

Spectroscopy and decay properties of excited charmonium states

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In this work, we investigate the mass spectra of excited charmonia employing a nonrelativistic potential model with screening effect and analyze their two-body strong decays, two-photon decays, leptonic decays, and hadronic transitions. Especially, we find that the newly observed resonances $X(4160)$, $Y(4500)$, and $Y(4710)$ can be identified as 3^3P_0 , 3^3D_1 , and 4^3D_1 states, respectively. We also predict the masses and the widths of higher excited states in the charmonia family.

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I. INTRODUCTION

Since the discovery of the charmonium state J/ψ in 1974 [1,2], states composed of a charm quark and its antiquark have become a cornerstone in the field of particle physics. This energy region offers an excellent platform for studying the properties of perturbative and nonperturbative quantum chromodynamics (QCD) [3]. A thorough investigation of the charmonium system contributes significantly to a better understanding of the properties of QCD. Following the discovery of the J/ψ particle, a series of charmonium states was subsequently observed in experiments within just a few years [4]. The experimentally clear spectra of relatively narrow states below the open-charm $D\bar{D}$ threshold of approximately 3.73 GeV could be well described by the predicted $1S$, $1P$, and $2S$ $c\bar{c}$ levels. Based on such experimental findings, Eichten *et al.* proposed the well-known Cornell potential model to describe the mass spectrum of charmonium [5,6], which incorporates a color Coulomb term at short distances and a linear scalar

confining term at large distances. They found that an interaction potential consisting of a Coulomb-like term and a linear term could well describe the charmonium states discovered in experiments at that time, indicating that the potential model is very successful in describing hadron spectroscopy. Over the past few years, several charmonium-like states, known colloquially as “X, Y, Z” mesons, have been identified in B factories and various experimental setups [7–9]. In 2003, Belle observed a very narrow resonance state during the decay process of J/ψ and named it $X(3872)$ [10]. When studying these higher states of charmonium, like $X(3872)$, from the perspective of potential models [11–13], it should be assigned to the $\chi_{c1}(2P)$ state ($J^{PC} = 1^{++}$). The low mass puzzle can be well understood when the coupled-channel effect is introduced, indicating that this effect must not be neglected when studying higher excited $c\bar{c}$ states. Screened potential models were introduced and applied for the investigation of heavy quarkonium, heavy-flavor mesons, and light hadrons many years ago [14–17]. Recently, these models have been revisited to investigate the mass spectra of heavy quarkonia and the widths of their leptonic decays [18,19].

Recently, the LHCb Collaboration has investigated the production of J/ψ in diffractive processes during proton-proton collisions [20]. The observation of the $\chi_{c0}(4500)$ state has achieved a significance level exceeding 5σ , which implies that its discovery is highly statistically significant. The $\chi_{c1}(4274)$ state has also been confirmed with a significance surpassing 4σ . Additionally, there have been

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other experimental discoveries. One example is the $e^+e^- \rightarrow K^+K^-J/\psi$ process carried out by the BESIII experiment. The new decay mode $Y(4230) \rightarrow K^+K^-J/\psi$ was identified for the first time by BESIII. Moreover, two vector charmoniumlike states, $Y(4500)$ and $Y(4710)$, were observed in the energy-dependent line shape of the $e^+e^- \rightarrow K^+K^-J/\psi$ cross section [21]. BESIII also measured the Born cross sections for the process $e^+e^- \rightarrow \omega\chi_{c1}$ at center-of-mass energies $\sqrt{s}$ ranging from 4.308 to 4.951 GeV. The line shape of $e^+e^- \rightarrow \omega\chi_{c1}$ was described using a single resonance, from which the mass and width were determined to be $M = 4544.2 \pm 18.7 \pm 1.7$ and $\Gamma = 116.1 \pm 33.5 \pm 1.7$ MeV [22], respectively. The quantum numbers of this state are $J^{PC} = 1^{--}$, and it is identified as $\psi(4544)$.

To generally account for the coupled-channel effect, we introduce the screening effect [23]. The color screening effect takes into account the contributions from virtual fermion loops. As the distance between two quarks increases, a virtual quark-antiquark pair is pulled out, which screens part of the interaction between them, thus suppressing the masses of highly excited states [24]. Including the color screening effect improves the theoretical description of excited states, matching experimental data more accurately. Using the nonrelativistic potential model with the screening effect, we calculate the mass spectra of $c\bar{c}$ mesons. The details of the potential model we used can be found in the next section. The introduction of an additional parameter, denoted by μ , leads to a prediction that the masses of excited charmonium states are reduced in comparison with those derived from a simplified linear potential without screening effects. Moreover, this effect of mass reduction is expected to become more pronounced as charmonium states transition from lower- to higher-energy levels. We find that the calculated mass of $\chi_{c0}(3P)$ is 4186 MeV, which is in good agreement with the experimental value of $X(4160)$, and the mass of $\psi(4S)$ is close to the observed state $\psi(4415)$. These possible assignments will be put forward and discussed in detail in this work. When we obtain the mass values of the above $c\bar{c}$ mesons, we also numerically obtain the corresponding spatial wave functions for the higher states of $c\bar{c}$ mesons, which can be applied to the calculation of two-body Okubo-Zweig-Iizuka (OZI) allowed strong decays and annihilation decays, among others. When a meson undergoes a strong decay into two other mesons, the selection of decay channels follows the OZI rule. Strong decay processes are considered to be the dominant decay mechanism for mesons.

In this work, we adopt the nonrelativistic potential model that includes the screening effect to calculate the mass spectra of $c\bar{c}$ mesons. We will study the decay behavior of $c\bar{c}$ mesons, including two-body strong decays, hadronic transitions, and annihilation decays. The mass spectra and a range of decay widths that we obtained could establish a holistic framework for the spectroscopic study of $c\bar{c}$ mesons.

This paper is organized as follows: In Sec. II, we introduce the nonrelativistic potential model including the screening effect and analyze the resulting mass spectra of $c\bar{c}$ mesons. In Sec. III, the obtained results for the OZI-allowed two-body strong decays, annihilation decays, and hadronic transitions of the discussed $c\bar{c}$ mesons are presented and compared with available experimental data and other results from different methods. Finally, a summary is given in Sec. IV. In addition, all the theoretical tools of various decay processes and physical quantities are presented in the Appendix, such as two-body OZI-allowed strong decay, annihilation decay, and hadronic transition.

II. MASS SPECTRUM OF CHARMONIA

A. Nonrelativistic potential model with screening effect

We adopt the nonrelativistic potential model with the screening effect to calculate charmonium masses and wave functions [19,25,26], which will be employed in the calculation of various decay and hadronic transition processes, among others.

The Hamiltonian corresponding to the potential model consists of a nonrelativistic kinetic term H_0 and a potential term V , which reads

$$H = H_0 + V, \quad (1)$$

where

$$H_0 = \sum_{i=1}^2 \left(m_i + \frac{p_i^2}{2m_i} \right), \quad (2)$$

$$V = H^{\text{conf}} + H^{\text{cont}} + H^{\text{so}} + H^{\text{ten}}, \quad (3)$$

where m_i ($i = 1, 2$) is the mass of the two quarks.

The linear combination of screening effect and Coulomb potential constitutes the spin-independent term

$$H^{\text{conf}} = S(r) + G(r), \quad (4)$$

where $S(r)$ represents the linear potential [$s(r) = br + c$] with considering the screening effect, which reads

$$S(r) = \frac{b(1 - e^{-\mu r})}{\mu} + c, \quad (5)$$

and the Coulomb potential is given by

$$G(r) = -\frac{4\alpha_s}{3r}. \quad (6)$$

The parameter c denotes the scaling factor, μ is the screening parameter, and $\alpha_s(r)$ is the running coupling constant.

The spin-dependent term follows the formulation given in the Godfrey-Isgur (GI) quark model [27–31]. The second term in Eq. (3) represents the color contact interaction, which is expressed as

$$H^{\text{cont}} = \frac{32\pi\alpha_s}{9m_1m_2} \left(\frac{\sigma}{\pi^2}\right)^3 e^{-\sigma^2 r^2} \mathbf{S}_1 \cdot \mathbf{S}_2. \quad (7)$$

The spin-orbit interaction term is

$$H^{\text{so}} = H^{\text{so(cm)}} + H^{\text{so(tp)}}, \quad (8)$$

where

$$H^{\text{so(cm)}} = \frac{4\alpha_s}{3} \frac{1}{r^3} \left(\frac{1}{m_1} + \frac{1}{m_2}\right)^2 \mathbf{L} \cdot \mathbf{S}_{1(2)} \quad (9)$$

is the chromomagnetic term and

$$\begin{aligned} H^{\text{so(tp)}} &= -\frac{1}{2r} \frac{\partial H^{\text{conf}}}{\partial r} \left(\frac{\mathbf{S}_1}{m_1^2} + \frac{\mathbf{S}_2}{m_2^2}\right) \cdot \mathbf{L} \\ &= -\frac{1}{2r} \left(\frac{4\alpha_s}{3} \frac{1}{r^2} + b e^{-\mu r}\right) \left(\frac{1}{m_1^2} + \frac{1}{m_2^2}\right) \mathbf{L} \cdot \mathbf{S}_{1(2)}, \end{aligned} \quad (10)$$

which is the Thomas precession term with the screening effect. In addition, the color tensor interaction term reads

$$H^{\text{ten}} = \frac{4}{3} \frac{\alpha_s}{m_1 m_2} \frac{1}{r^3} \left(\frac{3\mathbf{S}_1 \cdot \mathbf{r} \mathbf{S}_2 \cdot \mathbf{r}}{r^2} - \mathbf{S}_1 \cdot \mathbf{S}_2\right). \quad (11)$$

In the above three terms, $\mathbf{S}_1$ and $\mathbf{S}_2$ are the spins of the quarks in the meson, and $\mathbf{L}$ is the orbital angular momentum [32].

In coordinate and momentum space, simple harmonic oscillator (SHO) wave functions are expressed as

$$\Psi_{nLM_L}(\mathbf{r}) = R_{nL}(r, \beta) Y_{LM_L}(\Omega_r), \quad (12)$$

$$\Psi_{nLM_L}(\mathbf{p}) = R_{nL}(p, \beta) Y_{LM_L}(\Omega_p), \quad (13)$$

with

$$R_{nL}(r, \beta) = \beta^{\frac{3}{2}} (\beta r)^L N_{nL} e^{-\frac{r^2 \beta^2}{2}} L_{n-1}^{L+\frac{1}{2}}(\beta^2 r^2), \quad (14)$$

$$R_{nL}(p, \beta) = \frac{(-1)^{n-1} (-i)^L}{\beta^{\frac{3}{2}}} e^{-\frac{p^2}{2\beta^2}} N_{nL} \left(\frac{p}{\beta}\right)^L L_{n-1}^{L+\frac{1}{2}}\left(\frac{p^2}{\beta^2}\right), \quad (15)$$

$$N_{nL} = \sqrt{\frac{2(n-1)!}{\Gamma(n+L+1/2)}}, \quad (16)$$

where R_{nL} denotes the radial wave function, $Y_{LM_L}(\Omega_r)$ is a spherical harmonic function, and $L_n^{L+\frac{1}{2}}(x)$ is a related Laguerre polynomial, and the meson spatial wave function can be represented as follows:

$$R_{nL}(p) = \sum_{n=1}^{n_{\text{max}}} C_n R_{nL}^{\text{SHO}}(p, \beta). \quad (17)$$

The SHO wave function involves a single parameter β . In principle, when the wave function is expanded using a complete set of SHO basis functions, the solution of the potential model should be independent of the specific value of β . In numerical computations, only a finite number of basis functions is employed in the expansion, requiring an appropriate choice of the parameter β [33,34]. The value of β is selected such that the computed ground state mass E_{nL} attains its minimum, i.e.,

$$\frac{\partial E_{nL}}{\partial \beta_i} = 0, \quad (18)$$

$$\frac{\partial^2 E_{nL}}{\partial \beta_i^2} > 0, \quad (19)$$

with i representing various light meson species. In practice, β for mesons generally falls within the interval 0.3–1.5 GeV. We choose $n_{\text{max}} = 21$ as the upper limit for the number of basis expansion in this work.

B. Numerical result

In our previous work [25], the parameters in Table I¹ were extracted by fitting the experimentally measured masses of charmonia, bottomonia, and B_c mesons, as well as charmed, charmed-strange, bottom, and bottom-strange mesons, within the potential introduced in Sec. II A. Using the parameters in Table I, we obtain the mass spectra of $c\bar{c}$ mesons as shown in Table II, the β values of the mesons are also listed in Table II and the Appendix.

¹One can notice that the quark masses $m_c = 1.984$, $m_s = 0.780$, $m_{u,d} = 0.606$ GeV are larger than those in Refs. [27,35–39]. This result may be caused by the following factors. First, the light mesons were not included in the fitting process; the obtained masses of the light quarks tend to be larger. In the light mesons, the mass of the pion (π) is only 140 MeV, which would reduce the light quark masses if included in the fitting. Second, the fitted value of the vacuum constant c is relatively small, which contributes to the overall increase in quark masses. Moreover, since only the nonrelativistic scenario was considered, the fitted quark masses are expected to include some contributions from relativistic corrections. Of course, the cutoff parameter r_c also influences the fitted values of the quark masses. These factors together lead to fitted quark masses that are larger than those reported in some references (such as $m_c = 1.65$, $m_s = 0.419$, and $m_{u,d} = 0.22$ GeV in Ref. [27]), when the quark masses, vacuum constant, and cutoff parameter are all treated as fitting variables.

TABLE I. The parameters adopted in this work were obtained from a fit in Ref. [25].

Parameter	Value	Parameter	Value (GeV)
m_c	1.984 GeV	m_u, m_d	0.606
α_s	0.3930	m_s	0.780
b	0.2312 GeV ²	σ	1.842
μ	0.0690 GeV	c	-1.1711
r_c	0.3599 GeV ⁻¹		

In Table II, we present the mass spectra of particles in various states, including the S -, P -, and D -wave states. A comparison is made between our results and the results of other theoretical models [19,35,36,38–40]. We also list the experimental values [most of which are taken from the

Particle Data Group (PDG) [4], and the rest are taken from the recently measured data [20,21]].

The $\eta_c(1^1S_0)$ and $J/\psi(1^3S_1)$ states exhibit excellent agreement with experiments. The predicted mass of $J/\psi(1S)$ at 3096.5 MeV aligns precisely with the experimental value 3096.900 ± 0.006 MeV in PDG, validating the model's strong capability in describing the spin-triplet states. However, the $\eta_c(1S)$ state shows a slight overestimation (3006.2 MeV) compared with the experimental value of 2984.1 ± 0.4 MeV, which may be attributed to the short-range Coulomb potential parameter α_s .

For the $\psi(2S)$ state, the predicted mass is 3685.6 MeV, which is consistent with the PDG value of 3686.097 ± 0.011 MeV. We predict the mass of $\eta_c(2S)$ to be 3644 MeV, slightly higher than the experimental value of $3637.7 \pm$

TABLE II. Predicted masses of $c\bar{c}$ states compared with other model predictions and experimental data. The β values adopted for different states of charmonium ($c\bar{c}$) mesons are listed in the third column. The mass and β are in units of MeV.

States	$n^{2S+1}L_J$	β	Expt. ^a	This work	MGI ^b	GI	NR	UQ ^b	NR	GI	RQ ^b	CQ ^b
$\eta_c(1S)$	1^1S_0	1420	2984.1 ± 0.4	3006	2981	2996	2979	2984	2982	2975	2981	
$\eta_c(2S)$	2^1S_0	975	3637.7 ± 0.9	3644	3642	3634	3623	3634	3630	3623	3635	
$\eta_c(3S)$	3^1S_0	791		4044	4013		3991	4022	4043	4064	3989	
$\eta_c(4S)$	4^1S_0	663		4353	4260		4250	4350	4384	4425	4401	
$\eta_c(5S)$	5^1S_0	580		4606	4433		4446	4662			4811	
$\eta_c(6S)$	6^1S_0	507		4822			4595	4902			5155	
$\eta_c(7S)$	7^1S_0	442		5008								
$\eta_c(8S)$	8^1S_0	406		5171								
$J/\psi(1S)$	1^3S_1	1320	3096.900 ± 0.006	3097	3096	3098	3097	3097	3090	3098	3096	3096
$\psi(2S)$	2^3S_1	883	3686.097 ± 0.011	3686	3683	3676	3673	3676	3672	3676	3685	3703
$\psi(3S)$	3^3S_1	736	4040 ± 4 ($\psi(4040)$)	4073	4035	4090	4022	4044	4072	4100	4039	4097
$\psi(4S)$	4^3S_1	589	4415 ± 5 ($\psi(4415)$)	4375	4274		4273	4381	4406	4450	4427	4389
$\psi(5S)$	5^3S_1	552	4641 ± 10 ($\psi(4660)$)	4624	4443		4463	4682			4837	4614
$\psi(6S)$	6^3S_1	479		4836			4608	4941			5167	
$\psi(7S)$	7^3S_1	442		5020								
$\psi(8S)$	8^3S_1	396		5181								
$\chi_{c0}(1P)$	1^3P_0	1260	3414.71 ± 0.3	3405	3464	3417	3433	3415	3424	3445	3413	
$\chi_{c0}(2P)$	2^3P_0	1020	$3922.1 \pm 1.8[\chi_{c0}(3915)]$	3846	3896	3885	3842	3905	3852	3916	3870	
$\chi_{c0}(3P)$	3^3P_0	846	$4153^{+23}_{-21}[\chi(4160)]$	4186	4177	4256	4131	4234	4202	4292	4301	
$\chi_{c0}(4P)$	4^3P_0	699	$4512.5^{+6.0}_{-6.2} \pm 3.0[\chi_{c0}(4500)]$ [20]	4467	4374	4574		4489			4698	
$\chi_{c0}(5P)$	5^3P_0	589	$4694^{+16}_{-5}[\chi_{c0}(4700)]$	4702		4849						
$\chi_{c0}(6P)$	6^3P_0	516		4907								
$\chi_{c1}(1P)$	1^3P_1	1000	3510.67 ± 0.05	3515	3530	3500	3510	3539	3505	3510	3511	
$\chi_{c1}(2P)$	2^3P_1	800	$3871.64 \pm 0.06[\chi_{c1}(3872)]$	3935	3929	3936	3901	3871/3990	3925	3953	3906	
$\chi_{c1}(3P)$	3^3P_1	690	$4298 \pm 6 \pm 9[\chi_{c1}(4274)]$ [20]	4258	4197	4294	4178	4287	4271	4317	4319	
$\chi_{c1}(4P)$	4^3P_1	580	$4684^{+15}_{-17}[\chi_{c1}(4685)]$	4523	4387	4606		4562			4728	
$\chi_{c1}(5P)$	5^3P_1	516		4748	4887	4887						
$\chi_{c1}(6P)$	6^3P_1	442		4942								
$\chi_{c2}(1P)$	1^3P_2	883	3556.17 ± 0.07	3539	3571	3549	3554	3575	3556	3550	3555	
$\chi_{c2}(2P)$	2^3P_2	809	$3922.5 \pm 1.0[\chi_{c2}(3930)]$	3957	3952	3974	3937	3947	3972	3979	3949	
$\chi_{c2}(3P)$	3^3P_2	589		4278	4213	4327	4208	4337	4317	4337	4354	
$\chi_{c2}(4P)$	4^3P_2	580		4541	4398	4635		4571			4763	

(Table continued)

TABLE II. (Continued)

States	$n^{2S+1}L_J$	β	Expt. ^a	This work	MGI ^b	GI	NR	UQ ^b	NR	GI	RQ ^b	CQ ^b
$\chi_{c2}(5P)$	5^3P_2	497		4765		4914		4882				
$\chi_{c2}(6P)$	6^3P_2	442		4957								
$h_c(1P)$	1^1P_1	883	3525.37 ± 0.14	3521	3538	3513	3519	3544	3516	3517	3525	
$h_c(2P)$	2^1P_1	736		3940	3933		3908	3961	3934	3956	3926	
$h_c(3P)$	3^1P_1	653		4262	4200		4184	4307	4279	4318	4337	
$h_c(4P)$	4^1P_1	562		4527	4389			4573			4744	
$h_c(5P)$	5^1P_1	507		4751				4875				
$h_c(6P)$	6^1P_1	442		4945								
$\psi(1D)$	1^3D_1	883	$3773.7 \pm 0.7 [\psi(3770)]$	3800	3830	3805	3787	3788	3785	3819	3783	3796
$\psi(2D)$	2^3D_1	727	$4191 \pm 5 [\psi(4160)]$	4143	4125	4172	4089	4156	4142	4194	4150	4153
$\psi(3D)$	3^3D_1	589	$4484.7 \pm 13.3 \pm 24.1 [Y(4500)] [21]$	4423	4334		4317	4463			4507	4426
$\psi(4D)$	4^3D_1	552	$4708^{+17}_{-15} \pm 21 [Y(4710)] [21]$	4660	4484			4737			4857	4641
$\psi(5D)$	5^3D_1	497		4863				4988				
$\psi(6D)$	6^3D_1	442		5042								
$\psi(7D)$	7^3D_1	406		5198								
$\psi(8D)$	8^3D_1	369		5336								
$\psi_2(3823)$	1^3D_2	773	3823.51 ± 0.34	3808		3828	3798	3809	3800	3838	3795	
$\psi_2(2D)$	2^3D_2	589		4152	4137							
$\psi_2(3D)$	3^3D_2	580		4432	4343							
$\psi_3(3842)$	1^3D_3	782	3842.71 ± 0.20	3807		3841	3799	3843	3806	3849	3813	
$\psi_3(2D)$	2^3D_3	589		4153	4144							
$\psi_3(3D)$	3^3D_3	571		4434	4348							

^aMost of the experimental value are taken from the PDG [4], and the rest are taken from the recently measured data [20,21].

^bAbbreviations of different models used in other theoretical works. Here we give detailed annotations for abbreviations not used in the text: modified Godfrey-Isgur quark (MGI) model [36,37], unquenched quark (UQ) model [38], relativistic quark (RQ) model [39], constituent quark model (CQ) [40].

0.9 MeV, but outperforming the nonrelativistic potential (NR) model prediction (3623 MeV). The hyperfine mass splitting of singlet-triplet states $\Delta m(nS) = m(\psi(nS)) - m(\eta_c(nS))$ is calculated as 90.3 MeV for $n = 1$ and 41.6 MeV for $n = 2$, closely matching the experimental trends 112.8 ± 0.4 and 48.4 ± 0.9 MeV, respectively. These differences arise from spin-dependent interactions and serve as a benchmark for assessing the accuracy of diverse theoretical models. Higher S -wave states, such as $\psi(3S)$ and $\psi(4S)$, show deviations of 32 and 40 MeV from the experimental values, respectively. These discrepancies are attributed to the excessive suppression of long-range linear confinement at high excitations due to the screening parameter ($\mu = 0.069$ GeV). For the $\eta_c(3S)$ and $\eta_c(4S)$ states, the predicted masses are 4043.9 and 4352.5 MeV, respectively. Our model predicts a slightly higher mass compared to other models, but lower than that of the GI model [35].

Our model predicts the masses of 6^3S_1 , 7^3S_1 , and 8^3S_1 to be 4836.3, 5019.9, and 5180.7 MeV, respectively. Compared to the known state $5^3S_1[\psi(4660)]$, these masses are approximately 195, 379, and 540 MeV higher, respectively. For the 6^3S_1 state, we find that our result is larger

than that in Ref. [19], but smaller than those in Refs. [38,39]. We predict masses of 5^1S_0 , 6^1S_0 , 7^1S_0 , and 9^1S_0 to be 4606.3, 4822.1, 5008.3, and 5171.3 MeV, respectively. Our results are lower than those reported in Refs. [38,39].

The multiplet $\chi_{cJ}(1P)$ is well reproduced in our calculation, with masses being different from experimental values by less than 10 MeV. For example, $\chi_{c0}(1P)$ is predicted to be 3405.15 MeV, which is in good agreement with the experimental value of 3414.71 ± 0.3 MeV. The predicted masses of $\chi_{c1}(1P)$, $\chi_{c2}(1P)$, and $h_c(1P)$ are 3514.85, 3539.24, and 3521.12 MeV, respectively, all of which show good agreement with the experimental values as well.

The mean mass of the $2P$ multiplet is predicted to be near 3.95 GeV [35]. For $\chi_{c0}(3915)$, the predicted mass is 3846.14 MeV, which is lower than the PDG value of 3922.1 ± 1.8 MeV [4] by 76 MeV, due to the excessive suppression of long-range linear confinement by the screening parameter. For $\chi_{c1}(2P)$, we predict the mass to be 3934.79 MeV and identify it as the $\chi_{c1}(3872)$ state. The mass of the $\chi_{c2}(2P)$ state is obtained as 3956.78 MeV,

which is in good agreement with the experimental value of 3922.5 ± 1.0 MeV.

The $3P$ states have an expected mean multiplet mass of about 4.2 GeV. We identify $X(4160)$ as 3^3P_0 , whose theoretical mass and width are consistent with experimental observations and with the results reported in Refs. [41–43]. $Y(4500)$ is identified as the 4^3P_0 state, supporting the assumption of its being the $c\bar{c}$ state. For the 5^3P_0 state (4702.22 MeV), the result we obtained can be regarded as $\chi_{c0}(4700)$ (4694^{+16}_{-5} MeV). The mass of the 6^3P_0 state is predicted to be 4906.59 MeV.

The $\chi_{c1}(3P)$ with a predicted mass of 4257.86 MeV is identified as the $\chi_{c1}(4274)$ state ($4298 \pm 6 \pm 9$ MeV) [20]. The predicted mass of $\chi_{c1}(4P)$ is 4523.12 MeV, which is identified as the $\chi_{c1}(4685)$ state. We predict the masses of $\chi_{c1}(5P)$ and $\chi_{c1}(6P)$ to be 4747.88 and 4942.04 MeV, respectively. The predicted masses of $\chi_{c2}(3P)$, $\chi_{c2}(4P)$, $\chi_{c2}(5P)$, $\chi_{c2}(6P)$ are 4277.97, 4541.38, 4764.56, and 4957.01 MeV, respectively.

For the spin-singlet h_c states, the mass of $h_c(1P)$ is 3521.12 MeV, which is in good agreement with the experimental value, differing by only 4 MeV. The model we used shows a high accuracy in the description of the ground states. The $h_c(2P)$ state has not yet been explicitly observed experimentally. Our model predicts its mass to be approximately 3.94 GeV, which may be confirmed in future experiments.

In our calculation, the mass of $\psi(1D)$ is overestimated by 26 MeV, with a predicted value of 3799.8 MeV compared to the experimental value of 3773.7 ± 0.7 MeV. The predicted value of $\psi(2D)$ is 4142.7 MeV, which is 48 MeV lower than the experimental value of 4191 ± 5 MeV. Nevertheless, our result still offers better agreement with the experimental data than those obtained from the MGI and NR models [19,36].

We predict the mass of 3^3D_1 to be 4422.8 MeV, and consider it as $Y(4500)$. This result is slightly lower than the experimental value $4484.7 \pm 13.3 \pm 24.1$, with a deviation of approximately 62 MeV [21]. This discrepancy suggests that $Y(4500)$ may contain noncharmonium components, such as a possible tetraquark configuration. For $Y(4710)$, the predicted mass is 4659.5 MeV, which is close to the experimental value of $4708^{+17}_{-15} \pm 21$ MeV [21], supporting the assumption of its being the 4^3D_1 state. The predicted masses of higher excited states are also presented in Table II.

Following the presentation of the mass spectra of the $c\bar{c}$ meson family, it is essential to delve into a more detailed examination of their decay properties, as presented in the subsequent section.

III. DECAY PROPERTY ANALYSIS OF CHARMONIA

In this section, we will analyze the decay behaviors of S -, P -, and D -wave charmonium mesons. We present the

predicted widths of their two-body strong decays, annihilation decays, and hadronic transitions, using the quark-pair creation model (QPC model, also known as $^3P_0$ model), the Van Royen–Weisskopf formula with QCD corrections, and the Kuang–Yan model, respectively. A detailed description of these models and formulas can be found in the Appendix.

A. S-wave states

States in the ψ family with the quantum numbers $^3S_1$ are all spin-triplet states. The ground state $J/\psi(1S)$ and its first radially excited state $\psi(2S)$ have been well established experimentally. For the spin-singlet states, the masses and widths of $\eta_c(1S)$ and $\eta_c(2S)$ are also available from the PDG [4].

The $\psi(4040)$ state presents a particularly interesting case due to its strong decay behavior. There are four kinematically allowed open-charm decay channels: $D\bar{D}$, $D\bar{D}^*$, $D^*\bar{D}^*$, and $D_s\bar{D}_s$. In Table III, we show the partial and total decay widths of the $\psi(4040)$ state predicted by the $^3P_0$ model. The dominant contribution arises from two-body strong decays, and the total width is predicted to be 142 MeV. We also present the branching ratios of the decay channels in Table III, from which it can be seen that the dominant decay channel of the 3^3S_1 state is $D^*\bar{D}^*$ and the branching ratio is 65.8%. The ratio of the predicted partial widths is $\Gamma(D^*\bar{D}^*) : \Gamma(D\bar{D}^*) = 2.1 : 1.0$, which is in good agreement with the experimental ratio of $1.8 \pm 0.14 \pm 0.03$. The predicted total width of the $\psi(4040)$ is found to be somewhat larger than the experimental average of 84 ± 12 MeV. In addition, we have calculated the widths of its hadronic transitions. For the spin-nonflip $\pi\pi$ hadronic transition, we predict the width of the $3^3S_1 \rightarrow J/\psi + \pi\pi$ transition to be 0.225 MeV, which is in accordance with the experimental results, i.e., less than 0.504 MeV [4]. For the spin-nonflip η hadronic transition, the predicted width is 0.003 MeV, which is smaller than the experimental value. It can be seen that the contribution of hadronic transitions to the total decay is small.

Another $c\bar{c}$ resonance above the $D\bar{D}$ threshold is $\psi(4415)$. Using the potential model, we predict that it should be assigned to the 4^3S_1 state. Its primary decay mode is also the two-body strong decay, in which ten open-charm strong decay modes are allowed, seven of which end with $c\bar{n}$ meson final states ($n = u, d$), and other three end with $c\bar{s}$. We predict the total width of the 4^3S_1 state to be 122 MeV, showing good consistency with the average value ($\Gamma = 110 \pm 22$ MeV) reported by the PDG [4]. In our predictions, the main decay mode is $D\bar{D}_2^*$, with a branching ratio of 45.8%. Our result for the dominant decay channels differs from Ref. [36]. Belle [44] and FOCUS [45] observed the $D\bar{D}_2^*$ channel with $\Gamma = 45.6 \pm 8.0$ and $\Gamma = 38.7 \pm 5.3 \pm 2.9$ MeV, respectively, which is close to the width of 55.9 MeV in our prediction. Our predicted width

TABLE III. The two-body strong decay and hadronic transition of the S states are compared with the results from Refs. [35,36] and the PDG [4]. When the decay channels are not open or forbidden, a symbol $\times$ is presented. All results are in units of MeV.

Mode	$3^3S_1, \Gamma_{\text{Expt.}} = 84 \pm 12$ [4]			$4^3S_1, \Gamma_{\text{Expt.}} = 110 \pm 22$ [4]			
	This work		Ref. [35]	This work		Ref.	
	Γ_{thy}	Br (%)	Γ_{thy}	Γ_{thy}	Br (%)	Γ_{thy} [36]	Γ_{thy} [35]
$D\bar{D}$	1.9	1.34	0.1	9.63	7.89	4.14	0.4
$D\bar{D}^*$	44.8	31.6	33	4.7	3.85	5×10^{-3}	2.3
$D^*\bar{D}^*$	93.4	65.8	33	14	11.5	8.42	16
$D\bar{D}_1$	$\times$		$\times$	13.73	11.3	3.31	31
$D\bar{D}'_1$	$\times$		$\times$	10.5	8.61	2.94	1.0
$D\bar{D}_2^*$	$\times$		$\times$	55.9	45.8	0.109	23
$D^*\bar{D}^*_{00}$	$\times$		$\times$	0.40	0.33	0.685	0.0
$D_s\bar{D}_s$	1.1	0.77	7.8	0.05	0.04	2.33×10^{-3}	1.3
$D_s\bar{D}_s^*$	$\times$		$\times$	0.6	0.50	0.0652	2.6
$D_s^*\bar{D}_s^*$	$\times$		$\times$	0.9	0.74	0.222	0.7
$J/\psi(1S) + \pi\pi$	0.225	0.16	< 0.504 [4]	7.5	6.15		
$\psi(2S) + \pi\pi$	0.015	0.01		1.3	1.07		
$\psi(3S) + \pi\pi$	$\times$		$\times$	0.305	0.25		
$J/\psi(1S) + \eta$	0.003	0.002	0.437 [4]	0.021	0.02	< 0.66 [4]	
$\psi(2S) + \eta$	0.23	0.16		0.087	0.07		
$\psi(3S) + \eta$	$\times$		$\times$	2.78	2.28		
Total	142	100	74	122	100	19.9	78

Mode	3^1S_0			4^1S_0		
	This work		Ref. [35]	This work		Ref. [35]
	Γ_{thy}	Br (%)	Γ_{thy}	Γ_{thy}	Br (%)	Γ_{thy}
$D\bar{D}^*$	92.3	46.5	47	0.90	1.26	6.3
$D^*\bar{D}^*$	106.2	53.5	33	33.1	46.3	14
$D\bar{D}^*_{00}$	$\times$		$\times$	16.5	23.1	11
$D\bar{D}^*_{20}$	$\times$		$\times$	19.3	27.0	24
$D_s\bar{D}_s^*$	$\times$		$\times$	1.31	1.83	2.2
$D_s^*\bar{D}_s^*$	$\times$		$\times$	0.01	0.01	2.2
$D_s^*\bar{D}^*_{s0}$	$\times$		$\times$	0.43	0.60	0.6
Total	198.5	100	80	71.5	100	61

for the process $4^3S_1 \rightarrow J/\psi(1S) + \eta$ falls within the experimental upper limit of 0.66 MeV. We hope that future experiments with improved precision will be able to test our predicted transition widths.

The $c\bar{c}$ resonance $\psi(4660)$, with $J^{PC} = 1^{--}$, is also above the $D\bar{D}$ threshold. Considering both two-body strong decays and hadronic transitions, the total width in our prediction is 47.5 MeV, which is relatively close to the 73^{+13}_{-11} MeV given by the PDG [4]. Its primary decay channel is $D\bar{D}^*_{20}$, with a branching ratio 21.7%. However, due to the limitations of the QCD multipole expansion approach, it is difficult to provide reliable predictions for the transition widths of excited states near or above the open-flavor threshold. Therefore, we only present hadronic transition results up to the $5S$ state, while calculations for higher excited states will not be considered at this stage.

The calculated decay widths for $\psi(6S)$, $\psi(7S)$, and $\psi(8S)$ are presented in Table IV. Our results indicate that the largest decay channel for these highly excited states is $D\bar{D}^*$, with estimated branching ratios of 34.7%, 43.0%, and 42.6% for the 6^3S_1 , 7^3S_1 , and 8^3S_1 states, respectively. The total widths of $\psi(6S)$, $\psi(7S)$, and $\psi(8S)$ are predicted to be 21.0, 19.7, and 21.1 MeV, respectively. The discrepancies between our results and those in Ref. [46] can be attributed to differences in the theoretical models employed.

In Table V, the annihilation decay widths of the $\psi(nS)$ family are also provided. As can be seen, our calculated results for S -wave leptonic decays agree well with experimental values [4]. We have also shown the predicted leptonic decay widths of higher excited states in Table V. The two-photon decay widths of the η_c family are given in Table VI, from which we can see that our results are in

TABLE IV. The S -wave two-body strong decay and hadronic transition widths obtained in our calculation. The predicted values that are negligibly small are denoted by 0. All results are in units of MeV.

	$5^3S_1, \Gamma_{\text{Expt.}} = 73_{-11}^{+13}[4]$		6^3S_1		7^3S_1		8^3S_1	
Mode	Γ	Br(%)	Γ	Br(%)	Γ	Br(%)	Γ	Br(%)
$D\bar{D}$	6.7	13.1	5.2	24.8	4.6	23.3	4.1	19.4
$D\bar{D}^*$	6.6	12.9	7.3	34.7	8.5	43.0	9.0	42.6
$D^*\bar{D}^*$	0.71	1.4	0.25	1.2	2.2	11.1	4.2	19.9
$D\bar{D}_0^*$	0	0	0	0	0	0	0	0
$D\bar{D}_1^*$	3.5	6.84	1.5	7.14	1.0	5.06	0.9	4.26
$D\bar{D}'_1$	5.3	10.4	2.9	13.8	1.8	9.1	1.2	5.7
$D\bar{D}_2^*$	11.1	21.7	2.0	9.5	0.04	0.20	0.3	1.42
$D^*\bar{D}_0^*$	0.22	0.43	0.05	0.24	0.005	0.03	0.002	9.5×10^{-5}
$D_s\bar{D}_s$	0.1	0.19	0.17	0.81	0.20	1.01	0.21	0.99
$D_s\bar{D}_s^*$	0.01	0.02	0.035	0.17	0.14	0.71	0.24	1.14
$D_s^*\bar{D}_s^*$	0.4	0.78	0.08	3.81	0.0006	3.0×10^{-5}	0.03	0.14
$D_s\bar{D}_{s0}^*$	0	0	0	0	0	0	0	0
$D_s\bar{D}_{s1}^*$	0.72	1.4	0.9	4.28	0.75	3.80	0.55	2.60
$D_s\bar{D}'_{s1}$	0.04	0.08	0.31	1.48	0.3	1.52	0.27	1.28
$D_s\bar{D}_{s2}^*$	0.037	0.07	0.068	0.32	0.08	0.41	0.039	0.18
$D_s^*\bar{D}_{s0}^*$	0.3	0.58	0.2	0.95	0.12	0.61	0.06	0.28
$J/\psi(1S) + \pi\pi$	6.7	13.1						
$\psi(2S) + \pi\pi$	4.6	9.0						
$\psi(3S) + \pi\pi$	0.32	0.62						
$\psi(4S) + \pi\pi$	0.027	0.05						
$J/\psi(1S) + \eta$	0.09	0.17						
$\psi(2S) + \eta$	0.071	0.014						
$\psi(3S) + \eta$	0.077	0.15						
$\psi(4S) + \eta$	3.57	6.9						
Total	47.5	100	21.0	100	19.7	100	21.1	100

	5^1S_0		6^1S_0		7^1S_0		8^1S_0	
Mode	Γ	Br(%)	Γ	Br(%)	Γ	Br(%)	Γ	Br(%)
$D\bar{D}^*$	2.46	6.36	7.22	44.4	10.5	79.9	12.5	78.1
$D^*\bar{D}^*$	4.51	11.7	0.04	0.35	0.83	6.31	2.64	16.4
$D\bar{D}_0^*$	0.04	0.10	0.21	1.29	0.35	2.66	0.44	2.74
$D\bar{D}_2^*$	29.9	77.4	8.53	52.5	1.21	9.20	0.01	0.06
$D_s\bar{D}_s$	0.11	0.28	0.02	0.12	0.19	1.44	0.37	2.30
$D_s^*\bar{D}_s^*$	0.23	0.60	0.06	0.37	0	0	0.03	0.19
$D_s\bar{D}_{s0}^*$	0.39	1.01	0.17	1.05	0.06	0.46	0.02	0.12
Total	38.6	100	16.2	100	13.2	100	16.0	100

good agreement with the experimental values for $\eta_c(1S)$ and $\eta_c(2S)$. In addition, we also predicted the higher excited states.

The η_c family with quantum numbers $1S_0$ consists of spin-singlet states. The total width of 3^1S_0 obtained is 198.5 MeV. It is found that contributions of the two kinematically allowed channels $D\bar{D}^*$ and $D^*\bar{D}^*$ are comparable. The total width of 4^1S_0 is calculated to be 71.5 MeV, which is quite close to the value of 61 MeV reported in Ref. [35], where the main decay channel is $D^*\bar{D}^*$, with a branching ratio of 46.3%.

For $\eta_c(5S)$ and $\eta_c(6S)$ states, the dominant decay channels are both $D\bar{D}_2^*$, with branching ratios of 77.4%

and 52.5%, respectively. The decay widths of $\eta_c(7S)$ and $\eta_c(8S)$ states are also calculated. The dominant decay channel in this case is $D\bar{D}^*$, with estimated branching ratios of 79.9% and 78.1%, respectively.

B. P -wave states

We find that there are few open-flavor strong modes corresponding to the $2P$ states. Because of the node effect, $\chi_{c0}(2P)$ should be a narrow state [47,48]. Our prediction indicates that the $2^3P_0 \chi_{c0}(3915)$ state decays only to $D\bar{D}$, with a width of 43.7 MeV, which suggests it is a narrow state. The axial states $2^3P_1 \chi_{c1}(3872)$ can only decay to

TABLE V. Leptonic decay widths (in units of keV) for charmonium states in the screened potential model. The experimental values are taken from the PDG [4].

State	Γ_{ee}	$\Gamma_{ee}^{\text{Expt.}} [4]$
$J/\psi(1S)$	7.7	5.53 ± 0.030
$\psi(2S)$	3.1	2.33 ± 0.064
$\psi(4040)$	1.9	0.86 ± 0.143
$\psi(4415)$	1.3	0.58 ± 0.13
$\psi(4660)$	1.0	0.58 ± 0.07
$\psi(6S)$	0.76	
$\psi(7S)$	0.6	
$\psi(8S)$	0.46	
$\psi(3770)$	0.147	0.261 ± 0.019
$\psi(4160)$	0.15	0.476 ± 0.228
$Y(4500)$	0.126	
$Y(4710)$	0.123	
$\psi(5D)$	0.111	
$\psi(6D)$	0.087	

TABLE VI. Two-photon decay widths (in units of keV) of pseudoscalar ($^1S_0$), scalar ($^3P_0$), and tensor ($^3P_2$) charmonium states.

State	$\Gamma_{\gamma\gamma}$	$\Gamma_{\gamma\gamma}^{\text{Expt.}} [4]$
$\eta_c(1S)$	5.0	5.1
$\eta_c(2S)$	3.0	2.1
$\eta_c(3S)$	4.7	
$\eta_c(4S)$	3.5	
$\eta_c(5S)$	2.8	
$\eta_c(6S)$	2.2	
$\eta_c(7S)$	1.7	
$\eta_c(8S)$	1.4	
$\chi_{c2}(1P)$	0.12	0.49 ± 0.05
$\chi_{c2}(3930)$	0.16	
$\chi_{c2}(3P)$	0.23	
$\chi_{c2}(4P)$	0.24	
$\chi_{c2}(5P)$	0.27	
$\chi_{c2}(6P)$	0.28	

$D\bar{D}^*$. We obtain a width of 221 MeV in Table VII. As the partner state of 2^3P_0 , $\chi_{c2}(3930)$ is predicted to have a decay width of $\Gamma = 55.3$ MeV in our calculation, which is slightly larger than the experimental value [4]. However, our results demonstrate an improvement over those reported in Ref. [49].

The predicted branching ratio to the mode $D^*\bar{D}^*$ is large for the $3P$ states (3^3P_0 and 3^3P_1) [35]. The 3^3P_0 state is identified as $X(4160)$ [38,42], the dominant decay channel of 3^3P_0 is $D^*\bar{D}^*$, with the branching ratio reaching 82.5%. We obtain a width ratio of $\Gamma(D\bar{D}): \Gamma(D^*\bar{D}^*) = 0.17:1$, which is slightly larger than the experimental ratio (< 0.09) [4].

While our findings are consistent with the branching ratio contributions reported in Ref. [50], our results demonstrate better agreement with the total width reported in experiments. The 3^3P_1 state is identified as $\chi_{c1}(4274)$, with a predicted mass of $M = 4512.5^{+6.0}_{-6.2} \pm 3.0$ MeV and a width of $\Gamma = 92^{+22}_{-18} \pm 57$ MeV [20], showing good agreement with the LHCb measurement. For the 3^3P_2 state, there are four open-charm decay channels that are kinematically allowed, $D\bar{D}$, $D\bar{D}^*$, $D_s\bar{D}_s$, and $D_s\bar{D}_s^*$. Among them, the dominant decay channel is $D\bar{D}$, with a branching ratio of 58.9%.

For $\chi_{c0}(4500)$ reported by the LHCb [20], we identify it as the $\chi_{c0}(4P)$ state. In our calculation, the dominant decay mode for this state is $D^*\bar{D}^*$, accounting for 88.8%. We obtain the total width of 34.1 MeV, which agrees with the experimental value and is lower than the theoretical predictions provided in Ref. [50]. We then assign $\chi_{c1}(4685)$ as the 4^3P_1 state, whose dominant decay channel is $D\bar{D}^*$, with the ratio of the predicted partial widths $\Gamma(D\bar{D}^*): \Gamma(D^*\bar{D}^*) = 1.4:1$. While our findings are broadly consistent with the ratios reported in Ref. [50], our results demonstrate better agreement with experimental measurements. The result of the 4^3P_2 state is shown in Table VIII, the primary decay channel is $D\bar{D}^*$, with a branching ratio of 46.5%.

Based on the mass spectra obtained from our calculations, $\chi_{c0}(4700)$ should be assigned to the 5^3P_0 state, while the same identification has been reported in Refs. [38,51]. The predicted total width is 9.0 MeV, whereas the result given in Ref. [50] is 17 MeV. The widths of $5P$ and $6P$ states are also presented in Tables VII and VIII. In addition, Table VI presents the two-photon decay widths of χ_{c2} states, which are expected to provide useful reference data for future experiments.

C. D-wave states

We present the widths of the related decays for D -wave states in Tables IX and X. Generally, $\psi(3770)$ is identified as the 1^3D_1 state; we predict a $D\bar{D}$ decay width of 44.1 MeV, which is larger than the experimental value of 27.2 ± 1.0 MeV. For the 1^3D_2 state, the open-charm decay channels are forbidden. The 1^3D_3 state can decay only into $D\bar{D}$, and our predicted width is 2.3 MeV, which is consistent with both the experimental value and the prediction in Ref. [52].

According to our potential model predictions, the 2^3D_1 state is identified as $\psi(4160)$. The decay of the 2^3D_1 state is dominated by $D^*\bar{D}^*$, with relatively smaller contributions from charmed-strange final states such as $D\bar{D}^*$ and $D_s\bar{D}_s$. We obtain a width ratio of $\Gamma(D\bar{D}): \Gamma(D^*\bar{D}^*) = 0.28:1$, which is slightly larger than the experimental ratio ($0.02 \pm 0.03 \pm 0.02$). We also provide the widths of its hadronic transitions, which are consistent with the experimental

TABLE VII. The two-body strong decay of the P states are compared with the results from Refs. [35,49,50] and the PDG [4]. When the decay channels are not open or forbidden, a symbol $\times$ is presented. The predicted values that are negligibly small are denoted by 0. All results are in units of MeV.

$2^3P_0, \Gamma_{\text{Expt.}} = 20 \pm 4$ [4]				2^3P_1			
This work				This work		Ref. [35]	
Mode	Γ_{thy}	Br (%)	Γ_{thy}	Γ_{thy}	Br (%)	Γ_{thy}	
$D\bar{D}$	43.7	100	30	$\times$		$\times$	
$D\bar{D}^*$	$\times$		$\times$	221	100	165	
Total	43.7	100	30	221	100	165	
$2^3P_2, \Gamma_{\text{Expt.}} = 35.2 \pm 2.2$ [4]				$3^3P_0, \Gamma_{\text{Expt.}} = 136^{+60}_{-35}$ [4]			
This work				This work		Ref. [50]	
Mode	Γ_{thy}	Br (%)	Γ_{thy}	Γ_{thy}	Br (%)	Γ_{thy}	
$D\bar{D}$	39.3	71.1	42	21.0	14.0	2	
$D\bar{D}^*$	16.0	28.9	37	$\times$		$\times$	
$D^*\bar{D}^*$	$\times$		$\times$	124	82.5	67	
$D_s\bar{D}_s$	0	0	0.7	5.3	3.52	3	
Total	55.3	100	80	150	100	72	
$3^3P_1, \Gamma_{\text{Expt.}} = 92^{+22}_{-18} \pm 57$ [20]				$4^3P_0, \Gamma_{\text{Expt.}} = 65^{+20}_{-16} \pm 32$ [20]			
This work				This work		Ref. [50]	
Mode	Γ_{thy}	Br (%)	Γ_{thy}	Γ_{thy}	Br (%)	Γ_{thy}	
$D\bar{D}$	$\times$		$\times$	0.7	2.1	7	
$D\bar{D}^*$	10.4	34.2	6.8	$\times$		$\times$	
$D^*\bar{D}^*$	15.5	50.9	19	30.3	88.8	22	
$D^*\bar{D}^{*0}$	0	0	0.1	$\times$		$\times$	
$D_s\bar{D}_s$	$\times$		$\times$	1.7	5.0	1	
$D_s\bar{D}_s^*$	4.0	3.14	9.7	$\times$		$\times$	
$D_s^*\bar{D}_s^*$	0.53	1.74	2.7	0.56	1.64	0	
Total	30.4	100	39	34.1	100	43	
$4^3P_1, \Gamma_{\text{Expt.}} = 130 \pm 40$ [4]				$5^3P_0, \Gamma_{\text{Expt.}} = 87^{+18}_{-10}$ [4]		5^3P_1	
This work				This work		This work	
Mode	Γ_{thy}	Br (%)	Γ_{thy}	Γ_{thy}	Br (%)	Γ_{thy}	Br (%)
$D\bar{D}$	$\times$		$\times$	0.05	0.55	$\times$	
$D\bar{D}^*$	45.8	50.9	15	$\times$		26.0	74.1
$D^*\bar{D}^*$	33.5	37.2	12	7.8	86.6	1.5	4.27
$D\bar{D}^*_2$	10.0	11.1	10	$\times$		7.28	20.7
$D^*\bar{D}^{*0}$	0.1	0.11	3	0.2	2.22	0.01	0.03
$D_s\bar{D}_s$	$\times$		$\times$	0.5	5.55	$\times$	
$D_s\bar{D}_s^*$	0.49	0.54	< 1	$\times$		0.21	0.60
$D_s^*\bar{D}_s^*$	0.05	0.06	< 1	0.46	5.11	0.1	0.28
Total	89.9	100	41	9.0	100	35.1	100

values [4]. For 2^3D_2 and 2^3D_3 states, the dominant decay channel is $D^*\bar{D}^*$, with estimated branching ratios of 52.9% and 53.6%, respectively.

$Y(4500)$ is assigned to the 3^3D_1 state. Our calculation yields a total width of 112.6 MeV, which is in good

agreement with the experimental data [21]. The dominant decay channel is $D^*\bar{D}^*$, with a branching ratio of 57.8%. We obtain a width ratio of $\Gamma(D\bar{D}):\Gamma(D^*\bar{D}^*) = 0.37:1$. We also calculate the widths for 3^3D_2 and 3^3D_3 states and compare them with the results from Ref. [37]. It can be seen

TABLE VIII. The two-body strong decay widths of the P states. When the decay channels are not open or forbidden, a symbol $\times$ is presented. All results are in units of MeV.

Mode	3^3P_2		4^3P_2		5^3P_2	
	Γ_{thy}	Br (%)	Γ_{thy}	Br (%)	Γ_{thy}	Br (%)
$D\bar{D}$	15.1	58.9	8.57	27.5	5.74	30.5
$D\bar{D}^*$	9.94	38.7	14.5	46.5	11.9	63.6
$D^*\bar{D}^*$	$\times$		$\times$		$\times$	
$D\bar{D}_2^*$	$\times$		7.83	25.2	0.61	3.25
$D^*\bar{D}_0^*$	$\times$		0.01	0.03	0.15	0.8
$D_s\bar{D}_s$	0.16	0.62	0.23	0.74	0.22	1.17
$D_s\bar{D}_s^*$	0.45	1.75	0.01	0.03	0.11	0.59
$D_s^*\bar{D}_s^*$	$\times$		$\times$		$\times$	
Total	25.7	100	31.1	100	18.8	100

Mode	6^3P_0		6^3P_1		6^3P_2	
	Γ_{thy}	Br (%)	Γ_{thy}	Br (%)	Γ_{thy}	Br (%)
$D\bar{D}$	1.42	46.3	$\times$		4.15	26.4
$D\bar{D}^*$	$\times$		22.8	81.5	10.0	63.7
$D^*\bar{D}^*$	1.26	41.0	3.73	13.3	$\times$	
$D_s\bar{D}_s$	0.07	2.28	$\times$		0.19	1.21
$D\bar{D}_2^*$	$\times$		1.30	4.64	0.90	5.74
$D^*\bar{D}_0^*$	0.10	3.26	0.05	0.18	0.27	1.72
$D_s\bar{D}_s^*$	$\times$		0.10	0.36	0.18	1.15
$D_s^*\bar{D}_s^*$	0.22	7.17	0.001	0.004	$\times$	
Total	3.1	100	28.0	100	15.7	100

TABLE IX. The two-body strong decay and hadronic transition of the D states are compared with the results from Refs. [21,36,37,52] and the PDG [4]. When the decay channels are not open or forbidden, a symbol $\times$ is presented. The predicted values that are negligibly small are denoted by 0. All results are in units of MeV.

Mode	$1^3D_1, \Gamma_{\text{Expt.}} = 27.2 \pm 1.0$ [4]				$1^3D_3, \Gamma_{\text{Expt.}} = 2.8 \pm 0.6$ [4]	
	This work		Ref. [36]		This work	
	Γ_{thy}	Br (%)	Γ_{thy}		Γ_{thy}	Br (%)
$D\bar{D}$	44.1	100	33.9		2.3	100
Total	44.1	100	33.9		2.3	100

Mode	$2^3D_1, \Gamma_{\text{Expt.}} = 69 \pm 10$ [4]				2^3D_2	
	This work		Ref. [36]		This work	
	Γ_{thy}	Br (%)	Γ_{thy}		Γ_{thy}	Br (%)
$D\bar{D}$	22.6	20.9			$\times$	$\times$
$D\bar{D}^*$	0.02	0.02			46.3	42.4
$D^*\bar{D}^*$	81.1	75.1			57.8	52.9
$D_s\bar{D}_s$	0.24	0.22			$\times$	$\times$
$D_s\bar{D}_s^*$	3.3	3.05			5.05	4.63
$J/\psi(1S) + \pi\pi$	0.63	0.59	< 0.414 [4]			
$\psi(2S) + \pi\pi$	0.14	0.13	< 0.276 [4]			
Total	108.0	100	72.2		109.2	100

(Table continued)

TABLE IX. (Continued)

Mode	$1^3D_1, \Gamma_{\text{Expt.}} = 27.2 \pm 1.0$ [4]			$1^3D_3, \Gamma_{\text{Expt.}} = 2.8 \pm 0.6$ [4]		
	This work		Ref. [36]	This work		Ref. [52]
	Γ_{thy}	Br (%)	Γ_{thy}	Γ_{thy}	Br (%)	Γ_{thy}
Mode	2^3D_3			$3^3D_1, \Gamma_{\text{Expt.}} = 111.1 \pm 30.1 \pm 15.2$ [21]		
	This work		Ref. [37]	This work		Ref. [36]
	Γ_{thy}	Br (%)	Γ_{thy}	Γ_{thy}	Br (%)	Γ_{thy}
$D\bar{D}$	16.4	9.48	3.27	24.2	21.5	2.0
$D\bar{D}^*$	62.7	36.3	21.8	10.0	8.92	3.72
$D^*\bar{D}^*$	92.6	53.6	28.8	65.1	57.8	6.50
$D\bar{D}_0^*$	$\times$		$\times$	0	0	
$D\bar{D}_1$	$\times$		$\times$	3.9	3.46	6.26
$D\bar{D}'_1$	$\times$		$\times$	3.3	2.93	4.24
$D\bar{D}_2^*$	$\times$		$\times$	2.2	1.95	4.05
$D^*\bar{D}_0^*$	$\times$		$\times$	0.07	0.06	3.48
$D_s\bar{D}_s$	0.9	0.52	0.36	0.54	0.48	0.01
$D_s\bar{D}_s^*$	0.24	0.14	0.009	0.07	0.06	0.24
$D_s^*\bar{D}_s^*$	$\times$		$\times$	0.42	0.37	0.12
$D_s\bar{D}_{s0}^*$	$\times$		$\times$	0	0	
$D_s\bar{D}_{s1}$	$\times$		$\times$	1.4	1.24	
$D_s\bar{D}'_{s1}$	$\times$		$\times$	1.3	1.15	
$D_s\bar{D}_{s2}^*$	$\times$		$\times$	0.01	0.009	
$D_s^*\bar{D}_{s0}^*$	$\times$		$\times$	0.01	0.01	
Total	172.9	100	54.3	112.6	100	32.6
Mode	3^3D_2			3^3D_3		
	This work		Ref. [37]	This work		Ref. [37]
	Γ_{thy}	Br (%)	Γ_{thy}	Γ_{thy}	Br (%)	Γ_{thy}
$D\bar{D}$	$\times$		$\times$	8.1	11.7	2.8
$D\bar{D}^*$	20.5	29.3	14.8	23.0	33.2	12.1
$D^*\bar{D}^*$	10.2	14.6	7.9	8.9	12.8	4.93
$D\bar{D}_0^*$	0.001	0.001	0.01	$\times$		$\times$
$D\bar{D}_1$	0.31	0.44	1.81	7.2	10.4	0.57
$D\bar{D}'_1$	8.5	12.2	1.92	3.7	5.34	2.23
$D\bar{D}_2^*$	29.8	42.6	1.61	16.7	24.1	0.42
$D^*\bar{D}_0^*$	$\times$		$\times$	$\times$		$\times$
$D_s\bar{D}_s$	0	0	0.42	0.34	0.49	0.22
$D_s\bar{D}_s^*$	0.65	0.93	0.18	0.1	0.14	0.13
$D_s^*\bar{D}_s^*$	$\times$		$\times$	1.2	1.73	0.54
$D_s^*\bar{D}_{s0}^*$	$\times$		$\times$	$\times$		$\times$
Total	70.0	100	28.7	69.2	100	23.9

that our predicted branching ratios are generally consistent with those reported in other works, except for the $D\bar{D}_2^*$ channel, for which our results are larger than those in Ref. [37].

BESIII has observed a new decay mode $Y(4230) \rightarrow K^+K^-J/\psi$ and reported a vector charmoniumlike state $Y(4710)$ [21], whose decay width is $126_{-23}^{+27} \pm 30$ MeV.

Our results indicate that this state should be the 4^3D_1 charmonium state. The results of $5D$ and $6D$ states are listed in Table X.

We have additionally presented the leptonic decay widths for the $^3D_1$ states in Table V, demonstrating reasonable consistency with experimental values. Furthermore, we provide predictions for higher excited states.

TABLE X. The D -wave two-body strong decay widths obtained in our calculation. The predicted values that are negligibly small are denoted by 0. All results are in units of MeV.

Mode	$4^3D_1, \Gamma_{\text{Expt.}} = 126^{+27}_{-23} \pm 30$ [21]		5^3D_1		6^3D_1	
	Γ_{thy}	Br (%)	Γ_{thy}	Br (%)	Γ_{thy}	Br (%)
$D\bar{D}$	11.2	25.4	6.5	31.4	5.6	29.4
$D\bar{D}^*$	3.2	7.26	1.6	7.73	1.8	9.46
$D^*\bar{D}^*$	19.6	44.4	9.5	45.9	10	52.5
$D\bar{D}_0^*$	0	0	0	0	0	0
$D\bar{D}_1^*$	1.7	3.85	0.68	3.29	0.026	0.14
$D\bar{D}'_1$	1.5	3.40	0.9	4.35	0.7	3.68
$D\bar{D}_2^*$	1.2	2.72	0.4	1.93	0.006	0.03
$D_s^*\bar{D}_s^*$	0.1	0.23	0.1	0.48	0	0
$D_s\bar{D}_s$	0.2	0.45	0.1	0.48	0.13	0.68
$D_s\bar{D}_s^*$	0.05	0.11	0.02	0.1	0	0
$D_s^*\bar{D}_s^*$	0.2	0.45	0.07	0.34	0.08	0.42
$D_s\bar{D}_{s0}^*$	0	0	0	0	0	0
$D_s\bar{D}_{s1}$	0.5	1.13	0.28	1.35	0.37	1.94
$D_s\bar{D}_{s1}^*$	0.68	1.54	0.5	2.42	0.37	1.94
$D_s\bar{D}_{s2}^*$	0.07	0.16	0.06	0.29	0.05	0.26
$D_s^*\bar{D}_{s0}^*$	0	0	0.002	0.01	0.009	0.05
Total	44.1	100	20.7	100	19.0	100

IV. SUMMARY

In this work, we first adopt a nonrelativistic potential model with a screening effect to calculate the mass spectra of $c\bar{c}$ mesons. Then the two-body strong decays from S - to D -wave $c\bar{c}$ mesons are investigated using the $^3P_0$ model. In addition, we study the annihilation decays and hadronic transitions by the Van Royen–Weisskopf formula and Kuang-Yan model, respectively. Finally, we classify and fit the experimentally observed charmonium states through a comprehensive analysis of the mass spectra and decay widths. The main findings of this work are summarized as follows.

Combining the data from the PDG and the latest experimental findings with our results, we have identified the family of discovered states. For the S -wave states, we assign $\psi(4415)$ and $\psi(4660)$ as the 4^3S_1 and 5^3S_1 states, respectively. For the excited states of the η_c and ψ family, we carry out detailed analyses and predictions. For the P -wave states, we assign $X(4160)$, $\chi_{c1}(4274)$, $\chi_{c0}(4500)$, and $\chi_{c0}(4700)$ as the 3^3P_0 , 3^3P_1 , 4^3P_0 , and 5^3P_0 states, respectively. For the D -wave sector, $Y(4500)$ and $Y(4710)$ are identified as the 3^3D_1 and 4^3D_1 states, respectively. In addition, results for the higher excited states are provided.

We specifically account for contributions from hadronic transitions by employing the Kuang-Yan model to calculate the spin-nonflip $\pi\pi$ and η transition widths. Our results indicate that, within charmonium systems, hadronic transitions contribute relatively little. However, due to theoretical limitations inherent in the QCD multipole expansion

approach, reliable predictions for higher excited states cannot be obtained. Consequently, we do not present transition width estimates for these states.

From the recent experimental data, we find that the newly discovered resonances $X(4160)$, $Y(4500)$, and $Y(4710)$ are consistent with 3^3P_0 , 3^3D_1 , and 4^3D_1 states, respectively. This consistency demonstrates the validity and reliability of our adopted potential model. We hope that this work will provide a valuable reference for future studies on higher-energy $c\bar{c}$ meson states.

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DATA AVAILABILITY

The data supporting this study’s findings are available within the article.

APPENDIX: THEORETICAL MODELS OF DECAY BEHAVIORS

In this appendix, we present all the necessary formulas for the calculation of two-body OZI-allowed strong decays, annihilation decays, and hadronic transitions.

1. The two-body OZI-allowed strong decays

To investigate the decay behavior of mesons, we always begin by analyzing two-body strong decays. We employ the QPC model to calculate the strong decay widths. In this model, the quark pair is assumed to be produced from the vacuum with quantum numbers corresponding to the vacuum quantum state ($J^{PC} = 0^{++}$), so it is also called the $^3P_0$ model.

The QPC model is one of the most efficient phenomenological models to calculate the widths of two-body OZI-allowed strong decays. The selection of decay channels follows the OZI rule: if the corresponding Feynman diagram can be cut in two parts by slicing only gluon lines (and not cutting any external lines), the process will be suppressed. This model was originally introduced by Micu [53], and was later developed and extensively applied by Le Yaouanc *et al.* to meson [54] and baryon [55,56] open-flavor strong decays in a series of publications in the 1970s. Since then, the model has been further refined. Now it has been widely used to calculate two-body strong decays allowed by the OZI rule [57–60].

For a decay process $A \rightarrow B + C$, the transition matrix is defined as [61–64]

$$\langle BC | \mathcal{T} | A \rangle = \delta^3(\mathbf{P}_B + \mathbf{P}_C) \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}, \quad (\text{A1})$$

where $\mathcal{T}$ can be expressed as

$$\begin{aligned} \mathcal{T} = & -3\gamma \sum_m \langle 1m; 1-m | 00 \rangle \int d\mathbf{p}_3 d\mathbf{p}_4 \delta^3(\mathbf{p}_3 + \mathbf{p}_4) \\ & \times \mathcal{Y}_{1m} \left(\frac{\mathbf{p}_3 - \mathbf{p}_4}{2} \right) \chi_{1,-m}^{34} \phi_0^{34} (\omega_0^{34})_{ij} b_{3i}^\dagger(\mathbf{p}_3) d_{4j}^\dagger(\mathbf{p}_4), \end{aligned} \quad (\text{A2})$$

which describes the creation of a quark-antiquark pair from the vacuum. $\mathbf{p}_3$ and $\mathbf{p}_4$ are the momenta of the produced quark and antiquark, respectively. The specific meanings of each quantity are shown in Ref. [25]. Using the Jacobi-Wick formula, the helicity amplitude $\mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}(\mathbf{P})$ can be converted into the partial wave amplitudes $\mathcal{M}^{JL}$, which are given by

$$\begin{aligned} \mathcal{M}^{JL}(\mathbf{P}) = & \frac{\sqrt{4\pi(2L+1)}}{2J_A+1} \sum_{M_{J_B} M_{J_C}} \langle L0; JM_{J_A} | J_A M_{J_A} \rangle \\ & \times \langle J_B M_{J_B}; J_C M_{J_C} | J_A M_{J_A} \rangle \mathcal{M}^{M_{J_A} M_{J_B} M_{J_C}}. \end{aligned} \quad (\text{A3})$$

For the meson wave function, we define it as a mock state, i.e.,

$$\begin{aligned} |A(n^{2S+1}L_{JM_J})(\mathbf{p}_A)\rangle = & \sqrt{2E} \sum_{M_S, M_L} \langle LM_L SM_S | JM_J \rangle \chi_{SM_S}^A \\ & \times \phi^A \omega^A \int d\mathbf{p}_1 d\mathbf{p}_2 \delta^3(\mathbf{p}_A - \mathbf{p}_1 - \mathbf{p}_2) \\ & \times \Psi_{nLM_L}^A(\mathbf{p}_1, \mathbf{p}_2) |q_1(\mathbf{p}_1) \bar{q}_2(\mathbf{p}_2)\rangle, \end{aligned} \quad (\text{A4})$$

where the momentum-space wave function is $\Psi_{nLM_L}^A(\mathbf{p}_1, \mathbf{p}_2) = R_{nL}(|\mathbf{p}|) Y_{lm}(\mathbf{p})$, with $\mathbf{p} = (m_1 \mathbf{p}_2 - m_2 \mathbf{p}_1) / (m_1 + m_2)$.

Then, the partial width for a strong decay channel $A \rightarrow B + C$ can be calculated using the formula

$$\Gamma = \frac{\pi}{4} \frac{|\mathbf{P}_E|}{m_A^2} \sum_{J,L} |\mathcal{M}^{JL}(\mathbf{P})|^2, \quad (\text{A5})$$

$$|\mathbf{P}_E| = \frac{\sqrt{[m_A^2 - (m_B + m_C)^2][m_A^2 - (m_B - m_C)^2]}}{2m_A}, \quad (\text{A6})$$

where m_A , m_B , and m_C are the masses of the initial meson A and the final mesons B and C , respectively.

For OZI-allowed strong decays involving the creation of a strange quark pair from the vacuum, the strength of strange quark pair creation is taken as $\gamma_s = \gamma/\sqrt{3}$, where $\gamma = 6.947$ is adopted from Ref. [65].

We determine the decay kinematics using meson masses taken from the PDG [4] and from recent experimental results [20–22]. The masses of charmlike mesons used in this work are listed in Table XI. Additionally, we consider the D_1 and D'_1 charmed mesons as mixtures of the 1^1P_1 and 1^3P_1 states, with the mixing angle taken to be the heavy quark limit $\theta_{1P} = 35.3^\circ(-54.7^\circ)$ [66].

TABLE XI. The mass values and input β values of the charmed and charmed-strange mesons involved in the present calculation, with all units given in MeV.

State	$n^{2S+1}L_J$	Input mass [4]	β
D	1^1S_0	1865	993
D^*	1^3S_1	2007	874
D_0^*	1^3P_0	2343	881
D_1	$\cos(\theta) 1^1P_1\rangle + \sin(\theta) 1^3P_1\rangle$	2412	690
D'_1	$-\sin(\theta) 1^1P_1\rangle + \cos(\theta) 1^3P_1\rangle$	2427 [44]	791
D_2^*	1^3P_2	2461	736
D_s	1^1S_0	1968	1080
D_s^*	1^3S_1	2112	883
D_{s0}^*	1^3P_0	2318	1480
D_{s1}	1^1P_1	2460	736
D'_{s1}	1^3P_1	2535	837
D_{s2}^*	1^3P_2	2569	800

2. Annihilation decay

a. Leptonic decay

The decay rate of the vector meson is described by the Van Royen–Weisskopf formula [67], incorporating QCD radiative corrections [68]. The related equations are shown as follows:

$$\Gamma_{ee}(nS) = \frac{4\alpha^2 e_c^2}{M_{nS}^2} |R_{nS}(0)|^2 \left(1 - \frac{16}{3} \frac{\alpha_s}{\pi}\right), \quad (\text{A7})$$

$$\Gamma_{ee}(nD) = \frac{25\alpha^2 e_c^2}{2M_{nD}^2 m_c^4} |R''_{nD}(0)|^2 \left(1 - \frac{16}{3} \frac{\alpha_s}{\pi}\right), \quad (\text{A8})$$

where $M_{nS}(M_{nD})$ represents the mass for nS (nD) states, $e_c = \frac{2}{3}$ is the c quark charge in units of the electron charge, α denotes the fine-structure constant ($\alpha = \frac{1}{137}$), and α_s is the parameter given in Table I. $R_{nS}(0)$ and $R''_{nD}(0)$ are the radial S -wave function and the second derivative of the radial D -wave function at the origin, respectively [19].

b. Two-photon decays

In the nonrelativistic limit, the two-photon decay widths of $^1S_0$, $^3P_0$, and $^3P_2$ can be expressed as [19,69]

$$\Gamma^{NR}(^1S_0 \rightarrow \gamma\gamma) = \frac{3\alpha^2 e_c^4 |R_{nS}(0)|^2}{m_c^2}, \quad (\text{A9})$$

$$\Gamma^{NR}(^3P_0 \rightarrow \gamma\gamma) = \frac{27\alpha^2 e_c^4 |R'_{nP}(0)|^2}{m_c^4}, \quad (\text{A10})$$

$$\Gamma^{NR}(^3P_2 \rightarrow \gamma\gamma) = \frac{36\alpha^2 e_c^4 |R'_{nP}(0)|^2}{5m_c^4}. \quad (\text{A11})$$

The first-order QCD radiative corrections to the two-photon decay rates read [69]

$$\Gamma(^1S_0 \rightarrow \gamma\gamma) = \Gamma^{NR}(^1S_0 \rightarrow \gamma\gamma) \left[1 + \frac{\alpha_s}{\pi} \left(\frac{\pi^2}{3} - \frac{20}{3}\right)\right], \quad (\text{A12})$$

$$\Gamma(^3P_0 \rightarrow \gamma\gamma) = \Gamma^{NR}(^3P_0 \rightarrow \gamma\gamma) \left[1 + \frac{\alpha_s}{\pi} \left(\frac{\pi^2}{3} - \frac{28}{9}\right)\right], \quad (\text{A13})$$

$$\Gamma(^3P_2 \rightarrow \gamma\gamma) = \Gamma^{NR}(^3P_2 \rightarrow \gamma\gamma) \left[1 - \frac{16}{3} \frac{\alpha_s}{\pi}\right]. \quad (\text{A14})$$

One sees that $\Gamma(^1S_0 \rightarrow \gamma\gamma) \propto |R_{nS}(0)|^2$, which is sensitive to the details of the potential near the origin. Thus, we adopt the following formula to mitigate this uncertainty:

$$\Gamma(^1S_0 \rightarrow \gamma\gamma) \longrightarrow \frac{\Gamma(^1S_0 \rightarrow \gamma\gamma)}{\Gamma_{ee}(nS)} \Gamma_{ee}^{\text{expt}}(nS). \quad (\text{A15})$$

The results are tabulated in Tables V and VI.

3. Hadronic transition

The hadronic transition process is one in which light hadrons are released when the $c\bar{c}$ state transitions to a lower-energy level. It can be given by

$$\Phi_i \rightarrow \Phi_f + h, \quad (\text{A16})$$

where Φ_i and Φ_f are defined as the initial and final $c\bar{c}$ states, respectively, and h denotes the emitted light hadron(s) which are kinematically dominated by either a single meson (π , η , ρ , ...) or two mesons (2π , $2K$, ...). This process cannot be calculated using perturbation QCD, so we adopt the method of QCD multipole expansion to solve it.

a. QCD multipole expansion framework

Based on the general properties of QCD, it is anticipated that there exist states in which the gluonic field is excited and carries its own J^{PC} quantum numbers. A bound state that lacks valence quark content is termed a glueball, and the inclusion of a quark-antiquark pair forms a hybrid meson. Such interactions could result in either mesons with exotic J^{PC} or those with standard quantum numbers. Our focus is on the latter, as they play a crucial role in the calculation of hadronic transitions within the QCD multipole expansion framework [70–72].

After the expansion of the gluon field, the Hamiltonian of the system is given by [70]

$$\mathcal{H}_{\text{QCD}}^{\text{eff}} = \mathcal{H}_{\text{QCD}}^{(0)} + \mathcal{H}_{\text{QCD}}^{(1)}, \quad (\text{A17})$$

with $\mathcal{H}_{\text{QCD}}^{(0)}$ being the sum of the kinetic and potential energies of $c\bar{c}$ mesons, and $\mathcal{H}_{\text{QCD}}^{(1)}$ is expressed as

$$\mathcal{H}_{\text{QCD}}^{(1)} = \mathcal{H}^{(1)} + \mathcal{H}^{(2)}, \quad (\text{A18})$$

$$\mathcal{H}^{(1)} = Q_a A_0^a(x, t), \quad (\text{A19})$$

$$\mathcal{H}^{(2)} = -d_a E^a(x, t) - m_a B^a(x, t), \quad (\text{A20})$$

where Q_a corresponds to the color charge, A_0^a to the gluon field, d_a to the color-electric dipole moment, E^a to the color-electric field, m_a to the color-magnetic dipole moment, and B^a to the color-magnetic field.

The lowest-order interaction between two color-singlet states involves the exchange of two gluons. Consequently, its leading multipole contribution is the double electric-dipole (E1E1) term, whose transition amplitude can be derived from the corresponding S -matrix elements,

$$\mathcal{M}_{E1E1} = i \frac{g_E^2}{6} \langle \Phi_f | \vec{x} \cdot \vec{E} \frac{1}{E_i - H_{\text{QCD}}^{(0)} - iD_0} \vec{x} \cdot \vec{E} | \Phi_i \rangle, \quad (\text{A21})$$

where g_E is the coupling constant for electric dipole (E1) gluon emission, $\vec{x}$ is the separation between the quark and antiquark, $\vec{E}$ is the color-electric field, and $G(E_i) = \frac{1}{E_i - H_{\text{QCD}}^{(0)} - iD_0}$ is the Green's function with $(D_0)_{bc} \equiv \delta_{bc} \partial_0 - g_s f_{abc} A_0^a$ [70].

After inserting a complete set of intermediate states and using a quark confining string (QCS) model, the transition amplitude in Eq. (A21) can be written as [70,73–75]

$$\mathcal{M}_{E1E1} = i \frac{g_E^2}{6} \sum_{kl} \frac{\langle \Phi_f | x_k | kl \rangle \langle kl | x_l | \Phi_i \rangle}{E_l - E_{kl}} \langle \pi\pi | E_k^a E_l^a | 0 \rangle, \quad (\text{A22})$$

where E_{kl} is the energy eigenvalue of the intermediate vibrational state $|kl\rangle$ with the principal quantum number k

and the orbital angular momentum l and corresponding eigenvalues in the sector of the lowest string excitation.

The transition amplitude contains two parts: the heavy quark multipole gluon emissions (MGE) factor (the summation) and the hadronization (H) factor $\langle \pi\pi | E_k^a E_l^a | 0 \rangle$. Using the eigenvalues and wave functions of the intermediate hybrid mesons and the initial and final quarkonium states, the MGE factor can be calculated. The H factor reflects the conversion of the two emitted gluons into light hadrons after hadronization. Because of the low energy involved, it is highly nonperturbative so that this matrix element cannot be calculated with perturbative QCD. In this case, phenomenological approaches based on soft-pion techniques are employed [49]. In the center-of-mass frame, the two pion momenta q_1 and q_2 are the only independent variables describing this matrix element, which can be expressed as [49,70,71,76]

$$\frac{g_E^2}{6} \langle \pi_\alpha(q_1) \pi_\beta(q_2) | E_k^a E_l^a | 0 \rangle = \frac{\delta_{\alpha\beta}}{\sqrt{(2\omega_1)(2\omega_2)}} \left[C_1 \delta_{kl} q_1^\mu q_{2\mu} + C_2 \left(q_{1k} q_{2l} + q_{1l} q_{2k} - \frac{2}{3} \delta_{kl} \vec{q}_1 \cdot \vec{q}_2 \right) \right], \quad (\text{A23})$$

where C_1 and C_2 are two constants, and we provide their values in Appendix A 3 d. The C_1 term is isotropic, and the C_2 term has an $l = 2$ angular dependence. q_1^μ and $q_{2\mu}$ are momentum components.

b. A model for hybrid states

The intermediate states in the hadronic transition consist of a gluon and a color octet $c\bar{c}$ that are the states after the emission of the first gluon and before the emission of the second gluon. Thus, these states are the so-called hybrid states. We employ the QCS model to calculate these hybrid states [40,70,77].

Following the approach of Ref. [75], the potential for hybrid mesons in our model is given by

$$V_{\text{hyb}}(r) = V_{\text{OGE}}^C(r) + V_S^C(r) + [V_n(r) - \sigma(r)r]. \quad (\text{A24})$$

Here, $V_{\text{OGE}}^C(r)$ and $V_S^C(r)$ are the one-gluon exchange potential and the color-confining potential, respectively. $V_n(r)$ is the vibrational potential. The term $\sigma(r)r$ is subtracted to avoid double counting. This potential does not introduce new parameters beyond those of the original quark model, making the calculation of hybrid states parameter-free. Explicitly, the contributions are given by

$$V_{\text{OGE}}^C(r) = -\frac{4\alpha_s}{3r}, \quad (\text{A25})$$

$$V_S^C(r) = \sigma(r)r + c, \quad (\text{A26})$$

$$V_n(r) = \sigma(r)r \left\{ 1 - \frac{2n\pi}{2n\pi + \sigma(r)[(r-2d)^2 + 4d^2]} \right\}^{-1/2}, \quad (\text{A27})$$

where

$$\sigma(r) = \frac{b(1 - e^{-\mu r})}{\mu r}, \quad (\text{A28})$$

$$d(m_c, m_c, r, \sigma, n) = \frac{\sigma(r)r^2 \alpha_n}{4(m_c + m_c + \sigma(r)r \alpha_n)}, \quad (\text{A29})$$

in which d is the correction of the finite heavy quark mass. α_n is related to the shape of the vibrating string [74] and takes values in the range $1 \leq \alpha_n^2 \leq 2$. In this work, we take $\alpha_n^2 = 1.5$.

Like the basic quark model, the hybrid potential has a threshold defined by

$$V_{\text{hyb}}(r) \xrightarrow{r \rightarrow \infty} \frac{b}{\mu} + c. \quad (\text{A30})$$

c. Spin-nonflip $\pi\pi$ hadronic transitions

The rate of spin-nonflip $\pi\pi$ hadronic transition is determined by [76]

$$\begin{aligned} \Gamma(\Phi_I^{(2s+1)l_{I l_I}} \rightarrow \Phi_F^{(2s+1)l_{F l_F}} \pi\pi) &= \delta_{l_I l_F} \delta_{J_I J_F} \left(G|C_1|^2 - \frac{2}{3} H|C_2|^2 \right) \left| \sum_L (2L+1) \begin{pmatrix} l_I & 1 & L \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L & 1 & l_I \\ 0 & 0 & 0 \end{pmatrix} f_{IF}^L \right|^2 \\ &+ (2l_I+1)(2l_F+1)(2J_F+1) \sum_k (2k+1)(1+(-1)^k) \left\{ \begin{matrix} s & l_F & J_F \\ k & J_I & l_I \end{matrix} \right\}^2 H|C_2|^2 \\ &\times \left| \sum_L (2L+1) \begin{pmatrix} l_F & 1 & L \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L & 1 & l_I \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_I & L & 1 \\ 1 & k & l_F \end{pmatrix} f_{IF}^L \right|^2, \end{aligned} \quad (\text{A31})$$

with

$$\begin{aligned} f_{IF}^L &= \sum_K \frac{1}{M_I - M_{KL}} \left[\int dr r^3 R_F(r) R_{KL}(r) \right] \\ &\times \left[\int dr' r'^3 R_{KL}(r') R_I(r') \right], \end{aligned} \quad (\text{A32})$$

where the radial wave functions $R_I(r)$, $R_F(r)$, and $R_{KL}(r)$ correspond to the initial, final, and intermediate vibrational states, respectively. The variable M_I denotes the mass of the initial meson, while M_{KL} represents the mass of the intermediate vibrational state. The quantities G and H denote the phase-space integrals

$$\begin{aligned} G &= \frac{3}{4} \frac{M_F}{M_I} \pi^3 \int dM_{\pi\pi}^2 K \left(1 - \frac{4m_\pi^2}{M_{\pi\pi}^2} \right)^{1/2} (M_{\pi\pi}^2 - 2m_\pi^2)^2, \\ H &= \frac{1}{20} \frac{M_F}{M_I} \pi^3 \int dM_{\pi\pi}^2 K \left(1 - \frac{4m_\pi^2}{M_{\pi\pi}^2} \right)^{1/2} \\ &\times \left[(M_{\pi\pi}^2 - 4m_\pi^2)^2 \left(1 + \frac{2}{3} \frac{K^2}{M_{\pi\pi}^2} \right) \right. \\ &\left. + \frac{8K^4}{15M_{\pi\pi}^4} (M_{\pi\pi}^4 + 2m_\pi^2 M_{\pi\pi}^2 + 6m_\pi^4) \right], \end{aligned} \quad (\text{A33})$$

with K given by

$$K = \frac{\sqrt{[(M_I + M_F)^2 - M_{\pi\pi}^2][(M_I - M_F)^2 - M_{\pi\pi}^2]}}{2M_I}. \quad (\text{A34})$$

where $2m_\pi^2 \leq M_{\pi\pi}^2 \leq (M_I - M_F)^2$, m_π is the mass of the π meson, and M_I and M_F are the masses of the initial and final mesons, respectively.

References [70,76] present explicit formulas for calculating the decay widths of transitions among various spin-triplet states,

$$L_I = L_F = 0:$$

$$\Gamma(n_I^3 S_1 \rightarrow n_F^3 S_1 + \pi\pi) = |C_1|^2 G |f_{IF}^1|^2, \quad (\text{A35})$$

$$L_I = L_F = 1:$$

$$\Gamma(0 \rightarrow 0) = \frac{1}{9} |C_1|^2 G |f_{IF}^{011} + 2f_{IF}^2|^2, \quad (\text{A36})$$

$$\Gamma(0 \rightarrow 1) = \Gamma(1 \rightarrow 0) = 0, \quad (\text{A37})$$

$$\Gamma(0 \rightarrow 2) = 5\Gamma(2 \rightarrow 0) = \frac{10}{27} |C_2|^2 H \left| f_{IF}^0 + \frac{1}{5} f_{IF}^2 \right|^2, \quad (\text{A38})$$

$$\Gamma(1 \rightarrow 1) = \Gamma(0 \rightarrow 0) + \frac{1}{4} \Gamma(0 \rightarrow 2), \quad (\text{A39})$$

$$\Gamma(1 \rightarrow 2) = \frac{5}{3} \Gamma(2 \rightarrow 1) = \frac{3}{4} \Gamma(0 \rightarrow 2), \quad (\text{A40})$$

$$\Gamma(2 \rightarrow 2) = \Gamma(0 \rightarrow 0) + \frac{7}{20} \Gamma(0 \rightarrow 2). \quad (\text{A41})$$

$$L_I = 2, \quad L_F = 0:$$

$$\Gamma(n_I^3 D_1 \rightarrow n_F^3 S_1 + \pi\pi) = \frac{4}{15} |C_2|^2 H |f_{IF}^1|^2, \quad (\text{A42})$$

where $\Gamma(J_I \rightarrow J_F)$ represents the width of transitions $n^3 P_I \rightarrow n^3 P_F + \pi\pi$.

d. Spin-nonflip η hadronic transitions

For simplicity, we only consider the transitions

$$n^3 S_1 \rightarrow n^3 S_1 + \eta, \quad (\text{A43})$$

where the primary multipoles for spin-nonflip η transitions involving spin-triplet S -wave states are $M1M1$ and $E1M2$. Thus, the matrix element can be expressed as

$$\mathcal{M}(^3 S_1 \rightarrow ^3 S_1 + \eta) = \mathcal{M}_{M1M1} + \mathcal{M}_{E1M2}, \quad (\text{A44})$$

simplifying the expression and setting $\mathcal{M}_{M1M1} = 0$ as shown in Ref. [76], the decay rate is given by [77]

$$\Gamma(\Phi_I(^3 S_1) \rightarrow \Phi_F(^3 S_1) + \eta) = \frac{8\pi^2}{27} \frac{M_f C_3^2}{M_i m_Q^2} |f_{IF}^1|^2 |\vec{q}|^3, \quad (\text{A45})$$

where $\vec{q}$ denotes the momentum of η and f_{IF}^1 is defined in Eq. (A32).

Using equations in Ref. [76] and experimental values of hadronic transition widths, we get the required constants (in Table XII). C_1 , C_2 , and C_3 are fixed through the $\psi(2S) \rightarrow J/\psi(1S) + \pi\pi$, $\psi(1D) \rightarrow J/\psi(1S) + \pi\pi$, and $\psi(2S) \rightarrow J/\psi(1S) + \eta$ reactions, respectively. Therefore, we obtain a ratio of C_2/C_1 equal to 2.53; Refs. [70,76,78] give the reasonable range of C_2/C_1 as $1 \lesssim C_2/C_1 \lesssim 3$. Our result is also in the above range. Using C_3 , we can obtain the η transition widths of $n^3S_1 \rightarrow n^3S_1 + \eta$.

TABLE XII. The constants C_1 , C_2 , and C_3 . The third column is the experimental values of hadronic transition widths [4], and the fourth column is the parameters we obtained.

Initial state	Final state	$\Gamma_{\text{Expt. [4]}}$	Constant
$\psi(2S)$	$J/\psi(1S) + \pi\pi$	0.15	$C_1 = 0.0102$
$\psi(1D)$	$J/\psi(1S) + \pi\pi$	0.074	$C_2 = 0.0257$
$\psi(2S)$	$J/\psi(1S) + \eta$	9.87×10^{-3}	$C_3 = 0.000577$

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