

Extremal black hole with a unique ground state

Swapnamay Mondal 

*Department of Physics, Institute of Science, Banaras Hindu University, Varanasi, 221005, India
and Dublin Institute for Advanced Studies, 10 Burlington Road, Dublin, Ireland*



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Recent computations in gravity suggest that nonsupersymmetric extremal black holes lack any sizeable ground state degeneracy. We confirm this for a D-brane description of nonsupersymmetric four-charge extremal black holes in $N = 8$ string theory. The microscopic description comprises four stacks of D-branes wrapping various cycles of the internal six torus and intersecting at a point. The orientations of the stacks are such that supersymmetry is broken completely. We construct the low energy worldline Lagrangian for the brane system, which is seen to have 32 Goldstinos and 28 Goldstones. The Hamiltonian has a unique ground state, which carries a nonzero energy implying the absence of any truly extremal state. Our arguments rely on symmetry and hence should hold in the supergravity regime as well.

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I. INTRODUCTION

As per semiclassical gravity, extremal black holes carry vanishing temperature. Then standard statistical mechanics entails that the ground state degeneracy of extremal black holes should be exponential of its entropy. This is a huge degeneracy and begs the question: what stops this degeneracy from getting lifted? For supersymmetric black holes, the answer would be supersymmetry. But for nonsupersymmetric black holes, the answer is less than clear.

This issue has been debated for quite some time, e.g., Page has suggested there are no true degeneracies for nonsupersymmetric extremal black holes [1]. In fact the semiclassical analysis also indicates the presence of a gap and it has been suggested that Hawking's analysis itself may not be trustable below this gap [2,3]. Recently, for near-extremal Reissner-Nordström black holes, it has been shown that for low temperatures black hole entropy $S \sim \log T$, where T is the temperature and the proportionality constant is positive [4]. This relation implies a vanishing density of states near the ground state. These results are hard to reconcile with a sizeable ground state degeneracy.

Only a microscopic description of a black hole can settle this debate. Most well-known microscopic descriptions are two-dimensional conformal field theories (CFTs) capturing the dynamics of an effective string wrapping a circle [5,6]. Black holes, including nonsupersymmetric ones appear as

highly degenerate states in such CFTs. Entropies of a great many nonsupersymmetric black holes have been accounted for in this way [7–20]. However this degeneracy is likely a consequence of conformal symmetry, rather than extremality, e.g., even nonextremal states have exponential degeneracies in these settings [9,13,15]. Furthermore the relevant CFTs are free, which precludes any important effect interactions could have had [21]. If one could consider the full fledged sigma model describing the effective string (e.g., [22] for the CFT in [6]), perhaps the true origin of this degeneracy could be deciphered.

We take a different route. We consider a microscopic description where the low energy physics of the brane system is that of an effective particle, captured by a quantum mechanics. Interactions are captured in the potential. Unlike in a CFT, large degeneracy is atypical of quantum mechanics and hence such microscopic descriptions are perfect for settling this issue. Such a description was considered in [23], but the state counting amounted to counting brane intersections and hence was oblivious to the dynamics. Instead we exploit the $D2-D2-D2-D6$ black hole, for which the explicit microscopic Hamiltonian was constructed in [24,25]. Our strategy would be to flip the orientation of one of the constituent D-brane stacks, say the $D6$ stack, so as to break supersymmetry completely [26]. A closely related system was studied in [27], which realized non-Bogomol'nyi-Prasad-Sommerfield (BPS) extremal black holes as threshold bound states of BPS-branes. Also see [28]. But no microscopic Hamiltonian was constructed in these works. In this work, despite the lack of supersymmetry, we are able to determine the Hamiltonian to a significant extent, owing to supersymmetric nature of individual stacks. The resultant theory hosts 32 Goldstinos due to 32 broken supercharges

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of type IIA string theory. The theory makes the absence of any ground degeneracy explicit. Also, the nonzero energy of the ground state implies that quantum mechanically there are no truly extremal nonsupersymmetric states.

II. THE FOUR-CHARGE BPS BLACK HOLE

To begin with, we quickly gather the essentials of the $D2-D2-D2-D6$ system in type IIA string theory on $\mathbb{R}^{3,1} \times T^6$. The three D2-branes wrap three disjoint two cycles of the T^6 , say x^4-x^5, x^6-x^7 , and x^8-x^9 . The D6-branes wrap the entire T^6 . The branes intersect at a point, hence the low energy dynamics is that of an effective particle. The number of branes in various stacks amounts to charges carried by the black hole. The number of degrees of freedom of this effective particle grows quadratically with charges. For large charges the entropy of the corresponding black hole is $2\pi\sqrt{N_1N_2N_3N_4}$, where N_1, N_2, N_3, N_4 , respectively, are the number of branes in four stacks [29].

In order to write down the worldline theory of the effective particle, another crucial piece of information is the amount of supersymmetry preserved by the system. The ten-dimensional type IIA string theory has the supercharges $Q = \bar{\epsilon}_L Q_L + \bar{\epsilon}_R Q_R$, where $Q_{L,R}$ are the left/right moving supercharges and $\epsilon_{L,R}$ are ten-dimensional Majorana-Weyl spinors with opposite chirality. The four stack of D-branes imposes the following four conditions on $\epsilon_{L,R}$:

$$\begin{aligned} \epsilon_L &= s_{45}\Gamma^0\Gamma^4\Gamma^5\epsilon_R, \\ \epsilon_L &= s_{67}\Gamma^0\Gamma^6\Gamma^7\epsilon_R, \\ \epsilon_L &= s_{89}\Gamma^0\Gamma^8\Gamma^9\epsilon_R, \\ \epsilon_L &= s_{456789}\Gamma^0\Gamma^4\Gamma^5\Gamma^6\Gamma^7\Gamma^8\Gamma^9\epsilon_R, \end{aligned} \quad (1)$$

where s -s are signs and Γ -s are ten-dimensional gamma matrices. Simultaneous satisfaction of these conditions requires $s_{45}s_{67}s_{89} = -s_{456789}$, in which cases, four supercharges are preserved. In the remaining cases, no supercharges exist. Note that the individual signs are not very consequential for the worldline theory, only the relative signs are. To understand how the supersymmetry plays out at the level of the worldline Lagrangian, it is instructive to add stacks one by one. Since the theory is finally going to be reduced to $0+1$ dimension, we do so to start with.

A. A single stack of D-branes

A single stack preserves 16 supercharges. The dynamics of zero modes is the same as the dynamics of D0-branes, captured by the BFSS model [30]. Alternatively one can think of the dynamics as zero mode dynamics of a stack of D3-branes, captured by the $\mathcal{N} = 4$ super Yang Mills theory [31]. We find it more convenient to use the language of four dimensional supersymmetry, with the implicit assumption that the fields depend only on time.

A $\mathcal{N} = 4$ multiplet consists of one $\mathcal{N} = 1$ vector multiplet $V = (A^\mu, \lambda)$ and three $\mathcal{N} = 1$ chiral multiplets

$\Phi_i = (\phi_i, \psi_i)$, $i = 1, 2, 3$. Here we have expressed the on-shell field contents as (boson, fermion). The complex bosons ϕ_1, ϕ_2, ϕ_3 , respectively, describe the complex coordinate/Wilson line of/along the 45, 67, 89 two cycles [32]. The periodicity of the torus coordinates/Wilson lines will be ignored, as we assume the branes never fluctuate enough to see the compact nature of the two tori. In fact the six real scalars are related by $SO(6)$ rotations. The spatial components of the vector field A^μ describe the transverse coordinates of the branes along three flat directions.

The $\mathcal{N} = 4$ superalgebra has a $SU(4)_R$ R symmetry. This is reflected in the rotation of four fermions, coming from four $\mathcal{N} = 1$ multiplets as well as in the $SO(6) \sim SU(4)$ rotation of scalars. The overall $U(1)$ parts of the four fermions, coming from four $\mathcal{N} = 1$ multiplets, lead to 16 kappa symmetries signifying 16 broken supersymmetries.

The superpotential is $W_{\mathcal{N}=4} = \sqrt{2}\text{Tr}\Phi_1[\Phi_2, \Phi_3]$. Note the $\mathcal{N} = 4$ super Yang Mills is oblivious to the orientation of the brane, which becomes evident in its coupling to the Ramond-Ramond forms. But this would not concern us.

For rest of the paper, we will take each stack to contain a single brane, rendering the overall gauge group to be $U(1) \times U(1) \times U(1) \times U(1)$. Thus all fields will be numbers.

B. Two intersecting stacks of D-branes

Another stack of D-branes breaks half of the supersymmetries preserved by the first brane. Thus a total of eight supercharges are preserved, amounting to $\mathcal{N} = 2$ supersymmetry in four dimensions. The $\mathcal{N} = 4$ multiplet (say of the first brane) can be decomposed in one $\mathcal{N} = 2$ vector and another $\mathcal{N} = 2$ hypermultiplet. An $\mathcal{N} = 2$ vector multiplet contains the $\mathcal{N} = 1$ vector V and another $\mathcal{N} = 1$ chiral, which could be any one of Φ_1, Φ_2, Φ_3 . The remaining two $\mathcal{N} = 1$ chirals make a $\mathcal{N} = 2$ hyper. Clearly, there are three ways to decompose the $\mathcal{N} = 4$ multiplet in terms of $\mathcal{N} = 2$ ones. Which decomposition is realized depends on the specific D-brane pair. Common transverse directions fit into the $\mathcal{N} = 2$ vector, which features in the interaction.

Apart from the $\mathcal{N} = 4$ super Yang Mills fields, now there are additional fields coming from the strings stretched between two different stacks. The massless modes of these strings lead to a $\mathcal{N} = 2$ hypermultiplet in the bifundamental, containing two $\mathcal{N} = 1$ chiral multiplets. For the $(ij)^{th}$ pair of D-branes, we denote these $\mathcal{N} = 1$ chirals by $Z^{(ij)} = (z^{(ij)}, \chi^{(ij)})$, $Z^{(ji)} = (z^{(ji)}, \chi^{(ji)})$. They carry opposite charges.

For definiteness, let us consider the pair (12), where the first stack of D2-branes wraps the (45) cycle and the second stack wraps the (67) cycle. Then the relevant $\mathcal{N} = 2$ vector (of either branes) contains $V = (A^\mu, \lambda)$ and $\Phi_3 = (\phi_3, \psi_3)$. We look at the supersymmetry from the perspective of the first brane.

Transverse separations appear through the supermultiplets $V^{(1)} - V^{(2)}$ and $\Phi_3^{(1)} - \Phi_3^{(2)}$, where the superscript denotes the stack index. The combination $V^{(1)} - V^{(2)}$

appears in the kinetic term, whereas the superpotential $W_{12} = \sqrt{2}Z^{(12)}(\Phi_3^{(2)} - \Phi_3^{(1)})Z^{(21)}$ fashions $\Phi_3^{(2)} - \Phi_3^{(1)}$. The $\mathcal{N} = 2$ hypermultiplet comprising Φ_1, Φ_2 does not appear in the interaction.

The $\mathcal{N} = 2$ algebra in turn contains two $\mathcal{N} = 1$ subalgebras, related through the $SU(2)_R$ R symmetry, which in particular involves the exchange $z^{(12)} \leftrightarrow -i(z^{(21)})^\dagger$, $\psi_3^{(k)} \leftrightarrow \lambda^{(k)}$, $k=1,2$. In other words, the Lagrangian remains invariant, if one uses the following $\mathcal{N} = 1$ multiplets

$$\begin{aligned}\tilde{V}^{(k)} &= (A^{(k)}, \psi_3^{(k)}), & \tilde{\Phi}_3^{(k)} &= (\phi_3^{(k)}, \lambda^{(k)}), \\ \tilde{\Phi}_1^{(k)} &= (-i(\phi_2^{(k)})^\dagger, \psi_1^{(k)}), & \tilde{\Phi}_2^{(k)} &= (-i(\phi_1^{(k)})^\dagger, \psi_2^{(k)}), \\ \tilde{Z}^{(12)} &= (-i(z^{(21)})^\dagger, \chi^{(12)}), & \tilde{Z}^{(21)} &= (-i(z^{(12)})^\dagger, \chi^{(21)}),\end{aligned}\quad (2)$$

$k = 1, 2$, and A denotes the gauge field.

For other pairs, the gauge interactions are straightforward, whereas the superpotentials, respectively, read

$$\begin{aligned}W_{12} &= \sqrt{2}Z^{(12)}(\Phi_3^{(2)} - \Phi_3^{(1)})Z^{(21)}, \\ W_{23} &= \sqrt{2}Z^{(32)}(\Phi_1^{(2)} - \Phi_1^{(3)})Z^{(23)}, \\ W_{31} &= \sqrt{2}Z^{(13)}(\Phi_2^{(3)} - \Phi_2^{(1)})Z^{(31)}, \\ W_{14} &= \sqrt{2}Z^{(41)}(\Phi_1^{(1)} - \Phi_1^{(4)})Z^{(14)}, \\ W_{24} &= \sqrt{2}Z^{(42)}(\Phi_2^{(2)} - \Phi_2^{(4)})Z^{(24)}, \\ W_{34} &= \sqrt{2}Z^{(43)}(\Phi_3^{(3)} - \Phi_3^{(4)})Z^{(34)}.\end{aligned}\quad (3)$$

C. Three stacks of D-branes

Now let us add a third stack, say the D2-brane stack wrapping x^8-x^9 . Three stacks altogether preserve $\mathcal{N} = 1$ supersymmetry. There are two distinct ways this could be done: either by using the multiplets $(V^{(k)}, \Phi_3^{(k)}, Z^{(12)}, Z^{(21)})$, or by using the multiplets $(\tilde{V}^{(k)}, \tilde{\Phi}_3^{(k)}, \tilde{Z}^{(12)}, \tilde{Z}^{(21)})$, where $k = 1, 2$. Reduced supersymmetry opens the doors to new terms in the superpotentials W_{lin} , linear in Φ , and W_{cub} , cubic in Z . The choice of the particular $\mathcal{N} = 1$ subalgebra manifests in additional terms in the superpotential, where either $\Phi_3^{(k)}, Z^{(12)}, Z^{(21)}$ appears, or $\tilde{\Phi}_3^{(k)}, \tilde{Z}^{(12)}, \tilde{Z}^{(21)}$. Either way, the difference can be washed away by renaming fields. Note, unlike in the case of higher supersymmetries, these terms are not fixed by supersymmetry. Reduced supersymmetry also allows for Fayet Iliopoulos parameter $\zeta^{(i)}$ for the i th stack [33].

D. Four stacks of D-branes

The orientation of the fourth brane is no longer inconsequential. Depending on whether the condition (1) is satisfied or not, the whole system would preserve either $\mathcal{N} = 1$ or no supersymmetry.

Preservation of $\mathcal{N} = 1$ supersymmetry implies that the total Lagrangian would be expressible in terms of $\mathcal{N} = 1$

superfields. This can simply be achieved by using superfields $(V^{(k)}, \Phi_1^{(k)}, \Phi_2^{(k)}, \Phi_3^{(k)})$ for the k th stack and the superfields $(Z^{(kl)}, Z^{(lk)})$ for the (kl) th pair of stacks. Apart from the pairwise pieces (3), the superpotential comprises the following pieces

$$\begin{aligned}W_{lin} &= \sqrt{2}\left(c_{12}(\Phi_3^{(1)} - \Phi_3^{(2)}) + c_{23}(\Phi_1^{(2)} - \Phi_1^{(3)}) \right. \\ &\quad + c_{31}(\Phi_2^{(3)} - \Phi_2^{(1)}) + c_{14}(\Phi_1^{(1)} - \Phi_1^{(4)}) \\ &\quad \left. + c_{24}(\Phi_2^{(2)} - \Phi_2^{(4)}) + c_{34}(\Phi_3^{(3)} - \Phi_3^{(4)})\right),\end{aligned}\quad (4)$$

$$\begin{aligned}W_{cub} &= \sqrt{2}[Z^{(31)}Z^{(12)}Z^{(23)} + Z^{(13)}Z^{(32)}Z^{(21)} \\ &\quad + Z^{(12)}Z^{(24)}Z^{(41)} + Z^{(42)}Z^{(21)}Z^{(14)} \\ &\quad - Z^{(13)}Z^{(34)}Z^{(41)} + Z^{(31)}Z^{(14)}Z^{(43)} \\ &\quad + Z^{(34)}Z^{(42)}Z^{(23)} + Z^{(43)}Z^{(32)}Z^{(24)}].\end{aligned}\quad (5)$$

This system reproduces the correct degeneracies [24,25].

III. THE FOUR-CHARGE EXTREMAL NON-BPS BLACK HOLE

We set out by noting that the worldline Lagrangian should satisfy the following properties:

- (1) Over all there should be 28 Goldstones and 32 Goldstinos.
- (2)
 - (a) each stack should preserve 16 supercharges,
 - (b) each pair of stacks should preserve eight supercharges,
 - (c) each triplet of stacks should preserve four supercharges,
 - (d) four triplets of stacks should preserve four different supersymmetries.

The number of Goldstones is the same as the 1/8 BPS-brane system, as two systems differ only by change of orientation of some D-brane stacks. The number of Goldstinos reflects the fact that all 32 supersymmetries of type IIA string theory are broken. Having different number of Goldstones and Goldstinos is challenging since condition (2a), (2b) entails that the Lagrangian is still written in terms of superfields. Satisfaction of condition (2c) is straightforward, whereas condition (2d) is nontrivial. To appreciate this, let us denote the $SU(2)_R$ symmetry corresponding to the pair (kl) as R_{kl} . Let the (123) part of the Lagrangian fashion the fields $(V^{(k)}, \Phi^{(k)})$, $k = 1, 2, 3$ and $Z^{(kl)}$, $k, l = 1, 2, 3, k \neq l$. Then demanding that the triplet (124) preserves different supersymmetry than the triplet (123) implies the pairs (14) and (24) should involve fields $R_{12}(V^{(k)}, \Phi_1^{(k)}, \Phi_2^{(k)}, \Phi_3^{(k)})$, $k = 1, 2$. This in particular means that $R_{12}\Phi_1^{(1)}$ and $R_{12}\Phi_2^{(2)}$ should feature in the superpotentials $\tilde{W}_{14}$ and $\tilde{W}_{24}$, respectively. Here $\tilde{W}_{kl}$ stands for the analog of W_{kl} for the present case. On the other hand, demanding (123) and (134) preserves different

supersymmetries in particular entails that $\tilde{W}_{14}$ should involve $R_{13}\Phi_1^{(1)}$. Since $R_{12}\Phi_1^{(1)} \neq R_{13}\Phi_1^{(1)}$, simultaneous satisfaction of (2d) proves tricky.

The solution is to augment the $SU(2)_R$ rotations by appropriate $SU(4)_R$ rotations for the adjoint fields. To this end let us first define the following multiplets

$$\begin{aligned} V &= (A, \lambda), & \Phi_1 &= (\phi_1, \psi_1), & \Phi_2 &= (\phi_2, \psi_2), & \Phi_3 &= (\phi_3, \psi_3), \\ \tilde{V} &= (A, \psi_1), & \tilde{\Phi}_1 &= (\phi_1, \lambda), & \tilde{\Phi}_2 &= (-i\phi_2^\dagger, \psi_3), & \tilde{\Phi}_3 &= (-i\phi_3^\dagger, \psi_2), \\ \tilde{\tilde{V}} &= (A, \psi_2), & \tilde{\tilde{\Phi}}_1 &= (-i\phi_1^\dagger, \psi_3), & \tilde{\tilde{\Phi}}_2 &= (\phi_2, \lambda), & \tilde{\tilde{\Phi}}_3 &= (-i\phi_3^\dagger, \psi_1), \\ \tilde{\tilde{\tilde{V}}} &= (A, \psi_3), & \tilde{\tilde{\tilde{\Phi}}}_1 &= (-i\phi_1^\dagger, \psi_2), & \tilde{\tilde{\tilde{\Phi}}}_2 &= (-i\phi_2^\dagger, \psi_1), & \tilde{\tilde{\tilde{\Phi}}}_3 &= (\phi_3, \lambda). \end{aligned} \quad (6)$$

For the sake of brevity we shall specify the choice of these multiplets, respectively, as (V, Φ) , $(\tilde{V}, \tilde{\Phi})$, $(\tilde{\tilde{V}}, \tilde{\tilde{\Phi}})$, $(\tilde{\tilde{\tilde{V}}}, \tilde{\tilde{\tilde{\Phi}}})$. Note that the $\mathcal{N} = 4$ parts of the Lagrangian are agnostic to which of these supermultiplets are used. Let us denote the R symmetry that rotates the fermions of V and Φ_i as R_i . Define $\tilde{R}_i, \tilde{\tilde{R}}_i, \tilde{\tilde{\tilde{R}}}_i$ similarly. Then it can be checked that up to nomenclature

$$\begin{aligned} R_1(V, \Phi) &= (\tilde{V}, \tilde{\Phi}), & R_2(V, \Phi) &= (\tilde{\tilde{V}}, \tilde{\tilde{\Phi}}), & R_3(V, \Phi) &= (\tilde{\tilde{\tilde{V}}}, \tilde{\tilde{\tilde{\Phi}}}), \\ \tilde{R}_1(\tilde{V}, \tilde{\Phi}) &= (V, \Phi), & \tilde{R}_2(\tilde{V}, \tilde{\Phi}) &= (\tilde{\tilde{V}}, \tilde{\tilde{\Phi}}), & \tilde{R}_3(\tilde{V}, \tilde{\Phi}) &= (\tilde{\tilde{\tilde{V}}}, \tilde{\tilde{\tilde{\Phi}}}), \\ \tilde{\tilde{R}}_1(\tilde{\tilde{V}}, \tilde{\tilde{\Phi}}) &= (\tilde{\tilde{\tilde{V}}}, \tilde{\tilde{\tilde{\Phi}}}), & \tilde{\tilde{R}}_2(\tilde{\tilde{V}}, \tilde{\tilde{\Phi}}) &= (\tilde{V}, \tilde{\Phi}), & \tilde{\tilde{R}}_3(\tilde{\tilde{V}}, \tilde{\tilde{\Phi}}) &= (\tilde{V}, \tilde{\Phi}), \\ \tilde{\tilde{\tilde{R}}}_1(\tilde{\tilde{\tilde{V}}}, \tilde{\tilde{\tilde{\Phi}}}) &= (\tilde{\tilde{V}}, \tilde{\tilde{\Phi}}), & \tilde{\tilde{\tilde{R}}}_2(\tilde{\tilde{\tilde{V}}}, \tilde{\tilde{\tilde{\Phi}}}) &= (\tilde{V}, \tilde{\Phi}), & \tilde{\tilde{\tilde{R}}}_3(\tilde{\tilde{\tilde{V}}}, \tilde{\tilde{\tilde{\Phi}}}) &= (V, \Phi). \end{aligned} \quad (7)$$

Note $R_i, \tilde{R}_i, \tilde{\tilde{R}}_i, \tilde{\tilde{\tilde{R}}}_i$ are, respectively, associated with the brane pairs $(i, 4)$ and (j, k) where $i, j, k = 1, 2, 3$ and $i \neq j \neq k$.

The superpotentials associated to various pairs are $\tilde{W}_{12} = W_{12}$, $\tilde{W}_{23} = W_{23}$, $\tilde{W}_{31} = W_{31}$, $\tilde{W}_{14} = \sqrt{2}Z^{(14)}\tilde{\tilde{\Phi}}_1^{(1,4)}Z^{(41)}$, $\tilde{W}_{24} = \sqrt{2}Z^{(24)}\tilde{\tilde{\Phi}}_2^{(2,4)}Z^{(42)}$, and $\tilde{W}_{34} = \sqrt{2}Z^{(34)}\tilde{\tilde{\Phi}}_3^{(3,4)}Z^{(43)}$. In the gauge interaction of these pairs, we respectively have $V^{(1)} - V^{(2)}$, $V^{(2)} - V^{(3)}$, $V^{(3)} - V^{(1)}$, $\tilde{\tilde{V}}^{(1)} - \tilde{\tilde{V}}^{(4)}$, $\tilde{\tilde{V}}^{(2)} - \tilde{\tilde{V}}^{(4)}$, $\tilde{\tilde{V}}^{(3)} - \tilde{\tilde{V}}^{(4)}$.

The Goldstones are same as the supersymmetric case: $\tilde{X}^{(1)} + \tilde{X}^{(2)} + \tilde{X}^{(3)} + \tilde{X}^{(4)}$, $\phi_3^{(1)} + \phi_3^{(2)}$, $\phi_1^{(2)} + \phi_1^{(3)}$, $\phi_2^{(3)} + \phi_2^{(1)}$, $\phi_1^{(1)} + \phi_1^{(4)}$, $\phi_2^{(2)} + \phi_2^{(4)}$, $\phi_3^{(3)} + \phi_3^{(4)}$. Note the vector multiplet actually leads to two Goldstones, not three, since the Gauss law constraint negates one.

Coming to Goldstinos we first note that only the following combinations of fermions appear in interaction

$$\begin{aligned} \psi_3^{(1)} - \psi_3^{(2)}, & \quad \psi_1^{(2)} - \psi_1^{(3)}, & \quad \psi_2^{(3)} - \psi_2^{(1)}, \\ \psi_2^{(1)} - \psi_2^{(4)}, & \quad \psi_1^{(2)} - \psi_1^{(4)}, & \quad \psi_2^{(3)} - \psi_2^{(4)}, \\ \psi_3^{(1)} - \psi_3^{(4)}, & \quad \psi_3^{(2)} - \psi_3^{(4)}, & \quad \psi_1^{(3)} - \psi_1^{(4)}, \\ \lambda^{(1)} - \lambda^{(2)}, & \quad \lambda^{(2)} - \lambda^{(3)}, & \quad \lambda^{(3)} - \lambda^{(1)}, \end{aligned} \quad (8)$$

implying that the remaining combinations are Goldstinos

$$\begin{aligned} \psi_1^{(1)}, \psi_1^{(2)} + \psi_1^{(3)} + \psi_1^{(4)}, \\ \psi_2^{(2)}, \psi_2^{(1)} + \psi_2^{(3)} + \psi_2^{(4)}, \\ \psi_3^{(3)}, \psi_3^{(1)} + \psi_3^{(2)} + \psi_3^{(4)}, \\ \lambda^{(4)}, \lambda^{(1)} + \lambda^{(2)} + \lambda^{(3)}. \end{aligned} \quad (9)$$

We see there are 32 Goldstinos, signifying spontaneous breaking of all 32 supersymmetries.

At this stage, let us check that various supersymmetry requirements are met. Pairwise supersymmetry is there by construction. Coming to triplets, the supersymmetry of (123) pair is apparent. Supersymmetry of (124) is not apparent since $\tilde{W}_{12}$ fashions (V, Φ) superfields, whereas $\tilde{W}_{14}, \tilde{W}_{24}$ fashion $(\tilde{\tilde{V}}, \tilde{\tilde{\Phi}})$ superfields. Acting R_3 on $\tilde{W}_{12}$ expresses all of them in terms of $(\tilde{\tilde{V}}, \tilde{\tilde{\Phi}})$ superfields, exhibiting that the triplet (124) preserves $\mathcal{N} = 1$ supersymmetry, but a different one compared to the triplet (123). Note R_3 also acts on Z^{12}, Z^{21} .

To see the supersymmetry of (134), one has to $SU(2)_R$ rotate $\tilde{W}_{13}, \tilde{W}_{14}, \tilde{W}_{34}$, respectively, by $R_2, \tilde{\tilde{R}}_1, \tilde{\tilde{R}}_3$. These are precisely the $SU(2)_R$ rotations associated with those brane pairs. Similarly to see the supersymmetry of the triplet (234), one has to $SU(2)_R$ rotate $\tilde{W}_{23}, \tilde{W}_{24}$, respectively, by $R_1, \tilde{\tilde{R}}_2$. The relative $SU(2)_R$ rotation $\tilde{\tilde{R}}_3$ between (134) and (234) fields imply these triplets preserve different $\mathcal{N} = 1$ supersymmetries. To summarize, the triplets (123), (234),

(134), (124), make supersymmetry apparent, respectively, in terms of (V, Φ) , $(\tilde{V}, \tilde{\Phi})$, $(\tilde{\tilde{V}}, \tilde{\tilde{\Phi}})$, and $(\tilde{\tilde{\tilde{V}}}, \tilde{\tilde{\tilde{\Phi}}})$.

For the analog of the cubic superpotential (134), we note each piece corresponds to a triplet. Supersymmetry considerations of various triplets lead to the following cubic superpotential

$$\begin{aligned} \tilde{W}_{cub} = & \sqrt{2}[Z^{(31)}Z^{(12)}Z^{(23)} + Z^{(13)}Z^{(32)}Z^{(21)} \\ & + \tilde{Z}^{(12)}Z^{(24)}Z^{(41)} + Z^{(42)}\tilde{Z}^{(21)}Z^{(14)} \\ & - \tilde{Z}^{(13)}\tilde{Z}^{(34)}\tilde{Z}^{(41)} + \tilde{Z}^{(31)}\tilde{Z}^{(14)}\tilde{Z}^{(43)} \\ & + Z^{(34)}\tilde{Z}^{(42)}\tilde{Z}^{(23)} + Z^{(43)}\tilde{Z}^{(32)}\tilde{Z}^{(24)}], \end{aligned} \quad (10)$$

where $\tilde{Z}^{(ij)}$ denotes the appropriate $SU(2)_R$ rotated form of $Z^{(ij)}$. Clearly, the triplets (ijk) and (ijl) , $i, j, k, l = 1, 2, 3, 4, i \neq j \neq k \neq l$ preserve different supersymmetries.

Finding counterpart of the linear superpotential W_{lin} (4) proves tricky though, e.g., consider the piece $\sqrt{2}c_{12}(\Phi_3^{(1)} - \Phi_3^{(2)})$ associated with the pair (12). This piece explicitly preserves a particular $\mathcal{N} = 1$ subgroup of $\mathcal{N} = 2$ supersymmetry preserved by the pair (12). In the nonsupersymmetric case, this subgroup is preserved either by the triplet (123) or by (124) but not both. Therefore this piece is bound to be inconsistent with the supersymmetry of either (123) or (124). Similar problem exists for other pieces of (4). It is not clear which superfields should appear in the superpotential. We postulate the most general linear superpotential consistent with the Goldstones and Goldstinos

$$\begin{aligned} \tilde{W}_{lin} = & \sqrt{2}(c'_{12}\Phi_3^{(1,2)} + c'_{23}\Phi_1^{(2,3)} + c'_{31}\Phi_2^{(3,1)} + c'_{14}\tilde{\tilde{\Phi}}_1^{(1,4)} \\ & + c'_{24}\tilde{\tilde{\Phi}}_2^{(2,4)} + c'_{34}\tilde{\tilde{\Phi}}_3^{(3,4)} + c''_{12}\tilde{\tilde{\Phi}}_3^{(1,2)} + c''_{23}\tilde{\tilde{\Phi}}_1^{(2,3)} \\ & + c''_{31}\tilde{\tilde{\Phi}}_2^{(3,1)} + c''_{14}\tilde{\tilde{\Phi}}_1^{(1,4)} + c''_{24}\tilde{\tilde{\Phi}}_2^{(2,4)} + c''_{34}\tilde{\tilde{\Phi}}_3^{(3,4)}). \end{aligned}$$

A. The potential

The potential has the form

$$V = V_{\text{gauge}} + V_D + V_F. \quad (11)$$

Here V_{gauge} comes from the spatial part of the gauge kinetic term and is a sum of pairwise terms $|\vec{X}^{(i)} - \vec{X}^{(j)}|^2 |Z^{(ij)}|^2$. For generic values of Z fields, this can be set to zero by taking $\vec{X}^{(i)} - \vec{X}^{(j)} = \vec{0} \forall i \neq j$, i.e., all branes to coincide along the flat directions. Minimizations in F and D term potentials are not affected by this, since $\vec{X}$ fields do not appear there.

The D-term potential V_D is entirely fixed by the interaction of the gauge multiplets and hence is the same as in the BPS case. The Fayet-Iliopoulos parameters were determined solely in terms of the background fields in the BPS case [24,25]. We expect this to continue in the present case.

Finally we come to the F-term potential V_F [34]. The most important feature of F-term equations is that they are no longer holomorphic. This immediately requires a change of strategy in finding minima as compared to the supersymmetric case. In the supersymmetric case, the F-term equations are invariant under not only the $U(1) \times U(1) \times U(1)$ gauge symmetry, but an enhanced $\mathbb{C}^* \times \mathbb{C}^* \times \mathbb{C}^*$ symmetry. The effect of D-term equations can be incorporated as gauging this enhanced symmetry. Thus it suffices to solve the F-term equations alone subject to the complexified gauge invariance. But now due to lack of holomorphicity there is no such enhanced symmetry. So one must write down the whole potential and find its minima numerically.

B. The ground state

Semiclassical black hole thermodynamics suggests the presence of a highly degenerate ground state. For this, one would first require the potential to have large number of degenerate zero energy minima, and further the quantum ground state energies around each minima to be zero. For the BPS black holes, in harmonic approximation this comes from pairwise cancelation of bosonic and fermionic contributions, which in particular requires an equal number of bosonic and fermionic oscillators. Although the non-BPS worldline theory has an equal number of bosons and fermions, due to unequal number of Goldstones and Goldstinos, the number of interacting bosons and fermions differs by 4. This mismatch makes it impossible for the bosonic and fermionic ground state energies to cancel out. There is no reason why these positive ground state energies corresponding to different minima should coincide, except the $\mathbb{Z}_2$ symmetry due to sign change of the fields. Such twofold degeneracies would be lifted nonperturbatively. Hence the extremal black hole has a unique ground state, which is not quite extremal.

C. Large charges

The name “black hole” is really justified in the large charge limit, i.e., large N_i -s. For $N_i > 1$, various degrees of freedom are elevated to matrices, and the worldline theory becomes a non-Abelian one. In particular this introduces a new piece $W_{\mathcal{N}=4}$ in the superpotential, discussed earlier. However the number of Goldstones and Goldstinos remain the same as in the Abelian case. Thus the existence of a unique ground state holds true even in the large charge regime.

h. Turning gravity on: The D-brane description is valid in the limit of vanishing Newton’s constant. It has been argued that for CFT descriptions, the effect of turning gravity on is captured by $T\bar{T}$ deformations [35]. But for quantum mechanical descriptions, it is not clear how the theory (or even the degrees of freedom) changes. Whatever it may be, as no symmetry is being upset, the “black hole quantum mechanics” should continue to have the same number of

Goldstones and Goldstinos as the D-brane system. The number of Goldstinos is evident in the supergravity. As for the Goldstones, we note the bosonic part of the non-BPS and BPS black holes differs only by signs of certain gauge fields. Thus the key ingredient of our argument, namely the unequal number of interacting bosons and fermions, remains intact. Hence the black hole quantum mechanics, whatever it may be, can not have a degenerate ground state either.

IV. DISCUSSION

Reissner Nordström black holes have vanishing angular momentum. In the D-brane description angular momenta of individual microstates can be defined using Lefschetz $SU(2)$ [25,36–41] for supersymmetric black holes. But the extension of this notion to the present case is not straightforward, as the vacuum manifold is not complex.

The nonsupersymmetric nature of the system entails that individual theories on various brane worldvolumes will undergo nontrivial renormalization group flows, which in turn will modify the quantum mechanics capturing the zero mode sector. Symmetries, and therefore our conclusions, will not be affected by this though.

Coming to the elephant in the room, namely the extremal entropy, we note now that supersymmetry is gone, there are no protected quantities. Nevertheless given numerous instances of microstate counting accounting for nonsupersymmetric entropy one would be hopeful that some notion of

extremal entropy should persist in the microscopic description. Two extremal black holes differing only by the signs of the charges have the same Bekenstein-Hawking entropies, although subleading corrections showing up in the quantum entropy functions would differ. This implies that the extremal nonsupersymmetric entropy should not be too far from that of the corresponding supersymmetric black hole.

The sheer absence of any truly extremal state however makes a definition of the extremal entropy in the microscopic description tricky. Should the potential admit degenerate zero energy minima, the scenario envisaged in [42] would be an interesting possibility.

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DATA AVAILABILITY

No data were created or analyzed in this study.

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