

Quantum field theory and observers

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A generalization of the coadjoint orbit action describes the dynamics of an observer (or instrument). We consider how this fits in with the view of observables in field theory being correlations of readouts of instruments and show how one recovers the usual S -matrix formulas. A simple resolution of the Fermi paradox is also pointed out.

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I. INTRODUCTION

In recent works, Witten and collaborators brought in the observer in a nontrivial way in defining quantum field theory (QFT) on curved spacetimes [1,2]. In particular, Witten has argued that states corresponding to the observer can give a background-independent operator algebra which is important in the context of quantum gravity where the background geometry is subject to quantum fluctuations. For QFT on a flat or fixed background, i.e., without quantum gravity, the observer can be introduced but seems to play a fairly passive role. In fact, in quantum field theory as it is practiced, for example, for the calculation of scattering amplitudes and decay rates, we do introduce sources (or sinks) as a simple trick to collect correlators into a generating functional. Since the sources can be chosen or manipulated arbitrarily by agencies external to the system under study, they may be viewed as the mathematical expression of the observer in the practical version of QFT. (This is not a particularly new observation. Local sources have long been considered as the mathematical expression of instruments or detectors; see, for example, [3].) Many years ago, Schwinger tried to elevate sources as the primary objects of interest, fields arising only as an afterthought, as a convenient method, albeit immensely successful, of comparing or correlating sources [4]. No *a priori* reality is ascribed to fields. Unfortunately, Schwinger did not introduce any dynamics for the sources, and, as a result, the “source theory” he constructed has generally been viewed as nothing more than a slightly different rewriting of standard QFT. However, in the light of the recent emergence of the key role of the observer in quantum gravity, we

think it is worthwhile to reconsider this issue. We attempt a framework for augmenting QFT even on flat spacetime (or on a fixed background geometry) by introducing dynamics for sources and discuss the conclusions that might arise from it.¹

In the next section, we consider the probability that can be obtained or deduced from measurements which are recorded in terms of the settings of instruments. This is followed by a discussion of the dynamics of the observer or instrument. We then comment on describing the instrument using density matrices, and also on how the usual S -matrix formula fits into our formulation. Finally we comment on how the Fermi problem, which has been analyzed by several authors in the context of signaling and Einstein causality in field theory, has a fairly simple resolution.

II. WHAT IS OBSERVED

Physics is about comparing and correlating the readouts of different instruments. (Instrument, source/detector and observer are basically synonymous in what follows.) Therefore we start by considering the dynamics of an instrument. The instrument has a number of quantum states, say, N of them. (We will consider N to be very large eventually.) These states are characterized by a set of labels which can be viewed as the eigenvalues of a compatible set of observables. A compatible set in this context stands for a complete set of mutually commuting observables. This is basically tautological since, if we start from the point of view of giving primacy to the instruments

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¹Schwinger also had an earlier theory of measurements in quantum mechanics where any instrument is viewed as a filter $M(a, b)$, which returns the value “ a ” for an input “ b ,” with a certain probability [5]. A chain of reasoning then leads to the identification of $M(a, b)$ as $\langle a|b \rangle$, the inner product for the corresponding states on a Hilbert space. This has more recently been developed into a groupoid formulation of quantum mechanics [6]. This is not the subject of our discussion: we are focused on quantum field theory.

and accordingly we assign states with unique labels to it, the set of compatible observables can be taken as the set of all diagonal matrices in this basis of states.

We will assume that the readouts corresponding to these different states can be distinguished in an unambiguous way. This may require the use of techniques to amplify certain signals. For example, in the discussion in Doplicher [7], there are three parts to the measurement. There is the physical system being studied, there is the microscopic apparatus that is treated quantum mechanically, and there has to be a macroscopic apparatus whose job is to ensure that we can read out the initial and final states of the microscopic apparatus unambiguously. The macroscopic apparatus is also taken to be quantum mechanical, but with a large number of degrees of freedom, which provide sufficient amplification of the measured signal to produce an unambiguous readout. The Heisenberg cut, in the usual terminology of measurements in quantum mechanics, is somewhere in the transition from the microscopic apparatus to the macroscopic one. We consider these two together as constituting the instrument.

One of the advantages of considering just the instruments and their readouts, and physical theories as just a calculational framework for understanding correlations among such readouts, is that one evades the concept of the collapse of the wave function. This can be illustrated by one of the often-used examples involving the collapse of the wave function. Consider the measurement of the spin of the electron along the x_3 direction followed by the measurement along the x_1 direction. In the standard parlance, the wave function of the electron, in whatever state it was, collapses into an eigenstate of S_3 , say, corresponding to $S_3 = \frac{1}{2}$ due to the first measurement. The subsequent measurement then collapses this state into an eigenstate of S_1 , with probability $\frac{1}{2}$ for each of the two choices, $S_1 = \pm \frac{1}{2}$. The terminology used here makes a presumption that there is reality to the electron between the measurements and that it has some wave function (say corresponding to $S_3 = \frac{1}{2}$) into which it “collapsed” after the first measurement, and in which state it continues to be thereafter until the second measurement. None of this is directly observed. Therefore, stripping this down to the bare observations, what we really have is that, at a certain time, instrument 1 is in a certain state (which may be interpreted as corresponding to $S_3 = \frac{1}{2}$), and this evolves with instrument 2 showing readouts at a later time, the two possibilities for instrument 2 (which may be interpreted as corresponding to $S_1 = \pm \frac{1}{2}$) being of equal probability. There is a probability of $\frac{1}{2}$ that the initially obtained reading of instrument 1 leads to the readout corresponding to $S_1 = \frac{1}{2}$ for instrument 2, and there is a probability of $\frac{1}{2}$ that it will lead to a readout corresponding to $S_1 = -\frac{1}{2}$ for instrument 2. The correlation of readouts of instruments over time is probabilistic, but the readout for any measurement, even a single one, is definite. No state of the electron is presumed

in this observation and so the concept of “collapse” is not directly relevant.² The role of the physical theory is to understand the laws governing these correlations among instruments. Our aim is to express the standard calculational framework of quantum field theory in terms of correlations of readouts of instruments and their dynamics. But first, we will use the conventional formalism of field theory to define the quantity of interest.

A typical observation would run as follows. We start with some state $|\alpha\rangle$ as the state of the world, say at time $t = t_0$. At some point in time, say $t_1 > t_0$, an instrument is turned on with a certain setting $\{a\}$ for the readouts. Consider then the further time evolution of this setup to $t > t_1$. The state at time t is of the form

$$|\alpha, a, t\rangle = U(t, t_1) f_a(t_1) U(t_1, t_0) |\alpha\rangle, \quad (1)$$

where the insertion of $f_a(t_1)$ characterizes the turning-on of the instrument at time $t = t_1$ and U is the time-evolution operator. (If we think of the instrument as made of a collection of particles that can be described by quantum fields, we may think of f_a as a composite field operator creating the instrument in a state labeled by $\{a\}$.) At this time, the instrument can show a readout $\{b\}$ or another set of definite values $\{b'\}$, etc. We now ask the question: What is the probability that the setup in (1) is identical to the instrument showing readout $\{b\}$ at some time $t_2 < t$? The instrument showing readout $\{b\}$ at t_2 is described by

$$|\beta, b, t\rangle = U(t, t_2) f_b(t_2) U(t_2, t_0) |\beta\rangle. \quad (2)$$

Notice that the final state of the world in (1) could have changed in the process of measurement, so we should match with readout $\{b\}$ for the instrument, which is definite, but with any state $|\beta\rangle$ for the world. This is why we allow β to be different from α , in general. The probability amplitude for obtaining (2) from state (1) is

$$\begin{aligned} \mathcal{A}_{\beta b, \alpha a} &= \langle \beta | U^\dagger(t_2, t_0) f_b^*(t_2) U^\dagger(t, t_2) U(t, t_1) \\ &\quad \times f_a(t_1) U(t_1, t_0) | \alpha \rangle. \end{aligned} \quad (3)$$

If we view $f_a(t_1)$ as the operator creating the instrument at time t_1 , then the amplitude (3) is the propagator for the instrument on a time contour from t_0 to t and then back to t_0 , with f_b^* on the reverse branch of the contour. (The state of the world changes as well.) Notice that if $t_2 > t_1$, then we can use $U^\dagger(t, t_2) U(t, t_1) = U(t_2, t_1)$ and write

²One might argue that the “collapse” does occur for the microscopic apparatus, and it is what leads to definite readouts. For this reason, we have used the word “evade” regarding collapse for the system under study, it may not be eliminated but transferred to the apparatus. Anyway, this is not the main point for us. We are interested in an action for the instrument and how it fits into what we can observe.

$$\mathcal{A}_{\beta b, aa} = \langle \beta | U^\dagger(t_2, t_0) f_b^*(t_2) U(t_2, t_1) f_a(t_1) U(t_1, t_0) | \alpha \rangle. \quad (4)$$

The propagator for the instrument is then on the first branch. This simplification is not possible if we have several instruments making measurements at different times. For example, for two instruments, the amplitude is of the form

$$\begin{aligned} \mathcal{A}_{\beta b_1 b_2, aa_1 a_2} &= \langle \beta | U^\dagger(t_4, t_0) f_{b_1}^*(t_4) U^\dagger(t_3, t_4) f_{b_2}^*(t_3) U^\dagger(t, t_3) \\ &\times U(t, t_2) f_{a_2}(t_2) U(t_2, t_1) f_{a_1}(t_1) U(t_1, t_0) | \alpha \rangle. \end{aligned} \quad (5)$$

If $t_3 > t_2$, we may use $U^\dagger(t, t_3) U(t, t_2) = U(t_3, t_2)$, but the amplitude will still have propagators involving $f_{b_1}^*(t_4)$ on the reverse branch.

The time evolution in (1) and (2) will involve the time evolution of the instrument as well as the rest of the world. (“The rest of the world” would include any fields that interact with the instruments.) In general, there will be entanglement between the state of the instrument and the rest of the world as well. Further, in general, the state of the world must be described by a density matrix. Denoting the probability for $|\alpha\rangle$ as p_α , the probability we are interested in is given by

$$\mathcal{P}(b, a) = \sum_\beta \sum_\alpha |\mathcal{A}_{\beta b, aa}|^2 p_\alpha. \quad (6)$$

This is the quantity of interest. While the instruments can show readouts labeled as a and b , the rest of the world can evolve into any state, which is why we sum over β as well.

Each propagator for the instruments in $\mathcal{P}(b, a)$ can be expressed as a path integral over the world line of the instrument coupled to any fields that may exist in the ambient spacetime. Our aim is therefore to introduce the action for the instrument and rewrite $\mathcal{P}(b, a)$ as a functional integral. This will give a prescription for correlations of readouts without directly using states on a Hilbert space.

III. DYNAMICS OF THE OBSERVER

Turning to the question of how $\mathcal{P}(b, a)$ can be calculated, notice that in the language of quantum field theory, since the instrument is made of a number of particles that can be created from the vacuum by the action of creation operators, we can introduce the standard annihilation/creation operators c_i and $c_i^\dagger$, which are presumably composites of more fundamental particle operators, with $c_i^\dagger$ creating the instrument in the state $|i\rangle$ as $|i\rangle = c_i^\dagger |0\rangle$. However, we will simply postulate the existence of the instrument with its internal states, in the spirit of giving primacy to the observer.

The state $|i\rangle$ can presumably evolve to other states of the instrument, so the Hamiltonian is of the form $H = c_k^\dagger H_{kl} c_l$, where H_{kl} , $k, l = 1, 2, \dots, N$, is an $N \times N$ matrix. It is useful to consider the time evolution in terms of coherent states for c_i , $c_i^\dagger$, with $f_i(z) = \langle z | c_i^\dagger | 0 \rangle = \bar{z}_i e^{-\frac{1}{2} \bar{z}_i \cdot z}$. The path integral for the time evolution of the instrument then takes the form

$$\begin{aligned} \langle j | e^{-iHt} | i \rangle &= \int \frac{d^2 z}{\pi} \frac{d^2 z'}{\pi} f_j^*(z) \int [Dz D\bar{z}] e^{iS(z, z', t)} f_i(z'), \\ S(z, z', t) &= \int_0^t dt' \left[\frac{i}{2} (\dot{\bar{z}}_k z_k - \bar{z}_k \dot{z}_k) - \bar{z}_k H_{kl} z_l \right]. \end{aligned} \quad (7)$$

(Here we are considering the standard time evolution of a pure state generated by a Hamiltonian. In general, the instrument will be in a mixed state. We will consider some features of such a situation described by a density matrix in Sec. IV.) In (7), we have the functional integration over all $z, \bar{z}$ along the trajectory connecting $z', \bar{z}'$ at $t = 0$ to $z, \bar{z}$ at time t . This is indicated by the functional measure $[Dz D\bar{z}]$. The integrations over the initial point $z', \bar{z}'$ and the final point $z, \bar{z}$ have to include the choice of states $|i\rangle, |j\rangle$, with the coherent state wave functions $f_i(z')$, $f_j^*(z)$. This is separately indicated in (7).

With the given Hamiltonian, $c_k^\dagger c_k$ is conserved, and, since we are interested in the evolution of just one copy of the instrument, we can set $c_k^\dagger c_k = 1$ as a constraint. This is equivalent to $\bar{z}_k z_k - 1 = 0$. The conjugate constraint fixes an overall phase among the z_k 's, so that (7) reduces to a path integral on $\mathbb{CP}^{N-1} = SU(N)/U(N-1)$. Without loss of generality, we can parametrize this as $z_k = U_{k1}$, $U_{kl} \in SU(N)$. The result is

$$\begin{aligned} S &= \int dt [-i(U^\dagger \dot{U})_{11} - (U^\dagger H U)_{11}] \\ &= \int dt \text{Tr} \left[h U^\dagger \left(-i \frac{\partial}{\partial t} - H \right) U \right], \end{aligned} \quad (8)$$

where $h = |1\rangle\langle 1|$. The functional integration measure reduces to the standard volume element on $\mathbb{CP}^{N-1}$ at each point in time in defining (7) by time slicing.

The instrument is at some location in space, idealized as the point with spatial coordinates $\vec{x}$. The results (7) and (8), where we consider only transitions among the internal states of the instrument, apply to the rest frame of the instrument. In seeking the generalization of this to the fully relativistic case, there are two points to keep in mind. The states are not necessarily degenerate so the square of the mass is an $N \times N$ positive matrix. (There may be degeneracies among subsets of states.) This matrix can be taken to be diagonal. To motivate the possibility of including scalar fields in the ambient space that can interact with the instrument, it is useful momentarily to think of a field χ_i as representing the instrument with an action of the form

$$\mathcal{S} = \int \left[\frac{1}{2} (\partial\chi)^2 - \frac{1}{2} \chi(m^2)\chi - \frac{1}{2} \chi\Phi\chi \right], \quad (9)$$

where Φ is a scalar field (or composites thereof). Thus, $m^2 \rightarrow m^2 + \lambda\Phi$ can be used to include this type of interaction. $m^2 + \lambda\Phi$ is also an $N \times N$ matrix, not diagonal in general. We will need the square root of this expression for use in the action. In the coherent state basis, this leads to $\text{Tr}[hU^\dagger(\sqrt{m^2 + \lambda\Phi})U]$. Assuming there is a nonzero diagonal part m_0^2 (which will be the case of interest to us), we can write $m^2 = m_0^2 + \delta m^2$ and expand

$$\begin{aligned} & \text{Tr}[hU^\dagger(\sqrt{m^2 + \lambda\Phi})U] \\ & \equiv \text{Tr}[hU^\dagger M(z, \bar{z}, x)U] \\ & \approx \text{Tr}\left[hU^\dagger \left(m_0 + \frac{1}{2}m_0^{-1}(\delta m^2 + \lambda\Phi) + \dots\right)U\right]. \end{aligned} \quad (10)$$

The second point is about the need to maintain the invariance under reparametrization of $\tau \rightarrow \tau' = \tau'(\tau)$ for the world line $x^\mu(\tau)$ of the instrument. It is now easy to see that the relativistic generalization of (8) is given by

$$\mathcal{S}_{\text{instr}} = \int d\tau \text{Tr} \left[hU^\dagger \left(-i \frac{\partial U}{\partial \tau} - A_\mu \frac{dx^\mu}{d\tau} U - M \frac{ds}{d\tau} U \right) \right] \quad (11)$$

$$= \int d\tau \text{Tr} \left[hU^\dagger \left(-i \frac{\partial U}{\partial \tau} - A_\mu \frac{dx^\mu}{d\tau} U \right) \right] + \mathcal{S}_0, \quad (12)$$

where, in (12), we note that the M -dependent term in (11) can be rewritten using

$$\mathcal{S}_0 = - \int d\tau \left[\frac{\dot{x}^2}{4e} + (\text{Tr}(hU^\dagger M U))^2 e \right]. \quad (13)$$

Here e is a one-dimensional frame field; eliminating it leads to the usual form of the action given in (11), namely,

$$- \int \text{Tr}(hU^\dagger M U) \sqrt{\dot{x}^2} d\tau = - \int \text{Tr}(hU^\dagger M U) ds. \quad (14)$$

If we use the gauge fixing $x^0 = \tau$ for the reparametrization symmetry and neglect $\dot{x}^i$, as would be appropriate for large masses (or the rest frame of the instrument), Eq. (11) reduces to (8) as required. In this case H is a combination of A_0 and $\sqrt{m^2 + \lambda\Phi}$. (The term $\mathcal{S}_0$, for a single mass with no interaction term $\lambda\Phi$, is what was used in [2].) In the following, we regard A_μ and Φ as fields in the ambient spacetime.

The action in (11) is a generalization of the coadjoint orbit action used for a particle with $U(N)$ charges coupled to a gauge field A_μ , which leads to the Wong equations of motion [8]. If we neglect Φ and take all masses to be the same, it is exactly the action for the Wong particle; the generalization involves allowing for M to be a general

matrix, just as the A_μ are. In the general case, with nondegenerate masses and the coupling to Φ , the action (11) does not have the full gauge symmetry $(-iA)_\mu \rightarrow g(-iA_\mu)g^{-1} - \partial_\mu g g^{-1}$, $U \rightarrow gU$; it still has symmetry for a subgroup of products of $U(n)$'s corresponding to the invariance $g^\dagger(m^2 + \lambda\Phi)g = (m^2 + \lambda\Phi)$. [We may still consider A_μ as a $U(N)$ gauge field, but spontaneously broken in the ambient spacetime to the invariance subgroup of $m^2 + \lambda\Phi$.] Notice that the first term in (12) is a topological term independent of the background metric. We could view Mds as part of the connection one-form, but it is metric dependent via ds , namely, with a non-trivial metric tensor $g_{\mu\nu}$ for the ambient spacetime, $ds = \sqrt{g_{\mu\nu} dx^\mu dx^\nu}$. Also gauge transformations do not change M ; in this sense it is not exactly a connection one-form. The action (11) also shows that M is conjugate to the proper time s if we consider a canonical quantization. Our claim is that this generalization of the Wong action [8] is what is needed for an observer/instrument with internal degrees of freedom. The short-distance operator product expansion for the operators on the world line of the observer is the background-independent algebra considered in [2]. In the present context, these will be operator products like $A_\mu(x(\tau))A_\nu(x(\tau'))$ and $\Phi(x(\tau))\Phi(x(\tau'))$.

We can now consider the functional integral representation of $\mathcal{A}_{\beta b, aa}$ in (3). We will need functional integrations over A_μ and Φ , as these are fields in the ambient spacetime. But first, it is useful to recall that the path-integral representation of the usual Wilson line has the form [9]

$$\begin{aligned} W_{ba}(L) &= \left[P \exp \left(- \int_L A_\mu dx^\mu \right) \right]_{ba} \\ &= \int [Dz D\bar{z}] f_b^*(z, \bar{z}) e^{iS_L} f_a(z', \bar{z}'), \\ S_L &= \int_L \text{Tr} hU^\dagger \left(-i \frac{\partial U}{\partial \tau} - A_\mu \frac{dx^\mu}{d\tau} U \right). \end{aligned} \quad (15)$$

Here L denotes the path along which the integration is done, and P denotes path ordering of A_μ as usual. For our case, we also have Mds as part of the connection, as in (11), so, for the instrument we define the path-ordered exponential integral

$$\begin{aligned} I_{ba}(L) &= \left[P \exp \left(- \int_L (A_\mu dx^\mu + Mds) \right) \right]_{bx, ax'} \\ &= \int [Dz D\bar{z}] f_b^*(z, \bar{z}) e^{iS_{\text{instr}}(L)} f_a(z', \bar{z}'), \\ S_{\text{instr}}(L) &= \int_{x', z', \bar{z}'}^{x, z, \bar{z}} d\tau \text{Tr} \left[hU^\dagger \left(-i \frac{\partial U}{\partial \tau} - A_\mu \frac{dx^\mu}{d\tau} U - M \frac{ds}{d\tau} U \right) \right]. \end{aligned} \quad (16)$$

While $z, \bar{z}$ and $z', \bar{z}'$ are integrated over with f_b^* and f_a , we consider fixed values of the final and initial positions, x and x' , of the instrument.

The amplitude in (3) involves the evolution of the instrument starting at time t_1 to t and then evolution backward to t_2 , which is the time of the second readout. Thus, we will need to consider paths L in (3) as starting at $x^0 = t_1$, going forward to t and then back to t_2 . When this is used in (3), the time coordinate for the action integral for the fields A, Φ will go from t_0 to t and then back to t_0 , but the line integral in I_{ba} is only over a segment of this contour, from t_1 to t to t_2 . We can take $(t - t_2)$ to be large or as small as we like. Using a functional integration for the fields as well, we can now write the amplitude as

$$\begin{aligned} \mathcal{A}_{\beta b, aa} &= \int [DAD\Phi Dx] \langle \beta | A, \Phi \rangle e^{iS_C(A, \Phi)} I_{ba}(L) \langle A', \Phi' | \alpha \rangle \\ &= \int [DAD\Phi Dx D\mu] \\ &\quad \times \langle \beta | A, \Phi \rangle e^{iS_C(A, \Phi)} f_b^* e^{iS_{\text{instr}}} f_a \langle A', \Phi' | \alpha \rangle. \end{aligned} \quad (17)$$

Here $S_C(A, \Phi)$ is the action for the fields A, Φ in the ambient space. It connects A', Φ' to A, Φ over the range of the time integration. It is defined on a time contour from t_0 to t and back to t_0 as mentioned above. Although this is similar to the Schwinger-Keldysh (SK) contour, there is a distinction. Here the contour arises just for the amplitude, whereas the usual context for the SK contour is for time evolution of the density matrix. The functional integration over A should be defined using the product of the volume

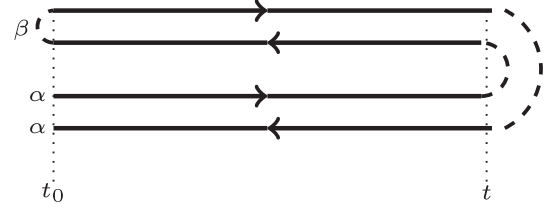


FIG. 1. The SK_4 contour. The inner two branches are for $\mathcal{A}$, and the outer two branches (3 and 4) are for $\mathcal{A}^*$. The time intervals are from t_0 to t ; the contours are extended slightly with arcs attached for clarity of the picture.

element of the gauge-orbit space at each point in time, in a time-sliced version of the functional integral, as is usually done. Further, $[D\mu]$ denotes integration over $z, \bar{z}$ with the volume element of $\mathbb{CP}^{N-1}$ at each point in time.

In the expression for the probability $\mathcal{P}(b, a)$, we have $\mathcal{A}_{\beta b, aa}^*$, which we can again write as a path integral of the form (17), but with reversal of the time contour (as needed for the conjugate amplitude). Thus, the full path integral for $\mathcal{P}(b, a)$ will be over a Schwinger-Keldysh-like contour, with four branches rather than two; we denote this contour by SK_4 (see Fig. 1). The inner two branches in the figure, say branches 1 and 2, correspond to $\mathcal{A}_{\beta b, aa}$, while the outer ones, say branches 3 and 4, correspond to the conjugate $\mathcal{A}_{\beta b, aa}^*$. We also have summation over β . This means that the fields A, Φ can be continued on to the third and fourth branches and the action $S(A, \Phi)$ is integrated over the full contour SK_4 . The expression for $\mathcal{P}(b, a)$ is thus

$$\begin{aligned} \mathcal{P}(b, a) &= \int [DAD\Phi] [Dx D\tilde{x}] e^{iS_{SK_4}(A, \Phi)} I_{ba}^*(\tilde{L}) I_{ba}(L) p(A', \Phi'; \tilde{A}', \tilde{\Phi}') \\ &= \left\langle \int [Dx D\tilde{x}] I_{ba}^*(\tilde{L}) I_{ba}(L) \right\rangle. \end{aligned} \quad (18)$$

For the third and fourth branches of the SK_4 contour, we designate the variables with a tilde. These are independent of the ones on the first two branches. The angular brackets in the second line of (18) indicate the integration over all the fields $A, \Phi, \tilde{A}, \tilde{\Phi}$, etc., with $e^{iS_{SK_4}(A, \Phi)} p(A', \Phi'; \tilde{A}', \tilde{\Phi}')$. Finally, we have

$$p(A', \Phi'; \tilde{A}', \tilde{\Phi}') = \sum_{\alpha} \langle A', \Phi' | \alpha \rangle p_{\alpha} \langle \alpha | \tilde{A}', \tilde{\Phi}' \rangle. \quad (19)$$

This gives the density matrix for the initial state of the world in terms of the values A', Φ' at the beginning and $\tilde{A}', \tilde{\Phi}'$ at the end of the SK_4 contour.

When we consider several observers or several instruments, say M of them, we get

$$\begin{aligned} \mathcal{P}(\{b\}, \{a\}) &= \left\langle \int \prod_i^M [Dx_i D\tilde{x}_i] I_{b_1 a_1}^*(\tilde{L}_1) \cdots I_{b_M a_M}^*(\tilde{L}_M) \right. \\ &\quad \left. I_{b_1 a_1}(L_1) \cdots I_{b_M a_M}(L_M) \right\rangle. \end{aligned} \quad (20)$$

The different instruments may have different numbers of internal states, say, $N_1, N_2, \dots, N_M$. If $N_{\max}$ is the highest of these values, the field A_{μ}, Φ will be in some subalgebra of $U(N_{\max})$, the choice of this subalgebra being a defining characteristic of the instrument.

Equation (20) expresses the basic observable $\mathcal{P}(\{b\}, \{a\})$ as a correlation among the readout histories or among a set of generalized Wilson lines of the instruments. No reference to states on a Hilbert space is needed at this stage. The correlators themselves are calculated using the action for the fields A_{μ}, Φ in the ambient spacetime. But

ultimately, as mentioned in the Introduction, we can view these fields merely as a calculational tool for the correlations of instrumental readouts, with no direct observable reality attributed to them. In this sense, Eq. (20) is the expression of Schwinger's goal in his "source theory." Equation (20) is the main result of this paper.

This equation expresses observables in terms of the world lines of the instruments, and in this sense it has some conceptual similarity to a Dirac-Feynman histories-based approach to quantum mechanics. Such an approach has also been argued to be the appropriate framework to include quantum gravity [10]. Also Eq. (20) has some similarity to the source-detector approach used in [11]. There, the authors consider the expectation values of local operators. The "sensitivity" of the detector to such a measurement is then shown to involve only retarded commutators, in agreement with Einstein causality. Our formula is different in making the instrument more central and also involves the SK_4 contour rather than the usual SK contour. However, if we consider all insertions of f 's and f^* 's to be on the first branch only, and hence the conjugate insertions are on the fourth branch, the second and third branches can be removed using the composition property of time evolution and the contour SK_4 will reduce to the usual SK contour.

In the rest of this paper, we make some comments related to (20). In general, the initial labels $\{a\}$, rather than being definite, will follow a probability distribution. Second, the standard S -matrix analyses can be obtained by suitable approximations to (20). We will comment on these in turn. We will also briefly indicate how Einstein causality is obtained in using (20) for the much-discussed Fermi problem/paradox.

$$\begin{aligned} \frac{\partial \rho_{ab}}{\partial t} = & 2\pi\lambda^2 \sum \frac{1}{2\omega_k V} [-T_{ac}^\alpha T_{cd}^\alpha \rho_{db} (n_k + 1) \delta(E_c - E_d + \omega_k) - T_{ac}^\alpha T_{cd}^\alpha \rho_{db} n_k \delta(E_c - E_d - \omega_k) \\ & - \rho_{ad} T_{dc}^\alpha T_{cb}^\alpha (n_k + 1) \delta(E_c - E_d + \omega_k) - \rho_{ad} T_{dc}^\alpha T_{cb}^\alpha n_k \delta(E_c - E_d - \omega_k) + T_{ac}^\alpha \rho_{cd} T_{db}^\alpha n_k \delta(E_d - E_b + \omega_k) \\ & + T_{ac}^\alpha \rho_{cd} T_{db}^\alpha (n_k + 1) \delta(E_d - E_b - \omega_k) + T_{ac}^\alpha \rho_{cd} T_{db}^\alpha n_k \delta(E_a - E_c - \omega_k) + T_{ac}^\alpha \rho_{cd} T_{db}^\alpha (n_k + 1) \delta(E_a - E_c + \omega_k)]. \end{aligned} \quad (21)$$

Here V is the spatial volume, and we can take the large volume limit in the usual way. The scalar field coupling is of the form $\Phi = 2m_0 \phi^\alpha T_{ab}^\alpha$, and it carries out the transitions $a \leftrightarrow b$ upon emission or absorption of the ϕ particles. T_{ab}^α are matrices effecting the transitions and can be viewed as the jump operators in the GKSL language. Also λ is the strength of coupling between ϕ^α and the instrument. We have taken the fields ϕ^α to obey thermal correlations, with

$$n_k = \frac{1}{e^{\omega_k/T} - 1}, \quad (22)$$

where T is the temperature. [We have not gone over the derivation of (21), since, apart from the fact that we use

IV. DENSITY MATRIX FOR THE OBSERVER

If we consider the instrument as a quantum mechanical system placed in an environment with various fields, it is clear that the state of the instrument has to be described by a mixed state. Even if we assume the initial settings can be controlled and distinguished from one another, there is a nonzero probability, perhaps very small, for settings other than the intended one. This can be due to interactions with fields in the environment or due to entanglement between the instrument and ambient fields, which can give a mixed state reduced density matrix for the instrument. Further, if we consider the background spacetimes with a horizon and a temperature, the instruments will reach thermal equilibrium, and hence they should again be described by a density matrix. Even though we refer to fields that may exist in the ambient spacetime, these fields, and even the nature of the spacetimes in which the instruments reside, are to be considered as an interpretation inferred from the correlations of records made by the instruments; this is the point of view we have tried to emphasize. Nevertheless, to take account of such possibilities, one should describe the initial state of the instruments by a density matrix, a mixed state in general.

The reduced density matrix, because of couplings to the ambient spacetime, will obey a version of the Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) equation [12]. Just to be specific, for example, for a heavy instrument coupled to a scalar field maintained at a temperature T in the ambient spacetime [for which we can use the approximation in the second line of (10)], the GKSL equation for the reduced density matrix is given by

thermal correlations for the fields ϕ^α , the logical steps in obtaining it follow the standard derivation of the GKSL equation.] It is easy to see that the choice

$$\rho_{ab} = \frac{1}{Z} e^{-E_b/T} \delta_{ab} = \rho_b \delta_{ab} \quad (23)$$

leads to the vanishing of all terms on the right-hand side of (21) leading to the equilibrium solution. This clearly shows the thermalization of the instrument and the need for using a density matrix for it unless the energy differences for the different readings of the instrument are large compared to T . Even if we do not consider fields at nonzero temperature, the state of the instrument will be described, in general, by a

mixed state due to entanglement and interactions with the environment.

The state of the instruments that is obtained upon completion of a set of measurements is more subtle. The results of the measurements are $\mathcal{P}(\{b\}, \{a\})$, where the labels $\{b\}$ correspond to a set of compatible observables (or a set of mutually commuting set of operators). In fact, $\mathcal{P}(\{b\}, \{a\})$ is inferred from measured values $\langle A_i \rangle$, which are taken to be $\langle A_i \rangle = \sum_{b,a} (A_i)_b \mathcal{P}(\{b\}, \{a\}) \rho_a$ for a set of mutually commuting observables A_i . By assigning a set of states $\{|b\rangle\}$ to the instrument, this may be expressed as $\langle A_i \rangle = \text{Tr}(A_i \rho')$, $\rho' = \sum_{b,a} |b\rangle \mathcal{P}(\{b\}, \{a\}) \rho_a \langle b|$. Thus, if we take the state of the instrument to be described by a density matrix, the diagonal elements of this density matrix, in the basis in which the set $\{A_i\}$ is diagonal, are determined by $\mathcal{P}(\{b\}, \{a\})$. No information about non-diagonal elements is obtained. One can consider measurements in different bases, i.e., using different sets of mutually commuting observables, to identify possible nondiagonal elements of ρ . This is often referred to as quantum tomography [13]. However, using a different set of compatible observables means that one is using a different set of instruments. If one uses a fixed set of instruments corresponding to a particular choice of the set of compatible observables, one gets just $\mathcal{P}(\{b\}, \{a\})$ (for that set). One may then consider the corresponding (diagonal) ρ as an approximate description of the state of the instruments after the conclusion of the measurements. One could then use $\rho' = \sum_{b,a} |b\rangle \mathcal{P}(\{b\}, \{a\}) \rho_a \langle b|$ as the input density matrix for any subsequent measurements. The natural question then is: How good is this ρ' as a description of the state of the instruments (or the world as seen via the instruments)?

If σ denotes the true density matrix of the instrument after measurements are completed, then a good measure of how close we are in positing the state to be described by $\rho' = \sum_{b,a} |b\rangle \mathcal{P}(\{b\}, \{a\}) \rho_a \langle b|$ is the fidelity given by [14]

$$F(\sigma, \rho') = \left[\text{Tr} \sqrt{\sqrt{\rho'} \sigma \sqrt{\rho'}} \right]^2. \quad (24)$$

However, since we do not know the true density matrix, this is not very useful in the present context.

Another measure is provided by the relative entropy [15]. A series of measurements, say via positive operator-valued measures, to determine the state would naturally lead to the relative entropy. (It is the quantum generalization of the Kullback-Liebler distance.) The determination of the state using measurements of $\mathcal{P}(\{b\}, \{a\})$ is a similar strategy. If ρ' is the diagonal density matrix inferred from the measurements of $\mathcal{P}(\{b\}, \{a\})$ and σ is a general nondiagonal density matrix with eigenvalues σ_k , i.e., $\sigma = U \sigma_k U^\dagger$ for some unitary matrix U , then since σ gives the same measured values for $\langle A_i \rangle$, we must have

$$\rho'_b = \sum_a \mathcal{P}(\{b\}, \{a\}) \rho_a = \sum_k |U_{bk}|^2 \sigma_k. \quad (25)$$

In other words, the diagonal elements of $\sigma = U \sigma_k U^\dagger$ should be the same as ρ'_b to give the same measured values $\langle A_i \rangle$ for a complete set of compatible observables. The relative entropy of σ and ρ' is then given by

$$\begin{aligned} S_{\text{rel}}(\sigma|\rho') &= \text{Tr}(\sigma \log \sigma - \sigma \log \rho') \\ &= -\text{Tr}(\rho' \log \rho') - [-\text{Tr}(\sigma \log \sigma)] \\ &= S(\rho') - S(\sigma). \end{aligned} \quad (26)$$

If the trace exists, this is positive. [ρ'_b are linear combinations of σ_k with positive coefficients, and since entropy is a concave function, $S(\rho') \geq S(\sigma)$.] This shows that ρ' which is diagonal has a higher entropy than any nondiagonal density matrix σ , which has the same diagonal elements as ρ'_b . Thus, ρ' maximizes the entropy subject to the diagonal values ρ'_b being determined by $\mathcal{P}(\{b\}, \{a\})$.

V. S-MATRIX FOR A SCALAR FIELD

We will now consider simplifying the expression (20) for what may be termed as a scalar field theory and show how it reproduces the usual field theoretic calculations. We have to make certain approximations that are also inherent in the usual approaches to the S -matrix.

First of all, the initial and final readouts are taken to be in the far past and far future, respectively, so that we may simplify the expression (3) for $\mathcal{A}$ as

$$\mathcal{A}_{\beta b, \alpha a} = \langle \beta | U^\dagger(t, t_0) f_b^*(t) U(t, t_0) f_a(t_0) | \alpha \rangle \quad (27)$$

with $t \rightarrow \infty$, $t_0 \rightarrow -\infty$ eventually. The probability thus takes the form

$$\mathcal{P}(b, a) = \sum_\gamma \sum_\alpha \langle \gamma | f_b^* U(t, t_0) f_a | \alpha \rangle p_\alpha \langle \alpha | f_a^* U^\dagger(t, t_0) f_b | \gamma \rangle. \quad (28)$$

In evaluating this with the functional integral over A, Φ , we get correlations among fields in U , separately among fields in $U^\dagger$, and also cross-correlations.³ Usually, one makes the further assumption that the state $|\alpha\rangle$ is restored over the long time interval so that only one state, namely $|\gamma\rangle = |\alpha\rangle$, contributes in the sum over γ .⁴ In calculating the correlations, this state is taken to be the ground (vacuum) state for the fields A, Φ . We also take $p_\alpha = 1$ for this state, zero for others, so that the starting state and the final state of the

³The result will be similar to the evolution of the density matrix in (21), with terms like $T_{ac}^\alpha \rho_{cd} T_{db}^\alpha$ representing the cross-correlations.

⁴This is not the case for evolution over a finite time interval, but may be a good approximation for large $(t - t_0)$.

world correspond to the vacuum state of the fields in ambient spacetime. In this case the sum over α has only one term. The probability thus reduces to $\mathcal{P}(b, a) = |\mathcal{A}_{ba}|^2$ with

$$\begin{aligned}\mathcal{A}_{ba} &= \langle \alpha | f_b^*(t) U(t, t_0) f_a(t_0) | \alpha \rangle \\ &= \int [DAD\Phi Dx] e^{iS(A, \Phi)} I_{ba}(L).\end{aligned}\quad (29)$$

To convert this description in terms of instruments to the more familiar expression for the S -matrix, some further reduction is needed. The instrument is taken to be sufficiently massive so that one can approximate its world line to be $\dot{x}^0 = 1$, $\dot{x}^i = 0$. In other words, the functional integrals $[Dx] e^{iS_{\text{instr}}}$ are localized on to these trajectories. Further, m^2 has a dominant term proportional to the identity, say $m_0^2 \mathbb{1}$, and the nondiagonal terms corresponding to the energy differences among the internal states $|a\rangle$ of the instrument (which are equivalent to mass differences for the instrument as a whole). Taking the masses to be large, we then approximate

$$\sqrt{\text{Tr}[hU^\dagger(m^2 + \lambda\Phi)U]} \sqrt{\dot{x}^2} \approx m + \frac{\lambda\Phi}{2m_0}, \quad (30)$$

where in the second term, the mass differences can be neglected. The nondiagonal terms with energy differences will be kept in the leading term since they are needed for the conservation of energy for absorption and emission of Φ particles. Focusing on just the scalar field for now, we also neglect the coupling of the gauge field A_μ . The action for the instrument is thus

$$\mathcal{S}_{\text{instr}} \approx \int d\tau \text{Tr} \left[hU^\dagger \left(-i \frac{\partial U}{\partial \tau} - \frac{\lambda\Phi}{2m_0} U \right) \right] - m \int d\tau. \quad (31)$$

A scalar field ϕ in the ambient spacetime corresponds to taking

$$\Phi(\vec{x}_p, x_p^0) = (2m_0)\phi(x_p)J(x_p), \quad (32)$$

where $J(x_p)$ is an operator (or matrix) that makes the appropriate transitions $a_p \rightarrow b_p$ for the apparatus. (The factor of $2m_0$ is just for convenience.) Some of these transitions can be interpreted as transitions producing or radiating ϕ -particles into the ambient space and some correspond to the absorption of the ϕ particles.

With these approximations, $\mathcal{A}$ takes the form

$$\mathcal{A}_{ba} = \int [D\Phi D\mu] e^{iS(\Phi)} \prod_i f_{b_i}^* e^{iS_{\text{instr}}} f_{a_i}. \quad (33)$$

We can expand $e^{iS_{\text{instr}}}$ in powers of Φ and, if we consider the lowest order term with a nontrivial $a \rightarrow b$ transition for

each of the instruments (that is, we need one factor of Φ for each instrument), we get

$$\mathcal{A}_{ba} = \int \prod_i dx_i^0 G(x_M, \dots, x_1) J(x_M)_{b_M a_M} \cdots J(x_1)_{b_1 a_1}, \quad (34)$$

$$\begin{aligned}G(x_M, \dots, x_1) &= \int [D\phi] e^{iS(\phi)} \phi(x_M) \cdots \phi(x_1), \\ J(x)_{ba} &= \int [D\mu] f_b^*(z) e^{iS_{0,\text{instr}}} f_a(z'),\end{aligned}\quad (35)$$

$$\mathcal{S}_{0,\text{instr}} = \int d\tau \text{Tr} \left[hU^\dagger \left(-i \frac{\partial U}{\partial \tau} \right) \right] - m \int d\tau. \quad (36)$$

The correlation of the ϕ 's is part of the usual S -matrix element. To see this more explicitly, notice that the correlation function $G(x_M, \dots, x_1)$ has a factorization of the form

$$\begin{aligned}G(x_M, \dots, x_1) &= \int_{z_1, \dots, z_M} G(x_M, z_M) \cdots G(x_1, z_1) V(z_1, \dots, z_M)\end{aligned}\quad (37)$$

where $G(x_i, z_i)$ are propagators for ϕ and $V(z_1, \dots, z_M)$ is the vertex function. By virtue of the choice of the labels b, a , the elements $J(x)_{ba}$ pick out the appropriate terms (referring to the production versus the detection of ϕ 's) in the mode expansion of the propagators $G(x_i, z_i)$. The amplitude $\mathcal{A}_{ba}$ then reduces to the usual textbook formula.

VI. THE FERMI PROBLEM

We will now consider the Fermi problem, which has been a much-studied case for signaling and Einstein causality in field theory [16]. The idea is to consider two atoms, say A and B , separated by a distance R . At time $t = 0$, A is in an excited state, and B is in its ground state. The question is to calculate the probability $P(t)$ that B is in an excited state at time t , to be interpreted as being due to B absorbing the photon from the spontaneous decay of A . Einstein causality would require that $P(t) = 0$ for $t < R$ (since it takes time R for the photon to propagate from A to B). However, the simple use of the (time-ordered) Feynman propagator leads to causality problems since it has correlations over spacelike separations. An analyticity argument by Hegerfeldt highlights some of the tricky issues involved in the analysis of this seemingly simple problem [17], suggesting that either atom B is never excited or it is in an excited state even before the decay of atom A . A factorization of the Hilbert space into locally defined Hilbert spaces for atom A and atom B , in the spirit of making local observations, is the premise of such an argument. The problem has a long history going back to Fermi's work in

1932, and has been resolved in different framings from straightforward calculations to the use of algebraic quantum field theory [18]. The factorization of the Hilbert space into locally defined Hilbert spaces for the atoms is not possible because of the type of von Neumann algebra for the observables, and this can help to resolve the problem. The direct use of probability, rather than amplitudes, can also be used to resolve this issue [19]. This latter method is

not the same but is fairly close to what we show below. We also refer to [19] for background historical information.

To set up this problem in our framework, we consider two instruments corresponding to A and B , where we have $I_{ba}^{(A)}$ with the transition $a \rightarrow b$ corresponding to the decay of A and $I_{dc}^{(B)}$ corresponding to B making a transition ($c \rightarrow d$) to the excited state. The amplitude (17) then takes the form

$$\mathcal{A}_{\beta bd, aac} = \int [DAD\Phi Dx D\mu] \langle \beta | A, \Phi \rangle e^{iS_c(A, \Phi)} [f_b^*(w_2^0) e^{iS_{\text{instr}} f_a(w_1^0)} f_d^*(v_2^0) e^{iS_{\text{instr}} f_c(v_1^0)}] \langle A', \Phi' | \alpha \rangle. \quad (38)$$

The actions are over the time contours as explained in relation to (17). We will consider the possible absorption and emission of a scalar particle, as this is simpler but captures the essence of the causality issues. This means that we consider a field ϕ in the ambient space. There are two possible world line choices or time contours corresponding to $v_2^0 > w_2^0$ and $w_2^0 < v_2^0$, as shown in Fig. 2. Using the expression for $\mathcal{A}_{\beta bd, aac}$ and its conjugate and integrating over Φ with $\exp(iS_{\text{SK}_4})$, we find the probability

$$\mathcal{P}(bd; ca) = \langle I_{ba}^{(A)*} I_{dc}^{(B)*} I_{dc}^{(B)} I_{ba}^{(A)} \rangle. \quad (39)$$

The integration over ϕ , indicated by the angular brackets in this equation, will produce correlators of ϕ . For ease of keeping track of the relevant terms, we can write the I 's in the usual field theory notation to the lowest order in ϕ as

$$\begin{aligned} I_{ba}^{(A)} &= \pm i \int_{w_1^0}^{w_2^0} \langle b | \rho(w) | a \rangle \phi(w), \\ I_{dc}^{(B)} &= \pm i \int_{v_1^0}^{v_2^0} \langle d | \rho(v) | c \rangle \phi(v), \end{aligned} \quad (40)$$

where ρ is an operator that effects the transitions $a \rightarrow b$ and $c \rightarrow d$ in the instruments. The positive sign in (40) applies when the fields are on the first branch, and the negative sign applies to the second branch. The instruments are separated by the spatial distance R , which is not explicitly indicated. We can now view the correlations of the fields ϕ as given by Wick contractions.

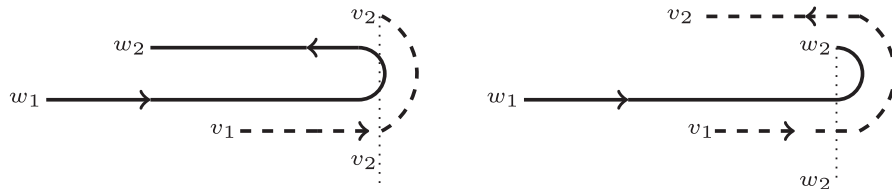


FIG. 2. The world lines for the two instruments A and B . The figure on the left applies to $v_2^0 > w_2^0$, and the one on the right to $w_2^0 > v_2^0$. All time labels for the left figure are $< v_2^0$, and $< w_2^0$ for the right one; arcs are added only to show continuity of the contours.

In (39), we will get four ϕ 's to the lowest order, so we need two correlators for ϕ to get a nonzero result in (39). There are then three possibilities given by

- (1) A correlator connecting $I_{dc}^{(B)}$ and $I_{ba}^{(A)}$, with another correlator connecting $I_{ba}^{(A)*}$ and $I_{dc}^{(B)*}$, the latter part being the conjugate of the first.
- (2) A correlator connecting $I_{ba}^{(A)*}$ and $I_{ba}^{(A)}$, with another correlator connecting $I_{dc}^{(B)*}$ and $I_{dc}^{(B)}$.
- (3) A correlator connecting $I_{dc}^{(B)*}$ and $I_{ba}^{(A)}$, with another correlator connecting $I_{ba}^{(A)*}$ and $I_{dc}^{(B)}$.

We will start by considering the second and third possibilities. For the second possibility, with the contour ordering we get $\langle \phi(\tilde{v}) \phi(v) \rangle$, where v refers to $I_{dc}^{(B)}$ (or $\langle d | \rho | c \rangle$) and $\tilde{v}$ to $I_{dc}^{(B)*}$; we will also have a factor $\langle \phi(\tilde{w}) \phi(w) \rangle$ for $I_{ba}^{(A)}$, $I_{ba}^{(A)*}$. To be compatible with B making a transition to an excited state (i.e., for the chosen d, c states of the instrument), $\phi(v)$ in $I_{dc}^{(B)}$ must correspond to absorption of the ϕ particle. However, $\langle \phi(\tilde{v}) \phi(v) \rangle$ for $\phi(v)$ with the exponential $e^{-i\omega t}$ corresponding to absorption is zero, if the ambient spacetime is taken to be in the vacuum state for the field ϕ . (If one has a different state for ϕ , then transitions like $c \rightarrow d$ are possible even without any input from atom A and the formulation of the Fermi problem does not apply.) Hence, the second possibility gives zero.

For the third case, the correlator connecting $I_{dc}^{(B)*}$ and $I_{ba}^{(A)}$ will be of the form $\langle \phi(\tilde{v}) \phi(w) \rangle$. Both ϕ 's need to have a positive exponential of the form $e^{i\omega t}$ to be compatible with the chosen states, i.e., decay of A and excitation of B . The

correlator $\langle \phi(\tilde{v})\phi(w) \rangle$ is zero for this case, so the third possibility gives zero as well.

Turning to the first possibility, consider the case $v_2^0 > w_2^0$ first. From Fig. 2, we see that $\phi(w)$, corresponding to $I_{ba}^{(A)}$, can be on the first branch, or the second, while $\phi(v)$, corresponding to $I_{dc}^{(B)}$, is on the first branch. For the fields on the first branch, we get the Feynman propagator $\langle \mathcal{T}\phi(w)\phi(v) \rangle$ while for $I_{ba}^{(A)}$ on the second branch we get $\langle \phi(w)\phi(v) \rangle$. In the amplitude, we thus get

$$\langle \mathcal{T}\phi(w)\phi(v) \rangle - \langle \phi(w)\phi(v) \rangle = \Theta(v^0 - w^0)[\phi(v), \phi(w)], \quad (41)$$

where $\Theta(v^0 - w^0)$ is the step function. [This is multiplied by the matrix elements for the instruments as in (40) and a similar term for the conjugate.] The minus sign for the second term arises from the fact that $\phi(w)$ in that term is from the time-reversed branch.

In a similar way, for the choice $w_2^0 > v_2^0$, we will get

$$\langle \mathcal{T}\phi(w)\phi(v) \rangle - \langle \phi(v)\phi(w) \rangle = \Theta(w^0 - v^0)[\phi(w), \phi(v)]. \quad (42)$$

Again, for $\mathcal{A}^*$ we have the conjugate of these expressions. Since we have the retarded commutators, the result is compatible with Einstein causality, vanishing over space-like separations for the two instruments. Notice also that we are considering the instruments to be sufficiently localized in space, and that the time of the readouts w_1^0, w_2^0 , etc., can be unambiguously defined, which means that the time separations $w_2^0 - w_1^0, v_2^0 - v_1^0$ are large compared to the time needed for the transitions to occur in the instruments.

The square of the retarded commutators is essentially the result obtained in [19] as well. This paper computed the probability for B to be in any excited state, not a specific one as we have done. (For us, there was a fixed choice, namely, d for B .) There were, therefore, some additional interference terms possible in the calculations in [19] that were crucial for the result.

VII. DISCUSSION

There are many situations in physics where it is necessary to sift out unphysical questions. For example, in the case of the double-slit interference experiment, what is measured is a two-instrument correlation function, namely, the correlation between the source of the particles and the detector at the screen, both placed in an ambient spacetime whose geometry carries information about the slits. Thus, while conjecturally we can consider the question of which slit the particle went through, this is not a properly posable question (within the grammar of quantum mechanics) since there is no detector placed at either slit to make the question answerable with observations. (If we place such a detector, it is a different problem of correlations among three instruments.) This example again illustrates that it will be useful to have a formulation where one is dealing only with what is actually observed with the instruments. This is the main point of this paper.

The dynamics of the instrument then becomes important. We have given a general form for this dynamics as a slight generalization of the coadjoint orbit action. The relevant probabilities are then naturally defined on a fourfold time contour. This can be expressed as the correlation function for a number of Wilson lines for the instruments, so the framework also ties in with the world line approach to calculations in field theory. As an example to illustrate the method, we considered the Fermi problem and showed how it is resolved in an elementary fashion.

The coadjoint orbit action is a generalization of the action used in [2] to the case of an observer/instrument with internal degrees of freedom. It can thus be applied to situations with a nontrivial background metric for the ambient spacetime. We plan to investigate this possibility further in the near future.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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