{(1 + \hat{f})^2 - 2\hat{g} + 1\} - 4\hat{a}_-^{-6}(\hat{\omega}_x^0)^2 - 4\hat{a}_-^{-6}(\hat{\omega}_y^0)^2] \\
&= \hat{a}_-^6[(1 + \hat{f})^4 - (2\hat{g} - 1)^2 - 4\hat{a}_-^{-6}(\hat{\omega}_x^0)^2 - 4\hat{a}_-^{-6}(\hat{\omega}_y^0)^2] \\
&= \hat{a}_-^6 \left[\left(1 + \sum_i \hat{f}_i\right)^4 - \left(2\sum_i \hat{g}_i - 1\right)^2 - 4\left(\sum_i \check{\omega}_{x,i}^0\right)^2 - 4\left(\sum_i \check{\omega}_{y,i}^0\right)^2 \right] \\
&\geq \hat{a}_-^6 \left[\sum_i (\hat{f}_i^4 - 4\hat{g}_i^2 - 4(\check{\omega}_{x,i}^0)^2 - 4(\check{\omega}_{y,i}^0)^2) + \sum_i (2\hat{f}_i^2 - 4|\hat{g}_i|) + \sum_{i \neq j} (3\hat{f}_i^2 \hat{f}_j^2 - 4\hat{g}_i \hat{g}_j - 4\check{\omega}_{x,i}^0 \check{\omega}_{x,j}^0 - 4\check{\omega}_{y,i}^0 \check{\omega}_{y,j}^0) \right]
\end{aligned} \tag{83}$$

with

$$\check{\omega}_{x,i}^0 := \hat{a}_-^{-3} \hat{\omega}_{x,i}^0 = \frac{j_i(\hat{y} - \hat{y}_i)}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^3}, \quad \check{\omega}_{y,i}^0 := \hat{a}_-^{-3} \hat{\omega}_{y,i}^0 = \frac{j_i(\hat{x} - \hat{x}_i)}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^3}, \tag{84}$$

where the first, second, and third summations can be shown to be positive under the conditions $|j_i| < 1/2$ ($i = 1, \dots, N$). Indeed,

$$\begin{aligned}
\hat{f}_i^4 - 4\hat{g}_i^2 - 4(\check{\omega}_{x,i}^0)^2 - 4(\check{\omega}_{y,i}^0)^2 &= \frac{1}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^4} - \frac{4j_i^2(\hat{z} - \hat{z}_i)^2}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^6} - \frac{4j_i^2(\hat{y} - \hat{y}_i)^2}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^6} - \frac{4j_i^2(\hat{x} - \hat{x}_i)^2}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^6} \\
&= \frac{1 - 4j_i^2}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^4} \\
&> 0,
\end{aligned} \tag{85}$$

$$2\hat{f}_i^2 - 4|\hat{g}_i| = 2\frac{1}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^2} - 4\frac{|j_i||\hat{z} - \hat{z}_i|}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^3} \quad (86)$$

$$\begin{aligned} &\geq 2\frac{1}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^2} - 4\frac{|j_i|}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^2} \\ &= 2\frac{1 - 2|j_i|}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^2} \\ &> 0, \end{aligned} \quad (87)$$

and

$$\begin{aligned} 3\hat{f}_i^2\hat{f}_j^2 - 4\hat{g}_i\hat{g}_j - 4\check{\omega}_{x,i}^0\check{\omega}_{x,j}^0 - 4\check{\omega}_{y,i}^0\check{\omega}_{y,j}^0 &= \frac{3}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^2|\hat{\mathbf{x}} - \hat{\mathbf{x}}_j|^2} - \frac{4j_i j_j (\hat{z} - \hat{z}_i)(\hat{z} - \hat{z}_j)}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^3|\hat{\mathbf{x}} - \hat{\mathbf{x}}_j|^3} - \frac{4j_i j_j [(\hat{\mathbf{x}} - \hat{\mathbf{x}}_i)(\hat{\mathbf{x}} - \hat{\mathbf{x}}_j) + (\hat{\mathbf{y}} - \hat{\mathbf{y}}_i)(\hat{\mathbf{y}} - \hat{\mathbf{y}}_j)]}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^3|\hat{\mathbf{x}} - \hat{\mathbf{x}}_j|^3} \\ &= \frac{3}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^2|\hat{\mathbf{x}} - \hat{\mathbf{x}}_j|^2} - \frac{4j_i j_j (\hat{\mathbf{x}} - \hat{\mathbf{x}}_i) \cdot (\hat{\mathbf{x}} - \hat{\mathbf{x}}_j)}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^3|\hat{\mathbf{x}} - \hat{\mathbf{x}}_j|^3} \\ &\geq \frac{3}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^2|\hat{\mathbf{x}} - \hat{\mathbf{x}}_j|^2} - \frac{12|j_i j_j|}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^2|\hat{\mathbf{x}} - \hat{\mathbf{x}}_j|^2} \\ &= \frac{3(1 - 4|j_i||j_j|)}{|\hat{\mathbf{x}} - \hat{\mathbf{x}}_i|^2|\hat{\mathbf{x}} - \hat{\mathbf{x}}_j|^2} \\ &> 0. \end{aligned} \quad (88)$$

The positivity of these equations implies that $\det g^{(4)}|_{(x,y)} > 0$. Therefore, no CTCs exist on or outside the horizons.

V. MYERS-PERRY BLACK HOLE BINARY

The symmetry of the multiblack hole solution is enhanced when all black holes are placed on the z axis, since the geometry then admits $U(1) \times U(1)$ isometry due to an additional $U(1)$ isometry generated by the rotation about the axis. We now give the explicit form of the solution in the case of two black holes. Without loss of generality, we take their positions to be $\mathbf{x}_1 = (0, 0, -a)$ and $\mathbf{x}_2 = (0, 0, a)$. Then, introducing cylindrical coordinates (ρ, ϕ, z) defined by $(x, y, z) = (\rho \cos \phi, \rho \sin \phi, z)$, we have

$$\begin{aligned} ds^2 &= \frac{H_-}{H_+} \left[d\psi - \frac{a_-^3(f^2 - 2g)}{2H_-} (dt + \omega_\phi^0 d\phi) + \omega_\phi^1 d\phi \right]^2 \\ &\quad - \frac{1}{H_-} (dt + \omega_\phi^0 d\phi)^2 \\ &\quad + H_+ (d\rho^2 + dz^2 + \rho^2 d\phi^2), \end{aligned} \quad (89)$$

with (48) and

$$f = \frac{1}{\sqrt{\rho^2 + (z+a)^2}} + \frac{1}{\sqrt{\rho^2 + (z-a)^2}}, \quad (90)$$

$$g = \frac{j_1(z+a)}{\sqrt{\rho^2 + (z+a)^2}} + \frac{j_2(z-a)}{\sqrt{\rho^2 + (z-a)^2}}, \quad (91)$$

$$\omega_\phi^0 = \frac{a_-^3 j_1 \rho^2}{\sqrt{\rho^2 + (z+a)^2}} + \frac{a_-^3 j_2 \rho^2}{\sqrt{\rho^2 + (z-a)^2}}, \quad (92)$$

$$\omega_\phi^1 = -\frac{z+a}{\sqrt{\rho^2 + (z+a)^2}} - \frac{z-a}{\sqrt{\rho^2 + (z-a)^2}}, \quad (93)$$

where ∂_ϕ is the rotational Killing vector around the z axis, together with the rotational Killing vector ∂_ψ . As shown in Sec. IV B, the asymptotic structure of the spacetime is locally 5D Minkowski spacetime, and the spatial infinity has the topology of the lens space $L(2; 1) = S^3/\mathbb{Z}_2$. This type of asymptotic structure for the time slice $t = \text{constant}$ is referred to as asymptotically locally Euclidean, as in the case of the Eguchi-Hanson space [21,22]. Multiblack hole solutions [23] and black ring solutions [24,25] on the Eguchi-Hanson space have been constructed as BPS solutions in 5D minimal supergravity (Einstein-Maxwell-Chern-Simons theory). In fact, in the limit where the rotation parameters a_- and j_i ($i = 1, 2$) vanish, the functions ω_ϕ^0 and g also vanish, and the metric reduces to

$$\begin{aligned} ds^2 &= -dt^2 + ds_4^2, \\ ds_4^2 &= f^{-1} (d\psi + \tilde{\omega}_\phi^1 d\phi)^2 + f (d\rho^2 + dz^2 + \rho^2 d\phi^2), \end{aligned} \quad (94)$$

where f and $\tilde{\omega}^1_\phi$ are given by Eqs. (90) and (93), respectively. The spatial metric ds^2_4 coincides with the Eguchi-Hanson metric written in the Gibbons-Hawking form [21,26]. Except for the horizons $z = \pm a$, the z axis corresponds to the rotational axis, where the spacelike Killing vectors with closed integral curves vanish.

Below, to show the absence of conical singularities on the z axis, we analyze the rod structure of the spacetime. Let η be an angular coordinate with period $\Delta\eta$, associated with a rotational Killing vector ∂_η . Then regularity on the axis (i.e., the absence of conical singularities) requires the condition [27]

$$\Delta\eta = 2\pi \lim_{\rho \rightarrow 0} \sqrt{\frac{\rho^2 g_{\rho\rho}}{g(\partial_\eta, \partial_\eta)}}. \quad (95)$$

To investigate the rod structure of this solution, it is more convenient to use the angular coordinate $\Psi = \psi/2$. Then, the metric at $\tilde{r} \rightarrow \infty$ behaves as

$$ds^2 \simeq -dt^2 + d\tilde{r}^2 + \frac{\tilde{r}^2}{4} [(d\Psi - \cos\theta d\phi)^2 + d\theta^2 + \sin^2\theta d\phi^2]. \quad (96)$$

This z axis can be divided into three rods:

- (i) the ϕ_- -rotational axis: $\Sigma_- = \{(\rho, z) | \rho = 0, z < -a\}$, with direction $\ell_- = \partial_{\phi_-} := -2\partial_\psi + \partial_\phi = -\partial_\Psi + \partial_\phi$,
- (ii) the ϕ -rotational axis: $\Sigma_\phi = \{(\rho, z) | \rho = 0, -a < z < a\}$, with direction $\ell = \partial_\phi$, and
- (iii) the ϕ_+ -rotational axis: $\Sigma_+ = \{(\rho, z) | \rho = 0, z > a\}$, with direction $\ell_+ = \partial_{\phi_+} := 2\partial_\psi + \partial_\phi = \partial_\Psi + \partial_\phi$.

This rod structure is the same as that of the Eguchi-Hanson space discussed in [28], except that our solution is 5D Lorentzian and contains horizons. In fact, the coordinates $(\phi, \Psi, \phi_-, \phi_+, z)$ used in this paper correspond to $(\psi, \phi, \tilde{\psi}, \tilde{\phi}, -z)$ in [28].

To ensure regularity, we assume that the orbits generated by $\{\ell_+, \ell\}$ are independently identified with period 2π , which implies

$$(\phi_+, \phi) \sim (\phi_+, \phi + 2\pi), \quad (\phi_+, \phi) \sim (\phi_+ + 2\pi, \phi). \quad (97)$$

Since the pair $\{\ell_+ - \ell, \ell\} = \{\partial_\Psi, \partial_\phi\}$ is related to $\{\ell_+, \ell\}$ by a $GL(2, \mathbb{Z})$ transformation, it can equally well be taken as a pair of independent 2π -periodic generators. Therefore,

$$(\Psi, \phi) \sim (\Psi, \phi + 2\pi), \quad (\Psi, \phi) \sim (\Psi + 2\pi, \phi). \quad (98)$$

Furthermore, since the pair $\{\ell_-, \ell\} = \{\partial_{\phi_-}, \partial_\phi\}$ is related to $\{\ell, \ell_+\}$ by a $GL(2, \mathbb{Z})$ transformation, the pair $\{\ell_-, \ell\}$ may also be chosen as a set of independent generators with period 2π . This leads to the identification

$$(\phi_-, \phi) \sim (\phi_-, \phi + 2\pi), \quad (\phi_-, \phi) \sim (\phi_- + 2\pi, \phi). \quad (99)$$

Indeed, we can show that, on the rods Σ_- , Σ_ϕ , and Σ_+ , the metric (89) satisfies the following conditions in accordance with the above identifications:

$$\left(\frac{\Delta\phi_-}{2\pi}\right)^2 = \lim_{\rho \rightarrow 0} \frac{\rho^2 g_{\rho\rho}}{g(\partial_{\phi_-}, \partial_{\phi_-})} = 1, \quad (100)$$

$$\left(\frac{\Delta\phi}{2\pi}\right)^2 = \lim_{\rho \rightarrow 0} \frac{\rho^2 g_{\rho\rho}}{g(\partial_\phi, \partial_\phi)} = 1, \quad (101)$$

$$\left(\frac{\Delta\phi_+}{2\pi}\right)^2 = \lim_{\rho \rightarrow 0} \frac{\rho^2 g_{\rho\rho}}{g(\partial_{\phi_+}, \partial_{\phi_+})} = 1, \quad (102)$$

which implies from Eq. (95) that there are no conical singularities on any of these rods.

In particular, on Σ_ϕ , since one Killing vector ∂_ϕ vanishes but the other Killing vector ∂_ψ does not vanish, Σ_ϕ is topologically cylinder. This structure is referred to as a “bubble.” In the vacuum case, two uncharged black holes do not seem to be in equilibrium, but they can be balanced by the presence of a bubble region between the two horizons.

VI. SUMMARY AND DISCUSSION

In this paper, we have constructed and analyzed a new class of multicentered rotating black hole solutions in 5D vacuum Einstein gravity. In the presence of two commuting Killing vector fields, the 5D Einstein theory can be reduced to an $SL(3, \mathbb{R})$ nonlinear sigma model coupled with 3D gravity, where the scalar sector consists of five fields—three inner products of the Killing vectors and two twist potentials. We have shown that the five scalar fields for the extremal Myers-Perry black hole can be written in terms of two harmonic functions on flat 3D space, and we then generalize these to multicentered harmonic functions to obtain an explicit family of multiple extremal Myers-Perry black holes located at arbitrary points in $\mathbb{E}^3$. We have proved that each center corresponds to a smooth S^3 Killing horizon provided the rotation parameters satisfy $|j_i| < 1/2$, and we have derived the near-horizon geometry and the horizon area. We have further established the regularity of the entire spacetime: all curvature singularities lie inside the horizons, and no singularities are present in the domain of outer communication. We have rigorously demonstrated the absence of closed timelike curves. The solutions are asymptotically flat up to $\mathbb{Z}_N$ quotient, leading to lens-space asymptotics, and we have also analyzed in detail a binary rotating black hole configuration, clarifying its rod structure and the emergence of a bubble region. These results provide the first explicit construction of multiple rotating 5D black holes without supersymmetry, extending classical

multiblack hole configurations to higher dimensions and revealing geometric features unique to 5D gravity.

Finally, we would like to discuss a further generalization of our solution. One may replace the harmonic function f in Eq. (49) with

$$\sum_i \frac{N_i}{|\mathbf{x} - \mathbf{x}_i|},$$

where each N_i is an integer greater than one. In this case, the topology of the horizon at $\mathbf{x} = \mathbf{x}_i$ becomes the lens space $L(N_i; 1)$, whereas the spatial infinity has the topology of $L(\sum_i N_i; 1)$. If, however, we take N_i to be a negative integer ($N_i = -1, -2, \dots$), curvature singularities or CTCs may occur outside the horizon. In our present

solution, each black hole rotates in the (x, y) plane as well as in the ψ direction. Allowing rotations in the (y, z) and (z, x) planes would require replacing the harmonic function g in Eq. (50) with a more general form. We leave these interesting extensions for future work.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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