

Parametrized spin-precessing inspiral-merger-ringdown waveform model for tests of general relativity

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The coalescence of binary black holes (BBHs) provides a unique arena to test general relativity (GR) in the dynamical, strong-field regime. To this end, we present pSEOBNRv5PHM, a parametrized, multipolar, spin-precessing waveform model for BBHs in quasicircular orbits, built within the effective-one-body formalism. Compared to its predecessor, pSEOBNRv4HM, our model introduces parametrized deviations from GR not only in the plunge-merger-ringdown stages, but also in the inspiral phase through modifications to the conservative dynamics. Additionally, it incorporates, for the first time, spin-precession effects. The free deviation parameters can be used to perform null tests of GR using current and future gravitational-wave observations. We validate pSEOBNRv5PHM through Bayesian parameter estimation, focusing on the quasinormal-mode frequency and damping time of the $(\ell, m, n) = (2, 2, 0)$ mode. Our analysis of synthetic signals from numerical-relativity (NR) simulations of highly precessing BH mergers shows that, while pSEOBNRv5PHM correctly recovers consistency with GR, neglecting spin precession can lead to false detections of deviations from GR even at current detector sensitivity. Conversely, when analyzing a synthetic signal from a NR simulation of a binary boson-star merger, the model successfully identifies a deviation from a GR BBH signal. Finally, we reanalyze 12 events from the third Gravitational Wave Transient Catalog. Using a hierarchical combination of these events, we constrain fractional deviations in the frequency and damping time of the $(2,2,0)$ quasinormal-mode to $\delta f_{220} = 0.00^{+0.06}_{-0.06}$ and $\delta \tau_{220} = 0.15^{+0.26}_{-0.24}$ at 90% credibility. These results are consistent with those from the LIGO-Virgo-KAGRA Collaboration, which did not account for spin-precession effects.

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I. INTRODUCTION

By the conclusion of the fourth observing run (O4) of the LIGO-Virgo-KAGRA (LVK) detectors [1–4], it is anticipated that more than 200 gravitational-wave (GW) events, predominantly binary black-hole (BBH) mergers, will have been observed [5,6]. The coalescence of BBH provides an unparalleled opportunity to test general relativity (GR) in the highly dynamical and strong-field regime [7–10], which is largely inaccessible to other experiments.

To probe GR through BBH events, both theory-specific [11–15] and theory-independent [16–27] frameworks have been developed, targeting all stages of binary coalescences:

the inspiral, merger, and ringdown. The ringdown phase, during which the perturbed remnant settles into a Kerr BH by emitting quasinormal modes (QNMs) with discrete frequencies and decay times determined solely by its mass and spin [28–32], provides a promising testing ground. Within GR, the “no-hair” conjecture [33–37] posits that an astrophysical, electrically neutral, BH is completely described by its mass and spin only. These two parameters uniquely determine the QNM frequencies. Therefore, the measurement of at least two QNMs allows to test the consistency between the estimates of mass and spin of the remnant object across multiple frequencies—this is the fundamental idea behind BH spectroscopy [9,10,22,24,25,38–53]. Any inconsistency between these estimate would potentially be indicative of a non-BH nature of the remnant object, or the incompleteness of GR as the underlying theory of gravity.

While the QNM frequencies and decay times depend solely on the remnant properties, the amplitude of each mode and the relative phases between them depend on the properties of the BHs in the binary and the binary dynamics. These quantities can be accurately extracted

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from numerical-relativity (NR) simulations [54–63]. Leveraging NR-informed amplitude parametrizations, while introducing additional assumptions about the nature of the coalescence, breaks degeneracies between remnant mass, spin, and GR-deviations in the QNM frequencies, that are otherwise present for a single QNM. This approach opens the possibility of measuring deviations from GR even with a single resolvable mode [10,22,25,64].

A consistent modeling of the merger-ringdown together with the inspiral takes full advantage of the entire signal power, and can further improve the ability to constrain deviations from GR during the ringdown, while also removing dependency on the predicted or estimated start time of the ringdown. This is the spirit of the parametrized SEOBNR (pSEOBNR) analysis, one of the flagship tests of GR performed within the LVK Collaboration [9,10]. This approach was first introduced in Refs. [22,25], that developed a parametrized model of the ringdown signal as part of the full inspiral-merger-ringdown (IMR) waveforms [65,66] in the effective-one-body (EOB) formalism [67,68], calibrated to NR simulations for spinning, nonprecessing BBHs. The model has been extended to parametrize the plunge-merger stages in Ref. [69], and to do theory-specific tests of GR in the ringdown in Ref. [12].

The pSEOBNR analysis has been applied to perform parametrized ringdown tests on 12 BBH GW signals observed by the LVK Collaboration, showing so far consistency with GR [9,10]. The signal-to-noise ratio (SNR) of the sources results in measurement errors for the frequency and decay time of the dominant QNM on the order of 10% and 20%, respectively, when combining events in a hierarchical way. Near-future upgrades to the LVK interferometers, and upcoming detectors on the ground, such as Einstein Telescope (ET) and Cosmic Explorer (CE) [70,71], and in space, such as LISA [72], will significantly increase the number of detected sources. Some events will be observed at SNR reaching thousands, enabling tests of GR with unprecedented precision. This increased sensitivity, however, poses a major challenge for waveform accuracy, as statistical uncertainties approach the systematic biases of the GW approximant models. Several studies predict severe biases in parameter estimation (PE) due to mismodeling errors with the upcoming fifth LVK observational run (O5) and next-generation detectors [73–76].

For tests of GR using the full IMR signal, such as the pSEOBNR approach, waveform systematics could lead to false indications of deviations from GR [69,77–80]. In Ref. [79], the pSEOBNR analysis was applied to massive BH binaries with LISA, showing that already for SNRs of ~ 100 , waveform mismodeling could erroneously indicate deviations from GR. The presence of biases depends strongly on the binary parameters. Merger-dominated signals, where the higher-order modes play a significant role, are especially sensitive to systematic errors [79]. For a

simulated stellar-mass BBH signal, detectable in O5 with an SNR of 75 and a large fraction of the SNR accumulated in the inspiral phase, the analysis found results consistent with GR [25]. Currently, the pSEOBNR analysis, as well as ringdown-only analyses based on similar templates [64], assume BBHs with spins aligned or antialigned with the orbital angular momentum of the binary (i.e., aligned spins). The analysis of the event GW200129_065458 in Ref. [69] reported a violation of GR in the merger amplitude of the waveform, while the QNM-deviation parameters remained consistent with GR predictions. This result was interpreted as a false violation of GR, originating either from waveform systematics [81] (mismodeling of spin precession that is absent in the baseline GR waveform model) or from data-quality issues [82]. Systematic biases in testing GR due to the absence or mismodeling of spin-precession effects have been highlighted in several other studies [83,84]. These results underscore the importance of improving waveform models and incorporating all relevant physical effects to perform reliable tests of GR.

In this work, we address two key advancements to improve the reliability and flexibility of the pSEOBNR tests of GR. First, we extend the pSEOBNR framework to spin-precessing binaries, by introducing a number of free parameters in the SEOBNRv5PHM model [85–88], a state-of-the-art multipolar, spin-precessing EOB waveform model for BBHs in quasicircular orbits. Additionally, we incorporate deviations across multiple stages of the coalescence. In the ringdown stage, we add fractional deviations to the frequency and damping time of the fundamental QNMs. For the plunge-merger stage, following Ref. [69], we add fractional deviations to the merger amplitude and frequency of the waveform, as well as the instant at which the amplitude peaks. Most notably, we introduce—for the first time—parametrized corrections in the inspiral stage by modifying the NR-calibration parameters of the model within the EOB Hamiltonian, which describes the conservative dynamics of the binary. This approach is similar in spirit to the TIGER framework [20,21], which also modifies phenomenological NR-calibration coefficients in the late inspiral stage of IMRPhenom models. Adding extra flexibility to our parametrized model allows for capturing more generic deviations from GR, which are generally expected to affect all stages of the coalescence. When suitable priors informed by NR uncertainties and fitting errors are employed, corrections to NR-calibration parameters can also be used to marginalize over waveform uncertainties [89–91]. We name the new parametrized waveform model pSEOBNRv5PHM.¹

¹pSEOBNRv5PHM is publicly available through the Python package pySEOBNR [92]. A tutorial notebook is available at: <https://waveforms.docs.ligo.org/software/pyseobnr>. Stable versions of pySEOBNR are published through the Python package index (PyPI), and can be installed via PIP `INSTALL pyseobnr`.

In this work, while we describe the implementation and morphology of all corrections, we validate our model through Bayesian PE for the ringdown of the $(\ell, m, n) = (2, 2, 0)$ mode, as it is the primary test currently performed in LVK analyses with pSEOBNR models. The $(2, 2, 0)$ mode is the dominant QNM in quasicircular mergers, except for highly precessing and asymmetric configurations [60, 93–95]. By recovering synthetic signals from NR simulations of highly precessing BH mergers [96], we demonstrate that pSEOBNRv5PHM correctly recovers consistency with GR. In contrast, neglecting spin-precession effects leads to false indications of deviations from GR, even at current detector sensitivity. Conversely, when analyzing a synthetic signal from a NR simulation of a binary boson-star merger [97], the model successfully identifies a deviation from a GR BBH signal.

Finally, we reanalyze 12 events from the third Gravitational-Wave Transient Catalog (GWTC-3) that were previously analyzed using an earlier version of the pSEOBNR model for aligned-spin binaries, pSEOBNRv4HM [25], by the LVK Collaboration [9, 10].

The paper is organized as follows. In Sec. II, we describe the construction of the pSEOBNRv5PHM model starting from the baseline SEOBNRv5PHM model, and introduce the non-GR parameters that describe potential deviations from GR during the different stages of the coalescence. In Sec. III, we conduct a detailed study of the morphology of the parametrized waveform to identify which parts of the waveform are affected when the non-GR parameters are varied individually. In Sec. IV, we perform synthetic-signal injection studies, using NR simulations of binary BHs and boson stars. We apply our parametrized model to real data in Sec. V, by reanalyzing 12 events from GWTC-3. Finally, we summarize our conclusions and future work in Sec. VI.

We use geometrical units $G = 1 = c$ unless stated otherwise.

II. THE PARAMETRIZED WAVEFORM MODEL

We start this section with a reminder of the construction of the multipolar spin-precessing SEOBNRv5PHM model [88]. We then describe, in Sec. II B, how we deform this baseline model by introducing deformations away from GR in the inspiral, merger, and ringdown stages, highlighting differences with respect to earlier iterations of the pSEOBNR models.

We consider a binary with masses m_1 and m_2 , with $m_1 \geq m_2$, and spins S_1 and S_2 . We define the following combinations of the masses

$$\begin{aligned} M &\equiv m_1 + m_2, & \mu &\equiv \frac{m_1 m_2}{M}, & \nu &\equiv \frac{\mu}{M}, \\ q &\equiv \frac{m_1}{m_2}, & \mathcal{M} &= \nu^{3/5} M. \end{aligned} \quad (2.1)$$

We also define the dimensionless spin vectors

$$\chi_i \equiv \frac{\mathbf{a}_i}{m_i} = \frac{\mathbf{S}_i}{m_i^2}, \quad (2.2)$$

the effective spin parameter χ_{eff} [98, 99],

$$\chi_{\text{eff}} = \frac{m_1 \chi_1^{\parallel} + m_2 \chi_2^{\parallel}}{m_1 + m_2}, \quad (2.3)$$

and the effective precessing-spin parameter χ_p [100],

$$\chi_p = \frac{1}{B_1 m_1^2} \max(B_1 m_1^2 \chi_1^{\perp}, B_2 m_2^2 \chi_2^{\perp}), \quad (2.4)$$

where $B_1 = 2 + 3m_2/(2m_1)$, $B_2 = 2 + 3m_1/(2m_2)$, and we have divided χ_i into its aligned-spin component ($\chi_i^{\parallel}$) and in-plane component ($\chi_i^{\perp}$).

A. Overview of the SEOBNRv5PHM waveform model

1. Effective-one-body dynamics of precessing-spin binary black holes

In the EOB formalism [67, 68, 101–103], the two-body conservative dynamics is described by an Hamiltonian H_{EOB} ,

$$H_{\text{EOB}} = M \sqrt{1 + 2\nu \left(\frac{H_{\text{eff}}}{\mu} - 1 \right)}, \quad (2.5)$$

where the effective Hamiltonian H_{eff} describes a test mass μ moving in a deformed Kerr spacetime of mass M , the deformation parameter being the symmetric mass ratio $0 \leq \nu \leq 1/4$.

A common strategy for building precessing-spin waveforms is to start from aligned-spin waveforms in the so-called *coprecessing frame*, in which the z -axis remains perpendicular to the instantaneous orbital plane, and then applying a suitable rotation to the inertial frame of the observer [104–108]. More specifically, SEOBNRv5PHM builds on previous studies that employed aligned-spin orbital dynamics in the coprecessing frame coupled to post-Newtonian (PN) expanded precessing-spin equations [109–111], to mitigate the computational expense of solving the equations of motion using the full precessing-spin EOB Hamiltonian [112–114].

To model precessional effects more accurately, SEOBNRv5PHM extends beyond this approach by using, in the coprecessing frame, an EOB Hamiltonian that includes partial precessional effects ($H_{\text{EOB}}^{\text{pprec}}$), in the form of orbit-averaged in-plane spin contributions for circular orbits [86]. The Hamiltonian $H_{\text{EOB}}^{\text{pprec}}$ reduces in the aligned-spin limit to the one used in the SEOBNRv5HM model [85]. This Hamiltonian features two higher-order PN terms (a_6 , d_{SO}) which are calibrated to 442 aligned-spin NR waveforms from the Simulating eXtreme Spacetimes (SXS)

Collaboration [96]. The parameter a_6 is a 5PN nonspinning coefficient and d_{SO} is a 4.5PN spin-orbit coefficient in H_{EOB} .

In the SEOBNRv5PHM model, the equations of motion in the coprecessing frame have the same form of the evolution equations for aligned-spin binaries, and read [88]:

$$\begin{aligned} \dot{r} &= \xi(r) \frac{\partial H_{\text{EOB}}^{\text{pprec}}}{\partial p_{r_*}}, & \dot{\phi} &= \frac{\partial H_{\text{EOB}}^{\text{pprec}}}{\partial p_\phi}, \\ \dot{p}_{r_*} &= -\xi(r) \frac{\partial H_{\text{EOB}}^{\text{pprec}}}{\partial r} + \mathcal{F}_r, & \dot{p}_\phi &= \mathcal{F}_\phi, \end{aligned} \quad (2.6)$$

where the dot indicates a time derivative. The evolution of the radial momentum is performed using the tortoise-coordinate $p_{r_*} = p_r \xi(r)$, with $\xi(r) = dr/dr_*$.

The radiation-reaction force is computed as [103]

$$\mathcal{F}_\phi = -\frac{\Phi_E}{\Omega}, \quad \mathcal{F}_r = \mathcal{F}_\phi \frac{p_r}{p_\phi}, \quad (2.7)$$

where $\Omega \equiv \dot{\phi}$ is the orbital frequency, and Φ_E is the energy flux radiated by the binary for quasicircular orbits. The energy flux is computed by summing over the contributions of the factorized PN modes (augmented with gravitational-self-force information [87]) $h_{\ell m}^{\text{F}}$ [115–118],

$$\Phi_E = \frac{\Omega^2}{16\pi} \sum_{\ell=2}^8 \sum_{m=-\ell}^{\ell} m^2 |d_L h_{\ell m}^{\text{F}}|^2, \quad (2.8)$$

where d_L is the luminosity distance from the binary to the observer.

For the evolution of the spins and angular momentum, SEOBNRv5PHM employs PN-expanded evolution equations that include higher-order PN information and use a spin-supplementary condition consistent with the Hamiltonian $H_{\text{EOB}}^{\text{pprec}}$ [86]. The spin and angular momentum evolution equations are used both for constructing the rotation between different reference frames during the inspiral, as summarized below, and for augmenting the coprecessing frame orbital dynamics, by using, into the Hamiltonian and aligned-spin modes, suitable projections of the spins onto the angular momentum at every time step of the evolution [88].

2. Inspiral-plunge waveforms

The gravitational polarizations can be written as

$$h_+ - ih_\times = \sum_{\ell, m} {}_{-2}Y_{\ell m}(\varphi, \iota) h_{\ell m}(t), \quad (2.9)$$

where ${}_{-2}Y_{\ell m}(\varphi, \iota)$ are the -2 spin-weighted spherical harmonics, with φ and ι being the azimuthal and polar angles to the observer. In the EOB framework [68], the GW modes in the coprecessing frame are decomposed as

$$\begin{aligned} h_{\ell m}(t) &= h_{\ell m}(t)^{\text{insp-plunge}} \theta(t_{\text{match}}^{\ell m} - t) \\ &+ h_{\ell m}(t)^{\text{merger-RD}} \theta(t - t_{\text{match}}^{\ell m}), \end{aligned} \quad (2.10)$$

where $\theta(t)$ is the Heaviside step function, $h_{\ell m}^{\text{insp-plunge}}$ corresponds to the inspiral-plunge part of the waveform, while $h_{\ell m}^{\text{merger-RD}}$ represents the merger-ringdown waveform. The matching time $t_{\text{match}}^{\ell m}$ is chosen to be the peak of the (2,2) mode amplitude for all (ℓ, m) modes except (5,5), for which it is taken as the peak of the (2,2) harmonic minus 10M [85].

The peak-time of the (2,2) mode (t_{peak}^{22}) is also calibrated to NR simulations. It is defined as

$$t_{\text{peak}}^{22} = t_{\text{ISCO}} + \Delta t_{\text{NR}}, \quad (2.11)$$

where t_{ISCO} is the time at which $r = r_{\text{ISCO}}$, and r_{ISCO} is the radius of the innermost-stable circular orbit (ISCO) [119] of a Kerr BH with the mass and spin of remnant [120,121]. The parameter Δt_{NR} is a free parameter calibrated to aligned-spin NR waveforms [85]. Note that this choice is different to the one employed in the SEOBNRv4 waveform models [66,114,122], which related the peak-time of the (2,2) mode to the peak of the orbital frequency, rather than the ISCO radius.

The inspiral-plunge modes in the coprecessing frame use the factorized, resummed expressions of the PN GW modes [115–118], with time-dependent projections of the spins, evaluated on the dynamics obtained from the EOB equations of motion [65,68]. Their accuracy during the plunge, when the radial motion dominates the dynamics, is further improved by applying the numerically tuned nonquasicircular (NQC) corrections $N_{\ell m}$ [115],

$$h_{\ell m}(t)^{\text{insp-plunge}} = h_{\ell m}^{\text{F}} N_{\ell m}, \quad (2.12)$$

which also allow for a smooth connection between the inspiral-plunge and merger-ringdown waveforms. Notably, the NQC corrections ensure that the amplitude of the EOB modes $|h_{\ell m}^{\text{insp-plunge}}|(t_{\text{match}}^{\ell m})$ and its first two derivatives, and the frequency of the EOB modes $\omega_{\ell m}^{\text{insp-plunge}}(t_{\text{match}}^{\ell m})$ and its first derivative, match those of the NR modes of at $t = t_{\text{match}}^{\ell m}$ (the so-called *input values*, $|h_{\ell m}^{\text{NR}}|$ and $\omega_{\ell m}^{\text{NR}}$). Parameter-space fits for the NR input values are provided in Appendix C of Ref. [85].

Following Ref. [85], SEOBNRv5PHM includes the following modes in the coprecessing frame,

$$(\ell, |m|) = \{(2, 2), (2, 1), (3, 3), (3, 2), (4, 4), (4, 3), (5, 5)\}. \quad (2.13)$$

Negative- m modes are derived from the positive- m ones using the reflection symmetry $h_{\ell m} = (-1)^\ell h_{\ell -m}^*$, which is

exact for aligned-spin binaries but not for precessing-spin binaries [123], even in the coprecessing frame.²

The GW polarizations in the inertial frame of the observer are those required for data-analysis applications. The SEOBNRv5PHM model makes use of three reference frames (see Fig. 1 of Ref. [88]):

- (1) The inertial frame of the observer (*source frame*). Quantities in this frame are denoted with the superscript I.
- (2) An inertial frame where the z -axis is aligned with the final angular momentum of the system ($\mathbf{J}_f$ -*frame*). Quantities in this frame are denoted with the superscript J. This frame facilitates the construction of the merger-ringdown modes. The QNMs are defined with respect to the direction of the final spin, and thus, the description of the ringdown signal as a linear combination of QNMs, is formally valid in this frame.
- (3) A noninertial frame which tracks the instantaneous motion of the orbital plane (the *coprecessing frame*). Quantities in this frame are denoted with the superscript CP.

The inertial-frame modes are related to the coprecessing-frame modes by a time-dependent rotation from the coprecessing frame to the $\mathbf{J}_f$ -frame, followed by a time-independent rotation from the $\mathbf{J}_f$ -frame to the final inertial frame

$$h_{\ell m}^I(t) = \sum_{m', m''} (\mathbf{R}^{J \rightarrow I})_{m, m'} (\mathbf{R}^{CP \rightarrow J})_{m', m''} h_{\ell m''}^{CP}(t), \quad (2.14)$$

where $\mathbf{R}^{X \rightarrow Y}$ denotes the rotation operator from the frame X to the frame Y , and m', m'' are summation indices over the modes available in the coprecessing frame. These rotations are implemented in pySEOBNR using quaternions, but can also be expressed in terms of Euler angles $\{\alpha(t), \beta(t), \gamma(t)\}$ [106]. Notably, spin precession induces mixing of modes with the same ℓ but different m , which can reorder the amplitudes of different modes as compared to the nonprecessing scenario [105, 125], leading to particularly large amplitudes for modes with $\ell = m \neq 2$ [60, 93, 95].

3. Merger-ringdown waveforms

After the merger, the EOB formalism models the transition to the ringdown stage using a phenomenological model [85, 126] based on the QNMs of the remnant BH. In SEOBNRv5PHM, the attachment of the merger-ringdown

waveform is performed in the coprecessing frame, using the merger-ringdown multipolar model developed for nonprecessing BBHs in Ref. [85].

For all harmonics, except for $(\ell, |m|) = (3, 2)$ and $(4, 3)$ which exhibit postmerger oscillations due to mode mixing [127, 128], the merger-ringdown waveform employs the following Ansatz [85, 126],

$$h_{\ell m}^{\text{merger-RD}}(t) = \nu \tilde{A}_{\ell m}(t) e^{i\tilde{\phi}_{\ell m}(t)} e^{-i\sigma_{\ell m 0}^{CP}(t - t_{\text{match}}^{\ell m})}, \quad (2.15)$$

where $\sigma_{\ell m 0}^{CP} = \sigma_{\ell m}^{R, CP} - i\sigma_{\ell m}^{I, CP}$ is the complex frequency of the least-damped QNM of the remnant BH, in the coprecessing frame. NR fits are employed to compute the final mass [120] and final spin [121] of the remnant. The real and imaginary parts of $\sigma_{\ell m 0}$, whether in the coprecessing or $\mathbf{J}_f$ -frame, are related to QNM oscillation frequency and damping time as follows:

$$f_{\ell m 0} = \frac{1}{2\pi} \text{Re}(\sigma_{\ell m 0}) \equiv \frac{1}{2\pi} \omega_{\ell m 0}^{\text{QNM}}, \quad (2.16a)$$

$$\tau_{\ell m 0} = -\frac{1}{\text{Im}(\sigma_{\ell m 0})}. \quad (2.16b)$$

The functions $\tilde{A}_{\ell m}$ and $\tilde{\phi}_{\ell m}$ in Eq. (2.15) are time-dependent amplitude and phase functions (see Sec. III of Ref. [85] for explicit expressions). Even though source-driven effects, overtones, and nonlinearities are not explicitly included in Eq. (2.15), these effects are effectively included in the merger-ringdown model, as the phenomenological *Ansätze* are calibrated against NR simulations.

To account for mode-mixing in the $(3, 2)$ and $(4, 3)$ modes, the same construction is applied to the corresponding *spheroidal harmonics* [129] $(3, 2, 0)$ and $(4, 3, 0)$, which maintain a monotonic amplitude and frequency evolution. The $(3, 2)$ and $(4, 3)$ spherical harmonics can be reconstructed by combining the $(3, 2, 0)$ and $(4, 3, 0)$ spheroidal harmonics with the $(2, 2)$ and $(3, 3)$ spherical harmonics, using appropriate mode-mixing coefficients [130].

The calculation of the waveform in the inertial observer's frame requires a description of the coprecessing frame Euler angles $\{\alpha(t), \beta(t), \gamma(t)\}$ which extends beyond merger. SEOBNRv5PHM makes use of a phenomenological prescription based on insights from NR simulations [131]. Specifically, it has been shown that the coprecessing frame continues to precess approximately around the direction of the final angular momentum with a precession frequency, ω_{prec} , proportional to the difference between the lowest overtone of the $(2, 2, 0)$ and $(2, 1, 0)$ QNM frequencies. This phenomenology leads to the following expressions for the merger-ringdown angles relating the $\mathbf{J}_f$ -frame and the coprecessing frame used in SEOBNRv5PHM,

$$\begin{aligned} \alpha^{\text{merger-RD}} &= \alpha(t_{\text{match}}) + \omega_{\text{prec}}(t - t_{\text{match}}), \\ \beta^{\text{merger-RD}} &= \beta(t_{\text{match}}), \\ \gamma^{\text{merger-RD}} &= \gamma(t_{\text{match}}) - \omega_{\text{prec}}(t - t_{\text{match}}) \cos \beta(t_{\text{match}}), \end{aligned} \quad (2.17)$$

²In parallel to this work, a NR-calibrated model for the antisymmetric mode contributions in the coprecessing frame was developed for SEOBNRv5PHM [124]. While incorporating these modes into the parametrized model is straightforward, we do not include them here, as the version of pSEOBNRv5PHM reviewed within the LVK Collaboration—intended for use in the O4a analyses—does not yet include these modes.

where $t_{\text{match}} = t_{\text{peak}}^{22}$ is the matching time of the merger-ringdown model. This rotation prescription provides an accurate approximation if the (2,0,0) QNM amplitude in the $\mathbf{J}_f$ -frame is negligible. However, for highly precessing binaries the (2,0,0) QNM can be strongly excited, and this assumption may be a source of systematics [60].

The behavior studied in Ref. [131] describes prograde-spin configurations, where the remnant spin is positively aligned with the orbital angular momentum at merger. Following Ref. [114], SEOBNRv5PHM extends the prescription to retrograde-spin cases by imposing simple precession around the final spin at a rate $\omega_{\text{prec}} \geq 0$

$$\omega_{\text{prec}} = \begin{cases} \omega_{220}^{\text{QNM},J}(\chi_f) - \omega_{210}^{\text{QNM},J}(\chi_f) & \chi_f \cdot \mathbf{L}_f > 0 \\ \omega_{210}^{\text{QNM},J}(\chi_f) - \omega_{220}^{\text{QNM},J}(\chi_f) & \chi_f \cdot \mathbf{L}_f < 0 \end{cases}, \quad (2.18)$$

that depends on whether the total angular momentum at merger $\chi_f \propto \mathbf{J}_f$ is aligned or not with the orbital angular momentum at merger $\mathbf{L}_f$. Here χ_f is a *signed* final spin with magnitude $|\chi_f|$, and the same sign of $\chi_f \cdot \mathbf{L}_f$. This prescription of the postmerger extension of the Euler angles in the *retrograde case* ($\chi_f \cdot \mathbf{L}_f < 0$) is significantly less tested than in the prograde case due to the limited availability of NR simulations covering the relevant region of parameter space—most notably high mass-ratio binaries—which also includes systems with transitional precession [132].

As mentioned earlier, the QNM frequencies obtained from BH perturbation theory are formally valid in the $\mathbf{J}_f$ -frame. Following recent insights from NR [133], SEOBNRv5PHM computes the coprecessing frame QNM frequencies from the ones in the $\mathbf{J}_f$ -frame as [see Refs. [88], Eq. (22) and [133], Eq. (35)],

$$\omega_{\ell m 0}^{\text{QNM,CP}} = \omega_{\ell m 0}^{\text{QNM},J} - m(1 - |\cos \beta(t_{\text{match}})|)\omega_{\text{prec}}. \quad (2.19)$$

B. Construction of the parametrized waveform model

The pSEOBNRv5PHM model introduces fractional deviation parameters to:

- (1) the frequency and damping time of the fundamental QNM frequencies,
 - (2) the instant at which the GW amplitude peaks, the instantaneous GW frequency at this time instant, and the value of the peak amplitude, and
 - (3) two calibration parameters in the EOB Hamiltonian.
- Items 1 and 2 can be used independently for each mode of the model in Eq. (2.13). This yields a total of 31 free parameters on top of the GR parameters. Specifically: there are 7×2 parameters related to the QNMs; 7×2 parameters related to the instantaneous GW frequency and amplitude; 1 parameter associated with the instant the GW amplitude peaks; and 2 EOB calibration parameters.

We introduce non-GR deformations to the QNMs, following the same strategy used in Refs. [22,25,69]. We perform the substitutions

$$f_{\ell m 0}^J \rightarrow f_{\ell m 0}^J(1 + \delta f_{\ell m 0}), \quad (2.20a)$$

$$\tau_{\ell m 0}^J \rightarrow \tau_{\ell m 0}^J(1 + \delta \tau_{\ell m 0}), \quad (2.20b)$$

where we impose that $\delta \tau_{\ell m 0} > -1$ to ensure that the remnant BH is stable; it rings down, instead of “ringing-up” exponentially. In the following, we will always refer to frequencies in the $\mathbf{J}_f$ -frame, and drop the superscript J to lighten the notation.

If the binary is spin-aligned, the foregoing discussion is sufficient to describe the parametrizations of the merger-ringdown part of the model. However, if the binary is spin-precessing, we have to care about the different reference frames used to model such systems.

In pSEOBNRv5PHM, we can add QNM deviations to the coprecessing frame waveform modes (2.15), and to the rotation angles that relate the $\mathbf{J}_f$ -frame and coprecessing frame after merger (2.17). As mentioned earlier, the QNM frequencies obtained from BH perturbation theory are formally valid in the $\mathbf{J}_f$ -frame. Therefore, it is most natural to add parametrized deviations to the frequencies in this frame. We see that by deforming the QNM frequencies in the $\mathbf{J}_f$ -frame, we also change the effective precession rate ω_{prec} (2.18), which gives an additional source of deviation from GR for the QNM frequencies in the coprecessing frame (2.19). Moreover, ω_{prec} also appears in the phenomenological equations for the Euler angles after t_{match} that relate the waveform modes at coprecessing and $\mathbf{J}_f$ -frames.

At the transition between prograde-spin and retrograde-spin configurations, a small discontinuity present in the rotations in the GR limit of the model is amplified when including nonzero QNM deviations: the same deviation in δf_{220} , or in δf_{210} , changes ω_{prec} in opposite directions depending on the sign of $\chi_f \cdot \mathbf{L}_f$ [see Eq. (2.18)]. This highlights the need to revisit the prescription for retrograde-spin configurations as more NR simulations covering the relevant parameter space become available [134]. Here, to safely avoid this problem, we introduce a boolean parameter `omega_prec_deviation` that propagates (if `True`) or not (if `False`) the QNM modifications to ω_{prec} . For PE applications, we choose to include QNM deviations in the rotations only if all posterior samples from a corresponding GR run are in a prograde-spin configuration. For posteriors that are entirely in a negative-spin configuration, no discontinuity arises; however, we still prefer not to include QNM deviations in the rotations since the prescription has not been extensively validated against NR simulations. We also note that such configurations, characterized by high mass ratios and negative spins, are uncommon among the observed events.

A notable difference with respect to the aligned-spin case comes from mixing of modes with the same ℓ and different m due to the rotation (2.14): when spin precession is present, even adding deviations to the coprecessing frame modes only, leads to corrections in a single QNM

propagating to different modes with the same ℓ in the inertial frame. Another source of mode-mixing—this time present already in the aligned-spin model—is the spherical-spheroidal mode mixing in the (3,2) and (4,3) modes. In this case, corrections from the (2,2) and (3,3) modes propagate to the (3,2) and (4,3) modes in a consistent way.

Plunge-merger deviations are included similarly as in Ref. [69]. We introduce fractional deviations to the NR-informed input values for the mode amplitudes and frequencies at $t = t_{\text{match}}^{\ell m}$, i.e.,

$$|h_{\ell m}^{\text{NR}}| \rightarrow |h_{\ell m}^{\text{NR}}|(1 + \delta A_{\ell m}), \quad (2.21a)$$

$$\omega_{\ell m}^{\text{NR}} \rightarrow \omega_{\ell m}^{\text{NR}}(1 + \delta \omega_{\ell m}), \quad (2.21b)$$

which are imposed via the NQC corrections. The factorized waveform modes and NQC corrections are directly used to compute coprecessing frame waveform during the inspiral-plunge phase (2.12). Consequently, the fractional deviations are applied to the amplitude and frequency of the modes in the coprecessing frame.

We also allow for changes to $t_{\text{match}}^{\ell m}$ by modifying the time-shift parameter Δt_{NR} , defined in Eq. (2.11) as,

$$\Delta t_{\text{NR}} \rightarrow \Delta t_{\text{NR}} - \delta \Delta t. \quad (2.22)$$

Unlike Ref. [69], we introduce additive (rather than fractional) deviations to the peak time. This choice is motivated by the different meaning of this parameter in the SEOBNRv4 and SEOBNRv5 models. In SEOBNRv4 the time-shift parameter related to the peak of the (2,2) mode to the peak of the orbital frequency, and had a constant sign across parameter space (i.e., the peak of the (2,2) mode always occurs earlier than to peak of Ω). In SEOBNRv5, however, the time-shift parameter relates the peak of the (2,2) mode to the ISCO radius of the remnant Kerr BH, and can take both positive and negative values (i.e., the merger-ringdown attachment time is either before or after the ISCO), including zero, making fractional deviation ill defined in such cases.

Inspirational deviations modify EOB calibration parameters in the Hamiltonian, a_6 and d_{SO} , as follows:

$$a_6 \rightarrow a_6 + \delta a_6, \quad (2.23a)$$

$$d_{\text{SO}} \rightarrow d_{\text{SO}} + \delta d_{\text{SO}}. \quad (2.23b)$$

Also in this case we apply parametrized corrections as additive deviations, since a_6 and d_{SO} can take on both positive and negative values across the parameter space. Unlike previous corrections, which only affected the waveform modes without impacting the binary's orbital dynamics, these deviations modify the orbital dynamics in the coprecessing frame, and consistently propagate to all the waveform modes.

III. MORPHOLOGY OF THE PARAMETRIZED WAVEFORM

Having introduced our waveform model, we now discuss how each of the parametrized deviations modifies the GW signal in GR, varying the parameters one at a time. We follow analogous explorations performed for aligned-spin binaries in Ref. [25] for the ringdown stage, and in Ref. [69] for the plunge-merger stage.

As an illustrative example, we consider a spin-precessing quasicircular binary system with the following mass ratio and spins:

$$q = 2.0, \quad \text{and} \quad \chi_1 = \chi_2 = [0.5, 0, 0.5], \quad (3.1)$$

where the spins are defined at a dimensionless orbital frequency $M\Omega = 0.015$, corresponding to a frequency of 20 Hz for a binary with total mass of approximately $50M_\odot$. We examine the impact of the deviations on both the rotation angles and the waveform in the coprecessing and inertial frames, focusing for clarity only on the (2,2) mode.

Let us start with the QNM deviation parameters $\delta f_{\ell m 0}$ and $\delta \tau_{\ell m 0}$. Figure 1 illustrates the impact of QNM frequency deviations $\delta f_{\ell m 0}$ on the postmerger extension of the Euler angles α (top rows) and γ (bottom rows), as described by Eq. (2.17). In the left panel, corrections are applied only to f_{220} , while in the right panel, corrections are applied only to f_{210} .

In all panels, the colored envelopes represent the range of deviations obtained by finely sampling the interval $\delta f_{\ell m 0} \in [-1.0, 1.0]$ in increments of 0.04, and the black curves correspond to the GR angles ($\delta f_{\ell m 0} = 0$) for the same binary parameters. The time $t = 0$ corresponds to the peak of the coprecessing (2,2) mode amplitude, which marks the attachment point of the merger-ringdown waveform.

The binary parameters in Eq. (3.1) describe a prograde-spin configuration ($\chi_f \cdot \mathbf{L}_f > 0$). The morphology of the angles aligns with the expected behavior from Eqs. (2.18) and (2.17): a positive (negative) value of δf_{220} increases (decreases) ω_{prec} , which subsequently increases (decreases) the slope of α and decreases (increases) the slope of γ after the attachment time. Conversely, for δf_{210} , the opposite behavior is observed.

We now examine the impact of the QNM deviations on the waveform. The top-left panel of Fig. 2 focuses on the frequency deviations δf_{220} , while the top-right panel focuses on the damping time deviations $\delta \tau_{220}$. For the frequency deviations, the top row shows the instantaneous GW frequency in the inertial frame ($M\omega_{22}^{\text{I}}$), while the bottom row shows the frequency in the coprecessing frame ($M\omega_{22}^{\text{CP}}$). For the damping time deviations, we instead examine the waveform amplitude in both inertial ($|h_{22}^{\text{I}}|$) and coprecessing ($|h_{22}^{\text{CP}}|$) frames. The colored envelopes represent the range of deviations obtained by finely

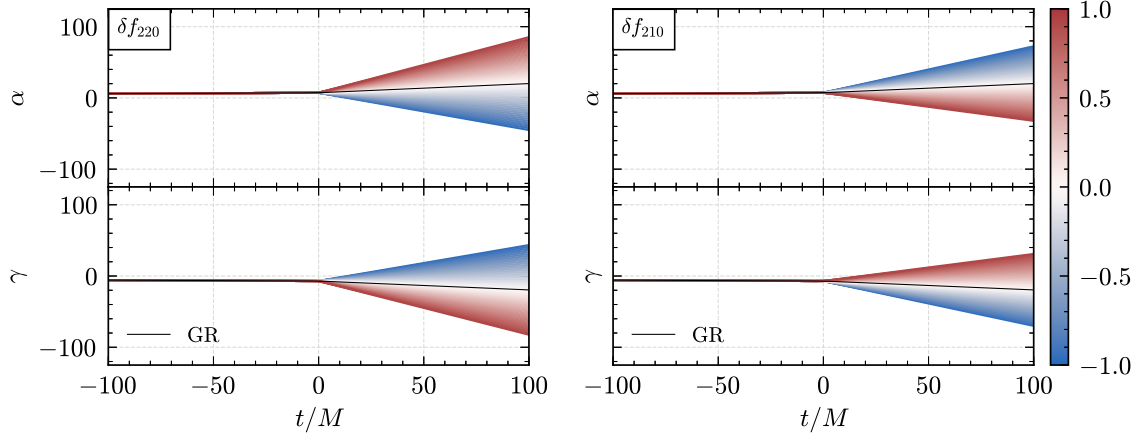


FIG. 1. Time evolution near the merger of the α and γ Euler angles for nonzero values of the QNM frequency deviations δf_{220} (left panel) and δf_{210} (right panel). The GR predictions ($\delta f_{\ell m 0} = 0$) are shown by the black curves.

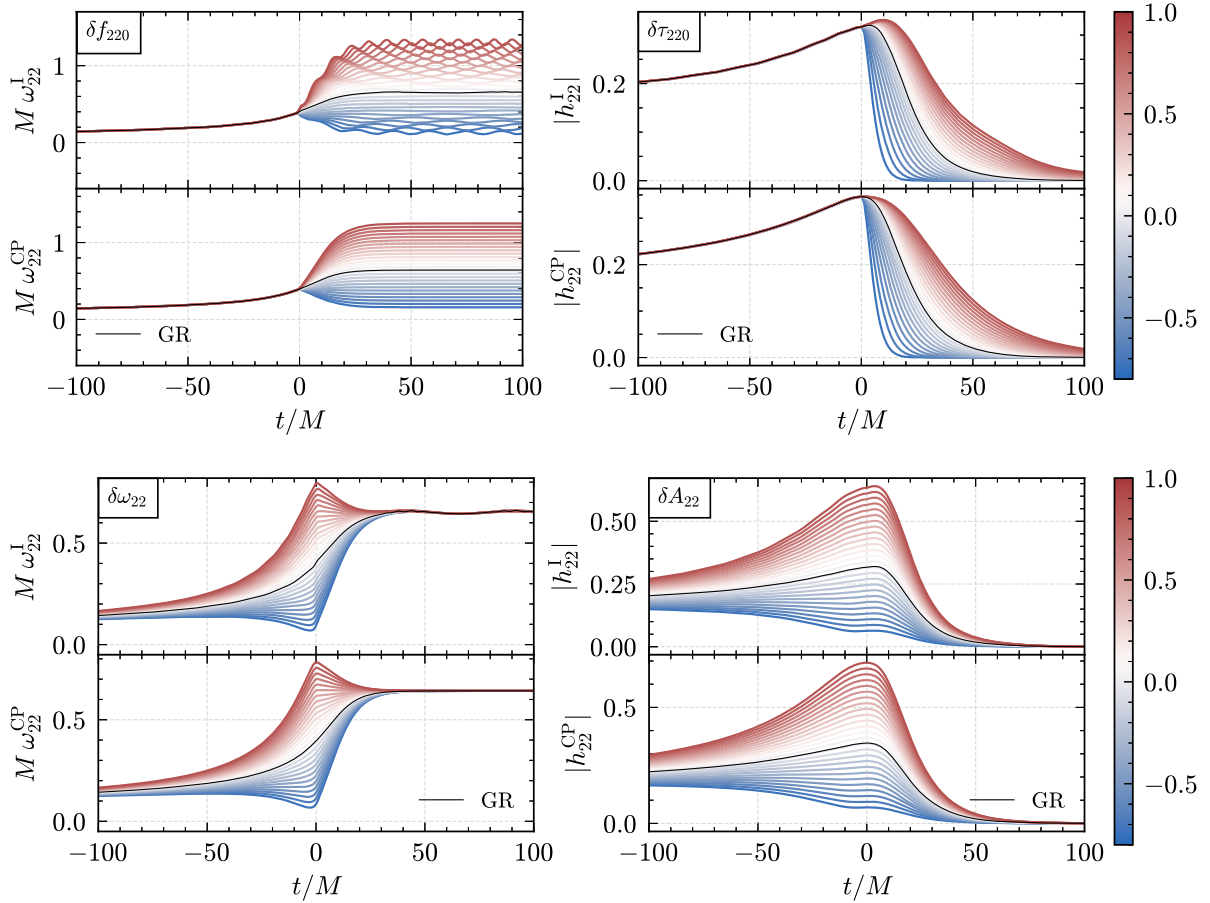


FIG. 2. Impact of QNM and merger deviations on the waveform morphology. The top-left panel shows the effect of frequency deviations δf_{220} on the instantaneous GW frequency in the inertial frame ($M\omega_{22}^I$) and the coprecessing frame ($M\omega_{22}^{CP}$). The top-right panel illustrates the effect of damping time deviations $\delta \tau_{220}$ on the waveform amplitude in both frames ($|h_{22}^I|$ and $|h_{22}^{CP}|$). The bottom-left and bottom-right panels explore the influence of merger frequency deviations $\delta \omega_{22}$ and merger amplitude deviations δA_{22} , respectively, on the GW frequency and amplitude in the inertial and coprecessing frames. Colored envelopes correspond to deviations sampled within the intervals $(\delta f_{220}, \delta \tau_{220}) \in [-0.8, 1.0]$ and $(\delta A_{22}, \delta \omega_{22}) \in [-0.8, 1.0]$, while black curves represent the GR prediction. The time $t = 0$ is defined as the peak amplitude of the coprecessing (2,2) mode. For precessing binaries, mode mixing is evident in the inertial frame, while the coprecessing frame exhibits a morphology consistent with aligned-spin binaries.

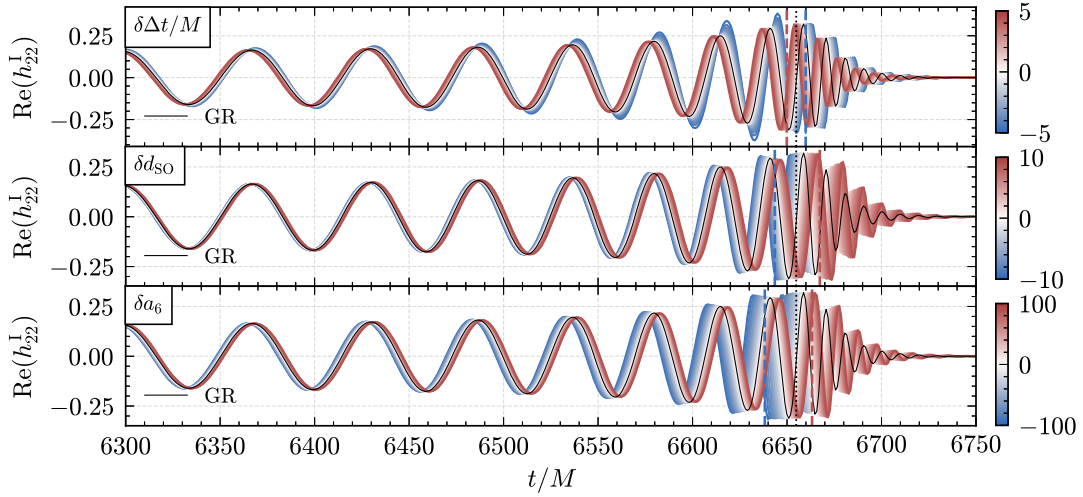


FIG. 3. Impact of inspiral-plunge deviations on the waveform morphology. The top panel shows the effect of the time-shift correction $\delta\Delta t$, while the middle and bottom panel show, respectively, the effect of the δd_{SO} and δa_6 corrections to calibration parameters in the EOB Hamiltonian. Colored envelopes represent deviations sampled within the intervals $\delta\Delta t \in [-5, 5]M$, $\delta d_{\text{SO}} \in [-10, 10]$, and $\delta a_6 \in [-100, 100]$, with the colored dashed lines denoting the merger-ringdown attachment time at the extremal values of the deviation parameters. Black curves correspond to the GR predictions, with the vertical dotted line marking the corresponding merger-ringdown attachment time.

sampling the interval $(\delta f_{220}, \delta \tau_{220}) \in [-0.8, 1.0]$ using 26 evenly spaced values, with the black curves corresponding to the GR predictions. The time $t = 0$ corresponds to the peak amplitude of the coprecessing (2,2) mode. For precessing binaries, modulations due to mode mixing between modes with the same ℓ in the coprecessing frame are evident in the inertial frame, particularly in the frequency. In the coprecessing frame, however, the morphology is analogous to that observed for aligned-spin binaries in previous studies [25]. Overall, the parametrized model provides smooth deviations from GR, even in the inertial frame. As with aligned-spin binaries, QNM corrections affect the waveform only after the merger time, which corresponds to the peak of the coprecessing (2,2) mode.

Turning to the merger deviations $(\delta A_{22}, \delta \omega_{22})$, the bottom-left panel of Fig. 2 examines the merger frequency deviation $\delta \omega_{22}$, while the bottom-right panel focuses on the merger amplitude deviation δA_{22} . For merger frequency deviations, we plot the instantaneous GW frequency, while for merger amplitude deviations, we analyze the waveform amplitude. In both cases, the plots include data from both the inertial and coprecessing frames. The deviation parameters are varied within the interval $(\delta A_{22}, \delta \omega_{22}) \in [-0.8, 1.0]$ using 26 evenly spaced values. In both inertial and coprecessing frames, the waveform phenomenology is consistent with that of aligned-spin binaries [69]. We note that for positive δA_{22} the peak amplitude increases while maintaining a monotonic amplitude evolution. Conversely, for negative δA_{22} , the amplitude decreases, resulting in a local minimum bordered by two maxima located before and after the minimum. Lastly, it is important to note that, although these parameters

are referred to as “merger parameters,” they also affect the late inspiral-plunge phase of the waveform (i.e., before the attachment time), with the NQC corrections ensuring a smooth transition across these phases.

Finally, Fig. 3 explores the impact of inspiral-plunge deviation parameters $(\delta\Delta t, \delta d_{\text{SO}}, \delta a_6)$ on the waveform morphology. In this figure, we present the real part of the (2,2) mode in the inertial frame, $\text{Re}(h_{22}^I)$. These corrections primarily affect the time to merger and the late-inspiral phasing of the binary. To better highlight their effects, the waveforms are aligned in the early inspiral, rather than setting the peak of the coprecessing (2,2) mode at $t = 0$. Colored envelopes represent deviations sampled within the intervals $\delta\Delta t \in [-5, 5]M$, $\delta d_{\text{SO}} \in [-10, 10]$, and $\delta a_6 \in [-100, 100]$, using 26 evenly spaced values each. The colored dashed lines denote the merger-ringdown attachment time at the extremal values of the deviation parameters. Black curves correspond to the GR predictions, with the vertical dotted line marking the corresponding merger-ringdown attachment time.

The top panel shows the effect of the time-shift correction $\delta\Delta t$. Unlike Ref. [69], we treat $\delta\Delta t$ as an additive, rather than fractional, deviation, allowing for a broader range of waveform morphologies. The parameter $\delta\Delta t$ directly modifies the time at which the merger-ringdown waveform is attached (2.11), affecting the binary’s time to merger. Negative (positive) values shift the merger to occur later (earlier), as dictated by the negative sign in Eq. (2.22). While $\delta\Delta t$ does not alter the inspiral dynamics, it still impacts the inspiral-plunge amplitude and phasing of the waveform via NQC corrections applied at the shifted attachment time. These corrections enforce the

NR-calibrated amplitude and frequency at the attachment time, but their influence on the waveform before that point is nontrivial and depends sensitively on the time at which they are applied. For example, we see that negative values of $\delta\Delta t$ may produce a maximum in amplitude before the attachment time. The middle and bottom panels illustrate the effects of δd_{SO} and δa_6 respectively. Both parameters, as high-order PN corrections to the EOB Hamiltonian, exhibit qualitatively similar impacts on the phasing of the waveform during the late inspiral. Specifically, positive (negative) values delay (advance) the merger in time. The prefactor of d_{SO} depends on the effective spin parameter χ_{eff} whereas a_6 is spin-independent. Consequently, the relative deviation with respect to GR caused by these corrections varies across the parameter space. The ranges explored are indicative of possible PE priors, with the broader range of δa_6 compared to δd_{SO} reflecting results from NR calibration and corresponding uncertainties [85,89].

IV. PARAMETER ESTIMATION ON SYNTHETIC SIGNALS

In this section, we summarize the Bayesian inference formalism used for PE of GW signals and synthetic-data studies. Our PE studies focus on the QNM deviation parameters of the $(\ell, m, n) = (2, 2, 0)$ mode, as it is the primary test currently performed in LVK analyses. In future work, we aim to extend this analysis to the higher modes in the ringdown, as well as the measurability of deviations in the inspiral and plunge-merger stages.

The goal of Bayesian PE is to infer the posterior distribution $p(\theta|d)$ for the parameters θ given the observed data d , using Bayes theorem

$$P(\theta|d) = \frac{\mathcal{L}(d|\theta)\pi(\theta)}{Z}, \quad (4.1)$$

where $\mathcal{L}(d|\theta)$ is the likelihood of the data d given the parameters θ , $\pi(\theta)$ is the prior on θ , and $Z \equiv \int d\theta \mathcal{L}(d|\theta)\pi(\theta)$ is the evidence [135]. To determine whether a model A is preferred over a model B , one can compute the Bayes factor, defined as the ratio of the evidence for the two different models $\mathcal{B}_B^A = Z_A/Z_B$.

We simulate and analyze signals using the Bilby package [136,137], and the nested sampler DYNESTY [138] using the acceptance-walk stepping method. We adopt sampler settings consistent with the latest LVK analyses [139] by using a number of accepted MCMC-chains $n_{\text{accept}} = 60$, number of live points $n_{\text{live}} = 1000$, while keeping the remaining sampling parameters to their default values. The pSEOBNRv5PHM waveforms are generated using Bilby TGR [140] and pySEOBNR [92], interfaced through GWSIGNAL. When analyzing simulated signals, we consider a three-detector (LIGO Hanford, LIGO Livingston and Virgo) network configuration, and use the LIGO power spectral

density (PSD) at O4 sensitivity [141] and Virgo PSD at advanced Virgo sensitivity [6]. The noise curves are named aLIGO_O4_high and AdV in Bilby.

We use standard priors for the GR parameters following Refs. [4,88]. Specifically, we sample the masses using the chirp mass ($\mathcal{M}$) and inverse mass ratio ($1/q$), with priors uniform in component masses. The priors on the dimensionless spin vectors are uniform in magnitude $a_i \in [0, 0.99]$, and isotropically distributed in the unit sphere for the spin directions. For the distance, we employ a prior uniform in the comoving-frame of the source, except in Sec. IV A where we use a prior uniform in distance to match the settings of the analysis in Ref. [88] for the same simulated signal. For the QNM deviation parameters, we use uniform priors in the ranges

$$\delta f_{220} \in [-0.8, 2.0] \quad \text{and} \quad \delta \tau_{220} \in [-0.8, 2.0]. \quad (4.2)$$

In cases exhibiting railing, we extend the prior range to $[-0.8, 4.0]$. The remaining priors are set according to Appendix C of Ref. [4].

A. Injection of a BBH signal in GR

In this section, we assess the importance of including spin-precession effects when performing tests of GR, by analyzing a synthetic NR signal of a BBH in GR. The injected signal corresponds to the NR waveform SXS: BBH:0165 from the public SXS catalog [96], with mass ratio $q = 6$, detector-frame total mass $M = 95M_{\odot}$ and BH's dimensionless spin vectors defined at 20 Hz of $\chi_1 = [-0.06, 0.78, -0.4]$ and $\chi_2 = [0.08, -0.17, -0.23]$. Notably, this BBH system exhibits strong spin precession, with a high mass ratio and a significant effective precessing-spin of $\chi_p \sim 0.78$. It stands out as one of the most challenging systems to model in the public SXS catalog [96]. We take the inclination with respect to the line of sight to be $\iota = \pi/2$ rad. The coalescence and polarization phases are $\phi = 1.2$ rad and $\psi = 0.7$ rad, respectively. The sky-position is defined by its right ascension of 0.33 rad and declination of -0.6 rad at a geocentric time of 1249852257 s. The luminosity distance to the source is chosen to be 1200 Mpc, which produces a network SNR of 18.1.

For this configuration, PE under the assumption of GR, using the SEOBNRv5PHM waveform model, yields posterior samples in both prograde-spin and retrograde-spin configurations. As discussed in Sec. II B, at the transition between these configurations, corrections to δf_{220} introduce a discontinuity in ω_{prec} . To address this, we exclude corrections to the ω_{prec} term in such cases. Nevertheless, we analyze the signal under both approaches to evaluate the impact of this choice.

Figure 4 presents the posterior probability distributions for the fractional deviations in the frequency and damping time of the $(2,2,0)$ QNM (δf_{220} and $\delta \tau_{220}$) for this synthetic

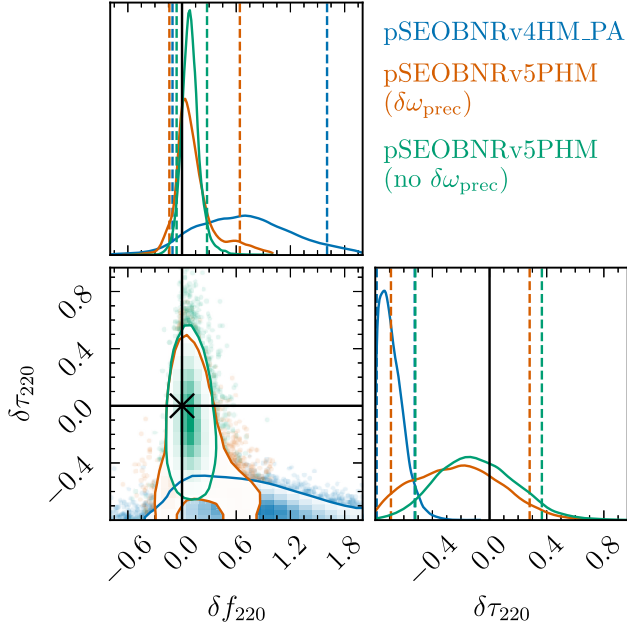


FIG. 4. Posterior probability distributions for the fractional deviations in the frequency and damping time of the (2,2,0) QNM (δf_{220} and $\delta \tau_{220}$), for a synthetic signal of a highly precessing BBH NR waveform from the public SXS catalog SXS:BBH:0165. The PE is performed with the pSEOBNRv4HM_PA model (blue) and with the pSEOBNRv5PHM model, including (orange) or excluding (green) the QNM corrections in the computation of the precession rate ω_{prec} (2.18). The 2D contours mark the 90% credible regions, while the dashed lines on the 1D marginalized distributions mark the 90% credible levels. The black vertical and horizontal lines mark the GR predictions ($\delta f_{220} = \delta \tau_{220} = 0$).

signal. We perform three parameter recoveries: with the pSEOBNRv4HM_PA model [69,142] (blue), and with the pSEOBNRv5PHM model, with (orange) and without (green) including QNM corrections in the computation of the precession rate ω_{prec} (2.18). The 2D contours mark the 90% credible regions, while the dashed lines on the 1D marginalized distributions mark the 90% credible levels. The black vertical and horizontal lines mark the GR predictions ($\delta f_{220} = \delta \tau_{220} = 0$).

First, we observe that the pSEOBNRv4HM_PA model, which does not include spin-precession effects, recovers a significant bias in the damping time away from the GR prediction. While some GR parameters (e.g., the masses) are still well-recovered, some parameters such as the binary's inclination are significantly biased. As highlighted in previous studies [69,84], waveform systematics, due to the absence of spin-precession effects, can lead to false indications of deviations from GR. On the other hand, at current detector sensitivity, the pSEOBNRv5PHM model demonstrates sufficient accuracy to recover GR predictions reliably, even for challenging binary configurations. This is true both with and without including QNM corrections in

ω_{prec} , although posterior distributions in the latter case appear more sensible, consistently with the avoidance of a discontinuous limit. This choice has no significant impact on other parameters, which are accurately recovered in both configurations of pSEOBNRv5PHM.

The natural log Bayes factor between pSEOBNRv5PHM without and with QNM corrections in ω_{prec} is $\ln \mathcal{B} \simeq 0.2$, which is comparable with the estimated error in the Bayes factor, indicating no significant preference over either model. On the other hand, the natural log Bayes factor between pSEOBNRv5PHM without QNM corrections in ω_{prec} and pSEOBNRv4HM is $\ln \mathcal{B} \simeq 5.7$ indicating a significant preference for the pSEOBNRv5PHM model. Finally, the natural log Bayes factor between pSEOBNRv5PHM without QNM corrections in ω_{prec} and SEOBNRv5PHM in GR is $\ln \mathcal{B} \simeq -1.5$, indicating overall preference for the GR hypothesis.

In spin-precessing binaries, the relative amplitudes of different modes can be reordered compared to the non-precessing scenario. In certain regions of the parameter space, the (2,2,0) mode may become subdominant, while modes with $\ell = m \neq 2$ can exhibit large amplitudes [60,93–95]. This observation motivates the investigation of whether the (2,1,0) mode can also be effectively constrained. For the SXS:BBH:0165 NR injection, we perform a recovery that allows for deviations in both the (2,2,0) and (2,1,0) QNMs. The results indicate that the posterior distribution for the (2,1,0) mode is only marginally informative: the damping time excludes large positive deviations, while the frequency remains compatible with the entire prior range, suggesting that, for this specific NR simulation, the (2,1,0) mode remains subdominant relative to the (2,2,0) mode.

While more detailed studies on model accuracy for future detectors remain to be conducted, we anticipate that improvements to the baseline GR model will be necessary for robust applications to LISA and next-generation ground-based detectors. This expectation aligns with similar findings for aligned-spin systems, especially for signals with SNRs reaching 100 or higher [79].

B. Injection of a beyond-GR signal

In this section, we study whether our model can identify a signal that does not originate from a BBH in GR.

In our first example, we simulate a signal using the pSEOBNRv5PHM model with nonzero QNM deviation parameters, $\delta f_{220} = \delta \tau_{220} = 0.5$. The simulated binary is a mass-asymmetric BBH with moderate spin-precession, with parameters $q = 4$, $(1+z)M = 75M_{\odot}$, $\chi_{\text{eff}} \simeq 0.15$, $\chi_p \simeq 0.6$, and a network SNR of 19.1. Figure 5 shows the posterior probability distributions for the fractional deviations in the frequency and damping time of the (2,2,0) QNM, recovered with pSEOBNRv5PHM. The 2D contours mark the 90% credible regions, while the dashed lines on the 1D marginalized distributions mark the 90% credible

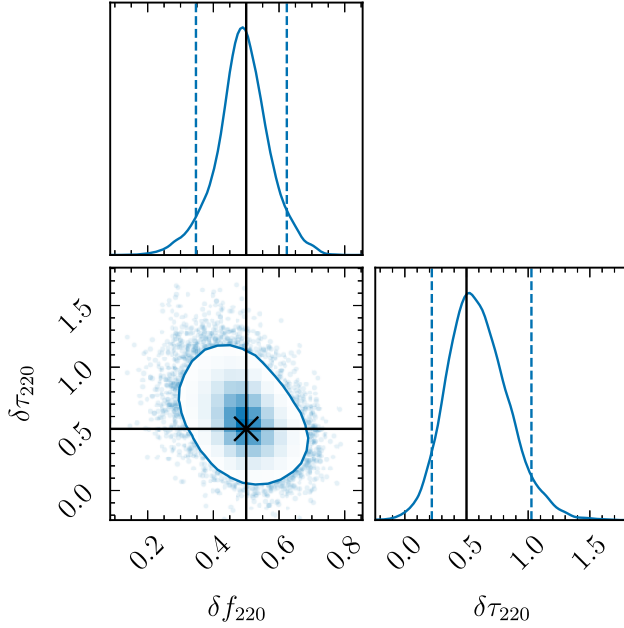


FIG. 5. Posterior probability distributions for the fractional deviations in the frequency and damping time of the $(2,2,0)$ QNM (δf_{220} and $\delta \tau_{220}$), for a synthetic signal modeled using pSEOBNRv5PHM with nonzero values for the QNM deviation parameters, and recovered with the same model. The 2D contours mark the 90% credible regions, while the dashed lines on the 1D marginalized distributions mark the 90% credible intervals. The black vertical and horizontal lines mark the injected values ($\delta f_{220} = \delta \tau_{220} = 0.5$).

levels. The black vertical and horizontal lines mark the injected values ($\delta f_{220} = \delta \tau_{220} = 0.5$), which are accurately recovered. The GR parameters are also well estimated. In contrast, recovering the same signal with the GR model SEOBNRv5PHM leads to biased intrinsic parameter estimates. In this case, the analysis incorrectly favors equal masses and a negative effective spin $\chi_{\text{eff}} \simeq -0.4$, with the injected values falling outside the 90% credible intervals. The natural log Bayes factor between pSEOBNRv5PHM and SEOBNRv5PHM is $\ln \mathcal{B} \simeq 15.7$, indicating strong evidence for a deviation from GR.

C. Injection of a binary boson-star signal

As a second example, we consider a synthetic signal from a publicly available NR simulation of a scalar-field solitonic boson star (BS) [143] merger, produced with the GRChombo code [144,145] and described in Ref. [97], where several high-precision, IMR waveforms spanning approximately 20 orbits were presented for equal-mass, quasicircular, nonspinning BS binaries of different compactness.

Boson stars are modeled by a complex scalar field, which can be decomposed into amplitude A and frequency ω , as $\varphi(t, r) = A(r)e^{i(\epsilon\omega t + \delta\phi)}$. Following Ref. [97] we introduce the parameter $\epsilon = \pm 1$ determining the rotation of the scalar field in the complex plane, and a phase offset $\delta\phi$. The

primary BS always has $\epsilon = 1$, $\delta\phi = 0$, and we consider the configuration with secondary parameters $\epsilon = 1$ and $\delta\phi = \pi$ (*antiphase*). The simulation we consider has central amplitude $A(0) = 0.17$, which gives rise to highly compact BSs with compactness $\mathcal{C} = 0.2$ and dimensionless tidal deformability $\Lambda \sim 10$. The coalescence results in the formation of a BH with final spin $\chi_f \simeq 0.7$, similar to the remnant of a nonspinning equal-mass BBH merger.

The injection setup matches the one for NR injections of BBHs, but we include only the $\ell = 2$ modes in both injection and recovery, since they are the only ones contained in the NR data. We take the same total mass and extrinsic parameters as for the SXS:BBH:0165 NR injection, except for the inclination angle which we take $\iota = \pi/3$ rad to give an optimal SNR of 31.5.

The left panel of Fig. 6 shows the posterior probability distributions for the fractional deviations in the frequency and damping time of the $(2,2,0)$ QNM (δf_{220} , $\delta \tau_{220}$) for the synthetic binary BS signal recovered with pSEOBNRv5PHM. The prediction for a BBH in GR ($\delta f_{220} = \delta \tau_{220} = 0$) is excluded at the 90% credible level, demonstrating that pSEOBNRv5PHM successfully identifies a deviation from the signal of a BBH in GR. Specifically, while δf_{220} remains consistent with zero, the posterior for $\delta \tau_{220}$ shows a preference for negative values, indicating a more strongly damped ringdown signal compared to what would be predicted from the component masses and spins of a BBH merger remnant that matches the inspiral signal.

When the BS signal is analyzed under the assumption of a BBH in GR, the reconstruction of the ringdown can be biased, because the inferred component masses and spins are themselves biased, and the final mass and spin are computed assuming a remnant from a BBH merger. For the simulation considered, the bias in the component masses and spins is the larger source of error, as the mass and spin of the remnant BH are nearly identical to those of a BBH merger remnant. The biases in the final mass and spin lead to incorrect estimates of the ringdown frequency and damping time. By allowing for corrections to the QNM frequencies, the pSEOBNR model is partially able to mitigate these biases and provide a better fit to the BS simulation. This is confirmed by the right panel of Fig. 6, which shows the detector-frame mass and spin of the remnant BH, estimated with the pSEOBNRv5PHM and SEOBNRv5PHM models. For pSEOBNRv5PHM, the remnant properties are estimated from the complex QNM frequencies by inverting the fitting formula provided in Ref. [40]. The SEOBNRv5PHM estimate is not compatible with the true values, indicated by the black vertical and horizontal lines. The inclusion of a damping time deviation in the pSEOBNRv5PHM model shifts the posterior toward the true values, particularly for the final spin, which are now recovered within the 90% credible region.

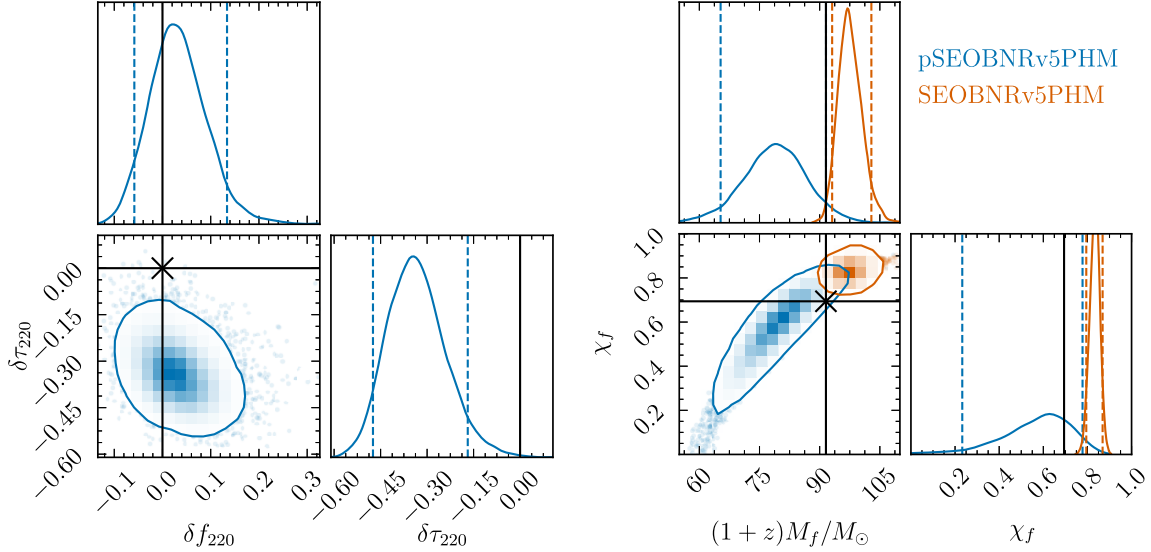


FIG. 6. Left panel: posterior probability distributions for the fractional deviations in the frequency and damping time of the (2,2,0) QNM (δf_{220} and $\delta \tau_{220}$), for a synthetic signal of a binary BS merger recovered with pSEOBNRv5PHM. The 2D contours mark the 90% credible regions, while the dashed lines on the 1D marginalized distributions mark the 90% credible intervals. The black vertical and horizontal lines mark the predictions for a BBH in GR ($\delta f_{220} = \delta \tau_{220} = 0$). Right panel: posterior probability distributions for the detector-frame mass and spin of the remnant BH, estimated with the pSEOBNRv5PHM and SEOBNRv5PHM models. For pSEOBNRv5PHM, the remnant properties are estimated from the complex QNM frequencies by inverting the fitting formula provided in Ref. [40]. The black vertical and horizontal lines mark the true values.

Note that the pSEOBNR model can infer that the signal does not originate from a BBH coalescence, even if the remnant is a BH in GR. In contrast, ringdown analyses limited to the postmerger stage might not reveal such discrepancies unless the ringdown results are compared to those from a complete IMR analysis based on the BBH assumption. In that scenario, the deviation could manifest either as a discrepancy between the recovered frequency and damping time and those predicted by the IMR analysis (when using an agnostic damped-sinusoid model) or as a mismatch between the IMR-inferred remnant mass and spin and those obtained from the ringdown stage (when employing a ringdown model that assumes a Kerr remnant).

The Bayes factor against the null hypothesis $\delta f_{220} = \delta \tau_{220} = 0$ can be estimated using the Savage-Dickey density ratio [146] without requiring an additional SEOBNRv5PHM run

$$\mathcal{B} = \frac{\pi(\delta f_{220} = \delta \tau_{220} = 0)}{P(\delta f_{220} = \delta \tau_{220} = 0|d)}. \quad (4.3)$$

For uniform priors in the range $[-0.8, 2.0]$, for both δf_{220} and $\delta \tau_{220}$, Eq. (4.3) yields a natural log Bayes factor of $\ln \mathcal{B} = 0.86$. It is important to note that the exact value of the Bayes factor depends on the choice of prior for the deviation parameters, which is somewhat arbitrary. For instance, using priors uniform in $[-0.8, 1.0]$ instead, the natural log Bayes factor becomes $\ln \mathcal{B} = 1.74$. These results indicate a slight preference for a deviation from

the null hypothesis, despite the inclusion of additional parameters.

We also observe biases in the GR parameters consistent with results reported in Ref. [97]. Specifically, the posteriors show significant support away from equal masses, where the true value $q = 1$ is excluded at the 90% credible level, leading to an overestimation of the primary mass. The spin magnitudes exhibit substantial support for high values ($a_1, a_2 > 0.5$), with partial alignment to the orbital angular momentum ($\chi_{\text{eff}} \simeq 0.5$). These biases in the mass ratio and effective spin allow the model to better reproduce the shallow chirp in the BS signal during the inspiral phase. Additionally, the luminosity distance is biased toward larger values.

V. PARAMETER ESTIMATION ON REAL DATA

In this section, we apply our model to real data by reanalyzing 12 events from GWTC-3, which were originally analyzed using the pSEOBNRv4HM model [25] in Ref. [10].

For the pSEOBNR test, a degeneracy exists between the fundamental ringdown frequency deviation parameter and the remnant mass [25], mostly for low-SNR events with negligible higher modes and for which only the postmerger is detectable. To mitigate this, a selection criterion is applied, requiring that both the inspiral and postinspiral regimes achieve $\text{SNR} > 8$, a criterion met by 12 binary systems from GWTC-3 that also satisfy the other selection criteria for tests of GR (detection in multiple

TABLE I. The median and symmetric 90% credible intervals of the one-dimensional marginalized posteriors of the fractional deviations in the frequency and damping time of the (2,2,0) QNM, $(\delta f_{220}, \delta \tau_{220})$, and of the remnant properties. The third and fourth columns list the frequency and damping time of the (2,2,0) QNM, as measured using the pSEOBNRv5 PHM model. The last two columns report the mass and spin of the remnant object, estimated from the complex QNM frequencies by inverting the fitting formula provided in Ref. [40].

Event	δf_{220}	$\delta \tau_{220}$	f_{220} (Hz)	τ_{220} (ms)	$(1+z)M_f/M_\odot$	χ_f
GW150914	$0.02^{+0.09}_{-0.07}$	$0.12^{+0.33}_{-0.27}$	$240.5^{+25.1}_{-25.6}$	$4.48^{+1.29}_{-1.06}$	$72.9^{+12.3}_{-12.8}$	$0.72^{+0.13}_{-0.28}$
GW170104	$-0.02^{+0.14}_{-0.13}$	$0.43^{+1.01}_{-0.66}$	$296.6^{+58.9}_{-54.5}$	$5.04^{+3.76}_{-2.37}$	$69.9^{+16.2}_{-20.6}$	$0.87^{+0.09}_{-0.42}$
GW190519_153544	$-0.14^{+0.20}_{-0.13}$	$0.17^{+0.48}_{-0.35}$	$120.4^{+17.2}_{-18.4}$	$8.36^{+3.96}_{-2.65}$	$140.7^{+34.8}_{-31.2}$	$0.67^{+0.20}_{-0.45}$
GW190521_074359	$0.06^{+0.17}_{-0.10}$	$-0.03^{+0.35}_{-0.26}$	$198.5^{+30.6}_{-28.9}$	$5.41^{+1.82}_{-1.41}$	$88.4^{+14.9}_{-16.6}$	$0.72^{+0.16}_{-0.37}$
GW190630_185205	$-0.06^{+0.12}_{-0.16}$	$0.00^{+0.56}_{-0.45}$	$247.6^{+34.9}_{-44.2}$	$3.96^{+2.32}_{-1.76}$	$69.4^{+16.5}_{-17.9}$	$0.70^{+0.20}_{-0.54}$
GW190828_063405	$0.11^{+0.11}_{-0.12}$	$0.18^{+0.52}_{-0.45}$	$226.7^{+40.1}_{-41.3}$	$6.18^{+2.67}_{-2.40}$	$88.4^{+15.6}_{-20.1}$	$0.85^{+0.09}_{-0.37}$
GW190910_112807	$0.01^{+0.11}_{-0.09}$	$0.60^{+0.63}_{-0.47}$	$175.0^{+23.7}_{-20.1}$	$9.45^{+3.48}_{-2.67}$	$122.6^{+18.1}_{-18.6}$	$0.90^{+0.05}_{-0.12}$
GW191109_010717	$1.31^{+0.65}_{-1.26}$	$-0.06^{+0.82}_{-0.53}$	$162.7^{+97.5}_{-80.3}$	$13.67^{+16.67}_{-10.46}$	$147.0^{+118.6}_{-72.3}$	$0.94^{+0.04}_{-0.39}$
GW200129_065458	$-0.01^{+0.06}_{-0.07}$	$0.18^{+0.42}_{-0.29}$	$259.4^{+30.0}_{-23.0}$	$5.30^{+1.97}_{-1.35}$	$76.5^{+11.0}_{-10.9}$	$0.85^{+0.08}_{-0.19}$
GW200208_130117	$0.25^{+1.65}_{-0.35}$	$-0.07^{+1.10}_{-0.43}$	$215.0^{+131.8}_{-56.9}$	$5.06^{+10.90}_{-2.33}$	$80.9^{+32.8}_{-25.6}$	$0.76^{+0.23}_{-0.56}$
GW200224_222234	$0.01^{+0.15}_{-0.11}$	$0.22^{+0.46}_{-0.33}$	$206.2^{+25.4}_{-18.4}$	$7.07^{+2.76}_{-1.94}$	$98.9^{+13.0}_{-15.2}$	$0.87^{+0.08}_{-0.17}$
GW200311_115853	$0.01^{+0.15}_{-0.07}$	$0.29^{+1.57}_{-0.54}$	$256.2^{+32.3}_{-24.3}$	$5.99^{+6.78}_{-2.55}$	$81.6^{+21.9}_{-21.4}$	$0.88^{+0.09}_{-0.35}$

interferometers and false-alarm rates $< 10^{-3} \text{ yr}^{-1}$). The final list of events used for this analysis is provided in Table I. We employ strain data from the Gravitational Wave Open Source Catalog (GWOSC) [147] and the released PSD and calibration envelopes.

As in previous studies [8–10], we present results combining information from multiple events, as this allows to place stronger constraints on the deviation parameters. Assuming that the fractional deviations $(\delta f_{220}, \delta \tau_{220})$ are constant across events, joint constraints can be obtained by multiplying the individual-event posteriors (given a flat prior on the deviation parameters) [8,148]. However, in most non-GR theories, the deviations parameters $(\delta f_{220}, \delta \tau_{220})$ are expected to vary depending on the source’s properties. Relaxing the assumption of constant deviations across all events requires a hierarchical inference framework, as originally proposed in Refs. [148,149]. This technique assumes that the non-GR parameters $(\delta f_{220}, \delta \tau_{220})$ are drawn from a common underlying distribution, whose properties are inferred from the population of events. Following Refs. [9,10,148,149] we model the population distribution with a Gaussian $\mathcal{N}(\mu, \sigma)$ of unknown mean μ and standard deviation σ (the *hyperparameters*). The goal is then to infer a posterior distribution $P(\mu, \sigma | \{d_j\})$ for μ and σ from a joint analysis of a set of events $\{d_j\}$, $j = 1, \dots, N$. If GR is correct, this posterior should be consistent with $\mu = 0$ and $\sigma = 0$. To perform this analysis, we use the STAN-based code [150] developed in Ref. [149] and employed in Refs. [9,10,25]. This code allows us to infer $P(\mu, \sigma | \{d_j\})$ across a set of events. From a posterior on the hyperparameters one can also infer population distributions for the original deviation parameters $(\delta f_{220}, \delta \tau_{220})$, by marginalizing over μ and σ . However, as originally developed in Ref. [149], this implementation is defined only for 1D

posteriors. Therefore, we restrict our presentation below to 1D hierarchical analyses (see Ref. [151] for a recent extension to multidimensional cases). Incorporating information about the underlying astrophysical population, such as simultaneously inferring the astrophysical population in the hierarchical analysis, would be important to mitigate the impact of unphysical prior assumptions on astrophysical parameters [152], which can impact non-GR deviations due to parameter correlations. However, we leave such an extension for future work, and use the same setup as Refs. [10,25].

The events GW191109_010717 and GW200208_130117 were not included in the computation of the combined bounds (hierarchical or joint posterior) in Ref. [10], as the posteriors on δf_{220} show multimodalities likely due to possibility of noise systematics not accounted for. We performed single-event analyses also for these events, finding consistent results with Ref. [10]. Therefore, we also do not include them in the combined results.

The results of the analysis are summarized in Fig. 7, which is based on Fig. 14 of Ref. [10]. The left panel of Fig. 7 shows the 2D posteriors (along with the marginalized 1D posteriors) of the frequency and damping time deviations for all the events listed in Table I. The contours are colored by the median detector-frame total mass $(1+z)M$ of the corresponding binary. We specifically highlight the posteriors from two events, GW150914 and GW200129_065458, which are among the loudest detected so far and provide strongest single-event bounds. The combined constraints are reported both by multiplying individual posteriors and by hierarchically combining events. In the right panel of Fig. 7 we also provide a summary of the 90% credible intervals on the 1D

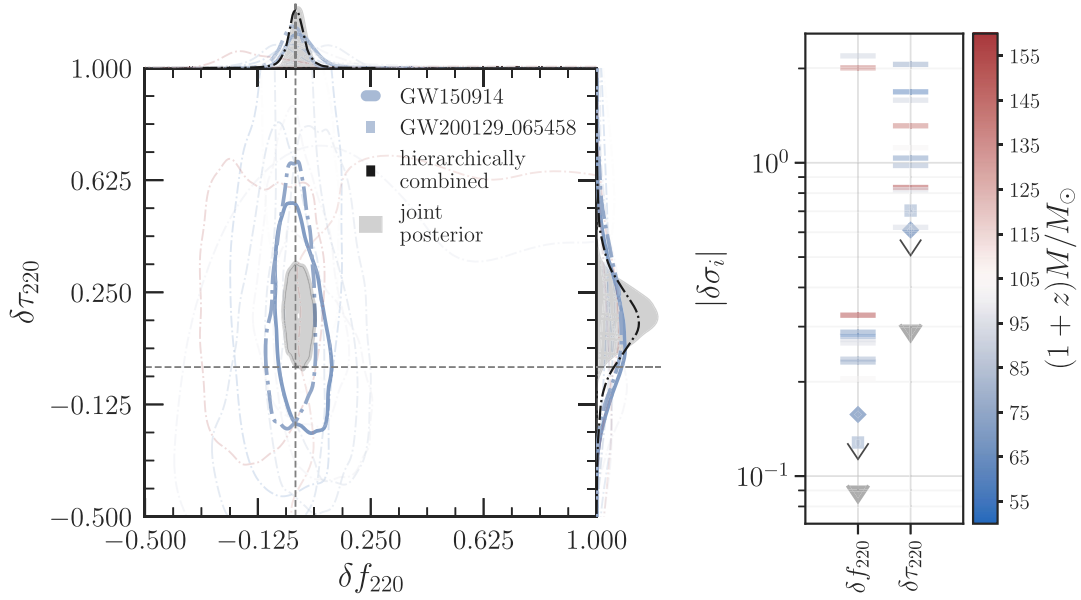


FIG. 7. Left panel: the 90% credible regions of the posterior probability distribution of the fractional deviations in the frequency and damping time of the (2,2,0) QNM, δf_{220} and $\delta \tau_{220}$, and their corresponding one-dimensional marginalized posterior distributions, for events from GWTC-3 passing a SNR threshold of 8 in both the inspiral and postinspiral signal. Posteriors for GW150914 and GW200129_065458 are separately shown. The filled gray contours denote the 90% credible regions on the joint constraints for $(\delta f_{220}, \delta \tau_{220})$ obtained by multiplying the individual event posteriors (given a flat prior), while the hierarchical method of combination yields the black dot dashed curves only shown in the 1D marginalized posteriors. The dashed gray lines mark the GR prediction $(\delta f_{220}, \delta \tau_{220}) = (0, 0)$. Right panel: 90% credible interval on the one-dimensional marginalized posteriors on $\delta \sigma_i = (\delta f_{220}, \delta \tau_{220})$, colored by the median detector-frame total mass $(1+z)M$, inferred assuming GR. Filled gray (unfilled black) triangles mark the constraints obtained when all the events are combined by multiplying likelihoods (hierarchically). The bounds from GW200129_065458 (square) and GW150914 (diamond) are indicated by the separate markers.

marginalized posteriors, color coded by the median detector-frame mass of the binary.

The results for GW150914 are broadly consistent with those reported in Ref. [10], with a Bayes factor between pSEOBNRv5PHM and pSEOBNRv4HM $\ln \mathcal{B} \simeq 0.6$.³ This is expected, as GW150914 is consistent with originating from a nonspinning binary, making the impact of waveform systematics subdominant [154]. In contrast, the posterior for GW200129_065458 differs more noticeably from the one in Ref. [10]. This event exhibits evidence of spin precession under the assumption of a binary in a quasicircular orbit [81]. However, uncertainties in glitch subtraction could affect the evidence for spin precession [82,155], and an alternative interpretation as an aligned-spin eccentric binary has been proposed [156].

³The original results with the pSEOBNRv4HM model from Ref. [10] were produced using the LALInference code [153] and do not report an estimate of the Bayesian evidence. For GW200129_065458 and GW150914 the Bayes factors are estimated from reruns using Bilby and DYNESTY, employing identical settings and priors as the pSEOBNR5PHM model (except for the spins, which are aligned), but using the pSEOBNRv4HM_PA model.

Compared to the result of Ref. [10] for GW200129_065458, we note that the posterior of $\delta \tau_{220}$ exhibits a larger tail towards positive values. This shift in $\delta \tau_{220}$ is driven by a slightly different recovery of the binary's luminosity distance and inclination angle when spin precession is included in the analysis. When analyzing the event with pSEOBNRv5HM under the aligned-spin assumption, we find results consistent with Ref. [10], indicating that the difference arises solely from the inclusion of spin precession. Overall, the pSEOBNRv5PHM model provides a significantly better fit to the data compared to pSEOBNRv4HM, with a natural log Bayes factor of $\ln \mathcal{B} \simeq 5.1$, favoring a spin-precessing quasicircular hypothesis over an aligned-spin quasicircular one. Under the assumption of a spin-precessing quasicircular binary, the impact of waveform systematics for this event remains non-negligible. A notable feature absent in pSEOBNRv5PHM is the inclusion of multipole asymmetries in the coprecessing frame, which are important for capturing evidence of spin precession in this event [157]. It would be valuable to reanalyze this event using future versions of pSEOBNRv5PHM that incorporate such asymmetries, as well as using a parametrized version of the recently developed aligned-spin, eccentric SEOBNRv5EHM

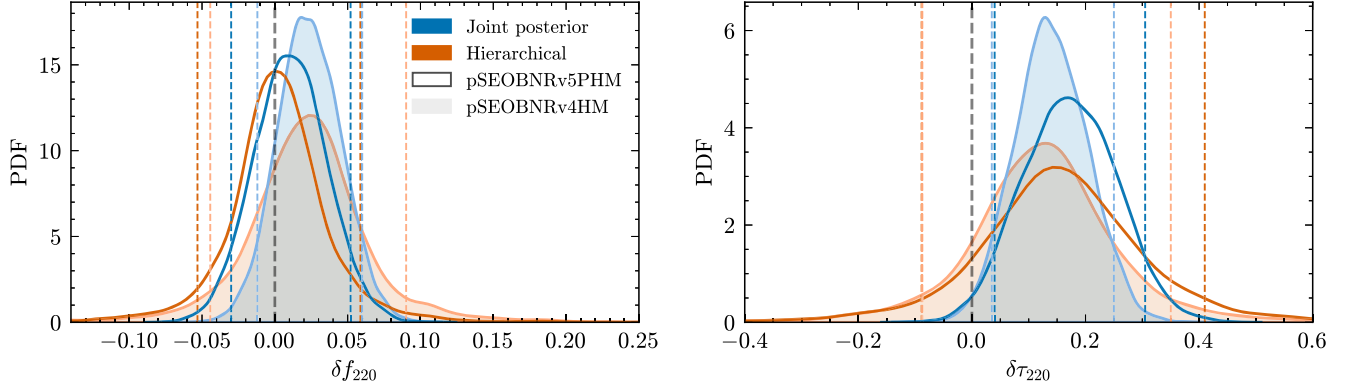


FIG. 8. The one-dimensional combined constraints on δf_{220} and $\delta \tau_{220}$, obtained by multiplying the posteriors from individual events (in blue), and using hierarchical combination (in orange), from GWTC-3 events. We compare results using the pSEOBNRv5PHM model (dark, unfilled curves) and the pSEOBNRv4HM model (light, filled curves). Vertical dashed lines represent the 90% credible intervals.

model [158,159], to further explore the impact of waveform assumptions on the results. We leave these studies for future work.

The posterior probability distributions for the fractional deviations in the frequency and damping time of the (2,2,0) QNM (δf_{220} and $\delta \tau_{220}$) for all 10 events from GWTC-3 used to produce combined results are reported in Appendix A. We specifically highlight the differences between the pSEOBNRv5PHM and pSEOBNRv4HM analyses. Overall, the two models produce broadly consistent results across all events. For several events, tighter constraints are achieved on δf_{220} with the pSEOBNRv5PHM model, particularly for GW190519_153544, which no longer shows a secondary mode in the posterior distribution for δf_{220} that was present in the pSEOBNRv4HM results. Results for $\delta \tau_{220}$ are also largely consistent, but a tail toward large $\delta \tau_{220}$ values is seen for GW200129_065458, as previously noted, and is even more pronounced for GW200311_115853.

Also for GW200311_115853, this tail is driven by a different recovery of the binary's distance and inclination angle when spin precession is included in the analysis, while using pSEOBNRv5HM under the aligned-spin assumption gives results consistent with Ref. [10]. The posteriors for the distance and inclination recovered using pSEOBNRv5HM are also in agreement with those obtained using SEOBNRv5PHM assuming GR. The maximum likelihood point recovered by pSEOBNRv5PHM corresponds to a positive $\delta \tau_{220}$, indicating a genuine correlation that provides a good fit to the data. This is confirmed not to be a sampling issue, as additional runs with more stringent sampler settings (nlive = 2000, maxmcmc = 10000) yield consistent results. Furthermore, the natural log Bayes factor of $\ln \mathcal{B} \simeq 1.7$ favors pSEOBNRv5PHM under the spin-precessing hypothesis over the aligned-spin scenario. We further investigate correlations between χ_p , d_L , ι , and QNM deviation parameters in Appendix B.

The combined bounds on the fractional deviations in the frequency and damping time of the (2,2,0) QNM using the pSEOBNRv5PHM model read

$$\delta f_{220} = 0.01^{+0.04}_{-0.04} \quad \text{and} \quad \delta \tau_{220} = 0.17^{+0.14}_{-0.13}, \quad (5.1)$$

by multiplying the posteriors and

$$\begin{aligned} \delta f_{220} &= 0.00^{+0.06}_{-0.06} \quad [\mu = 0.00^{+0.03}_{-0.03}, \sigma < 0.05] \\ \delta \tau_{220} &= 0.15^{+0.26}_{-0.24} \quad [\mu = 0.15^{+0.15}_{-0.15}, \sigma < 0.22] \end{aligned} \quad (5.2)$$

by combining hierarchically. The numbers in the square brackets are the hyper-parameter estimates. These results are broadly consistent with those reported by the LVK Collaboration from the analysis of the same 10 GW events from GWTC-3 [10], which used the pSEOBNRv4HM model without accounting for spin-precession effects.

In Fig. 8 we compare the combined constraints on δf_{220} and $\delta \tau_{220}$ from GWTC-3 events, obtained using the pSEOBNRv5PHM model (dark, unfilled curves) and the pSEOBNRv4HM model (light, filled curves). Constraints obtained by multiplying the posteriors from individual events are shown in blue, while results using hierarchical combination are shown in orange. Vertical dashed lines indicate the 90% credible intervals. The gray vertical dashed lines mark the GR predictions ($\delta f_{220}, \delta \tau_{220}$) = (0, 0).

The updated analysis provides slightly tighter constraints on δf_{220} compared to previous results, particularly when combining events hierarchically. This improvement is consistent with narrower posteriors obtained for several single-event results. Additionally, the peak of the posterior for δf_{220} is closer to zero in both the joint and hierarchically combined analyses. For $\delta \tau_{220}$, both the joint and hierarchically combined posteriors are broader and slightly shifted towards positive values. This is consistent with the presence of tails in single-event posteriors for specific events such as GW200311_115853 and GW200129_065458.

Similar to the results from GWTC-3, the joint posterior distribution for $\delta\tau_{220}$ places the GR prediction near the edge of the 90% credible level. This discrepancy could arise from a variety of factors, including noise fluctuations [9,25], parameter correlations [10], or intrinsic variance due to the limited number of events in the catalog [160]. Incorporating additional events from ongoing observing runs could help clarify this behavior.

Given a two-dimensional posterior distribution $P(x, y)$, the consistency with the null hypothesis can also be quantified by the GR quantile,

$$Q_0 = \int_{\{x,y\} \text{ where } P(x,y) \geq P(0,0)} P(x, y) dx dy. \quad (5.3)$$

The GR quantile corresponds to the fraction of the posterior enclosed by the isoprobability contour that passes through the GR value (0,0) [161], and is defined such that $Q_0 = 0$ ($Q_0 = 1$) indicates full consistency (full inconsistency) with the null hypothesis. For the joint posteriors, we consider $\{x, y\} = \{\delta f_{220}, \delta\tau_{220}\}$, while for the hierarchically results we have $\{x, y\} = \{\mu, \sigma^2\}$. We summarize the GR quantiles in Table II. Using the pSEOBv5PHM model, we find $Q_0 = 0.93$ when multiplying the posteriors, while the hierarchical combination yields $Q_0 = 0.003$ for δf_{220} and $Q_0 = 0.74$ for $\delta\tau_{220}$. For the pSEOBv4HM model, the GR quantile when multiplying the posteriors is $Q_0 = 0.97$, whereas the hierarchical combination gives $Q_0 = 0.32$ for δf_{220} and $Q_0 = 0.81$ for $\delta\tau_{220}$. Overall, the pSEOBv5PHM model shows slightly better consistency with GR in all cases.

The pSEOBv5PHM model can also be used to estimate the properties of the remnant BH. We compute effective values for the QNM frequency and damping time as follows:

$$f_{\ell m 0} = f_{\ell m 0}^{\text{GR}}(1 + \delta f_{\ell m 0}), \quad (5.4a)$$

$$\tau_{\ell m 0} = \tau_{\ell m 0}^{\text{GR}}(1 + \delta\tau_{\ell m 0}), \quad (5.4b)$$

where $f_{\ell m 0}^{\text{GR}}$ and $\tau_{\ell m 0}^{\text{GR}}$ are derived as functions of the component masses and spins using NR fits. The mass and spin of the remnant object can then be estimated from the complex QNM frequencies by inverting the fitting

formula provided in Ref. [40]. These results are summarized in Table I.

VI. CONCLUSIONS

We have presented pSEOBv5PHM, a parametrized, multipolar, waveform model for BBHs in quasicircular orbits, designed to perform null tests of GR across the inspiral, plunge-merger, and ringdown stages of compact binary coalescences. Notably, pSEOBv5PHM extends previous works which were limited to BBHs with aligned or antialigned spins [25,69,79] by incorporating spin-precession effects. After examining the morphology of the parametrized deviations, we employed pSEOBv5PHM to estimate deviations in the QNM frequency and damping time of the $(\ell, m, n) = (2, 2, 0)$ mode. Synthetic-signal studies using BBH NR waveforms highlight the importance of including spin-precession effects, even at current detector sensitivities, to robustly perform tests of GR and avoid biases or false deviations. By analyzing a synthetic signal from a publicly available NR simulation of scalar-field BS merger, we showed that pSEOBv5PHM can successfully identify that the signal does not originate from a BBH in GR. Finally, we have applied our model to real data by reanalyzing 12 events from GWTC-3. Using a hierarchical combination of these events, we constrained fractional deviations in the frequency and damping time of the (2,2,0) quasinormal-mode to $\delta f_{220} = 0.00_{-0.06}^{+0.06}$ and $\delta\tau_{220} = 0.15_{-0.24}^{+0.26}$ at 90% credibility. Our results are consistent with those from the LVK Collaboration, which did not account for spin-precession effects.

Similar to the results from GWTC-3, the joint posterior distribution for $\delta\tau_{220}$ places the GR prediction near the edge of the 90% credible level. Analyzing additional events from current and upcoming LVK observing runs is expected to refine these constraints and help better quantify the significance of the results. Furthermore, performing large-scale injection studies using NR or NR surrogate waveforms, along with incorporating Gaussian noise realizations or real-noise injections, would offer important insights into the potential influence of waveform systematics or noise fluctuations on the inferred deviation parameters. These effects should be thoroughly investigated and quantified in order to be able to claim a GR violation in GW observations [80].

In this work, while we described the implementation and morphology of all corrections, we validated our model through Bayesian PE for the ringdown of the dominant $(\ell, m, n) = (2, 2, 0)$ mode, as it is the primary test currently performed in LVK analyses. At current SNR, higher modes in the ringdown are not detectable with high statistical significance, but are expected to become confidently detectable with the improved SNR achievable through upcoming LVK upgrades [22,64]. In future work, we plan to explore higher modes in the ringdown, as well as the

TABLE II. Comparison of GR quantiles (Q_0) for joint and hierarchical results using pSEOBv4HM and pSEOBv5PHM. Smaller values of Q_0 indicate better consistency with GR.

Model	Joint Q_0	Hierarchical Q_0	
		δf_{220}	$\delta\tau_{220}$
pSEOBv4HM	0.97	0.32	0.81
pSEOBv5PHM	0.93	0.003	0.74

measurability of deviations in the inspiral and plunge-merger stages.

Given the modular nature of the `pySEOBNR` code, ongoing improvements to the baseline `SEOBNRv5PHM` model, such as the inclusion of multipole asymmetries in the coprecessing frame, and the calibration to spin-precessing NR simulations, can be immediately propagated to the parametrized `pSEOBNRv5PHM` model. It is also straightforward to add similar parametrized deviations to the eccentric, aligned-spin `SEOBNRv5EHM` waveform model [158,159] recently developed.

Massive BH binaries with masses from $10^4 M_\odot$ to $10^7 M_\odot$, detectable by the space-based LISA mission, are also prime candidates for BH spectroscopy tests [40]. The high SNR of these sources makes it especially important to incorporate all relevant physical effects in their analysis, including spin-precession, to mitigate potential biases arising from waveform systematics. We plan to use the `pSEOBNRv5PHM` model to extend the work of Ref. [79] to spin-precessing binaries, also including a realistic treatment of the LISA response function.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [147,163].

APPENDIX A: POSTERIOR DISTRIBUTIONS OF GWTC-3 EVENTS

We show in Fig. 9 the posterior probability distributions for the fractional deviations in the frequency and damping time of the (2,2,0) QNM (δf_{220} , $\delta \tau_{220}$), for the 10 events from GWTC-3 used to produce combined results for the `pSEOBNR` analysis. The PE is performed with the `pSEOBNRv4HM` model (blue) and with the `pSEOBNRv5PHM` model (orange). The results from `pSEOBNRv4HM` are taken from the data release associated with Ref. [10]. The 2D contours mark the 90% credible regions, while the dashed lines on the 1D marginalized distributions mark the 90% credible intervals. The black vertical and horizontal lines mark the GR predictions, $\delta f_{220} = \delta \tau_{220} = 0$.

APPENDIX B: CORRELATIONS BETWEEN GR PARAMETERS AND QNM DEVIATIONS FOR GW200311_115853

As mentioned in Sec. V of the main text, for GW200311_115853 we observe a tail in the posterior distribution for $\delta \tau_{220}$ towards positive values, when spin precession is included. Figure 10 shows the posterior probability distributions for the fractional deviations in the frequency and damping time of the (2,2,0) QNM (δf_{220} , $\delta \tau_{220}$), along with the effective precessing-spin parameter χ_p , the binary’s luminosity distance d_L , and inclination angle ι , highlighting the correlations among these parameters. The 2D contours mark the 90% credible regions. We show recoveries using the `pSEOBNRv5PHM` model (in orange) and its aligned-spin version `pSEOBNRv5HM` (in blue), as well the parameters inferred assuming GR with the `SEOBNRv5PHM` model (in green). The stars indicate the maximum likelihood parameters in each analysis.

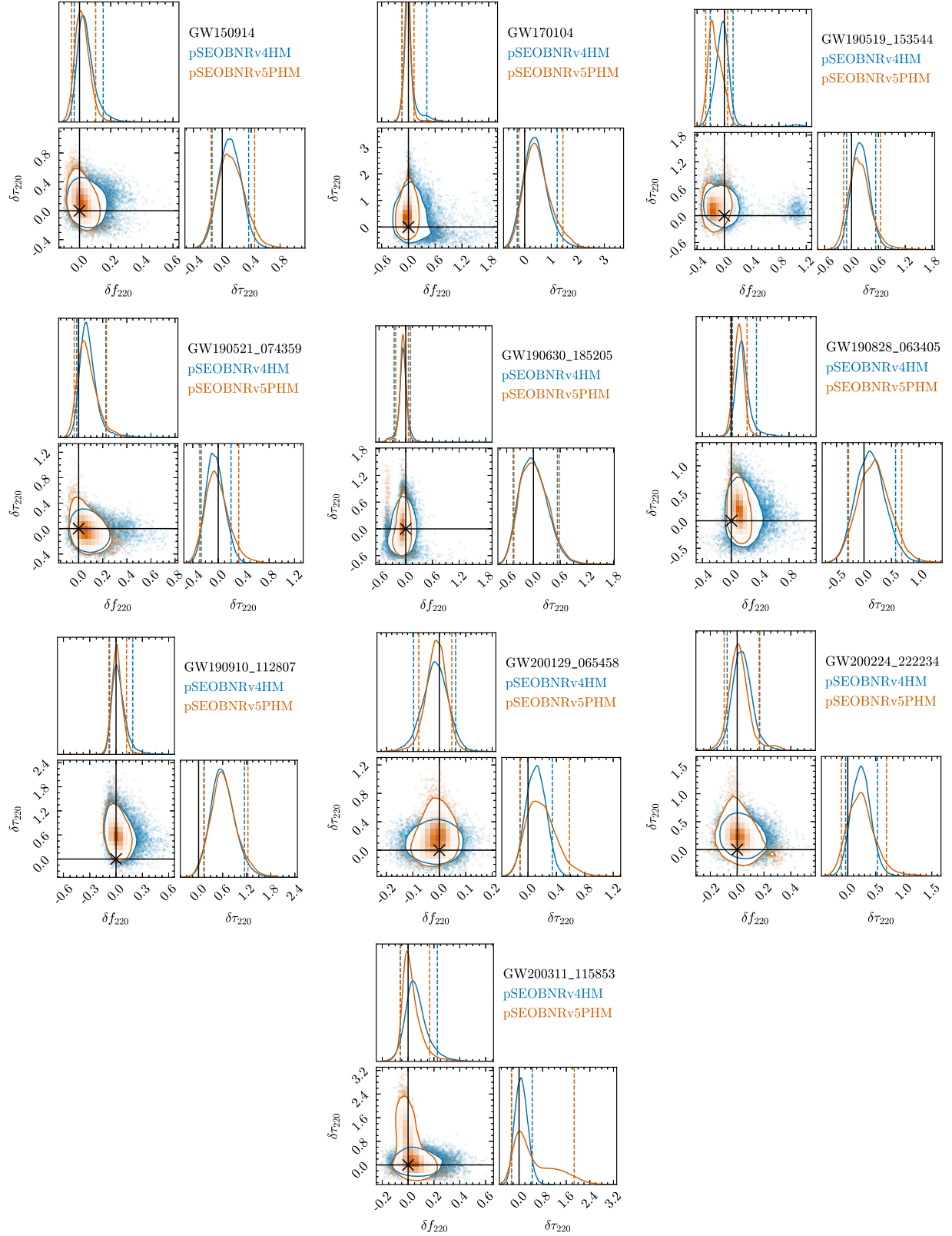


FIG. 9. Posterior probability distributions for the fractional deviations in the frequency and damping time of the (2,2,0) QNM (δf_{220} and $\delta \tau_{220}$), for the 10 events from GWTC-3 used to produce combined results for the pSEOBNR analysis. The PE is performed with the pSEOBNRv4HM model (blue) and with the pSEOBNRv5PHM model (orange). The 2D contours mark the 90% credible regions, while the dashed lines on the 1D marginalized distributions mark the 90% credible intervals. The black vertical and horizontal lines mark the GR predictions ($\delta f_{220} = \delta \tau_{220} = 0$).

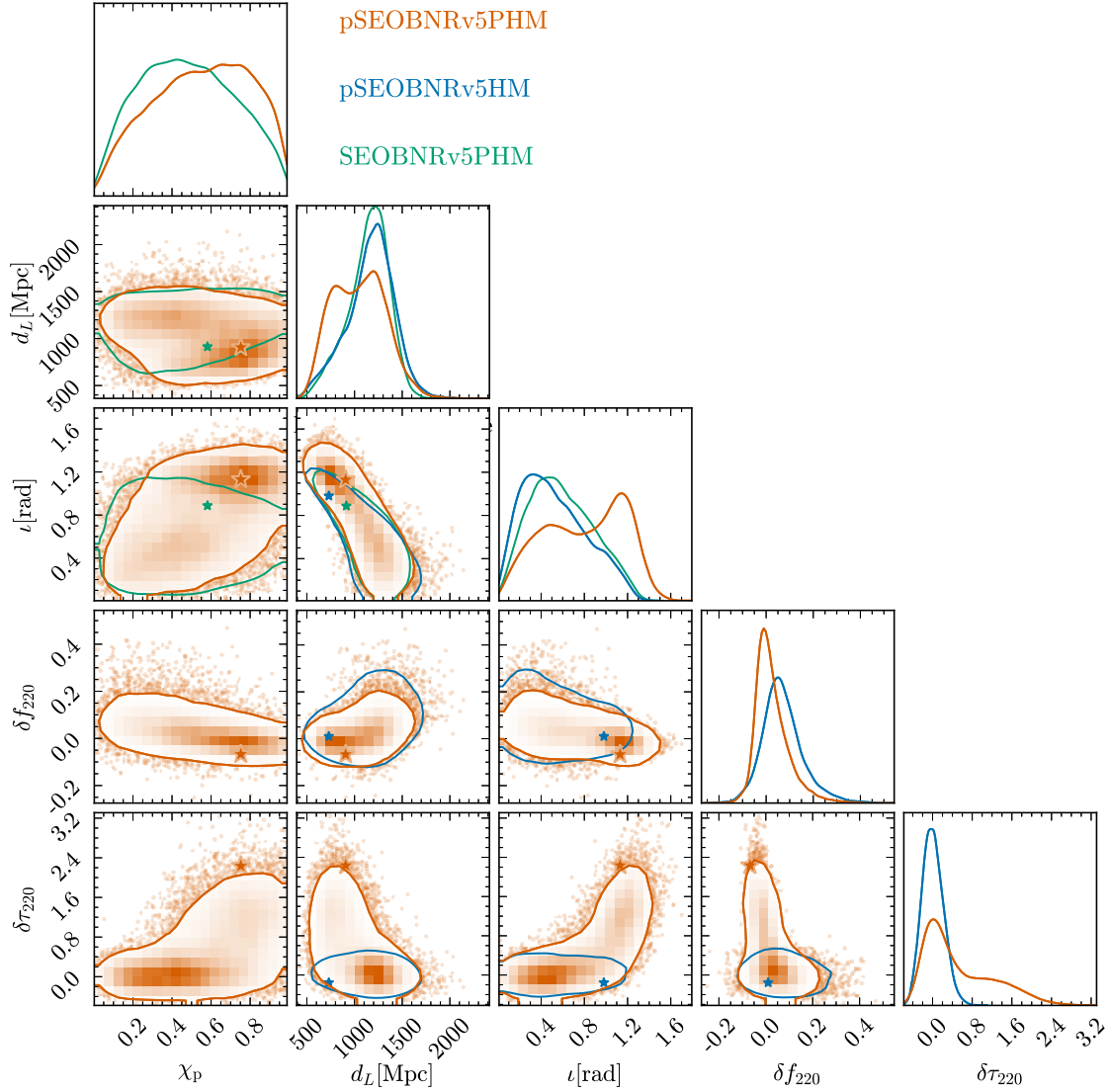


FIG. 10. Posterior probability distributions for the effective precessing-spin parameter χ_p , the binary's luminosity distance d_L and inclination angle ι , and the fractional deviations in the frequency and damping time of the (2,2,0) QNM (δf_{220} , $\delta \tau_{220}$), for the event GW200311_115853. We show recoveries using the pSEOBNRv5PHM model (in orange) and its aligned-spin version pSEOBNRv5HM (in blue), as well the parameters inferred assuming GR with the SEOBNRv5PHM model (in green). The 2D contours mark the 90% credible regions, and the stars indicate the maximum likelihood parameters recovered in each analysis.

Comparing the pSEOBNRv5PHM and pSEOBNRv5HM recoveries, we note that correlations between $\delta \tau_{220}$ and (d_L , ι) appear only when spin precession is included. In particular, the tail toward smaller distances and higher inclinations is correlated with χ_p , which in turn is correlated with $\delta \tau_{220}$. However, the SEOBNRv5PHM results show that correlations between (d_L , ι) and χ_p are not as pronounced in the GR recovery. When examining the two-dimensional posterior for distance and inclination, the recoveries from SEOBNRv5PHM and pSEOBNRv5HM are broadly consistent, while pSEOBNRv5PHM exhibits a secondary mode at

smaller distances and higher inclinations. Interestingly, the maximum likelihood for both SEOBNRv5PHM and pSEOBNRv5HM lies near the secondary mode of pSEOBNRv5PHM, despite most of the posteriors being centered elsewhere. This high likelihood indicates that even the SEOBNRv5PHM and pSEOBNRv5HM models fit the data well in that region; the increased flexibility provided by including both spin precession and QNM deviations allows for a broader range of configurations that match well the data around the region, at the cost of a shift in $\delta \tau_{220}$.

- [1] J. Aasi *et al.* (LIGO Scientific Collaboration), Advanced LIGO, *Classical Quantum Gravity* **32**, 074001 (2015).
- [2] F. Acernese *et al.* (Virgo Collaboration), Advanced Virgo: A second-generation interferometric gravitational wave detector, *Classical Quantum Gravity* **32**, 024001 (2015).
- [3] T. Akutsu *et al.* (KAGRA Collaboration), Overview of KAGRA: Detector design and construction history, *Prog. Theor. Exp. Phys.* **2021**, 05A101 (2021).
- [4] R. Abbott *et al.* (KAGRA, Virgo, and LIGO Scientific Collaborations), GWTC-3: Compact binary coalescences observed by LIGO and Virgo during the second part of the third observing run, *Phys. Rev. X* **13**, 041039 (2023).
- [5] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Observation of gravitational waves from a binary black hole merger, *Phys. Rev. Lett.* **116**, 061102 (2016).
- [6] B. P. Abbott *et al.* (KAGRA, LIGO Scientific, and Virgo Collaborations), Prospects for observing and localizing gravitational-wave transients with Advanced LIGO, Advanced Virgo and KAGRA, *Living Rev. Relativity* **19**, 1 (2016).
- [7] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Tests of general relativity with GW150914, *Phys. Rev. Lett.* **116**, 221101 (2016); **121**, 129902(E) (2018).
- [8] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Tests of general relativity with the binary black hole signals from the LIGO-Virgo catalog GWTC-1, *Phys. Rev. D* **100**, 104036 (2019).
- [9] R. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Tests of general relativity with binary black holes from the second LIGO-Virgo gravitational-wave transient catalog, *Phys. Rev. D* **103**, 122002 (2021).
- [10] R. Abbott *et al.* (LIGO Scientific, Virgo, and KAGRA Collaborations), Tests of general relativity with GWTC-3, [arXiv:2112.06861](https://arxiv.org/abs/2112.06861).
- [11] N. Yunes, K. Yagi, and F. Pretorius, Theoretical physics implications of the binary black-hole mergers GW150914 and GW151226, *Phys. Rev. D* **94**, 084002 (2016).
- [12] H. O. Silva, A. Ghosh, and A. Buonanno, Black-hole ringdown as a probe of higher-curvature gravity theories, *Phys. Rev. D* **107**, 044030 (2023).
- [13] A. Maselli, S. Yi, L. Pierini, V. Vellucci, L. Reali, L. Gualtieri, and E. Berti, Black hole spectroscopy beyond Kerr: Agnostic and theory-based tests with next-generation interferometers, *Phys. Rev. D* **109**, 064060 (2024).
- [14] F.-L. Julié, L. Pompili, and A. Buonanno, Inspiral-merger-ringdown waveforms in Einstein-scalar-Gauss-Bonnet gravity within the effective-one-body formalism, *Phys. Rev. D* **111**, 024016 (2025).
- [15] S. Maenaut, G. Carullo, P. A. Cano, A. Liu, V. Cardoso, T. Hertog, and T. G. F. Li, Ringdown analysis of rotating black holes in effective field theory extensions of general relativity, [arXiv:2411.17893](https://arxiv.org/abs/2411.17893).
- [16] L. Blanchet and B. S. Sathyaprakash, Detecting the tail effect in gravitational wave experiments, *Phys. Rev. Lett.* **74**, 1067 (1995).
- [17] K. G. Arun, B. R. Iyer, M. S. S. Qusailah, and B. S. Sathyaprakash, Probing the non-linear structure of general relativity with black hole binaries, *Phys. Rev. D* **74**, 024006 (2006).
- [18] N. Yunes and F. Pretorius, Fundamental theoretical bias in gravitational wave astrophysics and the parameterized post-Einsteinian framework, *Phys. Rev. D* **80**, 122003 (2009).
- [19] T. G. F. Li, W. Del Pozzo, S. Vitale, C. Van Den Broeck, M. Agathos, J. Veitch, K. Grover, T. Sidery, R. Sturani, and A. Vecchio, Towards a generic test of the strong field dynamics of general relativity using compact binary coalescence, *Phys. Rev. D* **85**, 082003 (2012).
- [20] M. Agathos, W. Del Pozzo, T. G. F. Li, C. Van Den Broeck, J. Veitch, and S. Vitale, TIGER: A data analysis pipeline for testing the strong-field dynamics of general relativity with gravitational wave signals from coalescing compact binaries, *Phys. Rev. D* **89**, 082001 (2014).
- [21] J. Meidam *et al.*, Parametrized tests of the strong-field dynamics of general relativity using gravitational wave signals from coalescing binary black holes: Fast likelihood calculations and sensitivity of the method, *Phys. Rev. D* **97**, 044033 (2018).
- [22] R. Brito, A. Buonanno, and V. Raymond, Black-hole spectroscopy by making full use of gravitational-wave modeling, *Phys. Rev. D* **98**, 084038 (2018).
- [23] V. Cardoso, M. Kimura, A. Maselli, E. Berti, C. F. B. Macedo, and R. McManus, Parametrized black hole quasinormal ringdown: Decoupled equations for nonrotating black holes, *Phys. Rev. D* **99**, 104077 (2019).
- [24] A. Maselli, P. Pani, L. Gualtieri, and E. Berti, Parametrized ringdown spin expansion coefficients: A data-analysis framework for black-hole spectroscopy with multiple events, *Phys. Rev. D* **101**, 024043 (2020).
- [25] A. Ghosh, R. Brito, and A. Buonanno, Constraints on quasinormal-mode frequencies with LIGO-Virgo binary-black-hole observations, *Phys. Rev. D* **103**, 124041 (2021).
- [26] A. K. Mehta, A. Buonanno, R. Cotesta, A. Ghosh, N. Sennett, and J. Steinhoff, Tests of general relativity with gravitational-wave observations using a flexible theory-independent method, *Phys. Rev. D* **107**, 044020 (2023).
- [27] E. M. Sängers *et al.*, Tests of general relativity with GW230529: A neutron star merging with a lower mass-gap compact object, [arXiv:2406.03568](https://arxiv.org/abs/2406.03568).
- [28] C. V. Vishveshwara, Scattering of gravitational radiation by a Schwarzschild black-hole, *Nature (London)* **227**, 936 (1970).
- [29] S. A. Teukolsky, Perturbations of a rotating black hole. 1. Fundamental equations for gravitational electromagnetic and neutrino field perturbations, *Astrophys. J.* **185**, 635 (1973).
- [30] S. Chandrasekhar and S. L. Detweiler, The quasi-normal modes of the Schwarzschild black hole, *Proc. R. Soc. A* **344**, 441 (1975).
- [31] K. D. Kokkotas and B. G. Schmidt, Quasinormal modes of stars and black holes, *Living Rev. Relativity* **2**, 2 (1999).
- [32] E. Berti, V. Cardoso, and A. O. Starinets, Quasinormal modes of black holes and black branes, *Classical Quantum Gravity* **26**, 163001 (2009).
- [33] W. Israel, Event horizons in static vacuum space-times, *Phys. Rev.* **164**, 1776 (1967).
- [34] S. W. Hawking, Black holes in general relativity, *Commun. Math. Phys.* **25**, 152 (1972).

- [35] B. Carter, Axisymmetric black hole has only two degrees of freedom, *Phys. Rev. Lett.* **26**, 331 (1971).
- [36] D. C. Robinson, Uniqueness of the Kerr black hole, *Phys. Rev. Lett.* **34**, 905 (1975).
- [37] P. O. Mazur, Proof of uniqueness of the Kerr-Newman black hole solution, *J. Phys. A* **15**, 3173 (1982).
- [38] S. L. Detweiler, Black holes and gravitational waves. III. The resonant frequencies of rotating holes, *Astrophys. J.* **239**, 292 (1980).
- [39] O. Dreyer, B. J. Kelly, B. Krishnan, L. S. Finn, D. Garrison, and R. Lopez-Aleman, Black hole spectroscopy: Testing general relativity through gravitational wave observations, *Classical Quantum Gravity* **21**, 787 (2004).
- [40] E. Berti, V. Cardoso, and C. M. Will, On gravitational-wave spectroscopy of massive black holes with the space interferometer LISA, *Phys. Rev. D* **73**, 064030 (2006).
- [41] S. Gossan, J. Veitch, and B. S. Sathyaprakash, Bayesian model selection for testing the no-hair theorem with black hole ringdowns, *Phys. Rev. D* **85**, 124056 (2012).
- [42] J. Meidam, M. Agathos, C. Van Den Broeck, J. Veitch, and B. S. Sathyaprakash, Testing the no-hair theorem with black hole ringdowns using TIGER, *Phys. Rev. D* **90**, 064009 (2014).
- [43] G. Carullo *et al.*, Empirical tests of the black hole no-hair conjecture using gravitational-wave observations, *Phys. Rev. D* **98**, 104020 (2018).
- [44] M. Isi, M. Giesler, W. M. Farr, M. A. Scheel, and S. A. Teukolsky, Testing the no-hair theorem with GW150914, *Phys. Rev. Lett.* **123**, 111102 (2019).
- [45] S. Bhagwat, M. Cabero, C. D. Capano, B. Krishnan, and D. A. Brown, Detectability of the subdominant mode in a binary black hole ringdown, *Phys. Rev. D* **102**, 024023 (2020).
- [46] G. Carullo, W. Del Pozzo, and J. Veitch, Observational black hole spectroscopy: A time-domain multimode analysis of GW150914, *Phys. Rev. D* **99**, 123029 (2019); **100**, 089903(E) (2019).
- [47] J. Calderón Bustillo, P. D. Lasky, and E. Thrane, Black-hole spectroscopy, the no-hair theorem, and GW150914: Kerr versus Occam, *Phys. Rev. D* **103**, 024041 (2021).
- [48] M. Isi and W. M. Farr, Analyzing black-hole ringdowns, [arXiv:2107.05609](https://arxiv.org/abs/2107.05609).
- [49] C. D. Capano, M. Cabero, J. Westerweck, J. Abedi, S. Kasta, A. H. Nitz, Y.-F. Wang, A. B. Nielsen, and B. Krishnan, Multimode quasinormal spectrum from a perturbed black hole, *Phys. Rev. Lett.* **131**, 221402 (2023).
- [50] R. Cotesta, G. Carullo, E. Berti, and V. Cardoso, Analysis of ringdown overtones in GW150914, *Phys. Rev. Lett.* **129**, 111102 (2022).
- [51] S. Ma, K. Mitman, L. Sun, N. Deppe, F. Hébert, L. E. Kidder, J. Moxon, W. Throwe, N. L. Vu, and Y. Chen, Quasinormal-mode filters: A new approach to analyze the gravitational-wave ringdown of binary black-hole mergers, *Phys. Rev. D* **106**, 084036 (2022).
- [52] V. Baibhav, M. H.-Y. Cheung, E. Berti, V. Cardoso, G. Carullo, R. Cotesta, W. Del Pozzo, and F. Duque, Agnostic black hole spectroscopy: Quasinormal mode content of numerical relativity waveforms and limits of validity of linear perturbation theory, *Phys. Rev. D* **108**, 104020 (2023).
- [53] H. Siegel, M. Isi, and W. M. Farr, Analyzing black-hole ringdowns. II. Data conditioning, *Phys. Rev. D* **111**, 044070 (2025).
- [54] I. Kamaretsos, M. Hannam, S. Husa, and B. S. Sathyaprakash, Black-hole hair loss: Learning about binary progenitors from ringdown signals, *Phys. Rev. D* **85**, 024018 (2012).
- [55] I. Kamaretsos, M. Hannam, and B. Sathyaprakash, Is black-hole ringdown a memory of its progenitor?, *Phys. Rev. Lett.* **109**, 141102 (2012).
- [56] L. London, D. Shoemaker, and J. Healy, Modeling ringdown: Beyond the fundamental quasinormal modes, *Phys. Rev. D* **90**, 124032 (2014); **94**, 069902(E) (2016).
- [57] L. T. London, Modeling ringdown. II. Aligned-spin binary black holes, implications for data analysis and fundamental theory, *Phys. Rev. D* **102**, 084052 (2020).
- [58] X. Jiménez Forteza, S. Bhagwat, P. Pani, and V. Ferrari, Spectroscopy of binary black hole ringdown using overtones and angular modes, *Phys. Rev. D* **102**, 044053 (2020).
- [59] M. H.-Y. Cheung, E. Berti, V. Baibhav, and R. Cotesta, Extracting linear and nonlinear quasinormal modes from black hole merger simulations, *Phys. Rev. D* **109**, 044069 (2024); **110**, 049902(E) (2024).
- [60] H. Zhu *et al.*, Black hole spectroscopy for precessing binary black hole coalescences, *Phys. Rev. D* **111**, 064052 (2025).
- [61] C. Pacilio, S. Bhagwat, F. Nobili, and D. Gerosa, Flexible mapping of ringdown amplitudes for nonprecessing binary black holes, *Phys. Rev. D* **110**, 103037 (2024).
- [62] L. Magaña Zertuche *et al.*, High-precision ringdown surrogate model for non-precessing binary black holes, [arXiv:2408.05300](https://arxiv.org/abs/2408.05300).
- [63] G. Carullo, Ringdown amplitudes of nonspinning eccentric binaries, *J. Cosmol. Astropart. Phys.* **10** (2024) 061.
- [64] V. Gennari, G. Carullo, and W. Del Pozzo, Searching for ringdown higher modes with a numerical relativity-informed post-merger model, *Eur. Phys. J. C* **84**, 233 (2024).
- [65] Y. Pan, A. Buonanno, M. Boyle, L. T. Buchman, L. E. Kidder, H. P. Pfeiffer, and M. A. Scheel, Inspiral-merger-ringdown multipolar waveforms of nonspinning black-hole binaries using the effective-one-body formalism, *Phys. Rev. D* **84**, 124052 (2011).
- [66] R. Cotesta, A. Buonanno, A. Bohé, A. Taracchini, I. Hinder, and S. Ossokine, Enriching the symphony of gravitational waves from binary black holes by tuning higher harmonics, *Phys. Rev. D* **98**, 084028 (2018).
- [67] A. Buonanno and T. Damour, Effective one-body approach to general relativistic two-body dynamics, *Phys. Rev. D* **59**, 084006 (1999).
- [68] A. Buonanno and T. Damour, Transition from inspiral to plunge in binary black hole coalescences, *Phys. Rev. D* **62**, 064015 (2000).
- [69] E. Maggio, H. O. Silva, A. Buonanno, and A. Ghosh, Tests of general relativity in the nonlinear regime: A parametrized plunge-merger-ringdown gravitational waveform model, *Phys. Rev. D* **108**, 024043 (2023).

- [70] M. Punturo *et al.*, The Einstein Telescope: A third-generation gravitational wave observatory, *Classical Quantum Gravity* **27**, 194002 (2010).
- [71] M. Evans *et al.*, A horizon study for cosmic explorer: Science, observatories, and community, [arXiv:2109.09882](#).
- [72] P. Amaro-Seoane *et al.* (LISA Collaboration), Laser Interferometer Space Antenna, [arXiv:1702.00786](#).
- [73] M. Pürrer and C.-J. Haster, Gravitational waveform accuracy requirements for future ground-based detectors, *Phys. Rev. Res.* **2**, 023151 (2020).
- [74] R. Gamba, M. Breschi, S. Bernuzzi, M. Agathos, and A. Nagar, Waveform systematics in the gravitational-wave inference of tidal parameters and equation of state from binary neutron star signals, *Phys. Rev. D* **103**, 124015 (2021).
- [75] A. Dhani, S. Völkel, A. Buonanno, H. Estelles, J. Gair, H. P. Pfeiffer, L. Pompili, and A. Toubiana, Systematic biases in estimating the properties of black holes due to inaccurate gravitational-wave models, [arXiv:2404.05811](#).
- [76] V. Kapil, L. Reali, R. Cotesta, and E. Berti, Systematic bias from waveform modeling for binary black hole populations in next-generation gravitational wave detectors, *Phys. Rev. D* **109**, 104043 (2024).
- [77] P. T. H. Pang, J. Calderón Bustillo, Y. Wang, and T. G. F. Li, Potential observations of false deviations from general relativity in gravitational wave signals from binary black holes, *Phys. Rev. D* **98**, 024019 (2018).
- [78] Q. Hu and J. Veitch, Accumulating errors in tests of general relativity with gravitational waves: Overlapping signals and inaccurate waveforms, *Astrophys. J.* **945**, 103 (2023).
- [79] A. Toubiana, L. Pompili, A. Buonanno, J. R. Gair, and M. L. Katz, Measuring source properties and quasinormal mode frequencies of heavy massive black-hole binaries with LISA, *Phys. Rev. D* **109**, 104019 (2024).
- [80] A. Gupta *et al.*, Possible causes of false general relativity violations in gravitational wave observations, *SciPost Phys. Commun. Rep.*, 5 (2025).
- [81] M. Hannam, C. Hoy, J. E. Thompson, S. Fairhurst, V. Raymond *et al.*, General-relativistic precession in a black-hole binary, *Nature (London)* **610**, 652 (2022).
- [82] E. Payne, S. Hourihane, J. Golomb, R. Udall, R. Udall, D. Davis, and K. Chatziioannou, Curious case of GW200129: Interplay between spin-precession inference and data-quality issues, *Phys. Rev. D* **106**, 104017 (2022).
- [83] C. Foo and E. Hamilton, Systematic bias due to mismodeling precessing binary black hole ringdown, *Phys. Rev. D* **110**, 104024 (2024).
- [84] R. S. Chandramouli, K. Prokup, E. Berti, and N. Yunes, Systematic biases due to waveform mismodeling in parametrized post-Einsteinian tests of general relativity: The impact of neglecting spin precession and higher modes, *Phys. Rev. D* **111**, 044026 (2025).
- [85] L. Pompili *et al.*, Laying the foundation of the effective-one-body waveform models SEOBNRv5: Improved accuracy and efficiency for spinning nonprecessing binary black holes, *Phys. Rev. D* **108**, 124035 (2023).
- [86] M. Khalil, A. Buonanno, H. Estelles, D. P. Mihaylov, S. Ossokine, L. Pompili, and A. Ramos-Buades, Theoretical groundwork supporting the precessing-spin two-body dynamics of the effective-one-body waveform models SEOBNRv5, *Phys. Rev. D* **108**, 124036 (2023).
- [87] M. van de Meent, A. Buonanno, D. P. Mihaylov, S. Ossokine, L. Pompili, N. Warburton, A. Pound, B. Wardell, L. Durkan, and J. Miller, Enhancing the SEOBNRv5 effective-one-body waveform model with second-order gravitational self-force fluxes, *Phys. Rev. D* **108**, 124038 (2023).
- [88] A. Ramos-Buades, A. Buonanno, H. Estelles, M. Khalil, D. P. Mihaylov, S. Ossokine, L. Pompili, and M. Shiferaw, Next generation of accurate and efficient multipolar precessing-spin effective-one-body waveforms for binary black holes, *Phys. Rev. D* **108**, 124037 (2023).
- [89] L. Pompili, A. Buonanno, and M. Pürrer, Accounting for numerical-relativity calibration uncertainty in gravitational-wave modeling and inference, [arXiv:2410.16859](#).
- [90] R. Bachhar, M. Pürrer, and S. R. Green, Incorporating waveform calibration error in gravitational-wave modeling and inference for SEOBNRv4, *Phys. Rev. D* **111**, 084050 (2025).
- [91] M. Breschi, S. Bernuzzi, K. Chakravarti, A. Camilletti, A. Prakash, and A. Perego, Kilohertz gravitational waves from binary neutron star mergers: Numerical-relativity informed postmerger model, *Phys. Rev. D* **109**, 064009 (2024).
- [92] D. P. Mihaylov, S. Ossokine, A. Buonanno, H. Estelles, L. Pompili, M. Pürrer, and A. Ramos-Buades, pySEOBNR: A software package for the next generation of effective-one-body multipolar waveform models, [arXiv:2303.18203](#).
- [93] S. A. Hughes, A. Apte, G. Khanna, and H. Lim, Learning about black hole binaries from their ringdown spectra, *Phys. Rev. Lett.* **123**, 161101 (2019).
- [94] H. Lim, S. A. Hughes, and G. Khanna, Measuring quasinormal mode amplitudes with misaligned binary black hole ringdowns, *Phys. Rev. D* **105**, 124030 (2022).
- [95] H. Siegel, M. Isi, and W. M. Farr, Ringdown of GW190521: Hints of multiple quasinormal modes with a precessional interpretation, *Phys. Rev. D* **108**, 064008 (2023).
- [96] M. Boyle *et al.*, The SXS Collaboration catalog of binary black hole simulations, *Classical Quantum Gravity* **36**, 195006 (2019).
- [97] T. Evstafyeva, U. Sperhake, I. M. Romero-Shaw, and M. Agathos, Gravitational-wave data analysis with high-precision numerical relativity simulations of boson star mergers, *Phys. Rev. Lett.* **133**, 131401 (2024).
- [98] E. Racine, Analysis of spin precession in binary black hole systems including quadrupole-monopole interaction, *Phys. Rev. D* **78**, 044021 (2008).
- [99] L. Santamaria *et al.*, Matching post-Newtonian and numerical relativity waveforms: Systematic errors and a new phenomenological model for non-precessing black hole binaries, *Phys. Rev. D* **82**, 064016 (2010).
- [100] P. Schmidt, F. Ohme, and M. Hannam, Towards models of gravitational waveforms from generic binaries II: Modelling precession effects with a single effective precession parameter, *Phys. Rev. D* **91**, 024043 (2015).

- [101] T. Damour, P. Jaranowski, and G. Schaefer, On the determination of the last stable orbit for circular general relativistic binaries at the third post-Newtonian approximation, *Phys. Rev. D* **62**, 084011 (2000).
- [102] T. Damour, Coalescence of two spinning black holes: An effective one-body approach, *Phys. Rev. D* **64**, 124013 (2001).
- [103] A. Buonanno, Y. Chen, and T. Damour, Transition from inspiral to plunge in precessing binaries of spinning black holes, *Phys. Rev. D* **74**, 104005 (2006).
- [104] A. Buonanno, Y.-b. Chen, and M. Vallisneri, Detecting gravitational waves from precessing binaries of spinning compact objects: Adiabatic limit, *Phys. Rev. D* **67**, 104025 (2003); **74**, 029904(E) (2006).
- [105] P. Schmidt, M. Hannam, S. Husa, and P. Ajith, Tracking the precession of compact binaries from their gravitational-wave signal, *Phys. Rev. D* **84**, 024046 (2011).
- [106] M. Boyle, R. Owen, and H. P. Pfeiffer, A geometric approach to the precession of compact binaries, *Phys. Rev. D* **84**, 124011 (2011).
- [107] R. O’Shaughnessy, B. Vaishnav, J. Healy, Z. Meeks, and D. Shoemaker, Efficient asymptotic frame selection for binary black hole spacetimes using asymptotic radiation, *Phys. Rev. D* **84**, 124002 (2011).
- [108] P. Schmidt, M. Hannam, and S. Husa, Towards models of gravitational waveforms from generic binaries: A simple approximate mapping between precessing and non-precessing inspiral signals, *Phys. Rev. D* **86**, 104063 (2012).
- [109] H. Estellés, S. Husa, M. Colleoni, D. Keitel, M. Mateu-Lucena, C. García-Quirós, A. Ramos-Buades, and A. Borchers, Time-domain phenomenological model of gravitational-wave subdominant harmonics for quasicircular nonprecessing binary black hole coalescences, *Phys. Rev. D* **105**, 084039 (2022).
- [110] S. Akçay, R. Gamba, and S. Bernuzzi, Hybrid post-Newtonian effective-one-body scheme for spin-precessing compact-binary waveforms up to merger, *Phys. Rev. D* **103**, 024014 (2021).
- [111] R. Gamba, S. Akçay, S. Bernuzzi, and J. Williams, Effective-one-body waveforms for precessing coalescing compact binaries with post-Newtonian twist, *Phys. Rev. D* **106**, 024020 (2022).
- [112] Y. Pan, A. Buonanno, A. Taracchini, L. E. Kidder, A. H. Mroué, H. P. Pfeiffer, M. A. Scheel, and B. Szilágyi, Inspiral-merger-ringdown waveforms of spinning, precessing black-hole binaries in the effective-one-body formalism, *Phys. Rev. D* **89**, 084006 (2014).
- [113] S. Babak, A. Taracchini, and A. Buonanno, Validating the effective-one-body model of spinning, precessing binary black holes against numerical relativity, *Phys. Rev. D* **95**, 024010 (2017).
- [114] S. Ossokine *et al.*, Multipolar effective-one-body waveforms for precessing binary black holes: Construction and validation, *Phys. Rev. D* **102**, 044055 (2020).
- [115] T. Damour and A. Nagar, Faithful effective-one-body waveforms of small-mass-ratio coalescing black-hole binaries, *Phys. Rev. D* **76**, 064028 (2007).
- [116] T. Damour and A. Nagar, Comparing effective-one-body gravitational waveforms to accurate numerical data, *Phys. Rev. D* **77**, 024043 (2008).
- [117] T. Damour, B. R. Iyer, and A. Nagar, Improved resummation of post-Newtonian multipolar waveforms from circularized compact binaries, *Phys. Rev. D* **79**, 064004 (2009).
- [118] Y. Pan, A. Buonanno, R. Fujita, E. Racine, and H. Tagoshi, Post-Newtonian factorized multipolar waveforms for spinning, non-precessing black-hole binaries, *Phys. Rev. D* **83**, 064003 (2011); **87**, 109901(E) (2013).
- [119] J. M. Bardeen, W. H. Press, and S. A. Teukolsky, Rotating black holes: Locally nonrotating frames, energy extraction, and scalar synchrotron radiation, *Astrophys. J.* **178**, 347 (1972).
- [120] X. Jiménez-Forteza, D. Keitel, S. Husa, M. Hannam, S. Khan, and M. Pürrer, Hierarchical data-driven approach to fitting numerical relativity data for nonprecessing binary black holes with an application to final spin and radiated energy, *Phys. Rev. D* **95**, 064024 (2017).
- [121] F. Hofmann, E. Barausse, and L. Rezzolla, The final spin from binary black holes in quasi-circular orbits, *Astrophys. J. Lett.* **825**, L19 (2016).
- [122] A. Bohé *et al.*, Improved effective-one-body model of spinning, nonprecessing binary black holes for the era of gravitational-wave astrophysics with advanced detectors, *Phys. Rev. D* **95**, 044028 (2017).
- [123] M. Boyle, L. E. Kidder, S. Ossokine, and H. P. Pfeiffer, Gravitational-wave modes from precessing black-hole binaries, [arXiv:1409.4431](https://arxiv.org/abs/1409.4431).
- [124] H. Estelles *et al.*, Adding equatorial-asymmetric effects for spin-precessing binaries into the SEOBNRv5PHM waveform model (2025) (to be published).
- [125] M. Boyle, Angular velocity of gravitational radiation from precessing binaries and the corotating frame, *Phys. Rev. D* **87**, 104006 (2013).
- [126] T. Damour and A. Nagar, A new analytic representation of the ringdown waveform of coalescing spinning black hole binaries, *Phys. Rev. D* **90**, 024054 (2014).
- [127] A. Buonanno, G. B. Cook, and F. Pretorius, Inspiral, merger and ring-down of equal-mass black-hole binaries, *Phys. Rev. D* **75**, 124018 (2007).
- [128] B. J. Kelly and J. G. Baker, Decoding mode mixing in black-hole merger ringdown, *Phys. Rev. D* **87**, 084004 (2013).
- [129] E. Berti, V. Cardoso, and M. Casals, Eigenvalues and eigenfunctions of spin-weighted spheroidal harmonics in four and higher dimensions, *Phys. Rev. D* **73**, 024013 (2006); **73**, 109902(E) (2006).
- [130] E. Berti and A. Klein, Mixing of spherical and spheroidal modes in perturbed Kerr black holes, *Phys. Rev. D* **90**, 064012 (2014).
- [131] R. O’Shaughnessy, L. London, J. Healy, and D. Shoemaker, Precession during merger: Strong polarization changes are observationally accessible features of strong-field gravity during binary black hole merger, *Phys. Rev. D* **87**, 044038 (2013).
- [132] T. A. Apostolatos, C. Cutler, G. J. Sussman, and K. S. Thorne, Spin induced orbital precession and its modulation of the gravitational wave forms from merging binaries, *Phys. Rev. D* **49**, 6274 (1994).
- [133] E. Hamilton, L. London, and M. Hannam, Ringdown frequencies in black holes formed from precessing black-hole binaries, *Phys. Rev. D* **107**, 104035 (2023).

- [134] E. Hamilton *et al.*, Catalog of precessing black-hole-binary numerical-relativity simulations, *Phys. Rev. D* **109**, 044032 (2024).
- [135] E. Thrane and C. Talbot, An introduction to Bayesian inference in gravitational-wave astronomy: Parameter estimation, model selection, and hierarchical models, *Publ. Astron. Soc. Aust.* **36**, e010 (2019); **37**, e036(E) (2020).
- [136] G. Ashton *et al.*, Bilby: A user-friendly Bayesian inference library for gravitational-wave astronomy, *Astrophys. J. Suppl. Ser.* **241**, 27 (2019).
- [137] I. M. Romero-Shaw *et al.*, Bayesian inference for compact binary coalescences with Bilby: Validation and application to the first LIGO–Virgo gravitational-wave transient catalogue, *Mon. Not. R. Astron. Soc.* **499**, 3295 (2020).
- [138] J. S. Speagle, DYNesty: A dynamic nested sampling package for estimating Bayesian posteriors and evidences, *Mon. Not. R. Astron. Soc.* **493**, 3132 (2020).
- [139] A. G. Abac *et al.* (LIGO Scientific, KAGRA, and Virgo Collaborations), Observation of gravitational waves from the coalescence of a $2.5 - 4.5M_{\odot}$ compact object and a neutron star, *Astrophys. J. Lett.* **970**, L34 (2024).
- [140] G. Ashton, C. Talbot, S. Roy, G. Pratten, T.-H. Pang, M. Agathos, T. Baka, E. Sanger, A. Mehta, J. Steinhoff, E. Maggio, A. Ghosh, and A. Vijaykumar, Bilby TGR (2024), <https://zenodo.org/records/10940210>.
- [141] Noise curves used for simulations in the update of the observing scenarios paper, LIGO Document No. T2000012-v2, 2022, <https://dcc.ligo.org/ligo-t2000012/public>.
- [142] D. P. Mihaylov, S. Ossokine, A. Buonanno, and A. Ghosh, Fast post-adiabatic waveforms in the time domain: Applications to compact binary coalescences in LIGO and Virgo, *Phys. Rev. D* **104**, 124087 (2021).
- [143] S. L. Liebling and C. Palenzuela, Dynamical boson stars, *Living Rev. Relativity* **26**, 1 (2023).
- [144] K. Clough, P. Figueras, H. Finkel, M. Kunesch, E. A. Lim, and S. Tunyasuvunakool, GRChombo: Numerical relativity with adaptive mesh refinement, *Classical Quantum Gravity* **32**, 245011 (2015).
- [145] T. Andrade *et al.*, GRChombo: An adaptable numerical relativity code for fundamental physics, *J. Open Source Software* **6**, 3703 (2021).
- [146] J. M. Dickey, The weighted likelihood ratio, linear hypotheses on normal location parameters, *Ann. Math. Stat.* **42**, 204 (1971).
- [147] R. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Open data from the first and second observing runs of Advanced LIGO and Advanced Virgo, *SoftwareX* **13**, 100658 (2021).
- [148] A. Zimmerman, C.-J. Haster, and K. Chatziioannou, On combining information from multiple gravitational wave sources, *Phys. Rev. D* **99**, 124044 (2019).
- [149] M. Isi, K. Chatziioannou, and W. M. Farr, Hierarchical test of general relativity with gravitational waves, *Phys. Rev. Lett.* **123**, 121101 (2019).
- [150] B. Carpenter, A. Gelman, M. D. Hoffman, D. Lee, B. Goodrich, M. Betancourt, M. Brubaker, J. Guo, P. Li, and A. Riddell, STAN: A probabilistic programming language, *J. Stat. Softw.* **76**, 1 (2017).
- [151] H. Zhong, M. Isi, K. Chatziioannou, and W. M. Farr, Multidimensional hierarchical tests of general relativity with gravitational waves, *Phys. Rev. D* **110**, 044053 (2024).
- [152] E. Payne, M. Isi, K. Chatziioannou, and W. M. Farr, Fortifying gravitational-wave tests of general relativity against astrophysical assumptions, *Phys. Rev. D* **108**, 124060 (2023).
- [153] J. Veitch *et al.*, Parameter estimation for compact binaries with ground-based gravitational-wave observations using the LALInference software library, *Phys. Rev. D* **91**, 042003 (2015).
- [154] B. P. Abbott *et al.* (LIGO Scientific and Virgo Collaborations), Effects of waveform model systematics on the interpretation of GW150914, *Classical Quantum Gravity* **34**, 104002 (2017).
- [155] R. Macas, A. Lundgren, and G. Ashton, Revisiting the evidence for precession in GW200129 with machine learning noise mitigation, *Phys. Rev. D* **109**, 062006 (2024).
- [156] N. Gupte *et al.*, Evidence for eccentricity in the population of binary black holes observed by LIGO–Virgo–KAGRA, *arXiv:2404.14286*.
- [157] P. Kolitsidou, J. E. Thompson, and M. Hannam, Impact of antisymmetric contributions to signal multipoles in the measurement of black-hole spins, *Phys. Rev. D* **111**, 024050 (2025).
- [158] A. Gamboa *et al.*, Accurate waveforms for eccentric, aligned-spin binary black holes: The multipolar effective-one-body model SEOBNRv5EHM, *arXiv:2412.12823*.
- [159] A. Gamboa, M. Khalil, and A. Buonanno, Third post-Newtonian dynamics for eccentric orbits and aligned spins in the effective-one-body waveform model SEOBNR-v5EHM, *arXiv:2412.12831*.
- [160] C. Pacilio, D. Gerosa, and S. Bhagwat, Catalog variance of testing general relativity with gravitational-wave data, *Phys. Rev. D* **109**, L081302 (2024).
- [161] A. Ghosh, N. K. Johnson-Mcdaniel, A. Ghosh, C. K. Mishra, P. Ajith, W. Del Pozzo, C. P. L. Berry, A. B. Nielsen, and L. London, Testing general relativity using gravitational wave signals from the inspiral, merger and ringdown of binary black holes, *Classical Quantum Gravity* **35**, 014002 (2018).
- [162] <https://gwosc.org/>.
- [163] R. Abbott *et al.*, Open data from the third observing run of LIGO, Virgo, KAGRA, and GEO, *Astrophys. J. Suppl. Ser.* **267**, 29 (2023).