

Spatial Wilson loops and energy loss for heavy quarks in a magnetized holographic QCD model

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We investigate the effective potential and the string tension for the spatial Wilson loop in hot dense quark-gluon plasma with two types of anisotropy, i.e., external magnetic field and spatial anisotropy, employing a holographic approach for the heavy quark model. In this approach, the string is extended in the fifth, holographic direction and has a turning point either on a dynamical wall (DW) configuration or on the horizon configuration in the fifth direction. We obtain the magnetic catalysis behavior for a phase transition between DW and horizon configuration of the string. The structure of the phase diagram does not depend on the boundary conditions choice for the dilaton field. Inclusion of the external magnetic field and spatial anisotropy enhance the string tension in the horizon configuration, namely drag force. For the spatially isotropic case $\nu = 1$ at different magnetic field values, the string tension is proportional to T^2 and is qualitatively consistent with lattice results. However, for the anisotropic case, $\nu = 4.5$, it deviates from the quadratic term.

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I. INTRODUCTION

The holographic approach is a nonperturbative method for investigating the strongly coupled regime of QCD. There are two different classes of holographic models: “top-down” and “bottom-up.” Holographic QCD is a suitable candidate for studying the quark-gluon plasma (QGP) observed in ultrarelativistic heavy-ion collisions (HIC) [1–6]. In particular, the phase diagram of QCD has received a lot of attention to study. In the “bottom-up” holographic approach, the choice of the metric warp factor has a crucial influence on the phase transition structure of QCD [7–18]. This choice is different for the light- and heavy-quark models and should correspond to the lattice results known as the Columbia plot [19,20]. In Refs. [21–34], different isotropic and anisotropic models were proposed to study QCD using holography.

Taking into account the anisotropy in the holographic model [35–46] is important because of QGP, which is

spatially anisotropic after HIC. The first Maxwell field in the model supports the chemical potential in the boundary theory. In addition, to consider two types of anisotropy due to the external magnetic field and spatial anisotropy, two Maxwell fields need to be introduced into the model [45]. The model that we use in this research was developed in Ref. [45] with modified warp factor. It produces the direct magnetic catalysis phenomenon for heavy quarks that is expected from lattice calculations. The Wilson loops calculations for the heavy-quarks model with a more simple warp factor [47] were done in Ref. [48]. The second and third Maxwell fields support the magnetic field and the spatial anisotropy defined by ν in the boundary theory, respectively. An anisotropic model with $\nu = 4.5$ [49] has been shown to be able to reproduce the energy dependence of the total multiplicity of particles created in HIC [50]. For the light-quark model, one can see Ref. [18], and for the heavy-quark model, see Ref. [14].

The experiments in the relativistic HIC have addressed the possibility of new parameters in the QCD phase diagram. A strong magnetic field $eB \sim 0.3 \text{ GeV}^2$ is created in the early stages of noncentral HIC [51–53]. Therefore, an external magnetic field can be considered the second anisotropy affecting the characteristics of QCD. Magnetized QCD has received a lot of attention in recent years. In particular, recently the magnetic field effect on the running coupling of QCD was investigated in

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Refs. [54–56]. Holographic models [47,57–63] studied the effect of magnetic field on the different properties of QCD.

Temporal Wilson loops and spatial Wilson loops (SWLs) in the holographic approach have been used to study different properties of QCD [39,64,65]. Temporal Wilson loops are used to investigate the confinement/deconfinement phase transition in QCD. The drag force acting on the heavy quark, moving in QGP, comes from its interaction with the plasma that was investigated in holographic QCD models [66–68]. For isotropic models, the SWL string tension is proportional to the corresponding drag force [69,70]. For anisotropic models, this relation takes place with a small modification; see Ref. [6]. Therefore, one can calculate SWLs to investigate the drag forces for a heavy quark moving in QGP. The drag forces defining energy loss are studied intensively in holographic models [71–76].

In this research, we consider the Einstein-three-Maxwell-dilaton action, set up in Ref. [45], to investigate the effective potential, string tension and to compare our results on the string tension with the drag force and lattice calculations. This model possesses two types of anisotropy, i.e., spatial anisotropy and external magnetic field. We study the effect of these anisotropies on the phase transition temperature obtained by Wilson loops. To do so, we considered SWLs in three different orientations, i.e., $\mathcal{W}_{xY_1}$, $\mathcal{W}_{xY_2}$, and $\mathcal{W}_{y_1Y_2}$, referred to as the first, second, and third orientations, respectively.

This paper is organized as follows: in Sec. II, we introduce a fully anisotropic five-dimensional holographic set up for the heavy quarks model. Section III presents the Born-Infeld action to get the dynamical wall (DW) equations. In Sec. IV, we obtain numerical results for particular SWL orientations. Section V investigates the connection between the string tension and the drag force and compares it with lattice results. In Sec. VI, the relation of the spatial string tensions and the experimental observables is discussed. In Sec. VII, we summarize our results. Appendix A describes the explicit forms of the equations of motion (EOMs). In Appendix B, arbitrary orientations of the Wilson loop are studied, and in Appendix C, the string tension at the DW and horizon configurations are derived.

II. HOLOGRAPHIC SETUP FOR HEAVY QUARKS

In this section, we consider the background with two types of anisotropy, i.e., spatial anisotropy and external magnetic field, that are found in Ref. [45], and the thermodynamical properties of this model.

A. Background

We consider the Lagrangian with three Maxwell fields in Einstein frame, used in Ref. [45], as follows:

$$\mathcal{L} = \sqrt{-g} \left[R - \frac{f_0(\phi)}{4} F_0^2 - \frac{f_1(\phi)}{4} F_1^2 - \frac{f_3(\phi)}{4} F_3^2 - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right], \quad (2.1)$$

where g is the determinant of the metric tensor, R is Ricci scalar, ϕ is the dilaton field, $V(\phi)$ is the dilaton field potential, and $f_0(\phi)$, $f_1(\phi)$, and $f_3(\phi)$ are the coupling functions associated with stresses F_0 , F_1 , and F_3 of the Maxwell fields, respectively. The electromagnetic tensor is defined as $F_{\rho\sigma} = \partial_\rho A_\sigma - \partial_\sigma A_\rho$ and the indexes ρ and σ numerate the spacetime coordinates (t, x_1, x_2, x_3, z) , with z being the radial holographic coordinate. Maxwell fields F_0 , F_1 , and F_3 are considered the first, the second, and the third, respectively.

We take the following ansatz for the metric [45] that in the Einstein frame is

$$ds^2 = \frac{L^2}{z^2} \mathfrak{b}(z) \left[-g(z) dt^2 + dx^2 + \left(\frac{z}{L} \right)^{2-\frac{2}{\nu}} dy_1^2 + e^{c_B z^2} \left(\frac{z}{L} \right)^{2-\frac{2}{\nu}} dy_2^2 + \frac{dz^2}{g(z)} \right], \quad (2.2)$$

$$\mathfrak{b}(z) = e^{2\mathcal{A}(z)}, \quad (2.3)$$

and for the matter fields:

$$\phi = \phi(z), \quad (2.4)$$

$$F_0 - \text{electric ansatz} \quad A_0 = A_t(z), \quad A_i = 0, \\ i = 1, 2, 3, 4,$$

$$F_k - \text{magnetic ansatz} \quad F_1 = q_1 dy^1 \wedge dy^2, \\ F_3 = q_3 dx \wedge dy^1, \quad (2.5)$$

where L is the anti-de Sitter radius, $\mathfrak{b}(z)$ is the warp factor in Einstein frame that is specified by a scale factor $\mathcal{A}(z)$; $g(z)$ is the blackening function; and ν is the parameter of primary anisotropy created by the nonsymmetry of HIC, such that $\nu = 1$ corresponds to spatially isotropic background and choice $\nu = 4.5$ for anisotropic background is in agreement with the experimental data on the energy dependence of the total multiplicity of particles produced in HIC [50]. The coefficient of secondary anisotropy c_B is related to the external magnetic field F_3 . Different choices of $\mathcal{A}(z)$ determine the description of the model with heavy/light quarks. For heavy quarks, we considered $\mathcal{A}(z) = -cz^2/4$ [14,16,47] and for light quarks, $\mathcal{A}(z) = -a \ln(bz^2 + 1)$ [18]. In this research, our choice of $\mathcal{A}(z)$ for the heavy quarks model is [45]

$$\mathcal{A}(z) = -cz^2/4 - (p - c_B q_3) z^4, \quad (2.6)$$

where $c = 4R_{gg}/3$, $R_{gg} = 1.16 \text{ GeV}^2$, $c = 1.547 \text{ GeV}^2$, and $p = 0.273 \text{ GeV}^4$ are parameters that can be fixed with lattice and experimental data for zero magnetic field, i.e., for the $c_B = 0$ case [10].

Varying the Lagrangian (2.1) by using the ansatz (2.2)–(2.5) leads to the EOMs for the Einstein, dilaton, and Maxwell fields that can be found in Appendix A [see Eqs. (A5)–(A10)]. The EOMs are solved with the boundary conditions

$$A_t(0) = \mu, \quad A_t(z_h) = 0, \quad (2.7)$$

$$g(0) = 1, \quad g(z_h) = 0. \quad (2.8)$$

For the dilaton field, different boundary conditions can be used. We denoted the boundary condition for $\phi(z, z_0)$, so that

$$\phi(z, z_0)|_{z=z_0} = 0. \quad (2.9)$$

We consider two types of boundary conditions by taking different z_0 [18,77–80]. The zero-boundary condition is chosen as

$$z_0 = \epsilon, \quad (2.10)$$

where $0 < \epsilon \ll 1$, and another boundary condition is

$$z_0 = \mathfrak{z}(z_h), \quad (2.11)$$

where $\mathfrak{z}(z_h)$ is a smooth function of z_h , different for light and heavy quarks. For heavy quarks, we take

$$z_0 = \mathfrak{z}(z_h) = e^{(-\frac{z_h}{4})} + 0.1, \quad (2.12)$$

known as a physical-boundary condition; for more details, see Refs. [18,77–80]. We note that in this paper, we considered the zero-boundary conditions, i.e., Eq. (2.10) in our calculations, and the physical-boundary condition (2.12) has just been used in Sec. IV A 2.

B. Thermodynamics

To investigate thermodynamics, we use the following formulas for temperature and entropy:

$$T = \frac{\sqrt{g_{tt}' g^{zz'}}}{4\pi} \Big|_{z=z_h} = \frac{\sqrt{g_{00}' g^{44'}}}{4\pi} \Big|_{z=z_h} = \frac{|g'(z)|}{4\pi} \Big|_{z=z_h}, \quad (2.13)$$

$$s = \frac{\sqrt{g_{xx} g_{y_1 y_1} g_{y_2 y_2}}}{4G_5} \Big|_{z=z_h} = \frac{\sqrt{g_{11} g_{22} g_{33}}}{4G_5} \Big|_{z=z_h}, \quad (2.14)$$

where G_5 is the gravitational constant that in our calculations we set as $8\pi G_5 = 1$, and $g(z)$ is the blackening function given by

$$g(z) = e^{c_B z^2} \left[1 - \frac{\tilde{I}_1(z)}{\tilde{I}_1(z_h)} + \frac{\mu^2 (2R_{gg} + c_B (q_3 - 1)) \tilde{I}_2(z)}{L^2 \left(1 - e^{(2R_{gg} + c_B (q_3 - 1)) \frac{z^2}{2}} \right)^2} \left(1 - \frac{\tilde{I}_1(z)}{\tilde{I}_1(z_h)} \frac{\tilde{I}_2(z_h)}{\tilde{I}_2(z)} \right) \right], \quad (2.15)$$

$$\tilde{I}_1(z) = \int_0^z e^{(2R_{gg} - 3c_B) \frac{\xi^2}{2} + 3(p - c_B q_3) \xi^4} \xi^{1+\frac{2}{\nu}} d\xi, \quad (2.16)$$

$$\tilde{I}_2(z) = \int_0^z e^{(2R_{gg} + c_B (\frac{q_3}{2} - 2)) \frac{\xi^2}{2} + 3(p - c_B q_3) \xi^4} \xi^{1+\frac{2}{\nu}} d\xi. \quad (2.17)$$

Using the metric (2.4) and the scale factor (2.6) with Eqs. (2.13) and (2.14), one can obtain the temperature and entropy, as follows:

$$T = \left| - \frac{e^{(2R_{gg} - c_B) \frac{z_h^2}{2} + 3(p - c_B q_3) \frac{z_h^4}{2}}}{4\pi \tilde{I}_1(z_h)} \left[1 - \frac{\mu^2 (2R_{gg} + c_B (q_3 - 1)) (e^{(2R_{gg} + c_B (q_3 - 1)) \frac{z_h^2}{2}} \tilde{I}_1(z_h) - \tilde{I}_2(z_h))}{L^2 \left(1 - e^{(2R_{gg} + c_B (q_3 - 1)) \frac{z_h^2}{2}} \right)^2} \right] \right|, \quad (2.18)$$

$$s = \frac{1}{4G_5} \left(\frac{L}{z_h} \right)^{1+\frac{2}{\nu}} e^{-(2R_{gg} - c_B) \frac{z_h^2}{2} - 3(p - c_B q_3) \frac{z_h^4}{2}}.$$

In Fig. 1, the temperature as a function of horizon $T(z_h)$ for different μ at fixed $\nu = 1$ (a), $\nu = 4.5$ (b), and different ν and c_B at fixed $\mu = 0$ (c), $\mu = 0.2 \text{ GeV}$ (d) is plotted. Figures 1(a) and 1(b) show that the temperature of the

Hawking-Page-like phase transition T_{HP} is lower for larger μ in both isotropic and anisotropic cases. Figures 1(c) and 1(d) show that primary anisotropy ν decreases T_{HP} for both zero and nonzero values of chemical potential.

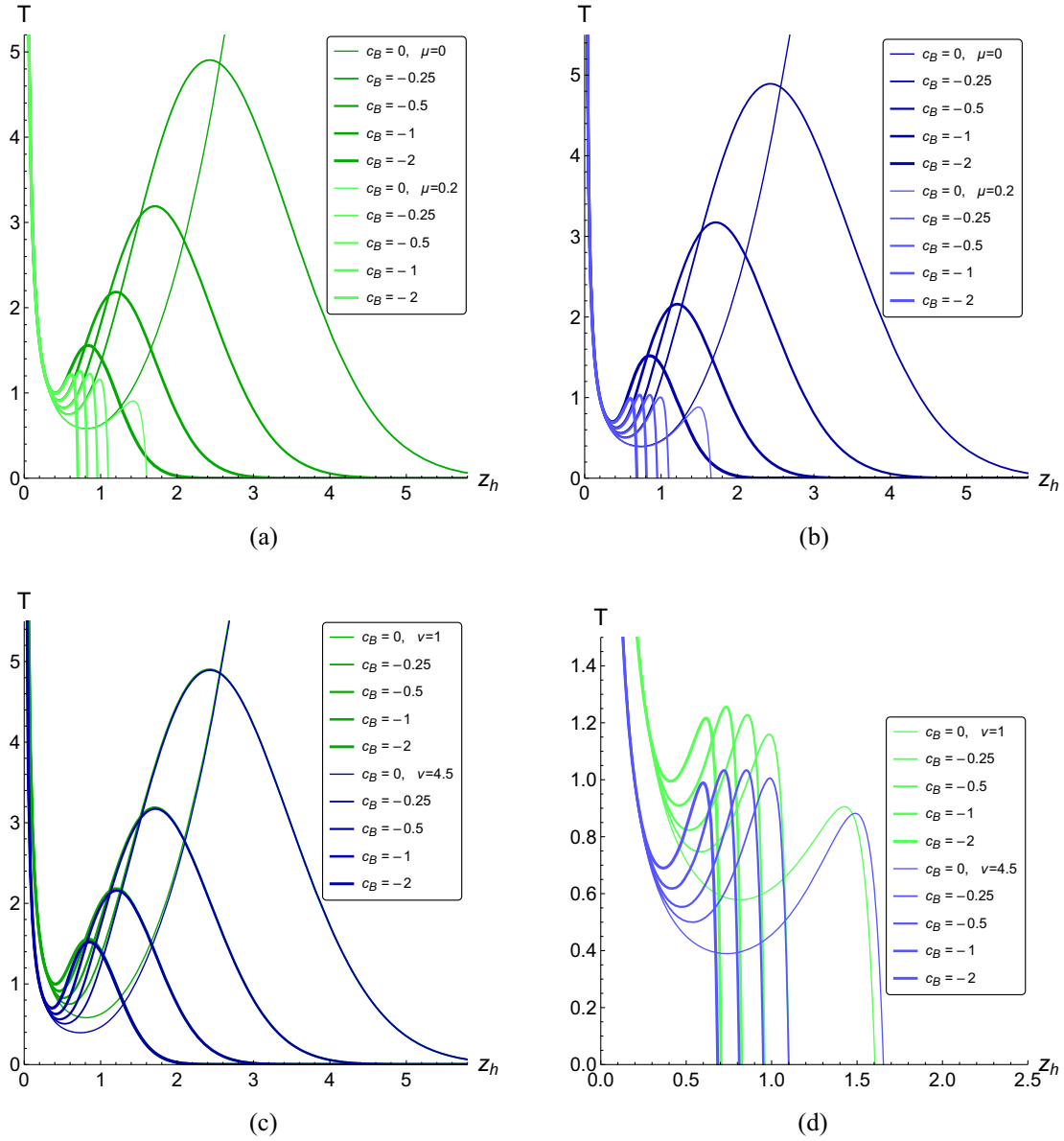


FIG. 1. Temperature as a function of horizon $T(z_h)$ for (a) different μ at fixed $\nu = 1$, (b) $\nu = 4.5$, and (c) different ν and c_B at fixed $\mu = 0$, (d) $\mu = 0.2$ GeV; $q_3 = 5$, $R_{gg} = 1.16$ GeV², $p = 0.273$ GeV⁴; $[T] = [\mu] = [z_h]^{-1} = [c_B]^{\frac{1}{2}} = \text{GeV}$.

III. WILSON LOOPS AND EFFECTIVE POTENTIAL

A. Born-Infeld type action

The actions that are considered for SWL, temporal Wilson loop, lightlike Wilson loop, and holographic entanglement entropy [4,16,48,81–83] are the specific cases of the Born-Infeld action:

$$\mathcal{S} = \int_{-\ell/2}^{\ell/2} M(z(\xi)) \sqrt{\mathcal{F}(z(\xi)) + (z'(\xi))^2} d\xi, \quad (3.1)$$

where ℓ is the interquark distance, ξ denotes the “time,” and $\mathcal{F}(z)$ is an arbitrary function. The action (3.1) describes the

dynamical system with a dynamic variable $z = z(\xi)$. The associated effective potential is

$$\mathcal{V}(z(\xi)) \equiv M(z(\xi)) \sqrt{\mathcal{F}(z(\xi))}. \quad (3.2)$$

After some algebra (for more detail, see Appendix C), one can obtain the action $\mathcal{S}$ and the length ℓ for DW and horizon configurations. The string tension can be obtained for two different configurations:

(i) the DW configuration

$$\sigma_{\text{DW}} = M(z_{\text{DW}}) \sqrt{\mathcal{F}(z_{\text{DW}})}, \quad (3.3)$$

(ii) the horizon configuration

$$\sigma_{z_h} = M(z_h) \sqrt{\mathcal{F}(z_h)}, \quad (3.4)$$

where σ_{DW} and σ_{z_h} are the spatial string tension in the DW and horizon configurations. The derivation of these formulas is presented in Appendix C. We will investigate the temperature dependence of the string tension for two configurations and three Wilson loop (WL) orientations in Sec. IV.

B. Dynamical wall equations

Obtaining general results for spatial Wilson loops and effective potential, we consider a fully anisotropic background:

$$\begin{aligned} ds^2 &= G_{\mu\nu} dx^\mu dx^\nu \\ &= \frac{L^2 \mathfrak{b}_s(z)}{z^2} \left[-g(z) dt^2 + \mathfrak{g}_1 dx^2 + \mathfrak{g}_2 dy_1^2 + \mathfrak{g}_3 dy_2^2 + \frac{dz^2}{g(z)} \right], \end{aligned} \quad (3.5)$$

and use it to calculate SWLs for different orientations. Here, $\mathfrak{b}_s(z) = \mathfrak{b}(z) e^{\sqrt{3}\phi(z)}$ is the warp factor in the string frame, and $\mathfrak{g}_1, \mathfrak{g}_2, \mathfrak{g}_3$ are anisotropic functions depending on z .

To get the DW equations, let us take the special cases of Eqs. (B12) and (B13), considering the particular orientations of the SWLs:

(i) The first orientation (xY_1 -plane): $\phi = 0, \theta = 0, \psi = 0; a_{11} = a_{22} = a_{33} = 1, a_{12} = a_{21} = a_{31} = a_{32} = a_{23} = 0$:

$$\mathcal{S}_{xY_1} = \frac{1}{2\pi\alpha'} \int_{\mathcal{P}} \left(\frac{L^2 \mathfrak{b}_s}{z^2} \right) \sqrt{\left(\mathfrak{g}_1 \mathfrak{g}_2 + \frac{z'^2}{g} \mathfrak{g}_2 \right)} d\xi^1 d\xi^2, \quad (3.6)$$

$$\mathcal{V}_1 \equiv \mathcal{V}_{xY_1}(z(\xi)) = \frac{1}{2\pi\alpha'} \left(\frac{L^2 \mathfrak{b}_s}{z^2} \right) \sqrt{\mathfrak{g}_1 \mathfrak{g}_2}. \quad (3.7)$$

(ii) The first orientation (Xy_1 -plane)¹: $\phi = \pi/2, \theta = 0, \psi = 0; a_{21} = a_{33} = -a_{12} = 1, a_{11} = a_{13} = a_{22} = a_{23} = a_{31} = a_{32} = 0$:

$$\mathcal{S}_{Xy_1} = \frac{1}{2\pi\alpha'} \int_{\mathcal{P}} \left(\frac{L^2 \mathfrak{b}_s}{z^2} \right) \sqrt{\left(\mathfrak{g}_1 \mathfrak{g}_2 + \frac{z'^2}{g} \mathfrak{g}_1 \right)} d\xi^1 d\xi^2, \quad (3.8)$$

¹The Nambu-Goto action for Xy_1 -plane coincides with Nambu-Goto action for xY_1 -plane.

$$\mathcal{V}_1 \equiv \mathcal{V}_{xY_1}(z(\xi)) = \frac{1}{2\pi\alpha'} \left(\frac{L^2 \mathfrak{b}_s}{z^2} \right) \sqrt{\mathfrak{g}_1 \mathfrak{g}_2}. \quad (3.9)$$

(iii) The second orientation (xY_2 -plane): $\phi = 0, \theta = \pi/2, \psi = 0; a_{11} = -a_{23} = a_{32} = 1, a_{12} = a_{13} = a_{21} = a_{22} = a_{31} = a_{33} = 0$:

$$\mathcal{S}_{xY_2} = \frac{1}{2\pi\alpha'} \int_{\mathcal{P}} \left(\frac{L^2 \mathfrak{b}_s}{z^2} \right) \sqrt{\left(\mathfrak{g}_1 \mathfrak{g}_3 + \frac{z'^2}{g} \mathfrak{g}_3 \right)} d\xi^1 d\xi^2, \quad (3.10)$$

$$\mathcal{V}_2 \equiv \mathcal{V}_{xY_2}(z(\xi)) = \frac{1}{2\pi\alpha'} \left(\frac{L^2 \mathfrak{b}_s}{z^2} \right) \sqrt{\mathfrak{g}_1 \mathfrak{g}_3}. \quad (3.11)$$

(iv) The third orientation (y_1Y_2 -plane): $\phi = \pi/2, \theta = \pi/2, \psi = -\pi/2; a_{22} = a_{31} = -a_{13} = 1, a_{11} = a_{12} = a_{21} = a_{23} = a_{32} = a_{33} = 0$:

$$\mathcal{S}_{y_1Y_2} = \frac{1}{2\pi\alpha'} \int_{\mathcal{P}} \left(\frac{L^2 \mathfrak{b}_s}{z^2} \right) \sqrt{\left(\mathfrak{g}_2 \mathfrak{g}_3 + \frac{z'^2}{g} \mathfrak{g}_2 \right)} d\xi^1 d\xi^2, \quad (3.12)$$

$$\mathcal{V}_3 \equiv \mathcal{V}_{y_1Y_2}(z(\xi)) = \frac{1}{2\pi\alpha'} \left(\frac{L^2 \mathfrak{b}_s}{z^2} \right) \sqrt{\mathfrak{g}_2 \mathfrak{g}_3}. \quad (3.13)$$

Note that the capital letters Y_1, X , and Y_2 in the WL orientations indicate the direction along which the loop contour has an infinite length.

The general form of the DW equation, if DW exists, is given by $\mathcal{V}'(z) = 0$ [5,84]. DW coordinates z_{DW} for SWLs in particular orientations are given by

$$\mathcal{DW}_{xY_1} = \mathcal{DW}_{Xy_1} \equiv \frac{2\mathfrak{b}'_s(z)}{\mathfrak{b}_s(z)} + \frac{\mathfrak{g}'_1(z)}{\mathfrak{g}_1(z)} + \frac{\mathfrak{g}'_2(z)}{\mathfrak{g}_2(z)} - \frac{4}{z} \Big|_{z=z_{\text{DW}}} = 0, \quad (3.14)$$

$$\mathcal{DW}_{xY_2} \equiv \frac{2\mathfrak{b}'_s(z)}{\mathfrak{b}_s(z)} + \frac{\mathfrak{g}'_1(z)}{\mathfrak{g}_1(z)} + \frac{\mathfrak{g}'_3(z)}{\mathfrak{g}_3(z)} - \frac{4}{z} \Big|_{z=z_{\text{DW}}} = 0, \quad (3.15)$$

$$\mathcal{DW}_{y_1Y_2} \equiv \frac{2\mathfrak{b}'_s(z)}{\mathfrak{b}_s(z)} + \frac{\mathfrak{g}'_2(z)}{\mathfrak{g}_2(z)} + \frac{\mathfrak{g}'_3(z)}{\mathfrak{g}_3(z)} - \frac{4}{z} \Big|_{z=z_{\text{DW}}} = 0. \quad (3.16)$$

From the metric (2.2), it follows that the anisotropic functions are given by

$$\mathfrak{g}_1(z) = 1, \quad \mathfrak{g}_2(z) = (z/L)^{2-2/\nu}, \quad \mathfrak{g}_3(z) = (z/L)^{2-2/\nu} e^{c_B z^2}. \quad (3.17)$$

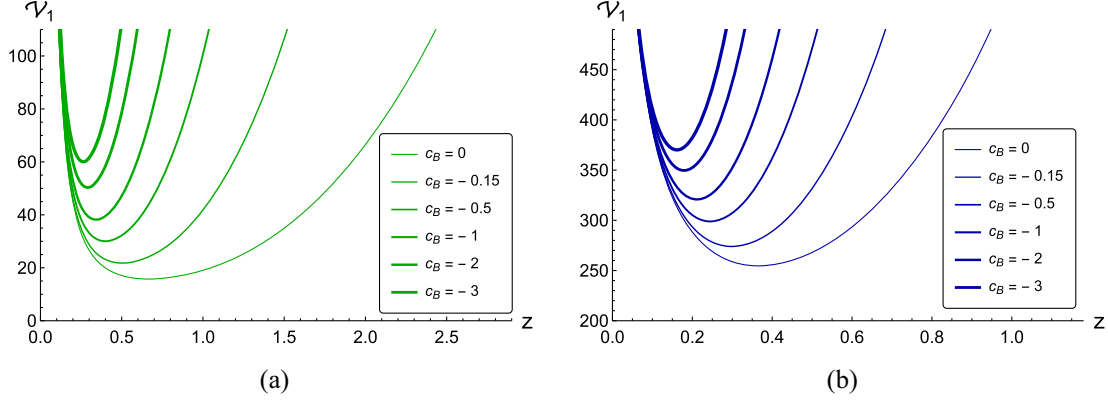


FIG. 2. Effective potential $\mathcal{V}_1(z)$ in the first orientation $\mathcal{W}_{xY_1}$ at (a) $\nu = 1$, and (b) $\nu = 4.5$ for different c_B considering the zero-boundary condition (2.10); $[z]^{-1} = [c_B]^{\frac{1}{2}} = \text{GeV}$.

In this case, spatial string tensions for three particular orientations, i.e., xY_1 , xY_2 , and y_1Y_2 , are given by

$$\begin{aligned}\sigma_1 \equiv \sigma_{xY_1} &= \frac{1}{2\pi\alpha'} \left(\frac{L^2 \mathfrak{b}_s(z)}{z^2} \right) \sqrt{\mathfrak{g}_1 \mathfrak{g}_2} \\ &= \frac{1}{2\pi\alpha'} \left(\frac{L^{1+1/\nu} \mathfrak{b}_s(z)}{z^{1+1/\nu}} \right),\end{aligned}\quad (3.18)$$

$$\begin{aligned}\sigma_2 \equiv \sigma_{xY_2} &= \frac{1}{2\pi\alpha'} \left(\frac{L^2 \mathfrak{b}_s(z)}{z^2} \right) \sqrt{\mathfrak{g}_1 \mathfrak{g}_3} \\ &= \frac{1}{2\pi\alpha'} \left(\frac{L^{1+1/\nu} \mathfrak{b}_s(z)}{z^{1+1/\nu}} \right) e^{c_B z^2/2},\end{aligned}\quad (3.19)$$

$$\begin{aligned}\sigma_3 \equiv \sigma_{y_1Y_2} &= \frac{1}{2\pi\alpha'} \left(\frac{L^2 \mathfrak{b}_s(z)}{z^2} \right) \sqrt{\mathfrak{g}_2 \mathfrak{g}_3} \\ &= \frac{1}{2\pi\alpha'} \left(\frac{L^{2/\nu} \mathfrak{b}_s(z)}{z^{2/\nu}} \right) e^{c_B z^2/2},\end{aligned}\quad (3.20)$$

where $z = z_h$ or $z = z_{\text{DW}}$.

IV. NUMERICAL RESULTS

We study the position of the DW for effective potentials to investigate the spatial string tension and phase transition of string at different configurations. The effective potentials depend on the orientation. For particular orientations xY_1 , xY_2 , and y_1Y_2 , the effective potentials denoted by $\mathcal{V}_1$, $\mathcal{V}_2$, and $\mathcal{V}_3$ are given by Eqs. (3.18)–(3.20), respectively. Although the effective potential depends on the parameters c , z_0 , z_h , and c_B , it does not depend on the chemical potential μ .

A. Spatial Wilson loop $\mathcal{W}_{xY_1}$

1. Zero-boundary condition

In this section, the behavior of the effective potential $\mathcal{V}_1$ as a function of the holographic coordinate z corresponding

to the first orientation of SWL, i.e., $\mathcal{W}_{xY_1}$, for different values of magnetic coefficients c_B is presented in Fig. 2 using the zero-boundary condition (2.10). Figure 2 shows that $\mathcal{V}_1$ has local minimums, thus confirming $\mathcal{V}'(z) = 0$, i.e., the dynamical walls exist in the isotropic case $\nu = 1$ and in the anisotropic case $\nu = 4.5$ with different values of the magnetic coefficient c_B .

The locations of the dynamical walls for $\nu = 1$ and $\nu = 4.5$ with different values of c_B and are given in Tables I and II, respectively.

The SWL string tension can be obtained by calculating the minimal value of the effective potential at the DW $z = z_{\text{DW}}$ or at the horizon $z = z_h$. Figures 3 and 4 show the spatial string tension σ_1 as a function of temperature T for the first orientation of SWLs, i.e., $\mathcal{W}_{xY_1}$, at $\mu = 0$, $\mu = 0.5$, and $\mu = 1$ GeV for different c_B in the isotropic case $\nu = 1$, and the anisotropic case $\nu = 4.5$, respectively. Both figures were obtained considering the zero-boundary condition (2.10), and the cyan lines present the dependence of $\sigma(z_{\text{DW}})$ on temperature in the DW configuration, while the green

TABLE I. Locations of the dynamical walls z_{DW} for $\mathcal{V}_1$ in the first orientation $\mathcal{W}_{xY_1}$ at $\nu = 1$, considering the zero-boundary condition (2.10); $[z]^{-1} = [c_B]^{\frac{1}{2}} = \text{GeV}$.

$\mathcal{V}_1$	$\nu = 1$					
$-c_B$	0	0.15	0.5	1	2	3
z_{DW}	0.665	0.500	0.400	0.341	0.285	0.261

TABLE II. Locations of the dynamical walls z_{DW} for $\mathcal{V}_1$ in the first orientation $\mathcal{W}_{xY_1}$ at $\nu = 4.5$, considering the zero-boundary condition (2.10); $[z]^{-1} = [c_B]^{\frac{1}{2}} = \text{GeV}$.

$\mathcal{V}_1$	$\nu = 4.5$					
$-c_B$	0	0.15	0.5	1	2	3
z_{DW}	0.366	0.300	0.245	0.212	0.180	0.162

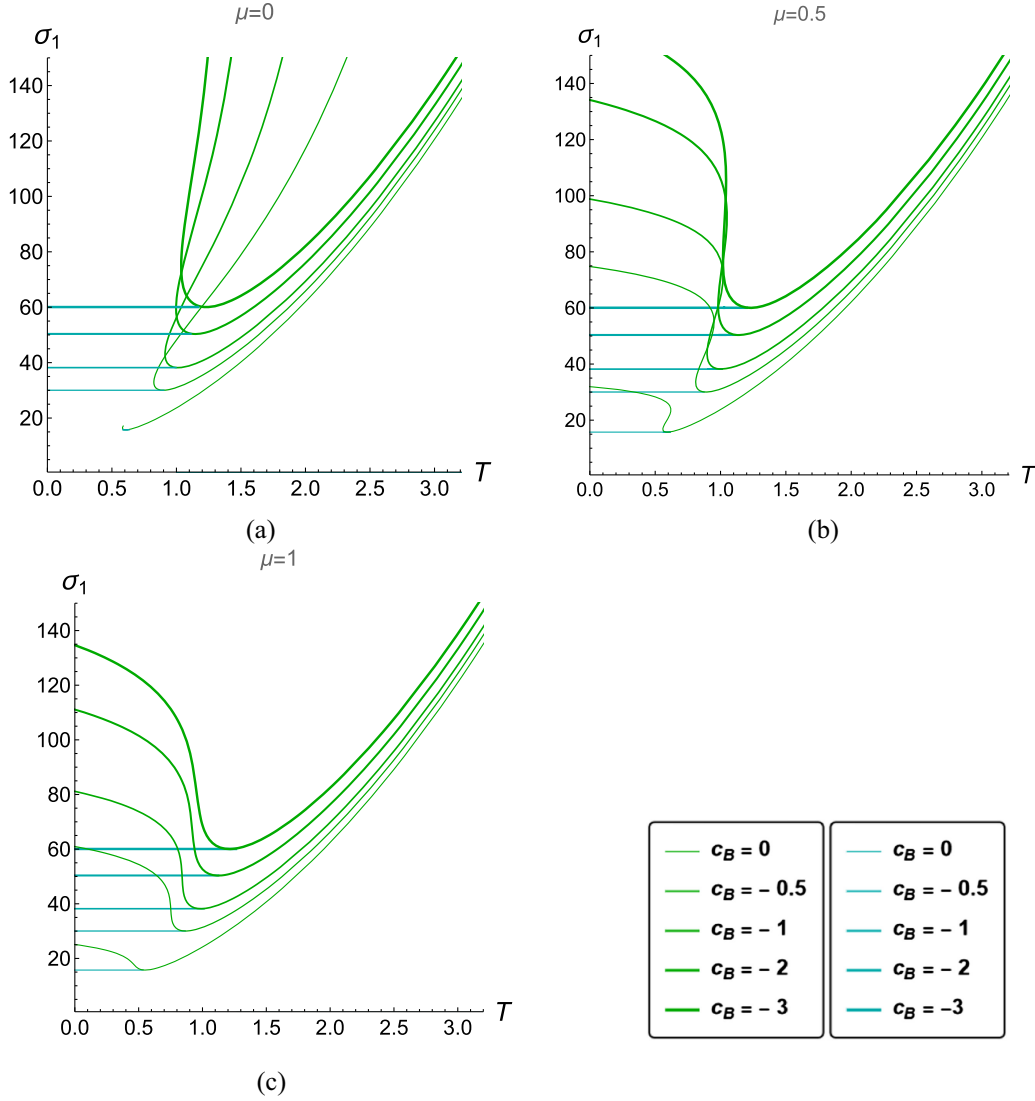


FIG. 3. Spatial string tension σ_1 in the first orientation $\mathcal{W}_{xY_1}$ as a function of temperature T for isotropic case $\nu = 1$ at (a) $\mu = 0$ GeV, (b) $\mu = 0.5$ GeV, and (c) $\mu = 1$ GeV with different c_B considering the zero-boundary condition (2.10). The cyan line and green curve show the DW and horizon configuration, respectively; $[\sigma]^\frac{1}{2} = [T] = [\mu] = [c_B]^\frac{1}{2} = \text{GeV}$.

and blue lines present the dependence of $\sigma(z_h)$ on temperature in the horizon configuration.

The phase transition occurs at critical temperature T_{cr} between two connected string configurations, i.e., DW and horizon configurations (see Fig. 25), with different values of spatial string tension: $\sigma(z_{DW})$ and $\sigma(z_h)$, respectively (see also Sec. III A). Note that this is a continuous phase transition. For this phase transition, the string tension receives a large change in its value at the critical temperature T_{cr} . To put it in another way, the second derivative of the spatial string tension with respect to temperature $\partial^2 \sigma / \partial T^2$ undergoes a jump. To compare this phase transition obtained from SWLs with confinement/deconfinement and first-order phase transitions, see Sec. VII and Fig. 24.

Figure 4 shows, that the spatial string tension σ value increases with the spatial anisotropy ν and the external magnetic field c_B . Here and below, “larger magnetic field” means “larger absolute value of c_B .” In addition, Figs. 3 and 4 show that at lower temperature $T < T_{cr}$, i.e., in the DW configuration, the spatial string tension does not depend on the temperature and gets a constant value such as $\sigma_1 = 60 \text{ GeV}^2$ for $\mu = 0$, $c_B = -3 \text{ GeV}^2$. Note also that the DW coordinates are independent from the chemical potential. However, for $\mu = 0$, $c_B = 0$, Figs. 3(a) and 4(a) show that for very low T , the σ_1 does not exist due to lack of the DW coordinate.

Figure 5 shows, that at fixed chemical potential, the increasing magnetic coefficient c_B increases the transition temperature between the DW and the horizon

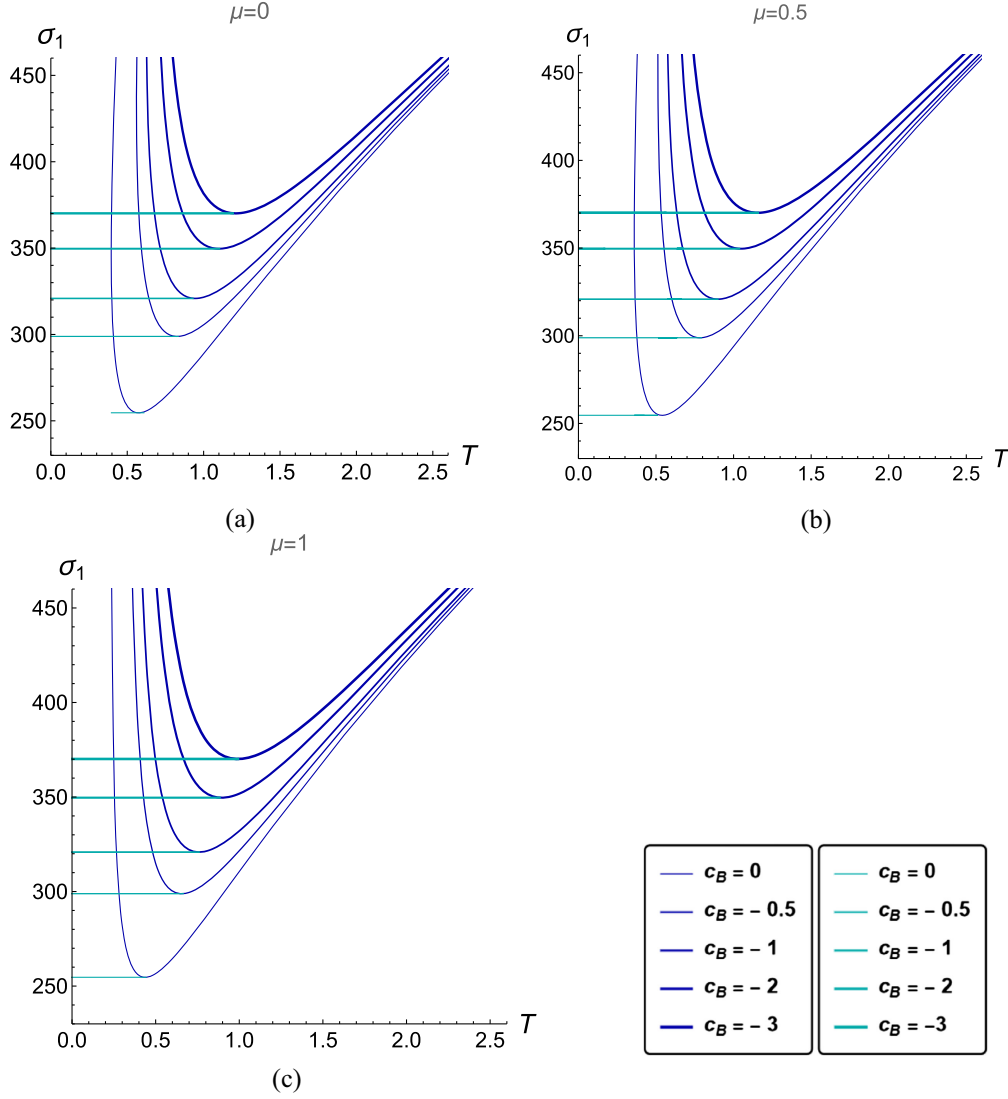


FIG. 4. Spatial string tension σ_1 in the first orientation $\mathcal{W}_{xY_1}$ as a function of temperature T for anisotropic case $\nu = 4.5$ at (a) $\mu = 0$ GeV, (b) $\mu = 0.5$ GeV, and (c) $\mu = 1$ GeV with different c_B considering the zero-boundary condition (2.10). The cyan line and blue curve show the DW and horizon configuration, respectively; $[\sigma]^\frac{1}{2} = [T] = [\mu] = [c_B]^\frac{1}{2} = \text{GeV}$.

configuration, thus confirming the magnetic catalysis phenomenon. Furthermore, inclusion of spatial anisotropy $\nu = 4.5$ decreases the critical transition temperature T_{cr} between the DW and the horizon configuration for all values of the magnetic field coefficient c_B in our model.

2. Physical-boundary condition

In this section, we apply the physical-boundary condition (2.12) to the dilaton field to calculate the DW coordinate and the spatial string tension. We would like to compare the results of this subsection and those of Sec. IV A 1.

The behavior of the effective potential $\mathcal{V}_1$ as a function of holographic coordinate z for $\nu = 1$ and $\nu = 4.5$ corresponding to the first SWL orientation $\mathcal{W}_{xY_1}$ for different values of the magnetic field coefficients c_B , considering the physical-boundary condition (2.12), is presented in Fig. 6.

This figure shows that there is a minimum effective potential not only for the zero-boundary condition (2.10), but also for physical-boundary conditions (2.12).

The locations of the dynamical walls for different values of the magnetic coefficient c_B in the isotropic case $\nu = 1$ and the anisotropic case $\nu = 4.5$ for the physical-boundary condition are given in Tables III and IV, respectively.

Figures 7 and 8 show spatial string tension σ_1 as a function of temperature T at $\mu = 0$, $\mu = 0.5$, and $\mu = 1$ GeV with different c_B for the isotropic case $\nu = 1$ and anisotropic case $\nu = 4.5$, respectively. In this case, we considered the physical-boundary condition (2.10). The cyan line shows the dependence of σ in the DW configuration $\sigma(z_{DW})$ on temperature, and the green curve shows the dependence of $\sigma(z_h)$ in the horizon configuration on temperature.

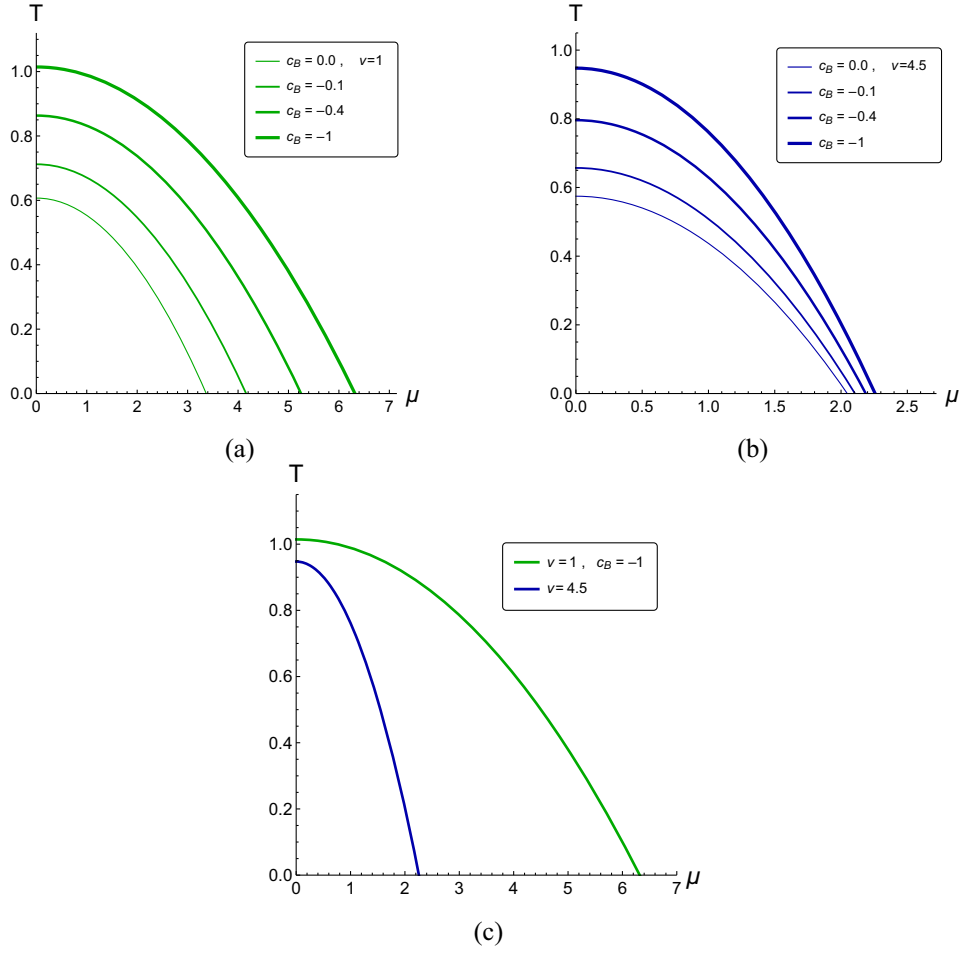


FIG. 5. Phase transition diagrams in (μ, T) -plane in the first orientation $\mathcal{W}_{xY_1}$ at (a) $\nu = 1$, (b) $\nu = 4.5$, and (c) comparison between $\nu = 1$ and $\nu = 4.5$ at fixed $c_B = -1$ GeV² corresponding to the transition from the DW configuration to the horizon configuration of σ_1 considering the zero-boundary condition (2.10); $[T] = [\mu] = [c_B]^{\frac{1}{2}} = \text{GeV}$.

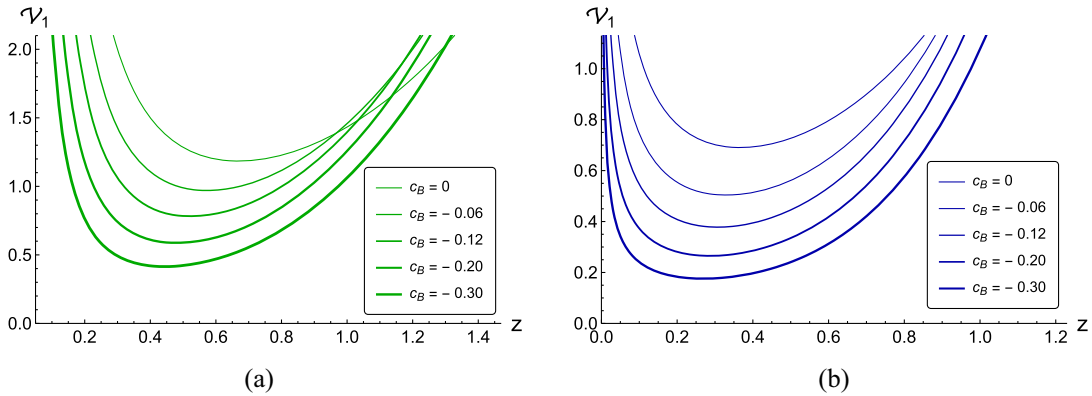


FIG. 6. Effective potential $\mathcal{V}_1(z)$ in the first orientation $\mathcal{W}_{xY_1}$ at (a) $\nu = 1$, and (b) $\nu = 4.5$ for different c_B considering physical-boundary condition (2.12); $[z]^{-1} = [c_B]^{\frac{1}{2}} = \text{GeV}$.

In contradistinction to the zero-boundary condition (2.10), for the physical-boundary condition, Figs. 7 and 8 show that inclusion of spatial anisotropy $\nu = 4.5$ and increasing the magnetic field c_B decreases the spatial string

tension. However, for both zero- and physical-boundary conditions, the magnetic catalysis behavior was obtained. The DW coordinates are independent from the chemical potential. However, in the case of $\mu = 0$ for very low T in

TABLE III. Locations of dynamical walls z_{DW} for $\mathcal{V}_1$ in the first orientation $\mathcal{W}_{xY_1}$ at $\nu = 1$ with different c_B for the physical-boundary condition (2.12); $[z]^{-1} = [c_B]^{\frac{1}{2}} = \text{GeV}$.

$\mathcal{V}_1$	$\nu = 1$				
$-c_B$	0	0.06	0.12	0.2	0.3
z_{DW}	0.665	0.572	0.521	0.480	0.442

TABLE IV. Locations of dynamical walls z_{DW} for $\mathcal{V}_1$ in the first orientation $\mathcal{W}_{xY_1}$ at $\nu = 4.5$ with different c_B for the physical-boundary condition (2.12); $[z]^{-1} = [c_B]^{\frac{1}{2}} = \text{GeV}$.

$\mathcal{V}_1$	$\nu = 4.5$				
$-c_B$	0	0.06	0.12	0.2	0.3
z_{DW}	0.366	0.330	0.309	0.288	0.270

all values of c_B , Figs. 7(a) and 9(a) show that σ_1 does not exist due to lack of the DW coordinate. In the horizon configuration $T > T_{cr}$, for both the isotropic case $\nu = 1$ and the anisotropic case $\nu = 4.5$, the spatial string tension increases monotonically with temperature for the zero- and physical-boundary conditions.

Comparison of Figs. 5 and 9 clearly shows that the phase diagram does not depend on the boundary conditions. This result is confirmed by other models; see Ref. [77].

B. Spatial Wilson loop $\mathcal{W}_{xY_2}$

The behavior of the effective potential $\mathcal{V}_2$ as a function of the holographic coordinate z corresponding to the second orientation of the SWL, i.e., $\mathcal{W}_{xY_2}$ for different values of magnetic coefficients c_B is presented in Fig. 10 using the

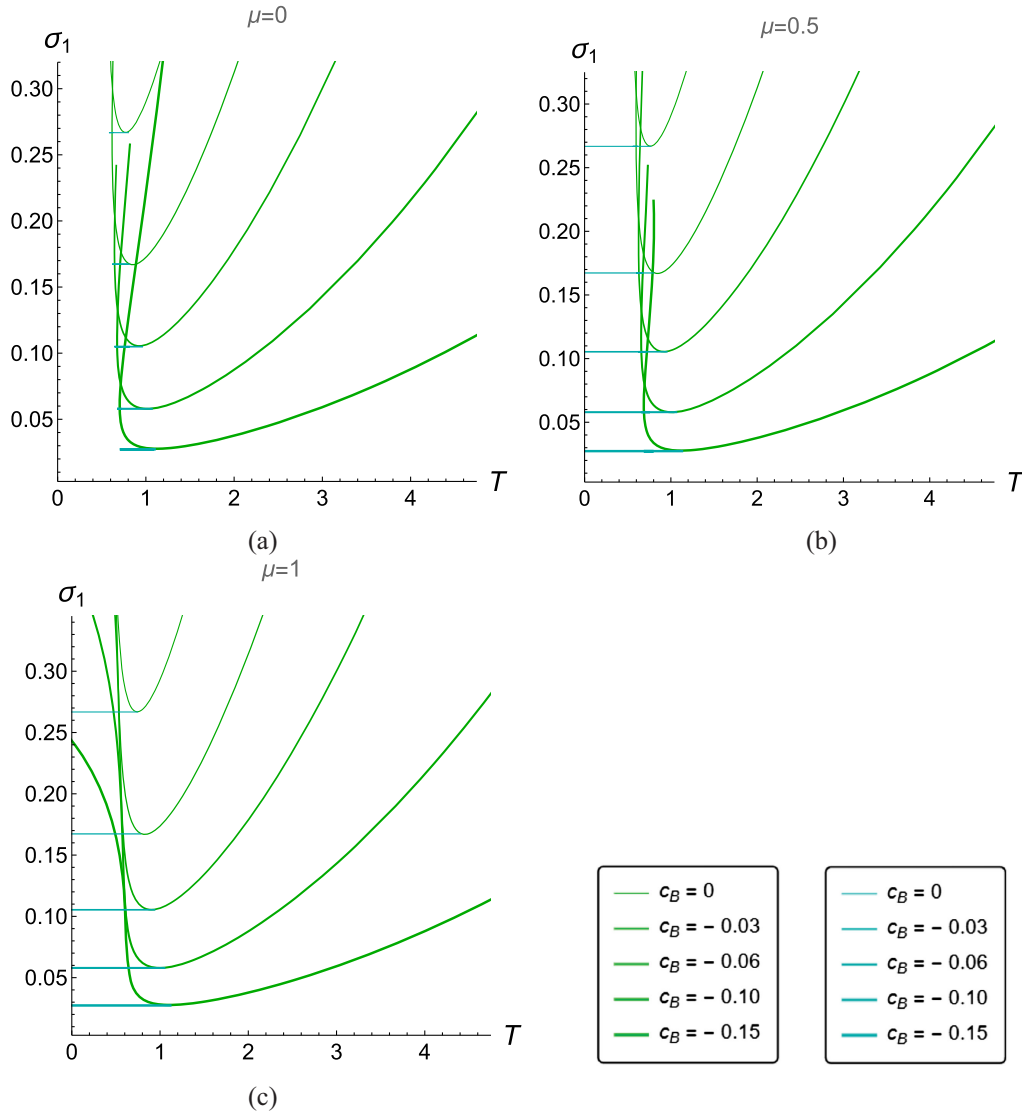


FIG. 7. Spatial string tension σ_1 in the first orientation $\mathcal{W}_{xY_1}$ as a function of temperature T for isotropic case $\nu = 1$ at (a) $\mu = 0$ GeV, (b) $\mu = 0.5$ GeV, and (c) $\mu = 1$ GeV with different c_B considering the physical-boundary condition (2.10). The cyan line and green curve show the DW and horizon configuration, respectively; $[\sigma]^{\frac{1}{2}} = [T] = [\mu] = [c_B]^{\frac{1}{2}} = \text{GeV}$.

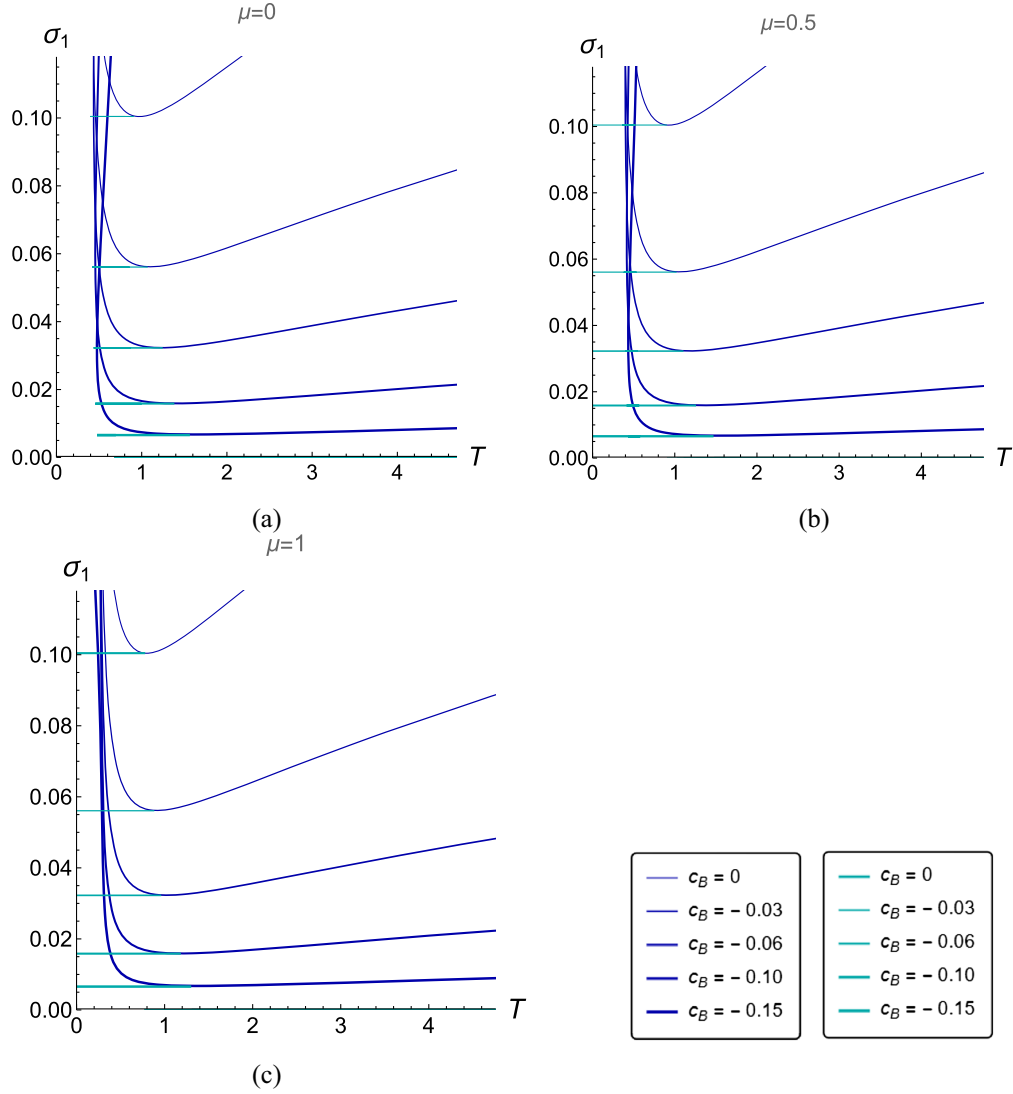


FIG. 8. Spatial string tension σ_1 in the first orientation $\mathcal{W}_{xY_1}$ as a function of temperature T for anisotropic case $\nu = 4.5$ at (a) $\mu = 0$ GeV, (b) $\mu = 0.5$ GeV, and (c) $\mu = 1$ GeV with different c_B considering the physical-boundary condition (2.10). The cyan line and blue curve show the DW and horizon configuration, respectively; $[\sigma]^\frac{1}{2} = [T] = [\mu] = [c_B]^\frac{1}{2} = \text{GeV}$.

zero-boundary condition (2.10). Figure 10 shows that $\mathcal{V}_2$ has local minimums, thus confirming $\mathcal{V}'(z) = 0$, i.e., the dynamical walls exist at different values of the magnetic coefficient c_B for the isotropic and anisotropic cases $\nu = 1$ and $\nu = 4.5$, respectively.

Tables V and VI show the DW locations at different values of the magnetic field c_B for $\nu = 1$ and $\nu = 4.5$, respectively.

Figures 11 and 12 depict the spatial string tension σ_2 as a function of temperature T in $\mu = 0$, $\mu = 0.5$, and $\mu = 1$ GeV with different c_B for the isotropic case $\nu = 1$ and anisotropic case $\nu = 4.5$, respectively. The cyan lines present the dependence of $\sigma(z_{\text{DW}})$ on temperature in the DW configuration, while the green and blue lines present the dependence of $\sigma(z_h)$ on temperature in the horizon configuration.

Figures 13(a) and 13(b) show the phase diagrams in the (μ, T) -plane for σ_2 in the second orientation $\mathcal{W}_{xY_2}$ corresponding to the transition from the DW configuration to the horizon configuration at the zero-boundary condition (2.10) for $\nu = 1$ and $\nu = 4.5$, respectively. Figure 13(c) shows that inclusion of spatial anisotropy decreases T_{cr} .

The results of the spatial string tension as a function of T in the second SWL orientation $\mathcal{W}_{xY_2}$ shown in Figs. 11 and 12 are the following:

- (i) It exhibits magnetic catalytic behavior, meaning that T_{cr} increases with increasing magnetic field c_B .
- (ii) The spatial string tension σ increases as the spatial anisotropy ν , and the external magnetic field c_B increases.
- (iii) At lower temperature $T < T_{cr}$ in the DW configuration, the string tension does not depend on the

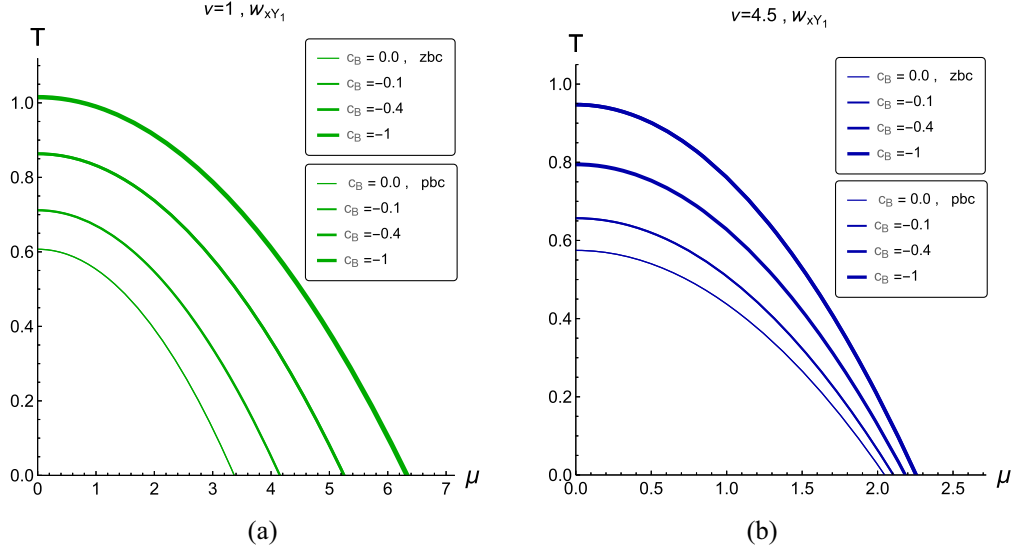


FIG. 9. Phase transition diagrams in the (μ, T) -plane in the first orientation $\mathcal{W}_{xy1}$ corresponding to the transition from the DW configuration to the horizon configuration of σ_1 at (a) $\nu = 1$ and (b) $\nu = 4.5$ with different c_B comparing the zero-boundary condition (zbc) (2.10) and physical-boundary condition (pbc) (2.12); $[T] = [\mu] = [c_B]^{\frac{1}{2}} = \text{GeV}$.

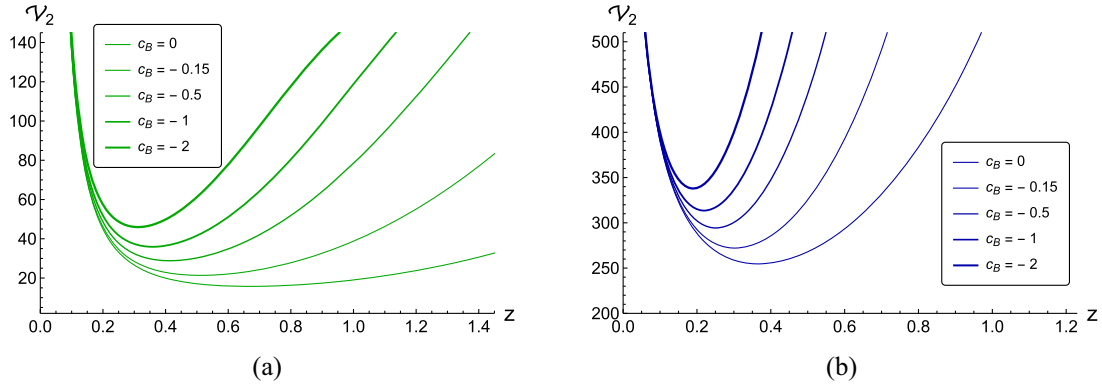


FIG. 10. Effective potential $\mathcal{V}_2(z)$ in the second orientation $\mathcal{W}_{xy2}$ at (a) $\nu = 1$, and (b) $\nu = 4.5$ for different c_B considering the zero-boundary condition (2.10); $[z]^{-1} = [c_B]^{\frac{1}{2}} = \text{GeV}$.

temperature and gets a constant value such as $\sigma_2 = 46 \text{ GeV}^2$ at $\mu = 0$ and $c_B = -2 \text{ GeV}^2$.

- (iv) At higher temperature $T > T_{cr}$ in the horizon configuration, the string tension increases monotonically.

- (v) The DW coordinates are independent of the chemical potential.

Note that comparing the phase diagrams between the first SWL orientation $\mathcal{W}_{xy1}$ and the second orientation $\mathcal{W}_{xy2}$ is investigated in Sec. IV D.

TABLE V. Locations of dynamical walls z_{DW} for $\mathcal{V}_2$ in the second orientation $\mathcal{W}_{xy2}$ at $\nu = 1$ considering the zero-boundary condition (2.10); $[z]^{-1} = [c_B]^{\frac{1}{2}} = \text{GeV}$.

$\mathcal{V}_2$	$\nu = 1$				
$-c_B$	0	0.15	0.5	1	2
z_{DW}	0.665	0.515	0.415	0.360	0.315

TABLE VI. Locations of dynamical walls z_{DW} for $\mathcal{V}_2$ in the second orientation $\mathcal{W}_{xy2}$ at $\nu = 4.5$ considering the zero-boundary condition (2.10); $[z]^{-1} = [c_B]^{\frac{1}{2}} = \text{GeV}$.

$\mathcal{V}_2$	$\nu = 4.5$				
$-c_B$	0	0.15	0.5	1	2
z_{DW}	0.366	0.305	0.250	0.220	0.190

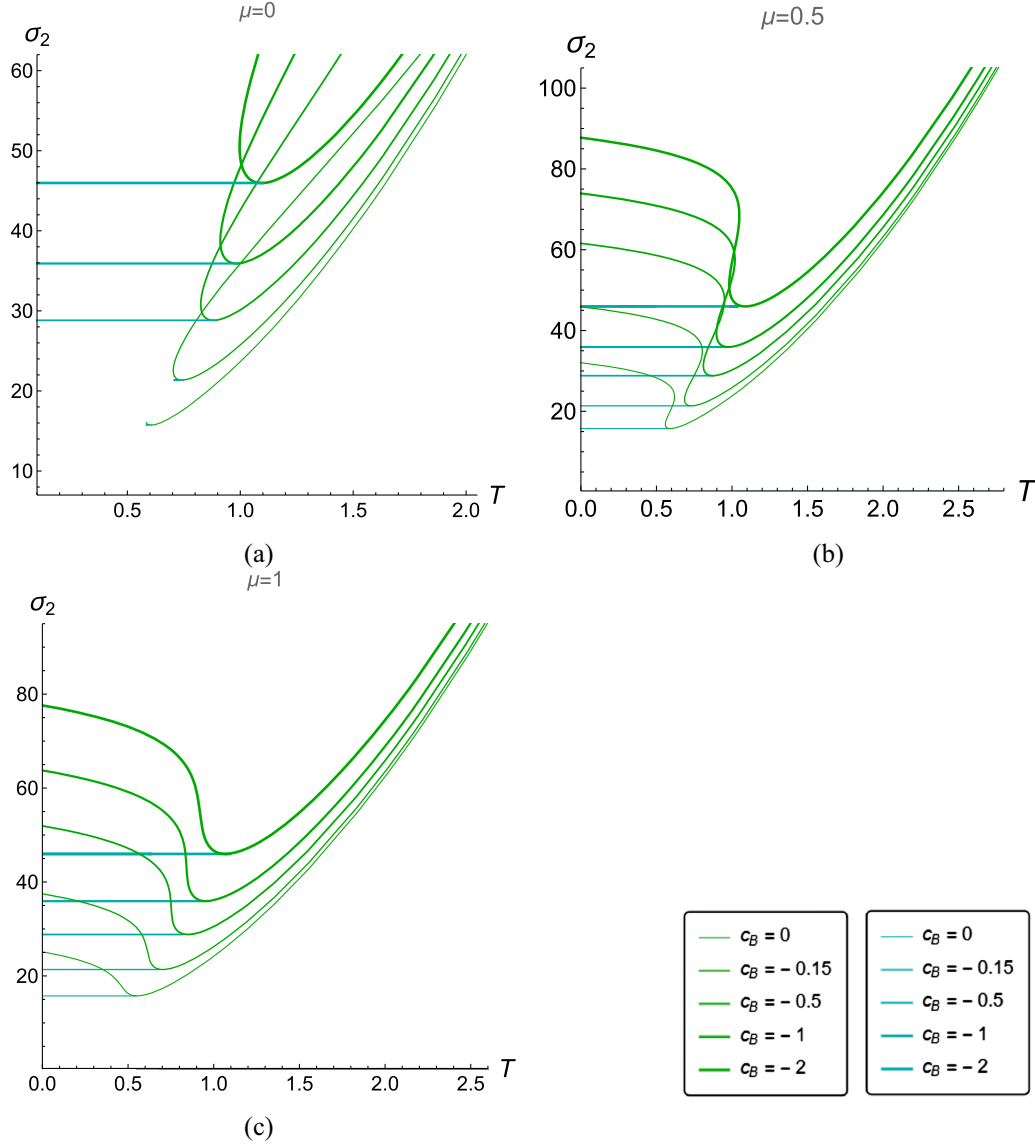


FIG. 11. Spatial string tension σ_2 in the second orientation $\mathcal{W}_{xY_2}$ as a function of temperature T for anisotropic case $\nu = 1$ at (a) $\mu = 0$ GeV, (b) $\mu = 0.5$ GeV, and (c) $\mu = 1$ GeV with different c_B considering the zero-boundary condition (2.10). The cyan line and blue curve show the DW and horizon configuration, respectively; $[\sigma]^\frac{1}{2} = [T] = [\mu] = [c_B]^\frac{1}{2} = \text{GeV}$.

C. Spatial Wilson loop $\mathcal{W}_{y_1Y_2}$

The third SWL orientation, i.e., $\mathcal{W}_{y_1Y_2}$, has very peculiar characteristics compared to the first orientation, $\mathcal{W}_{xY_1}$, and the second orientation, $\mathcal{W}_{xY_2}$.

The effective potential $\mathcal{V}_3(z)$ in the third orientation $\mathcal{W}_{y_1Y_2}$ for $\nu = 1$ and $\nu = 4.5$ in different c_B considering the zero-boundary condition (2.10) is shown in Fig. 14. In $\nu = 1$, Fig. 14(a), the effective potential curves and the DW coordinates are exactly the same as the results of the second SWL orientation $\mathcal{W}_{xY_2}$ as is clear from Eqs. (3.19) and (3.20). However, for $\nu = 1$ there are DW coordinates, but for $\nu = 4.5$, Fig. 14(b), the curves of the effective potential have no minimum and therefore no DW coordinates.

It is important to note that the third SWL orientation $\mathcal{W}_{y_1Y_2}$ in the anisotropic system has a different characteristic compared to other orientations. We found that there is a critical value of anisotropy $\nu_{cr} = 2.5$, where the DW coordinates disappear. To see this feature, Fig. 15(a) shows the effective potential $\mathcal{V}_3(z)$ in the third orientation $\mathcal{W}_{y_1Y_2}$ for fixed value of the magnetic field $c_B = -1 \text{ GeV}^2$ and different ν considering the zero-boundary condition (2.10). In Fig. 15(b), the critical value of anisotropy $\nu_{cr} = 2.5$ is depicted as an enlargement. For $\nu \geq 2.5$, there are no DW coordinates in the system and therefore no spatial string tension.

Due to the existence of ν_{cr} in the third SWL orientation $\mathcal{W}_{y_1Y_2}$, we should consider anisotropy $\nu < 2.5$ to

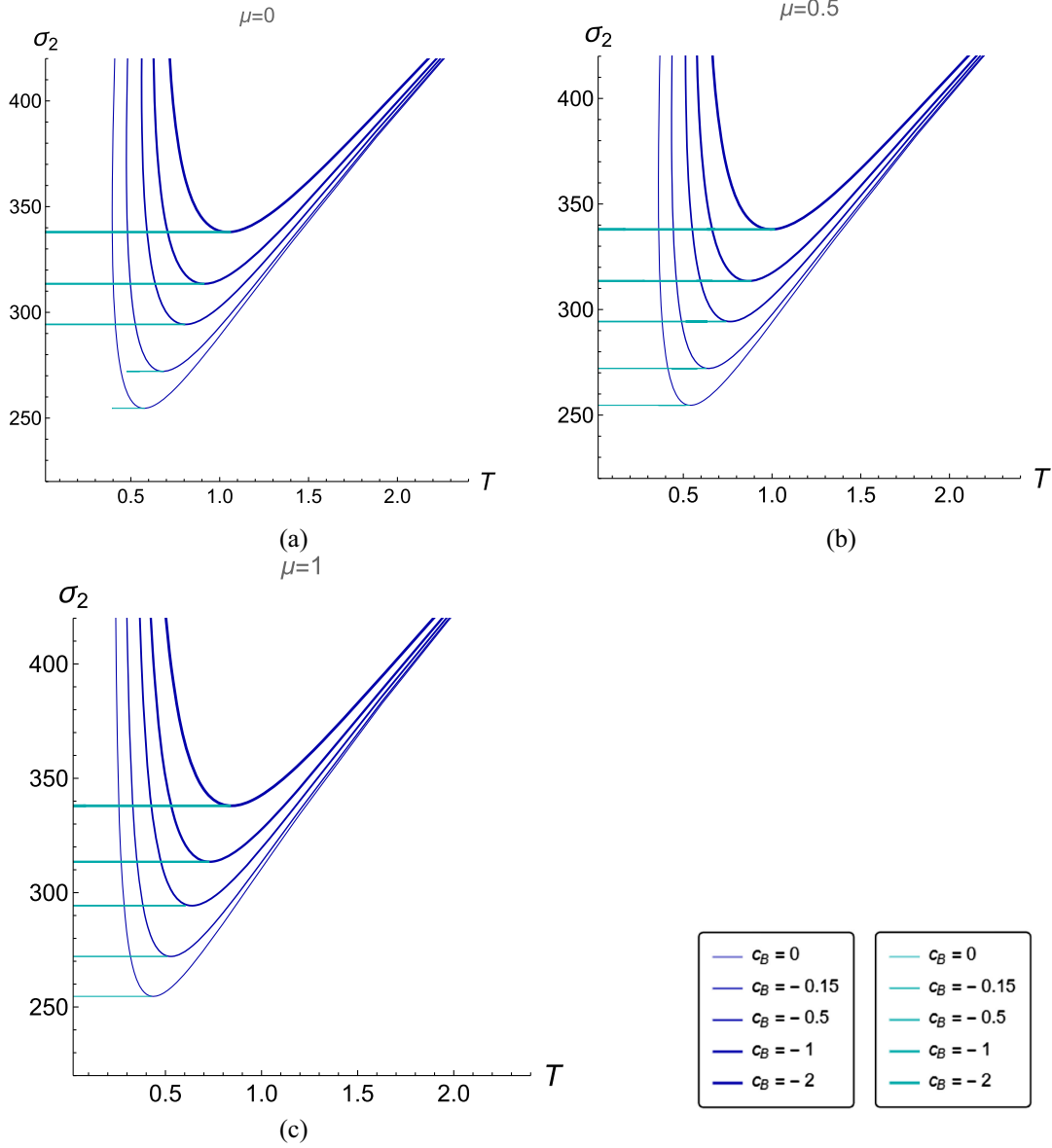


FIG. 12. Spatial string tension σ_2 in the second orientation $\mathcal{W}_{xY_2}$ as a function of temperature T for isotropic case $\nu = 4.5$ at (a) $\mu = 0$ GeV, (b) $\mu = 0.5$ GeV, and (c) $\mu = 1$ GeV with different c_B considering the zero-boundary condition (2.10). The cyan line and green curve show the DW and horizon configuration, respectively; $[\sigma]^\frac{1}{2} = [T] = [\mu] = [c_B]^\frac{1}{2} = \text{GeV}$.

investigate the DW coordinate and spatial string tension. We choose $\nu = 2$ and depict the potential $\mathcal{V}_3(z)$ at different values of magnetic field c_B considering the zero-boundary condition (2.10) in Fig. 16. The minimum of the curves in Fig. 16 determines the DW coordinates presented in Table VII for $\nu = 2$.

Figure 17 shows the spatial string tension σ_3 as a function of temperature T for the anisotropic case $\nu = 2$ in the third SWL orientation $\mathcal{W}_{y_1Y_2}$ at $\mu = 0$, $\mu = 0.5$ and $\mu = 1$ GeV and different magnetic fields c_B considering the zero-boundary condition (2.10). The cyan lines present the dependence of $\sigma(z_{\text{DW}})$ on temperature in the DW configuration, while the orange lines present the dependence of

$\sigma(z_h)$ on temperature in the horizon configuration. The spatial string tension exhibits a constant value in the DW configuration at $T < T_{cr}$ such as $\sigma_3 = 438 \text{ GeV}^2$ at $\mu = 0$ and $c_B = -2 \text{ GeV}^2$. However, in the horizon configuration at $T > T_{cr}$ it increases monotonically.

Figure 18(a) shows the phase diagrams in the (μ, T) -plane at $\nu = 2$, and Fig. 18(b) presents the comparison between $\nu = 1$ and $\nu = 2$ with fixed magnetic field $c_B = -1 \text{ GeV}^2$ in the third SWL orientation $\mathcal{W}_{y_1Y_2}$, corresponding to the transition from the DW configuration to the horizon configuration of σ_3 , considering the zero-boundary condition (2.10). Figure 18(a) confirms the magnetic catalysis behavior at $\nu = 2$, and Fig. 18(b) shows that

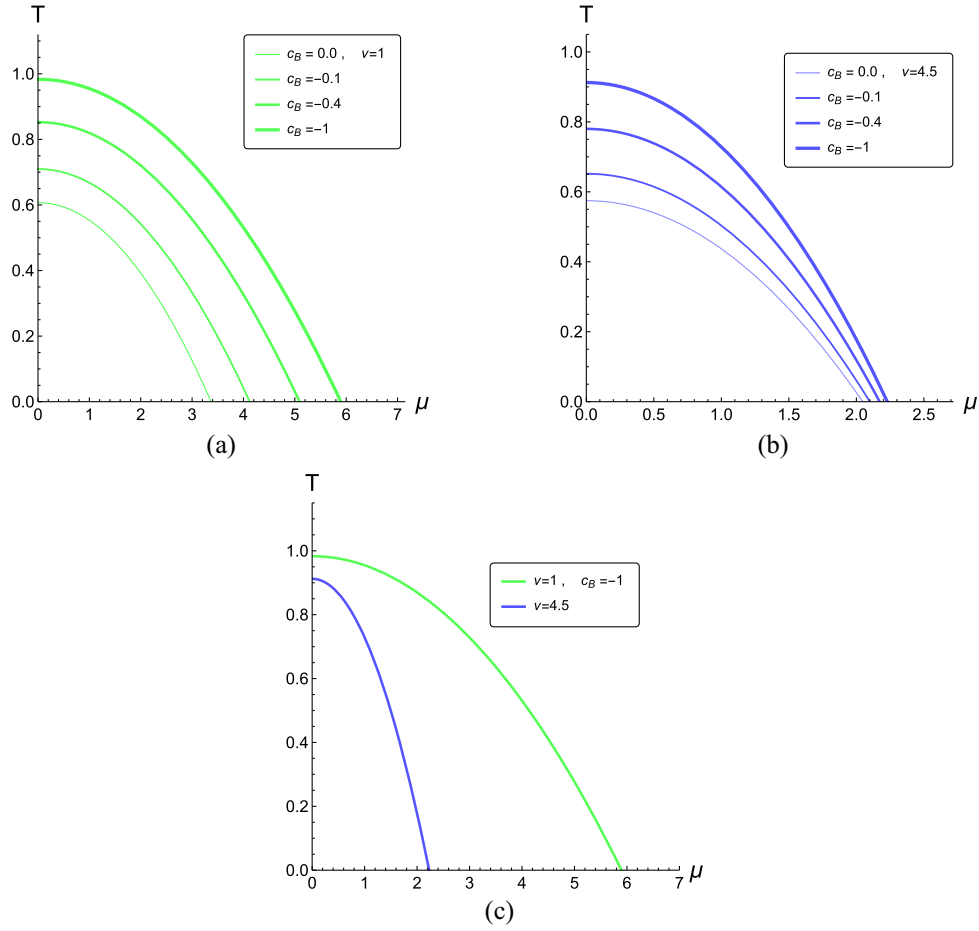


FIG. 13. Phase diagrams in (μ, T) -plane at (a) $\nu = 1$, (b) $\nu = 4.5$, and (c) comparison between $\nu = 1$ and $\nu = 4.5$ at fixed $c_B = -1$ GeV² in the second orientation $\mathcal{W}_{y_1 y_2}$, corresponding to the transition from the DW configuration to the horizon configuration of σ_2 considering the zero-boundary condition (2.10); $[T] = [\mu] = [c_B]^{\frac{1}{2}} = \text{GeV}$.

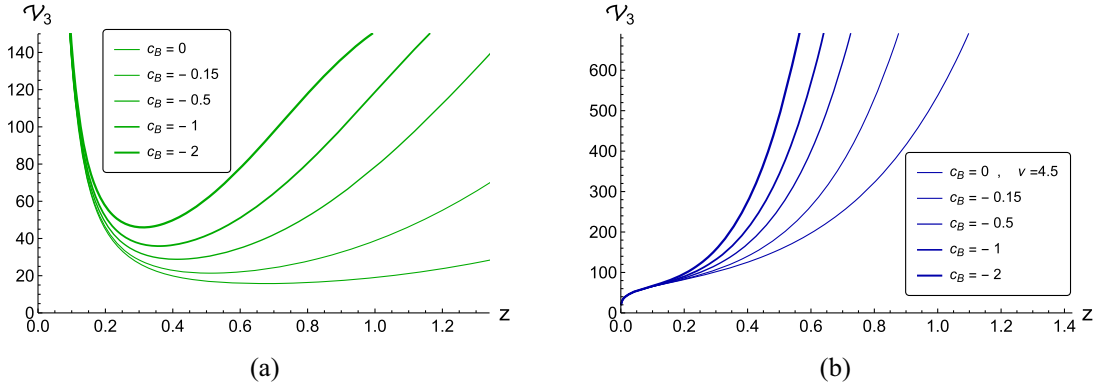


FIG. 14. Effective potential $\mathcal{V}_3(z)$ in the third orientation $\mathcal{W}_{y_1 y_2}$ for different anisotropies (a) $\nu = 1$, and (b) $\nu = 4.5$ at different c_B considering the zero-boundary condition (2.10); $[z]^{-1} = [c_B]^{\frac{1}{2}} = \text{GeV}$.

inclusion of anisotropy increases or decreases the transition temperature depending on μ in this case.

D. Phase diagrams in particular SWL orientations

Different orientations of SWLs mean different embeddings of the string in the bulk. In addition, different

embeddings lead to different physics [1], as presented in Fig. 19. The figure compares the phase diagrams in the (μ, T) -plane in the first $\mathcal{W}_{xY_1}$ and second orientation $\mathcal{W}_{xY_2}$ at (a) $\nu = 1$, and (b) $\nu = 4.5$, respectively. The transition temperature characterizes the phase transition between the DW configuration and the horizon configuration of σ_3

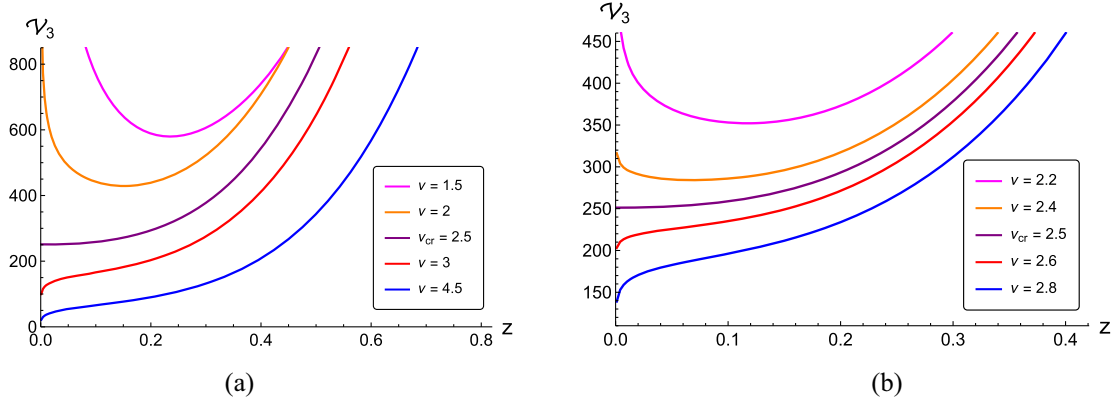


FIG. 15. Effective potential $\mathcal{V}_3(z)$ in the third orientation $\mathcal{W}_{y_1 y_2}$ at fixed value of magnetic field $c_B = -1$ for (a) different anisotropies ν , and (a) enlargement of panel (a) around the critical value of spatial anisotropy $\nu_{cr} = 2.5$ considering the zero-boundary condition (2.10); $[z]^{-1} = [c_B]^{\frac{1}{2}} = \text{GeV}$.

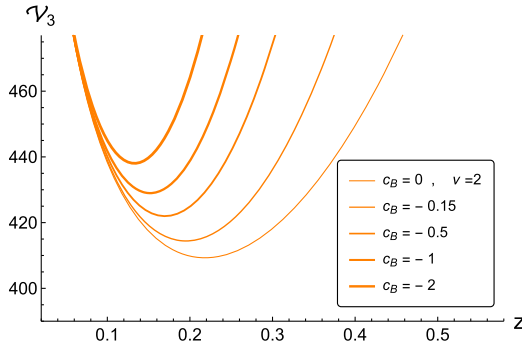


FIG. 16. Effective potential $\mathcal{V}_3(z)$ in the third orientation $\mathcal{W}_{y_1 y_2}$ for $\nu = 2$ at different values of magnetic field c_B considering the zero-boundary condition (2.10); $[z]^{-1} = [c_B]^{\frac{1}{2}} = \text{GeV}$.

considering the zero-boundary condition (2.10). Our results reveal that for the isotropic $\nu = 1$ and anisotropic $\nu = 4.5$ cases, the transition temperature has lower values for the second SWL orientation $\mathcal{W}_{xY_2}$ compared to the first orientation. Note that for $c_B = 0$, the phase transition curves coincide exactly, as the values of σ for both orientations are the same; see Eqs. (3.18) and (3.19).

Figure 20 compares phase transition diagrams in the (μ, T) -plane at $\nu = 2$ for all three orientations, i.e., the first $\mathcal{W}_{xY_1}$, the second $\mathcal{W}_{xY_2}$, and the third orientation $\mathcal{W}_{y_1 y_2}$, corresponding to the transition of σ_3 from the DW configuration to the horizon configuration at the

TABLE VII. Locations of dynamical walls z_{DW} for $\mathcal{V}_3$ in the third orientation $\mathcal{W}_{y_1 y_2}$ at $\nu = 2$ considering the zero-boundary condition (2.10); $[z]^{-1} = [c_B]^{\frac{1}{2}} = \text{GeV}$.

$\mathcal{V}_3$	$\nu = 2$				
$-c_B$	0	0.15	0.5	1	2
z_{DW}	0.218	0.195	0.170	0.152	0.133

zero-boundary condition (2.10). The result of Fig. 19 is confirmed for the first $\mathcal{W}_{xY_1}$ and the second $\mathcal{W}_{xY_2}$ orientations. In addition, Fig. 20 shows that the transition temperature has higher values for the third SWL orientation $\mathcal{W}_{y_1 y_2}$ in comparison to the first and second ones for all values of the magnetic field c_B .

V. COMPARING THE SPATIAL STRING TENSION WITH LATTICE CALCULATIONS

The connection between spatial string tensions and drag forces is remarkable. The drag force specifies the resistance that the heavy quark undergoes while moving through the hot and dense QGP. The drag force on the stretched string can describe the energy loss of the heavy quark [66,67]. It was shown that in the strong coupling regime, the drag force is proportional to the spatial string tension $\sigma \propto T^2$, where T denotes temperature [69,70]. In addition, the temperature dependence of the spatial string tension is studied in more detail in Ref. [85].

Note that for the general metric (3.5) considering $\mathbf{g}_1 = 1$, the drag forces on the heavy quarks in the x , y_1 , and y_2 directions have already been calculated in the anisotropic model [6] as

$$p_x = v_x \frac{\mathfrak{b}_s(z_h)}{z_h^2}, \quad (5.1)$$

$$p_{y_1} = v_{y_1} \frac{\mathfrak{b}_s(z_h)}{z_h^2} \mathfrak{g}_2(z_h), \quad (5.2)$$

$$p_{y_2} = v_{y_2} \frac{\mathfrak{b}_s(z_h)}{z_h^2} \mathfrak{g}_3(z_h). \quad (5.3)$$

If we take into account $L^2/\alpha' = \sqrt{\lambda}$ and identify

$$v_x = v\sqrt{\mathfrak{g}_2}, \quad (5.4)$$

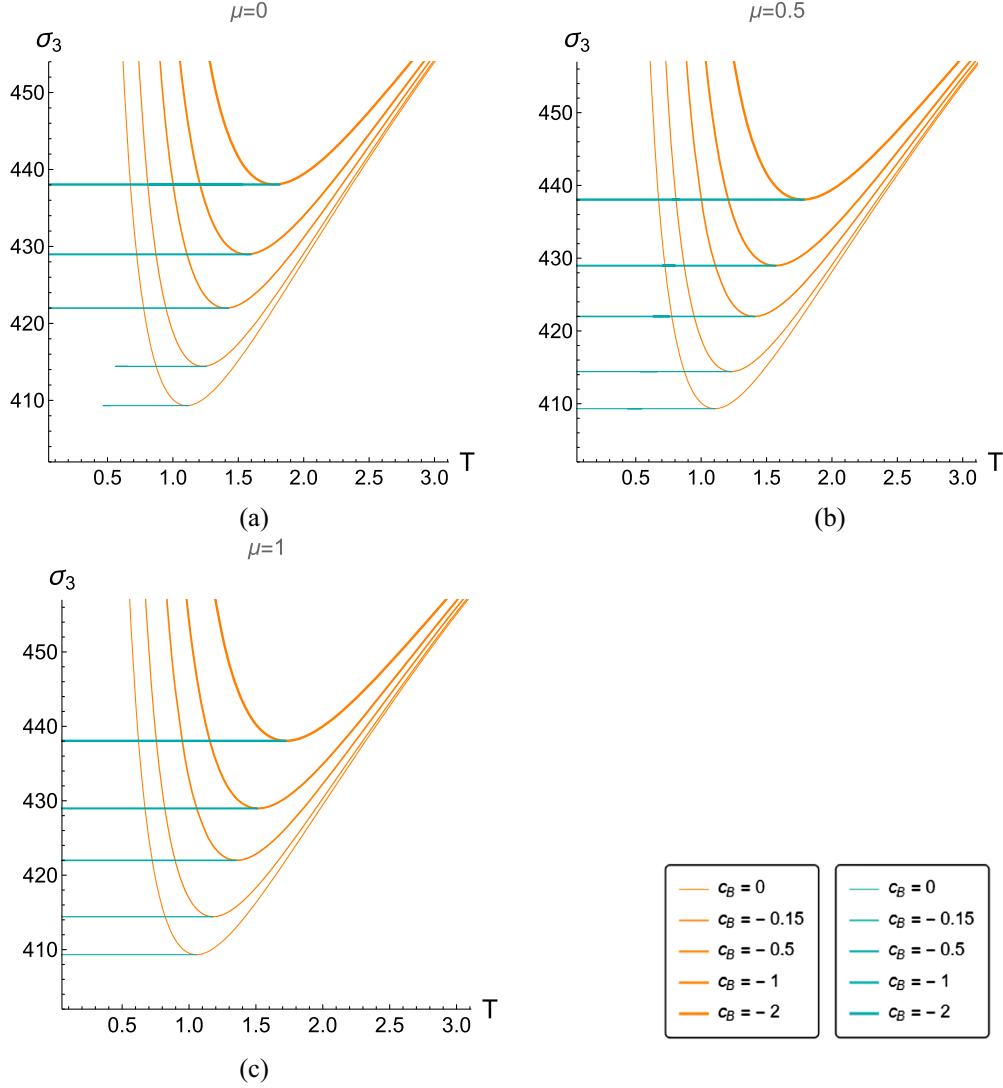


FIG. 17. Spatial string tension σ_3 in the third orientation $\mathcal{W}_{y_1 y_2}$ as a function of temperature T for isotropic case $\nu = 2$ at (a) $\mu = 0$ GeV, (b) $\mu = 0.5$ GeV, and (c) $\mu = 1$ GeV with different c_B considering the zero-boundary condition (2.10). The cyan line and orange curve show the DW and horizon configuration, respectively; $[\sigma]^\frac{1}{2} = [T] = [\mu] = [c_B]^\frac{1}{2} = \text{GeV}$.

$$v_{y_1} = v \frac{\sqrt{\mathfrak{g}_3}}{\mathfrak{g}_2}, \quad (5.5)$$

$$v_{y_2} = v \frac{\sqrt{\mathfrak{g}_2}}{\sqrt{\mathfrak{g}_3}}, \quad (5.6)$$

the drag forces (5.1)–(5.3) can reproduce spatial string tensions (3.18)–(3.20) at $z = z_h$, as follows:

$$\sigma_{xY_1}|_{z=z_h} = \frac{\sqrt{\lambda}}{2\pi} \frac{p_x}{v} = \frac{\sqrt{\lambda}}{2\pi} \left(\frac{\mathfrak{b}_s(z_h)}{z_h^2} \right) \sqrt{\mathfrak{g}_2}, \quad (5.7)$$

$$\sigma_{xY_2}|_{z=z_h} = \frac{\sqrt{\lambda}}{2\pi} \frac{p_{y_1}}{v} = \frac{\sqrt{\lambda}}{2\pi} \left(\frac{\mathfrak{b}_s(z_h)}{z_h^2} \right) \sqrt{\mathfrak{g}_3}, \quad (5.8)$$

$$\sigma_{y_1 Y_2}|_{z=z_h} = \frac{\sqrt{\lambda}}{2\pi} \frac{p_{y_2}}{v} = \frac{\sqrt{\lambda}}{2\pi} \left(\frac{\mathfrak{b}_s(z_h)}{z_h^2} \right) \sqrt{\mathfrak{g}_2 \mathfrak{g}_3}, \quad (5.9)$$

for some constant v and $\mathfrak{g}_1 = 1$. Therefore, the spatial string tension is proportional to the drag force in the anisotropic medium.

It is interesting to investigate the temperature dependence of the spatial string tension. Taking $\mathfrak{g}_1 = 1$, $\mathfrak{g}_2(z_h) = (z_h/L)^{2-2/\nu}$, and $\mathfrak{g}_3(z_h) = (z_h/L)^{2-2/\nu} e^{c_B z_h^2}$, we have

$$\sigma_1 \equiv \sigma_{xY_1}|_{z=z_h} = \frac{1}{2\pi\alpha'} \left(\frac{L^{1+1/\nu} \mathfrak{b}_s(z_h)}{z_h^{1+1/\nu}} \right), \quad (5.10)$$

$$\sigma_2 \equiv \sigma_{xY_2}|_{z=z_h} = \frac{1}{2\pi\alpha'} \left(\frac{L^{1+1/\nu} \mathfrak{b}_s(z_h)}{z_h^{1+1/\nu}} \right) e^{c_B z_h^2/2}, \quad (5.11)$$

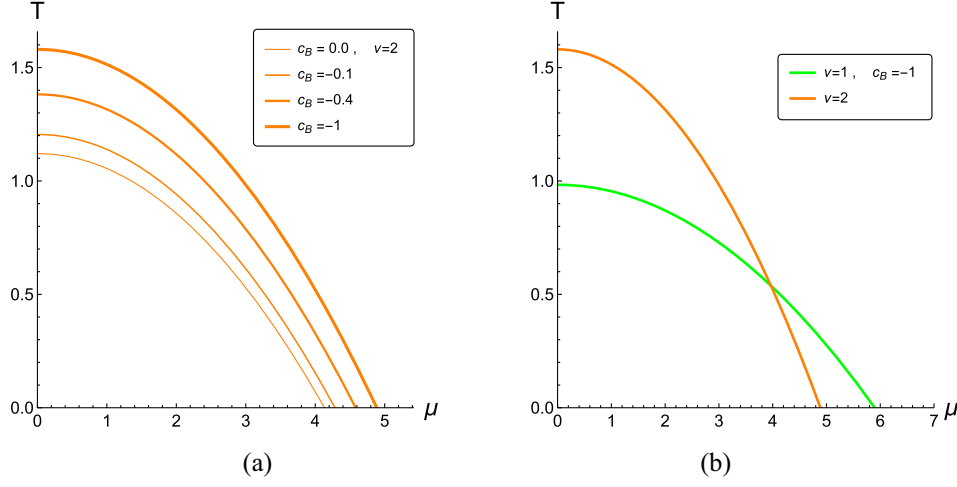


FIG. 18. Phase diagrams in (μ, T) -plane at (a) $\nu = 2$, and (b) comparison between $\nu = 1$ and $\nu = 2$ at fixed $c_B = -1$ GeV² in the third orientation $\mathcal{W}_{y_1 y_2}$, corresponding to the transition of σ_3 from the DW configuration to the horizon configuration, considering the zero-boundary condition (2.10); $[T] = [\mu] = [c_B]^{\frac{1}{2}} = \text{GeV}$.

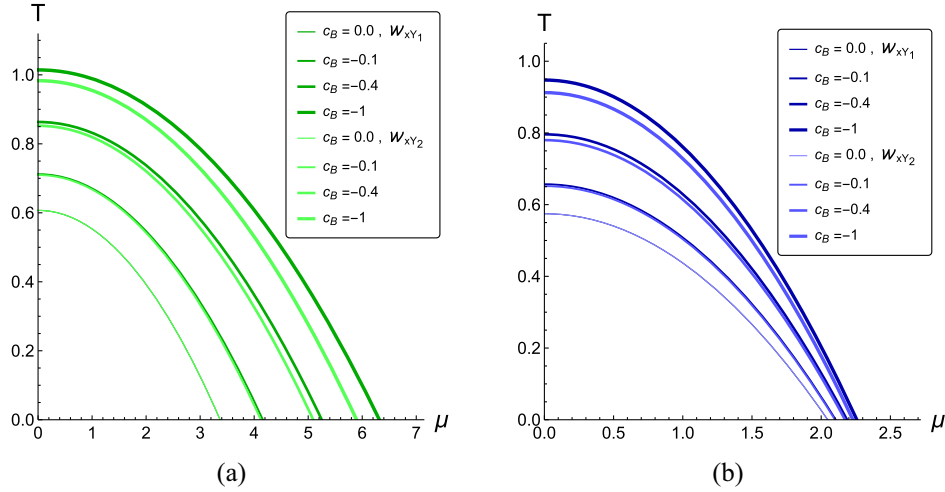


FIG. 19. Phase diagrams in (μ, T) -plane at (a) $\nu = 1$, and (b) $\nu = 4.5$ in the first orientation $\mathcal{W}_{xy1}$ and the second orientation $\mathcal{W}_{xy2}$, corresponding to the transition of σ_3 from the DW configuration to the horizon configuration considering the zero-boundary condition (2.10); $[T] = [\mu] = [c_B]^{\frac{1}{2}} = \text{GeV}$.

$$\sigma_3 \equiv \sigma_{y_1 y_2} \big|_{z=z_h} = \frac{1}{2\pi\alpha'} \left(\frac{L^{2/\nu} \mathbf{b}_s(z_h)}{z_h^{2/\nu}} \right) e^{c_B z_h^2/2}. \quad (5.12)$$

Figure 21(a) presents the string tension σ_1 in the first orientation $\mathcal{W}_{xy1}$ as a function of the temperature T at $\mu = 0$ for the isotropic case $\nu = 1$. The fitted solid red curve is $11.2 + 12.4T^2$, and the fitted dashed red curve is $33.1 + 10.9T^2$. Figure 21(b) depicts the anisotropic case $\nu = 4.5$ at $\mu = 0$. The fitted solid red curve is $226.1 + 91.3T^2 - 31.9T^3 + 3.7T^4$, and the fitted dashed red curve is $288.2 + 31.9T^2 + 0.8T^3 - 1.4T^4$. Our result shows that for the fully isotropic case $\nu = 1$ and $c_B = 0$, we have $\sigma_1 \propto T^2$ which is compatible with the result reported

in Refs. [67,69,85] and also is qualitatively consistent with the lattice results [86,87]. Inclusion of spatial anisotropy $\nu = 4.5$ clearly shows that the behavior of $\sigma(T)$ deviates from the quadratic term and requires higher terms.

Figure 22(a) shows the temperature dependence of the string tension σ_2 in the second orientation $\mathcal{W}_{xy2}$ at $\mu = 0$ for the isotropic case $\nu = 1$. In Fig. 22(b), the fitted solid red curve is $18.9 + 11.7T^2$ and the fitted dashed red curve is $30.2 + 11.1T^2$ for the anisotropic case $\nu = 4.5$; at $\mu = 0$, the fitted solid red curve is $258.3 + 58.1T^2 - 13.6T^3 + 0.9T^4$ and the fitted dashed red curve is $303.6 + 26.7T^2 + 1.3T^3 - 1.1T^4$. In the isotropic case $\nu = 1$ at different values of the magnetic field c_B we get $\sigma_2 \propto T^2$. Note that our results for nonzero magnetic field

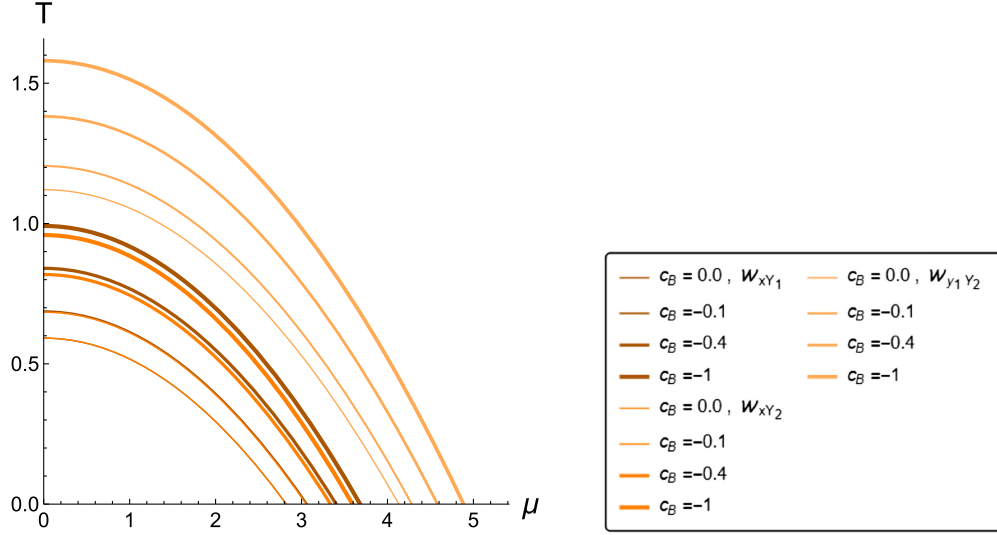


FIG. 20. Phase transition diagrams in (μ, T) -plane at $\nu = 2$ for the first $\mathcal{W}_{xY_1}$, and the second $\mathcal{W}_{xY_2}$ with the third orientation $\mathcal{W}_{y_1Y_2}$, corresponding to the transition of σ_3 from the DW configuration to the horizon configuration at the zero-boundary condition (2.10); $[T] = [\mu] = [c_B]^{\frac{1}{2}} = \text{GeV}$.

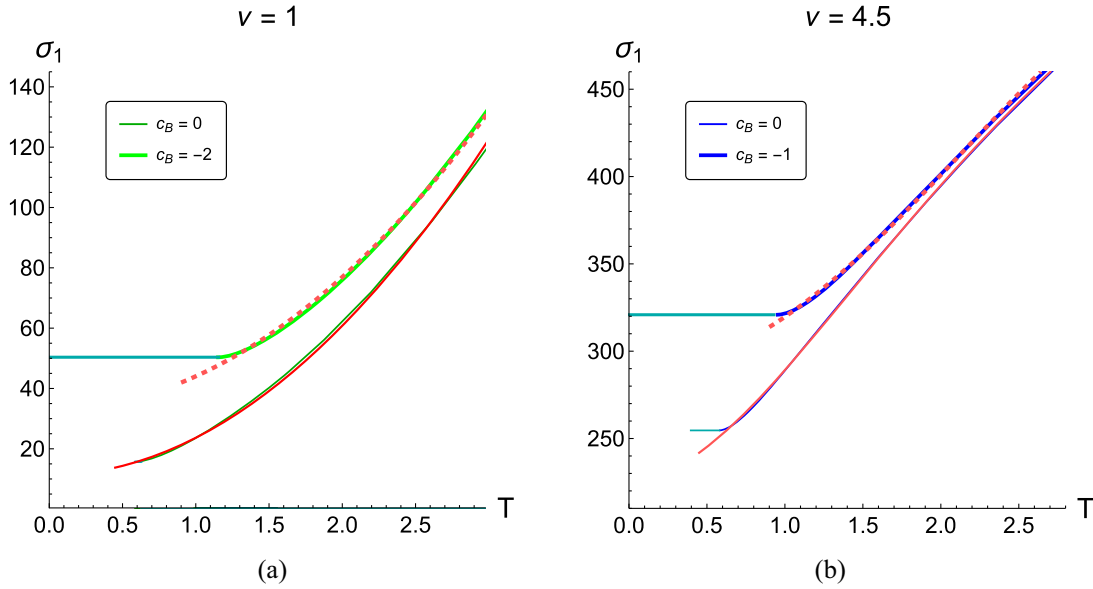


FIG. 21. Spatial string tension σ_1 in the first orientation $\mathcal{W}_{xY_1}$ as a function of temperature T at $\mu = 0$ for (a) isotropic case $\nu = 1$; the solid red curve is $11.2 + 12.4T^2$ and the dashed red curve is $33.1 + 10.9T^2$, and for (b) anisotropic case $\nu = 4.5$; the solid red curve is $226.1 + 91.3T^2 - 31.9T^3 + 3.7T^4$ and the dashed red curve is $288.2 + 31.9T^2 + 0.8T^3 - 1.4T^4$; $[\sigma]^{\frac{1}{2}} = [T] = [\mu] = [c_B]^{\frac{1}{2}} = \text{GeV}$.

$c_B \neq 0$ are our prediction and can probably be checked with lattice calculations. For the anisotropic case, $\nu = 4.5$ $\sigma(T)$ is proportional to the higher terms of T .

Applying Eqs. (5.10)–(5.12) to the third orientation $\mathcal{W}_{y_1Y_2}$ shows that in the isotropic case $\nu = 1$ with different c_B , the results for $\sigma(T)$ are the same as in the second orientation. Therefore, in Fig. 23, the temperature dependence of σ_3 is investigated only for the anisotropic case $\nu = 2$ at $\mu = 0$ and different c_B . The solid gray curve is

$409.6 + 4.5T^2 + 0.9T^3 - 0.3T^4$, and the dashed gray curve is $433.8 - 4.3T^2 + 3.9T^3 - 0.6T^4$. Therefore, in this case, $\sigma(T)$ deviates from the quadratic behavior in the presence of spatial anisotropy.

It is important to note that for all orientations inclusion of the external magnetic field and the spatial anisotropy enhances the string tension in the horizon configuration, i.e., the drag force. Our results reveal that for $T < T_{cr}$, the string tension value is temperature independent and gets a

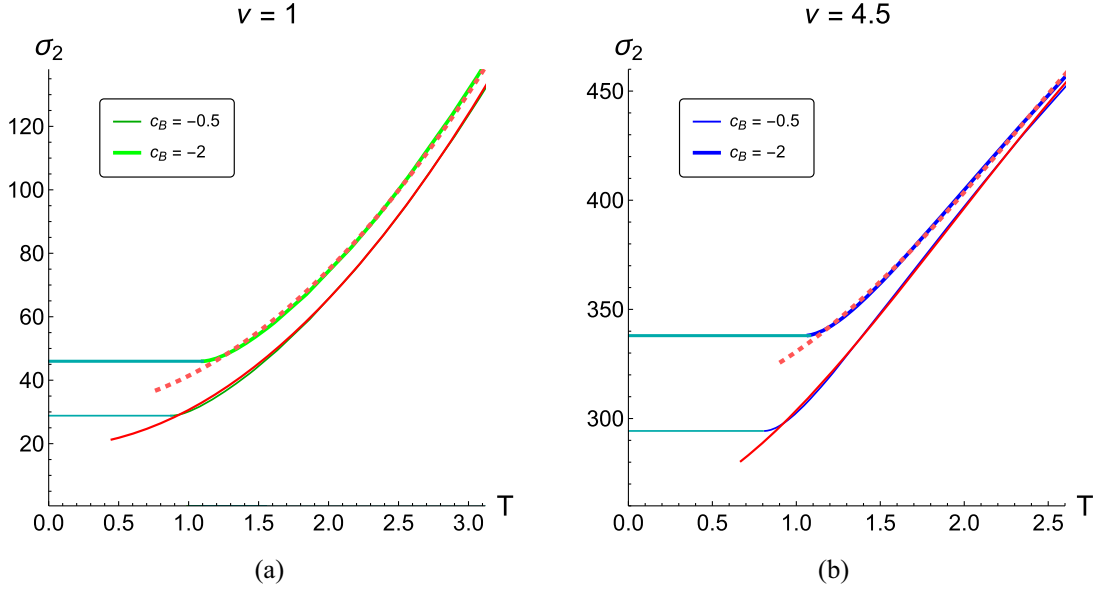


FIG. 22. Spatial string tension σ_2 in the second orientation $\mathcal{W}_{xY_2}$ as a function of temperature T at $\mu = 0$ for (a) isotropic case $\nu = 1$; the solid red curve is $18.9 + 11.7T^2$ and the dashed red curve is $30.2 + 11.1T^2$, and for (b) anisotropic case $\nu = 4.5$; the solid red curve is $258.3 + 58.1T^2 - 13.6T^3 + 0.9T^4$ and the dashed red curve is $303.6 + 26.7T^2 + 1.3T^3 - 1.1T^4$; $[\sigma]^{1/2} = [T] = [\mu] = [c_B]^{1/2} = \text{GeV}$.

constant value. Similar behavior is reported in Ref. [85] and also in lattice calculations [86,87]. In our model, for all orientations at $T < T_{cr}$, the string tension profile is realized on the DW configuration, and at $T > T_{cr}$, it is realized on the horizon configuration and it increases monotonically.

VI. SPATIAL STRING TENSIONS AND EXPERIMENTAL OBSERVABLES

Strong suppression of high transverse-momentum p_T hadrons and electrons from heavy-quark decays in central collisions compared to scaled proton-proton collisions, combined with the elliptic flow (v_2), constitute important experimental evidence for the creation of a quark-gluon plasma in heavy-ion collisions.²

There are two main experimental observables related to these evidences of QGP creation in HIC:

- (i) The nuclear modification factor R_{AA} [88–90], which specifies the modification of the production yield in nucleus-nucleus collisions relative to pp collisions due to hard partonic scattering processes,

$$R_{AA}(p_T) = \frac{1}{\langle N_{\text{coll}} \rangle} \cdot \frac{dN_{AA}/dp_T}{dN_{pp}/dp_T}, \quad (6.1)$$

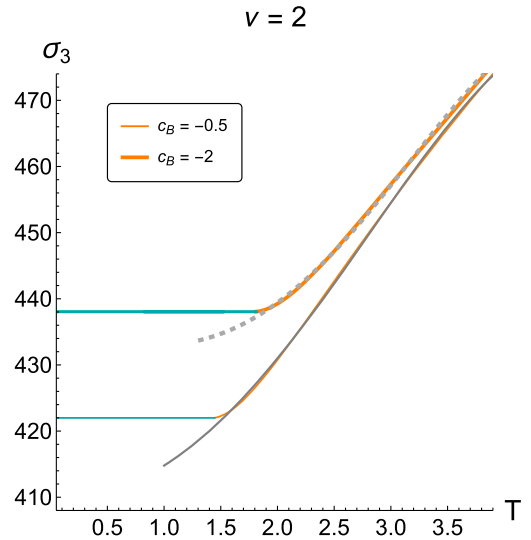


FIG. 23. Spatial string tension σ_3 in the third orientation $\mathcal{W}_{y_1Y_2}$ as a function of temperature T at $\mu = 0$ for anisotropic case $\nu = 2$; the solid gray curve is $409.6 + 4.5T^2 + 0.9T^3 - 0.3T^4$ and the dashed gray curve is $433.8 - 4.3T^2 + 3.9T^3 - 0.6T^4$; $[\sigma]^{1/2} = [T] = [\mu] = [c_B]^{1/2} = \text{GeV}$.

²In fact, the experimental evidence for the creation of a QGP in HICs comes from several complementary signatures observed at facilities like Relativistic Heavy Ion Collider (RHIC) and LHC. These signals reflect the formation of a hot dense medium with properties, consistent with deconfined quarks and gluons. Besides two signatures, mentioned above, and concerning high- p_T suppression and elliptic flow, there are more signatures: increased production of strange and multi-strange hadrons (e.g., Λ , Σ , Ω) in A + A collisions compared to p + p or p + A collisions is due to the fact that the QGP facilitates rapid equilibration of strange quarks, which is difficult in a purely hadronic medium; excess of low- and intermediate-mass dileptons and direct photons beyond what hadronic decays predict, that is related with photons and dileptons escape the medium without strong interaction; suppression of bound states like J/Ψ and Υ in HIC compared to expectations from p + p collisions, since color screening in the QGP prevents quarkonium binding (“Debye screening”), reducing their yield.

where p_T is the transverse momentum, dN_{AA}/dp_T is the p_T spectrum measured in AA collisions, and dN_{pp}/dp_T is the p_T -differential yield in pp collisions. Strong suppression of high-transverse-momentum hadrons and electrons in heavy quark decays in central collisions compared to scaled proton-proton collisions, numerically expressed as an inequality $R_{AA} \ll 1$ for nuclear modification factor for high- p_T particles.

- (ii) The elliptic flow v_2 [91–93], which is a measure of the anisotropy in the azimuthal distribution of particle production. The distribution is typically written as

$$\frac{dN}{d\phi} \propto 1 + 2 \sum_{n=1}^{\infty} v_n \cos(n(\phi - \Psi_n)), \quad (6.2)$$

where ϕ is the azimuthal angle of a particle, Ψ_n is the orientation of the n th harmonic symmetry plane, and v_2 corresponds to $n = 2$. A positive v_2 indicates a preference for emission in the plane perpendicular to the beam direction (the reaction plane), reflecting the almond-shaped overlap region in noncentral collisions.

Both of them are directly related with the drag coefficient η_D . The factor $R_{AA}(p_T)$ is expected to follow the approximate scaling relation which depends on the drag coefficient η_D , as follows:

$$R_{AA}(p_T) \propto \left(1 - \frac{C\eta_D L}{p_T}\right)^{n-2}, \quad (6.3)$$

where C is a constant encoding medium density and kinematic factors, η_D is the drag coefficient, L is the path length in the medium (see, e.g., Ref. [94]), and $n \sim 5-8$ is a good approximation at high p_T at RHIC and LHC energies [95–97]. The factor $R_{AA}(p_T)$ also includes a term associated with the momentum diffusion coefficient, which in its turn is directly related to the momentum broadening coefficient κ . In the anisotropic plasma,

$$R_{AA}(p_T, \phi) \approx \left(1 - \frac{C\eta(\phi)L}{p_T}\right)^{n-2} + \frac{n(n+1)}{2} \frac{L}{p_T^2} \kappa(\phi), \quad (6.4)$$

where

$$\eta(\phi) = \eta_{\parallel} \cos^2 \phi + \eta_{\perp} \sin^2 \phi, \quad (6.5)$$

$$\kappa(\phi) = \kappa_{\parallel} \cos^2 \phi + \kappa_{\perp} \sin^2 \phi. \quad (6.6)$$

The first term represents suppression due to anisotropic drag force, and the second term represents momentum

broadening from anisotropic diffusion. Here, ϕ is the angle in the transverse plane (the plane perpendicular to the beam axis) between $\vec{p}_T$ and the impact vector.

The elliptic flow v_2 is also related to the drag force and momentum broadening

$$v_2(p_T) \simeq -\frac{(n-2)CL}{4p_T}(\eta_{\parallel} - \eta_{\perp}) + \frac{n(n+1)L}{8p_T^2}(\kappa_{\parallel} - \kappa_{\perp}). \quad (6.7)$$

If the medium is in thermal equilibrium at temperature T , drag coefficient and momentum broadening are not independent. They satisfy the Einstein relation

$$\eta_{\parallel} = \frac{\kappa_{\parallel}}{2ET}, \quad \eta_{\perp} = \frac{\kappa_{\perp}}{2ET}. \quad (6.8)$$

In this paper, we have calculated the spatial string tensions in the anisotropic holographic model. According to Eqs. (5.8) and (5.9), the drag forces p_{y_1} and p_{y_2} are related to the spatial string tensions σ_{xY_2} and $\sigma_{y_1Y_2}$, respectively, as follows:

$$p_{y_1} = \frac{2\pi v}{\sqrt{\lambda}} \sigma_{xY_2}, \quad (6.9)$$

$$p_{y_2} = \frac{2\pi v}{\sqrt{\lambda}} \sigma_{y_1Y_2}. \quad (6.10)$$

These drag forces p_{y_1} and p_{y_2} define the corresponding drag coefficients $\eta_{\parallel} = p_{y_1}/v$ and $\eta_{\perp} = p_{y_2}/v$, which, according to Eqs. (6.5) and (6.6), enter the main phenomenological formulas (6.4) and (6.7).

Note that since the nuclear modification factor R_{AA} and the elliptic flow coefficient v_2 depend only on the transverse momentum p_T , motion in the transverse plane is relevant; longitudinal motion along the beam axis does not contribute. Consequently, the resulting expressions for R_{AA} and v_2 simply involve the projection of the anisotropic transport coefficients onto the direction of motion in the transverse plane.

To summarize, our analysis reveals the dependence of the string tensions on the temperature and chemical potential, as well as on the magnetic field strengths. These dependencies inevitably lead to the dependence of the observed R_{AA} and v_2 on temperature, density, and magnetic field. A detailed connection with the experimental data on R_{AA} and v_2 warrants a separate publication.

VII. CONCLUSION

In this paper, we investigate the effective potential and the spatial string tension σ and its phase transitions between the DW configuration and the horizon configuration using spatial Wilson loops. To do this, we used the

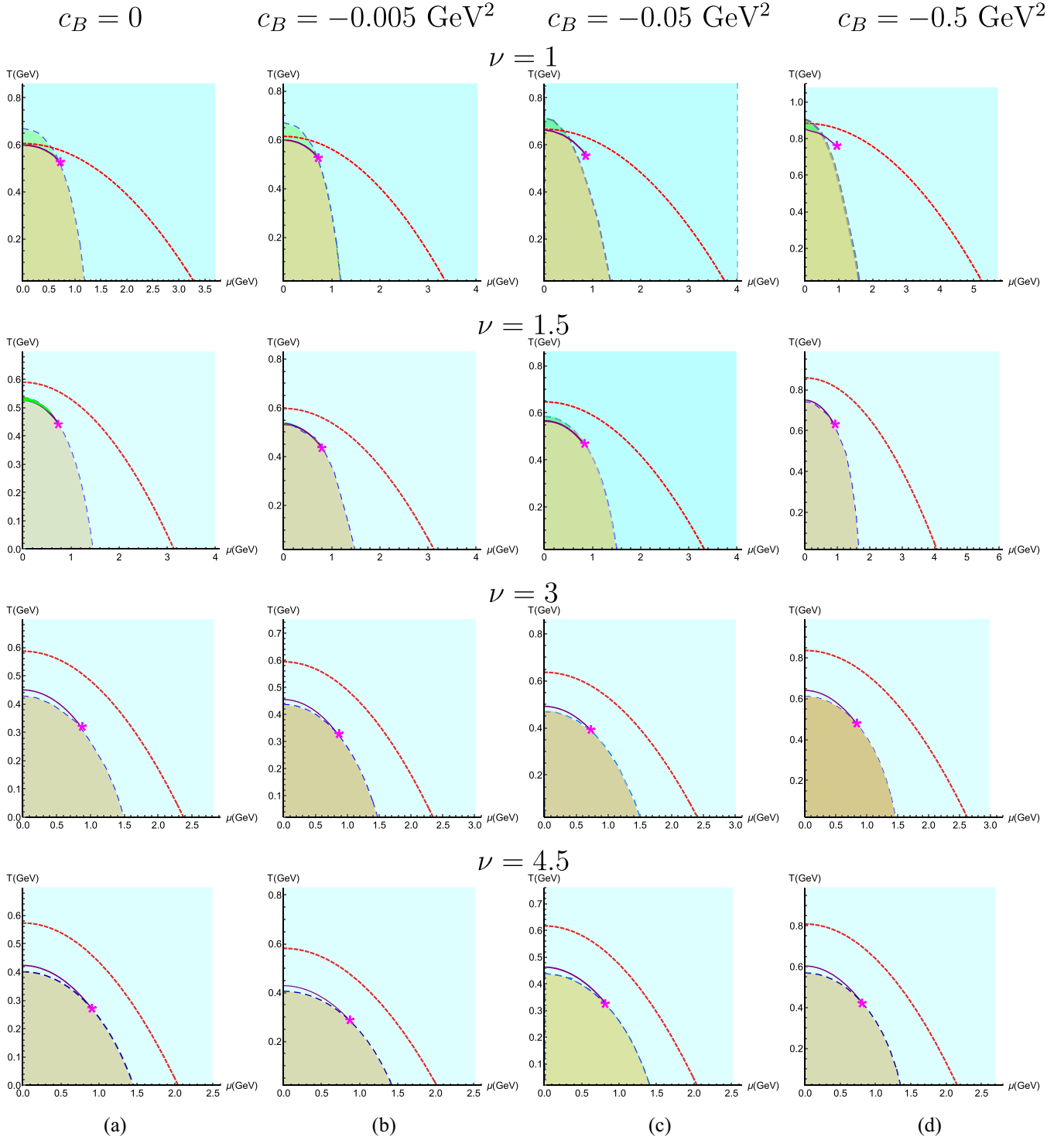


FIG. 24. Phase diagrams in (μ, T) -plane for Heavy Quark models in columns (a) $c_B = 0 \text{ GeV}^2$, (b) $c_B = -0.005 \text{ GeV}^2$, (c) $c_B = -0.05 \text{ GeV}^2$, and (d) $c_B = -0.5 \text{ GeV}^2$. The first, second, third, and fourth rows show $\nu = 1$, $\nu = 1.5$, $\nu = 3$, and $\nu = 4.5$, respectively.

Einstein-three-Maxwell-dilaton action and incorporated a five-dimensional metric with a special warp factor to construct an anisotropic holographic model under a strong magnetic field for the heavy-quark case [45]. In addition, the effect of spatial anisotropy ν , that reflects the

noncentrality of HIC, on the string tension for three particular orientations is studied.

We obtained the SWL and the effective potential for general orientations, namely the dependence of σ on temperature T is studied for three particular SWL

orientations, i.e., σ_i , $i = 1, 2, 3$ corresponding to $\mathcal{W}_{xY_1}$, $\mathcal{W}_{xY_2}$, and $\mathcal{W}_{y_1Y_2}$, respectively.

Let us summarize our findings, depending on different orientations.

(i) *The first SWL orientation $\mathcal{W}_{xY_1}$:*

some similarities between the zero-boundary condition (2.10) and the physical-boundary condition (2.12):

the phase diagram and the DW coordinates do not depend on the boundary condition, inclusion of spatial anisotropy $\nu = 4.5$ decreases the values of the critical transition T between the DW and the horizon configuration in our model; the main difference between the zero- and physical-boundary conditions:

inclusion of spatial anisotropy $\nu = 4.5$ and increasing the magnetic field c_B causes the spatial string tension σ_1 to increase/decrease at zero/physical-boundary conditions,

at very low T in the case of $\mu = 0$ with $c_B \neq 0$, the σ_1 exists/does not exist at zero/physical-boundary conditions due to the absence of the DW coordinate.

(ii) *The second SWL orientation $\mathcal{W}_{xY_2}$:*

the spatial string tension σ increases with the spatial anisotropy ν and the external magnetic field c_B ; the transition temperature occurs at lower values in the second SWL orientation $\mathcal{W}_{xY_2}$, compared to the first orientation $\mathcal{W}_{xY_1}$, for both the isotropic case $\nu = 1$ and the anisotropic case $\nu = 4.5$.

(iii) *The third SWL orientation $\mathcal{W}_{y_1Y_2}$:*

for $\nu = 1$ the effective potential values and the DW coordinates are exactly the same as the results for the second SWL orientation $\mathcal{W}_{xY_2}$; see Eqs. (3.19) and (3.20); for $\nu = 4.5$, there are no DW coordinates; in fact, there is a critical value of anisotropy $\nu_{cr} = 2.5$, i.e. at $\nu \geq 2.5$ the DW coordinates disappear; for $\nu = 2$, the transition temperature has higher values in the third SWL orientation $\mathcal{W}_{y_1Y_2}$, compared to the first $\mathcal{W}_{xY_1}$ and second $\mathcal{W}_{xY_2}$ orientations for all values of the magnetic field c_B .

There also are some properties, that are common and independent of the different SWL orientations choice, which are summarized as follows.

Universal characteristics

- (i) The phase transition has magnetic catalysis behavior, which means that the critical temperature T_{cr} increases as the magnetic field c_B increases.
- (ii) As the magnetic field c_B increases, the value of the DW coordinate decreases, meaning the phase transition occurs at a smaller holographic coordinate z closer to the boundary.
- (iii) DW coordinates are independent of the chemical potential value.

- (iv) At lower temperature $T < T_{cr}$ in the DW configuration, the string tension is independent of the temperature.
- (v) At higher temperature $T > T_{cr}$ in the horizon configuration, the string tension increases monotonically.
- (vi) Inclusion of the external magnetic field and spatial anisotropy enhance the string tension in the horizon configuration, i.e., the drag force.
- (vii) For the fully isotropic case $\nu = 1$ and $c_B = 0$, the temperature dependence of the string tension $\sigma \propto T^2$ is qualitatively consistent with the lattice results [86,87].

It is very interesting to compare the phase transition between the DW and the horizon configuration, obtained by SWL, with the confinement/deconfinement phase transition line, obtained from temporal WL and the first-order phase transition line following from the thermodynamics of the model. Figure 25 shows this comparison for different values of magnetic field $c_B = 0, -0.005, -0.05, -0.5 \text{ GeV}^2$, and different spatial anisotropies $\nu = 1, 1.5, 3, 4.5$. In this comparison the second orientation of SWL, xY_2 , is used, while for temporal WL, tY_2 orientation is considered. The phase transition line obtained by SWL is depicted by a red dashed line, the confinement/deconfinement phase transition is depicted by a blue dashed line, and the first-order phase transition is depicted by a solid magenta line. The blue regions represent the QGP phase, the green regions correspond to the quarkyonic phase, and the brown regions correspond to the confined phase.

Figure 24 depicts that at small μ and for spatially isotropic case $\nu = 1$, the transition temperature via SWL is lower than the confinement/deconfinement transition temperature and higher than the first-order phase transition temperature. However, including anisotropies $\nu = 1.5, \nu = 3$, and $\nu = 4.5$, our result shows that for any μ the transition temperature via SWL is always higher than the confinement/deconfinement and the first-order transition temperature.

Note that diffusion coefficient is studied near the critical point in terms of temperature [70], and also temperature dependence of diffusion coefficient is investigated in Ref. [98] near the phase transition in isotropic backgrounds. Studying and extending the diffusion coefficient in the anisotropic background that we introduced in this research will be considered for future research.

Furthermore, it would be interesting to take into account other anisotropic models with different warp factors $\mathfrak{h}_s(z)$, corresponding to the light quark model [18] or the heavy quark model [40], to study the DW coordinates and string tension. Recently, this background has been used in holographic QCD to investigate the jet quenching parameter [82].

ACKNOWLEDGMENTS

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DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

The explicit forms of EOMs are given by

$$A''_t + A'_t \left(\frac{\mathfrak{b}'}{2\mathfrak{b}} + \frac{f'_0}{f_0} + \frac{\nu - 2}{\nu z} + c_B z \right) = 0, \quad (\text{A5})$$

$$g'' + g' \left(\frac{3\mathfrak{b}'}{2\mathfrak{b}} - \frac{\nu + 2}{\nu z} + c_B z \right) - \left(\frac{z}{L} \right)^2 \frac{f_0 (A'_t)^2}{\mathfrak{b}} - \left(\frac{z}{L} \right)^{\frac{2}{\nu}} \frac{q_3^2 f_3}{\mathfrak{b}} = 0, \quad (\text{A6})$$

$$\mathfrak{b}'' - \frac{3(\mathfrak{b}')^2}{2\mathfrak{b}} + \frac{2\mathfrak{b}'}{z} - \frac{4\mathfrak{b}}{3\nu z^2} \left(1 - \frac{1}{\nu} + \left(1 - \frac{3\nu}{2} \right) c_B z^2 - \frac{\nu c_B^2 z^4}{2} \right) + \frac{\mathfrak{b}(\phi')^2}{3} = 0, \quad (\text{A7})$$

$$2g' \left(1 - \frac{1}{\nu} \right) + 3g \left(1 - \frac{1}{\nu} \right) \left(\frac{\mathfrak{b}'}{\mathfrak{b}} - \frac{4(\nu + 1)}{3\nu z} + \frac{2c_B z}{3} \right) + \left(\frac{L}{z} \right)^{1-\frac{4}{\nu}} \frac{L e^{-c_B z^2} q_1^2 f_1}{\mathfrak{b}} = 0, \quad (\text{A8})$$

$$2g' \left(1 - \frac{1}{\nu} + c_B z^2 \right) + 3g \left[\left(1 - \frac{1}{\nu} + c_B z^2 \right) \left(\frac{\mathfrak{b}'}{\mathfrak{b}} - \frac{4}{3\nu z} + \frac{2c_B z}{3} \right) - \frac{4(\nu - 1)}{3\nu z} \right] + \left(\frac{L}{z} \right)^{1-\frac{4}{\nu}} \frac{L e^{-c_B z^2} q_1^2 f_1}{\mathfrak{b}} - \left(\frac{z}{L} \right)^{1+\frac{2}{\nu}} \frac{L q_3^2 f_3}{\mathfrak{b}} = 0, \quad (\text{A9})$$

$$\begin{aligned} & \frac{\mathfrak{b}''}{\mathfrak{b}} + \frac{(\mathfrak{b}')^2}{2\mathfrak{b}^2} + \frac{3\mathfrak{b}'}{\mathfrak{b}} \left(\frac{g'}{2g} - \frac{\nu + 1}{\nu z} + \frac{2c_B z}{3} \right) - \frac{g'}{3zg} \left(5 + \frac{4}{\nu} - 3c_B z^2 \right) + \frac{8}{3z^2} \left(1 + \frac{3}{2\nu} + \frac{1}{2\nu^2} \right) - \frac{4c_B}{3} \left(1 + \frac{3}{2\nu} - \frac{c_B z^2}{2} \right) \\ & + \frac{g''}{3g} + \frac{2}{3} \left(\frac{L}{z} \right)^2 \frac{\mathfrak{b}V}{g} = 0. \end{aligned} \quad (\text{A10})$$

For more details, we refer the interested reader to Ref. [45].

APPENDIX B: ARBITRARY ORIENTATIONS OF WILSON LOOPS

In the holographic approach, the Nambu-Goto action in the background (3.5) for the probe string is

$$S = \frac{1}{2\pi\alpha'} \int d\xi^1 d\xi^2 \sqrt{-\det h_{\alpha\beta}}, \quad (\text{B1})$$

APPENDIX A: SOLUTION OF EOMs

Varying Lagrangian (2.1) over the metric, we get the Einstein EOM

$$G_{\mu\nu} = T_{\mu\nu}, \quad (\text{A1})$$

where

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R, \quad \frac{\delta S_m}{\delta g^{\mu\nu}} = T_{\mu\nu} \sqrt{-g}, \quad (\text{A2})$$

and varying the Lagrangian over the fields, we get field equations

$$D_\mu D^\mu \phi + V'(\phi) + \sum_{i=0,1,3} \frac{f'_i(\phi)}{4} F_{(i)}^2 = 0, \quad (\text{A3})$$

$$\partial_\mu (\sqrt{-g} f_i F_{(i)}^{\mu\nu}) = 0. \quad (\text{A4})$$

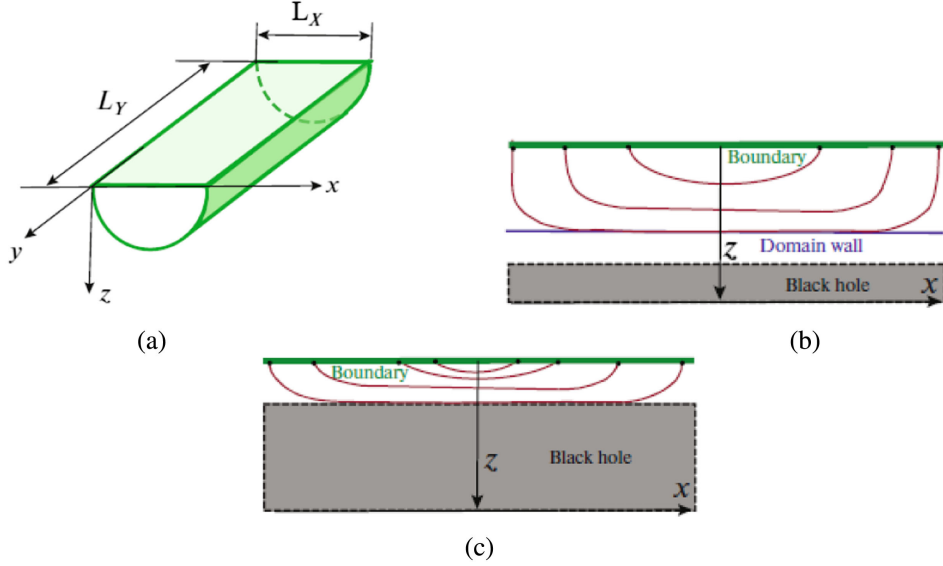


FIG. 25. (a) Spatial Wilson loop and its world sheet. (b) describing the DW configuration and the horizon configuration. (c) after the phase transition from DW configuration to horizon configuration [6]; $[T] = [\mu] = \text{GeV}$.

where the induced metric $h_{\alpha\beta}$ is given by

$$h_{\alpha\beta} = G_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu \quad (\text{B2})$$

and is orientation dependent. Here, we use the same orientation to parametrize the rectangular Wilson loop as was considered to calculate the entanglement entropy using the rectangular parallelepiped in Ref. [81].

To parametrize the world sheet of string, we use the rotation matrix $M(\phi, \theta, \psi)$ in a three-dimensional space with the representation

$$x^i = \sum_{j=1,2,3} a_{ij}(\phi, \theta, \psi) \zeta^j, \quad i = 1, 2, 3, \quad (\text{B3})$$

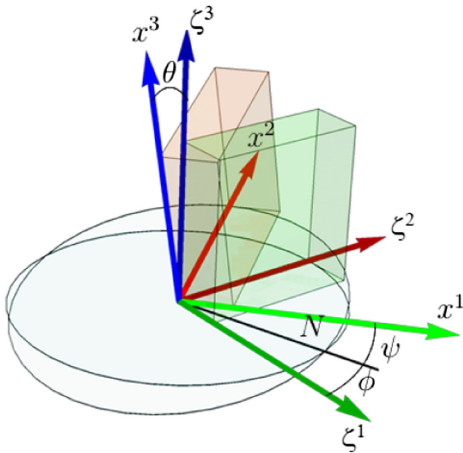


FIG. 26. Rotating the green rectangular cube by the Euler angles (ϕ, θ, ψ) , we get the pink cube oriented along the axes (x^1, x^2, x^3) . The node line is denoted by N , and $(\zeta^1, \zeta^2, \zeta^3)$ are axes before rotation [81].

where a_{ij} in terms of the Euler angles ϕ, θ, ψ are the components of the rotation matrix

$$M(\phi, \theta, \psi) = \begin{pmatrix} a_{11}(\phi, \theta, \psi) & a_{12}(\phi, \theta, \psi) & a_{13}(\phi, \theta, \psi) \\ a_{21}(\phi, \theta, \psi) & a_{22}(\phi, \theta, \psi) & a_{23}(\phi, \theta, \psi) \\ a_{31}(\phi, \theta, \psi) & a_{32}(\phi, \theta, \psi) & a_{33}(\phi, \theta, \psi) \end{pmatrix}, \quad (\text{B4})$$

where

$$\begin{aligned} a_{11}(\phi, \theta, \psi) &= \cos \phi \cos \psi - \cos \theta \sin \phi \sin \psi, \\ a_{12}(\phi, \theta, \psi) &= -\cos \psi \sin \phi - \cos \phi \cos \theta \sin \psi, \\ a_{13}(\phi, \theta, \psi) &= \sin \theta \sin \psi, \\ a_{21}(\phi, \theta, \psi) &= \cos \theta \cos \psi \sin \phi + \cos \phi \sin \psi, \\ a_{22}(\phi, \theta, \psi) &= \cos \phi \cos \theta \cos \psi - \sin \phi \sin \psi, \\ a_{23}(\phi, \theta, \psi) &= -\cos \psi \sin \theta, \\ a_{31}(\phi, \theta, \psi) &= \sin \phi \sin \theta, \\ a_{32}(\phi, \theta, \psi) &= \cos \phi \sin \theta, \\ a_{33}(\phi, \theta, \psi) &= \cos \theta. \end{aligned} \quad (\text{B5})$$

Here, ϕ is the angle between the ζ^1 -axis and the node line (N), θ is the angle between the ζ^3 and x^3 -axes, and ψ is the angle between the node line N and x^1 -axis; see Fig. 26.

We use the following embedding to describe the two-dimensional world sheet of string in the five-dimensional bulk:

$$\begin{aligned}
X^0(\xi) &= \text{const}, \\
X^i(\xi) &= \sum_{\alpha=1,2} a_{i\alpha}(\phi, \theta, \psi) \xi^\alpha, \quad i = 1, 2, 3, \quad \alpha = 1, 2, \\
X^4(\xi) &= z(\xi^1),
\end{aligned} \tag{B6}$$

where X^i are spatial coordinates.

The line element of the induced metric on the world sheet is

$$ds_{ws}^2 = g_{\alpha\beta} d\xi^\alpha d\xi^\beta, \quad \alpha, \beta = 1, 2. \tag{B7}$$

$$\begin{aligned}
ds_{ws}^2 &= \frac{L^2 \mathbf{b}_s(z)}{z^2} \left(\sum_{i=1,2,3} \mathbf{g}_i(z) d(x^i)^2 + \frac{d(x^4)^2}{g} \right) \\
&= \frac{L^2 \mathbf{b}_s(z)}{z^2} \left(\sum_{i=1,2,3} \mathbf{g}_i(z) \left(\sum_{j=1,2} a_{ij}(\phi, \theta, \psi) d\xi^j \right)^2 \right. \\
&\quad \left. + z'^2 \frac{d(\xi^1)^2}{g(z)} \right).
\end{aligned} \tag{B8}$$

We have

$$g_{\alpha\beta} = \frac{L^2 \mathbf{b}_s(z)}{z^2} \bar{g}_{\alpha\beta}, \tag{B9}$$

After applying the embedding relations (B3), the induced metric can be written as

$$\begin{aligned}
\bar{g}_{11}(z, \phi, \theta, \psi) &= \mathbf{g}_1 a_{11}^2 + \mathbf{g}_2 a_{21}^2 + \mathbf{g}_3 a_{31}^2 + \frac{z'^2}{g}, \\
\bar{g}_{22}(z, \phi, \theta, \psi) &= \mathbf{g}_1 a_{12}^2 + \mathbf{g}_2 a_{22}^2 + \mathbf{g}_3 a_{32}^2, \\
\bar{g}_{12}(z, \phi, \theta, \psi) &= \mathbf{g}_1 a_{11} a_{12} + \mathbf{g}_2 a_{21} a_{22} + \mathbf{g}_3 a_{31} a_{32}, \\
\bar{g}_{21} &= \bar{g}_{12}.
\end{aligned} \tag{B10}$$

The determinant of the induced metric is given by

$$\begin{aligned}
\det g_{\alpha\beta} &= \left(\frac{L^2 \mathbf{b}_s}{z^2} \right)^2 \left(\left(\mathbf{g}_1 a_{11}^2 + \mathbf{g}_2 a_{21}^2 + \mathbf{g}_3 a_{31}^2 + \frac{z'^2}{g} \right) (\mathbf{g}_1 a_{12}^2 + \mathbf{g}_2 a_{22}^2 + \mathbf{g}_3 a_{32}^2) \right. \\
&\quad \left. - (\mathbf{g}_1 a_{11} a_{12} + \mathbf{g}_2 a_{21} a_{22} + \mathbf{g}_3 a_{31} a_{32})^2 \right).
\end{aligned} \tag{B11}$$

The Nambu-Goto action for SWL is given by

$$S_{SWL} = \frac{1}{2\pi\alpha'} \int_{\mathcal{W}} \left(\frac{L^2 \mathbf{b}_s}{z^2} \right) \sqrt{\left(\mathbf{g}_1 \mathbf{g}_2 a_{33}^2 + \mathbf{g}_1 \mathbf{g}_3 a_{23}^2 + \mathbf{g}_2 \mathbf{g}_3 a_{13}^2 + \frac{z'^2}{g} \bar{g}_{22} \right) d\xi^1 d\xi^2}, \tag{B12}$$

where the integration is over the world sheet $\mathcal{W}$ components and $g, \mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3$ are functions of z and $\bar{g}_{22}, \bar{g}_{33}, \bar{g}_{23}$ are functions of z and Euler angles. The effective potential can be obtained as

$$\mathcal{V}(z(\xi)) = \frac{1}{2\pi\alpha'} \left(\frac{L^2 \mathbf{b}_s}{z^2} \right) \sqrt{\mathbf{g}_1 \mathbf{g}_2 a_{33}^2 + \mathbf{g}_1 \mathbf{g}_3 a_{23}^2 + \mathbf{g}_2 \mathbf{g}_3 a_{13}^2}. \tag{B13}$$

Note that the action (B12) and the effective potential (B13) depend on the angles of rotation and anisotropy.

APPENDIX C: STRING TENSION AT DW AND HORIZON CONFIGURATIONS

Using Eq. (3.2), the first integral related to the action (3.1) is given by

$$\frac{M(z(\xi)) \mathcal{F}(z(\xi))}{\sqrt{\mathcal{F}(z(\xi)) + (z'(\xi))^2}} = \mathcal{I}. \tag{C1}$$

At the turning point $z = z_*$, where the minimal surface has the closest distance to the horizon, we have $z'(\xi) = 0$. Then, from Eq. (C1), one can obtain

$$M(z_*) \sqrt{\mathcal{F}(z_*)} = \mathcal{I}. \tag{C2}$$

Utilizing Eqs. (C1) and (C2) to find z' , the length ℓ and the action $\mathcal{S}$ (3.1) are given by general formulas

$$\frac{\ell}{2} = \int_{\epsilon}^{z_*} \frac{1}{\sqrt{\mathcal{F}(z)}} \frac{dz}{\sqrt{\frac{\mathcal{V}^2(z)}{\mathcal{V}^2(z_*)} - 1}}, \quad (\text{C3})$$

$$\frac{\mathcal{S}}{2} = \int_{\epsilon}^{z_*} \frac{M(z)dz}{\sqrt{1 - \frac{\mathcal{V}^2(z_*)}{\mathcal{V}^2(z)}}}, \quad (\text{C4})$$

where ϵ is the regulator in gravity theory and corresponds to the UV cutoff in gauge theory. Taking into account the limit $\ell \rightarrow \infty$, we have two categories.

(i) There is a stationary point of $\mathcal{V}(z)$ that

$$\mathcal{V}'|_{z_{\text{DW}}} = 0, \quad (\text{C5})$$

where $z = z_{\text{DW}}$ is a DW coordinate that can be considered equal to the turning point $z_* = z_{\text{DW}}$. Near the turning point we have

$$\sqrt{\frac{\mathcal{V}^2(z)}{\mathcal{V}^2(z_{\text{DW}})} - 1} = \sqrt{\frac{\mathcal{V}''(z_{\text{DW}})}{\mathcal{V}(z_{\text{DW}})}}(z - z_*) + \mathcal{O}((z - z_*)^2). \quad (\text{C6})$$

Then, using Eqs. (C3) and (C4), one gets

$$\ell \underset{z \rightarrow z_*}{\sim} \frac{1}{\sqrt{\mathcal{F}(z_{\text{DW}})}} \sqrt{\frac{\mathcal{V}(z_{\text{DW}})}{\mathcal{V}''(z_{\text{DW}})}} \log(z - z_*), \quad (\text{C7})$$

$$\mathcal{S} \underset{z \rightarrow z_*}{\sim} M(z_{\text{DW}}) \sqrt{\frac{\mathcal{V}(z_{\text{DW}})}{\mathcal{V}''(z_{\text{DW}})}} \log(z - z_*). \quad (\text{C8})$$

Therefore,

$$\mathcal{S} \sim M(z_{\text{DW}}) \sqrt{\mathcal{F}(z_{\text{DW}})} \ell. \quad (\text{C9})$$

Then, we have

$$\sigma_{\text{DW}} = M(z_{\text{DW}}) \sqrt{\mathcal{F}(z_{\text{DW}})}, \quad (\text{C10})$$

where σ_{DW} is the spatial string tension in the DW configuration.

(ii) There is no stationary point in the region $0 < z < z_h$ for $\mathcal{V}(z)$. We suppose the near-horizon expansion

$$\mathcal{F}(z) = \mathfrak{F}(z_h)(z - z_h) + \mathcal{O}((z - z_h)^2). \quad (\text{C11})$$

In this configuration, the string stretches on the horizon and we take $z_* = z_h$. We propose two items:

(i) if $M(z_h) \neq \infty$, we have

$$\ell \rightarrow \infty, \quad (\text{C12})$$

$$\mathcal{S} \rightarrow 0; \quad (\text{C13})$$

(ii) if $M(z) \xrightarrow{z \rightarrow z_h} \infty$ as

$$M(z) \underset{z \rightarrow z_h}{\sim} \frac{\mathbf{m}(z_h)}{\sqrt{z - z_h}}; \quad (\text{C14})$$

then, using Eqs. (C3) and (C4), we have

$$\ell \underset{z \rightarrow z_h}{\sim} \frac{1}{\sqrt{\mathfrak{F}(z_h)}} \frac{1}{\sqrt{\frac{2\mathcal{V}'(z_h)}{\mathcal{V}(z_h)}}} \log(z - z_h), \quad (\text{C15})$$

$$\mathcal{S} \underset{z \rightarrow z_h}{\sim} \mathbf{m}(z_h) \frac{1}{\sqrt{\frac{2\mathcal{V}'(z_h)}{\mathcal{V}(z_h)}}} \log(z - z_h), \quad (\text{C16})$$

and therefore,

$$\sigma_{z_h} = \mathbf{m}(z_h) \sqrt{\mathfrak{F}(z_h)} = M(z_h) \sqrt{\mathcal{F}(z_h)}, \quad (\text{C17})$$

where σ_{z_h} is the spatial string tension in the horizon configuration.

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