

Hadronic decay $D^+ \rightarrow \pi^+ \eta \eta$ and the $a_0(980)$ and $f_0(1370)$ contributions

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Motivated by the recent BESIII Collaboration measurements of the decay $D^+ \rightarrow \pi^+ \eta \eta$, we investigate this process by considering the contribution from the resonance $a_0(980)$, which is generated by S -wave pseudoscalar meson-pseudoscalar meson interaction, and the contribution from the intermediate scalar meson $f_0(1370)$. Our results could give a good description of the $\pi^+ \eta$ and $\eta \eta$ invariant mass distributions, which implies that the contribution from the intermediate $f_0(1370)$, neglected in BESIII analysis, could play a role in this process. Furthermore, the predicted Dalitz plots are also consistent with the BESIII measurements. The more precise measurements about this process in the future could shed light on the mechanism of this process and the properties of $f_0(1370)$ and $a_0(980)$.

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I. INTRODUCTION

In the traditional quark model, hadrons are categorized into mesons, which are composed of a quark-antiquark pair, and baryons, which are composed of three quarks [1,2]. Although most of the mesons could be well described within the traditional quark models, some mesons have exotic properties that are difficult to explain. In 2003, the Belle Collaboration reported an exotic state $X(3872)$ [3], and since that, many candidates of the exotic states were reported by experiments, and garnered much attention [4–10]. In the scalar meson sector, there are also many candidates for exotic states, for instance, $a_0(980)$, $f_0(980)$, $f_0(500)$, $f_0(1370)$, and $f_0(1710)$. As we know, the identification of the light scalar mesons is very difficult, resulting from their large decay widths, and there are many different interpretations for the structures of the scalar mesons, such as traditional $q\bar{q}$ states, multiquark states, hadronic molecules, glueballs, or the mixing of different components, etc. [11–21].

Several years ago, the BESIII Collaboration observed the process $D^+ \rightarrow \pi^+ \eta \eta$ using an integrated luminosity of 2.93 fb^{-1} collected at the center-of-mass energy of 3.773 GeV and reported the branching fraction

$\mathcal{B} = (2.96 \pm 0.24 \pm 0.10) \times 10^{-3}$ [22]. Later, the authors of Ref. [23] studied the process $D^+ \rightarrow \pi^+ \eta \eta$ by considering the $a_0(980)$ dynamically generated from the pseudoscalar meson-pseudoscalar meson interaction, and predicted the signal of $a_0(980)$ in the $\pi^+ \eta$ invariant mass distribution. Recently, the BESIII Collaboration has updated the measurements of the process $D^+ \rightarrow \pi^+ \eta \eta$ using 20.3 fb^{-1} of e^+e^- collision data at the center-of-mass energy 3.773 GeV , and found a clear cusp structure in the $\pi^+ \eta$ invariant mass distribution, which should be associated with the scalar $a_0(980)$ [24], as predicted in the theoretical work [23]. The BESIII amplitude analysis of this process suggests that the $a_0(980)$ line shape cannot be well described by a three-channel coupled Flatté formula for the $a_0(980)$, and could arise from the triangle loop rescattering of $D^+ \rightarrow \bar{K}_0^*(1430)K^+ \rightarrow a_0(980)^+\eta$ and $D^+ \rightarrow K_0^*(1430)^+\bar{K}^0 \rightarrow a_0(980)^+\eta$ with a significance of 5.8σ [24]. However, with Eq. (18) of Ref. [25], one can see that the triangle singularity of $D^+ \rightarrow \bar{K}_0^*(1430)K^+ \rightarrow a_0(980)^+\eta$ appears at $M_{\pi^+\eta} = 1073 \text{ MeV}$, about 80 MeV away from the nominal $a_0(980)$ mass, which implies that the triangle diagram proposed by BESIII [24] is far from developing a triangle singularity. Indeed, the $a_0(980)$ could be dynamically generated from the S -wave interaction of the coupled channels $K\bar{K}$ and $\pi\eta$ within the chiral unitary approach [16,26], which has been widely used in many theoretical studies [27–41]. Therefore, we need to further investigate the mechanism of the process $D^+ \rightarrow \pi^+ \eta \eta$ by considering the $a_0(980)$, dynamically generated from the S -wave pseudoscalar meson-pseudoscalar meson interaction.

In this work, we will investigate the process $D^+ \rightarrow \pi^+ \eta \eta$ by considering the contribution from the resonance

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$a_0(980)$, which is generated by S -wave pseudoscalar meson-pseudoscalar meson interaction. In addition, we will take into account the contribution of the light scalar $f_0(1370)$. Then, we will calculate the $\pi^+\eta$ and $\eta\eta$ invariant mass distributions in the process $D^+ \rightarrow \pi^+\eta\eta$, respectively, and contrast the results with experiment with or without $f_0(1370)$. The work done here should be an incentive for a more accurate experimental analysis to be performed.

This paper is organized as follows. In Sec. II, we present the theoretical formalism of the process $D^+ \rightarrow \pi^+\eta\eta$. Numerical results and discussions are shown in Sec. III. Finally, we give a short summary in the last section.

II. FORMALISM

In this section, we will present the theoretical formalism. We first consider the process from the perspective that the $a_0(980)$ is dynamically generated by the S -wave pseudoscalar meson-pseudoscalar meson in Sec. II A, and the mechanism of the intermediate state $f_0(1370)$ is given in Sec. II B. Finally, we give the formalism for the invariant mass distributions for the process $D^+ \rightarrow \pi^+\eta\eta$ in Sec. II C.

A. Contribution from $a_0(980)$

Taking into account that $a_0(980)$ could be generated via $\pi^+\eta$ and $K^+\bar{K}^0$ final state interaction, as done in Refs. [36–38], we first show the quark level diagram for the $D^+ \rightarrow \pi^+\eta\eta$ process via the W^+ external emission in Fig. 1. The c quark from the initial D^+ meson weakly decays into a W^+ boson and a d quark, then the W^+ boson decays into $u\bar{d}$ quark pair. The $u\bar{d}$ quark pair from the W^+ boson will hadronize into π^+ , while the d quark and the $\bar{d}$ quark of the initial D^+ meson, together with the antiquark-quark pair $\bar{d}d$, which is created from the vacuum with the quantum numbers $J^{PC} = 0^{++}$, will hadronize into η and η mesons, as follows,

$$\begin{aligned} D^+ &= c\bar{d} \\ &\Rightarrow W^+ d\bar{d} \\ &\Rightarrow u\bar{d}d(\bar{d}d)\bar{d} \\ &\Rightarrow \pi^+ d(\bar{d}d)\bar{d} \\ &\Rightarrow \frac{1}{3}\pi^+\eta\eta, \end{aligned} \quad (1)$$

where we use the meson flavor wave functions $D^+ = c\bar{d}$, $\pi^+ = u\bar{d}$, and $\eta = \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d - \bar{s}s)$ as Refs. [42,43].

Similar, one has another quark level diagram for the $D^+ \rightarrow K^+\bar{K}^0\eta$ process via the W^+ external emission in Fig. 2. The c quark from the initial D^+ meson weakly decays into a W^+ boson and an s quark, then the W^+ boson decays into a $u\bar{s}$ quark pair. The $u\bar{s}$ quark pair from the W^+ boson will hadronize into K^+ , while the s quark and the $\bar{d}$ quark of the initial D^+ meson, together with the

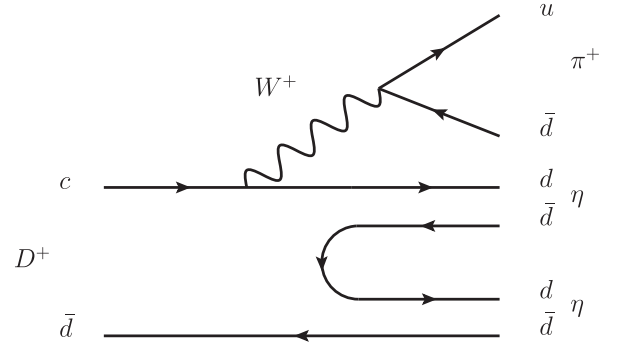


FIG. 1. Quark level diagram for the process $D^+ \rightarrow \pi^+\eta\eta$ via the W^+ external emission.

antiquark-quark pair $\bar{d}d$ created from the vacuum, will hadronize into $\bar{K}^0$ and η mesons, as follows,

$$\begin{aligned} D^+ &= c\bar{d} \\ &\Rightarrow W^+ s\bar{d} \\ &\Rightarrow u\bar{s}s(\bar{d}d)\bar{d} \\ &\Rightarrow K^+ s(\bar{d}d)\bar{d} \\ &\Rightarrow \frac{1}{\sqrt{3}}K^+\bar{K}^0\eta, \end{aligned} \quad (2)$$

where we also use the meson flavor wave functions $D^+ = c\bar{d}$, $K^+ = u\bar{s}$, $\bar{K}^0 = s\bar{d}$, and $\eta = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} - s\bar{s})$ as in Refs. [42,43].

Next, we could write all final meson-meson components after the hadronization,

$$D^+ \Rightarrow \frac{1}{\sqrt{3}}\eta\left(\frac{1}{\sqrt{3}}\pi^+\eta + K^+\bar{K}^0\right). \quad (3)$$

Then, the decay $D^+ \rightarrow \pi^+\eta\eta$ could happen through the tree level and S -wave pseudoscalar meson-pseudoscalar meson interaction, which will dynamically generate the $a_0(980)$ state, as show in Figs. 3(a) and 3(b), respectively. Since there are two η mesons in the final states, one can write the decay amplitudes as follows,

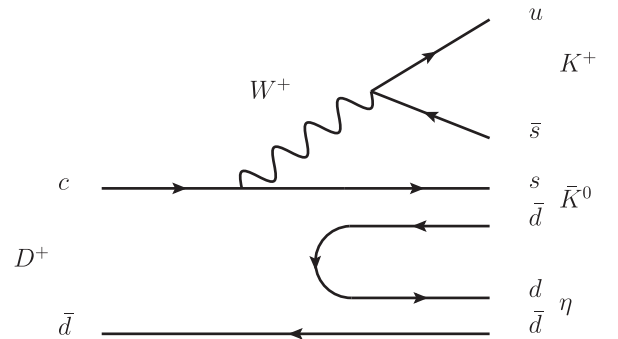


FIG. 2. Quark level diagram for the process $D^+ \rightarrow K^+\bar{K}^0\eta$ via the W^+ external emission.

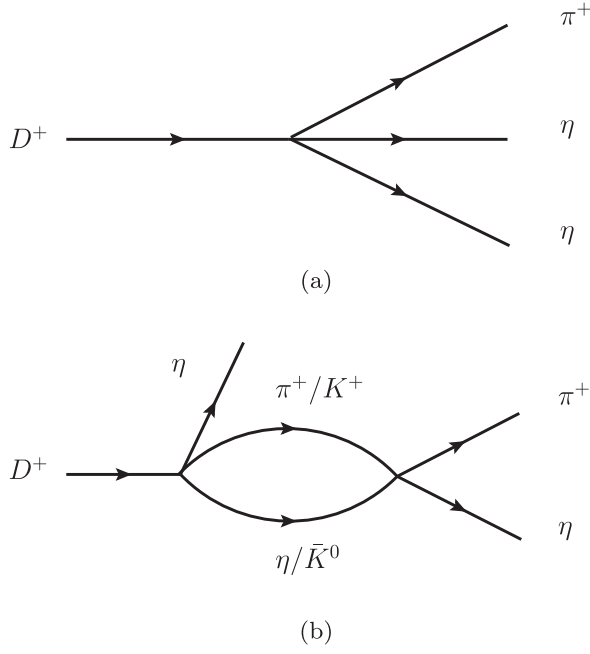


FIG. 3. Mechanisms of the process $D^+ \rightarrow \pi^+ \eta \eta$. (a) tree level, (b) the S -wave pseudoscalar meson-pseudoscalar meson interaction.

$$\mathcal{T}^{\text{Tree}} = V_p h_{\pi^+ \eta}, \quad (4)$$

$$\mathcal{T}^{a_0} = V_p \left[\sum_i h_i G_i(\pi^+ \eta_1) t_{i \rightarrow \pi^+ \eta}(\pi^+ \eta_1) + \sum_i h_i G_i(\pi^+ \eta_2) t_{i \rightarrow \pi^+ \eta}(\pi^+ \eta_2) \right], \quad (5)$$

where V_p is a global normalization constant,¹ and $i = 1, 2$ correspond to the $\pi^+ \eta$ and $K^+ \bar{K}^0$ channels, respectively. The coefficients h_i could be obtained from Eq. (3),

$$h_{\pi^+ \eta} = \frac{1}{3}, \quad h_{K^+ \bar{K}^0} = \sqrt{\frac{1}{3}}. \quad (6)$$

The G_i in Eq. (5) is the loop function of the meson-meson system [37],

$$G_i = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(P - q)^2 - m_{1i}^2 + i\epsilon} \frac{1}{q^2 - m_{2i}^2 + i\epsilon}, \quad (7)$$

where m_{1i} and m_{2i} are the masses of two mesons in i -th coupled channel, respectively, P is the four-momentum of the meson-meson system, and q is the four-momentum of

¹Here the parameter V_p contains all dynamical factors of the weak production vertex of Figs. 3(a) and 3(b). Since in this work we mainly focused on the final state interactions of this process, we assume V_p to be constant, as done in Refs. [44–46].

meson in the center-of-mass frame. For the meson-meson loop function, we take the cutoff method [47], and the loop function could be written as,

$$G(s) = \frac{1}{16\pi^2 s} \left\{ \sigma \left(\arctan \frac{s + \Delta}{\sigma \lambda_1} + \arctan \frac{s - \Delta}{\sigma \lambda_2} \right) - \left[(s + \Delta) \ln \frac{q_{\max}(1 + \lambda_1)}{m_1} + (s - \Delta) \ln \frac{q_{\max}(1 + \lambda_2)}{m_2} \right] \right\}, \quad (8)$$

where

$$\sigma = [- (s - (m_1 + m_2)^2)(s - (m_1 - m_2)^2)]^{1/2}, \quad (9)$$

$$\Delta = m_1^2 - m_2^2, \quad (10)$$

$$\lambda_1 = \sqrt{1 + \frac{m_1^2}{q_{\max}^2}}, \quad \lambda_2 = \sqrt{1 + \frac{m_2^2}{q_{\max}^2}}, \quad (11)$$

with $s = P^2 = M_{\pi^+ \eta}^2$. In order to dynamically generate the $a_0(980)$ state, we adopt the cutoff parameter $q_{\max} = 600$ MeV as used in Refs. [36–38].

The $t_{i \rightarrow \pi^+ \eta}$ in Eq. (5) are the transition amplitudes of the coupled channels, and $t_{i \rightarrow \pi^+ \eta}$ could be obtained through the Bethe-Salpeter equation,

$$T = [1 - VG]^{-1} V. \quad (12)$$

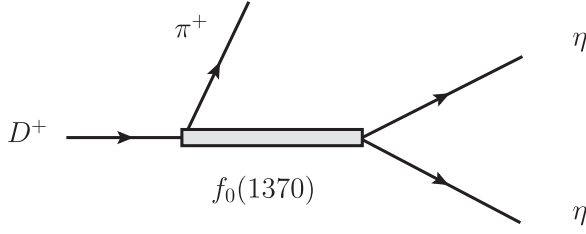
In our calculation, we take into account two coupled channels $\pi^+ \eta$ and $K^+ \bar{K}^0$. The transition potential V_{ij} is obtained from Refs. [36,37],

$$\begin{aligned} V_{\pi^+ \eta \rightarrow \pi^+ \eta} &= -\frac{2m_\pi^2}{3f^2}, \\ V_{K^+ \bar{K}^0 \rightarrow K^+ \bar{K}^0} &= -\frac{s}{4f^2}, \\ V_{K^+ \bar{K}^0 \rightarrow \pi^+ \eta} &= -\frac{3s - 2m_K^2 - m_\eta^2}{3\sqrt{3}f^2}, \end{aligned} \quad (13)$$

where the decay constant $f = f_\pi = 93$ MeV. m_π and m_K represent the averaged masses of the pion and kaon.

B. Contribution from $f_0(1370)$

According to BESIII measurements of the process $D^+ \rightarrow \pi^+ \eta \eta$, one can find that the $\eta \eta$ invariant mass distribution is significantly different from the phase space distribution in the region 1300–1500 MeV [24]. Considering that the scalar meson $f_0(1370)$ could decay into the $\eta \eta$ [48], there could be the contribution from the $f_0(1370)$ state. Therefore, we can also consider the contribution from $f_0(1370)$, and the mechanism of the process

FIG. 4. Mechanism of the intermediate $f_0(1370)$.

$D^+ \rightarrow \pi^+ f_0(1370) \rightarrow \pi^+ \eta \eta$ is depicted in Fig. 4. We can write the amplitude for the intermediate $f_0(1370)$ state,

$$\mathcal{T}^{f_0(1370)} = V_p \frac{\alpha M_{f_0(1370)} \Gamma_{f_0(1370)}}{M_{\eta\eta}^2 - M_{f_0(1370)}^2 + i M_{f_0(1370)} \Gamma_{f_0(1370)}}, \quad (14)$$

where the α is the relative strength of the contribution from the intermediate $f_0(1370)$ with respect to the one of $a_0(980)$. Here the $M_{f_0(1370)}$ and $\Gamma_{f_0(1370)}$ are the mass and width of the $f_0(1370)$. Since the $f_0(1370)$ is a broad state, and its mass and width have not yet been established [48], there are also many studies about the nature of the $f_0(1370)$ [49–52].

C. Invariant mass distribution

We can write the total amplitude of the $D^+ \rightarrow \pi^+ \eta \eta$ reaction as,

$$\mathcal{T}^{\text{Total}} = \mathcal{T}^{\text{Tree}} + \mathcal{T}^{a_0} + \mathcal{T}^{f_0(1370)} e^{i\phi}, \quad (15)$$

where the phase angle ϕ is free parameter. Applying the standard formula of the three-body decay width from review of particle physics (RPP) [48], we have

$$\frac{d^2\Gamma}{dM_{\pi^+\eta_1} dM_{\eta_1\eta_2}} = \frac{1}{(2\pi)^3} \frac{M_{\pi^+\eta_1} M_{\eta_1\eta_2}}{8M_{D^+}^3} |\mathcal{T}^{\text{Total}}|^2. \quad (16)$$

Given a specific value of the invariant mass M_{12} , the corresponding range for the invariant mass M_{23} is determined according to the RPP [48],

$$\begin{aligned} (m_{23}^2)_{\min} &= (E_2^* + E_3^*)^2 - \left(\sqrt{E_2^{*2} - m_2^2} + \sqrt{E_3^{*2} - m_3^2} \right)^2, \\ (m_{23}^2)_{\max} &= (E_2^* + E_3^*)^2 - \left(\sqrt{E_2^{*2} - m_2^2} - \sqrt{E_3^{*2} - m_3^2} \right)^2, \end{aligned} \quad (17)$$

where E_2^* and E_3^* are the energies of particles 2 and 3 in the M_{12} rest frame, respectively,

$$\begin{aligned} E_2^* &= \frac{M_{12}^2 - m_1^2 + m_2^2}{2M_{12}}, \\ E_3^* &= \frac{M_{D^+}^2 - M_{12}^2 - m_3^2}{2M_{12}}, \end{aligned} \quad (18)$$

where m_1 , m_2 , and m_3 denote the masses of particles 1, 2, and 3, respectively. The masses and widths of the particles are taken from the RPP [48]. Permutation of the indices allows us to evaluate all three mass distributions, using M_{12} , M_{23} as independent variables, and the property $M_{12}^2 + M_{13}^2 + M_{23}^2 = M_{D^+}^2 + m_{\pi^+}^2 + m_{\eta}^2 + M_{\eta}^2$ to get M_{13} from them.

It is notable that there are two η mesons in this process so that they have $\pi^+\eta_1$ and $\pi^+\eta_2$ invariant mass distributions. In order to better compare with the experimental data, we take,

$$\frac{d\Gamma}{dM_{\pi^+\eta}} = \frac{d\Gamma}{dM_{\pi^+\eta_1}} + \frac{d\Gamma}{dM_{\pi^+\eta_2}}. \quad (19)$$

III. RESULTS AND DISCUSSIONS

As we discussed above, we can get $d\Gamma/dM_{12}$ by integrating $d^2\Gamma/(dM_{12}dM_{23})$ over M_{23} with the limits of Eq. (17). Firstly, we have only considered the contribution from $a_0(980)$ state for comparing with the BESIII measurements [24]. It is worth mentioning that, in this case, there is only one free parameter, i.e., the global normalization constant V_p . The parameter of the fit is given in Table I, and we have $\chi^2/\text{d.o.f.} = 137.93/(126 - 1) = 1.10$. We show our results of the $\pi^+\eta$ invariant mass distributions in Fig. 5(a). The blue dotted curve shows the contributions from $a_0(980)$, which includes the contributions by exchanging two η . The blue dashed curve shows the contribution from tree level. The black solid curve shows the total contribution. One can find a significant cusp structure around 980 MeV in the $\pi^+\eta$ invariant mass distribution, which could be associated with the $a_0(980)$ state. However, one can see that, in the 1000–1200 MeV region, our results are lower than the BESIII measurements [24], which implies that there could be some contributions from other intermediate states. On the other hand, we show the $\eta\eta$ invariant mass distribution in Fig. 5(b). The blue dotted curve shows the contribution

TABLE I. Fitted parameters of this work. The mass of $f_0(1370)$ is in units of MeV.

Parameters	V_p	α	$M_{f_0(1370)}$	ϕ	$\chi^2/\text{d.o.f.}$
without $f_0(1370)$	1026.9 ± 8.5				1.10
with $f_0(1370)$	1024.2 ± 29.7	0.07 ± 0.01	1331.5 ± 95.1	$(1.22 \pm 0.26)\pi$	0.96

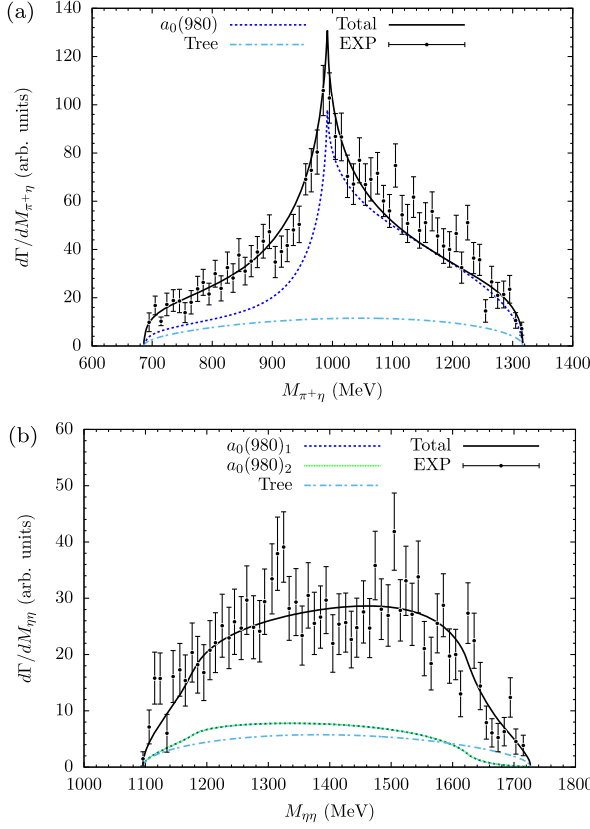


FIG. 5. $\pi^+\eta$ (a) and $\eta\eta$ (b) invariant mass distributions of the $D^+ \rightarrow \pi^+\eta\eta$ reaction without the contribution of $f_0(1370)$.

from $a_0(980)_1$ ($\pi^+\eta_1$) and the green solid curve shows the contribution from $a_0(980)_2$ ($\pi^+\eta_2$). The blue dashed curve shows the contribution from tree level. The black solid curve shows the total contribution. One can also find that, in the whole energy region, our results are in fair agreement with the BESIII measurements [24].

Furthermore, we have considered the contribution from the intermediate $f_0(1370)$ state, which can decay into $\eta\eta$ in S -wave. In this case, we have considered the $f_0(1370)$ state and the effect of interference between $f_0(1370)$ and other terms. Since the BESIII measurements have larger uncertainties, we fix the $f_0(1370)$ width $\Gamma_{f_0(1370)} = 350$ MeV in order to reduce the number of the free parameters. In our calculation, there are four free parameters, (1) the strength V_p for the contribution from the tree diagram and the $a_0(980)$; (2) the relative strength α for the contribution from $f_0(1370)$; (3) the $f_0(1370)$ mass $M_{f_0(1370)}$; (4) the interference phase angle ϕ . The parameters of the fit are given in Table I. One can find that the fitted $\chi^2/\text{d.o.f.} = 117.34/(126 - 4) = 0.96$ is smaller than the results (1.10) without the contribution from the intermediate $f_0(1370)$.

We present our results of the $\pi^+\eta$ invariant mass distributions in Fig. 6(a). The blue dotted curve shows

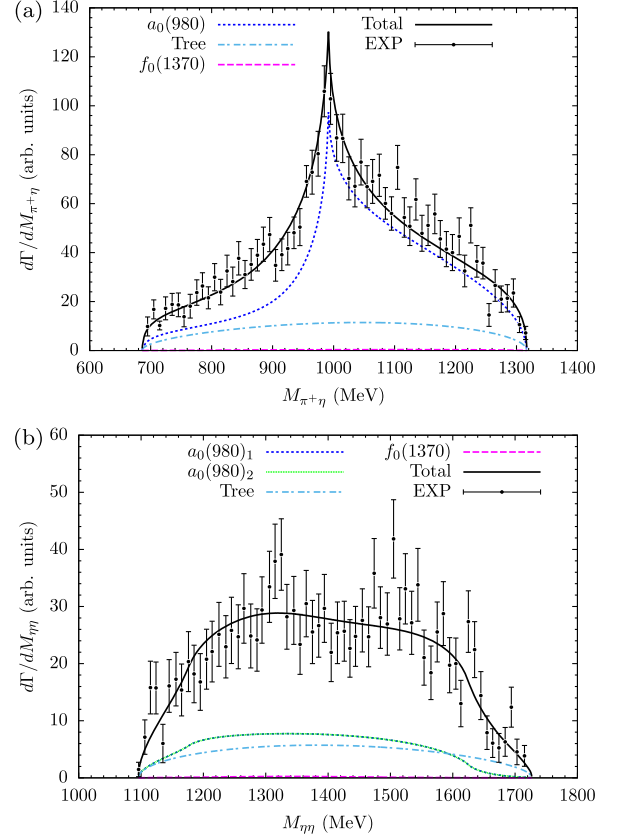


FIG. 6. $\pi^+\eta$ (a) and $\eta\eta$ (b) invariant mass distributions of the $D^+ \rightarrow \pi^+\eta\eta$ reaction with the contribution of $f_0(1370)$.

the contributions from $a_0(980)$. The blue dashed curve shows the contribution from tree level. The black solid curve shows the total contribution. One can see that, in the 1000–1200 MeV region, the results considering the contribution from $f_0(1370)$ are in good agreement with the BESIII measurements [24]. Meanwhile, we show the $\eta\eta$ invariant mass distribution in Fig. 6(b). The blue dotted curve shows the contributions from $a_0(980)_1$ ($\pi^+\eta_1$). The green solid curve shows the contributions from $a_0(980)_2$ ($\pi^+\eta_2$). The blue dashed curve shows the contribution from tree level. The black solid curve shows the total contribution. One can find that, although the contribution from the $f_0(1370)$ is small, the $\eta\eta$ invariant mass distribution with the interference effect of $f_0(1370)$ is significantly different from the previous results shown in Fig. 5(b). Indeed, the BESIII measured $\pi^+\eta$ and $\eta\eta$ still have a large fluctuation; the more precise measurements in the future could be used to constrain the contribution from the $f_0(1370)$, and determine the mass and width of $f_0(1370)$.

Finally, we show the Dalitz plots of “ $M_{\pi^+\eta}^2$ ” vs “ $M_{\pi^+\eta}^2$ ” and “ $M_{\pi^+\eta}^2$ ” vs “ $M_{\eta\eta}^2$ ” with the contribution from $f_0(1370)$ state in Figs. 7 and 8, respectively. One can find the clear signal of the $a_0(980)$ around $M_{\pi^+\eta}^2 = 0.96 \text{ GeV}^2$.

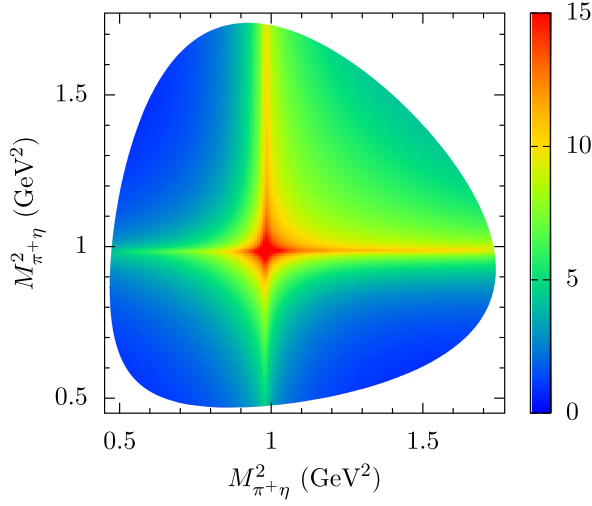


FIG. 7. Dalitz plot of “ $M_{\pi^+\eta}^2$ ” vs “ $M_{\pi^+\eta}^2$ ” for the double differential decay width $d^2\Gamma/dM_{\pi^+\eta}dM_{\pi^+\eta}$ of the process $D^+ \rightarrow \pi^+\eta\eta$.

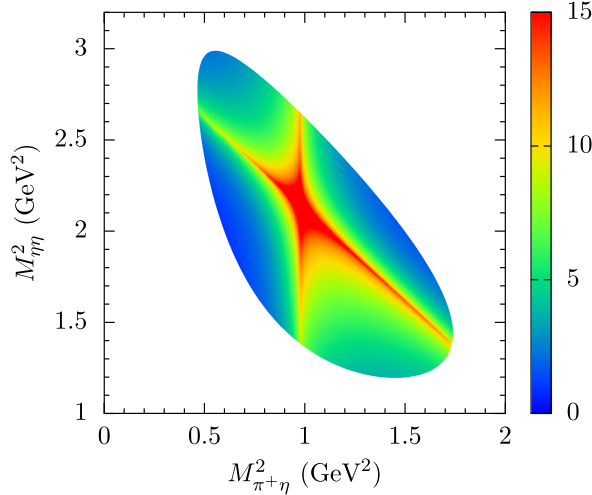


FIG. 8. Dalitz plot of “ $M_{\pi^+\eta}^2$ ” vs “ $M_{\eta\eta}^2$ ” for the double differential decay width $d^2\Gamma/dM_{\pi^+\eta}dM_{\eta\eta}$ of the process $D^+ \rightarrow \pi^+\eta\eta$.

IV. CONCLUSIONS

Recently, the BESIII Collaboration has measured the process $D^+ \rightarrow \pi^+\eta\eta$ using 20.3 fb^{-1} of e^+e^- collision data at the center-of-mass energy 3.773 GeV , and found a clear cusp structure in the $\pi^+\eta$ invariant mass distribution, which should be associated with the scalar $a_0(980)$ [24]. The BESIII amplitude analysis of this process suggests that the $a_0(980)$ line shape cannot be well described by the

three-channel coupled Flatté formula for the $a_0(980)$, and could arise from the triangle loop rescattering of $D^+ \rightarrow \bar{K}_0^*(1430)K^+ \rightarrow a_0(980)^+\eta$ with a significance of 5.8σ [24]. However, the triangle singularity of $D^+ \rightarrow \bar{K}_0^*(1430)K^+ \rightarrow a_0(980)^+\eta$ appears at $M_{\pi^+\eta} = 1073 \text{ MeV}$, about 80 MeV away from the nominal $a_0(980)$ mass, which implies that the triangle diagram proposed by BESIII [24] is far away from developing a triangle singularity.

According to BESIII measurements of the process $D^+ \rightarrow \pi^+\eta\eta$, one can find that the $\eta\eta$ invariant mass distribution is significantly different from the phase space distribution in the region $1300\text{--}1500 \text{ MeV}$ [24]. Considering that the scalar meson $f_0(1370)$ could decay into $\eta\eta$ [48], there could be a contribution from the $f_0(1370)$ state.

Therefore, in this work, we have investigated this process by considering the contribution from the resonance $a_0(980)$, which is generated by S -wave pseudoscalar meson-pseudoscalar meson interaction within the chiral unitary approach, and the contribution from the intermediate scalar meson $f_0(1370)$.

One can find that our results including the contribution from the intermediate $f_0(1370)$, neglected in BESIII analysis, give a good description of the $\pi^+\eta$ and $\eta\eta$ invariant mass distributions, which supports the molecular nature of the $a_0(980)$, and implies that the $f_0(1370)$ plays a sizeable role. Since the properties of the scalar $f_0(1370)$ have not been well established and the BESIII data have large fluctuation, the more precise measurements about this process by Belle II and the proposed STCF could shed light on the mechanism of this process and the nature of $f_0(1370)$.

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DATA AVAILABILITY

The data are not publicly available. The data are available from the authors upon reasonable request.

- [1] M. Gell-Mann, *Phys. Lett.* **8**, 214 (1964).
- [2] G. Zweig, [10.17181/CERN-TH-401](#).
- [3] S. K. Choi *et al.* (Belle Collaboration), *Phys. Rev. Lett.* **91**, 262001 (2003).
- [4] H. X. Chen, W. Chen, X. Liu, Y. R. Liu, and S. L. Zhu, *Rep. Prog. Phys.* **86**, 026201 (2023).
- [5] N. Brambilla, S. Eidelman, C. Hanhart, A. Nefediev, C. P. Shen, C. E. Thomas, A. Vairo, and C. Z. Yuan, *Phys. Rep.* **873**, 1 (2020).
- [6] L. Meng, B. Wang, G. J. Wang, and S. L. Zhu, *Phys. Rep.* **1019**, 1 (2023).
- [7] D. Guo, Q. H. Yang, L. Y. Dai, and A. P. Szczepaniak, *Eur. Phys. J. C* **85**, 389 (2025).
- [8] M. Z. Liu, Y. W. Pan, Z. W. Liu, T. W. Wu, J. X. Lu, and L. S. Geng, *Phys. Rep.* **1108**, 1 (2025).
- [9] Q. H. Yang, L. Y. Dai, and U. G. Meißner, [arXiv:2412.07599](#).
- [10] E. Wang, L. S. Geng, J. J. Wu, J. J. Xie, and B. S. Zou, *Chin. Phys. Lett.* **41**, 101401 (2024).
- [11] F. E. Close and N. A. Tornqvist, *J. Phys. G* **28**, R249 (2002).
- [12] C. Amsler and N. A. Tornqvist, *Phys. Rep.* **389**, 61 (2004).
- [13] D. V. Bugg, *Phys. Rep.* **397**, 257 (2004).
- [14] E. Klempt and A. Zaitsev, *Phys. Rep.* **454**, 1 (2007).
- [15] J. R. Pelaez, *Phys. Rep.* **658**, 1 (2016).
- [16] J. Nieves and E. Ruiz Arriola, *Phys. Lett. B* **455**, 30 (1999).
- [17] G. Janssen, B. C. Pearce, K. Holinde, and J. Speth, *Phys. Rev. D* **52**, 2690 (1995).
- [18] T. Wolkanowski, F. Giacosa, and D. H. Rischke, *Phys. Rev. D* **93**, 014002 (2016).
- [19] R. L. Jaffe, *Phys. Rev. D* **15**, 267 (1977).
- [20] R. L. Jaffe, *The Gregory Breit Centennial Symposium* (World Scientific, Singapore, 2001).
- [21] R. L. Jaffe, *AIP Conf. Proc.* **964**, 1 (2007); *Prog. Theor. Phys. Suppl.* **168**, 127 (2007).
- [22] M. Ablikim *et al.* (BESIII Collaboration), *Phys. Rev. D* **101**, 052009 (2020).
- [23] N. Ikeno, M. Bayar, and E. Oset, *Eur. Phys. J. C* **81**, 377 (2021).
- [24] M. Ablikim *et al.* (BESIII Collaboration), [arXiv:2505.12086](#).
- [25] M. Bayar, F. Aceti, F. K. Guo, and E. Oset, *Phys. Rev. D* **94**, 074039 (2016).
- [26] J. A. Oller and E. Oset, *Nucl. Phys. A* **620**, 438 (1997); **A652**, 407(E) (1999).
- [27] Z. Wang, Y. Y. Wang, E. Wang, D. M. Li, and J. J. Xie, *Eur. Phys. J. C* **80**, 842 (2020).
- [28] J. J. Xie, L. R. Dai, and E. Oset, *Phys. Lett. B* **742**, 363 (2015).
- [29] E. Oset, W. H. Liang, M. Bayar, J. J. Xie, L. R. Dai, M. Albaladejo, M. Nielsen, T. Sekihara, F. Navarra, L. Roca *et al.*, *Int. J. Mod. Phys. E* **25**, 1630001 (2016).
- [30] M. Y. Duan, J. Y. Wang, G. Y. Wang, E. Wang, and D. M. Li, *Eur. Phys. J. C* **80**, 1041 (2020).
- [31] X. Zhu, H. N. Wang, D. M. Li, E. Wang, L. S. Geng, and J. J. Xie, *Phys. Rev. D* **107**, 034001 (2023).
- [32] X. C. Feng, L. L. Wei, M. Y. Duan, E. Wang, and D. M. Li, *Phys. Lett. B* **846**, 138185 (2023).
- [33] G. Y. Wang, N. C. Wei, H. M. Yang, E. Wang, L. S. Geng, and J. J. Xie, *Phys. Rev. D* **106**, 056001 (2022).
- [34] J. Y. Wang, M. Y. Duan, G. Y. Wang, D. M. Li, L. J. Liu, and E. Wang, *Phys. Lett. B* **821**, 136617 (2021).
- [35] W. H. Liang, J. J. Xie, and E. Oset, *Eur. Phys. J. C* **76**, 700 (2016).
- [36] J. X. Lin, J. T. Li, S. J. Jiang, W. H. Liang, and E. Oset, *Eur. Phys. J. C* **81**, 1017 (2021).
- [37] M. Y. Duan, W. T. Lyu, C. W. Xiao, E. Wang, J. J. Xie, D. Y. Chen, and E. Oset, *Phys. Rev. D* **111**, 016004 (2025).
- [38] M. Y. Li, W. T. Lyu, L. J. Liu, and E. Wang, *Phys. Rev. D* **111**, 034046 (2025).
- [39] W. T. Lyu, S. C. Zhang, G. Y. Wang, J. J. Wu, E. Wang, L. S. Geng, and J. J. Xie, *Phys. Rev. D* **110**, 054020 (2024).
- [40] X. H. Zhang, H. Zhang, B. C. Ke, L. J. Liu, D. M. Li, and E. Wang, *Phys. Rev. D* **110**, 114050 (2024).
- [41] W. H. Liang, H. X. Chen, E. Oset, and E. Wang, *Eur. Phys. J. C* **79**, 411 (2019).
- [42] F. Close, *An Introduction to Quarks and Partons* (Academic Press, London, 1979), ISBN 978-0-12-175152-4.
- [43] K. Miyahara, T. Hyodo, M. Oka, J. Nieves, and E. Oset, *Phys. Rev. C* **95**, 035212 (2017).
- [44] H. Zhang, Y. H. Lyu, L. J. Liu, and E. Wang, *Chin. Phys. C* **47**, 043101 (2023).
- [45] X. Zhu, D. M. Li, E. Wang, L. S. Geng, and J. J. Xie, *Phys. Rev. D* **105**, 116010 (2022).
- [46] Y. W. Peng, W. Liang, X. Xiong, and C. W. Xiao, *Phys. Rev. D* **110**, 036013 (2024).
- [47] F. K. Guo, R. G. Ping, P. N. Shen, H. C. Chiang, and B. S. Zou, *Nucl. Phys. A* **773**, 78 (2006).
- [48] S. Navas *et al.* (Particle Data Group), *Phys. Rev. D* **110**, 030001 (2024).
- [49] D. V. Bugg, *Eur. Phys. J. C* **52**, 55 (2007).
- [50] W. Ochs, *J. Phys. G* **40**, 043001 (2013).
- [51] R. Molina, D. Nicmorus, and E. Oset, *Phys. Rev. D* **78**, 114018 (2008).
- [52] L. S. Geng and E. Oset, *Phys. Rev. D* **79**, 074009 (2009).