

Thermodynamical uncertainties for primordial black holes from cosmological phase transitions

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(Received 10 July 2025; revised 19 September 2025; accepted 13 April 2026; published 18 May 2026)

Strongly supercooled first-order phase transitions have been proposed as a primordial black hole (PBH) production mechanism. While previous works rely on simplified models with limited thermodynamic precision, we stress that reliable theoretical PBH predictions require precise nucleation dynamics within realistic extensions of the Standard Model. By employing high-temperature dimensional reduction and computing the one-loop fluctuation determinants, we provide a state-of-the-art thermodynamic analysis and obtain a universal lower bound on the transition timescale, $\beta/H_* \simeq 5$. Then, we estimate the corresponding PBH abundance for classically conformal gauge-Higgs theories. Accounting for constraints from successful percolation and QCD chiral symmetry breaking, the parameter space where PBHs are viable dark matter candidates is severely limited.

DOI: [10.1103/nj27-ltyg](https://doi.org/10.1103/nj27-ltyg)

The detection of astrophysical gravitational waves (GWs) by the LIGO-VIRGO [1] Collaboration and the first evidence of a stochastic gravitational wave background signal from pulsar timing array experiments [2–20] may open a new window into the early universe.

In the hot primordial plasma, symmetries are restored and eventually broken as the universe cools, potentially via first-order phase transitions (FOPTs). In the Standard Model (SM), both the electroweak (EW) and QCD transitions are crossovers [21,22]. Many beyond the SM (BSM) scenarios predict a strong first-order EW phase transition, sourcing GWs in the sensitivity range of LISA [23,24]. A compelling class of BSM theories are classically conformal extensions of the SM, which yield strong GW signals due to significant supercooling [25–54]. In slow, strongly supercooled FOPTs, each Hubble patch hosts a few large true vacuum bubbles, enclosing remnant false vacuum

regions. These late-decaying regions induce large-scale inhomogeneities from statistical fluctuations during nucleation. If density perturbations are large enough at horizon reentry, they can collapse into primordial black holes (PBHs) [55–86]. The resulting PBH abundance critically depends on the transition timescale.

Previous studies of PBHs induced by supercooled FOPTs rely on simplified approaches to compute the PT dynamics. This article presents a state-of-the-art thermodynamic analysis of classically conformal SM extensions. Following [51], we examine strongly supercooled FOPTs in the $U(1)_{\text{CW}}$ and $SU(2)_X$ models by incorporating higher-order thermal corrections through the three-dimensional effective field theory (3D EFT) of high-temperature dimensional reduction [87–89]. In flat spacetime, this allows us to compute the next-to-leading order (NLO) thermal bubble nucleation rate [49,90,91] including fluctuation determinants [51,92,93]. We find the slowest transitions to have a timescale of $\beta/H_p = 5.50$ for $SU(2)_X$ and $\beta/H_p = 5.99$ for $U(1)_{\text{CW}}$, respectively. Employing the EFT framework enables a mapping to a wide class of BSM theories, lending universality to our findings.

Finally, we employ numerical simulations utilizing `deltaPT` [77,94] and semianalytic fitting formulas [76–78] to estimate the PBH abundance in these models. We demonstrate that no region of parameter space allows for a sizable PBH abundance, which is strongly susceptible to

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the theoretical thermodynamic uncertainties of super-cooled FOPTs.

I. MODEL REVIEW

Classically conformal models [25–42,44–54] do not exhibit a mass term in the tree-level potential. Therefore, the SM Higgs mass squared μ_{SM}^2 is replaced by a portal coupling to a new scalar field, such that the potential at tree level becomes

$$V_{\text{tree}} = \lambda_h H^4 + \lambda_\phi \Phi^4 - \lambda_p H^2 \Phi^2, \quad (1)$$

where $H = (G_+, (h + iG_0)/\sqrt{2})$ is the SM Higgs doublet. The BSM scalar Φ is charged under a new gauge symmetry, which is spontaneously broken by radiative effects. As Φ acquires a vacuum expectation value (VEV), the EW vacuum is generated via the negative portal coupling in Eq. (1), $\mu_{\text{SM}}^2 = \lambda_p \langle \Phi \rangle^2$. To keep our findings generic for gauge-Higgs classically conformal theories, in this article, we consider two such model implementations.

II. CONFORMAL $U(1)_{\text{CW}}$ MODEL

This model extends the SM with the $U(1)_{\text{CW}}$ Coleman-Weinberg (CW) [95] dark sector.¹ Besides the SM particle content, this model [34] includes a scalar $\Phi = (\bar{\varphi} + iG)/\sqrt{2}$ charged under $U(1)_{\text{CW}}$, and an Abelian Z' gauge boson with gauge coupling g_{CW} . We decompose the radial mode as $\bar{\varphi} = \phi + s$, so that the scalar VEV is given by $v_\phi = m_{Z'}/(Q_{\text{CW}} g_{\text{CW}})$ with $Q_{\text{CW}} = 1$ the $U(1)_{\text{CW}}$ charge. In the $U(1)_{\text{B-L}}$ model, e.g., $Q_{\text{B-L}} = 2$ [34]. Given input parameters $\{g_{\text{CW}}, m_{Z'}\}$, the quartic λ_ϕ and portal coupling λ_p are fixed by requiring the correct EW scale; see Appendix A and [47,49]. We further set the kinetic mixing parameter to zero at the EW scale, $g_{\text{mix}}(\bar{\mu} = m_{Z'}) = 0$. Experimental Z' bounds [97,98] imply $m_{Z'} \gtrsim \mathcal{O}(\text{TeV})$, i.e., $v_\phi \gg v_h$ for moderate g_{CW} .

III. CONFORMAL $SU(2)_X$ MODEL

This model features a local $SU(2)_X$ symmetry in addition to the SM gauge groups [34,47,49,51,99,100]. The scalar $\Phi = (G_X^+, (\bar{\varphi} + iG_X^0)/\sqrt{2})$, with $\bar{\varphi} = \phi + s$, is a doublet under $SU(2)_X$ and a singlet under the SM. The X boson mass relates to the zero-temperature VEV as $v_\phi = 2m_X/g_X$. For given $\{g_X, m_X\}$, the remaining parameters are fixed as in $U(1)_{\text{CW}}$. Reproducing the EW scale requires $m_X \gtrsim \mathcal{O}(\text{TeV})$, so $v_\phi \gg v_h$ in both models. Hence, we restrict to tunneling in the new-scalar direction [47,100].

¹Conformal $U(1)$ models generically exhibit an ultraviolet Landau pole for the coupling constant. For certain values of parameters, they can also be stable up to the Planck scale; cf., e.g., [34,96].

To scrutinize the robustness of predictions of PBH abundance, in this article, we consider the following *theoretical uncertainties*:

A. Effective action at NLO

By utilizing the derivative expansion, we show the form of the effective action of an arbitrary background field ϕ . This is a starting point for calculating the nucleation rate, and it illustrates the general setup for soft, static, infrared modes described by the dimensionally reduced 3D EFT at high temperatures [89].

In the soft 3D leading-order (LO) effective potential,

$$V_{\text{eff}}^{\text{LO}}(\phi) = \frac{\mu_3^2}{2} \phi^2 + \frac{\lambda_{\phi,3}}{4} \phi^4 - \eta_1 \phi^3 - (y_D + \eta_2^{\frac{2}{3}} \phi^2)^{\frac{3}{2}}, \quad (2)$$

a barrier is induced by vector and temporal-scalar ultra-violet (UV) contributions; see, e.g., [51,101]. The barrier is parametrized by $\eta_1 \sim g^3$, the Debye screening mass $y_D \sim m_D^2$, and the interaction between temporal and Lorentz scalars $\eta_2 \sim h_3^{3/2} \sim \eta_1$ where in three dimensions $[\phi] = \frac{1}{2}$, $[\eta_1] = \frac{3}{2}$, $[m_D] = [\lambda_{\phi,3}] = 1$. The effective parameters are listed in Appendix A.

To compute the LO action,

$$S_3^{\text{LO}} = \int_{\mathbf{x}} \left[\frac{1}{2} (\partial_i \phi)^2 + V_{\text{eff}}^{\text{LO}}(\phi) \right], \quad (3)$$

where $\int_{\mathbf{x}} = \int d^d \mathbf{x}$, we first focus on a simpler setup and set $m_D \rightarrow 0$, yielding

$$V_{\text{eff}}^{\text{LO}}(\phi) = \frac{\mu_3^2}{2} \phi^2 + \frac{\lambda_{\phi,3}}{4} \phi^4 - \eta \phi^3, \quad (4)$$

with $\eta = \eta_1 + \eta_2$, which isolates the soft-enhanced contribution to the barrier [101]. At LO, the barriers take the following form:

$$\eta = \frac{g_{\text{CW},3}^3}{4\pi}, \quad \text{for } U(1)_{\text{CW}}, \quad (5)$$

$$\eta = \frac{3g_{X,3}^3}{32\pi}, \quad \text{for } SU(2)_X, \quad (6)$$

revealing that, for fixed model parameters, the transition weakens with increasing the gauge group rank.

This form of $V_{\text{eff}}^{\text{LO}}$ illustrates the universality of the EFT approach, as different dark sectors yield the same expression with appropriately modified 3D couplings. By utilizing the dimensionless parameters [92,102–106] $x = \lambda_{\phi,3}/\eta^{2/3}$, $y = \mu_{\phi,3}^2/\eta^{4/3}$, and $\phi \rightarrow \eta^{\frac{1}{3}} \varphi$, we can recast the potential in the generic form

$$\tilde{V}_{\text{eff}}^{\text{LO}}(\varphi) = \frac{V_{\text{eff}}^{\text{LO}}(\phi)}{\eta^2} = \frac{y}{2}\varphi^2 + \frac{x}{4}\varphi^4 - \varphi^3. \quad (7)$$

After redefining $\mathbf{x} \rightarrow \tilde{\mathbf{x}}y^{-\frac{1}{2}}\eta^{-\frac{2}{3}}$ and $\varphi \rightarrow y\varphi$, the LO action (3) takes a dimensionless two-parameter form

$$S_3^{\text{LO}} = \kappa \int_{\tilde{\mathbf{x}}} \left[\frac{1}{2}(\partial_i \varphi)^2 + \frac{1}{2}\varphi^2 + \frac{\gamma}{4}\varphi^4 - \varphi^3 \right] \equiv \kappa \tilde{S}_3(\gamma), \quad (8)$$

where $\kappa = y^{\frac{3}{2}}$ and $\gamma = xy$. Such a form allows for fitting the LO 3D action directly [92,107–109].

The above procedure extends directly to computing the action for the full LO soft potential (2). There, we restore a nonvanishing m_D to include the nonanalytic temporal-scalar contribution to the barrier. At LO, one can either fit the action (3) with multiple parameters [101], similar to Eq. (8), or compute it numerically [49], as employed in this article.

At NLO, the action receives a two-loop correction from the effective potential, *viz.* $S_3^{\text{NLO}} \supset \int_{\mathbf{x}} V_3^{\text{NLO}}(\phi)$. The explicit forms of V_3^{NLO} for the considered models are found in [49,90] or Appendix A.

B. Bubble nucleation

The full thermal bubble nucleation rate factorizes into statistical and dynamical parts [92,110]:

$$\Gamma(T) = A_{\text{dyn}} \times A_{\text{stat}}. \quad (9)$$

The statistical part incorporates equilibrium effects and can be computed using the imaginary-time formalism and 3D EFT. In the saddle-point approximation, A_{stat} is given by the Euclidean $O(3)$ effective action evaluated on the bounce solution $\varphi_b(r)$, where r is the radius of the critical bubble. We compute A_{stat} up to NLO in the soft expansion following [51]. The LO background φ_b solves $\nabla_r^2 \varphi_b = \frac{\partial V_3^{\text{LO}}}{\partial \varphi_b}$ with boundary conditions $\varphi_b(\infty) = 0$ and $\partial_r \varphi_b(0) = 0$.

To extend the nucleation rate to full one-loop order, corrections from fluctuations around the critical bubble φ_b must be included. These are encoded in the fluctuation determinants [92]. For two-loop effects, we employ the zeroth order of the derivative expansion, *i.e.*, the correction to the effective potential V_3^{NLO} . Evaluating the effective action on φ_b , the statistical part becomes

$$A_{\text{stat}} = \det_S \det_V e^{-S_3^{\text{LO}}[\varphi_b] + C[\varphi_b]} e^{-\int_{\mathbf{r}} V_3^{\text{NLO}}[\varphi_b]}, \quad (10)$$

where $\det_S$ contains the one-loop scalar fluctuation determinant and $\det_V$ accounts for vector fluctuations. While nonzero Matsubara modes are integrated out during dimensional reduction and affect the effective parameters entering into the potential, only the zero modes remain dynamical and contribute to the fluctuation determinants. Relevant contributions are discussed in Appendix A. To avoid

double counting, the zero-momentum vector mode must be removed from the LO action [111]. This is achieved by adding the term

$$C[\varphi_b] = -\frac{1}{12\pi} \int_{\mathbf{x}} [2n_V m_V^3 + n_V m_{V,0}^3 - n_V m_D^3]. \quad (11)$$

Here, n_V is the number of gauge modes, m_V is the mass of spatial gauge modes, $m_{V,0}$ is the mass of temporal gauge modes, and m_D denotes the Debye mass. For $SU(2)_X$, $n_V = 3$ and for $U(1)_{\text{CW}}$, $n_V = 1$.

The dynamical part captures nonequilibrium effects of bubble nucleation [110,112–114], which we approximate without damping as $A_{\text{dyn}} \simeq \sqrt{\lambda_-/(2\pi)}$, where λ_- is the negative eigenvalue of the scalar fluctuation operator [93,115–117]. This approximation often yields a lower bound on β/H in comparison to the full out-of-equilibrium contributions as seen in, *e.g.*, [118]. As a consequence, the resulting PBH abundance is likely overestimated, strengthening our conclusion of a negligible PBH abundance.

C. Percolation criterion

Strongly supercooled FOPTs require particular attention during the computation of the relevant temperature scales. The PT parameters are computed at the *percolation temperature* T_p , defined via the probability, $P = \exp(-I(T))$, for a point in spacetime to remain in the false vacuum [119–122]. Utilizing the quantity

$$I(T) = \frac{4\pi}{3} \int_T^{T_c} \frac{dT'}{T'^4} \frac{\Gamma(T')}{H(T')} \left(\int_T^{T'} d\tilde{T} \frac{v_w}{H(\tilde{T})} \right)^3, \quad (12)$$

the moment of percolation, when the PT completes, is determined by imposing that a certain fraction of the Universe has been converted to the true vacuum.² The typically employed values

$$I(T_{p,1}) = 1 \quad \text{and} \quad I(T_{p,2}) = 0.34 \quad (13)$$

correspond to $P_1 \approx 0.34$ and $P_2 \approx 0.71$. To additionally avoid eternal inflation [124–126], we verify that the false vacuum fraction of the entire spacetime volume decreases around T_p [127],

$$\frac{1}{V_{\text{false}}} \frac{dV_{\text{false}}}{dt} = H(T) \left(3 + T \frac{dI(T)}{dT} \right) \Big|_{T=T_p} < 0. \quad (14)$$

If this is not the case, the transition cannot finish successfully at the estimated T_p . This is particularly relevant for slow, or conversely strong PTs. In parts of the parameter

²In Eq. (12), we fix the bubble wall velocity to $v_w = 1$, as we investigate very strong transitions that often correspond to detonations. For a general perturbative determination of v_w , see [123].

space, the false vacuum volume starts decreasing at a temperature $T_{\text{shrink}} < T_p$, which still allows for a successful transition below T_{shrink} . In such cases, it remains unclear at which temperature the transition parameters should be evaluated [43,128,129].

We investigate the regime of extreme supercooling, where the Universe enters a period of thermal inflation at the temperature T_V , defined by

$$\frac{\pi^2}{30} g_e T_V^4 = \Delta V_{\text{eff}}(T_V), \quad (15)$$

where $\Delta V_{\text{eff}}(T_V)$ is the false vacuum energy and g_e denotes the relativistic degrees of freedom related to the energy density. Since $T_p \ll T_V$ in the entire parameter space we consider, the Hubble parameter in Eqs. (12) and (14) is well approximated by

$$H(T) = H_V(T) = \left(\frac{\Delta V_{\text{eff}}(T_V)}{3M_{\text{Pl}}^2} \right)^{\frac{1}{2}}, \quad (16)$$

with M_{Pl} being the reduced Planck mass.

This approximation neglects the contribution to radiation energy density from the propagating bubble walls³ [76,78]. Accounting for it requires iteratively solving Eq. (12) coupled to the Friedmann equation, typically lowering the Hubble rate at T_p and increasing β/H_p , thereby suppressing PBH formation. However, since the no-percolation region from Eq. (14) also shrinks, this mainly induces an overall shift in parameter space. We thus adopt the simplified approach of Eq. (16).

D. QCD-sourced transitions

In a large part of the parameter spaces of both models, the conformal PT is sufficiently supercooled to delay EW symmetry breaking to temperatures below the QCD scale. Then, the cosmic QCD transition occurs first with six massless flavors around $T_{\text{QCD}} \approx 100$ MeV [130]. The breaking of the chiral symmetry through the formation of quark condensates induces a QCD-scale VEV for the SM Higgs [131,132], $v_{h,\text{QCD}} \approx 100$ MeV. This breaks classical scale invariance (CSI) and induces an additional contribution to the effective potential via the portal term in Eq. (1),

$$V_{\text{tree}} \supset V_{\text{QCD}} = -\frac{\lambda_p}{4} v_{h,\text{QCD}}^2 \phi^2. \quad (17)$$

Since this negative mass term counteracts the thermal barrier, it accelerates the conformal PT [48,50]. As a consequence, the inverse timescale β/H_p is enhanced.

³Despite having its source from the potential energy, the energy stored in ultrarelativistic walls redshifts as radiation.

IV. PRIMORDIAL BLACK HOLE ABUNDANCE

Black holes form when an energy overdensity is confined within a spacetime region smaller than its Schwarzschild radius. We examine the impact of thermodynamical uncertainties (A)–(D) on PBH formation during strongly supercooled FOPTs [55–86].

PBHs form when the density contrast in a Hubble patch exceeds a critical threshold,

$$\delta_c = \frac{\rho(t_k) - \bar{\rho}(t)}{\bar{\rho}(t)}, \quad (18)$$

where $\rho(t_k) = \rho_r(t_k) + \rho_v(t_k)$, $\bar{\rho}(t) = \rho_r(t) + \rho_v(t)$, and t_k denotes the time at which we evaluate the density contrast when the mode k reenters the horizon. The threshold δ_c is determined from numerical relativity using the compaction function at Hubble crossing [133]. In a radiation-dominated universe, $\delta_c \approx 0.55$ [134].

PBH formation requires sufficiently large late-blooming regions of false vacuum, screened by surrounding regions of true vacuum, to collapse [62]. As pointed out in [70], an overdensity within a false vacuum region is not enough, and one should consider the curvature perturbation that renders the universe locally nonflat. To estimate the PBH abundance, one considers the mass fraction at formation using the density contrast distribution $P(\delta_c)$ evaluated at $t = t_k \geq t_p$ where t_p is the time of percolation:

$$\begin{aligned} \beta_k(M) &= \int_{\delta_c}^{\infty} d\delta \frac{M}{M_k} P_k(\delta) \delta_D \left(\ln \frac{M}{M(\delta)} \right) \\ &= \frac{\kappa}{\gamma} \left(\frac{M}{\kappa M_k} \right)^{\frac{\gamma+1}{\gamma}} P_k(\delta(M)). \end{aligned} \quad (19)$$

Here, M_k denotes the mass within a Hubble horizon, δ_D is the Dirac δ -function, M is the PBH mass, and $\delta(M) = \delta_c + (M/(\kappa M_k))^{1/\gamma}$. For a radiation fluid, $\gamma = 0.38$ and $\kappa = 4.2$ [134], provided one considers a scale-invariant curvature power spectrum where γ depends on the equation of state, and κ depends on the initial profile of the curvature perturbation. The present-day PBH mass function is

$$\psi(M) = \int d \ln k \beta_k(M) \frac{\rho_r(T_k) s(T_0)}{\rho_c s(T_k)}, \quad (20)$$

where the integration is over all k -modes close to the inverse Hubble horizon. Here, ρ_c is the critical energy density of the Universe, T_k denotes the temperature when the mode k reenters the horizon, and $s(T)$ is the entropy density with T_0 being the present cosmic microwave background temperature. The total PBH abundance normalized to the dark matter (DM) energy density then reads

$$f_{\text{PBH}} = \frac{\Omega_{\text{PBH}}}{\Omega_{\text{DM}}}, \quad \text{with} \quad \Omega_{\text{PBH}} = \int d \ln M \psi(M), \quad (21)$$

where the integration range is determined by the initial mass within the horizon (16) and its subsequent evolution given the transition timescale. The PBH fraction of DM is well described by the fit [76–78]

$$f_{\text{PBH}} = b_1 \exp(-b_2 e^{b_3(\beta/H_p)}) \frac{T_{\text{rh}}}{\text{GeV}}, \quad (22)$$

where T_{rh} is defined in Eq. (23).

For $\beta/H_p \sim 7.5$, the correct DM abundance is obtained when $b_1 \approx 5.5 \times 10^6$, $b_2 \approx 0.064$, and $b_3 \approx 0.806$ [77]. However, it was recently shown that these values severely overestimate the resulting PBH abundance [79]. Since b_3 impacts the abundance double-exponentially (cf. [135] for a similar argument), one can rescale it and instead explore a different target β/H_p ; see the results below.

The PBH abundance depends on the post-transition reheating temperature T_{rh} , computed assuming instantaneous reheating by equating

$$\Delta V_{\text{eff}}(T_p) = \frac{\pi^2}{30} g_{\text{e,th}} T_{\text{rh}}^4, \quad (23)$$

where $g_{\text{e,th}}$ denotes the relativistic degrees of freedom in the broken phase and $\Delta V_{\text{eff}}(T_p)$ is the false vacuum energy released during the transition.

V. RESULTS

Figure 1 shows scans in $m_{Z'} - g_{\text{CW}}$ space for $U(1)_{\text{CW}}$ and $m_X - g_X$ for $SU(2)_X$. Rows correspond to models, columns to the chosen percolation criterion (C). Color indicates β/H_p , with gray regions showing QCD-sourced transitions at small gauge couplings. Light gray areas mark unsuccessful percolation at T_p , where the false vacuum fraction fails to decrease; see Eqs. (12) and (14). In $SU(2)_X$, we exclude regions with nonperturbative g_X near T_p [47].

We find minimum inverse timescales $\beta/H_p = 5.99$ [$U(1)_{\text{CW}}$] and $\beta/H_p = 5.50$ [$SU(2)_X$], respectively, which increases with decreasing gauge boson mass or increasing gauge coupling. Achieving a given β/H_p requires larger g_X in $SU(2)_X$ than g_{CW} in $U(1)_{\text{CW}}$, as seen from Eqs. (5) and

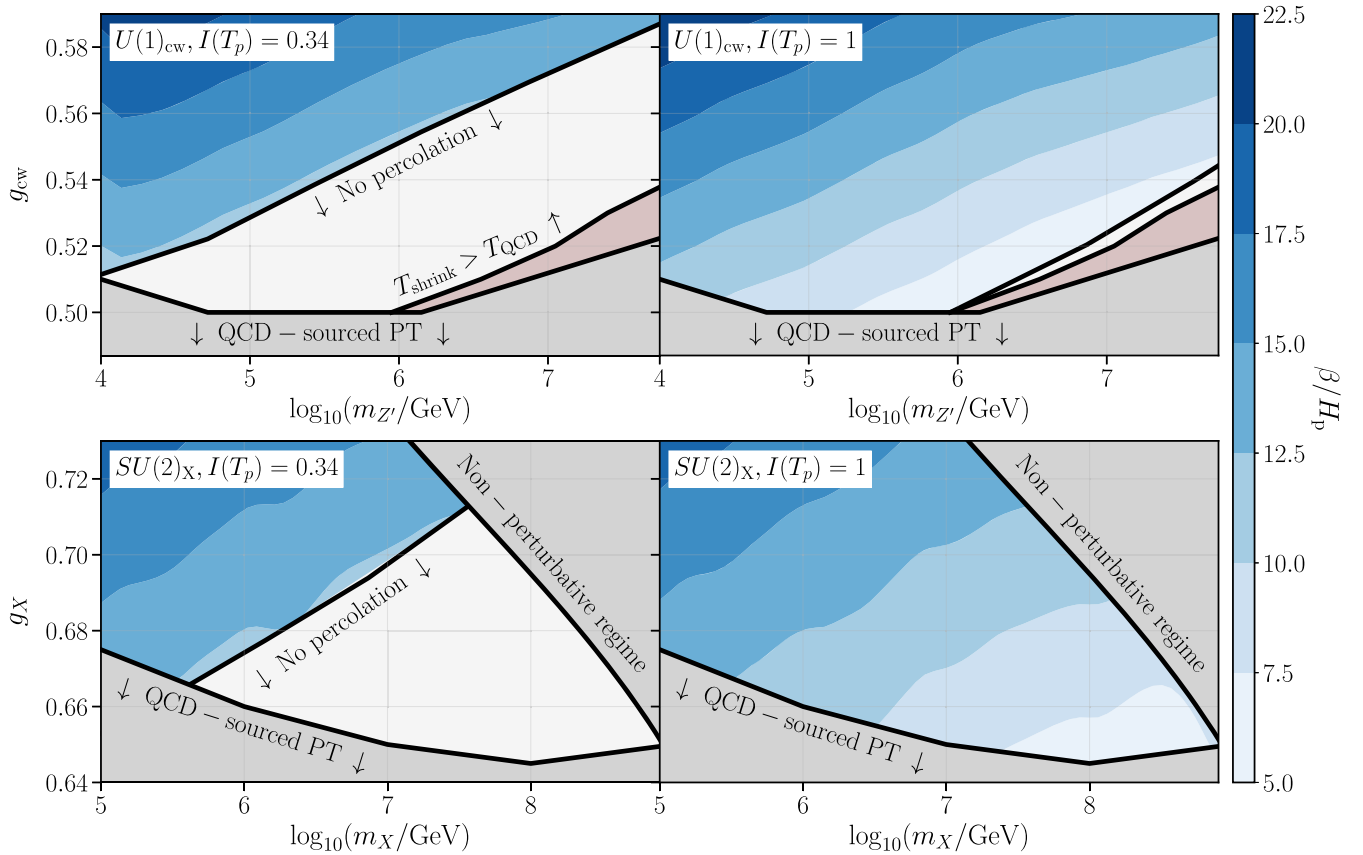


FIG. 1. Contour plots of the inverse transition timescale β/H_p in the $U(1)_{\text{CW}}$ and $SU(2)_X$ models, shown as a function of the gauge boson masses $m_X, m_{Z'}$ and couplings g_X, g_{CW} at the input scale defined by the gauge boson masses. The values $I(T_p)$ for the percolation criteria (13) are indicated, and the no-percolation region is shown in light gray. Gray regions mark where the PT completes only via QCD catalysis or enters a nonperturbative regime for $SU(2)_X$. The gray-red area corresponds to $T_{\text{shrink}} < T_{\text{QCD}}$. The oscillations in the contours are an artifact of finite grid size in the parameter space scans.

(6), which relate the barrier height to gauge group rank at fixed temperature.

For $I(T_{p,2}) = 0.34$ (left panels), the minimum inverse timescale we find is $\beta/H_p \simeq \mathcal{O}(10)$, as benchmark points predicting slower transitions lie deep in the no-percolation region where the standard formalism for estimating T_p [cf. uncertainty (C)] breaks down. Instead, the transition is expected to finish below $T_{\text{shrink}} < T_p$, where the false vacuum fraction starts to decrease, as dictated by Eq. (14). If $T_{\text{shrink}} < T_{\text{QCD}}$, QCD effects accelerate the conformal PT. Here, chiral symmetry breaking induces CSI breaking in the ϕ potential via the QCD-scale Higgs VEV (17). The negative portal term counteracts the thermal barrier, facilitating tunneling. This accelerates the PT, reflected in a steep drop of the bounce action S_3 or, equivalently, an enhanced inverse timescale β/H_p [45,50]. Thus, the minimum $\beta/H_p \simeq 5$ obtained using $I(T_{p,1}) = 1$ represents a universal lower bound.

To our knowledge, all previous PBH-related studies relied on simplified approaches to compute the thermal nucleation rate. The most common is “Daisy resummation” (cf., e.g., [63,66,82]), which typically underestimates β/H_p by $\mathcal{O}(50\%)$ compared to the methods used in this article [49,51,136]. To reduce large uncertainties from missing higher-order corrections and large logarithms in the effective potential, employing the 3D EFT is essential. It is also crucial to minimize uncertainties in the nucleation rate computation, such as the breakdown of the derivative expansion [51,111].

Employing the fit (22), we identify a narrow region for $I(T_{p,1}) = 1$ where PBHs can constitute all of DM. However, a recent gauge-covariant study [79] found that this approach significantly overestimates the PBH production rate. A sizable abundance would thus require even smaller values $\beta/H_p \ll 4$. Such a scenario can be estimated by rescaling $b_3 \rightarrow \chi b_3$ in Eq. (22). Adjusting χ such that, e.g., $f_{\text{PBH}}(\beta/H_p = 3) = 1$, we find $f_{\text{PBH}} = 0$ for our most optimistic scenario $\beta/H_p \simeq 5$. We have explicitly verified this by using the updated version of `deltaPT` [77].

Having investigated two concrete models by employing state-of-the-art thermodynamic methods and estimating the PBH abundance using the updated overdensity criterion [79], we find no parameter space where PBHs are viable DM candidates. Note that [79] neglects, e.g., the energy flux carried by the bubble walls. Therefore, the required β/H_p to achieve a sizable PBH population may change in upcoming studies. We stress that any new threshold value, however, should be above $\beta/H_p \simeq 5$, which is a universal lower limit allowed by thermodynamics.

VI. SUMMARY

Theoretical uncertainties in computations of strongly supercooled FOPTs impact PBH formation as follows:

- (i) Using state-of-the-art thermodynamics, we find the slowest transitions in the $U(1)_{\text{CW}}$ and $SU(2)_X$ models yield $\beta/H_p = 5.99$ and $\beta/H_p = 5.50$, respectively. Successful percolation depends sensitively on the criterion used; in particular, criterion $I(T_{p,2})$ predicts a large no-percolation region.
- (ii) For benchmark points with incomplete percolation at T_p , the transition terminates at $T_{\text{shrink}} < T_p$. In $U(1)_{\text{CW}}$, this may precede the cosmic QCD transition, while in $SU(2)_X$, $T_{\text{shrink}} < T_{\text{QCD}}$ always. QCD-sourced transitions always enhance β/H_p due to CSI breaking by chiral condensates.
- (iii) A sizable PBH abundance requires $\beta/H_p \lesssim 7.5$ using the criterion of [77] and is possible only if the percolation criterion $I(T_{p,1})$ is satisfied. With the updated criterion [79], we do not find the required $\beta/H_p \ll 4$ anywhere in the viable parameter space.

Since small values of β/H_p imply bubble sizes comparable to the Hubble radius, future work will include effects of spacetime curvature on bubble nucleation and growth, and also incorporation of the energy transfer of bubbles modifying the transition timescale [137,138]. Classically conformal models may also feature a prolonged matter-dominated era [45], which impacts PBH formation.

ACKNOWLEDGMENTS

The authors thank Bogumiła Świeżewska, Marek Lewicki, Piotr Toczek, Tuomas V. I. Tenkanen, and Jorinde van de Vis for fruitful discussions and comments on the manuscript. The authors also thank George Parker for illuminating discussions regarding the notion of fine failing. D. S. acknowledges support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through the CRC-TR 211 “Strong-interaction matter under extreme conditions”—Project No. 315477589–TRR 211. N. R. acknowledges support in part by INFN TASP. P. S. was supported by the Swiss National Science Foundation (SNSF) under Grant No. PZ00P2-215997. M. K. and this work were supported by Polish National Science Centre under Grants No. 2023/49/B/ST2/02782 and No. 2023/50/E/ST2/00177. N. R., P. S., and D. S. acknowledge the hospitality of the University of Warsaw during the final stages of this work. M. K. acknowledges the hospitality of SISSA during the finalization of this work.

DATA AVAILABILITY

The data that support the findings of this article are openly available [139].

APPENDIX

1. Matching relations for 3D EFTs

This section reports the corresponding matching relations for the $U(1)_{\text{CW}}$ and $SU(2)_X$ conformal theories with a

portal coupling to the SM. For the $U(1)_{\text{CW}}$ model, this effectively extends the relations of [90]. The corresponding relations were both computed using in-house FORM [140] diagrammatic software, as well as DRalgo [141].

a. Conformal $U(1)$ model

The renormalization group equations listed below are associated with the parameters of the $U(1)_{\text{CW}}$ model and encode their running with respect to the four-dimensional $\overline{\text{MS}}$ renormalization scale $\bar{\mu}$ via the β -functions. To this end, we use

$$t \equiv \ln \bar{\mu}^2 \quad (\text{A1})$$

and find at one-loop level

$$\partial_t g_{\text{CW}}^2 = \frac{1}{(4\pi)^2} \frac{g_{\text{CW}}^4}{3}, \quad (\text{A2})$$

$$\partial_t \lambda_\phi = \frac{1}{(4\pi)^2} (10\lambda_\phi^2 - 6g_{\text{CW}}^2 \lambda_\phi + 3g_{\text{CW}}^4). \quad (\text{A3})$$

The matching relations in the nonconformal limit are given in [90] and are computed here for the conformal $U(1)_{\text{CW}}$ model

$$g_{\text{CW},3}^2 = g_{\text{CW}}^2 T \left[1 - \frac{g_{\text{CW}}^2}{(4\pi)^2} \frac{L_b}{3} \right], \quad (\text{A4})$$

$$m_{\text{D}}^2 = \frac{g_{\text{CW}}^2 T^2}{3} \left[1 - \frac{1}{(4\pi)^2} \left(\frac{L_b - 7}{3} g_{\text{CW}}^2 - 4\lambda_\phi \right) \right], \quad (\text{A5})$$

where we define a shorthand notation

$$L_b \equiv 2 \ln \frac{\bar{\mu}}{T} - 2(\ln(4\pi) - \gamma_E), \quad L_f \equiv L_b + 4 \ln 2, \quad (\text{A6})$$

$$c = \frac{1}{2} \left(\ln \frac{8\pi}{9} + (\ln \zeta_2)' - 2\gamma_E \right), \quad (\text{A7})$$

with γ_E being the Euler-Mascheroni constant, $\zeta_s = \zeta(s)$ for $\text{Re}(s) > 1$ the Riemann zeta function, and $(\ln \zeta_s)' = \zeta'(s)/\zeta(s)$. The thermal mass of the complex $U(1)$ singlet and its quartic couplings take the form

$$\begin{aligned} \mu_3^2 = & \frac{T^2}{12} (4\lambda_\phi + 3g_{\text{CW}}^2) - \frac{T^2}{(4\pi)^2} \frac{g_{\text{CW}}^2}{9} (2g_{\text{CW}}^2 - 6\lambda_\phi) \\ & - \frac{T^2}{(4\pi)^2} L_b \left(\frac{13}{12} g_{\text{CW}}^4 - 2g_{\text{CW}}^2 \lambda_\phi + \frac{10}{3} \lambda_\phi^2 \right) \\ & - \left(c + \ln \frac{3T}{\bar{\mu}_{3d}} \right) \frac{4g_{\text{CW},3}^4 - 8g_{\text{CW},3}^2 \lambda_{\phi,3} + 8\lambda_{\phi,3}^2 + h_3^2}{(4\pi)^2}, \end{aligned} \quad (\text{A8})$$

$$\lambda_{\phi,3} = T \left[\lambda_\phi + \frac{(2 - 3L_b)g_{\text{CW}}^4 + L_b(6g_{\text{CW}}^2 \lambda_\phi - 10\lambda_\phi^2)}{(4\pi)^2} \right], \quad (\text{A9})$$

$$h_3 = 2g_{\text{CW}}^2 T \left[1 - \frac{1}{(4\pi)^2} \left(\frac{L_b - 4}{3} g_{\text{CW}}^2 - 8\lambda_\phi \right) \right], \quad (\text{A10})$$

$$\kappa_3 = 16 \frac{g_{\text{CW}}^4 T}{(4\pi)^2}. \quad (\text{A11})$$

The corresponding effective potential at the nucleation scale after integrating out simultaneously $m_{Z'} \sim g_{\text{CW},3} \phi_3$ and $m_{Z'_0} \sim g_{\text{CW},3} T$ is given by

$$V_{\text{eff}}^{\text{LO}} = \frac{1}{2} \mu_3^2 \phi_3^2 + \frac{1}{4} \lambda_{\phi,3} \phi_3^4 - \frac{1}{12\pi} (2m_{Z'}^3 + m_{Z'_0}^3), \quad (\text{A12})$$

where the corresponding mass eigenvalues in the 3D EFT are given by

$$\begin{aligned} m_H^2 &= \mu_3^2 + 3\lambda_{\phi,3} \phi_3^2, & m_G^2 &= \mu_3^2 + \lambda_{\phi,3} \phi_3^2, \\ m_{Z'}^2 &= g_{\text{CW},3}^2 \phi_3^2, & m_{Z'_0}^2 &= m_{\text{D}}^2 + \frac{1}{2} h_3 \phi_3^2. \end{aligned} \quad (\text{A13})$$

The corresponding NLO effective potential consists of temporal and vector contributions and is given by

$$\begin{aligned} V_{\text{eff}}^{\text{NLO}} = & -\frac{1}{(4\pi)^2} \left[h_3 \frac{m_{Z'_0}^2 - m_{\text{D}}^2}{4} \left(1 + \ln \frac{\bar{\mu}_{3d}^2}{4m_{Z'_0}^2} \right) \right. \\ & \left. + g_{\text{CW},3}^2 m_{Z'}^2 \left(1 + \ln \frac{\bar{\mu}_{3d}^2}{4m_{Z'}^2} \right) - \frac{\kappa_3}{8} m_{Z'_0}^2 \right], \end{aligned} \quad (\text{A14})$$

where $\bar{\mu}_{3d}$ is the 3D EFT renormalization scale. In practice, we then subtract the field-independent contributions as in [49]. Finally, we report the field normalization factor, or kinetic term, for the nucleating scalar

$$Z_{\phi_3} = \frac{1}{48\pi} \left[-22 \frac{g_{\text{CW},3}}{\phi_3} + \frac{1}{4} \frac{h_3^2 \phi_3^2}{m_{Z'_0}^3} \right]. \quad (\text{A15})$$

b. Conformal $SU(2)$ model

The matching relations for constructing both the action and the effective potential of the conformal $SU(2)_X$ can be found in [49,51]. Here, we recollect them for compactness.

The one-loop β -functions are

$$\partial_t g_X^2 = -\frac{g_X^4}{(4\pi)^2} \frac{43}{6}, \quad (\text{A16})$$

$$\partial_t \lambda_\phi = \frac{1}{(4\pi)^2} \left(12\lambda_\phi^2 - \frac{9}{2} g_X^2 \lambda_\phi + \frac{9}{16} g_X^4 \right). \quad (\text{A17})$$

The corresponding matching relations are

$$g_{X,3}^2 = g_X^2 T \left[1 + \frac{g_X^2}{(4\pi)^2} \frac{4 + 43L_b}{6} \right], \quad (\text{A18})$$

$$m_D^2 = g_X^2 T^2 \left[\frac{5}{6} + \frac{1}{(4\pi)^2} \left(\frac{430L_b + 207}{72} g_X^2 + \lambda_\phi \right) \right]. \quad (\text{A19})$$

The thermal mass of the dark $SU(2)_X$ doublet and its quartic couplings take the form⁴

$$\begin{aligned} \mu_3^2 = & \frac{T^2}{16} (8\lambda_\phi + 3g_X^2) + \frac{T^2}{(4\pi)^2} \frac{g_X^2}{4} \left(\frac{167}{24} g_X^2 + 3\lambda_\phi \right) \\ & + \frac{T^2}{(4\pi)^2} L_b \left(\frac{17}{16} g_X^4 + \frac{9}{4} g_X^2 \lambda_\phi - 6\lambda_\phi^2 \right) \\ & - \left(c + \ln \frac{3T}{\bar{\mu}_{3d}} \right) \frac{1}{(4\pi)^2} \left[\frac{39}{16} g_{X,3}^4 + 9g_{X,3}^2 \lambda_{\phi,3} - 12\lambda_{\phi,3}^2 \right. \\ & \left. + 6g_{X,3}^2 h_3 - \frac{3}{2} h_3^2 \right], \end{aligned} \quad (\text{A20})$$

$$\lambda_{\phi,3} = T \left[\lambda_\phi + \frac{3}{(4\pi)^2} \left(\frac{2 - 3L_b}{16} g_X^4 + L_b \frac{3g_X^2 \lambda_\phi - 8\lambda_\phi^2}{2} \right) \right], \quad (\text{A21})$$

$$h_3 = \frac{g_X^2 T}{2} \left[1 + \frac{1}{(4\pi)^2} \left(\frac{43L_b + 51}{12} g_X^2 + 6\lambda_\phi \right) \right], \quad (\text{A22})$$

$$\kappa_3 = \frac{34}{3} \frac{g_X^4 T}{(4\pi)^2}. \quad (\text{A23})$$

The corresponding effective potential at the nucleation scale after integrating out simultaneously $m_X \sim g_{X,3} \phi_3$ and $m_{X_0} \sim g_{X,3} T$ is given by

$$V_{\text{eff}}^{\text{LO}} = \frac{1}{2} \mu_3^2 \phi_3^2 + \frac{1}{4} \lambda_{\phi,3} \phi_3^4 - \frac{3}{12\pi} (2m_X^3 + m_{X_0}^3), \quad (\text{A24})$$

where the corresponding mass eigenvalues in the 3D EFT are given by

$$\begin{aligned} m_H^2 &= \mu_3^2 + 3\lambda_{\phi,3} \phi_3^2, & m_G^2 &= \mu_3^2 + \lambda_{\phi,3} \phi_3^2, \\ m_X^2 &= \frac{g_{X,3}^2 \phi_3^2}{4}, & m_{X_0}^2 &= m_D^2 + \frac{1}{2} h_3 \phi_3^2. \end{aligned} \quad (\text{A25})$$

The corresponding NLO effective potential consists of temporal and vector contributions and is given by

$$\begin{aligned} V_{\text{eff}}^{\text{NLO}} = & \frac{3}{(4\pi)^2} \left[-h_3 \frac{m_{X_0}^2 - m_D^2}{4} \left(1 + \ln \frac{\bar{\mu}_{3d}^2}{4m_{X_0}^2} \right) + \frac{5\kappa_3}{8} m_{X_0}^2 \right. \\ & + g_{X,3}^2 \left(m_X m_{X_0} + \frac{3}{2} m_{X_0}^2 \right) \\ & + \frac{4m_{X_0}^2 - m_X^2}{2} \ln \frac{\bar{\mu}_{3d}}{m_X^2 + 2m_{X_0}} \\ & \left. + \frac{m_X^2}{16} \left(7 + 8 \ln 2 + 21 \ln \frac{\bar{\mu}_{3d}^2}{9m_X^2} \right) \right]. \end{aligned} \quad (\text{A26})$$

In practice, we then subtract the field-independent contributions as in [49]. Finally, we report the field normalization factor, or kinetic term, for the nucleating scalar

$$Z_{\phi_3} = \frac{1}{16\pi} \left[-11 \frac{g_{X,3}}{\phi_3} + \frac{1}{4} \frac{h_3^2 \phi_3^2}{m_{X_0}^3} \right]. \quad (\text{A27})$$

2. Fixing input parameters

In the 3D EFT, we use tree-level vacuum values for the above couplings. This procedure can be extended to one-loop corrected relations, as described in Appendix A of [142] and [89,143,144].

The parameters of both classically conformal models are fixed by the following scheme [for details in the $SU(2)_X$ model, see Appendix C of [47]]:

In: Gauge coupling g_{CW} for $U(1)_{\text{CW}}$ and g_X for $SU(2)_X$ as well as the corresponding pole masses of $m_{Z'}$ or m_X . Here, we associate pole masses with $\overline{\text{MS}}$ masses. The parameters are defined at the $\overline{\text{MS}}$ -scale $\bar{\mu} = m_X$.

- (i) Minimize the scalar potential at the loop level to determine the new scalar VEV; this fixes scalar quartic λ_ϕ .
- (ii) Run the parameters to and compute the scalar VEV at the $\overline{\text{MS}}$ -scale $\bar{\mu} = m_Z$; match the portal coupling λ_p to generate the EW vacuum.
- (iii) Run the parameters to the matching scale $\bar{\mu} = X\bar{T}$ with $\bar{T} = \pi T$ using one-loop β -functions (A2) and (A3) as well as (A16) and (A17). Here, X is a constant typically varied to quantify the importance of higher-order corrections. We set $X = 1$ throughout the analysis.

Out: $\overline{\text{MS}}$ -parameters as functions of physical parameters and $\bar{T}$; 3D EFT parameters.

Note the minimization of the loop-corrected potential to fix the input parameters can be improved by using the full one-loop vacuum renormalization. By relating pole masses to physical two-point functions, the improved scheme ensures that higher-order corrections in the renormalization conditions can be included and the momentum dependence of the pole masses is respected. See, e.g., [89,142,143].

⁴In comparison to [49], we display the corrected μ_3^2 .

3. Fluctuation determinants

In our computation of the thermal bubble nucleation rate, we evaluated the fluctuation determinants for one-loop corrections from the scalar and gauge modes. For the $SU(2)_X$ conformal model, we reuse the results from [51]. To compute the fluctuation determinant in the conformal $U(1)_{CW}$ model, we adapt results obtained in [51,92]. Here, we briefly summarize the relevant operators.

The scalar-mode determinant containing one-loop scalar mode corrections is given by

$$\det_S = \mathcal{I}_\phi \sqrt{\left| \frac{\det \mathcal{O}_\phi(\varphi_F)}{\det' \mathcal{O}_\phi(\varphi_b)} \right|}, \quad \mathcal{I}_\phi = \left(\frac{S_3^{LO}[\varphi_b]}{2\pi} \right)^{3/2},$$

$$\mathcal{O}_\phi = -\partial^2 + (V_{\text{eff}}^{LO})'', \quad (\text{A28})$$

where $\mathcal{I}_\phi$ is the Jacobian resulting from going to collective coordinates (see, e.g., [92,93]), φ_b denotes the LO bounce solution, while φ_F is a false vacuum solution, which in our case is given by $\varphi_F = 0$. The primed $\det'$ denotes a determinant without negative eigenvalues. This contribution can be evaluated numerically using BubbleDet [93]. The explicit form of the operator $\mathcal{O}_\phi$ is model-dependent, as it includes the second derivative of the LO effective potential.

For the $SU(2)_X$ model, the vector determinant contains contributions from mixed spatial gauge modes and

Goldstone $\det_{X_G}$, transverse gauge modes $\det_{X_T}$, temporal gauge modes $\det_{X_0}$, and ghosts $\det_g$:

$$\det_V = \det_{X_0} \det_g \det_{X_T} \det_{X_G}. \quad (\text{A29})$$

For the detailed form of these determinants and the illustration of their numerical evaluation, we refer to [51]. For the $U(1)_{CW}$ model, the overall form of the vector determinant is identical, and differs only in the details of the gauge fluctuation operators that contain different expressions for the gauge modes masses. The gauge determinants can also be evaluated by using a derivative expansion, *viz.*

$$\det_V \simeq C[\varphi_b] + \int_{\mathbf{r}} \frac{Z_{\phi_3}[\varphi_b]}{2} (\partial_i \varphi_b)^2 + \mathcal{O}(\partial^4), \quad (\text{A30})$$

where C is the zero-momentum part of the one-loop gauge fluctuations, which is a correction to the effective potential and was introduced in Eq. (11). The explicit form of Z_{ϕ_3} can be found in Eqs. (A27) and (A15). This expansion behaves well for the temporal gauge modes, as they are massive even at φ_F . On the other hand, the expansion always breaks down for the gauge-Goldstone determinant as the corresponding modes become inevitably massless at the tail of the bounce solution [51,111].

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