

Bulk reconstruction in 2D multihorizon black hole

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The goal of the bulk reconstruction program is to construct boundary representations of fields in asymptotically anti-de Sitter spacetimes. In this paper, we extend the program by computing the boundary representation of massless fields in an Achúcarro-Ortiz black hole spacetime. We obtain analytic expressions for smearing functions in both the exterior and interior of the black hole. We also obtain expressions for Papadodimas-Raju mirror operators.

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I. INTRODUCTION

According to the AdS/CFT correspondence, all physical information of a $d + 1$ -dimensional anti-de Sitter spacetime is fully encoded in a d -dimensional conformal field theory. Figuring out the complete dictionary between bulk and boundary is a key open problem.

A first step toward this goal is translating bulk fields in the bulk theory to operators in the boundary CFT. The program of mapping bulk fields to boundary operators has come to be known as bulk reconstruction [1–25]. For a recent review, we refer to [26].

When considering the mapping between the entire bulk and the boundary, one finds a boundary representation of the form:

$$\phi(r, \mathbf{x}) = \int d^d \mathbf{x}' K(r, \mathbf{x}, \mathbf{x}') \mathcal{O}(\mathbf{x}').$$

Here $(r, \mathbf{x})$ are the bulk coordinates, and $\mathbf{x}'$ are the boundary coordinates.

Here, $K(r, \mathbf{x}, \mathbf{x}')$ is the smearing function, and the integration is over the entire boundary. This is known as HKLL construction in the literature.¹

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¹For subregion reconstructions, this applies for causal wedges. To reconstruct a bulk field outside of causal wedge but still in the entanglement wedge, a different construction is needed [27,28]. We do not consider subregion reconstruction in this paper.

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In this paper, we have constructed boundary representation for fields in a two-dimensional black hole background. First, let us explain the motivation for this.

The question of bulk reconstruction in black hole backgrounds has been of interest for a long time. In AdS/CFT, the smoothness of the black hole horizon is equivalent to the existence of boundary representation for fields in the interior of a black hole [9,29]. For an eternal black hole, the boundary representations of fields inside the horizon can in principle be constructed via HKLL construction, with the resulting operator having support in both the boundary CFTs. For a collapsing black hole situation, where there is only one asymptotic boundary, HKLL construction does not work in the interior. Establishing horizon smoothness is more subtle in this case. One can still construct boundary representations for fields at late times by following the “mirror operator” construction of Papadodimas and Raju [30–32]. However, there is no example of an analytic mirror operator smearing function in the literature.

Recently there has been much interest in the “island solution” of the black hole information paradox. While it is understood that information escapes the black holes, it is not entirely clear how. It would be helpful if one could track the information of fields in the bulk from the boundary theory at all times. In order to do this, one would need to know the boundary representations of bulk fields (see, for instance, [33]). Explicit boundary representation formulas for black hole backgrounds would therefore be useful in developing a better understanding of information recovery from black holes.

Another interesting question pertains to the inner horizons of multi-horizon black holes. These inner horizons can be Cauchy horizons, in which case the evolution of fields beyond the inner horizon cannot be determined from data at the inner horizon. It requires boundary conditions at the

timelike singularities. But does the boundary CFT “know” how fields evolve beyond the inner horizon? It remains an open question if the region beyond the inner horizon is accessible to the boundary CFT(s) [34]. Boundary representation of fields in multihorizon spacetimes would be a helpful step in developing such an understanding.

Despite these points of interest and much work on the subject (see, for instance, [5,7,11,14,16,21]), analytic HKLL constructions of fields in the interior of a black hole are almost absent in the literature, even for double-sided black holes. Indeed, the only example is for pure AdS_2 , viewed as a “black hole” in AdS -Rindler coordinates. The issue in dimensions greater than two is that the smearing function in a black hole background does not have an analytic expression but is distributional in nature [11,35].² This makes it harder to gain insights, although some progress can be made in reconstructing delocalised wave packets instead of fields [14,21]. This problem, however, does not arise for a two-dimensional black hole, and one can hope to find analytic representations of the smearing function in this case.

All these points lead us to the problem considered in this paper: the construction of boundary representation for Achúcarro-Ortiz black holes. These are two-dimensional asymptotically anti-de Sitter black holes obtained by dimensional reduction from rotating BTZ black holes [36] (for an analysis on the stability of their Cauchy horizon, see [37]). For this background, we obtained analytic expressions for boundary representations of minimally coupled massless fields.³ We also obtain expressions for mirror operators. However, the smearing function in this case turns out to be divergent. Hence, we construct smearing functions for wave packets instead.

We have thus filled the gap in the literature for analytic expressions of boundary representations in a black hole background. The main results of the paper are given in the Eqs. (3.9), (3.19), and (4.6). These should prove useful in answering the kind of questions we have outlined above.

The paper is organized as follows. In Sec. II, we provide a brief sketch of how boundary representations are constructed in black holes. Section III deals with the case of the massless scalar. Mirror operators for massless

scalars are constructed in Sec. IV. We conclude with a summary in Sec. V.

II. BULK RECONSTRUCTION IN BLACK HOLE BACKGROUNDS

First, let us sketch the HKLL construction for pure AdS in d spacetime dimensions. Consider a scalar field $\phi(r, \mathbf{x})$ in the bulk. Let $\mathcal{O}(\mathbf{x})$ be the CFT primary dual to $\phi(r, \mathbf{x})$. From the extrapolate dictionary [38,39], the correlation function of $\phi(r, \mathbf{x})$ is related to that of $\mathcal{O}(\mathbf{x})$ as

$$\lim_{r \rightarrow \infty} r^{n\Delta} \langle \phi(r, \mathbf{x}_1) \phi(r, \mathbf{x}_2) \dots \phi(r, \mathbf{x}_n) \rangle = \langle 0 | \mathcal{O}(\mathbf{x}_1) \mathcal{O}(\mathbf{x}_2) \dots \mathcal{O}(\mathbf{x}_n) | 0 \rangle,$$

where Δ is the conformal dimension of $\mathcal{O}$. Within correlation functions, we may then write

$$\lim_{r \rightarrow \infty} r^\Delta \phi(r, \mathbf{x}) = \mathcal{O}(\mathbf{x}). \quad (2.1)$$

The idea behind HKLL construction is to solve the bulk Klein-Gordon equation with (2.1) as a “boundary condition.”

One way of doing this in practice is to start with the mode expansion:

$$\phi(r, \mathbf{x}) = \int d^d k \frac{1}{N_{\mathbf{k}}} f_{\mathbf{k}}(r) e^{i\mathbf{k} \cdot \mathbf{x}} a_{\mathbf{k}} + \text{hermitian conjugate}, \quad (2.2)$$

where $f_{\mathbf{k}}(r) e^{i\mathbf{k} \cdot \mathbf{x}}$ are the *normalizable* mode solutions of the Klein-Gordon equation, and $N_{\mathbf{k}}$ is the normalization constant. Substituting (2.1) into (2.2), we first obtain boundary representations for the creation and annihilation operators:

$$a_{\mathbf{k}} = \frac{N_{\mathbf{k}}}{M_{\mathbf{k}}} \int d^d x \mathcal{O}(\mathbf{x}) e^{-i\mathbf{k} \cdot \mathbf{x}} \equiv \frac{N_{\mathbf{k}}}{M_{\mathbf{k}}} \mathcal{O}_{\mathbf{k}}, \quad (2.3)$$

where

$$M_{\mathbf{k}} = \lim_{r \rightarrow \infty} r^\Delta f_{\mathbf{k}}(r). \quad (2.4)$$

Then substituting (2.3) in (2.2), one obtains the boundary representation of the bulk scalar, which can be expressed in the form:

$$\phi(r, \mathbf{x}) = \int d^d x' K(r, \mathbf{x}; \mathbf{x}') \mathcal{O}(\mathbf{x}'), \quad (2.5)$$

where

²One can get around this by complexifying the boundary [5]. In the complexified boundary approach, expressions for the smearing function in the interior have also been constructed. However, it is unclear that this is the right thing to do. There is no way to recover an expression on the real boundary in this approach (no analog of the $i\epsilon$ prescription), and the physical meaning of working with a complex time, with regards to unitarity, is unclear.

³Field equations for massive fields in this background do not have analytic solutions. One can also consider the case of probe fields coupled with the dilaton, but this corresponds to the zero mode of the rotating BTZ black hole, and the smearing function can be read off from the rotating BTZ case [21].

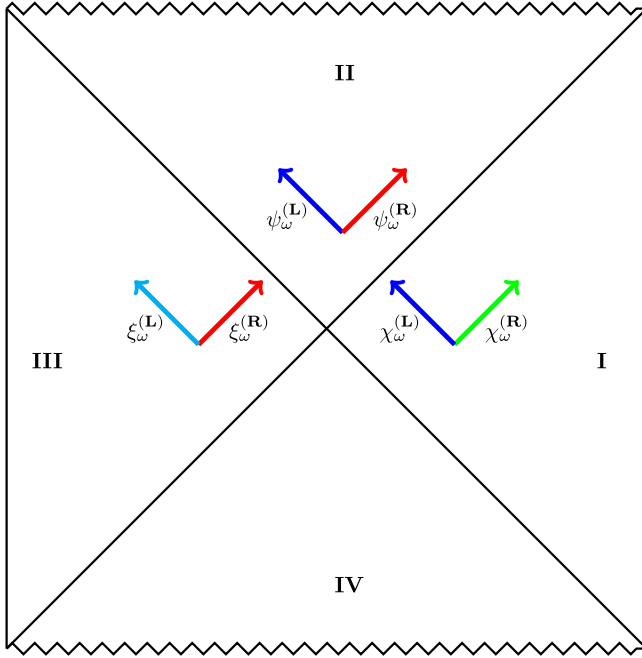


FIG. 1. Interior and exterior modes in a black hole. Left moving exterior modes $\chi_\omega^{(L)}$ from region I continue into region II. Similarly, right moving modes $\xi_\omega^{(R)}$ from region III continue into region II. These match with the interior modes $\psi_\omega^{(L)}, \psi_\omega^{(R)}$ at the horizons.

$$K(r, \mathbf{x}; \mathbf{x}') = \int d^d k \frac{1}{M_k} f_k(r) e^{i\mathbf{k} \cdot (\mathbf{x} - \mathbf{x}')} \quad (2.6)$$

is called the smearing function.

For an eternal black hole, there are two asymptotic boundary regions. In the exterior regions of the black hole (regions I and III in Fig. 1), the construction works exactly as in the case of pure AdS. So in region I, we will obtain:

$$\phi_I(r, \mathbf{x}) = \int d^d x' K_I(r, \mathbf{x}; \mathbf{x}') O(\mathbf{x}'), \quad (2.7)$$

where

$$K_I(r, \mathbf{x}; \mathbf{x}') = \int d^d k \frac{1}{M_k} f_k(r) e^{i\mathbf{k} \cdot (\mathbf{x} - \mathbf{x}')} \quad (2.8)$$

and $f_k(r)$ are the normalizable modes that die down at the asymptotic boundary of I.

The situation is different in the interior of the black hole (or white hole). The modes of the region II $\psi_k(r, \mathbf{x}) = f_k(r) e^{i\mathbf{k} \cdot \mathbf{x}}$ can be written as a linear combination of left-moving and right-moving modes:

$$\psi_k(r, \mathbf{x}) = c_k \psi_k^L(r, \mathbf{x}) + d_k \psi_k^R(r, \mathbf{x}).$$

The unknown constants are to be fixed by demanding that interior modes match the exterior normalizable modes at

the horizon. This is illustrated in Fig. 1. As is evident from the figure, the left(right) moving modes from region I(III) cross the horizon and enter the interior region.

Once c_k, d_k are fixed, the boundary representation can be computed:

$$\phi_{II}(r, \mathbf{x}) = \int d^d \mathbf{x}' K_L(r, \mathbf{x}; \mathbf{x}') \mathcal{O}_L(\mathbf{x}') + \int d^d \mathbf{x}'' K_R(r, \mathbf{x}; \mathbf{x}'') \mathcal{O}_R(\mathbf{x}''), \quad (2.9)$$

where

$$K_R(r, \mathbf{x}; \mathbf{x}') = \int d^d \mathbf{k} \frac{c_k}{M_k} \psi_k^R(r, \mathbf{x}) e^{-i\mathbf{k} \cdot \mathbf{x}'} + \text{H.c.} \quad (2.10)$$

Here, $\mathcal{O}_L, \mathcal{O}_R$ refer to operators on the left and right CFT, respectively.

$K_L(r, \mathbf{x}; \mathbf{x}')$ is obtained similarly or directly from K_R via a *CPT* transformation. The boundary representation for ϕ_{II} is a sum of operators on left and right CFT.

One can also find a boundary representation with support on only one boundary, using the Papadodimas-Raju mirror operator construction.

For a CFT state $|\psi\rangle$ dual to a two-sided black hole, the starting point of the mirror operator construction is the observation [32]:

$$\mathcal{O}_k^L |\psi\rangle = e^{-\omega} (\mathcal{O}_k^R)^\dagger |\psi\rangle, \quad (2.11)$$

$$(\mathcal{O}_k^L)^\dagger |\psi\rangle = e^{\omega} \mathcal{O}_k^R |\psi\rangle, \quad (2.12)$$

where ω is the time component of $\mathbf{k}$.

This allows us to translate between $\mathcal{O}_L$ and $\mathcal{O}_R$ as long as we are working with the state $|\psi\rangle$ (or small excitations around it). We can then find a boundary representation supported in only boundary (say, right):

$$\phi_{II} = \int d^d \mathbf{x}' (K_R(r, \mathbf{x}; \mathbf{x}') \mathcal{O}_R(\mathbf{x}') + K_{\text{mirror}}(r, \mathbf{x}; \mathbf{x}') \mathcal{O}_R(\mathbf{x}')). \quad (2.13)$$

where

$$K_{\text{mirror}}(r, \mathbf{x}; \mathbf{x}') = \int d^d k \frac{d_k}{M_k} (e^{\omega} \psi_k^L(r, \mathbf{x}) e^{-i\mathbf{k} \cdot \mathbf{x}} + e^{-\omega} (\psi_k^L(r, \mathbf{x}))^* e^{i\mathbf{k} \cdot \mathbf{x}}), \quad (2.14)$$

The mirror operator construction is state dependent. This representation will be valid on a subspace of the CFT that contains the state $|\psi\rangle$ and small excitations of $|\psi\rangle$. This is the subspace corresponding to bulk effective field theory on this background.

For collapsing black holes, regions III and IV are absent. In this case, HKLL construction fails for the interior of the black hole [40]. However, the mirror operator construction provides a boundary representation that is valid at late times.

III. BOUNDARY REPRESENTATION FOR MASSLESS SCALAR

Now we will use the formulas from the previous section to obtain boundary representations for the Achucarro-Ortiz black hole.

The metric of an Achucarro-Ortiz black hole is given by [36]

$$ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)},$$

$$F(r) = \frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2}. \quad (3.1)$$

Here r_+ , r_- denote the outer and inner horizons, respectively. The surface gravities corresponding to the two horizons are

$$\kappa_{\pm} = \frac{r_+^2 - r_-^2}{r_{\pm}}. \quad (3.2)$$

It is obtained by dimensional reduction of rotating BTZ black holes. The metric (3.1) approaches the 2D AdS metric for large values of r , but it is not 2D AdS itself as the Ricci scalar is not constant [36]. The Penrose diagram for this spacetime is shown in Fig. 2.

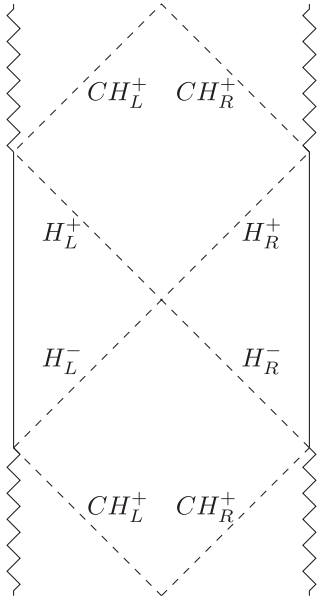


FIG. 2. Penrose diagram of Achucarro-Ortiz black hole. H_L^+, H_R^+ are the future right and left horizons, respectively. H_L^-, H_R^- are the past right and left horizons. CH_L^+, CH_R^+ are the left and right inner horizons.

The massless Klein-Gordon equation

$$\square\Phi = 0, \quad (3.3)$$

translates, after substituting $\Phi(r, t) = e^{i\omega t}\phi_\omega(r)$, to the following equation:

$$F(r)\frac{\partial}{\partial r}\left(F(r)\frac{\partial f_\omega(r)}{\partial r}\right) + \omega^2 f_\omega(r) = 0. \quad (3.4)$$

This equation has the general solution:

$$f_\omega(r) = A_\omega \cos\left(\frac{\omega \log\left(\frac{r-r_+}{r+r_+}\frac{r+r_-}{r-r_-}\right)r_-/r_+}{2\kappa_+}\right) + B_\omega \sin\left(\frac{\omega \log\left(\frac{r-r_+}{r+r_+}\frac{r+r_-}{r-r_-}\right)r_-/r_+}{2\kappa_+}\right), \quad (3.5)$$

where A_ω, B_ω are undetermined constants.

A. Region I

To obtain the boundary representation for a scalar in region I, we only need the normalizable mode solution. Demanding that the solution vanishes as $r \rightarrow \infty$ fixes the solution (up to normalization constant) to be

$$f_\omega^{\text{norm}}(r) = \sin\left(\frac{\omega \log\left(\frac{r-r_+}{r+r_+}\frac{r+r_-}{r-r_-}\right)r_-/r_+}{2\kappa_+}\right). \quad (3.6)$$

For a massless field in $d = 2$, $\Delta = 1$. Taking the boundary limit, of the above, we find

$$M_\omega = \lim_{r \rightarrow \infty} r f_\omega(r) = \omega. \quad (3.7)$$

We can now use (2.8) to find the smearing function for this region. This gives

$$K_I(r; t, t') = \int_{-\infty}^{\infty} d\omega \frac{1}{\omega} \sin\left(\frac{\log\left(\frac{r-r_+}{r+r_+}\frac{r+r_-}{r-r_-}\right)r_-/r_+}{2\kappa_+}\right) \times \cos \omega|t - t'|. \quad (3.8)$$

Performing this integral, one obtains the expression

$$K_I(r; t, t') = \frac{\pi}{4} \theta\left(\frac{\log\left(\frac{r-r_+}{r+r_+}\frac{r+r_-}{r-r_-}\right)r_-/r_+}{2\kappa_+} - |t - t'|\right). \quad (3.9)$$

The expression for the boundary representation of a massless bulk field in region I is

$$\phi(r, t) = \frac{\pi}{4} \int dt' \theta \left(\frac{\log \left(\frac{r-r_+}{r+r_+} \left(\frac{r+r_-}{r-r_-} \right)^{r_-/r_+} \right)}{2\kappa_+} - (t-t') \right) \mathcal{O}(t'). \quad (3.10)$$

If we rewrite the smearing function in terms of the Eddington-Finkelstein radial coordinate r_* , which is defined via

$$dr_* = \frac{dr}{F(r)} \Rightarrow r_* = \frac{\log \left(\frac{r-r_+}{r+r_+} \left(\frac{r+r_-}{r-r_-} \right)^{r_-/r_+} \right)}{2\kappa_+}, \quad (3.11)$$

the smearing function can be written as

$$K_I(r, t; t') = \frac{\pi}{4} \theta((r_* + |t - t'|)). \quad (3.12)$$

This smearing function has support only on spacelike separated points, as expected. In the limit $r_- \rightarrow 0$, it matches with the smearing function for AdS₂-Rindler [22].

We can compute the bulk-boundary two-point function using this smearing function:

$$\langle \phi(r_1, t_1) \mathcal{O}(t_2) \rangle = \int dt'_1 K(r_1, t_1; t'_1) \langle \mathcal{O}(t'_1) \mathcal{O}(t_2) \rangle. \quad (3.13)$$

The CFT two-point function for a primary operator of conformal dimension 1 is given by:

$$\langle \mathcal{O}(t'_1) \mathcal{O}(t_2) \rangle = \frac{1}{(t'_1 - t_2 - i\epsilon)^2}. \quad (3.14)$$

Using the smearing function (3.12), we then obtain

$$\langle \phi(r_1, t_1) \mathcal{O}(t_2) \rangle = \frac{\pi}{2(r_{1*}^2 - (t_1 - t_2 - i\epsilon)^2)}, \quad (3.15)$$

where r_{1*} is the Eddington-Finkelstein radial coordinate corresponding to r_1 .

B. Region II

To construct the boundary representation in region II, it is useful to use (3.5) and write

$$f_\omega(r) = C_\omega e^{i \frac{\log \left(\frac{r-r_+}{r+r_+} \left(\frac{r+r_-}{r-r_-} \right)^{r_-/r_+} \right)}{2\kappa_+}} + D_\omega e^{-i \frac{\log \left(\frac{r-r_+}{r+r_+} \left(\frac{r+r_-}{r-r_-} \right)^{r_-/r_+} \right)}{2\kappa_+}}. \quad (3.16)$$

These correspond to the left and right moving modes once the $e^{i\omega t}$ factor is multiplied.

The constants C_ω , D_ω are determined by matching the solution at the horizon to the normalizable modes in the exterior. For the horizon between region I and II, the interior modes must match with the normalizable modes

in I at the horizon. The left moving modes match at the horizon.⁴

Matching the left moving mode with the left moving part of the normalizable mode in region I given in (3.6) at the horizon, we obtain the constant C_ω :

$$C_\omega = 1/2i. \quad (3.17)$$

We now use compute the smearing function K_R supported in the asymptotic boundary of region I using (2.10):

$$K_R(r, t, t') = \int d\omega \frac{1}{2i\omega} \left(e^{i \frac{\log \left(\frac{r-r_+}{r+r_+} \left(\frac{r+r_-}{r-r_-} \right)^{r_-/r_+} \right)}{2\kappa_+} + t - t'} \right) + \text{H.c.} \quad (3.18)$$

This simplifies to

$$K_R(r, t, t') = \int d\omega \frac{1}{\omega} \sin \left(\frac{\log \left(\frac{r-r_+}{r+r_+} \left(\frac{r+r_-}{r-r_-} \right)^{r_-/r_+} \right)}{2\kappa_+} + t - t' \right).$$

Performing the above integration, we finally get the result:

$$K_R(r, t, t') = \frac{2\pi}{\frac{\log \left(\frac{r-r_+}{r+r_+} \left(\frac{r+r_-}{r-r_-} \right)^{r_-/r_+} \right)}{2\kappa_+} + t - t'} = \frac{2\pi}{r_* + t - t'}, \quad (3.19)$$

where r_* is given by (3.11). $K_L(r, t, t')$ can be obtained by transforming $t \rightarrow -t$, $t' \rightarrow -t'$.

We find that the smearing function has a simple algebraic form in this case.

IV. MIRROR OPERATOR CONSTRUCTION

If we consider a scenario where an Achucarro-Ortiz black hole is formed via collapse, then only one asymptotic boundary will be present. In this case, the HKLL construction above fails. However, the Papadodimas-Raju mirror operator construction can be used to obtain a state-dependent boundary representation, which will be valid at late times.

Let us consider the case where we have only regions I and II. To find the mirror, we first fix the constant C_ω in (3.16) in the same way as above.

Then using the formula (2.14), we find the smearing function:

⁴The way to see this technically is to write the modes as a function Kruskal coordinates U , V . The right moving modes $U^{i\omega}$ drop out of the mode expansion at the horizon $U \rightarrow 0$ between I and II, and one needs to match only the left moving modes. See Appendix A2 of [7] for a detailed example.

$$K_{\text{mirror}}(r, t; t') = \int d\omega \frac{1}{\omega} \left(e^{\pi\omega} e^{i\omega(r_* - (t-t'))} + e^{-\pi\omega} e^{-i\omega(r_* - (t-t'))} \right). \quad (4.1)$$

This integral is divergent. The remedy of complexifying the boundary would not work in this case—the choice that would make one integral convergent would make the other divergent.

We can follow [14,21] in constructing a smearing function for a bulk wave packet instead of a bulk field at a single point. We smear the bulk field with a wave packet peaked around a point t_0 in the time direction:

$$\Phi(r, t_0) = \int dt \phi^+(r, t) \xi_{\omega_0, t_0}(t) + \text{H.c.}, \quad (4.2)$$

where ϕ^+ denotes the positive frequency part of the field. Considering the wave packet [14]

$$\xi_{\omega_0, t_0}(t) = e^{i\omega(t-t_0)} \frac{\sin(\epsilon(t-t_0))}{\sqrt{\epsilon(t-t_0)}}, \quad (4.3)$$

we obtain

$$\Phi(r, t_0) = \int dt' K_{\omega_0}(r, t_0; t') \mathcal{O}(t'), \quad (4.4)$$

where

$$K_{\omega_0}(r, t_0; t') = \int dt \xi_{\omega_0, t_0}(t) K_+(r, t; t') + \text{H.c.}, \quad (4.5)$$

where K_+ is the positive frequency part of the smearing function.

For the mirror operator smearing function we obtain

$$K_{\omega_0}^{\text{mirror}}(r, t_0; t') = \int_{\omega_0 - \epsilon/2}^{\omega_0 + \epsilon/2} d\omega \frac{1}{\omega} \left(e^{\pi\omega} e^{i\omega(r_* - (t-t'))} + e^{-\pi\omega} e^{-i\omega(r_* - (t-t'))} \right).$$

This integrates to give the result

$$K_{\omega_0}^{\text{mirror}}(r, t_0; t') = \frac{1}{2} i \left(-\text{Ei} \left(-(i(r_* - (t-t')) + \pi) \left(\frac{\epsilon}{2} + \omega_0 \right) \right) + \text{Ei} \left((i(r_* - (t-t')) + \pi) \left(\frac{\epsilon}{2} + \omega_0 \right) \right) - \text{Ei} \left((i(r_* - (t-t')) + \pi) \left(\omega_0 - \frac{\epsilon}{2} \right) \right) + \text{Ei} \left(\frac{1}{2} (i(r_* - (t-t')) + \pi) (\epsilon - 2\omega_0) \right) \right), \quad (4.6)$$

where Ei is the exponential integral function. We can use (4.6) to compute (smeared) correlation functions for a late stage one-sided black hole.

V. SUMMARY

In this paper, we constructed boundary representations for a massless scalar in an Achucarro-Ortiz black hole background. As noted in the introduction, there are several open questions in black hole physics for which boundary representations of bulk fields should be helpful. However, analytic expressions for smearing functions in black hole backgrounds are scarce. Our result fills a gap in the literature by providing such an example.

We considered a massless free scalar and obtained its boundary representation following the HKLL construction. By employing the mode sum approach, we constructed the smearing functions in both the interior and exterior of the black hole. The results are given in (3.9) and (3.19), respectively. We find that while the smearing function in the exterior has the familiar form of a θ function, the expression for the interior region is a simple algebraic one.

In the case of a collapsing black hole, there is only one asymptotic boundary, and the HKLL construction fails for the interior. However, one can still construct a boundary representation by following the Papadodimas-Raju mirror operator construction. We followed this construction and found an expression for the corresponding smearing function, which is divergent. Hence, we constructed the smearing function for a wave packet. This is given in (4.6). Thus, we obtained boundary representations for a massless scalar for both eternal and collapsing Achucarro-Ortiz black holes.

It is straightforward to generalize the results to free gravitons. In the holographic gauge, a graviton field can be recast as a set of free massless scalars [10], and our results will apply.

In the future, we will be interested in applying our results to answer some of the outstanding questions about black hole physics in holography.

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