

Condensate phases of nuclear matter from AdS hardwall models

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This work develops our previous study of confined phases at finite densities in AdS/QCD by systematically exploring the possibility of baryonic condensates. Using phenomenologically motivated boundary conditions in an AdS hardwall model, we show that both baryonic and quark-type condensates dominate the phase diagram at low temperatures. We also undertake a careful scan of the parameter space to extract robust conclusions.

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I. INTRODUCTION

Compact stars are remnants of old massive stars that have gone through a supernova explosion, providing a unique opportunity to study matter at high density and low temperature. Inside these stars, matter ranges from nuclei at low densities in the crust through highly neutron-rich uniform matter in the outer core leading to a variety of possible condensates, such as baryon superfluids (and/or superconductors), meson condensates, and even quark pairs in the inner core (a fairly modern summary can be found in the excellent book [1]). All of these can have a considerable impact on the characteristics of the compact star [2,3]. QCD matter at high densities and low temperatures thus presents an exciting opportunity to explore a novel condensed matter system with multiple mass scales. However, quantitative analysis of the QCD phase diagram and/or equation of state (EOS) in this regime is a hard problem with multiple unanswered questions [4]. Perturbative QCD analysis is only reliable at high densities or temperatures where the effective degrees of freedom are nearly free quarks. Lattice QCD and effective theory methods (such as chiral perturbation theory) and mean-field theory approximations [5] are employed at low densities and temperatures. However, it is difficult to even assess their validity at mid-to-high density. Therefore, it is important to study this

system using other techniques in the hope that a consensual understanding of the phase diagram will emerge.

Since the formulation of gauge/gravity duality (or holography) in the seminal work by Maldacena [6], the AdS/CFT correspondence has been used to study the nonperturbative strong coupling effects in the field theory using the gravitational description. Significant effort has been devoted to studying QCD-like boundary dual theories through modified bulk gravity theories going far beyond the original conformal duality. This area of research, referred to as AdS/QCD, has been extensively explored using both phenomenologically motivated bottom-up approach that captures the essential features of the boundary theory, such as hardwall [7], softwall [8], and V-QCD models [9], or a more rigorous top-down approach based on the first principle string theory (see [10–16]). Anti-de Sitter (AdS) holography has been applied in a variety of contexts, including the QCD plasma [17], the Regge behavior of hadrons [18], and chiral symmetry breaking [7]. In particular, significant amount of work has been done in obtaining the EOS of dense QCD matter using holography [19–35]. However, the possibility of condensation instabilities has not been evaluated in QCD related contexts.

From the phenomenology of QCD at high densities, we can expect three kinds of condensates to occur in such systems. Firstly, mesons such as pions, kaons, etc., can undergo Bose condensation restricted by charge conservation and beta equilibrium. Secondly, neutron, proton, and quark condensates associated with superfluidity and superconductivity have been suggested at super nuclear densities. At higher densities, quark pairs forming color flavor locked (CFL)-type phases are also possible. Finally, we have the all-pervasive chiral condensate associated with the QCD vacuum and its vanishing at deconfinement.

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These condensed phases are plausibly present in the interior of neutron stars (for instance, see [36] and citations thereof). However, the properties of these phases, studied using effective field theory, show a strong dependence on the parameter values and the operators included in the model. Additionally, these approaches do not necessarily respect causality and stability constraints, particularly when extrapolated beyond empirically constrained regions. For instance, in certain extrapolated regimes, the pressure may increase while the energy density remains constant, leading to a divergent speed of sound $c_s^2 = \partial P / \partial \epsilon \rightarrow \infty$, as discussed in [37]. Similar superluminal behavior may appear, such as in [38], where $c_s^2 > 1$ in specific density intervals. Moreover, standard parametric representations of the neutron star equation of state, including piecewise polytropes and spectral fits, typically satisfy thermodynamic stability but do not enforce causality by construction, see [39].

On the other hand, the holographic approach is rather easier and is surprisingly consistent with QCD phenomenology in the regions (of temperature, density, and couplings) where it has been compared. The equations of state obtained from AdS duals respect causality at least as long as the bulk matter obeys standard positive energy conditions. Such studies will present us with a range of possibilities that will help us consolidate our expectations about QCD proper.

The hardwall approach to QCD is particularly interesting in this context since the hardwall geometry is expected to capture features common to other holographic QCD approaches in the region between the IR and UV cutoffs, as we will explain in more detail below. Phase diagrams using the hardwall model at finite baryon density were studied in [40,41], with the latter including confined phases at low density and temperatures.

We therefore attempt to construct a simple holographic model which gives a complete phase diagram *including quark condensates* and backreaction. This study should be viewed as an intermediate step in building a *complete* holographic model for realistic nuclear matter at high baryonic and isospin densities.

A key step to studying QCD condensates was the identification of a dual to a confined phase at low temperatures. This dual, known as the charged AdS (CAdS) geometry, was explored at length in [41]. This work develops the previous exploration by adding bulk degrees of freedom dual to condensates in the boundary theory.

This document is organized as follows. The first section explains the new features arising from the addition of a complex scalar dual to the condensing operator. In particular, we discuss the choices of the scaling dimension and baryonic charge. Following a short summary of the finite density phases without condensates, we discuss the phase diagrams obtained with the simplest boundary condition to understand the effect of the various parameters separately. Using this knowledge, we construct phase diagrams using

phenomenologically motivated boundary conditions and show that condensed phases dominate at low temperatures. These include condensates in confined phase (charged AdS) at low densities, giving way to condensates in deconfined phase (charged black hole) at higher densities. We end with a discussion motivating further studies.

II. ADS HARDWALL MODELS

The AdS hardwall models attempt to extend AdS holography to systems such as QCD which are not scale invariant. These involve imposing IR cutoff(s) on the bulk radial coordinate z which breaks conformal symmetry and enables the possibility of a confinement deconfinement transition [42]. In a recent study [41], we further explored these models at finite baryon density by analyzing a class of solutions we call CAdS. Incorporating ‘phenomenological’ boundary conditions on the fields of the bulk theory at the IR boundary, we were able to obtain phase diagrams in the $\mu_B - T$ plane which reasonably reflect expectations about the QCD phase diagram. In that work however, we did not consider thermodynamic instabilities that could lead to condensates. In this work, we expand on the previous work by including a complex scalar field that allows for the possibility of breaking the $U(1)_B$ symmetry spontaneously. The work of [43] showed that in an AdS black hole background, at sufficiently high density, this indeed happens. The question, then, is whether this spontaneous breakdown occurs in the hardwall models which could indicate that QCD is prone to such condensation.

Thus, we write a minimally coupled Einstein-Maxwell-Scalar action in five dimensions,

$$S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} (R - 2\Lambda_G) - \frac{1}{g_5^2} \int d^5x \sqrt{-g} \frac{F^2}{4} - \lambda_s \int d^5x \sqrt{-g} (|D\psi|^2 + m^2|\psi|^2), \quad (1)$$

which has been shown to lead to symmetry breaking and condensates [43]. It is remarkable that the scalar field condenses without the need for a potential. The coupling constants in the action are

$$\kappa^2 = \frac{4\pi^2 L^3}{N_c^2}; \quad g_5^2 = \frac{24\pi^2 L}{\theta N_f N_c}; \quad \lambda_s = 1. \quad (2)$$

The cosmological constant in the Einstein-Hilbert part of the action is set to be $2\Lambda_G = -\frac{12}{L^2}$, so that the asymptotic solution is AdS_5 with a length scale L . N_c , N_f are the number of colors and flavors, respectively. We emphasize that our use of holographic techniques involves an extrapolation from the large N_c limit, where the AdS/CFT correspondence is theoretically well-established, to the case of $N_c = 3$ relevant for real-world QCD. While this extrapolation is not strictly justified from first principles,

it is often employed with the expectation that certain qualitative features of strongly coupled gauge theories persist at finite N_c . Accordingly, our results should be interpreted as providing qualitative insights rather than precise quantitative predictions. The parameter θ can be motivated, from a top-down 10D perspective, as arising from the volumes of internal manifolds. It can also be regarded as arising from dilaton fields which are introduced as being dual to the running coupling of QCD in AdS/QCD models such as the softwall. In QCD terms, this parameter captures a deviation of the current-current two-point function from the perturbative expression which corresponds to $\theta = 1$. A ratio of these coupling constants,

$$\zeta = \frac{2\kappa^2}{3g_5^2} = \frac{\theta N_f L^2}{9N_c}, \quad (3)$$

controls the effect of the gauge field on the background and large N_c corresponds to the probe approximation. We will present our results for a few values of ζ around $\zeta = 0.77$ (which is the value that matches a Boltzmann gas of nuclear matter at high temperatures [23]).

Note that if the scalar field action is also multiplied by N_f , then we may interpret it as dual to the condensate of all flavors of quark rather than a single flavor. In our work, we shall regard the scalar field as representing the condensates of only the lightest quarks (or baryons). Since the action of the scalar field ψ is quadratic, by rescaling ψ we can set $\lambda_s = 1$. In the action, while all coordinates have length dimensions, those of the various fields are $[F] = [l]^{-2}$, $[\psi] = [l]^{-1}$, $[\lambda_s] = [l]^{-1}$, and $[m] = [l]^{-1}$.

The standard AdS/CFT dictionary states that the mass m of the scalar field controls the scaling dimension Δ of the scalar operator in the boundary (or scaling dimension of the condensate operator) [44,45] via the equation $m^2 L^2 = \Delta(\Delta - 4)$. The gauge transformations,

$$\psi \rightarrow e^{iq\alpha(x)}\psi, \quad (4)$$

$$A_\mu \rightarrow A_\mu + \partial_\mu \alpha, \quad (5)$$

fix the covariant derivative to be $D_\mu = \partial_\mu - iqA_\mu$.

The charge q of the scalar field controls the interpretation of the scalar field as a quark or baryon condensate in the following sense. Since the charge associated with the gauge field A_μ is identified with the *quark* number, if the *scalar* field ψ has charge 2, we may interpret ψ to be the dual to a quark bilinear operator. In the physically relevant case of $N_c = 3$, such an operator may correspond to a color-singlet diquark, formed by contracting two quark fields with an appropriate color and flavor structure to ensure gauge invariance (see, e.g., [3]). The color neutrality of this operator is captured by an N_c prefactor in the action of the scalar field, expressing that all colors are equally condensed. On the other hand, if it has a charge $2N_c$ under

the gauge field, we can say that it is dual to a baryon condensate since a baryon involves N_c quarks.

The mass of the scalar field is not as easily fixed. From a fundamental perspective, the mass of the bulk scalar field can be $m \sim \frac{1}{L}$ (for instance, it can arise by Kaluza-Klein reduction from 10D) or $m \sim \frac{N_c}{L_s} = \frac{\lambda_s^{1/4} N_c}{L}$ if it is an essentially “stringy” mode. Both these choices make sense from the QCD perspective as well. We can regard the operator dual to ψ as composed of $2N_c$ quarks and hence with scaling dimension $\Delta \sim N_c$ or as an effective bosonic operator with scaling dimension $\Delta \sim \mathcal{O}(1)$. This assignment of the mass need not be related to the charge assignment either: for weakly interacting quarks, the charge and scaling dimension will both be proportional to N_c , but when they are strongly interacting, the mass can be much smaller with significant binding energy. We will revisit these options in Sec. VII.

A. Equations of motion

We assume a diagonal metric Ansatz,

$$ds^2 = -\frac{g(z)}{h(z)} L^2 dt^2 + \frac{L^2}{z^2} d\vec{x}^2 + \frac{L^2}{g(z)} dz^2. \quad (6)$$

The coordinates and fields are now rescaled by L so that they are dimensionless. In particular, the metric functions g , h , the scalar gauge potential $A_0 = \phi(z)$, and the scalar field ψ will also become dimensionless. The AdS radius L can then be set to unity without any loss of generality. In these units, the (conformal) boundary is taken to be located at $z = \epsilon = 10^{-4}$ in the numerical computations. Finally, we use the gauge fixing condition $A_z = 0$ which combines with rotation and translation invariance to force $A_i = 0$, $i = 1, 2$, and 3. Finally, the residual gauge symmetry is fixed by requiring that $\psi(z)$ be real. The dimensionless equations of motion with these Ansätze,

$$g' - g\left(\frac{4}{z} + \frac{h'}{2h}\right) + 4z = \frac{2\zeta z}{L^2} \left(\frac{h\phi'^2}{4} + \frac{m^2 L \lambda_s g_5^2 \psi^2}{2}\right), \quad (7)$$

$$\frac{h'}{h} - \frac{4}{z} = \frac{2\zeta \lambda_s g_5^2 z}{L^3} \left(\psi'^2 + \frac{q^2 \psi^2 \phi^2}{g^2} h\right), \quad (8)$$

$$\phi'' - \phi' \left(\frac{3}{z} - \frac{h'}{2h}\right) = \lambda_s g_5^2 \frac{2q^2 \psi^2}{g(z)L} \phi, \quad (9)$$

$$\psi'' + \psi' \left(-\frac{3}{z} + \frac{g'}{g} - \frac{h'}{2h}\right) = m_{\text{eff}}^2 \psi, \quad (10)$$

where, $t \equiv \frac{d}{dz}$ and, the effective mass squared, defined as

$$m_{\text{eff}}^2 = \frac{m^2}{g(z)} - \frac{q^2 h(z) \phi(z)^2}{g(z)^2} \quad (11)$$

is a function of the radial coordinate z . It has been shown in [46] that, for condensation to occur, it is not only necessary that $m_{\text{eff}}^2(z) < 0$, but it must remain negative over a sufficiently extended region in z .

These equations are seen to be invariant under the following transformations:

$$(\vec{x}, t, z) \rightarrow \alpha(\vec{x}, t, z), \quad \phi \rightarrow \frac{\phi}{\alpha}, \quad g \rightarrow \alpha^2 g, \quad h \rightarrow \alpha^4 h, \quad (12)$$

$$h \rightarrow \frac{h}{\beta^2}, \quad \phi \rightarrow \phi\beta. \quad (13)$$

The former corresponds to a scale symmetry, while the latter is expected from time rescaling invariance. These can be fully eliminated by a hardwall cutoff $z_0 = 1$ and setting $h(\epsilon) = 1$, respectively.

B. Boundary conditions and parameters

As the differential equations indicate, we need two boundary conditions on ϕ, ψ [(9), (10)] and one each on h, g [(8), (7)] to obtain a unique solution. Three of these are determined by physical requirements imposed at the UV boundary, $z = \epsilon$. The Frobenius analysis of the equations of motion near the UV boundary ($z \rightarrow \epsilon$) gives the expansion:

- (i) $\phi \sim \mu - Qz^2 + \dots$;
- (ii) $\psi \sim \psi_+ z^{\Delta_+} + \psi_- z^{\Delta_-} + \dots$ where, $\Delta_{\pm} = 2 \pm \sqrt{4 + m^2 L^2}$;
- (iii) $g \sim z^2(1 + cz^4 + \dots)$;
- (iv) $h \sim z^4(1 + \tilde{h}\psi_-^2 z^{2\Delta_-} + \dots)$;

defining the quark chemical potential $\mu_Q = \mu = \mu_B/N_c$. Additionally, since we require that the $U(1)_B$ symmetry is *spontaneously* broken, the solutions must satisfy $\psi_- = 0$ [46]. Finally, we fix the scaling of the boundary time coordinate by requiring that $h(\epsilon) = 1$ *after* fixing the hardwall cutoff z_0 . The remaining three boundary conditions will be specified by fixing the fields g, ϕ, ψ at the IR boundary.

III. CONFINED PHASES AT FINITE DENSITY: A REVIEW

We will first summarize the results for the phase diagram obtained in [41] at finite chemical potential in the *absence* of condensates. For a trivial scalar field $\psi = 0$, the equations of motion (7), (8), (9) can be solved analytically. At finite temperature and chemical potential, three possible solutions are

- (i) Thermal AdS (thAdS)

$$g(z) = z^2; \quad h(z) = z^4; \quad \phi(z) = \mu \quad (14)$$

with zero charge density;

- (ii) Charged black hole (CBH) with density $\rho = \frac{\mu}{z_H}$

$$g(z) = z^2 \left(1 - cz^4 + \frac{\zeta \mu^2}{z_H^4 L^2} z^6 \right),$$

$$h(z) = z^4, \quad \phi(z) = \mu \left(1 - \frac{z^2}{z_H^2} \right). \quad (15)$$

where z_H is the black hole horizon. The parameter c is fixed by the condition that z_H is the smallest root of $g(z_H)$ for each μ ;

- (iii) Charged AdS (CAdS)

$$g(z) = z^2 \left(1 - cz^4 + \frac{\zeta Q}{L^2} z^6 \right),$$

$$h(z) = z^4, \quad \phi(z) = \mu - Qz^2. \quad (16)$$

with $\frac{\zeta^2 Q^4}{L^4 c^4} > \frac{4}{27}$ implying that the function $g(z)$ never has a root.

Here, Q and c [or equivalently, $g(z_0)$] are free parameters that have to be fixed by applying boundary conditions to determine the solutions uniquely. The finite charge density Q in this class of solutions were attributed to sources for the gauge field that are localized behind the cutoff surface. In this view, we imagine the IR region $z > z_0$ in the bulk to involve a distribution of baryon sources such as wrapped branes or fundamental strings. Such sources have been shown to be “pulled” to the far IR regions [47–49] in AdS geometries which justifies the expectation (the last reference importantly points out that the chiral condensate prevents the baryons from “falling all the way”). The distribution of these branes along the radial and transverse directions [29] can be expected to capture the pressure and energy density of the baryon distribution in the boundary theory. In the hardwall approach, this distribution will determine the gravity solution in the region $0 < z < z_0$ via the boundary conditions at the IR cutoff.

The thermodynamic pressure computed in the hardwall approach turns out to be

$$P = \frac{N_c^2}{8\pi^2} \left(\frac{g_0 + g_B(\mu_B) + z_0^2}{z_0^6} + \frac{\zeta Q^2 z_0^2}{L^2} \right), \quad (17)$$

where $g_B(\mu_B)$ and the last term depending on Q will be attributed to the baryon distribution. We fix the constant $g_0 = 6z_0^2$ by computing the ρ -meson mass (see [41] for details). The parameter $2\epsilon Q \equiv \phi'(\epsilon)$ is fixed by the quark density ρ_Q (determined by other means as a function of μ_B),

$$Q = \frac{12\pi^2 L}{\theta N_f N_c} \rho_Q, \quad (18)$$

where the prefactor is the standard AdS/CFT normalization. Finally, equating the total pressure $p = p_B + \frac{N_c^2}{8\pi^2} \frac{g_0 + z_0^2}{z_0^6}$,

allows us to fix

$$g(\mu_B) = \frac{8\pi^2}{N_c^2} p_B(\mu_B) - \frac{\zeta Q^2 z_0^2}{L^2} \quad (19)$$

with the baryonic contribution p_B determined by other means such as the dynamics of the sources that are localized behind the cutoff. Postponing an analysis of the holographic source distribution, in [41], we modeled the answers using *boundary* phenomenology. The reason for this choice was twofold. Firstly, in QCD literature, phenomenological models such as the Nambu–Jona-Lasinio (NJL) or chiral perturbation theory are widely used to compute the equations of state at finite baryon density under the assumption that these *do* accurately model the baryon distribution. Thus, the AdS source distribution should agree with these models in their regions of validity. Secondly, our focus is on investigating condensation instability due to additional operators being switched on using the hardwall approach. This work is a preliminary in how to use the hardwall model to study condensation. To that extent, using phenomenology derived boundary conditions seem a reasonable choice.

The IR boundary condition for the gauge potential is also specified by phenomenology,

$$\phi'(z_0) = \frac{z_0^3}{\sqrt{h(z_0)}} \frac{24\pi^2 L}{\theta N_f N_c} \rho_Q. \quad (20)$$

This equation is nothing but Gauss' law which attributes the electric flux at the cutoff surface on the rhs to the baryonic number density ρ_Q located behind the cutoff surface (the normalization is determined by the usual AdS/CFT). The chemical potential, defined as the UV value,

$$\phi(\epsilon) = \mu_Q = \frac{\mu_B}{N_c}, \quad (21)$$

allows us to fix the cutoff value of the gauge potential by integrating the solution

$$\phi(\epsilon) - \phi(z_0) = \int dz \phi'(z). \quad (22)$$

The solutions can then be found by using a shooting method starting from the IR cutoff $z_0 = 1$ (This sets $g_0 = 6$.) to the UV cutoff ϵ with these boundary conditions.

A. Phenomenology

At densities around the nuclear saturation density $\rho_0 = 0.157 \text{ fm}^{-3}$, in-medium chiral perturbation theory (χ PT) [50] indicates that baryons form a strongly interacting gas of nucleons described by a van der Waals type of equation of state including a gas-liquid transition. This

calculation is sensible only at densities below $3.3\rho_0$ and temperature $T \lesssim 50 \text{ MeV}$, while the liquid gas critical point occurs below ρ_0 and $T \sim 16 \text{ MeV}$ (these numbers are fits to the data mentioned in [50]).

At higher densities, due to strong overlap among the baryons, a ‘quarkyonic’ phase is expected to be a better description of dense nuclear matter. This phase is described as confined nuclear matter where the effective degrees of freedom are nevertheless quarklike [51,52] with a *quark* Fermi surface. But, the chiral condensate is still nonzero indicating significant binding energy among these quarks. In our project, we simply use the NJL model as capturing this phase since the quarks in this model, which interact via four and six Fermi interactions in the presence of a chiral condensate, are effective degrees of freedom. Note that it is by no means obvious that the NJL description ought to be ‘better’ at higher densities per the logic presented. The NJL model is often variously augmented by including a variety of condensates and possibly the Polyakov loop to include effects of confinement. In our work, we use the two flavor NJL without the Polyakov loop as a description of the IR physics. The idea being that using the AdS geometry together with the g_0 parameter will capture some aspects of confinement dynamics represented by the Polyakov loop. We note that the NJL model includes a UV cutoff Λ and can be expected to be sensible only for chemical potentials $\mu_Q < \Lambda$. The reason for using the *two* flavor model is that the in-medium chiral perturbation theory which results in the van der Waals (vdW) gas description does not consider the strange quark baryons. It seems appropriate to confront the two flavor EFT with the two flavor NJL.

We are therefore assuming that as we increase the density, the effective degrees of freedom change from strongly interacting baryons to strongly interacting quarks prompting the use of two different descriptions for the same nuclear matter system. Therefore, we will use both these equations of state to determine the boundary conditions. From the point of view of holography, these two models (i.e., χ PT and NJL model pressures and density) maybe regarded as arising from two different baryonic distributions in the deep IR. As the density increases, the spatial distribution can be expected to change. Therefore the saddle point of the AdS path integral will change leading to the change in pressure.

Note that using an effective field theory (EFT) description to capture the region $IR \ z > z_0$ in AdS is self-consistent with the idea that those holographic modes which are dual to low energy degrees of freedom are localized in the deep interior of AdS. If so, excising this region $z > z_0$ and supplying boundary conditions produced by a low-energy EFT is the correct thing to do.

The two questions of interest are which description is ‘better’ at higher densities (in the sense of leading to larger pressures) and whether these models possess an instability to forming condensates.

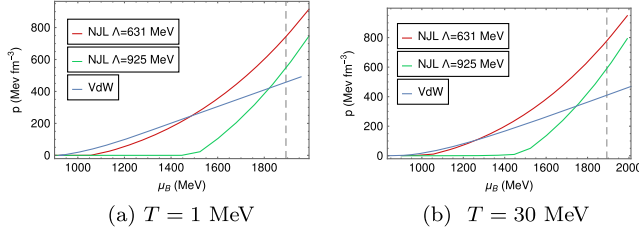


FIG. 1. Pressure comparison of NJL and vdW. The gray dashed line depicts the limit of validity for $\Lambda = 631$ MeV.

B. Phase diagram without condensates

Using p_B and ρ_Q as computed from these models (i.e., χ PT and NJL model) in the boundary conditions, we plot the pressures computed from the hardwall model in Fig. 1. While the blue curve represents the vdW EOS, the green and red curves represent pressures computed from an NJL model with UV cutoffs $\Lambda = 925$ MeV and $\Lambda = 631$ MeV, respectively. These choices of parameters are the ones discussed in the primary reference [53].

The first point to observe is the strong parameter dependence of the pressure computed from the NJL model. If the NJL model were to reliably predict the pressure of baryonic matter, we must expect the red and green curves to overlap in some region of density. However, these two already differ in their predictions for the chiral transition and meson melting as shown in the reference cited. Thus, we see that the two NJL models produce significantly different pressures at low densities. Additionally, at very low temperatures, only the pressures computed using the NJL model with $\Lambda = 631$ MeV agrees reasonably well with the vdW EOS. The agreement improves with temperature although the range of agreement decreases sharply.

Nevertheless, we see that beyond some density, the NJL description has larger pressure thereby vindicating our choice of the NJL description for higher densities. The “phase” transition point, shown in Fig. 1 for two different values of temperature, however, depends strongly on the choice of the parameters in the NJL model. For either choice of the NJL cutoff, we see that the quark description leads to higher pressures at large densities. While the transition for $\Lambda = 631$ MeV, which occurs at around $\mu_B = 1500$ MeV, is quite close to the 3Λ limit of validity, for $\Lambda = 925$ MeV, the change occurs well within the region of validity. But, it is noteworthy that the transition chemical potential is significantly altered by temperature and already at $T = 30$ MeV, even the NJL model with $\Lambda = 631$ MeV cutoff predicts a change in description at $\mu_B \sim 1300$ MeV.

From this comparison, we draw the conclusion that, while at low densities, QCD is a strongly interacting vdW gas/liquid, already at intermediate densities, the constituent quark degrees of freedom play a significant role. We may also infer that as temperature increases, this change in description occurs at lower densities.

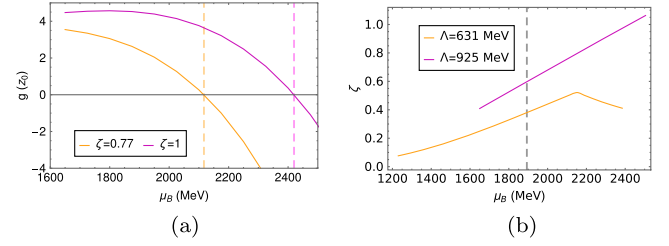


FIG. 2. (a) $g(z_0)$ as function of μ_B for $\Lambda = 925$ MeV. The vertical dashed line represents the presence of the horizon in the NJL phase. (b) ζ vs. μ_B for $g(z_0) = 0$ for different values of Λ . The vertical gray dashed line represents the region of validity for $\Lambda = 631$ MeV.

1. Deconfinement

At even higher densities, a *deconfined* phase of quarks may be expected and is usually modeled, holographically, by a CBH. Thus, we can search for a transition to a higher density phase modeled by a charged black hole in AdS.

Interestingly, as illustrated in Fig. 2(a), the metric function $g(z)$ develops a horizon as we increase the baryon chemical potential μ_B within the NJL phase for $\Lambda = 925$ MeV. That is to say, the blackening factor, $g(z)$, vanishes at the IR cutoff z_0 (this does not occur for $\Lambda = 631$ MeV). This observation is consistent with the expectation that the ‘quarkyonic’ phase deconfines at high density. The value of μ_B required for the horizon to form depends on the parameter ζ , with larger values of ζ requiring a greater μ_B .

In Fig. 2(b), we plot ζ as a function of μ_B such that $g(z_0) = 0$. These curves illustrate how the position of the horizon depends on ζ . Notably, for $\Lambda = 631$ MeV, the system does not develop a horizon in the NJL phase without a condensate when $\zeta \gtrsim 0.5$. In contrast, for $\Lambda = 925$ MeV, a horizon is always present for $\zeta \gtrsim 0.5$.

In view of this behavior and our previous observations about agreement with the vdW EOS, we will henceforth proceed with the NJL model using parameter set corresponding to $\Lambda = 631$ MeV.

The phase transition between charged black hole and NJL for $\zeta = 0.77$ and 1 is shown in Fig. 3. The red curves represent NJL with $\Lambda = 631$ MeV. The blue curve

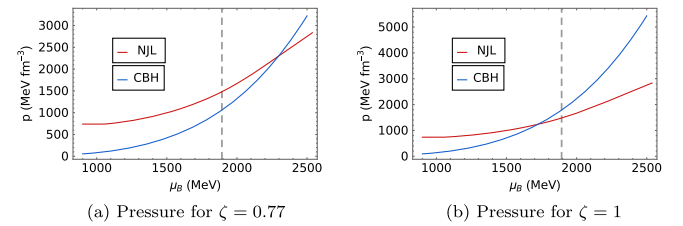


FIG. 3. Phase transition between charged black hole and NJL for $T = 1$ MeV. The dashed vertical gray line depicts the limit of validity for $\Lambda = 631$ MeV.

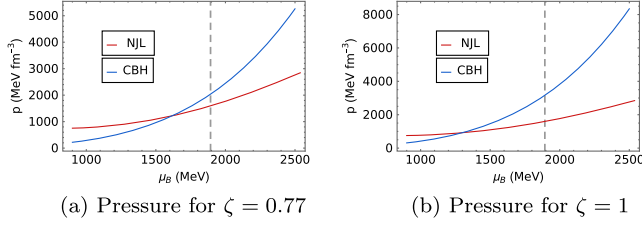


FIG. 4. Phase transition between charged black hole and NJL for $T = 60$ MeV. The dashed vertical gray line depicts the limit of validity for $\Lambda = 631$ MeV.

represents the CBH phase, whereas the gray dashed line denotes the region of validity for $\Lambda = 631$ MeV. We see that the pressure of the black hole geometry is larger at high chemical potential. While the location of the first-order phase transition is dependent on ζ , this comparison suggests that the highest density phase is always better modeled by the black hole which allows for a density of about 3 baryons per fm^3 . For $\zeta = 0.77$, the black hole transition occurs well beyond the region of validity of the NJL model with $\Lambda = 631$ MeV. For $\zeta = 1$, the transition between the CBH and NJL phase lies within the validity range of the NJL model. Additionally, as shown in Fig. 4, at slightly higher temperature values, the phase transition shifts to within the region of validity. Therefore, the occurrence of the confinement/deconfinement transition may be treated as a robust prediction from these approaches.

The pressure of the vdW phase is lower than that of the NJL phase at $T = 1$ MeV when $\mu_B \gtrsim 1500$ MeV. Consequently, in the absence of the NJL phase, the deconfinement transition from nuclear matter to the CBH phase would occur at a lower value of the chemical potential. This behavior is clearly illustrated in Fig. 5. If we assume that only the vdW and CBH phases are present, the deconfinement transition occurs around $\mu_B = 1950$ MeV. For $\zeta = 0.77$, this transition occurs very close to the validity limit of the vdW description, around $\rho \sim 3.3\rho_0 \approx 0.48 \text{ fm}^{-3}$, beyond which the effective nuclear matter model may no longer be reliable. At the transition, the density appears to jump by a very large amount, from around 0.45 baryons per fm^3 to around 2.7 baryons per fm^3 , while for NJL to CBH transition the density jumps from

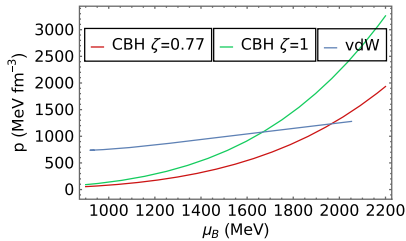


FIG. 5. Phase transition between the charged black hole and vdW phase at $T = 1$ MeV.

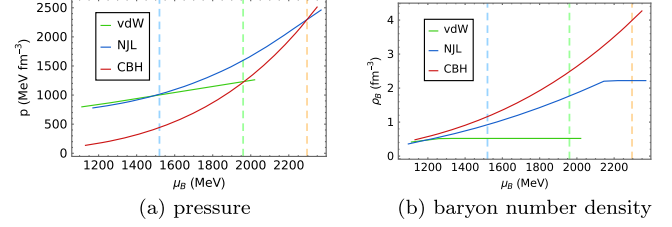


FIG. 6. (a) Illustrates the pressure, (b) Displays the baryon number density, for the vdW, NJL, and CBH phases. The phase transitions between the vdW, NJL, and CBH phases are marked by vertical dashed lines; cyan for the transition from vdW to NJL, green for vdW to CBH, and orange for NJL to CBH.

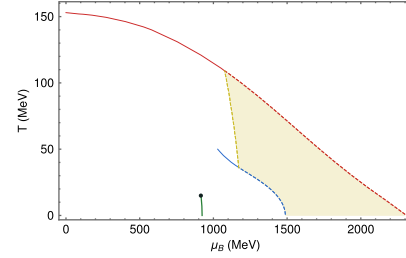


FIG. 7. Phase diagram for $\zeta = 0.77$ and $\Lambda = 631$ MeV.

around 2 baryons per fm^3 to around 4 baryons per fm^3 as maybe inferred from the second panel of Fig. 6. This figure represents (a) the pressure and (b) the baryon density of the vdW, NJL, and CBH phases represented by green, blue, and red curves, respectively. The phase transitions between the vdW, NJL, and CBH phases are marked by vertical dashed lines. The cyan for the transition from vdW to NJL, green for vdW to CBH, and orange for NJL to CBH. Therefore, we venture that the sequence $\text{vdW} \rightarrow \text{NJL} \rightarrow \text{CBH}$ is possibly the better description of the phases of QCD as shown in Fig. 7. A more detailed analysis of the phase diagram with different parameters can be found in [16]. We will now proceed to switch on the complex scalar fields and investigate the possibility of condensation in the relevant phases.

IV. CONDENSATE GEOMETRIES

Baryonic matter at high density can, in general, be expected to exhibit various levels of clustering due to various attractive interactions. At intermediate densities, this can lead to the formation of condensates involving dibaryons. These superfluid/superconducting phases have been argued to occur in the range of densities $950 < \mu_B < 1100$ MeV [54] albeit for a much smaller range of temperatures ($0 < T < 10$ MeV). At somewhat higher densities, *quark* pairing can also occur in various forms, including the diquarks [55] and CFL phases [56]. The pairs formed will be between particle-type and hole-type degrees of freedom mediated by the dense medium but these attractive forces

are likely to be strongly spin dependent. It is thus conceivable that spontaneous breaking of baryon number symmetry occurs as evinced by the corresponding condensates.

In the AdS bulk, such symmetry breaking condensates have been extensively studied by introducing a charged scalar, which takes over the phase diagram at high densities. However, nearly all of these studies have been conducted in the black hole background, which, as summarized in the preceding section, can be expected to appear only at the highest densities.

A. Charged black hole

At finite temperature and chemical potential in *four* bulk dimensions, scalar instabilities of charged black hole solutions [46] was discussed by [43] as leading to a superfluid phase of the boundary theory. We will revisit these in *five* dimensions [57] but in the presence of an IR cutoff, which can be expected to contribute a few new ingredients.

At the high chemical potentials applicable to the black hole phase, we expect that the condensate operator at the boundary to be made up of a pair of quarks. Thus, following the discussion in Sec. II, we set $\Delta = 3$ and $q = 2$. Since the bulk geometry contains a black hole, the scalar field Eq. (10) imposes a regularity condition $g'\psi' - m^2\psi = 0$ at the horizon z_H while the gauge potential vanishes. Using these, we can numerically solve the equations for different values of ζ and plot the condensate and relative condensate number density as a function of ratio $\tilde{\mu}_B/\tilde{t}$ in Fig. 8. The charged black hole background exhibits a scale symmetry, which naturally motivates expressing thermodynamic quantities in terms of dimensionless ratios such as μ_B/T . Thus, the onset of condensation occurs at a fixed value of the ratio μ_B^c/T_c for a given ζ . This ratio in the $\mu_B - T$ plane corresponds to the slope of the curve at the onset of condensation. However, for different values of the parameter ζ , the onset of condensation does not occur at the same value of this ratio. To enable a meaningful comparison across different ζ , we normalize the chemical potential and temperature by the onset ratio μ_B^c/T_c , such that all curves begin at a common point: $\tilde{\mu}_B/\tilde{t} = 1$.

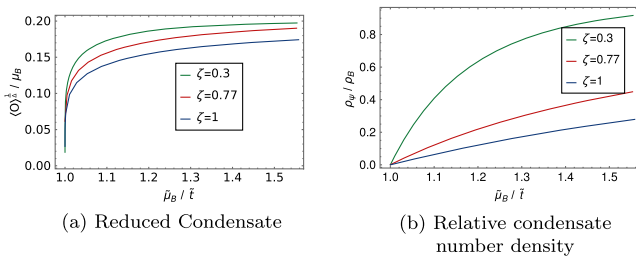


FIG. 8. Thermodynamics quantities for different ζ . The horizontal axis is explained in the text below.

Here, the *condensate* number density can be computed as the charge stored in the scalar field,

$$\begin{aligned} \rho_\psi &= iqg_5^2 \int_\epsilon^{z_0} \sqrt{-g} g^{00} (\psi^* D_0 \psi - \psi D_0 \psi^*) dz \\ &= \frac{48\pi^2 L}{\theta N_f N_c} q^2 L^2 \int_\epsilon^{z_0} \frac{\sqrt{h} \phi}{z^3 g} \psi^2 dz, \end{aligned} \quad (23)$$

where D_0 is the time component of the covariant derivative defined below Eq. (5).

As the ratio $\tilde{\mu}_B/\tilde{t}$ increases, the fraction of baryon density in the condensate also rises, as shown in the second panel.

The gauge coupling ζ does not qualitatively change the nature of the solution or the behavior of the condensate. However, the amount of matter inside the condensate increases much faster for smaller values of ζ .

Figure 9 represents the thermodynamic quantities for the charged black hole with and without condensate as a function of μ_B for $T = 15$ MeV, represented by the red and blue curves, respectively. As in the case of holographic superconductors in four dimensions, even in the five-dimensional model, condensed phases are preferred at sufficiently high densities, as seen in the first panel of Fig. 9—that is to say, condensation leads to increased pressure. The range for μ_B shown in the figure was chosen in anticipation that these phases will be relevant at high density. Entropy is decreased due to condensation and the total density is correspondingly increased and stored in the condensate. By comparing with the black hole NJL transition in Fig. 3, already we see that we can expect quark condensates at $\mu_B \approx 1750$ MeV rather than quarks.

B. Charged AdS

We now turn to the CAdS backgrounds that describe the confined phase. We will first undertake a detailed study of the solutions with a nontrivial scalar field with $q = 2$, $\Delta = 3$, and $\zeta = 0.77$ using the boundary condition $\phi(z_0) = 0$. Then we will focus on the qualitative effects of varying the parameter ζ in our model. This will inform our expectations when we study the system using the phenomenological boundary conditions of the previous section. Note that this phase will remain temperature independent.

Given an initial value $\psi(z_0) = \psi_0$, we can numerically solve the differential equation for the scalar field to obtain a

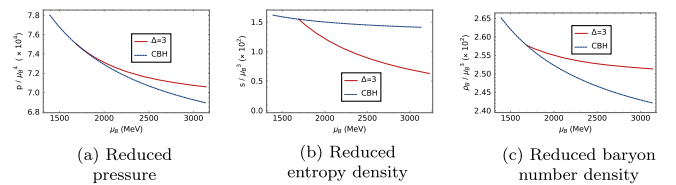


FIG. 9. Reduced thermodynamic quantities as a function of baryon chemical potential at $T = 15$ MeV.

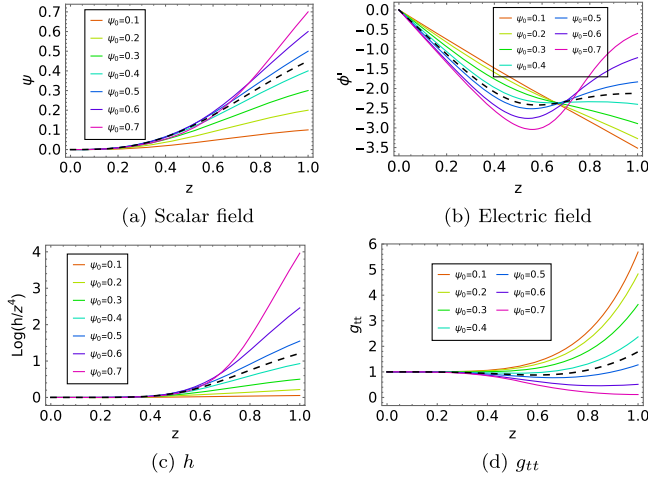


FIG. 10. Numerical solution for different values of ψ_0 at $\mu_B = 1566$ MeV. The black dashed lines show the minimum energy solution.

solution without a boundary source term. Figure 10 shows numerical solutions for $\mu_B = 1566$ MeV. Different colors represent various values of ψ_0 . The black dashed curve denotes the solution that maximizes the pressure (maximum pressure solutions will be discussed below).

For the solution to describe a *ground state*, the scalar field profile should not have a node (this is because of general expectations from Sturm-Liouville theory). We will solve the equation for a range of z beyond the cutoff $z > z_0$ and require that the solution must not have a node even in the extended geometry. This is under the expectation that once a node appears, it is likely to persist in an IR complete geometry because it is a topological feature and hence will not represent a valid *ground-state* description.

The grounds for this expectation is as follows. The scalar field equations are linear, and for small amplitudes, these can be studied in the probe approximation. In this situation, the scalar field equations define a Sturm-Liouville problem. General theorems then suggest that the eigenfunction corresponding to the lowest eigenvalue of the Sturm-Liouville operator must not have a node. These theorems are stable under small changes in the operator which, in our language, will correspond to changing the background by a little (for instance a IR complete background) or by adding other fields. These reasons form the basis for our expectation that the absence of a node in the extended situation provides a robust *definition* of condensation.

In Fig. 11, we show that scalar field solutions with a node indeed exist at large enough chemical potential. The log-log plot of the scalar field shows that beyond the IR cutoff, for a fixed value of ψ_0 , as we increase μ_B , a node appears in the scalar field as shown by curves that sharply dip downwards. These solutions share the same parameter values as those in Fig. 10, except for μ_B , which varies for a range from 2436 MeV (the orange curve) to

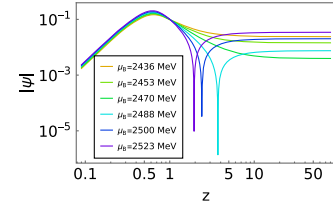


FIG. 11. The log-log plot displays the function $|\psi(z)|$ for a fixed $\psi_0 = 0.1$ extended beyond the cutoff z_0 .

2523 MeV (the purple curve). Clearly, for a given value of μ_B , there are several values of ψ_0 which give acceptable profiles of the condensate. It is important to note that the scalar field profile exhibits a maxima which is not obvious from Fig. 10. This means that requiring the absence of a node in the *full* geometry is a strong requirement which restricts the class of condensate solutions. In particular, restricting attention to the $0 < z < z_0$ region could lead to an overestimate of the range of the condensed phases. Also, the location of the maxima in ψ moves towards the boundary as the chemical potential increases which suggests that at higher chemical potentials, additional terms in the action such as $|\psi|^4$ will become important.

As depicted in Fig. 10, $h(z_0) \rightarrow \infty$ at the IR cutoff with increasing ψ_0 . This singularity is a naked singularity at which $\det(g) \rightarrow 0$ because g_{tt} vanishes (but g_{zz} does not). Thus, there appears to be an upper limit to ψ_0 beyond which there are no solutions to the scalar field (at the least, these are numerically difficult to find).

We can fix the IR value of the scalar field $\psi_0 = \psi(z_0)$ in two ways. We can fix the value by using additional physics input, such as the dependence of the pole mass of a meson on the density. Since mesons are fluctuations in the gauge field, their propagator will depend on ψ_0 . We will however proceed differently.

To motivate our choice, let us consider the form (A1) of the on shell free energy presented in the Appendix. If we regard the IR region $z > z_0$ as being described by another set of gauge fields and scalar fields, then z_0 is like a UV cutoff for this region, and hence the on-shell action can be written in one of these forms. This shows that the pressure of the matter in the region $z > z_0$ can be expressed in terms of only the fields g, ϕ, ϕ', h all evaluated at the IR surface z_0 . In particular, even if this region also contains a condensate, as we may expect, the on shell action does not depend on the values of the scalar field at the IR cutoff. Hence, the natural choice for ψ_0 is the one that extremizes the on shell action. Since we are working in the grand canonical ensemble, this is equivalent to maximizing the pressure. This has the result that the pole mass of the mesons is dynamically determined. The first panel of Fig. 12 shows that the pressure attains a maximum (represented by a black dot) at an intermediate value of the cutoff value ψ_0 . This means that the solution is not beset by issues such as large derivatives, curvature etc., which

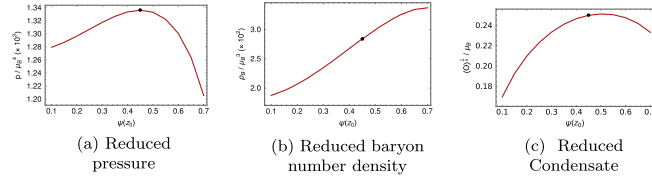


FIG. 12. Thermodynamic variables as a function of ψ_0 for $\mu_B = 1566$ MeV.

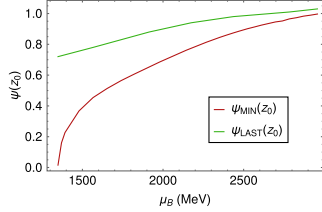


FIG. 13. $\psi(z_0)$ as a function of μ_B .

would have raised additional validity issues since these are likely to occur only in extreme conditions. In Fig. 10, the gravity solutions corresponding to maximum pressure (shown by dashed lines) are also seen to be intermediate, in this sense. In particular, the issues arising from large h field are avoided. Note that the condensate also has a maximum as a function of ψ_0 , which may not be the same as the pressure maximum. At the onset of condensation, the ψ_0 corresponding to maximum pressure is vanishingly small as illustrated by the red curve in Fig. 13. As we increase the chemical potential, ψ_0 shifts to larger values. The green curve represents the initial condition corresponding to the solution which leads to $g_{tt}(z_0) \rightarrow 0$. Clearly, at larger densities, the maximum pressure tends to this singular geometry. However, as we will see below, as we increase the chemical potential, the CAdS condensate solution will make a transition to condensate solutions in CBH geometry. Thus, we can conclude that condensate solutions exist at all values of $\mu_B > \mu_B^*$, where μ_B^* is the onset of condensation in the CAdS geometry with the $\phi_0 = 0$ boundary condition.

We can now compare the pressure of the CAdS geometry with and without condensation, as shown in Fig. 14. The dashed blue line represents the pressure of CAdS without the scalar condensate. We observe that the presence of condensation leads to an increase in pressure, indicated by

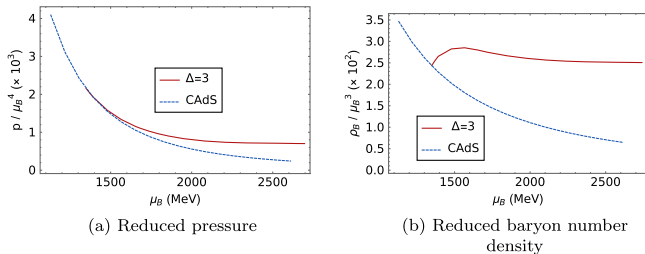


FIG. 14. Thermodynamic variables as a function of μ_B .

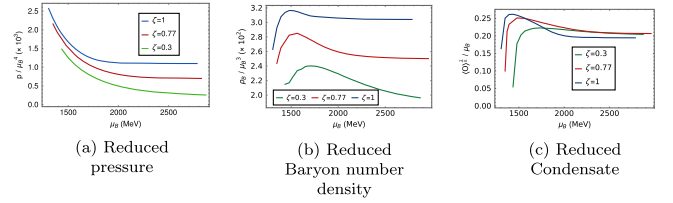


FIG. 15. Thermodynamic variables as a function of μ_B .

the solid red curve. As we increase the chemical potential, the condensate solutions continue to exhibit higher pressure. The second panel shows that baryon density increases sharply upon condensation.

Since the value ζ is not fixed by any physical condition, we need to study its effect on the condensation instability. Increasing the value of ζ results in a rise in both pressure and baryon density. Furthermore, we note that the onset of condensation occurs earlier for larger values of ζ . Additionally, the condensate saturates more rapidly as a function of μ_B for higher values of ζ . These observations are illustrated in Fig. 15. Since ζ controls the interaction between the quarks (gauge fields) and the glue (metric fields), we see that increasing this interaction strength actually leads to higher pressure and densities.

C. Phase diagrams

We can now identify the solutions with the largest pressure and thereby construct the phase diagram in the $\mu_B - T$ plane. We will present our results for a few choices of the parameters with a view toward the more phenomenological cases to be discussed in the upcoming section. The dependence of the phase diagram on the parameter ζ can be interpreted in terms of phenomenological quantities that this parameter determines [7]. As mentioned in the previous section, once the condensate appears in the CAdS geometry, it always remains the lower free-energy solution. Since the pressure for the horizonless geometries with the boundary condition $\phi_0 = 0$ does not depend on temperature, the transition line is independent of temperature and is shown by a black dotted curve in Fig. 16. As the chemical potential

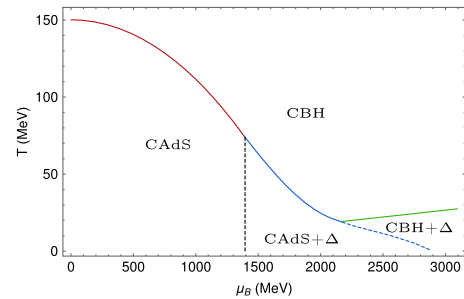
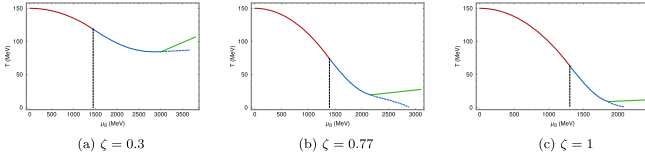


FIG. 16. Phase diagram: Schematic representation of the different phases of the system, used for labeling purposes in the discussion.

FIG. 17. Phase diagram for various values of ζ .

increases, the CADS with condensate makes a first-order phase transition to the charged black hole with condensate, represented by the blue dashed line. As the temperature increases sufficiently high, the charged black hole phase becomes favored. These transitions are illustrated by the red, blue, and green solid curves, which represent the transition from CADS, CADS with condensate, and charged black hole with condensate phases, respectively.

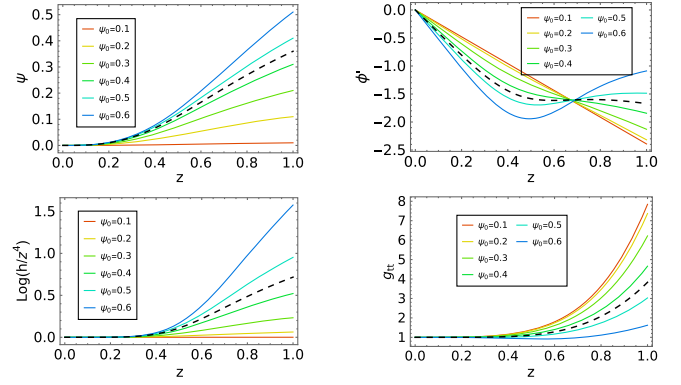
We illustrate the effect of the coupling parameter ζ on the phase diagram in Fig. 17. The first observation from this figure is that the onset of the condensate in CADS backgrounds shifts to smaller values of μ_B as ζ increases. Additionally, increasing ζ significantly destabilizes the condensate in the black hole phase shown by the downward shift of the green coexistence line in the phase diagrams. We can also note that increasing ζ decreases the size of the region described by the confined charged AdS type geometries. One may note that the phase diagram for $\zeta = 0.3$ resembles the phase diagram presented in [58].

V. PHENOMENOLOGICAL BOUNDARY CONDITIONS

In this section, we will study the possibility of symmetry-breaking condensates when the baryonic matter is described by the phenomenology of χ PT and the NJL model. As demonstrated in previous sections, the coupling constant ζ significantly influences the formation of condensates. We will explore the behavior of these condensates within both the vdW phase and the NJL phase, focusing on two specific values of ζ , namely $\zeta = 0.77$ and $\zeta = 1$.

A. vdW description

As discussed in Sec. III B, the vdW phase is thermodynamically preferred at intermediate baryon chemical potential. We therefore analyze the condensate in this phase for intermediate values of the chemical potential. Since the vdW phase, described by in-medium chiral perturbation theory, contains baryonic degrees of freedom, the natural condensing operator would carry baryon charge $q = 2N_c$ and scaling dimension $\Delta = 3N_c$. However, in a fundamental description, we also have quark bilinear operators with $q = 2, \Delta = 3$ which can be expected to condense in the deconfined phase. In the present setup, we study the latter case, as in the charged black hole case of Sec. IV A. The effects of baryon pair operators with $q = 2N_c = 6$ and various scaling dimensions in the vdW

FIG. 18. Numerical solution for different values of ψ_0 for $\zeta = 0.77$, $\mu_B = 1566$ MeV, and $T = 0$ MeV. The black dashed lines show the minimum energy solution.

phase are presented in Sec. VII. In a complete description of QCD, both these operators will be present simultaneously, and it is interesting to explore the simultaneous condensation of these operators.¹

We numerically solve the coupled differential equations to obtain the condensate profile, ensuring that the scalar field solution has no boundary source term. As before, we scan over different values of $\psi(z_0) = \psi_0$ and select the configuration that maximizes the pressure. We also verify that the scalar field solutions contain no nodes, even for $z > z_0$. An upper bound exists for the allowed initial values of the scalar field as g_π approaches 0; however, the minimum-energy solution lies well within the physically valid range of initial conditions.

Figure 18 displays numerical solutions for $\zeta = 0.77$, $T = 0$ MeV, and $\mu_B = 1566$ MeV. Different colors correspond to different values of ψ_0 , while the black dashed curve denotes the solution yielding the maximum pressure.

We now present the thermodynamic properties of these solutions as functions of chemical potential and temperature. Figure 19 shows the thermodynamic quantities in the vdW phase, with and without a condensate, at $T = 0$ MeV. The former is represented by the solid red curve, while the latter is shown by the blue dotted curve.

The first panel demonstrates that the pressure in the phase with a nonzero condensate exceeds that of the phase without a condensate, indicating that the former is thermodynamically favored. The condensation transition is continuous. Consider the derivative $\frac{\partial p}{\partial \mu}$ evaluated on the condensate solutions and subtract it from the corresponding uncondensed solution,

$$\left. \frac{\partial p}{\partial \mu} \right|_{\psi} - \left. \frac{\partial p}{\partial \mu} \right|_{\psi=0} \sim \frac{\partial}{\partial \mu} \left(\frac{\partial p}{\partial \psi_0} \right). \quad (24)$$

¹Preliminary results suggest that only one of the two operators condenses.

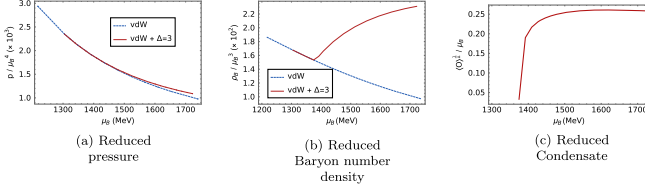


FIG. 19. Thermodynamical quantities with and without condensation for $\zeta = 0.77$ and $T = 0$ MeV.

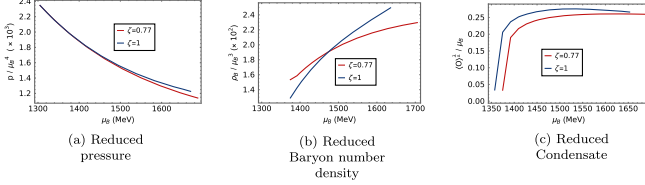


FIG. 20. Thermodynamics quantities as function of chemical potential at $T = 0$ MeV for $\zeta = 0.77$ and 1.

In the limit $\mu \rightarrow \mu_c$, the partial derivative with respect to ψ_0 vanishes as seen in the first panel of Fig. 19. This is inferred from the tangency of the two curves near the onset of condensation.

The second panel shows the reduced baryon number density, which exhibits a significant enhancement in the presence of the condensate. The last panel displays the reduced condensate as a function of μ_B . It rises sharply near the onset and then varies slowly at larger chemical potentials.

We show the thermodynamic quantities as functions of chemical potential at zero temperature for two values of ζ in Fig. 20. The red curve corresponds to $\zeta = 0.77$ and the blue curve to $\zeta = 1$.

From the figure, we see that both the pressure and the condensate are higher for larger ζ at the same T and μ_B . The second panel of Fig. 20 shows that the baryon density is larger for smaller ζ at low chemical potential, but becomes larger for higher ζ at large chemical potential.

We next study how the vdW phase changes with temperature, both with and without the condensate. The results are similar to the $T = 0$ case, as the chemical potential increases, the vdW phase with a condensate is still the thermodynamically preferred phase. Since the behavior is the same, we do not show a direct comparison between the two phases here. Instead, we show in Fig. 21 the thermodynamic quantities of the condensate phase as functions of chemical potential for $\zeta = 0.77$ and several temperatures.

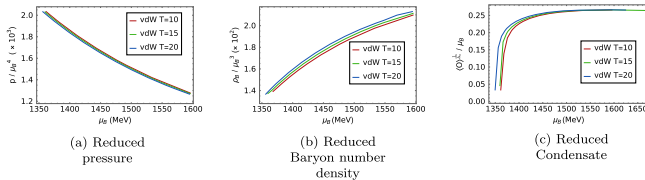


FIG. 21. Thermodynamical quantities as a function of μ_B for different T and $\zeta = 0.77$.

The figure includes results for $T = 10, 15$, and 20 MeV, represented by red, green, and blue curves, respectively. The first panel shows that the reduced pressure becomes smaller as the temperature increases, while the second and third panels show that the reduced baryon number density and the reduced condensate both increase with temperature. This behavior is in contrast to the charged black hole phase, where the relative condensate number density is larger at lower temperatures.

B. NJL description

At slightly higher densities, we have argued that the vdW description gives way to a situation involving a gas of quarks with a gradually vanishing chiral condensate and described it using an NJL model. Since the NJL baryon density is in the form of quarks, the baryon charge of the scalar field will remain $q = 2$ as befits a Cooper pair of quarks. In this sense, these phases can be thought of as either a superconducting/superfluid phase of quarks or even a CFL phase. Correspondingly, the scaling dimension of the dual operator is $\Delta = 3$. Using the pressure and baryon density computed from the NJL model to set the IR boundary conditions, we can numerically solve the equations to obtain solutions with nonzero scalar field profiles.

These solutions for $\zeta = 0.77$, $\mu_B = 1566$ MeV, and $T = 1$ MeV are shown in Fig. 22 and clearly show that the solutions are well-behaved and under control. As before, the maximum pressure solution occurs at an intermediate value of the initial condition ψ_0 as shown in Fig. 23. The maximum pressure solution once again approaches the last solution as g_H approaches 0 for large values of μ_B .

We will now present the thermodynamic properties of these configurations as we vary the chemical potential and temperature. Figure 24 shows the thermodynamic properties of the NJL phase, with and without condensate, indicated by the solid red and dashed blue curves, respectively, as functions of μ_B for $T = 1$ MeV and $\zeta = 0.77$. The

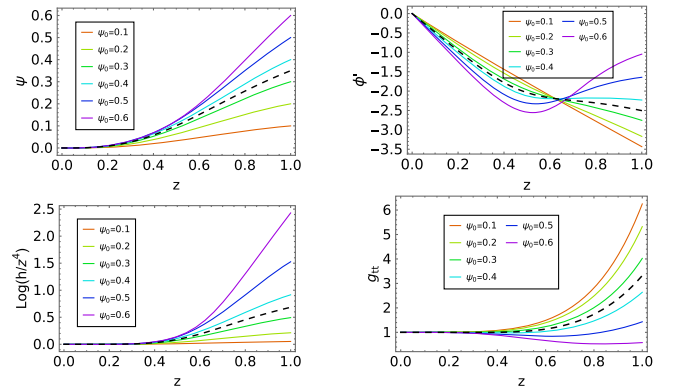


FIG. 22. Numerical solution for $\zeta = 0.77$, $\mu_B = 1566$ MeV, and $T = 1$ MeV and different values of ψ_0 . The black dashed lines show the minimum energy solution.

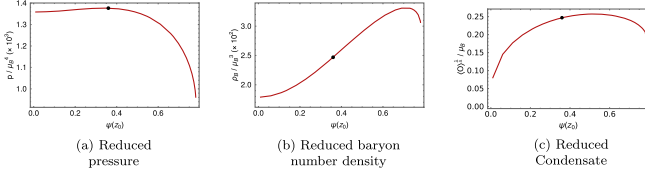


FIG. 23. Thermodynamic variables as function of ψ_0 for $\zeta = 0.77$, $\mu_B = 1566$ MeV. The minimum energy solution is represented by the black dot.

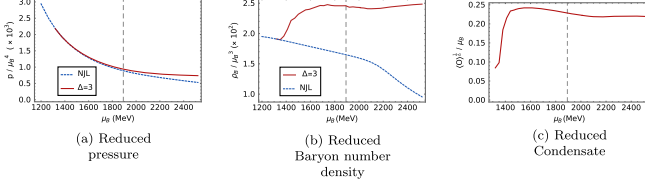


FIG. 24. Thermodynamical quantities as function of μ_B for $\zeta = 0.77$ and $T = 15$ MeV.

first panel in Fig. 24 shows that condensation occurs at $\mu_B \approx 1300$ MeV. A comparison of the pressure between the NJL model with and without condensate shows that condensate phases always have higher pressure. Again, the transition is continuous. The second and the third panel show the variation of the total baryon density and the condensate as a function of μ_B in comparison with the situation sans condensate. Concomitant with the increase in the condensate, the total baryon density increases sharply with condensation. The figure above is not reliable beyond $\mu_B \approx 1900$ MeV (represented by the gray dashed vertical line) since we are close to the NJL cutoff $\Lambda = 631$ MeV.

We plot the variation of thermodynamic quantities as a function of μ_B for two different values of ζ in Fig. 25. As expected from our previous analysis with $\phi_0 = 0$, the pressure and number density increase for larger ζ values. The onset of condensation occurs later for smaller ζ . These plots are computed for $q = 2$, $\Delta = 3$, and $T = 15$ MeV.

Temperature dependence is shown in Fig. 26. The solid red, green, and blue curves represent the thermodynamic variables for $T = 15$, 25, and 50 MeV, respectively. The scenario is quite different from that of a condensate in the Van der Waals phase at finite temperature. As the temperature increases, the pressure increases while the baryon number density decreases, as might be expected.

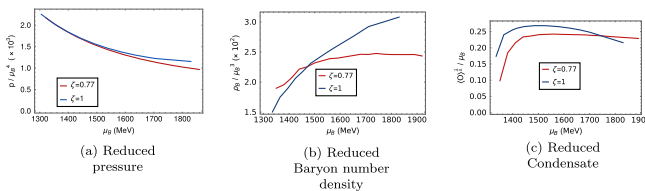


FIG. 25. Thermodynamics quantities as function of chemical potential at $T = 15$ MeV for $\zeta = 0.77$ and 1.

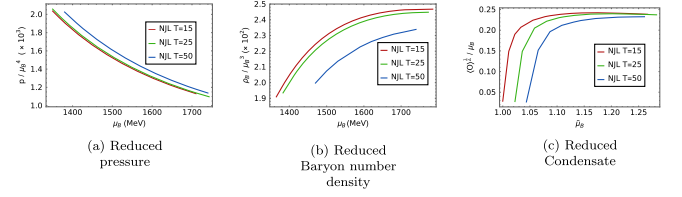


FIG. 26. Thermodynamical quantities as a function of μ_B for different T and $\zeta = 0.77$.

VI. PHASE DIAGRAMS

In this section, we will construct the phase diagram of the boundary theory in the $\mu_B - T$ plane, now including the condensate phases. The phase boundaries are obtained by comparing the pressures of the various solutions.

A. Phase transition with NJL

Figure 27 compares the pressure of the condensate phase obtained with NJL boundary conditions and with vdW boundary conditions for two values of temperature and $\zeta = 0.77$. The vdW phase with condensate is shown by the green curve, and the NJL phase with condensate by the blue dashed curve.

In the first panel, at $T = 1$ MeV, the pressure is higher for the vdW phase at low chemical potential, but a transition to the NJL phase occurs at $\mu_B \approx 1570$ MeV, where the NJL pressure becomes larger. Above this temperature, the vdW nuclear liquid is expected to condense into a diquark superfluid described by the NJL condensate, although the phase remains confined. In the second panel, at $T = 15$ MeV, the transition chemical potential decreases to $\mu_B \approx 1440$ MeV. This trend continues in the full phase diagram discussed later.

B. Phase transition with CBH phase

The previous section demonstrated that the NJL condensate phase exhibits a higher pressure at increased chemical potentials. However, we have another possible phase described by a condensate in the charged black hole geometry. Comparing the pressure of these phases, in Fig. 28 we see that the NJL model seems to evolve smoothly to the charged black hole at high densities. The charged black hole with the condensate is shown by a red curve, and the NJL with the condensate is shown by a blue dotted curve. The

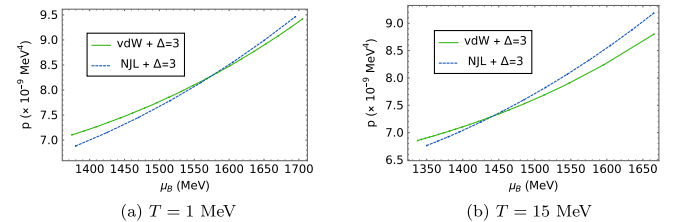


FIG. 27. The phase transition between vdW and NJL for $\zeta = 0.77$.

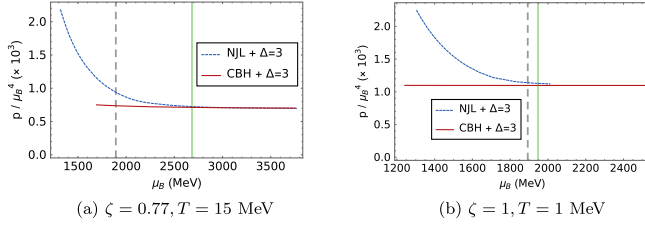


FIG. 28. Reduced pressure as a function of baryon chemical potential.

gray dashed curve represents a region of validity for the NJL equations of state. The solid green line marks the transition chemical potential $\mu_B = 2685$ MeV at $T = 15$ MeV and $\zeta = 0.77$ in the first panel, while the second panel shows $\mu_B = 1950$ for $\zeta = 1$ and $T = 1$ MeV.

An interesting point to note is that the solutions corresponding to different initial conditions, $\psi(z_0) = \psi_0$, at the given temperature and high chemical potential, Fig. 29, show a limited range of solutions for ψ_0 . From the figure, it is clear that the solution corresponding to the maximum pressure, denoted by the black dotted curve, exhibits the development of a singularity as it approaches the IR cutoff, z_0 , where $g_{tt} \rightarrow 0$. This suggests that the solution becomes increasingly singular near the IR limit.

We now summarize these observations in the form of a phase diagram shown in Fig. 30 where the phases have been labeled. We will first describe these phases and then discuss the effects of parameter ζ and their validity in subsequent figures. Built on the phase diagram of [41] which has baryon gas, baryon liquid, quarkyonic and deconfined quarks as phases, we observe that there are condensates at low temperatures. At intermediate chemical potential, we find condensates in the vdW phase. This phase appears in the region enclosed by the red dashed lines in the phase diagram, and its onset is marked by a second-order transition. As the chemical potential increases, the vdW condensate undergoes a first-order phase transition to the NJL condensate phase, shown in the region enclosed by the purple curves. At even higher chemical potential, the charged black hole condensate becomes favored over the NJL condensate, marking another second-order phase transition. We must note here that the complexity of QCD suggests that including more fields and/or modifying the bulk model can change these transitions. Therefore,

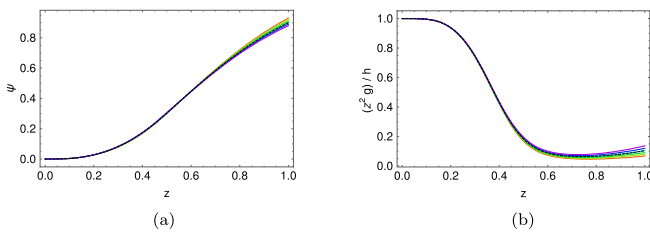


FIG. 29. Solutions for $\mu_B = 2685$ MeV and $T = 15$ MeV.

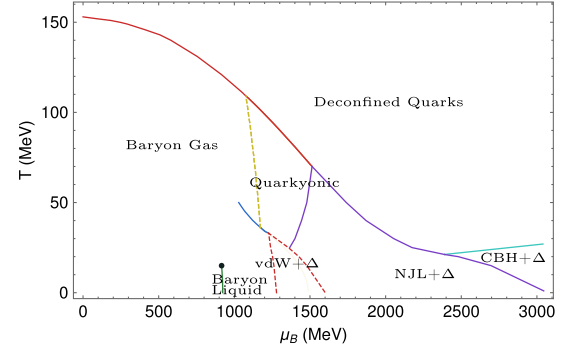


FIG. 30. Phase diagram: Schematic representation of the different phases of the system, used for labeling purposes in the discussion.

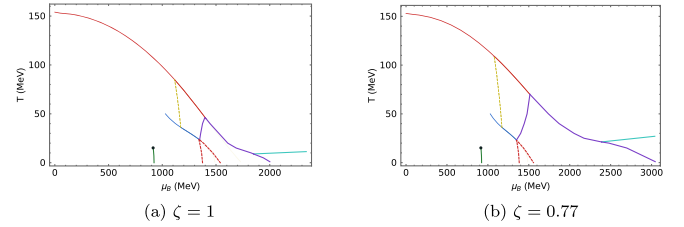


FIG. 31. Phase diagram with condensate for two values of ζ computed for $q = 2$, $\Delta = 3$, using phenomenological boundary conditions.

perhaps the only take home message is that baryonic condensates are highly likely at even intermediate densities and any effective field theory description of QCD phases must consider these from the get-go.

The phase diagram changes significantly with the parameter ζ . In Fig. 31, we illustrate the effect of ζ on the phase diagram. The condensates in both NJL and CBH are less favored as we increase ζ . Whereas the vdW condensates span a larger region on the phase diagram.

We must remind the reader that for the smaller value of ζ , we have seen that the transition from NJL to the black hole phase is not very reliable at very low temperatures (see Fig. 28). However, we do expect that the results are not qualitatively changed by an improved treatment by continuity with the slightly larger value of ζ and continuity of the phase boundary as a function of μ_B, T .

VII. BARYONIC CONDENSATES

In the preceding analysis, we restricted our attention to operators with charge $q = 2$ and scaling dimension $\Delta = 3$. However, other possibilities naturally arise in QCD, where operators with different charges and scaling dimensions may also condense [59–61]. In particular, a condensate composed of baryon pairs is expected, corresponding physically to proton superconductivity or neutron superfluidity. Holographically, this is minimally captured by a bulk complex scalar field dual to a baryon-bilinear operator

which carries bulk charge $q = 2N_c = 6$, reflecting the baryon number of the dibaryon condensate states.

Since the baryonic medium is strongly interacting, the operator describing a baryon-pair condensate will in general be highly renormalized, and its scaling dimension Δ may deviate significantly from the naive perturbative estimate $\Delta = 2 \times \frac{3}{2}N_c = 9$. A value close to $\Delta \approx 3$ suggests that the condensate behaves like a fermionic pair, consistent with intact baryons pairing without being destabilized by the medium. At intermediate densities, however, large- N_c arguments support the possibility of quark-hadron continuity, in which baryons dissolve into their quark constituents. In such a regime, we can expect that Δ approaches the perturbative value $\Delta \approx 3N_c = 9$, reflecting loosely bound quark pairs and nearly dissolved baryonic states. We therefore turn to the study of these higher-charge operators in order to explore both possibilities. However, restricting to the $\zeta = 0.77$ case only.

A. vdW operators

In this context, we first examine the vdW phase. Within this phase, we again find condensate-type solutions, now associated with operators of larger charge and scaling dimension. As in the $q = 2, \Delta = 3$ case, the physically relevant solutions are those obtained by maximizing the pressure with respect to the boundary value ψ_0 , and these lie within a family of bulk profiles free of any pathology. From the bulk perspective, the large charge of the complex scalar field generates strong electric fields, leading to a significant energy cost. This motivates imposing $m^2 > 0$ for the charged scalar, which suppresses excessive condensation. For this reason, we restrict attention to representative operators with scaling dimension $\Delta > 4$.

Table I lists the chemical potential at which a condensate operator with charge q and scaling dimension Δ first appears at $T = 0$ MeV in the vdW phase. The general pattern is as follows. Operators with larger q condense earlier, while for a fixed q , smaller values of Δ also favor earlier condensation. However, if Δ is too small, condensation would set in already at $\mu_B < M_B$, which is physically inconsistent. In the vdW phase, this requires $\Delta \gtrsim 5$, so only operators above this threshold are admissible.

At the same time, very large values of Δ delay condensation. If Δ is sufficiently large, the onset occurs later than for the $q = 2, \Delta = 3$ operator studied previously

(shown in blue in Table I). Thus, in the vdW phase, there exists only a finite window of higher-dimension operators that can meaningfully condense. They must be heavy enough to avoid unphysical onset, but not so heavy as to be delayed beyond the $q = 2, \Delta = 3$ case.

B. NJL operators

If, after the vdW phase, an NJL phase does indeed intervene before deconfinement, then dibaryonic operators with $q = 6$ and large scaling dimensions may also condense in this phase. Motivated by this, we have examined the same class of operators in the NJL phase and find that similar qualitative features persist.

Table II lists the chemical potential at which a condensate operator with charge q and scaling dimension Δ first appears at $T = 1$ MeV in the NJL phase. From the table, we find that in the NJL phase, the onset of condensation follows the same trend as in the vdW phase. Operators with larger charge q begin to condense at smaller values of the chemical potential, meaning they condense earlier than those with smaller charge. While increasing the scaling dimension Δ delays the onset, making condensation harder to achieve.

A key difference, however, is that the physical lower bound on Δ shifts upward in the NJL phase. Specifically, we require $\Delta \gtrsim 6$ to make sure that condensation only happens once $\mu_B > M_B$, which is physically sensible. The upper bound also changes. In the vdW phase, operators with $\Delta \gtrsim 7$ condensed too late to play any role, since their onset occurred at a larger chemical potential than the $q = 2, \Delta = 3$ operator. In the NJL phase, however, we find that this is no longer the case: even the $\Delta = 7$ operator condenses earlier than the $q = 2, \Delta = 3$ operator. This means that in the NJL phase, heavier operators with larger scaling dimensions can become relevant, opening up a wider window of possible condensates.

C. CBH operators

Given the trends observed in the vdW and NJL phases, it is natural to ask whether similar operators may condense in the CBH background. Such operators can indeed be constructed, but the analysis differs because the CBH background has a scale symmetry, and the onset of condensation is controlled only by the ratio T/μ_B .

To illustrate this, Fig. 32 shows the onset lines in the temperature-chemical potential plane. Each curve represents

TABLE I. Onset of condensation μ_B at $T = 0$ MeV and $\zeta = 0.77$.

q	Δ	μ_B (MeV)
6	6.3	1157.1
6	6.5	1226.7
6	7.0	1435.5
2	3.0	1383.3

TABLE II. Onset of condensation μ_B at $T = 1$ MeV and $\zeta = 0.77$.

q	Δ	μ_B (MeV)
6	6.3	1052.7
6	6.5	1078.8
6	7.0	1226.7
2	3.0	1357.2

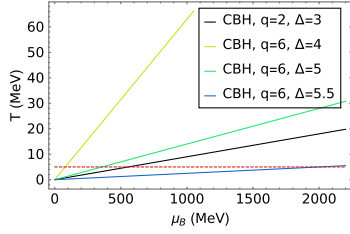


FIG. 32. The onset T/μ_B of condensate operator with different charge q and scaling dimension Δ in CBH phase. The red dotted horizontal line represents a $T = 5$ MeV.

the value of T/μ_B at which condensation occurs, with different colors corresponding to operators of different charge and scaling dimension. The yellow, green, and blue curves correspond to $q = 6$ operators with $\Delta = 4, 5$, and 5.5 , respectively. For comparison, the black curve shows the $q = 2, \Delta = 3$ operator analyzed in the previous sections. A horizontal red dotted line marks $T = 5$ MeV, which serves as a reference. At this fixed temperature, the intersections indicate the value of μ_B where each operator condenses.

From the figure, we find that the $q = 6, \Delta = 4$ and $\Delta = 5$ operators condense at $\mu_B < M_B$, which is unphysical and rules them out. For slightly larger scaling dimension, $\Delta = 5.5$, the onset occurs well after the $q = 2, \Delta = 3$ operator. Thus, in the CBH background, no dibaryon-type operator condenses in a physically sensible window.

VIII. DISCUSSION

We will present a summary of our results and an outlook towards the determination of the equation of state of nuclear matter at high densities.

In this work which uses the AdS-hardwall models, we have shown that that nuclear matter at high densities, either in the form of baryons or in the form of quarks, is unstable to condensation. We have analyzed the system using boundary conditions inspired by QCD phenomenology, as well as a simpler boundary condition where $\phi_0 = 0$. The thermodynamically preferred description always involves condensates except possibly at very low densities. The maximum pressure condition for fixing the scalar field ψ at IR cutoff z_0 not only ensures that the onset of condensation is always a continuous second-order phase transition but also eliminates the possibility of offset of the condensate, which would be indicated by the appearance of a node in the scalar field profile. These conclusions seem to be robust over a range of the coupling ζ , which controls the interaction between the matter and gravity in the bulk.

We attempted to describe phases of nuclear matter more realistically by using phenomenological boundary conditions. Uniformly, we find that condensed phases are always preferred at low temperatures for large enough densities. Remarkably, our results suggest that the CAdS geometry with NJL boundary conditions evolves smoothly into the

deconfined quark phase, holographically realized by the charged black hole geometry.

For the most part, we presented results pertaining to the condensation of a diquark operator with $q = 2$ and $\Delta = 3$. Beyond the di-quark operators, an important question is whether dibaryon-type operators can also condense and thus play a significant role in dense QCD matter. To explore this, we examined operators with charge $q = 6$ and varying scaling dimensions, as discussed in Sec. VII. We find that these operators do condense and in fact condense before the di-quark operators. Moreover, these latter operators are restricted to a certain range of Δ , depending on the phase under consideration on the grounds of phenomenological consistency. Then natural question is what happens if the scalar fields corresponding to both these operators are turned on in the bulk. A systematic study of such competition between operators with different (q, Δ) is currently in progress and will be reported in future work.

We have also clarified how these hardwall models can be thermodynamically consistent with the total baryon number density, given by $\rho_B = \frac{\partial p}{\partial \mu_B} = \bar{\rho} + \rho_\psi$. However, while the entropy density, must be computed as $s = \frac{\partial p}{\partial T}$, the Euler relation must be used to compute the energy density, as the standard holographic method for obtaining the energy density from the boundary stress-energy tensor does not apply in the hard-wall framework.

We have obtained phase diagrams, now including condensates, within the hardwall framework. Post-transitions, the density is seen to *increase* sharply after condensation. Additionally, the phase diagram has been shown to vary sharply with temperature. These two results are likely to be of much significance for the structure of compact stars which involve nuclear matter at high densities.

Even within the hardwall approach, our study does not explore all the aspects that are crucial for nuclear matter proper. For instance, we do not consider the effects of running coupling at all, and we did not include the field dual to the chiral condensate in the bulk. Additionally, we must study isospin chemical potentials, which will allow for yet another axis to the phase diagram. Some of these questions are clearly straightforward extensions that are likely to be of phenomenological interest.

A key input to our study is the manner in which we incorporate the phenomenological equations of state in the hardwall model. This gives rise to two separate questions. Firstly, one can hope to study the fields sourced by a distribution of baryons in the AdS bulk. In earlier studies based on the Witten-Sakai-Sugimoto model, the baryons were represented by a dilute gas of instantons localized on D8 branes [62]. It will be interesting to revisit these studies to determine an approximate truncated bulk solution which nevertheless treats the interactions among the instantons better. This could enable a better handling of the strong interactions among the baryons and avoid the questions arising from the choice of boundary conditions.

On the other hand, one can ask whether quantum gravity restricts the boundary conditions that can be applied to such truncated hardwall models. This might be natural by considering a path integral formulation of the bulk theory and focusing on gauge and diffeomorphism invariances and anomalies.

DATA AVAILABILITY

The data that support the findings of this article are openly available [63].

APPENDIX A: HOLOGRAPHIC RENORMALIZATION

To extract thermodynamic properties, we note that the bulk action in (1) following [64] evaluates to

$$S_{\text{bulk}} = \frac{1}{2\kappa^2} \int \frac{d}{dz} \left(\frac{1}{z} \sqrt{-g} g^{zz} \right) d^5x. \quad (\text{A1})$$

This form is particularly convenient for us because it expresses the on shell action in terms of $g(z_0) = g^{zz}(z_0)$.

This action, however, yields divergences when evaluated on shell, both from the gravity and matter components. To address this, we have implemented a holographic renormalization scheme [65], which adds extra terms that we discuss below,

$$S_{\text{ren}} = S_{\text{bulk}} + S_M + S_{GH} + S_{ct1} + S_{ct2}, \quad (\text{A2})$$

$$S_{GH} = -\frac{2}{2\kappa^2} \int d^4x \sqrt{\gamma} K. \quad (\text{A3})$$

$$S_{ct1} = \frac{2}{2\kappa^2} \int d^4x \sqrt{\gamma} \frac{3}{L}, \quad (\text{A4})$$

$$S_{ct2} = \lambda_s (4 - \Delta_+) \int d^4x \sqrt{\gamma} \frac{\psi^2}{L}. \quad (\text{A5})$$

- (i) The Gibbons-Hawking term is added for the well-definedness of the variational principle. We do not add a GH term at the IR boundary. Here γ is the induced metric on the UV surface $z = \epsilon$ and K is the trace of the extrinsic curvature associated with this surface.
- (ii) The counterterm, S_{ct1} , cancels divergences from the gravity part S_g as well as from Gibbons-Hawking term S_{GH} .
- (iii) In the matter part S_M , the field strength F^2 never diverges because of the asymptotic nature of the gauge field. However, the scalar field part diverges for $\Delta \leq 2.5$.

- (iv) We need another counterterm, S_{ct2} , which cancels the divergences from scalar field action. It is to be noted that the coefficient is always Δ_+ , independent of the choice of normalizable mode.

APPENDIX B: ROBUSTNESS OF NUMERICAL SOLUTIONS

We have employed a numerical shooting method to obtain solutions, using a UV cutoff parameter ϵ to regulate the boundary near the AdS boundary. We have varied ϵ in the range $10^{-1} < \epsilon < 10^{-6}$ and observed that thermodynamic quantities converge rapidly for $\epsilon < 10^{-2}$. Based on this observation, we fix $\epsilon = 10^{-4}$ for computations involving condensate solutions in the charged black hole geometry and the CAdS geometry with NJL-type boundary conditions.

However, solving for condensate solutions in Sec. VII for dibaryon type operators is significantly more challenging at small ϵ due to numerical instabilities. In particular, for large values of the scaling dimension Δ , it becomes difficult to maintain the non-normalizable mode at zero using the shooting method, which is sensitive to initial conditions. To manage this issue and ensure stable numerical integration, we choose a larger cutoff value, $\epsilon = 10^{-1}$, for this class of solutions.

This difficulty stems from the fact that we transform the boundary value problem into an initial value problem when applying the shooting method. Although alternative numerical strategies may help to improve accuracy at lower cutoffs, we expect that such refinements would not significantly alter the thermodynamic results or the overall phase structure of the system.

To quantify the impact of the chosen UV cutoff, in Fig. 33, we plot the percentage error in thermodynamic quantities as a function of ϵ in the vdW phase with condensate, for a representative case with scaling dimension $\Delta = 6.3$ and charge $q = 6$.

As shown in Fig. 33, the percentage error in the pressure remains small for $\epsilon = 10^{-1}$, indicating that the resulting phase diagram is not significantly affected by this choice. Nonetheless, developing more robust numerical techniques will be important for studying more realistic holographic models, particularly those incorporating features such as isospin asymmetry or a running coupling.

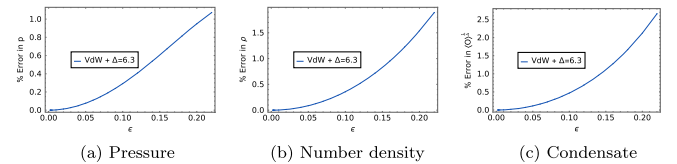


FIG. 33. Percentage error in thermodynamic quantities as a function of the UV cutoff ϵ .

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