

Unitarity limits on triple gauge boson production

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(Received 20 January 2026; accepted 16 March 2026; published 21 April 2026)

We derive the unitarity constraints on anomalous quartic gauge couplings originating from the $f\bar{f}' \rightarrow VV'V''$ channel with $V^{(')(\prime\prime)} = \gamma, Z, W^\pm$, and H . We also assess the importance of these bounds on the present and future experimental searches.

DOI: 10.1103/n529-w187

I. INTRODUCTION

The production of electroweak gauge bosons provides a window into beyond the Standard Model (SM) physics at the CERN Large Hadron Collider (LHC) because triple (TGC) and quartic (QGC) gauge boson couplings are determined by the SM gauge symmetry. Presently, the studies of QGC are carried out in the exclusive production of two [1,2] or three [3–11] gauge bosons as well as in the vector-boson-scattering production of electroweak gauge boson pairs [12–15].

Up to this moment, there is no indication of the existence of any new state or departure from the SM dynamics. Hence, it is natural to assume that there is a mass gap between the ultraviolet extension of the SM and the electroweak scale. In this scenario, new physics effects can be parametrized using effective field theory (EFT) and they manifest themselves in the tails of kinematic distributions. In general, the most stringent constraints on Wilson coefficients of effective operators that contain both TGC and QGC originate from the study of their TGC component in the pair production of gauge bosons. Therefore, most of the present LHC searches for effects of QGC focus on the so-called genuine QGC operators—that is, operators that generate QGC but do not have any TGC associated with them.

It is well known that scattering amplitudes containing EFT operators grow as the center-of-mass energy increases, eventually, presenting violation of partial-wave unitarity. This indicates that the perturbative use of EFT has a limited validity range. Thus, when probing anomalous QGC one must verify whether perturbative partial-wave unitarity is satisfied in the signal simulations to guarantee consistency of the analyses. Previously, unitarity constraints on genuine QGC operators were obtained analyzing two-to-two scattering of electroweak gauge bosons, here denoted as $VV \rightarrow VV$ [16,17]. In this work we complement the existing literature on the subject by systematically deriving the unitarity limits on QGC originating from the $f\bar{f}' \rightarrow VV'V''$ channel with $V^{(')(\prime\prime)} = \gamma, Z, W^\pm$, and H , here denoted as $FF \rightarrow VVV$.

This paper is organized as follows: in Sec. II we present the QGC operators that we consider in our analyses, as well as the basic expressions of the three-particle partial-wave projections needed for our studies. Section III contains our results using the unitarity violating partial-wave projections presented in the Appendix while Sec. IV is dedicated to the discussion of the impact of the $FF \rightarrow VVV$ unitarity limits on present and future experimental analyses.

II. THEORETICAL FRAMEWORK

A. Effective field theory

Low-energy EFTs are based on the particle content as well as the symmetries of theory. Here, we assume that the observed Higgs boson is the SM one, i.e., it belongs to an electroweak scalar doublet. Therefore, we can construct a low-energy effective theory where the $SU(2)_L \otimes U(1)_Y$ gauge symmetry is linearly realized [18–24] which takes the form

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$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_{n=5}^{\infty} \sum_i \frac{f_i^{(n)}}{\Lambda^{n-4}} \mathcal{O}_i^{(n)}, \quad (2.1)$$

where the dimension- n operators $\mathcal{O}_i^{(n)}$ contain the SM fields and their covariant derivatives. We denote by $f_i^{(n)}$ the corresponding Wilson coefficient and by Λ the ultraviolet energy scale. The lowest dimension containing genuine QGC operators is eight [25–27]. Assuming C and P conservation, there are 20 dimension-eight genuine QGC operators that we classify by their number of gauge boson field strengths. There are three QGC operators formed of four covariant derivatives of the Higgs field

$$\begin{aligned} \mathcal{O}_{S,0} &= [(D_\mu \Phi)^\dagger D_\nu \Phi] \times [(D^\mu \Phi)^\dagger D^\nu \Phi], \\ \mathcal{O}_{S,1} &= [(D_\mu \Phi)^\dagger D^\mu \Phi] \times [(D_\nu \Phi)^\dagger D^\nu \Phi], \\ \mathcal{O}_{S,2} &= [(D_\mu \Phi)^\dagger D_\nu \Phi] \times [(D^\nu \Phi)^\dagger D^\mu \Phi], \end{aligned} \quad (2.2)$$

where Φ stands for the Higgs doublet, the covariant derivative is given by $D_\mu \Phi = (\partial_\mu + igW_\mu^j \frac{\sigma^j}{2} + ig'B_\mu \frac{1}{2})\Phi$, and σ^j ($j = 1, 2, 3$) represent the Pauli matrices. In addition, there are seven QGC operators that exhibit two covariant derivatives of the Higgs field as well as two field strengths:

$$\begin{aligned} \mathcal{O}_{M,0} &= \text{Tr}[\hat{W}_{\mu\nu} \hat{W}^{\mu\nu}] \times [(D_\beta \Phi)^\dagger D^\beta \Phi], \\ \mathcal{O}_{M,1} &= \text{Tr}[\hat{W}_{\mu\nu} \hat{W}^{\nu\beta}] \times [(D_\beta \Phi)^\dagger D^\mu \Phi], \\ \mathcal{O}_{M,2} &= [B_{\mu\nu} B^{\mu\nu}] \times [(D_\beta \Phi)^\dagger D^\beta \Phi], \\ \mathcal{O}_{M,3} &= [B_{\mu\nu} B^{\nu\beta}] \times [(D_\beta \Phi)^\dagger D^\mu \Phi], \\ \mathcal{O}_{M,4} &= [(D_\mu \Phi)^\dagger \hat{W}_{\beta\nu} D^\mu \Phi] \times B^{\beta\nu}, \\ \mathcal{O}_{M,5} &= [(D_\mu \Phi)^\dagger \hat{W}_{\beta\nu} D^\nu \Phi] \times B^{\beta\mu} + \text{H.c.}, \\ \mathcal{O}_{M,7} &= [(D_\mu \Phi)^\dagger \hat{W}_{\beta\nu} \hat{W}^{\beta\mu} D^\nu \Phi], \end{aligned} \quad (2.3)$$

where $\hat{W}_{\mu\nu} \equiv W_{\mu\nu}^j \frac{\sigma^j}{2}$ is the $SU(2)_L$ field strength while $B_{\mu\nu}$ stands for the $U(1)_Y$ one. Finally, we can construct ten operators with four field strengths:

$$\begin{aligned} \mathcal{O}_{T,0} &= \text{Tr}[\hat{W}_{\mu\nu} \hat{W}^{\mu\nu}] \times \text{Tr}[\hat{W}_{\alpha\beta} \hat{W}^{\alpha\beta}], \\ \mathcal{O}_{T,1} &= \text{Tr}[\hat{W}_{\alpha\nu} \hat{W}^{\mu\beta}] \times \text{Tr}[\hat{W}_{\mu\beta} \hat{W}^{\alpha\nu}] \\ \mathcal{O}_{T,2} &= \text{Tr}[\hat{W}_{\alpha\mu} \hat{W}^{\mu\beta}] \times \text{Tr}[\hat{W}_{\beta\nu} \hat{W}^{\nu\alpha}], \\ \mathcal{O}_{T,3} &= \text{Tr}[\hat{W}_{\mu\nu} \hat{W}_{\alpha\beta}] \times \text{Tr}[\hat{W}^{\alpha\nu} \hat{W}^{\mu\beta}] \\ \mathcal{O}_{T,4} &= \text{Tr}[\hat{W}_{\mu\nu} \hat{W}_{\alpha\beta}] \times B^{\alpha\nu} B^{\mu\beta}, \\ \mathcal{O}_{T,5} &= \text{Tr}[\hat{W}_{\mu\nu} \hat{W}^{\mu\nu}] \times B_{\alpha\beta} B^{\alpha\beta} \\ \mathcal{O}_{T,6} &= \text{Tr}[\hat{W}_{\alpha\nu} \hat{W}^{\mu\beta}] \times B_{\mu\beta} B^{\alpha\nu}, \\ \mathcal{O}_{T,7} &= \text{Tr}[\hat{W}_{\alpha\mu} \hat{W}^{\mu\beta}] \times B_{\beta\nu} B^{\nu\alpha} \\ \mathcal{O}_{T,8} &= B_{\mu\nu} B^{\mu\nu} B_{\alpha\beta} B^{\alpha\beta}, \quad \mathcal{O}_{T,9} = B_{\alpha\mu} B^{\mu\beta} B_{\beta\nu} B^{\nu\alpha}. \end{aligned} \quad (2.4)$$

The above 20 genuine QGC operators give rise to modifications to vertices $VVVV$, $VVVH$, and $VVHH$ ($V = W^\pm$, Z , and A) that are compatible with electric charge, C and P conservation [26].

B. Three-particle partial waves

In order to obtain the unitarity bounds from the process leading to the production of three electroweak gauge bosons

$$f_{1,\lambda_1} \bar{f}_{2,\lambda_2} \rightarrow V_{3,\lambda_3} V_{4,\lambda_4} V_{5,\lambda_5} \quad (2.5)$$

we project the corresponding helicity amplitude into a kinematic basis $\{B_{\lambda_1,\lambda_2;\lambda_3,\lambda_4,\lambda_5}^{J,a}\}$ with definite total angular momentum J . The index a represents additional quantum numbers needed to completely characterize the states besides helicities and total angular momentum. The partial-wave amplitudes $T^{J,a}(f_{1,\lambda_1} \bar{f}_{2,\lambda_2} \rightarrow V_{3,\lambda_3} V_{4,\lambda_4} V_{5,\lambda_5})$ are obtained by expanding the scattering amplitude in this basis:

$$\begin{aligned} \mathcal{M}(f_{1,\lambda_1} \bar{f}_{2,\lambda_2} \rightarrow V_{3,\lambda_3} V_{4,\lambda_4} V_{5,\lambda_5}) \\ = \sum_{J,a} T^{J,a}(f_{1,\lambda_1} \bar{f}_{2,\lambda_2} \rightarrow V_{3,\lambda_3} V_{4,\lambda_4} V_{5,\lambda_5}) \times B_{\lambda_1,\lambda_2;\lambda_3,\lambda_4,\lambda_5}^{J,a}. \end{aligned} \quad (2.6)$$

It is convenient to normalize the partial-wave basis such that

$$\begin{aligned} \int d\Phi_{f_1,f_2} d\Phi_{V_3,V_4,V_5} B_{\lambda_1,\lambda_2;\lambda_3,\lambda_4,\lambda_5}^{J,a} (B_{\lambda_1,\lambda_2;\lambda_3,\lambda_4,\lambda_5}^{J',b})^* \\ = (2J+1) \delta^{J,J'} \delta^{a,b}, \end{aligned} \quad (2.7)$$

where Φ_S is the phase space for states S . The normalization allows us to write the partial-wave amplitudes as

$$\begin{aligned} T^{J,a}(f_{1,\lambda_1} \bar{f}_{2,\lambda_2} \rightarrow V_{3,\lambda_3} V_{4,\lambda_4} V_{5,\lambda_5}) \\ = \frac{1}{2J+1} \int d\Phi_{f_1,f_2} d\Phi_{V_3,V_4,V_5} \mathcal{M}_{\lambda_1,\lambda_2;\lambda_3,\lambda_4,\lambda_5} (B_{\lambda_1,\lambda_2;\lambda_3,\lambda_4,\lambda_5}^{J,a})^*. \end{aligned} \quad (2.8)$$

The partial-wave basis can be constructed from the kinematic polynomials obtained using spinor-helicity techniques [28]. The kinematic polynomials $\mathcal{P}_{\lambda_1,\lambda_2;\lambda_3,\lambda_4,\lambda_5}^{J,a}$ relevant for $2 \rightarrow 3$ processes are given in the Appendix A of Ref. [28]. For the process (2.5) with massless fermions the relevant polynomials are

$\lambda_1, \lambda_2; \lambda_3, \lambda_4, \lambda_5$	J	a	$\mathcal{P}^{J,a}$
$(-\frac{1}{2}, \frac{1}{2}; 0, 0, 0)$	1	1	$192\sqrt{2}\pi^2 S^{-3/2} \langle 14 \rangle [42]$
	1	2	$192\sqrt{2}\pi^2 S^{-3/2} \langle 15 \rangle [52]$
$(-\frac{1}{2}, \frac{1}{2}; 0, 0, 1)$	1	1	$384\sqrt{\frac{5}{7}}\pi^2 S^{-2} \langle 13 \rangle [52][53]$
	1	2	$384\sqrt{\frac{5}{7}}\pi^2 S^{-2} \langle 14 \rangle [52][54]$
$(-\frac{1}{2}, \frac{1}{2}; 0, 1, 1)$	1	1	$576\pi^2 S^{-5/2} \langle 15 \rangle [52][54]^2$
	1	2	$576\pi^2 S^{-5/2} \langle 14 \rangle [42][54]^2$
	1	3	$288\sqrt{10}\pi^2 S^{-5/2} \langle 13 \rangle [42][53][54]$
$(-\frac{1}{2}, \frac{1}{2}; -1, 0, 1)$	1	1	$288\sqrt{10}\pi^2 S^{-5/2} \langle 13 \rangle \langle 34 \rangle [52][54]$
$(-\frac{1}{2}, \frac{1}{2}; -1, 1, 1)$	1	1	$1152\sqrt{\frac{35}{31}}\pi^2 S^{-3} \langle 13 \rangle \langle 35 \rangle [52][54]^2$
	1	2	$1152\sqrt{\frac{35}{31}}\pi^2 S^{-3} \langle 13 \rangle \langle 34 \rangle [42][54]^2$
$(-\frac{1}{2}, \frac{1}{2}; 1, 1, 1)$	1	1	$1152\sqrt{\frac{35}{31}}\pi^2 S^{-3} \langle 14 \rangle [32][43][54]^2$
	1	2	$1152\sqrt{\frac{35}{31}}\pi^2 S^{-3} \langle 15 \rangle [32][53][54]^2$
	1	3	$1152\sqrt{\frac{35}{31}}\pi^2 S^{-3} \langle 13 \rangle [42][43][53]^2$
	1	4	$1152\sqrt{\frac{35}{31}}\pi^2 S^{-3} \langle 15 \rangle [42][53]^2[54]$
	1	5	$576\sqrt{14}\pi^2 S^{-3} \langle 14 \rangle [42][43][53][54]$

(2.9)

where $S = (p_1 + p_2)^2$. In addition, the polynomials with $\lambda_i \rightarrow -\lambda_i$ are obtained with the substitution $\langle ij \rangle \leftrightarrow [ji]$ for all i, j . We use the conventions for the square and angle brackets of Ref. [28] while the phase space integrals were carried out using spinor helicity variables as defined in the Appendix A of Ref. [29].

The kinematic polynomials in Eq. (2.9) are normalized to one, however, they are not orthogonal for different values of the additional quantum numbers “ a .” Hence, to define the partial-wave amplitudes we construct the corresponding normalized and orthogonal kinematic basis as

$$\mathcal{B}_{\lambda_1\lambda_2;\lambda_3,\lambda_4,\lambda_5}^{J,a} = \sum_b \mathcal{R}_{ab}^J \times \mathcal{P}_{\lambda_1\lambda_2;\lambda_3,\lambda_4,\lambda_5}^{J,b} \quad (2.10)$$

where the “rotation” matrices are given by

$$\begin{aligned} \mathcal{R}_{-\frac{1}{2},\frac{1}{2};0,0,0}^1 &= \begin{pmatrix} 1 & 1 \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{pmatrix}, \quad \mathcal{R}_{-\frac{1}{2},\frac{1}{2};0,0,1}^1 = \frac{1}{2} \begin{pmatrix} \sqrt{\frac{7}{5}} & \sqrt{\frac{7}{5}} \\ \sqrt{\frac{7}{2}} & -\sqrt{\frac{7}{2}} \end{pmatrix}, \quad \mathcal{R}_{-\frac{1}{2},\frac{1}{2};0,1,1}^1 = \begin{pmatrix} 0.442 & 0.405 & 0.317 \\ 0.175 & -0.596 & 1.00 \\ 2.25 & 1.89 & 0.730 \end{pmatrix} \\ \mathcal{R}_{-\frac{1}{2},\frac{1}{2};-1,0,1}^1 &= \frac{1}{2} \begin{pmatrix} \sqrt{\frac{31}{21}} & \sqrt{\frac{31}{21}} \\ \sqrt{\frac{31}{10}} & -\sqrt{\frac{31}{10}} \end{pmatrix}, \quad \mathcal{R}_{-\frac{1}{2},\frac{1}{2};-1,1,1}^1 = \begin{pmatrix} 0.203 & 0.429 & -0.0671 & 0.372 & 0.280 \\ 0.320 & -0.0197 & 0.606 & -0.331 & 0.384 \\ -0.940 & 0.125 & 0.431 & 0.220 & 0.300 \\ -0.174 & -0.361 & -0.781 & -0.396 & 1.019 \\ 0.684 & -2.84 & 0.597 & 2.58 & 0.572 \end{pmatrix}. \end{aligned} \quad (2.11)$$

We can then construct the partial-wave amplitudes as the corresponding linear combination

$$T^{J,a}(f_{1,\lambda_1}\bar{f}_{2,\lambda_2} \rightarrow V_{3,\lambda_3}V_{4,\lambda_4}V_{5,\lambda_5}) = \sum_b (\mathcal{R}_{ab}^J)^* \times \tilde{T}^{J,b}(f_{1,\lambda_1}\bar{f}_{2,\lambda_2} \rightarrow V_{3,\lambda_3}V_{4,\lambda_4}V_{5,\lambda_5}) \quad (2.12)$$

in terms of the projections over the nonorthogonal basis

$$\begin{aligned} \tilde{T}^{J,b}(f_{1,\lambda_1}\bar{f}_{2,\lambda_2} \rightarrow V_{3,\lambda_3}V_{4,\lambda_4}V_{5,\lambda_5}) \\ = \frac{1}{2J+1} \int d\Phi_{f_1,f_2} d\Phi_{V_3,V_4,V_5} \\ \times \mathcal{M}_{\lambda_1\lambda_2;\lambda_3,\lambda_4,\lambda_5}(\mathcal{P}_{\lambda_1,\lambda_2;\lambda_3,\lambda_4,\lambda_5}^{J,b})^*. \end{aligned} \quad (2.13)$$

Finally, the optical theorem can be generalized leading to

$$T_{i \rightarrow f}^{J,a} - (T_{i \rightarrow f}^{J,a})^* = i \sum_X T_{i \rightarrow X}^{J,a} (T_{f \rightarrow X}^{J,a})^* \quad (2.14)$$

that leads to the partial-wave unitarity bound

$$|T_{i \rightarrow f}^{J,a}(f_{1,\lambda_1}\bar{f}_{2,\lambda_2} \rightarrow V_{3,\lambda_3}V_{4,\lambda_4}V_{5,\lambda_5})| < 1. \quad (2.15)$$

III. UNITARITY LIMITS ON GENUINE QGC AT TRIPLE GAUGE BOSON PRODUCTION

Many experimental studies of anomalous QGCs are carried out studying the exclusive production of three electroweak gauge bosons, i.e., $FF \rightarrow VVV$. In this section we analyze the unitarity limits associated with the production of three gauge and/or Higgs bosons. In doing so, we focus on the leading processes that present the fastest growth as a function of the three gauge boson center-of-mass energy $\sqrt{S_{VVV}}$.

A. $\mathcal{O}_{S,i}$ operators

For operators $\mathcal{O}_{S,i}$, the partial-wave amplitudes displaying the fastest increase with $\sqrt{S_{VVV}}$ are the ones with three longitudinally polarized gauge bosons and they are shown in Table II. These results are presented for the nonorthogonal partial-wave states.¹ The high-energy behavior of the leading helicity amplitudes is

$$T_{-\frac{1}{2},\frac{1}{2};0,0,0}^{1,a} = C_{S,i}^{VVV} \times \frac{f_{S,i}}{\Lambda^4} S_{VVV}^{3/2} v, \quad (3.1)$$

where v stands for the Higgs vacuum expectation value. This high-energy behavior is to be expected from simple dimensional analysis. For $\mathcal{O}_{S,i}$ operators the four-vector-boson coupling originates from the four Higgs covariant derivatives and therefore it is proportional to v^4 while the presence of three polarized gauge bosons leads to a factor v^{-3} resulting in a scattering amplitude proportional to vS_{VVV} since the scattering amplitude has inverse mass dimension. Taking into account that the two-to-three phase

¹For the sake of concreteness we show the results for initial left-handed currents, that is, with *incoming* helicities $\lambda_1 = -\lambda_2 = -\frac{1}{2}$ (as they are assumed to be massless). Furthermore the results for initial right-handed fermion currents can be obtained by projecting over the parity flipped basis, and changing the chirality of the corresponding fermion weak charge involved in the process.

space scales as S_{VVV} and the partial-wave basis as $S_{VVV}^{-1/2}$, we obtain a partial-wave amplitude proportional to $vS_{VVV}^{3/2}$.

Explicitly, after rotating to the partial waves on the orthogonal basis with Eq. (2.10), we find the largest partial-wave coefficients for each coupling to be

$$|C_{S,i,\max}^{VVV}| = (1.8, 0.90, 0.90) \times 10^{-5} \quad (3.2)$$

for $i = 0, 1, 2$. They arise from the charged current (CC) processes $f_d\bar{f}_u \rightarrow W^-W^+W^-$. The corresponding unitarity bounds read

$$\begin{aligned} \left| \frac{f_{S,0}}{\Lambda^4} \right| &\leq 2.3 \times 10^5 \left(\frac{\text{TeV}^2}{S_{VVV}} \right)^{\frac{3}{2}}, \\ \left| \frac{f_{S,1}}{\Lambda^4} \right| &\leq 4.5 \times 10^5 \left(\frac{\text{TeV}^2}{S_{VVV}} \right)^{\frac{3}{2}}, \\ \left| \frac{f_{S,2}}{\Lambda^4} \right| &\leq 4.5 \times 10^5 \left(\frac{\text{TeV}^2}{S_{VVV}} \right)^{\frac{3}{2}}. \end{aligned} \quad (3.3)$$

For instance, assuming that $f_{S,i}/\Lambda^4 = 1 \text{ TeV}^{-4}$ unitarity is violated in $FF \rightarrow VVV$ at the center-of-mass energies of (61, 76, 76) TeV for $i = 0, 1, 2$, respectively.

B. $\mathcal{O}_{M,i}$ operators

All $\mathcal{O}_{M,i}$ operators contain two field strengths, therefore, their contributions to scattering amplitudes vanish when all vector boson polarizations are longitudinal. In this case the amplitudes with fastest increase with $\sqrt{S_{VVV}}$ are the ones with two longitudinally polarized and one transversely polarized gauge bosons. In this case the amplitude contains a factor v^2 originating from the two covariant derivatives which cancels out the factor v^{-2} due to the two longitudinally polarized gauge bosons, resulting in a scattering amplitude that grows as $S_{VVV}^{3/2}$. Again, taking into account that the two-to-three phase space scales as S_{VVV} and the partial-wave basis as $S_{VVV}^{-1/2}$, we find leading contributions of these operators depending upon S_{VVV} as

$$T_{-\frac{1}{2},\frac{1}{2};0,0,1}^{1,a} = C_{M,i}^{VVV} \times \frac{f_{M,i}}{\Lambda^4} S_{VVV}^2. \quad (3.4)$$

The $\mathcal{O}_{M,i}$ nonorthogonal partial-wave amplitudes for the relevant processes are presented in Table III. Using these results and Eq. (2.10), we can evaluate the fastest growing partial wave for each Wilson coefficient:

$$|C_{M,i,\max}^{VVV}| = (2.3, 1.6, 2.3, 1.5, 1.5, 0.97, 0.78) \times 10^{-5} \quad (3.5)$$

for $i = 0, 1, 2, 3, 4, 5, 7$, respectively. They arise from the CC processes $f_d\bar{f}_u \rightarrow W^-W^+W^-$, ZZW^- , HHW^- for coupling $i = 0, 1, 7$, from CC processes $f_d\bar{f}_u \rightarrow ZW^-\gamma$, $HW^-\gamma$ for coupling $i = 4, 5$, and from neutral

current (NC) processes $\ell^-\ell^+ \rightarrow W^+W^-\gamma$ and $\nu_\ell\bar{\nu}_\ell \rightarrow W^+W^-\gamma$ for $i = 2, 3$. The corresponding unitarity bounds are

$$\begin{aligned} \left| \frac{f_{M,0}}{\Lambda^4} \right| &< 4.2 \times 10^4 \frac{\text{TeV}^4}{S_{VVV}^2}, & \left| \frac{f_{M,1}}{\Lambda^4} \right| &< 6.4 \times 10^4 \frac{\text{TeV}^4}{S_{VVV}^2}, \\ \left| \frac{f_{M,2}}{\Lambda^4} \right| &< 4.4 \times 10^4 \frac{\text{TeV}^4}{S_{VVV}^2}, & \left| \frac{f_{M,3}}{\Lambda^4} \right| &< 6.6 \times 10^4 \frac{\text{TeV}^4}{S_{VVV}^2}, \\ \left| \frac{f_{M,4}}{\Lambda^4} \right| &< 6.8 \times 10^4 \frac{\text{TeV}^4}{S_{VVV}^2}, & \left| \frac{f_{M,5}}{\Lambda^4} \right| &< 1.0 \times 10^5 \frac{\text{TeV}^4}{S_{VVV}^2}, \\ \left| \frac{f_{M,7}}{\Lambda^4} \right| &< 1.3 \times 10^5 \frac{\text{TeV}^4}{S_{VVV}^2}. \end{aligned} \quad (3.6)$$

We find that the growth of the scattering amplitudes is as fast as S_{VVV}^2 . However the numerical coefficients are small compared to those from $2 \rightarrow 2$ amplitudes in Ref. [16] (see also next section). This is so because of the higher partial wave ($J = 1$) and larger normalization factors of partial waves for three-body final states as well as from the additional gauge coupling constant in the ffV vertex. We can assess that the impact of such small numerical coefficients by obtaining the center-of-mass energy for which unitarity is violated in $FF \rightarrow VVV$ taking $f_{M,i}/\Lambda^4 = 1 \text{ TeV}^{-4}$:

$$(14, 16, 14, 16, 16, 18, 19) \text{ TeV} \quad \text{for } i = 0, \dots, 7. \quad (3.7)$$

C. $\mathcal{O}_{T,i}$ operators

All $\mathcal{O}_{T,i}$ operators possess four field strength tensors, consequently their leading contributions appear in amplitudes where all gauge bosons are transversely polarized. Thus, in this case there are no v factors in the amplitude and, as expected from dimensional analysis, the $\mathcal{O}_{T,i}$ operators lead to a growth of the scattering amplitude proportional to $S_{VVV}^{3/2}$. Hence, after accounting for phase space and partial-wave S_{VVV} dependence, we find

$$T_{-\frac{1}{2}, \pm 1, \pm 1, \pm 1}^{1,a} = C_{T,i}^{VVV} \times \frac{f_{T,i}}{\Lambda^4} S_{VVV}^2. \quad (3.8)$$

We present in Tables IV and V the $\mathcal{O}_{T,i}$ contributions to the nonorthogonal partial-wave amplitudes for all the relevant processes. After rotating to the projections on the orthogonal basis with Eq. (2.10), we find the largest partial-wave amplitudes to have coefficients

$$\begin{aligned} |C_{T,i,\max}^{VVV}| &= (1.9, 2.0, 0.61, 0.74, 0.34, 0.94, \\ &0.68, 0.23, 1.9, 0.55) \times 10^{-4} \end{aligned} \quad (3.9)$$

for $i = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$, respectively. They arise from $f_d\bar{f}_u \rightarrow W^-W^+W^-$ for $i = 0, 1, 2, 3$, $f_d\bar{f}_u \rightarrow \gamma\gamma W^-$ for $i = 4, 5, 6, 7$, and $\ell\bar{\ell} \rightarrow \gamma\gamma\gamma$ and $\nu_\ell\bar{\nu}_\ell \rightarrow \gamma\gamma\gamma$ for $i = 8, 9$. This leads to the unitarity constraints

$$\begin{aligned} \left| \frac{f_{T,0}}{\Lambda^4} \right| &< 5.2 \times 10^3 \frac{\text{TeV}^4}{S_{VVV}^2}, & \left| \frac{f_{T,1}}{\Lambda^4} \right| &< 5.0 \times 10^3 \frac{\text{TeV}^4}{S_{VVV}^2}, \\ \left| \frac{f_{T,2}}{\Lambda^4} \right| &< 1.6 \times 10^4 \frac{\text{TeV}^4}{S_{VVV}^2}, & \left| \frac{f_{T,3}}{\Lambda^4} \right| &< 1.3 \times 10^4 \frac{\text{TeV}^4}{S_{VVV}^2}, \\ \left| \frac{f_{T,4}}{\Lambda^4} \right| &< 3.0 \times 10^4 \frac{\text{TeV}^4}{S_{VVV}^2}, & \left| \frac{f_{T,5}}{\Lambda^4} \right| &< 1.1 \times 10^4 \frac{\text{TeV}^4}{S_{VVV}^2}, \\ \left| \frac{f_{T,6}}{\Lambda^4} \right| &< 1.5 \times 10^4 \frac{\text{TeV}^4}{S_{VVV}^2}, & \left| \frac{f_{T,7}}{\Lambda^4} \right| &< 4.3 \times 10^4 \frac{\text{TeV}^4}{S_{VVV}^2}, \\ \left| \frac{f_{T,8}}{\Lambda^4} \right| &< 5.2 \times 10^3 \frac{\text{TeV}^4}{S_{VVV}^2}, & \left| \frac{f_{T,9}}{\Lambda^4} \right| &< 1.8 \times 10^4 \frac{\text{TeV}^4}{S_{VVV}^2}. \end{aligned} \quad (3.10)$$

So despite the $\mathcal{O}_{T,i}$ operators leading to a growth of the scattering amplitude proportional to S_{VVV}^2 , again, as for the case of $\mathcal{O}_{M,i}$ operators, the corresponding numerical coefficients are small because of phase space, partial-wave, and gauge coupling factors, which result into weak unitarity bounds. For instance, assuming $f_{T,i}/\Lambda^4 = 1 \text{ TeV}^{-4}$, unitarity violation takes place in $FF \rightarrow VVV$ at $\sqrt{S_{VVV}}$ of

$$(8.5, 8.4, 11, 11, 13, 10, 11, 14, 8.5, 12) \text{ TeV} \quad \text{for } i = 0, \dots, 9. \quad (3.11)$$

IV. DISCUSSION

For a fixed value of a Wilson coefficient, the unitarity bound indicates the energy scale where perturbative unitarity breaks down, signaling the need to modify the theoretical approach at higher energies. In particular, the study of the $FF \rightarrow VVV$ process indicates the maximum validity of the approach as a function of the three gauge boson invariant mass. From the previous section we have learned that the QGC contributions to $FF \rightarrow VVV$ helicity amplitudes grow as S_{VVV}^2 with $\sqrt{S_{VVV}}$ being the three gauge boson invariant mass; see Table I.

Analogously, the analysis of the anomalous QGC contributions to the $VV \rightarrow VV$ amplitudes marks the energy limit of the applicability of EFT as a function of the gauge boson pair invariant mass. Reference [16] contains a detailed study of unitarity limits on QGCs stemming from $VV \rightarrow VV$ scattering processes. These bounds were obtained from the $J = 0$ and $J = 1$ partial-wave amplitudes for the scattering of gauge and Higgs bosons. Considering just one operator at a time the tightest bounds arise from the analysis of the $J = 0$ partial wave whose fastest energy growing behavior is of the form

$$T^{J=0} = C_i^{VV} \times \frac{f_i}{\Lambda^4} S_{VV}^2, \quad (4.1)$$

where C_i^{VV} are numerical constants and $\sqrt{S_{VV}}$ stands for the diboson center-of-mass energy. We found the largest coefficients for one operator at a time to be

TABLE I. QGC contributions to $FF \rightarrow VVV$ that present a S_{VVV}^2 growth of the helicity amplitude.

Process	$\mathcal{O}_{M,0}$	$\mathcal{O}_{M,1}$	$\mathcal{O}_{M,2}$	$\mathcal{O}_{M,3}$	$\mathcal{O}_{M,4}$	$\mathcal{O}_{M,5}$	$\mathcal{O}_{M,7}$	$\mathcal{O}_{T,0}$	$\mathcal{O}_{T,1}$	$\mathcal{O}_{T,2}$	$\mathcal{O}_{T,3}$	$\mathcal{O}_{T,4}$	$\mathcal{O}_{T,5}$	$\mathcal{O}_{T,6}$	$\mathcal{O}_{T,7}$	$\mathcal{O}_{T,8}$	$\mathcal{O}_{T,9}$
$f\bar{f} \rightarrow \gamma\gamma\gamma$								✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$f\bar{f} \rightarrow \gamma ZZ$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$f\bar{f} \rightarrow \gamma\gamma Z$								✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$f\bar{f} \rightarrow ZZZ$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
$f\bar{f} \rightarrow W^- W^+ \gamma$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		
$f\bar{f} \rightarrow W^- W^+ Z$	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		
$f_d \bar{f}_u \rightarrow W^- W^+ W^-$	✓	✓					✓	✓	✓	✓	✓						
$f_d \bar{f}_u \rightarrow \gamma\gamma W^-$								✓	✓	✓	✓	✓	✓	✓	✓		
$f_d \bar{f}_u \rightarrow ZZ W^-$	✓	✓			✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		
$f_d \bar{f}_u \rightarrow \gamma Z W^-$					✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓		

$$\begin{aligned}
|C_{S,i,\max}^{VV}| &= \left(\frac{1}{32\pi}, \frac{7}{96\pi}, \frac{5}{96\pi} \right) \quad \text{for } i = 0, 1, 2 \\
|C_{M,i,\max}^{VV}| &= \left(\frac{\sqrt{6}}{32\pi}, \frac{\sqrt{6}}{128\pi}, \frac{\sqrt{2}}{16\pi}, \frac{\sqrt{2}}{64\pi}, \frac{1}{32\pi}, \frac{1}{64\pi}, \frac{\sqrt{6}}{256\pi} \right), \quad \text{for } i = 0, 1, 2, 3, 4, 5, 7 \\
|C_{T,i,\max}^{VV}| &= \left(\frac{5}{12\pi}, \frac{5}{24\pi}, \frac{13}{96\pi}, \frac{3}{32\pi}, \frac{1}{16\pi}, \frac{\sqrt{3}}{8\pi}, \frac{7}{48\pi}, \frac{\sqrt{3}}{32\pi}, \frac{2}{3\pi}, \frac{7}{24\pi} \right) \quad \text{for } i = 0 \dots 9.
\end{aligned} \tag{4.2}$$

As a reference, for $f_{X,i}/\Lambda^4 = 1 \text{ TeV}^{-4}$ unitarity violation in $VV \rightarrow VV$ amplitudes takes place at diboson invariant masses, M_{VV} , in excess of 1.5–4.7 TeV depending on the QGC operator. Therefore, the question which arises in the experimental searches of genuine QGC in triple gauge boson production is how to implement the unitarity constraints from gauge boson pair scattering. One conservative prescription, sometimes referred to as “clipping method” in the experimental literature [6,7], consists of restricting the EFT contribution to phase space regions where the invariant mass of any of the gauge bosons pairs is smaller than a cutoff value ($M_{VV,\text{clip}}$). Next, the constraints on the Wilson coefficient are derived from data as a function of $M_{VV,\text{clip}}$. The limits are more stringent the higher $M_{VV,\text{clip}}$ is due to the growth of the anomalous cross section, and the validity of the limits are verified by comparing them to the corresponding unitarity bounds from $VV \rightarrow VV$. In some cases of the LHC Run I and Run II data, the results show sensitivity of the $FF \rightarrow VVV$ processes to QGC without violation of the $VV \rightarrow VV$ unitarity bounds [6,7] for $M_{VV,\text{clip}}$ of the order 1–2 TeV.

Within this approach it is straightforward to clarify if the $FF \rightarrow VVV$ unitarity limits which we have derived in Sec. III imposed any further constraints on the QGC analyses of three gauge boson production performed within the clipping method. This can be systematically addressed by determining the maximum S_{VVV} implied by the $FF \rightarrow VVV$ unitarity limits for the limit value of the coupling obtained after employing the clipping method.

In brief, with the clipping method one obtains the bounds on the anomalous coefficients from the three gauge boson process P compatible with the $VV \rightarrow VV$ unitarity bounds to be

$$\frac{f_X}{\Lambda^4} \leq \frac{1}{|C_{X,\max}^{VV}| M_{VV,\text{clip},P}^4} \tag{4.3}$$

where $|C_{X,\max}^{VV}|$ are the numerical coefficients in Eq. (4.2). Now, for this value of the anomalous Wilson coefficient, $FF \rightarrow VVV$ unitarity requires that

$$\sqrt{S_{VVV,P}} \leq \left(\frac{\Lambda^4}{f_X |C_{X,\max}^{VVV}|} \right)^{\frac{1}{4}}. \tag{4.4}$$

The most constraining case will correspond to the maximal value of the Wilson coefficient in Eq. (4.3) which implies that the maximum three gauge boson invariant mass for which $FF \rightarrow VVV$ unitarity is not violated in process P is

$$\sqrt{S_{VVV,P}^{\max}} = \left(\frac{|C_{X,\max}^{VV}|}{|C_{X,\max}^{VVV}|} \right)^{\frac{1}{4}} M_{VV,\text{clip},P} \sim 4 - 6 M_{VV,\text{clip},P}, \tag{4.5}$$

where we used the values of the coefficients $C_{X,\max}^{VV}$ in Eq. (4.2) and the coefficients $C_{X,\max}^{VVV}$ in Eqs. (3.5) and (3.9).

On the other hand, simple kinematics implies that for any process P with the three final gauge bosons with four-momenta p_1 , p_2 , and p_3 ,

$$\begin{aligned}
\sqrt{S_{VVV,P}} &\equiv \sqrt{(p_1 + p_1 + p_3)^2} \\
&\leq \sqrt{(p_1 + p_2)^2 + (p_1 + p_3)^2 + (p_2 + p_3)^2} \\
&= \sqrt{M_{12}^2 + M_{13}^2 + M_{23}^2} \leq \sqrt{3} M_{VV\text{clip},P}, \quad (4.6)
\end{aligned}$$

where M_{ij} stands for the invariant mass of the particles i and j . So $\sqrt{S_{VVV,P}}$ is always smaller than $\sqrt{S_{VVV,P}^{\text{max}}}$ by at least a factor of 2.

All in all we conclude that the $FF \rightarrow VVV$ unitarity bounds do not alter the present [3–11] and future anomalous QGC analyses in the exclusive production of three gauge bosons obtained with the clipping method. In brief, our findings demonstrate the consistency of the clipping method.

ACKNOWLEDGMENTS

We thank Gabriel M. Salla for enlightening discussions. O.J.P.E. is partially supported by CNPq Grant No. 302120/2025-4. This project is funded by USA-NSF Grant No. PHY-2210533. It has also received support from the European Union’s Horizon 2020 research and

innovation program under the Marie Skłodowska–Curie Grant Agreement No. 860881-HIDDeN, and Horizon Europe research and innovation programme under the Marie Skłodowska–Curie Staff Exchange Grant Agreement No. 101086085 “ASYMMETRY”. It also receives support from Grant No. PID2019–105614 GB-C21, and “Unit of Excellence Maria de Maeztu 2020–2023” award to the ICC-UB CEX2019–000918-M, funded by MCIN/AEI/10.13039/501100011033, and from Grant No. 2021-SGR-249 (Generalitat de Catalunya).

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

APPENDIX: HELICITY AMPLITUDES

Here, we present the helicity amplitudes in Tables II–V that contain the fastest growth with $\sqrt{S}$ for the processes $FF \rightarrow VVV$ ($V = \gamma, Z, W^\pm$, and H) used to obtain the $2 \rightarrow 3$ unitarity constraints.

TABLE II. Partial-wave nonorthogonal projections for $FF \rightarrow VVV$ generated by $\mathcal{O}_{S,i}$. In this table, $f = u, d, \ell$, or ν_ℓ , while we denote by $f_{u(d)} = u(d)$ or $\nu_\ell(\ell)$. The NC scattering amplitude dependence on the chosen fermion is encoded in the couplings $g_L = T_3 - Qs_W^2$ where T_3 and Q are the third component of isospin and charge of the corresponding fermion with s_W (c_W) being the sine (cosine) of the weak mixing angle. Moreover, $g_L = 1$ for the remaining CC processes where we omit, for simplicity, the Cabibbo–Kobayashi–Maskawa (CKM) factors.

		$\tilde{T}_{-\frac{1}{2},\frac{1}{2},0,0}^{1,a}$ in units of $\frac{e^2}{3840\pi^2} \times g_L^f \times \frac{f_{S,i}}{\Lambda^4} S_{VVV}^{3/2} v \times \frac{1}{\sqrt{N_{id}}}$		
	a/i	0	1	2
$f\bar{f} \rightarrow W^+W^-Z$	1	$-i \frac{1}{2\sqrt{2}c_W^2s_W^2}$	$i \frac{1}{\sqrt{2}c_W^2s_W^2}$	$-i \frac{1}{2\sqrt{2}c_W^2s_W^2}$
	2	$i \frac{1}{\sqrt{2}c_W^2s_W^2}$	$-i \frac{\sqrt{2}}{c_W^2s_W^2}$	$i \frac{1}{\sqrt{2}c_W^2s_W^2}$
$f\bar{f} \rightarrow ZHH$	1	$-i \frac{\sqrt{2}}{c_W^2s_W^2}$	$i \frac{1}{\sqrt{2}c_W^2s_W^2}$	$i \frac{1}{\sqrt{2}c_W^2s_W^2}$
	2	$-i \frac{\sqrt{2}}{c_W^2s_W^2}$	$i \frac{1}{\sqrt{2}c_W^2s_W^2}$	$i \frac{1}{\sqrt{2}c_W^2s_W^2}$
$f_d\bar{f}_u \rightarrow W^-W^+W^-$	1	$i \frac{1}{s_W^2}$	$-i \frac{1}{2s_W^2}$	$-i \frac{1}{2s_W^2}$
	2	$i \frac{1}{s_W^2}$	$-i \frac{1}{2s_W^2}$	$-i \frac{1}{2s_W^2}$
$f_d\bar{f}_u \rightarrow ZZW^-$	1	$-i \frac{1}{4s_W^2}$	$i \frac{1}{2s_W^2}$	$-i \frac{1}{4s_W^2}$
	2	$i \frac{1}{2s_W^2}$	$-i \frac{1}{2s_W^2}$	$i \frac{1}{2s_W^2}$
$f_d\bar{f}_u \rightarrow HHW^-$	1	$-i \frac{1}{4s_W^2}$	$i \frac{1}{2s_W^2}$	$-i \frac{1}{4s_W^2}$
	2	$i \frac{1}{2s_W^2}$	$-i \frac{1}{2s_W^2}$	$i \frac{1}{2s_W^2}$
$f_d\bar{f}_u \rightarrow ZHW^-$	1	$\frac{3}{4s_W^2}$	0	$-\frac{3}{4s_W^2}$
	2	0	0	0

TABLE III. Partial wave non-orthogonal projections for $FF \rightarrow VVV$ generated by $\mathcal{O}_{M,i}$. In the table $f = u, d, \ell$, or ν_ℓ , while we denote by $f_{u(d)} = u(d)$ or $\nu_\ell(\ell)$. The scattering amplitude dependence on the chosen fermion is encoded in the couplings $g_L = T_3 - Qs_W^2 = T_3c_W^2 - \frac{Y}{2}s_W^2$ with the hypercharge $Y = 2(Q - T_3)$, and the we have defined the combination $\tilde{g} \equiv T_3c_W^2 + \frac{Y}{2}s_W^2$. The amplitude for the CC processes are the same for quarks and leptons up to the CKM factor which we are omitting for simplicity.

		$\tilde{T}_{\frac{1}{2}, \frac{1}{2}, \lambda_3, \lambda_4, \lambda_5}^{1,a}$ in units of $\frac{e}{\pi^2} \frac{1}{192\sqrt{70}} \times \frac{f_{M,i}}{\Lambda^4} S_{VVV}^2 \times \frac{1}{\sqrt{N_{id}}}$						
	a/i	0	1	2	3	4	5	7
$\lambda_3, \lambda_4, \lambda_5 = 0, 0, 1$								
$f\bar{f} \rightarrow ZZ\gamma$	1	$-T_3$	$\frac{2T_3}{3}$	Y	$-\frac{2Y}{3}$	$-\frac{\tilde{g}}{2c_Ws_W}$	$\frac{\tilde{g}}{3c_Ws_W}$	$\frac{T_3}{3}$
	2	$-T_3$	$\frac{2T_3}{3}$	Y	$-\frac{2Y}{3}$	$-\frac{\tilde{g}}{2c_Ws_W}$	$\frac{\tilde{g}}{3c_Ws_W}$	$\frac{2T_3}{6}$
$f\bar{f} \rightarrow ZZZ$	1	$-\frac{T_3c_W}{s_W}$	$\frac{2T_3c_W}{3s_W}$	$\frac{Ys_W}{c_W}$	$-\frac{2Ys_W}{3c_W}$	$-\frac{2T_3-Y}{4}$	$-\frac{2T_3-Y}{6}$	$-\frac{2T_3c_W}{6s_W}$
	2	$-\frac{T_3c_W}{s_W}$	$\frac{2T_3c_W}{3s_W}$	$\frac{Ys_W}{c_W}$	$-\frac{2Ys_W}{3c_W}$	$-\frac{2T_3-Y}{4}$	$-\frac{2T_3-Y}{6}$	$-\frac{2T_3c_W}{6s_W}$
$f\bar{f} \rightarrow W^+W^-\gamma$	1	$-T_3$	$\frac{2T_3}{3}$	$-Y$	$\frac{2Y}{3}$	$-\frac{\tilde{g}}{3c_Ws_W}$	$-\frac{10T_3c_W^2+3Ys_W^2}{24c_Ws_W}$	$-\frac{T_3}{3}$
	2	$-T_3$	$\frac{2T_3}{3}$	$-Y$	$\frac{2Y}{3}$	$-\frac{\tilde{g}}{2c_Ws_W}$	$-\frac{6T_3c_W^2+5Ys_W^2}{24c_Ws_W}$	$-\frac{T_3}{3}$
$f\bar{f} \rightarrow W^+W^-Z$	1	$-\frac{T_3c_W}{s_W}$	$\frac{2T_3c_W}{3s_W}$	$\frac{Ys_W}{c_W}$	$-\frac{2Ys_W}{3c_W}$	$\frac{2T_3-Y}{4}$	$-\frac{-10T_3+3Y}{24}$	$-\frac{T_3c_W}{3s_W}$
	2	$-\frac{T_3c_W}{s_W}$	$\frac{2T_3c_W}{3s_W}$	$\frac{Ys_W}{c_W}$	$-\frac{2Ys_W}{3c_W}$	$\frac{2T_3-Y}{4}$	$-\frac{-10T_3+3Y}{24}$	$-\frac{T_3c_W}{3s_W}$
$f\bar{f} \rightarrow HH\gamma$	1	$-T_3$	$\frac{2T_3}{3}$	$-Y$	$\frac{2Y}{3}$	$\frac{\tilde{g}}{2c_Ws_W}$	$\frac{\tilde{g}}{3c_Ws_W}$	$-\frac{T_3}{3}$
	2	$-\frac{2T_3}{2}$	$\frac{2T_3}{3}$	$-Y$	$\frac{2Y}{3}$	$\frac{\tilde{g}}{2c_Ws_W}$	$\frac{\tilde{g}}{3c_Ws_W}$	$-\frac{T_3}{3}$
$f\bar{f} \rightarrow HHZ$	1	$-\frac{T_3c_W}{s_W}$	$\frac{2T_3c_W}{3s_W}$	$\frac{Ys_W}{c_W}$	$-\frac{2Ys_W}{3c_W}$	$-\frac{2T_3-Y}{4}$	$-\frac{2T_3-Y}{6}$	$-\frac{T_3c_W}{3s_W}$
	2	$-\frac{T_3c_W}{s_W}$	$\frac{2T_3c_W}{3s_W}$	$\frac{Ys_W}{c_W}$	$-\frac{2Ys_W}{3c_W}$	$-\frac{2T_3-Y}{4}$	$-\frac{2T_3-Y}{6}$	$-\frac{T_3c_W}{3s_W}$
$f\bar{f} \rightarrow HW^+W^-$	1	0	0	0	0	$-i\frac{Y}{4c_W}$	$-i\frac{5Y}{24c_W}$	$i\frac{2T_3}{24s_W}$
	2	0	0	0	0	$-i\frac{Y}{4c_W}$	$-i\frac{3Y}{24c_W}$	$i\frac{2T_3}{24s_W}$
$f_d\bar{f}_u \rightarrow W^-W^+W^-$	1	$-\frac{1}{\sqrt{2}s_W}$	$\frac{\sqrt{2}}{3s_W}$	0	0	0	0	$-\frac{1}{4\sqrt{2}s_W}$
	2	$-\frac{1}{\sqrt{2}s_W}$	$\frac{\sqrt{2}}{3s_W}$	0	0	0	0	$-\frac{5}{12\sqrt{2}s_W}$
$f_d\bar{f}_u \rightarrow ZZW^-$	1	$-\frac{1}{\sqrt{2}s_W}$	$\frac{\sqrt{2}}{3s_W}$	0	0	0	0	$-\frac{1}{3\sqrt{2}s_W}$
	2	$-\frac{1}{\sqrt{2}s_W}$	$\frac{\sqrt{2}}{3s_W}$	0	0	0	0	$-\frac{1}{3\sqrt{2}s_W}$
$f_d\bar{f}_u \rightarrow ZW^-\gamma$	1	0	0	0	0	$\frac{c_W}{2\sqrt{2}s_W}$	$\frac{5c_W}{12\sqrt{2}s_W}$	$\frac{1}{12\sqrt{2}}$
	2	0	0	0	0	$\frac{c_W}{2\sqrt{2}s_W}$	$\frac{c_W}{4\sqrt{2}s_W}$	$-\frac{1}{12\sqrt{2}}$
$f_d\bar{f}_u \rightarrow HHW^-$	1	$-\frac{1}{\sqrt{2}s_W}$	$\frac{\sqrt{2}}{3s_W}$	0	0	0	0	$-\frac{1}{3\sqrt{2}s_W}$
	2	$-\frac{1}{\sqrt{2}s_W}$	$\frac{\sqrt{2}}{3s_W}$	0	0	0	0	$-\frac{1}{3\sqrt{2}s_W}$
$f_d\bar{f}_u \rightarrow HW^-\gamma$	1	0	0	0	0	$i\frac{c_W}{2\sqrt{2}s_W}$	$i\frac{5c_W}{12\sqrt{2}s_W}$	$i\frac{1}{12\sqrt{2}}$
	2	0	0	0	0	$i\frac{c_W}{2\sqrt{2}s_W}$	$i\frac{c_W}{4\sqrt{2}s_W}$	$-i\frac{1}{12\sqrt{2}}$
$f_d\bar{f}_u \rightarrow HW^-Z$	1	0	0	0	0	$-i\frac{1}{2\sqrt{2}}$	$-i\frac{5}{12\sqrt{2}}$	$i\frac{c_W}{12\sqrt{2}s_W}$
	2	0	0	0	0	$-i\frac{1}{2\sqrt{2}}$	$-i\frac{1}{4\sqrt{2}}$	$-i\frac{c_W}{12\sqrt{2}s_W}$
$\lambda_3, \lambda_4, \lambda_5 = 0, 1, 0$								
$f\bar{f} \rightarrow W^+W^-Z$	1	0	0	0	0	$\frac{Y}{4c_W}$	$\frac{Y}{8c_W}$	$\frac{T_3}{12s_W}$
	2	0	0	0	0	$\frac{Y}{4c_W}$	$\frac{5Y}{24c_W}$	$-\frac{T_3}{12s_W}$
$f\bar{f} \rightarrow HW^+W^-$	1	0	0	0	0	$i\frac{Y}{4c_W}$	$i\frac{Y}{8c_W}$	$-i\frac{T_3}{12s_W}$
	2	0	0	0	0	$i\frac{Y}{4c_W}$	$i\frac{5Y}{24c_W}$	$i\frac{T_3}{12s_W}$
$f_d\bar{f}_u \rightarrow ZZW^-$	1	0	0	0	0	$-\frac{1}{2\sqrt{2}}$	$-\frac{5}{12\sqrt{2}}$	$\frac{c_W}{12\sqrt{2}s_W}$
	2	0	0	0	0	$-\frac{1}{2\sqrt{2}}$	$-\frac{1}{4\sqrt{2}}$	$-\frac{c_W}{12\sqrt{2}s_W}$
$f_d\bar{f}_u \rightarrow HW^-Z$	1	0	0	0	0	0	0	$-i\frac{1}{12s_W}$
	2	0	0	0	0	0	0	$i\frac{1}{12s_W}$

TABLE IV. Partial-wave non-orthogonal projections for $FF \rightarrow VVV$ generated by $\mathcal{O}_{T,i}$ with final polarizations $++-$ (and permutations). Here, $f = u, d, \ell$, or ν_ℓ , while we denote by $f_{u(d)} = u(d)$ or $\nu_\ell(\ell)$. The scattering amplitude dependence on the chosen fermion is encoded in the couplings $g_L = T_3 - Qs_W^2 = T_3c_W^2 - \frac{Y}{2}s_W^2$ with the hypercharge $Y = 2(Q - T_3)$, and the we have defined the combination $\tilde{g} \equiv T_3c_W^2 + \frac{Y}{2}s_W^2$. The amplitude for the CC processes are the same for quarks and leptons up to the CKM factor which we are omitting for simplicity.

	$\tilde{T}_{-\frac{1}{2}, \frac{1}{2}, \lambda_3, \lambda_4, \lambda_5}^{1,a} = \tilde{T}_{-\frac{1}{2}, \frac{1}{2}, \lambda_3, \lambda_4, \lambda_5}^{1,2}$ in units of $\frac{e}{\pi^2} \sqrt{\frac{7}{155}} \times \frac{f_{T,i}}{\Lambda^4} S_{VVV}^2 \times \frac{1}{\sqrt{N_{id'!}}}$									
	0	1	2	3	4	5	6	7	8	9
$\lambda_3, \lambda_4, \lambda_5 = 1, 1, -1$										
$f\bar{f} \rightarrow \gamma\gamma\gamma$	$-\frac{T_3s_W^2}{48\sqrt{2}}$	$-\frac{T_3s_W^2}{48\sqrt{2}}$	$-\frac{T_3s_W^2}{64\sqrt{2}}$	$-\frac{T_3s_W^2}{64\sqrt{2}}$	$-\frac{\tilde{g}}{64\sqrt{2}}$	$-\frac{\tilde{g}}{48\sqrt{2}}$	$-\frac{\tilde{g}}{48\sqrt{2}}$	$-\frac{\tilde{g}}{48\sqrt{2}}$	$-\frac{Yc_W^2}{24\sqrt{2}}$	$-\frac{Yc_W^2}{32\sqrt{2}}$
$f\bar{f} \rightarrow \gamma ZZ$	$-\frac{T_3c_W^2}{48\sqrt{2}}$	$-\frac{T_3c_W^2}{48\sqrt{2}}$	$-\frac{T_3c_W^2}{64\sqrt{2}}$	$-\frac{T_3c_W^2}{64\sqrt{2}}$	$-\frac{Q-3\tilde{g}}{192\sqrt{2}}$	$\frac{\tilde{g}}{48\sqrt{2}}$	$\frac{(c_W^2-s_W^2)(2T_3-Y)}{192\sqrt{2}}$	$-\frac{Q-3\tilde{g}}{192\sqrt{2}}$	$-\frac{Ys_W^2}{24\sqrt{2}}$	$-\frac{Ys_W^2}{32\sqrt{2}}$
$f\bar{f} \rightarrow \gamma\gamma Z$	$-\frac{T_3c_Ws_W}{48\sqrt{2}}$	$-\frac{T_3c_Ws_W}{48\sqrt{2}}$	$-\frac{T_3c_Ws_W}{64\sqrt{2}}$	$-\frac{T_3c_Ws_W}{64\sqrt{2}}$	$-\frac{(1-3s_W^2)(g_L)}{192\sqrt{2}c_Ws_W}$	$-\frac{2c_W^4T_3+s_W^4Y}{96\sqrt{2}c_Ws_W}$	$\frac{(2T_3-Y)s_Wc_W}{96\sqrt{2}}$	$-\frac{(1-3s_W^2)(g_L)}{192\sqrt{2}c_Ws_W}$	$\frac{Yc_Ws_W}{24\sqrt{2}}$	$\frac{Yc_Ws_W}{32\sqrt{2}}$
$f\bar{f} \rightarrow ZZZ$	$-\frac{T_3c_W^3}{48\sqrt{2}s_W}$	$-\frac{T_3c_W^3}{48\sqrt{2}s_W}$	$-\frac{T_3c_W^3}{64\sqrt{2}s_W}$	$-\frac{T_3c_W^3}{64\sqrt{2}s_W}$	$-\frac{(2T_3-Y)c_Ws_W}{128\sqrt{2}}$	$-\frac{(2T_3-Y)c_Ws_W}{96\sqrt{2}}$	$-\frac{(2T_3-Y)c_Ws_W}{96\sqrt{2}}$	$-\frac{(2T_3-Y)c_Ws_W}{128\sqrt{2}}$	$\frac{Ys_W^3}{24\sqrt{2}c_W}$	$\frac{Ys_W^3}{32\sqrt{2}c_W}$
$f\bar{f} \rightarrow W^-W^+\gamma$	$-\frac{T_3}{48\sqrt{2}}$	0	$-\frac{T_3}{192\sqrt{2}}$	$-\frac{T_3}{192\sqrt{2}}$	$-\frac{Y}{384\sqrt{2}}$	$-\frac{Y}{96\sqrt{2}}$	0	$-\frac{Y}{384\sqrt{2}}$	0	0
$f\bar{f} \rightarrow W^-W^+Z$	$-\frac{T_3c_W}{48\sqrt{2}s_W}$	0	$-\frac{T_3c_W}{192\sqrt{2}s_W}$	$-\frac{T_3c_W}{192\sqrt{2}s_W}$	$\frac{Yc_W}{384\sqrt{2}s_W}$	$\frac{Ys_W}{96\sqrt{2}c_W}$	0	$\frac{Ys_W}{384\sqrt{2}c_W}$	0	0
$f_d\bar{f}_u \rightarrow W^-W^+W^-$	$-\frac{1}{96s_W}$	$-\frac{1}{96s_W}$	$-\frac{1}{192s_W}$	$-\frac{1}{192s_W}$	0	0	0	0	0	0
$f_d\bar{f}_u \rightarrow \gamma\gamma W^-$	$-\frac{s_W}{96}$	0	$-\frac{s_W}{384}$	$-\frac{s_W}{384}$	$-\frac{c_W^2}{384s_W}$	$-\frac{c_W^2}{95s_W}$	0	$-\frac{c_W^2}{384s_W}$	0	0
$f_d\bar{f}_u \rightarrow ZZW^-$	$-\frac{c_W^2}{96s_W}$	0	$-\frac{c_W^2}{384s_W}$	$-\frac{c_W^2}{384s_W}$	$-\frac{s_W}{384}$	$-\frac{s_W}{96}$	0	$-\frac{s_W}{384}$	0	0
$f_d\bar{f}_u \rightarrow \gamma ZW^-$	$-\frac{c_W}{96}$	0	$-\frac{c_W}{384}$	$-\frac{c_W}{384}$	$\frac{c_W}{384}$	$\frac{c_W}{96}$	0	$\frac{c_W}{384}$	0	0
$\lambda_3, \lambda_4, \lambda_5 = 1, -1, 1$										
$f\bar{f} \rightarrow \gamma\gamma\gamma$	$-\frac{T_3s_W^2}{48\sqrt{2}}$	$-\frac{T_3s_W^2}{48\sqrt{2}}$	$-\frac{T_3s_W^2}{64\sqrt{2}}$	$-\frac{T_3s_W^2}{64\sqrt{2}}$	$-\frac{\tilde{g}}{64\sqrt{2}}$	$-\frac{\tilde{g}}{48\sqrt{2}}$	$-\frac{\tilde{g}}{48\sqrt{2}}$	$-\frac{\tilde{g}}{64\sqrt{2}}$	$-\frac{Yc_W^2}{24\sqrt{2}}$	$-\frac{Yc_W^2}{32\sqrt{2}}$
$f\bar{f} \rightarrow \gamma ZZ$	$-\frac{T_3c_W^2}{48\sqrt{2}}$	$-\frac{T_3c_W^2}{48\sqrt{2}}$	$-\frac{T_3c_W^2}{64\sqrt{2}}$	$-\frac{T_3c_W^2}{64\sqrt{2}}$	$-\frac{Q-3\tilde{g}}{192\sqrt{2}}$	$\frac{\tilde{g}}{48\sqrt{2}}$	$\frac{(c_W^2-s_W^2)(2T_3-Y)}{192\sqrt{2}}$	$-\frac{Q-3\tilde{g}}{192\sqrt{2}}$	$-\frac{Ys_W^2}{24\sqrt{2}}$	$-\frac{Ys_W^2}{32\sqrt{2}}$
$f\bar{f} \rightarrow \gamma\gamma Z$	$-\frac{T_3c_Ws_W}{48\sqrt{2}}$	$-\frac{2T_3c_Ws_W}{48\sqrt{2}}$	$-\frac{T_3c_Ws_W}{64\sqrt{2}}$	$-\frac{T_3c_Ws_W}{64\sqrt{2}}$	$-\frac{(1-3s_W^2)(g_L)}{192\sqrt{2}c_Ws_W}$	$\frac{(2T_3-Y)s_Wc_W}{96\sqrt{2}}$	$-\frac{(\tilde{g})(c_W^2-s_W^2)}{96\sqrt{2}c_Ws_W}$	$-\frac{(1-3s_W^2)(g_L)}{192\sqrt{2}c_Ws_W}$	$\frac{Yc_Ws_W}{24\sqrt{2}}$	$\frac{Yc_Ws_W}{32\sqrt{2}}$
$f\bar{f} \rightarrow ZZZ$	$-\frac{T_3c_W^3}{48\sqrt{2}s_W}$	$-\frac{2T_3c_W^3}{48\sqrt{2}s_W}$	$-\frac{T_3c_W^3}{64\sqrt{2}s_W}$	$-\frac{T_3c_W^3}{64\sqrt{2}s_W}$	$-\frac{(2T_3-Y)c_Ws_W}{128\sqrt{2}}$	$-\frac{(2T_3-Y)c_Ws_W}{96\sqrt{2}}$	$-\frac{(2T_3-Y)c_Ws_W}{96\sqrt{2}}$	$-\frac{(2T_3-Y)c_Ws_W}{128\sqrt{2}}$	$\frac{Ys_W^3}{24\sqrt{2}c_W}$	$\frac{Ys_W^3}{32\sqrt{2}c_W}$
$f\bar{f} \rightarrow W^-W^+\gamma$	0	$-\frac{T_3}{96\sqrt{2}}$	$-\frac{T_3}{192\sqrt{2}}$	$-\frac{T_3}{192\sqrt{2}}$	$-\frac{Y}{384\sqrt{2}}$	0	$-\frac{Y}{192\sqrt{2}}$	$-\frac{Y}{384\sqrt{2}}$	0	0
$f\bar{f} \rightarrow W^-W^+Z$	0	$-\frac{T_3c_W}{96\sqrt{2}s_W}$	$-\frac{T_3c_W}{192\sqrt{2}s_W}$	$-\frac{T_3c_W}{192\sqrt{2}s_W}$	$\frac{Yc_W}{384\sqrt{2}s_W}$	0	$\frac{Ys_W}{192\sqrt{2}c_W}$	$\frac{Ys_W}{384\sqrt{2}c_W}$	0	0
$f_d\bar{f}_u \rightarrow W^-W^+W^-$	$-\frac{1}{96s_W}$	$-\frac{1}{192s_W}$	$-\frac{1}{192s_W}$	$-\frac{1}{192s_W}$	0	0	0	0	0	0
$f_d\bar{f}_u \rightarrow \gamma\gamma W^-$	0	$-\frac{s_W}{192}$	$-\frac{s_W}{384}$	$-\frac{s_W}{384}$	$-\frac{c_W^2}{384s_W}$	0	$-\frac{c_W^2}{192s_W}$	$-\frac{c_W^2}{384s_W}$	0	0
$f_d\bar{f}_u \rightarrow ZZW^-$	0	$-\frac{c_W^2}{192s_W}$	$-\frac{c_W^2}{384s_W}$	$-\frac{c_W^2}{384s_W}$	$-\frac{s_W}{384}$	0	$-\frac{s_W}{192}$	$-\frac{s_W}{384}$	0	0
$f_d\bar{f}_u \rightarrow \gamma ZW^-$	0	$-\frac{c_W}{192}$	$-\frac{c_W}{384}$	$-\frac{c_W}{384}$	$\frac{c_W}{384}$	0	$\frac{c_W}{192}$	$\frac{c_W}{384}$	0	0
$\lambda_3, \lambda_4, \lambda_5 = -1, 1, 1$										
$f\bar{f} \rightarrow \gamma\gamma\gamma$	$-\frac{T_3s_W^2}{48\sqrt{2}}$	$-\frac{T_3s_W^2}{48\sqrt{2}}$	$-\frac{T_3s_W^2}{64\sqrt{2}}$	$-\frac{T_3s_W^2}{64\sqrt{2}}$	$-\frac{\tilde{g}}{64\sqrt{2}}$	$-\frac{\tilde{g}}{48\sqrt{2}}$	$-\frac{\tilde{g}}{48\sqrt{2}}$	$-\frac{\tilde{g}}{64\sqrt{2}}$	$-\frac{Yc_W^2}{24\sqrt{2}}$	$-\frac{Yc_W^2}{32\sqrt{2}}$
$f\bar{f} \rightarrow \gamma ZZ$	$-\frac{T_3c_W^2}{48\sqrt{2}}$	$-\frac{T_3c_W^2}{48\sqrt{2}}$	$-\frac{T_3c_W^2}{64\sqrt{2}}$	$-\frac{T_3c_W^2}{64\sqrt{2}}$	$-\frac{Q-3\tilde{g}}{192\sqrt{2}}$	$\frac{\tilde{g}}{48\sqrt{2}}$	$\frac{(c_W^2-s_W^2)(2T_3-Y)}{192\sqrt{2}}$	$-\frac{Q-3\tilde{g}}{192\sqrt{2}}$	$-\frac{Ys_W^2}{24\sqrt{2}}$	$-\frac{Ys_W^2}{32\sqrt{2}}$
$f\bar{f} \rightarrow \gamma\gamma Z$	$-\frac{T_3c_Ws_W}{48\sqrt{2}}$	$-\frac{T_3c_Ws_W}{48\sqrt{2}}$	$-\frac{T_3c_Ws_W}{64\sqrt{2}}$	$-\frac{T_3c_Ws_W}{64\sqrt{2}}$	$-\frac{(1-3s_W^2)(g_L)}{192\sqrt{2}c_Ws_W}$	$\frac{(2T_3-Y)s_Wc_W}{96\sqrt{2}}$	$-\frac{(\tilde{g})(c_W^2-s_W^2)}{96\sqrt{2}c_Ws_W}$	$-\frac{(1-3s_W^2)(g_L)}{192\sqrt{2}c_Ws_W}$	$\frac{Yc_Ws_W}{24\sqrt{2}}$	$\frac{Yc_Ws_W}{32\sqrt{2}}$
$f\bar{f} \rightarrow ZZZ$	$-\frac{T_3c_W^3}{48\sqrt{2}s_W}$	$-\frac{T_3c_W^3}{48\sqrt{2}s_W}$	$-\frac{T_3c_W^3}{64\sqrt{2}s_W}$	$-\frac{T_3c_W^3}{64\sqrt{2}s_W}$	$-\frac{(2T_3-Y)c_Ws_W}{128\sqrt{2}}$	$-\frac{(2T_3-Y)c_Ws_W}{96\sqrt{2}}$	$-\frac{(2T_3-Y)c_Ws_W}{96\sqrt{2}}$	$-\frac{(2T_3-Y)c_Ws_W}{128\sqrt{2}}$	$\frac{Ys_W^3}{24\sqrt{2}c_W}$	$\frac{Ys_W^3}{32\sqrt{2}c_W}$
$f\bar{f} \rightarrow W^-W^+\gamma$	0	$-\frac{T_3}{96\sqrt{2}}$	$-\frac{T_3}{192\sqrt{2}}$	$-\frac{T_3}{192\sqrt{2}}$	$-\frac{Y}{384\sqrt{2}}$	0	$-\frac{Y}{192\sqrt{2}}$	$-\frac{Y}{384\sqrt{2}}$	0	0
$f\bar{f} \rightarrow W^-W^+Z$	0	$-\frac{T_3c_W}{96\sqrt{2}s_W}$	$-\frac{T_3c_W}{192\sqrt{2}s_W}$	$-\frac{T_3c_W}{192\sqrt{2}s_W}$	$\frac{Yc_W}{384\sqrt{2}s_W}$	0	$\frac{Ys_W}{192\sqrt{2}c_W}$	$\frac{Ys_W}{384\sqrt{2}c_W}$	0	0
$f_d\bar{f}_u \rightarrow W^-W^+W^-$	0	$-\frac{1}{96s_W}$	$-\frac{1}{192s_W}$	$-\frac{1}{192s_W}$	0	0	0	0	0	0
$f_d\bar{f}_u \rightarrow \gamma\gamma W^-$	0	$-\frac{s_W}{192}$	$-\frac{s_W}{384}$	$-\frac{s_W}{384}$	$-\frac{c_W^2}{384s_W}$	0	$-\frac{c_W^2}{192s_W}$	$-\frac{c_W^2}{384s_W}$	0	0
$f_d\bar{f}_u \rightarrow ZZW^-$	0	$-\frac{c_W^2}{192s_W}$	$-\frac{c_W^2}{384s_W}$	$-\frac{c_W^2}{384s_W}$	$-\frac{s_W}{384}$	0	$-\frac{s_W}{192}$	$-\frac{s_W}{384}$	0	0
$f_d\bar{f}_u \rightarrow \gamma ZW^-$	0	$-\frac{c_W}{192}$	$-\frac{c_W}{384}$	$-\frac{c_W}{384}$	$\frac{c_W}{384}$	0	$\frac{c_W}{192}$	$\frac{c_W}{384}$	0	0

TABLE V. Partial-wave non-orthogonal projections for $FF \rightarrow VVV$ generated by $\mathcal{O}_{T,i}$ with final polarizations $+++$. Conventions are the same as in Table IV.

		$\tilde{T}^{1,a}_{-\frac{1}{2},\frac{1}{2},1,1,1}$ in units of $\frac{e}{\pi^2 576} \sqrt{\frac{70}{31}} \times \frac{f_{T,i}}{\Lambda^4} \mathcal{S}_{VVV}^2 \times \frac{1}{\sqrt{N_{d!}}}$									
	i	0	1	2	3	4	5	6	7	8	9
$f\bar{f} \rightarrow \gamma\gamma\gamma$	a										
	1	$-2T_3 s_W^2$	$-2T_3 s_W^2$	$-\frac{T_3}{2} s_W^2$	$-\frac{T_3}{2} s_W^2$	$-\frac{\tilde{g}}{2}$	$-2\tilde{g}$	$-2\tilde{g}$	$-\frac{\tilde{g}}{2}$	$-4Y c_W^2$	$-Y c_W^2$
	2	$-2T_3 s_W^2$	$-2T_3 s_W^2$	$-\frac{T_3}{2} s_W^2$	$-\frac{T_3}{2} s_W^2$	$-\frac{\tilde{g}}{2}$	$-2\tilde{g}$	$-2\tilde{g}$	$-\frac{\tilde{g}}{2}$	$-4Y c_W^2$	$-Y c_W^2$
	3	$2T_3 s_W^2$	$2T_3 s_W^2$	$\frac{T_3}{2} s_W^2$	$-\frac{T_3}{2} s_W^2$	$\frac{\tilde{g}}{2}$	$2\tilde{g}$	$2\tilde{g}$	$\frac{\tilde{g}}{2}$	$4Y c_W^2$	$Y c_W^2$
	4	$-2T_3 s_W^2$	$-2T_3 s_W^2$	$-\frac{T_3}{2} s_W^2$	$-\frac{T_3}{2} s_W^2$	$-\frac{\tilde{g}}{2}$	$-2\tilde{g}$	$-2\tilde{g}$	$-\frac{\tilde{g}}{2}$	$-4Y c_W^2$	$-Y c_W^2$
	5	0	0	0	0	0	0	0	0	0	0
$f\bar{f} \rightarrow \gamma ZZ$	1	$-2T_3 c_W^2$	$-2T_3 c_W^2$	$-\frac{T_3}{2} c_W^2$	$-\frac{T_3}{2} c_W^2$	$\frac{Q+5\tilde{g}}{10}$	$-\frac{6Q+10\tilde{g}}{5}$	$-\frac{2Q-10\tilde{g}}{5}$	$-\frac{-3Q+5\tilde{g}}{10}$	$-4Y s_W^2$	$-Y s_W^2$
	2	$-2T_3 c_W^2$	$-2T_3 c_W^2$	$-\frac{T_3}{2} c_W^2$	$-\frac{T_3}{2} c_W^2$	$\frac{Q+5\tilde{g}}{10}$	$-\frac{6Q+10\tilde{g}}{5}$	$-\frac{2Q-10\tilde{g}}{5}$	$-\frac{-3Q+5\tilde{g}}{10}$	$-4Y s_W^2$	$-Y s_W^2$
	3	$-2T_3 c_W^2$	$2T_3 c_W^2$	$\frac{T_3}{2} c_W^2$	$-\frac{T_3}{2} c_W^2$	$\frac{Q-\tilde{g}}{2}$	$-(2\tilde{g})$	$-\frac{(2T_3-Y)(c_W^2-s_W^2)}{2}$	$-\frac{\tilde{g}}{2}$	$4Y s_W^2$	$Y s_W^2$
	4	$-2T_3 c_W^2$	$-2T_3 c_W^2$	$-\frac{T_3}{2} c_W^2$	$-\frac{T_3}{2} c_W^2$	$-\frac{Q-5\tilde{g}}{10}$	$-\frac{4Q-10\tilde{g}}{5}$	$-\frac{6Q-20\tilde{g}}{10}$	$-\frac{2Q-5\tilde{g}}{10}$	$-4Y s_W^2$	$-Y s_W^2$
	5	$-2T_3 c_W^2$	0	0	0	$\frac{Q}{5} \sqrt{\frac{31}{10}}$	$-\frac{Q}{5} \sqrt{\frac{31}{10}}$	$\frac{Q}{5} \sqrt{\frac{31}{10}}$	$-\frac{Q}{5} \sqrt{\frac{31}{10}}$	0	0
$f\bar{f} \rightarrow \gamma\gamma Z$	1	$-2T_3 c_W s_W$	$-2T_3 c_W s_W$	$-\frac{T_3 c_W s_W}{2}$	$-\frac{T_3 c_W s_W}{2}$	$-\frac{2g_L-5s_W^2 c_W^2 (2T_3-Y)}{20c_W s_W}$	$-\frac{4g_L-5s_W^2 c_W^2 (2T_3-Y)}{5s_W c_W}$	$-\frac{3g_L-5s_W^2 c_W^2 (2T_3-Y)}{5s_W c_W}$	$-\frac{4g_L-5s_W^2 c_W^2 (2T_3-Y)}{20c_W s_W}$	$4Y c_W s_W$	$Y c_W s_W$
	2	$-2T_3 c_W s_W$	$-2T_3 c_W s_W$	$-\frac{T_3 c_W s_W}{2}$	$-\frac{T_3 c_W s_W}{2}$	$\frac{2T_3 c_W^4 + Y s_W^4}{4s_W c_W}$	$(2T_3 - Y) s_W c_W$	$-\frac{(c_W^2 - s_W^2)(2\tilde{g})}{2s_W c_W}$	$\frac{(2T_3 - Y) s_W c_W}{4}$	$4Y c_W s_W$	$Y c_W s_W$
	3	$2T_3 c_W s_W$	$2T_3 c_W s_W$	$\frac{T_3 c_W s_W}{2}$	$\frac{T_3 c_W s_W}{2}$	$\frac{2g_L-5s_W^2 c_W^2 (2T_3-Y)}{20c_W s_W}$	$\frac{4g_L-5s_W^2 c_W^2 (2T_3-Y)}{5s_W c_W}$	$\frac{3g_L-5s_W^2 c_W^2 (2T_3-Y)}{5s_W c_W}$	$-\frac{4g_L-5s_W^2 c_W^2 (2T_3-Y)}{20c_W s_W}$	$-4Y c_W s_W$	$-Y c_W s_W$
	4	$-2T_3 c_W s_W$	$-2T_3 c_W s_W$	$-\frac{T_3 c_W s_W}{2}$	$-\frac{T_3 c_W s_W}{2}$	$\frac{2T_3 c_W^4 + Y s_W^4}{4s_W c_W}$	$(2T_3 - Y) s_W c_W$	$-\frac{(c_W^2 - s_W^2)(\tilde{g})}{s_W c_W}$	$\frac{(2T_3 - Y) s_W c_W}{4}$	$4Y c_W s_W$	$Y c_W s_W$
	5	0	0	0	0	$-\sqrt{\frac{31}{10}} \frac{2g_L}{10c_W s_W}$	$\sqrt{\frac{31}{10}} \frac{2g_L}{5c_W s_W}$	$-\sqrt{\frac{31}{10}} \frac{2g_L}{10c_W s_W}$	$\sqrt{\frac{31}{10}} \frac{2g_L}{20c_W s_W}$	0	0
$f\bar{f} \rightarrow ZZZ$	1	$-\frac{2T_3 c_W^3}{s_W}$	$-\frac{2T_3 c_W^3}{s_W}$	$-\frac{T_3 c_W^3}{2s_W}$	$-\frac{T_3 c_W^3}{2s_W}$	$-\frac{(2T_3-Y) s_W c_W}{4}$	$-(2T_3 - Y) s_W c_W$	$-(2T_3 - Y) s_W c_W$	$-\frac{(2T_3-Y) s_W c_W}{4}$	$4 \frac{Y s_W^3}{c_W}$	$-\frac{Y s_W^3}{c_W}$
	2	$-\frac{2T_3 c_W^3}{s_W}$	$-\frac{2T_3 c_W^3}{s_W}$	$-\frac{T_3 c_W^3}{2s_W}$	$-\frac{T_3 c_W^3}{2s_W}$	$-\frac{(2T_3-Y) s_W c_W}{4}$	$-(2T_3 - Y) s_W c_W$	$-(2T_3 - Y) s_W c_W$	$-\frac{(2T_3-Y) s_W c_W}{4}$	$4 \frac{Y s_W^3}{c_W}$	$-\frac{Y s_W^3}{c_W}$
	3	$\frac{2T_3 c_W^3}{s_W}$	$\frac{2T_3 c_W^3}{s_W}$	$\frac{T_3 c_W^3}{2s_W}$	$\frac{T_3 c_W^3}{2s_W}$	$\frac{(2T_3-Y) s_W c_W}{4}$	$(2T_3 - Y) s_W c_W$	$(2T_3 - Y) s_W c_W$	$\frac{(2T_3-Y) s_W c_W}{4}$	$-4 \frac{Y s_W^3}{c_W}$	$-\frac{Y s_W^3}{c_W}$
	4	$-\frac{2T_3 c_W^3}{s_W}$	$-\frac{2T_3 c_W^3}{s_W}$	$-\frac{T_3 c_W^3}{2s_W}$	$-\frac{T_3 c_W^3}{2s_W}$	$-\frac{(2T_3-Y) s_W c_W}{4}$	$-(2T_3 - Y) s_W c_W$	$-(2T_3 - Y) s_W c_W$	$-\frac{(2T_3-Y) s_W c_W}{4}$	$4 \frac{Y s_W^3}{c_W}$	$-\frac{Y s_W^3}{c_W}$
	5	0	0	0	0	0	0	0	0	0	0
$f\bar{f} \rightarrow W^+ W^- \gamma$	1	$-\frac{2T_3}{5}$	$-\frac{3T_3}{5}$	$-\frac{T_3}{5}$	$-\frac{T_3}{10}$	$-\frac{Y}{20}$	$-\frac{2Y}{5}$	$-\frac{3Y}{10}$	$-\frac{Y}{10}$	0	0
	2	0	$-T_3$	0	$-\frac{T_3}{2}$	$-\frac{Y}{4}$	0	$-\frac{Y}{2}$	0	0	0
	3	$\frac{4T_3}{5}$	$\frac{3T_3}{5}$	$\frac{T_3}{5}$	$\frac{T_3}{10}$	$\frac{Y}{20}$	$\frac{2Y}{5}$	$\frac{3Y}{10}$	$\frac{Y}{10}$	0	0
	4	0	$-T_3$	0	$-\frac{T_3}{2}$	$-\frac{Y}{4}$	0	$-\frac{Y}{2}$	0	0	0
	5	$\frac{2T_3}{5} \sqrt{\frac{31}{10}}$	$-\frac{T_3}{5} \sqrt{\frac{31}{10}}$	$\frac{T_3}{10} \sqrt{\frac{31}{10}}$	$-\frac{T_3}{5} \sqrt{\frac{31}{10}}$	$-\frac{Y}{10} \sqrt{\frac{31}{10}}$	$\frac{Y}{5} \sqrt{\frac{31}{10}}$	$-\frac{Y}{10} \sqrt{\frac{31}{10}}$	$\frac{Y}{20} \sqrt{\frac{31}{10}}$	0	0
$f\bar{f} \rightarrow W^+ W^- Z$	1	$-\frac{4T_3 c_W}{5s_W}$	$-\frac{3T_3 c_W}{5s_W}$	$-\frac{T_3 c_W}{5s_W}$	$-\frac{T_3 c_W}{10s_W}$	$\frac{Y s_W}{20c_W}$	$\frac{2Y s_W}{5c_W}$	$\frac{3Y s_W}{10c_W}$	$\frac{Y s_W}{10c_W}$	0	0
	2	0	$-\frac{T_3 c_W}{s_W}$	0	$-\frac{T_3 c_W}{2s_W}$	$\frac{Y s_W}{4c_W}$	0	$\frac{Y s_W}{2c_W}$	0	0	0
	3	$\frac{4T_3 c_W}{5s_W}$	$\frac{3T_3 c_W}{5s_W}$	$\frac{T_3 c_W}{5s_W}$	$\frac{T_3 c_W}{10s_W}$	$-\frac{Y s_W}{20c_W}$	$-Y \frac{2s_W}{5c_W}$	$-\frac{3Y s_W}{10c_W}$	$-\frac{Y s_W}{10c_W}$	0	0
	4	0	$-\frac{T_3 c_W}{s_W}$	0	$-\frac{T_3 c_W}{2s_W}$	$\frac{Y s_W}{4c_W}$	0	$\frac{Y s_W}{2c_W}$	0	0	0
	5	$\sqrt{\frac{31}{10}} \frac{2T_3 c_W}{5s_W}$	$-\sqrt{\frac{31}{10}} \frac{T_3 c_W}{5s_W}$	$\sqrt{\frac{31}{10}} \frac{T_3 c_W}{10s_W}$	$-\sqrt{\frac{31}{10}} \frac{T_3 c_W}{5s_W}$	$\sqrt{\frac{31}{10}} \frac{Y s_W}{10c_W}$	$-\sqrt{\frac{31}{10}} \frac{Y s_W}{5c_W}$	$\sqrt{\frac{31}{10}} \frac{Y s_W}{10c_W}$	$-\sqrt{\frac{31}{10}} \frac{Y s_W}{20c_W}$	0	0
$f d \bar{f}_u \rightarrow W^- W^+ W^-$	1	$-\frac{2\sqrt{2}}{5s_W}$	$-\frac{4\sqrt{2}}{5s_W}$	$-\frac{1}{5\sqrt{2}s_W}$	$-\frac{3}{5\sqrt{2}s_W}$	0	0	0	0	0	0
	2	$-\frac{2\sqrt{2}}{5s_W}$	$-\frac{4\sqrt{2}}{5s_W}$	$-\frac{1}{5\sqrt{2}s_W}$	$-\frac{3}{5\sqrt{2}s_W}$	0	0	0	0	0	0
	3	$\frac{\sqrt{2}}{s_W}$	$1\sqrt{2}s_W$	$\frac{1}{2\sqrt{2}s_W}$	0	0	0	0	0	0	0
	4	$-\frac{3\sqrt{2}}{5s_W}$	$-\frac{7}{5\sqrt{2}s_W}$	$-\frac{3}{10\sqrt{2}s_W}$	$-\frac{\sqrt{2}}{5s_W}$	0	0	0	0	0	0
	5	$\sqrt{\frac{31}{5}} \frac{1}{5s_W}$	$-\sqrt{\frac{31}{5}} \frac{1}{10s_W}$	$\sqrt{\frac{31}{5}} \frac{1}{20s_W}$	$-\sqrt{\frac{31}{5}} \frac{1}{10s_W}$	0	0	0	0	0	0
$f d \bar{f}_u \rightarrow \gamma\gamma W^-$	1	$-\frac{2\sqrt{2}s_W}{5}$	$-\frac{3s_W}{5\sqrt{2}}$	$-\frac{s_W}{5\sqrt{2}}$	$-\frac{s_W}{10\sqrt{2}}$	$-\frac{c_W^2}{10\sqrt{2}s_W}$	$-\frac{2\sqrt{2}c_W^2}{5s_W}$	$-\frac{3c_W^2}{5\sqrt{2}s_W}$	$-\frac{c_W^2}{5\sqrt{2}s_W}$	0	0
	2	0	$-\frac{s_W}{\sqrt{2}}$	0	$-\frac{s_W}{2\sqrt{2}}$	$-\frac{c_W^2}{2\sqrt{2}s_W}$	0	$-\frac{c_W^2}{\sqrt{2}s_W}$	0	0	0
	3	$\frac{2\sqrt{2}s_W}{5}$	$\frac{3s_W}{5\sqrt{2}}$	$\frac{s_W}{5\sqrt{2}}$	$\frac{s_W}{10\sqrt{2}}$	$\frac{c_W^2}{10\sqrt{2}s_W}$	$\frac{2\sqrt{2}c_W^2}{5s_W}$	$\frac{3c_W^2}{5\sqrt{2}s_W}$	$\frac{c_W^2}{5\sqrt{2}s_W}$	0	0
	4	0	$-\frac{s_W}{\sqrt{2}}$	0	$-\frac{s_W}{2\sqrt{2}}$	$-\frac{c_W^2}{2\sqrt{2}s_W}$	0	$-\frac{c_W^2}{\sqrt{2}s_W}$	0	0	0
	5	$\sqrt{\frac{31}{5}} \frac{s_W}{5}$	$-\sqrt{\frac{31}{5}} \frac{s_W}{10}$	$\sqrt{\frac{31}{5}} \frac{s_W}{20}$	$-\sqrt{\frac{31}{5}} \frac{s_W}{10}$	$-\sqrt{\frac{31}{5}} \frac{10c_W^2}{s_W}$	$\sqrt{\frac{31}{5}} \frac{c_W^2}{s_W}$	$-\sqrt{\frac{31}{5}} \frac{10c_W^2}{s_W}$	$\sqrt{\frac{31}{5}} \frac{c_W^2}{20s_W}$	0	0

(Table continued)

TABLE V. (Continued)

		$\tilde{T}_{-\frac{1}{2},\frac{1}{2},1,1,1}^{1,a}$ in units of $\frac{e}{\pi^2 576} \sqrt{\frac{70}{31}} \times \frac{f_{T,i}}{\Lambda^4} S_{VV}^2 \times \frac{1}{\sqrt{N_d!}}$									
	i	0	1	2	3	4	5	6	7	8	9
$f_d \tilde{f}_u \rightarrow ZZW^-$	1	$-\frac{2\sqrt{2}c_W^2}{5s_W}$	$-\frac{3c_W^2}{5\sqrt{2}s_W}$	$-\frac{c_W^2}{5\sqrt{2}s_W}$	$-\frac{c_W^2}{10\sqrt{2}s_W}$	$-\frac{s_W}{10\sqrt{2}}$	$-\frac{2\sqrt{2}s_W}{5}$	$-\frac{3s_W}{5\sqrt{2}}$	$-\frac{s_W}{5\sqrt{2}}$	0	0
	2	0	$-\frac{c_W^2}{\sqrt{2}s_W}$	0	$-\frac{c_W^2}{2\sqrt{2}s_W}$	$-\frac{s_W}{2\sqrt{2}}$	0	$-\frac{s_W}{\sqrt{2}}$	0	0	0
	3	$\frac{2\sqrt{2}c_W^2}{5s_W}$	$\frac{3c_W^2}{5\sqrt{2}s_W}$	$\frac{c_W^2}{5\sqrt{2}s_W}$	$\frac{c_W^2}{10\sqrt{2}s_W}$	$\frac{s_W}{10\sqrt{2}}$	$\frac{2\sqrt{2}s_W}{5}$	$\frac{3s_W}{5\sqrt{2}}$	$\frac{s_W}{5\sqrt{2}}$	0	0
	4	0	$-\frac{c_W^2}{\sqrt{2}s_W}$	0	$-\frac{c_W^2}{2\sqrt{2}s_W}$	$-\frac{s_W}{2\sqrt{2}}$	0	$-\frac{s_W}{\sqrt{2}}$	0	0	0
	5	$\sqrt{\frac{31}{5}} \frac{c_W^2}{5s_W}$	$-\sqrt{\frac{31}{5}} \frac{c_W^2}{10s_W}$	$\sqrt{\frac{31}{5}} \frac{c_W^2}{20s_W}$	$-\sqrt{\frac{31}{5}} \frac{c_W^2}{10s_W}$	$-\sqrt{\frac{31}{5}} \frac{s_W}{10}$	$\sqrt{\frac{31}{5}} \frac{s_W}{5}$	$-\sqrt{\frac{31}{5}} \frac{s_W}{10}$	$\sqrt{\frac{31}{5}} \frac{s_W}{20}$	0	0
$f_d \tilde{f}_u \rightarrow \gamma ZW^-$	1	$-\frac{2\sqrt{2}c_W}{5}$	$-\frac{3c_W}{5\sqrt{2}}$	$-\frac{c_W}{5\sqrt{2}}$	$-\frac{c_W}{10\sqrt{2}}$	$\frac{c_W}{10\sqrt{2}}$	$\frac{2\sqrt{2}c_W}{5}$	$\frac{3c_W}{5\sqrt{2}}$	$\frac{c_W}{5\sqrt{2}}$	0	0
	2	0	$-\frac{c_W}{\sqrt{2}}$	0	$-\frac{c_W}{2\sqrt{2}}$	$\frac{c_W}{2\sqrt{2}}$	0	$\frac{c_W}{\sqrt{2}}$	0	0	0
	3	$\frac{2\sqrt{2}c_W}{5}$	$\frac{3c_W}{5\sqrt{2}}$	$\frac{c_W}{5\sqrt{2}}$	$\frac{c_W}{10\sqrt{2}}$	$-\frac{c_W}{10\sqrt{2}}$	$-\frac{2\sqrt{2}c_W}{5}$	$-\frac{3c_W}{5\sqrt{2}}$	$-\frac{c_W}{5\sqrt{2}}$	0	0
	4	0	$-\frac{c_W}{\sqrt{2}}$	0	$-\frac{c_W}{2\sqrt{2}}$	$\frac{c_W}{2\sqrt{2}}$	0	$\frac{c_W}{\sqrt{2}}$	0	0	0
	5	$\sqrt{\frac{31}{5}} \frac{c_W}{5}$	$-\sqrt{\frac{31}{5}} \frac{c_W}{10}$	$\sqrt{\frac{31}{5}} \frac{c_W}{20}$	$-\sqrt{\frac{31}{5}} \frac{c_W}{10}$	$\sqrt{\frac{31}{5}} \frac{c_W}{10}$	$-\sqrt{\frac{31}{5}} \frac{c_W}{5}$	$\sqrt{\frac{31}{5}} \frac{c_W}{10}$	$-\sqrt{\frac{31}{5}} \frac{c_W}{20}$	0	0

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