

Polarization analysis in $e^+e^- \rightarrow J/\psi \rightarrow$ baryon-antibaryon pairs

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We present a comprehensive polarization analysis for the process $e^+e^- \rightarrow J/\psi \rightarrow$ baryon-antibaryon pairs $B_1\bar{B}_2$. All possible helicity amplitudes for $e^+e^- \rightarrow J/\psi$ and $J/\psi \rightarrow B_1\bar{B}_2$ are provided, along with their transformation properties under parity and CP operations. Taking full account of beam polarization, we analyze the polarization of the J/ψ produced in e^+e^- annihilation and show that all eight independent polarization components can be nonzero. By introducing a polarization transfer matrix, we propose a novel method to compute the polarization transfer in the decay $J/\psi \rightarrow B_1\bar{B}_2$. Considering both hadronic and radiative baryon decay models, we derive the joint angular distribution of the final-state particles. Focusing on the process $J/\psi \rightarrow \Lambda\bar{\Sigma}^0$, we provide statistical uncertainty estimates for probing CP violation in associated production of different baryons. This work establishes a complete framework for studying the parity and CP violations present in baryon productions and decays within the helicity formalism.

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I. INTRODUCTION

Verifying the symmetries of physical laws and their violation across different energy scales has played a central role in advancing our understanding of particle interactions. Parity violation, originating from weak interactions, was first proposed by Lee and Yang [1] and soon confirmed by the beta decay experiment of polarized ^{60}Co nuclei [2] and subsequent meson decay experiments [3,4]. Meanwhile, CP violation, a necessary condition for matter-antimatter asymmetry in the universe as proposed by Sakharov [5], still faces many unresolved issues.

The CP violation was first experimentally discovered in meson decays. In 1964, Brookhaven National Laboratory

observed CP violation in the decay of neutral K mesons, marking the first observation of CP violation in a process involving the strange quark [6]. Later, CP violation was also observed in the decays of B mesons, which involve b quarks, by the BABAR Collaboration at SLAC [7] and the BELLE Collaboration at KEK [8]. More recently, the LHCb Collaboration reported CP violation in the decays of D^0 mesons, which involve c quarks [9]. These CP violations effects can be explained within the standard model (SM) via the weak interaction and the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [10,11]. However, the amount of CP violation predicted by the SM is insufficient to explain the observed matter-antimatter imbalance in the universe [12]. Theoretical models have also proposed the possibility of CP violation in the strong interaction [13–16], but experimental evidence has not yet been found [17]. As a result, searching for signals of strong CP violation and for new sources of CP violation beyond the SM across various energy scales and in different reaction processes remains a key focus in modern particle physics.

The first search for CP violation in hyperon decays was conducted by comparing the branching ratios of the decays of Σ^+ and $\bar{\Sigma}^-$ [18]. Although no signal of CP violation was observed, this search sparked significant theoretical interest in exploring CP violation through hyperon decays [19–24].

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Recently, the LHCb Collaboration reported the first observation of CP violation with a significance of 3.1σ in decay $\Lambda_b^0 \rightarrow \Lambda K^+ K^-$ [25] and 5.2σ in decay $\Lambda_b^0 \rightarrow p K^- \pi^+ \pi^-$ [26], respectively. However, no evidence of CP violation has been found in ordinary hyperons [17].

Electron-positron annihilation at the J/ψ mass peak energy benefits from a significant enhancement in the cross section, offering high-statistics opportunities for studying baryon properties through processes such as $J/\psi \rightarrow B_1 \bar{B}_2$ [27]. Theoretical studies on the polarization correlations of baryon pairs in such processes provide valuable opportunities to probe baryon CP violation [28–38]. The BESIII Collaboration, which has accumulated the world’s largest J/ψ sample, has provided precise measurements of the decay parameters for baryon hadronic decays such as Λ [39,40], Σ [41,42], and Ξ [43–45]. Recently, BESIII has also reported hints on CP violation measurements in associated production of different baryons, $J/\psi \rightarrow \Lambda \bar{\Sigma}$ [46], and baryon radiative decays [47–50]. The proposed STCF experiment [51] will significantly expand the data samples, increasing J/ψ statistics by 2 orders of magnitude in one year compared to BESIII.

Theoretical studies of CP violation in hyperons have been extended to investigations of CP violation in baryon production processes [52–54], including scenarios with polarized beams [55–60]. A key focus of these studies is to establish the joint angular distribution of the final-state products. As the amount of data grows (for example, in future STCF experiments), precise measurements of CP violation need to account for specific experimental conditions [55–60]. In the BEPCII/BESIII experiment, because the electron and positron beams cross at a small angle of 11 milliradians, the polarization of the J/ψ particle is slightly different in the lab frame compared to its center-of-mass frame. This difference must be considered when measuring CP violation. On the other hand, the effects of the electron mass and the structure of the coupling vertex between the electron and the J/ψ have often been neglected. In CP violation studies, these factors can introduce critical corrections and must be carefully considered.

This paper presents a refined polarization analysis for the process $e^+e^- \rightarrow J/\psi \rightarrow B_1 \bar{B}_2$. We perform a detailed polarization analysis in both the production and decay subprocesses of J/ψ , as well as the decay of baryons, within the helicity formalism developed by Jacob and Wick [61]. This helicity-based approach is equivalent to describing the production and decay subprocess of J/ψ in terms of particle form factors and baryon decays in terms of partial wave amplitudes. We explicitly establish the correspondence between helicity amplitudes and particle form factors or partial-wave amplitudes.

A comprehensive polarization analysis for the subprocess $e^+e^- \rightarrow J/\psi$ is provided with all possible helicity amplitudes, including parity violating and CP violating ones. These amplitudes are connected to the anomalous magnetic

moment form factor and the electric dipole moment of the electron. We also take into account the spin-flip helicity amplitude arising from the electron mass. By matching the helicity amplitudes with the electron form factors, we estimate their magnitudes. We further analyze the effects influencing beam polarization, particularly emphasizing the mixing of transverse and longitudinal polarization when transforming from the laboratory frame to the center-of-mass frame. Based on these, we provide a detailed analysis of the resulting polarization of J/ψ .

The complete polarization analyses of the subprocess of J/ψ decay to both baryon-antibaryon pairs $B\bar{B}$ and to different baryons $B_1 \bar{B}_2$ are presented. The helicity amplitudes corresponding to parity conservation, parity violation, and CP violation are introduced. For the process $J/\psi \rightarrow B\bar{B}$, the helicity amplitudes are associated with the form factors of the baryon B , and for the process $J/\psi \rightarrow B_1 \bar{B}_2$, they correspond to the transition form factors of $B_1 \bar{B}_2$. We establish the correspondence between the helicity amplitudes and the form factors and provide the method to search for CP violation within the helicity formalism. Furthermore, a novel approach to represent the polarization transfer in the $J \rightarrow B_1 \bar{B}_2$ subprocess is introduced via the polarization transfer matrix.

We provide the complete expression for the joint angular distribution of the final-state particles. The polarization of hyperons can be measured through their decay processes. We focus on the hadronic and radiative decay of hyperons, providing the helicity amplitudes and the polarization transfer matrix for these decay processes. By combining polarization analyses of J/ψ production subprocess, its decay into baryons, and the decays of baryons, we present the complete expression for the joint angular distribution of all final-state products. Focusing on the process $J/\psi \rightarrow \Lambda \bar{\Sigma}^0$, we provide sensitivity predictions for the helicity amplitude parameters and their associated transition form factors. When combined with measurements from the conjugate channel $J/\psi \rightarrow \bar{\Lambda} \Sigma^0$, this allows the search for potential signals of CP violation. These isospin-violating processes are particularly sensitive to electroweak interactions, making them valuable for investigating CP violations.

The structure of this paper is as follows. In Sec. II, we present the polarization analysis of the subprocess $e^+e^- \rightarrow J/\psi$. In Sec. III, we provide the polarization analysis of the subprocess $J/\psi \rightarrow B_1 \bar{B}_2$. In Sec. IV, we discuss baryon decays and provide the complete joint angular distribution. Focusing on the $J/\psi \rightarrow \Lambda \bar{\Sigma}^0$ process, the sensitivities to the transition form factors are provided. Finally, we present a short summary in Sec. V.

II. POLARIZATION ANALYSIS IN PRODUCTION OF J/ψ

In high-energy reactions, the polarization state of particles is described by their spin density matrix. We begin with a systematic analysis of the beam polarization and

derive the spin density matrix for J/ψ in the helicity formalism. As the spin-1 particle, J/ψ has eight independent polarization components. For the subprocess $e^+e^- \rightarrow J/\psi$, the polarization of J/ψ is primarily determined by electron (anomalous) form factors and the polarization of the initial beams. Taking both effects into account, we derive the full expression for the polarization components of J/ψ .

A. Spin density matrix

In the helicity formalism, the spin density matrix of the particle with spin s can be expressed as [30,34]

$$\rho_s = \sum_{\mu=0}^{4s(s+1)} S_\mu \Sigma_\mu, \quad (1)$$

where S_μ are the polarization expansion coefficients, and Σ_μ are the corresponding polarization projection matrices.

For the spin-1/2 particle, the spin density matrix is given by

$$\begin{aligned} S_\mu &= \{S_0, S_1, S_2, S_3\}, \\ \Sigma_\mu &= \frac{1}{2} \{I, \sigma_x, \sigma_y, \sigma_z\}, \end{aligned} \quad (2)$$

where σ_x , σ_y , and σ_z are the Pauli matrices, S_0 corresponds to the cross-section term, and S_i represents the polarization terms. These polarization terms are related to the Cartesian spin components through the following relation:

$$\{S_1, S_2, S_3\}/S_0 = \{S_T^x, S_T^y, S_L\}. \quad (3)$$

Then, the spin density matrix for the initial-state positron and electron can be expressed as

$$\begin{aligned} \rho^+ &= \frac{1}{2} \begin{pmatrix} 1 + \bar{S}_L & \bar{S}_T^x - i\bar{S}_T^y \\ \bar{S}_T^x + i\bar{S}_T^y & 1 - \bar{S}_L \end{pmatrix} \quad \text{for } e^+, \\ \rho^- &= \frac{1}{2} \begin{pmatrix} 1 + S_L & S_T^x - iS_T^y \\ S_T^x + iS_T^y & 1 - S_L \end{pmatrix} \quad \text{for } e^-. \end{aligned} \quad (4)$$

Under realistic experimental conditions, various factors contribute to beam polarization. For circular accelerators, charged particles experience spontaneous polarization due to their motion in magnetic fields, a phenomenon known as the Sokolov-Ternov effect [62]. The direction and magnitude of the positron and electron spontaneous polarization depend on factors such as the magnetic field strength, beam

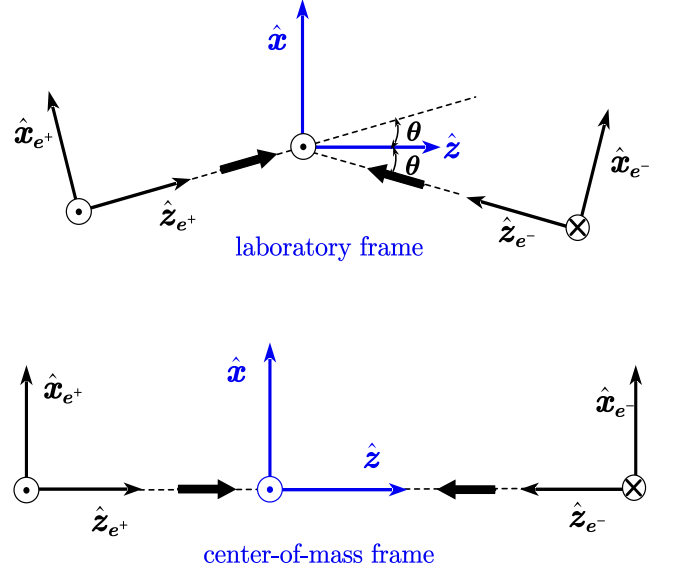


FIG. 1. Comparison of the laboratory frame (upper) and center-of-mass frame (lower) for positron-electron collisions.

energy, beam lifetime, and so on. Furthermore, experiments such as the proposed STCF [51] aim to prepare a polarized beam as the key design feature. For collider experiments at circular accelerators, the two beams typically intersect at a small angle at the interaction point [63]. As a result, the laboratory frame generally does not coincide with the center-of-mass frame. When polarization vectors are transformed between these frames, transverse and longitudinal polarization components can mix.

The coordinate systems of the laboratory and center-of-mass frames are shown in Fig. 1. In the laboratory frame, the z axis is defined as the bisector of the angle between the direction of the positron beam and the opposite direction of the electron beam. The plane formed by the two beam directions is taken as the x - z plane. The momenta of the positron and electron in this frame can be expressed as follows:

$$\begin{aligned} P_{e^+} &= \frac{m}{\sqrt{1-\beta^2}} (1, \beta \sin \theta, 0, \beta \cos \theta), \\ P_{e^-} &= \frac{m}{\sqrt{1-\beta^2}} (1, \beta \sin \theta, 0, -\beta \cos \theta), \end{aligned} \quad (5)$$

where β is the velocity of the beams. The polarization vectors of the positron and electron in the laboratory frame are then given by

$$\begin{aligned} S_{e^+} &= \left(\frac{\beta}{\sqrt{1-\beta^2}} \bar{S}_{L,\text{lab}}, \frac{\sin \theta}{\sqrt{1-\beta^2}} \bar{S}_{L,\text{lab}} + \bar{S}_{T,\text{lab}}^x \cos \theta, \bar{S}_{T,\text{lab}}^y \frac{\cos \theta}{\sqrt{1-\beta^2}} \bar{S}_{L,\text{lab}} - \bar{S}_{T,\text{lab}}^x \sin \theta \right), \\ S_{e^-} &= \left(\frac{\beta}{\sqrt{1-\beta^2}} S_{L,\text{lab}}, \frac{\sin \theta}{\sqrt{1-\beta^2}} S_{L,\text{lab}} + S_{T,\text{lab}}^x \cos \theta, -S_{T,\text{lab}}^y \frac{\cos \theta}{\sqrt{1-\beta^2}} S_{L,\text{lab}} + S_{T,\text{lab}}^x \sin \theta \right). \end{aligned} \quad (6)$$

To transform from the laboratory frame to the center-of-mass frame, we apply the following Lorentz boost matrix to the beams:

$$\Lambda_{\mathcal{V}}^{\mu} = \begin{pmatrix} \frac{1}{\sqrt{1-\beta^2\sin^2\theta}} & -\frac{\beta\sin\theta}{\sqrt{1-\beta^2\sin^2\theta}} & 0 & 0 \\ -\frac{\beta\sin\theta}{\sqrt{1-\beta^2\sin^2\theta}} & \frac{1}{\sqrt{1-\beta^2\sin^2\theta}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (7)$$

Using this transformation, the momenta of the positron and electron in the center-of-mass frame become

$$\begin{aligned} P_{e^+} &= \frac{m}{\sqrt{1-\beta^2}} \left(\sqrt{1-\beta^2\sin^2\theta}, 0, 0, \beta\cos\theta \right), \\ P_{e^-} &= \frac{m}{\sqrt{1-\beta^2}} \left(\sqrt{1-\beta^2\sin^2\theta}, 0, 0, -\beta\cos\theta \right). \end{aligned} \quad (8)$$

Correspondingly, the polarization vectors of the positron and electron in the center-of-mass frame are

$$\begin{aligned} S_{e^+} &= \left(\frac{\beta\cos\theta}{\sqrt{1-\beta^2\sin^2\theta}} \left(\frac{\bar{S}_{L,\text{lab}}}{\sqrt{1-\beta^2}} \cos\theta - \bar{S}_{T,\text{lab}}^x \sin\theta \right), \right. \\ &\quad \left. \frac{\bar{S}_{T,\text{lab}}^x \cos\theta + \sqrt{1-\beta^2}\bar{S}_{L,\text{lab}} \sin\theta}{\sqrt{1-\beta^2\sin^2\theta}}, \bar{S}_{T,\text{lab}}^y, \frac{\bar{S}_{L,\text{lab}} \cos\theta}{\sqrt{1-\beta^2}} - \bar{S}_{T,\text{lab}}^x \sin\theta \right), \\ S_{e^-} &= \left(\frac{\beta\cos\theta}{\sqrt{1-\beta^2\sin^2\theta}} \left(\frac{S_{L,\text{lab}}}{\sqrt{1-\beta^2}} \cos\theta - S_{T,\text{lab}}^x \sin\theta \right), \right. \\ &\quad \left. \frac{S_{T,\text{lab}}^x \cos\theta + \sqrt{1-\beta^2}S_{L,\text{lab}} \sin\theta}{\sqrt{1-\beta^2\sin^2\theta}}, -S_{T,\text{lab}}^y, S_{T,\text{lab}}^x \sin\theta - \frac{S_{L,\text{lab}} \cos\theta}{\sqrt{1-\beta^2}} \right). \end{aligned} \quad (9)$$

When further transformed into the helicity frame of the positron and electron, one can extract their polarization components as

$$\begin{aligned} \bar{S}_T^x &= \frac{\bar{S}_{T,\text{lab}}^x \cos\theta + \sqrt{1-\beta^2}\bar{S}_{L,\text{lab}} \sin\theta}{\sqrt{1-\beta^2\sin^2\theta}}, \\ \bar{S}_T^y &= \bar{S}_{T,\text{lab}}^y, \\ \bar{S}_L &= \frac{\bar{S}_{L,\text{lab}} \cos\theta - \sqrt{1-\beta^2}\bar{S}_{T,\text{lab}}^x \sin\theta}{\sqrt{1-\beta^2\sin^2\theta}}, \\ S_T^x &= \frac{S_{T,\text{lab}}^x \cos\theta - \sqrt{1-\beta^2}S_{L,\text{lab}} \sin\theta}{\sqrt{1-\beta^2\sin^2\theta}}, \\ S_T^y &= S_{T,\text{lab}}^y, \\ S_L &= \frac{S_{L,\text{lab}} \cos\theta + \sqrt{1-\beta^2}S_{T,\text{lab}}^x \sin\theta}{\sqrt{1-\beta^2\sin^2\theta}}. \end{aligned} \quad (10)$$

Since the initial beam polarization is generated in the laboratory frame, while theoretical polarization analysis is

typically performed in the center-of-mass frame, experimental observables must also be transformed into the center-of-mass frame for consistency. This transformation leads to a mixing of longitudinal and transverse polarization components, which ultimately affects the joint angular distribution of the final-state particles. Therefore, such effects can manifest themselves in experimental measurements and should not be neglected.

For spin-1 particles, such as J/ψ , their spin density matrix can be expressed as

$$\rho_1 = \sum_{\mu=0}^8 S_{\mu} \Sigma_{\mu}. \quad (11)$$

The polarization expansion coefficients S_{μ} and the polarization projection matrix Σ_{μ} can be determined according to the approach described in [34]. For S_{μ} , we adopt the following scheme:

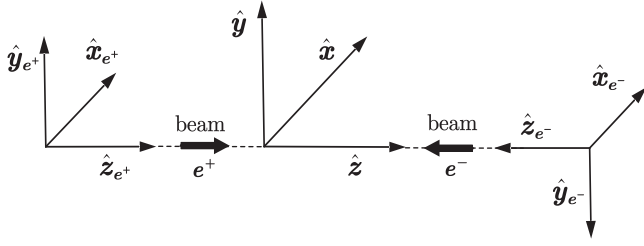


FIG. 2. The $e^+e^- \rightarrow J/\psi$ process in c.m. frame. The coordinate systems $x_{e^+} - y_{e^+} - z_{e^+}$, $x_{e^-} - y_{e^-} - z_{e^-}$, and $x - y - z$ are the helicity frames for positron, electron, and J/ψ .

$$\frac{1}{S_0} \begin{Bmatrix} S_0 & S_1 & S_2 \\ S_3 & S_4 & S_5 \\ S_6 & S_7 & S_8 \end{Bmatrix} = \begin{Bmatrix} 1 & S_L & S_T^x \\ S_T^y & S_{LL} & S_{LT}^x \\ S_{LT}^y & S_{TT}^{xx} & S_{TT}^{xy} \end{Bmatrix}, \quad (12)$$

where $S_L, \dots, S_{TT}^{xy}$ are the spin components of the spin-1 particle in Cartesian form, as detailed in Appendix A. The explicit expression of Σ_μ in this scheme is provided in Appendix B.

B. Production amplitudes for the subprocess $e^+e^- \rightarrow J/\psi$

For the subprocess $e^+e^- \rightarrow J/\psi$, as showed in Fig. 2, the helicity amplitudes can be written as

$$A_{\lambda_1 \lambda_2}^{ee} = \begin{pmatrix} h_1^{ee} + h_3^{ee} & h_2^{ee} - h_4^{ee} \\ h_2^{ee} + h_4^{ee} & h_1^{ee} - h_3^{ee} \end{pmatrix}, \quad (13)$$

where λ_1 and λ_2 are the helicities of the positron and electron, respectively. Then, we have

$$\begin{aligned} h_1^{ee} &= \frac{1}{2} \left(A_{\frac{1}{2}, \frac{1}{2}}^{ee} + A_{-\frac{1}{2}, -\frac{1}{2}}^{ee} \right), \\ h_2^{ee} &= \frac{1}{2} \left(A_{-\frac{1}{2}, \frac{1}{2}}^{ee} + A_{\frac{1}{2}, -\frac{1}{2}}^{ee} \right), \\ h_3^{ee} &= \frac{1}{2} \left(A_{\frac{1}{2}, \frac{1}{2}}^{ee} - A_{-\frac{1}{2}, -\frac{1}{2}}^{ee} \right), \\ h_4^{ee} &= \frac{1}{2} \left(A_{-\frac{1}{2}, \frac{1}{2}}^{ee} - A_{\frac{1}{2}, -\frac{1}{2}}^{ee} \right). \end{aligned} \quad (14)$$

These helicity amplitudes are correlated to the scattering amplitudes expressed in the (anomalous) form factors of the electron, which are given by

$$\begin{aligned} \mathcal{M}_{\lambda_1 \lambda_2} &= \bar{v}(p_1, \lambda_1) \left(F_1(q^2) \gamma^\mu + F_2(q^2) \frac{i\sigma^{\mu\nu} q_\nu}{2m} + F_3(q^2) \gamma^\mu \gamma_5 \right. \\ &\quad \left. + F_4(q^2) \frac{\sigma^{\mu\nu} q_\nu}{2m} \gamma_5 \right) u(p_2, \lambda_2) \varepsilon_\mu(q, \kappa), \end{aligned} \quad (15)$$

where $\bar{v}$ and u are the spinors of the positron and electron, respectively. $p_1(p_2)$, $\lambda_1(\lambda_2)$, and m represent their

momenta, helicities, and mass. ε_μ and $q = p_1 + p_2$ are the polarization vector and momentum of J/ψ , and $\kappa = \lambda_1 - \lambda_2$ is the helicity of J/ψ . The form factors F_1, F_2 , and F_4 correspond to the charge, the anomalous magnetic dipole moment [64], and the electric dipole moment form factor of the electron, respectively, and F_3 represents the parity violation form factor. In the timelike region, these form factors can be complex.

At one-loop order, the anomalous magnetic moment of the electron, see, for instance, in [65], is given by

$$\begin{aligned} F_2(q^2) &= \frac{\alpha m^2}{\pi} \int_0^1 dx \int_0^{1-x} dy \frac{(x+y)(1-x-y)}{m^2(x+y)^2 - Q^2 xy} \\ &= \frac{\alpha m^2}{\pi Q \sqrt{Q^2 - 4m^2}} \ln \frac{Q - \sqrt{Q^2 - 4m^2}}{Q + \sqrt{Q^2 - 4m^2}}, \end{aligned} \quad (16)$$

with $Q = \sqrt{q^2}$ and $\alpha = e^2/(4\pi)$. The parity violation form factor F_3 arises primarily from the contribution of Z-boson exchange,

$$F_3(q^2) = \frac{E_c^2(4 \cos 2\theta_W - 1)}{M_Z^2(1 - \cos 4\theta_W) - 3E_c^2(3 - 2 \cos 2\theta_W)}, \quad (17)$$

where $E_c = \sqrt{q^2}/2$ represents the beam energy in the center-of-mass frame, M_Z is the mass of the Z boson, and θ_W is the weak coupling angle. The electric dipole moment form factor F_4 is related to the electron's dipole moment d_e by

$$d_e = -\frac{F_4}{2m}. \quad (18)$$

According to the experimental constraint $d_e < 4.1 \times 10^{-30}$ ecm [17], we obtain

$$|F_4| < 6.43 \times 10^{-20}. \quad (19)$$

It is evident that the contribution from the electric dipole moment is much smaller than the contributions from the other terms.

Since Eqs. (13) and (15) are equivalent, i.e., $A_{\lambda_1 \lambda_2}^{ee} = \mathcal{M}_{\lambda_1 \lambda_2}$, we derive the relationship between the helicity amplitude and form factors,

$$\begin{aligned} h_1^{ee} &= 2mF_1(q^2) - \frac{2E_c^2}{m} F_2(q^2), \\ h_2^{ee} &= 2\sqrt{2}E_c(F_1(q^2) - F_2(q^2)), \\ h_3^{ee} &= i\frac{2E_c}{m} \sqrt{E_c^2 - m^2} F_4(q^2), \\ h_4^{ee} &= 2\sqrt{2}\sqrt{E_c^2 - m^2} F_3(q^2). \end{aligned} \quad (20)$$

We normalize these amplitudes with h_2^{ee} and parametrize the helicity amplitudes in the following scheme:

$$\begin{aligned}\frac{h_4^{ee}}{h_2^{ee}} &= \sqrt{a_1} e^{i\varphi_1}, \\ \frac{h_1^{ee}}{h_2^{ee}} &= \sqrt{a_2} e^{i\varphi_2}, \\ \frac{h_3^{ee}}{h_2^{ee}} &= \sqrt{a_3} e^{i\varphi_3}.\end{aligned}\quad (21)$$

At leading order, we obtain

$$\begin{aligned}\sqrt{a_1} &= \frac{E_c \sqrt{E_c^2 - m^2} (4 \cos 2\theta_W - 1)}{M_z^2 (1 - \cos 4\theta_W) - 3E_c^2 (3 - 2 \cos 2\theta_W)} \approx 2.33 \times 10^{-4}, \\ \sqrt{a_2} &= \frac{m}{\sqrt{2}E_c} \left(1 - \frac{E_c^2}{m^2} F_2(q^2) \right) \approx 2.36 \times 10^{-4}, \\ \sqrt{a_3} &= \left| i \frac{\sqrt{E_c^2 - m^2}}{\sqrt{2}m} F_4(q^2) \right| < 1.38 \times 10^{-16}.\end{aligned}\quad (22)$$

We find that although the parameter a_3 , associated with the electric dipole moment of the electron, is negligibly small

and can be safely ignored, the parameters a_1 and a_2 , which are related to the electron mass, anomalous magnetic moment, and Z -boson exchange, remain important for polarization analysis, especially in investigations of CP violation.

C. The polarization components of J/ψ

The production density matrix of J/ψ for the subprocess $e^- e^+ \rightarrow J/\psi$ can be expressed as

$$\begin{aligned}\rho_{\kappa, \kappa'}^{J/\psi} &= 2 \sum_{\lambda_1, \lambda_1', \lambda_2, \lambda_2'} \mathcal{D}_{\kappa, \lambda_1 - \lambda_2}^{1*} (0, 0, 0) \mathcal{D}_{\kappa', \lambda_1' - \lambda_2'}^1 (0, 0, 0) \\ &\times \rho_{\lambda_1, \lambda_1'}^+ \rho_{\lambda_2, \lambda_2'}^- A_{\lambda_1, \lambda_2} A_{\lambda_1', \lambda_2'}^*,\end{aligned}\quad (23)$$

where $\mathcal{D}_{\kappa, \lambda}^J(\Omega) = \mathcal{D}_{\kappa, \lambda}^J(0, 0, 0)$ is the Wigner- $\mathcal{D}$ matrix. We can then obtain the polarization components of J/ψ , which are given by

$$S_\mu = \frac{\text{Tr}[\rho_{\kappa, \kappa'}^{J/\psi} \Sigma_\mu]}{\text{Tr}[\Sigma_\mu \Sigma_\mu]}.\quad (24)$$

Substituting Eqs. (13) and (21) into the above expression, we obtain

$$\begin{aligned}S_0 &= 1 + a_1 + a_2 + a_3 - (1 + a_1 - a_2 - a_3) \bar{S}_L S_L \\ &\quad + (a_2 - a_3) (\bar{S}_T^x S_T^x - \bar{S}_T^y S_T^y) - 2\sqrt{a_1} (\bar{S}_L - S_L) \cos \varphi_1 \\ &\quad + 2\sqrt{a_2 a_3} [(\bar{S}_L + S_L) \cos(\varphi_2 - \varphi_3) - (\bar{S}_T^x S_T^y + \bar{S}_T^y S_T^x) \sin(\varphi_2 - \varphi_3)], \\ S_1 &= (1 + a_1) (\bar{S}_L - S_L) - 2\sqrt{a_1} (1 - \bar{S}_L S_L) \cos \varphi_1, \\ S_2 &= \sqrt{2} \{ \sqrt{a_2} [(\bar{S}_T^x + S_T^x) \cos \varphi_2 + (\bar{S}_L S_T^y + S_L \bar{S}_T^y) \sin \varphi_2] \\ &\quad + \sqrt{a_3} [(\bar{S}_L S_T^x + S_L \bar{S}_T^x) \cos \varphi_3 + (\bar{S}_T^y + S_T^y) \sin \varphi_3] \\ &\quad - \sqrt{a_1 a_2} [(\bar{S}_L S_T^x - S_L \bar{S}_T^x) \cos(\varphi_1 - \varphi_2) + (\bar{S}_T^y - S_T^y) \sin(\varphi_1 - \varphi_2)] \\ &\quad - \sqrt{a_1 a_3} [(\bar{S}_T^x - \bar{S}_T^x) \cos(\varphi_1 - \varphi_3) + (S_L \bar{S}_T^y - \bar{S}_L S_T^y) \sin(\varphi_1 - \varphi_3)] \}, \\ S_3 &= \sqrt{2} \{ \sqrt{a_2} [(\bar{S}_T^y - S_T^y) \cos \varphi_2 + (\bar{S}_L S_T^x - S_L \bar{S}_T^x) \sin \varphi_2] \\ &\quad + \sqrt{a_3} [(S_L \bar{S}_T^y - \bar{S}_L S_T^y) \cos \varphi_3 + (S_T^x - \bar{S}_T^x) \sin \varphi_3] \\ &\quad + \sqrt{a_1 a_2} [(\bar{S}_L S_T^y + S_L \bar{S}_T^y) \cos(\varphi_1 - \varphi_2) + (\bar{S}_T^x + S_T^x) \sin(\varphi_1 - \varphi_2)] \\ &\quad + \sqrt{a_1 a_3} [(\bar{S}_T^y + S_T^y) \cos(\varphi_1 - \varphi_3) + (\bar{S}_L S_T^x + S_L \bar{S}_T^x) \sin(\varphi_1 - \varphi_3)] \}, \\ S_4 &= \frac{1}{2} \{ 1 + a_1 - 2a_2 - 2a_3 - (1 + a_1 + 2a_2 + 2a_3) \bar{S}_L S_L \\ &\quad - 2\sqrt{a_1} (\bar{S}_L - S_L) \cos \varphi_1 - 2(a_2 - a_3) (\bar{S}_T^x S_T^x - \bar{S}_T^y S_T^y) \\ &\quad - 4\sqrt{a_2 a_3} [(\bar{S}_L + S_L) \cos(\varphi_2 - \varphi_3) - (\bar{S}_T^x S_T^y + \bar{S}_T^y S_T^x) \sin(\varphi_2 - \varphi_3)] \}, \\ S_5 &= \sqrt{2} \{ \sqrt{a_2} [(\bar{S}_L S_T^x - S_L \bar{S}_T^x) \cos \varphi_2 + (S_T^y - \bar{S}_T^y) \sin \varphi_2] \\ &\quad + \sqrt{a_3} [(S_T^x - \bar{S}_T^x) \cos \varphi_3 + (\bar{S}_L S_T^y - S_L \bar{S}_T^y) \sin \varphi_3] \\ &\quad - \sqrt{a_1 a_2} [(\bar{S}_T^x + S_T^x) \cos(\varphi_1 - \varphi_2) - (\bar{S}_L S_T^y + S_L \bar{S}_T^y) \sin(\varphi_1 - \varphi_2)] \\ &\quad - \sqrt{a_1 a_3} [(\bar{S}_L S_T^x + S_L \bar{S}_T^x) \cos(\varphi_1 - \varphi_3) - (\bar{S}_T^y + S_T^y) \sin(\varphi_1 - \varphi_3)] \}, \\ S_6 &= \sqrt{2} \{ -\sqrt{a_2} [(\bar{S}_L S_T^y + S_L \bar{S}_T^y) \cos \varphi_2 - (\bar{S}_T^x + S_T^x) \sin \varphi_2] \end{aligned}$$

$$\begin{aligned}
& -\sqrt{a_3}[(\bar{S}_T^y + S_T^y) \cos \varphi_3 - (\bar{S}_L S_T^x + S_L \bar{S}_T^x) \sin \varphi_3] \\
& -\sqrt{a_1 a_2}[(\bar{S}_T^y - S_T^y) \cos(\varphi_1 - \varphi_2) + (S_L \bar{S}_T^x - \bar{S}_L S_T^x) \sin(\varphi_1 - \varphi_2)] \\
& -\sqrt{a_1 a_3}[(S_L \bar{S}_T^y - \bar{S}_L S_T^y) \cos(\varphi_1 - \varphi_3) + (\bar{S}_T^x - S_T^x) \sin(\varphi_1 - \varphi_3)], \\
S_7 &= (1 - a_1)(\bar{S}_T^x S_T^x + \bar{S}_T^y S_T^y) - 2\sqrt{a_1}(\bar{S}_T^y S_T^x - \bar{S}_T^x S_T^y) \sin \varphi_1, \\
S_8 &= (1 - a_1)(\bar{S}_T^y S_T^x - \bar{S}_T^x S_T^y) + 2\sqrt{a_1}(\bar{S}_T^x S_T^x + \bar{S}_T^y S_T^y) \sin \varphi_1.
\end{aligned} \tag{25}$$

Based on Eq. (22), and by neglecting the complex phase of the helicity amplitude and the small contributions from the terms related to the electric dipole moment, we obtain

$$\begin{aligned}
S_0 &= 1 + a_1 + a_2 - (1 + a_1 - a_2)\bar{S}_L S_L - 2\sqrt{a_1}(\bar{S}_L - S_L) + a_2(\bar{S}_T^x S_T^x + \bar{S}_T^y S_T^y), \\
S_1 &= (1 + a_1)(\bar{S}_L - S_L) - 2\sqrt{a_1}(1 - \bar{S}_L S_L), \\
S_2 &= \sqrt{2}[\sqrt{a_2}(\bar{S}_T^x + S_T^x) - \sqrt{a_1 a_2}(\bar{S}_L S_T^x - S_L \bar{S}_T^x)], \\
S_3 &= \sqrt{2}[\sqrt{a_2}(\bar{S}_T^y - S_T^y) + \sqrt{a_1 a_2}(\bar{S}_L S_T^y + S_L \bar{S}_T^y)], \\
S_4 &= \frac{1}{2}[1 + a_1 - 2a_2 - (1 + a_1 + 2a_2)\bar{S}_L S_L - 2\sqrt{a_1}(\bar{S}_L - S_L) - 2a_2(\bar{S}_T^x S_T^x + \bar{S}_T^y S_T^y)], \\
S_5 &= \sqrt{2}[\sqrt{a_2}(\bar{S}_L S_T^x - S_L \bar{S}_T^x) - \sqrt{a_1 a_2}(\bar{S}_T^x + S_T^x)], \\
S_6 &= \sqrt{2}[-\sqrt{a_2}(\bar{S}_L S_T^y + S_L \bar{S}_T^y) - \sqrt{a_1 a_2}(\bar{S}_T^y - S_T^y)], \\
S_7 &= (1 - a_1)(\bar{S}_T^x S_T^x + \bar{S}_T^y S_T^y), \\
S_8 &= (1 - a_1)(\bar{S}_T^y S_T^x - \bar{S}_T^x S_T^y).
\end{aligned} \tag{26}$$

By substituting the expressions of the polarization components above into Eq. (12), one can obtain the spin components of J/ψ in Cartesian form. We retain the leading-order contributions from a_1 and a_2 to analyze these Cartesian polarization components. Notably, two polarization components remain nonzero even when the beam is unpolarized,

$$\begin{aligned}
S_L^\psi &= -2\sqrt{a_1}, \\
S_{LL}^\psi &= \frac{1}{2}.
\end{aligned} \tag{27}$$

When the beam is polarized, these components are modified to

$$\begin{aligned}
S_L^\psi &= \frac{\bar{S}_L - S_L}{1 - \bar{S}_L S_L} - 2\sqrt{a_1} \left[1 - \left(\frac{\bar{S}_L - S_L}{1 - \bar{S}_L S_L} \right)^2 \right], \\
S_{LL}^\psi &= \frac{1}{2} \left[1 - \frac{3a_2(1 + \bar{S}_L S_L + \bar{S}_T^x S_T^x - \bar{S}_T^y S_T^y)}{1 - \bar{S}_L S_L} \right].
\end{aligned} \tag{28}$$

The longitudinal polarization component S_L^ψ of J/ψ is sensitive to both beam polarization and weak coupling at the electron- J/ψ vertex. This makes it a potential observable for probing the weak mixing angle θ_W .

The two components of the transverse tensor polarization are significantly affected by beam polarization:

$$\begin{aligned}
S_{TT}^{xx,\psi} &= \left(1 + 2\sqrt{a_1} \frac{\bar{S}_L - S_L}{1 - \bar{S}_L S_L} \right) \frac{\bar{S}_T^x S_T^x + \bar{S}_T^y S_T^y}{1 - \bar{S}_L S_L}, \\
S_{TT}^{xy,\psi} &= \left(1 + 2\sqrt{a_1} \frac{\bar{S}_L - S_L}{1 - \bar{S}_L S_L} \right) \frac{\bar{S}_T^y S_T^x - \bar{S}_T^x S_T^y}{1 - \bar{S}_L S_L}.
\end{aligned} \tag{29}$$

The remaining four polarization components are slightly influenced by the beam polarization. Their contributions are suppressed by the smallness of the electron mass and the anomalous magnetic moment form factor,

$$\begin{aligned}
S_T^{x,\psi} &= \frac{\sqrt{2a_2}(\bar{S}_T^x + S_T^x)}{1 - \bar{S}_L S_L}, \\
S_T^{y,\psi} &= \frac{\sqrt{2a_2}(\bar{S}_T^y - S_T^y)}{1 - \bar{S}_L S_L}, \\
S_{LT}^{x,\psi} &= \frac{\sqrt{2a_2}(\bar{S}_L S_T^x - S_L \bar{S}_T^x)}{1 - \bar{S}_L S_L}, \\
S_{LT}^{y,\psi} &= -\frac{\sqrt{2a_2}(\bar{S}_L S_T^y + S_L \bar{S}_T^y)}{1 - \bar{S}_L S_L}.
\end{aligned} \tag{30}$$

When the electron mass and form factors are neglected, the polarization of J/ψ reduces to the form discussed in Refs. [56,59]. Furthermore, if beam polarization is also ignored, the J/ψ polarization simplifies to the case presented in Ref. [30], where $S_{LL}^\psi = 1/2$ and all other polarization components vanish. Under conditions of beam transverse polarization, these specific polarization

components of J/ψ provide access to the anomalous magnetic moment of the electron in the timelike region.

III. POLARIZATION ANALYSIS IN DECAY OF J/ψ

In the helicity formalism, we present a complete polarization analysis of the decay of J/ψ into baryon-antibaryon pairs. The analysis is divided into two types: baryon-antibaryon pair production and associated production of different baryons. By introducing helicity amplitudes that account for parity and CP violation, we define observables that can be used to search for CP violation signals. Finally, we propose a novel expression to represent the polarization transfer for the subprocess $J/\psi \rightarrow B_1 \bar{B}_2$.

A. Decay amplitudes for the subprocess $J/\psi \rightarrow B\bar{B}$

For the subprocess $J/\psi \rightarrow B\bar{B}$ (see Fig. 3), where $\bar{B}$ is the antiparticle of B (e.g., $J/\psi \rightarrow \Lambda \bar{\Lambda}$), the helicity amplitudes can be written as

$$A_{\lambda_1, \lambda_2} = \begin{pmatrix} h_1 + h_3 & h_2 + h_4 \\ h_2 - h_4 & h_1 - h_3 \end{pmatrix}, \quad (31)$$

where λ_1 and λ_2 denote the helicities of B and $\bar{B}$, respectively. Then, we have

$$\begin{aligned} h_1 &= \frac{1}{2} \left(A_{\frac{1}{2}, \frac{1}{2}} + A_{-\frac{1}{2}, -\frac{1}{2}} \right), \\ h_2 &= \frac{1}{2} \left(A_{\frac{1}{2}, -\frac{1}{2}} + A_{-\frac{1}{2}, \frac{1}{2}} \right), \\ h_3 &= \frac{1}{2} \left(A_{\frac{1}{2}, \frac{1}{2}} - A_{-\frac{1}{2}, -\frac{1}{2}} \right), \\ h_4 &= \frac{1}{2} \left(A_{\frac{1}{2}, -\frac{1}{2}} - A_{-\frac{1}{2}, \frac{1}{2}} \right). \end{aligned} \quad (32)$$

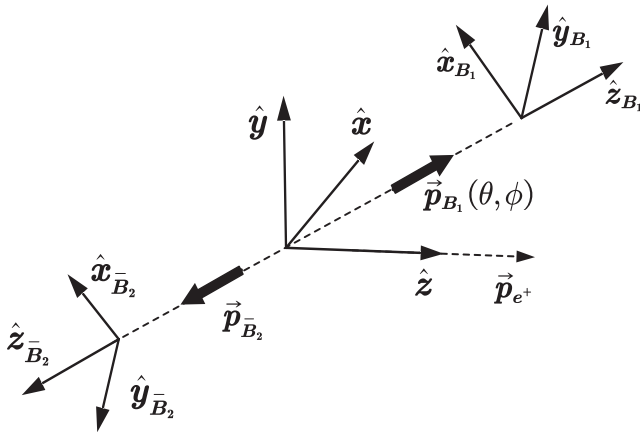


FIG. 3. The $J/\psi \rightarrow B_1 \bar{B}_2$ process in J/ψ rest frame (helicity frame). The coordinate systems $x_{B_1} - y_{B_1} - z_{B_1}$, $x_{B_2} - y_{B_2} - z_{B_2}$, and $x - y - z$ represent the helicity frame for B_1 , $\bar{B}_2$, and J/ψ , respectively. θ and ϕ denote the production angles of the B_1 baryon in the J/ψ rest frame.

Under parity and CP transformations, the helicity amplitude follows the transformation relations:

$$\begin{aligned} A_{\lambda_1, \lambda_2}^{B\bar{B}} &\xrightarrow{\text{Parity Transform}} P_B P_{\bar{B}} P_{J/\psi} (-1)^{J-s_1-s_2} A_{-\lambda_1, -\lambda_2}^{B\bar{B}}, \\ A_{\lambda_1, \lambda_2}^{B\bar{B}} &\xrightarrow{\text{Charge Conjugation}} \eta_{J/\psi} (-1)^J A_{\lambda_1, \lambda_2}^{\bar{B}B}, \\ A_{\lambda_1, \lambda_2}^{B\bar{B}} &\xrightarrow{CP \text{ Transform}} P_B P_{\bar{B}} P_{J/\psi} (-1)^{J-s_1-s_2} \eta_{J/\psi} (-1)^J A_{-\lambda_1, -\lambda_2}^{\bar{B}B}, \end{aligned} \quad (33)$$

where P_B , $P_{\bar{B}}$, and $P_{J/\psi}$ represent the parity of B , $\bar{B}$, and J/ψ , and J , s_1 , and s_2 represent their spins, $\eta_{J/\psi}$ is the charge parity of J/ψ , and $(-1)^J$ is the charge parity of the fermion-antifermion system $B\bar{B}$. We obtain that h_1 and h_2 correspond to amplitudes that conserve both parity and CP , h_3 corresponds to an amplitude with both parity and CP violation, and h_4 corresponds to an amplitude with parity violation but CP conservation.

These helicity amplitudes are related to the form factors of the baryon B , which are introduced by

$$\begin{aligned} \mathcal{M}_{\lambda_1, \lambda_2} &= \bar{u}(p_1, \lambda_1) \left(F_1(q^2) \gamma^\mu + F_2(q^2) \frac{i\sigma^{\mu\nu} q_\nu}{2M} + F_3(q^2) \gamma^\mu \gamma_5 \right. \\ &\quad \left. + F_4(q^2) \frac{\sigma^{\mu\nu} q_\nu}{2M} \gamma_5 \right) v(p_2, \lambda_2) \varepsilon_\mu(q, \kappa), \end{aligned} \quad (34)$$

where $\bar{u}$ and v are the spinors of B and $\bar{B}$, p_1 and p_2 denote their momenta, and M is the baryon mass. ε_μ denotes the polarization vector of J/ψ , and $q = p_1 + p_2$, $\kappa = \lambda_1 - \lambda_2$ are its momentum and helicity, respectively. The form factors F_1 , F_2 , and F_4 correspond to the charge, magnetic dipole moment, and electric dipole moment form factors of the baryon B , and F_3 represents the parity violation form factor. These form factors can be complex in the timelike region. The relation between the electric dipole moment form factor and the electric dipole moment d_B is given by [52,53]

$$\frac{1}{2M} F_4(q^2) = \frac{2e}{3M_{J/\psi}^2} g_V d_B, \quad (35)$$

where $M_{J/\psi}$ is the mass of J/ψ , and $g_V = 1.286 \text{ GeV}^2$ is the effective coupling constant between the charm quark and J/ψ [54].

According to Eqs. (31) and (34), we derive the correspondence between the helicity amplitudes and the baryon form factors:

TABLE I. Based on the measurements in [40,42,44,45,49] and the calculations in [54], along with Eqs. (36) and (37), we calculate the size of the parity-violating parameters for the $J/\psi \rightarrow B\bar{B}$ processes.

Process	α_2	ϕ_2
$J/\psi \rightarrow p\bar{p}$	2.39×10^{-7}	π
$J/\psi \rightarrow n\bar{n}$	2.91×10^{-7}	0
$J/\psi \rightarrow \Lambda\bar{\Lambda}$	1.22×10^{-7}	0
$J/\psi \rightarrow \Sigma^0\bar{\Sigma}^0$	1.98×10^{-13}	0
$J/\psi \rightarrow \Sigma^+\bar{\Sigma}^-$	1.02×10^{-7}	π
$J/\psi \rightarrow \Sigma^-\bar{\Sigma}^+$	$\dots$	0
$J/\psi \rightarrow \Xi^0\bar{\Xi}^0$	1.23×10^{-7}	0
$J/\psi \rightarrow \Xi^-\bar{\Xi}^+$	1.90×10^{-7}	0

$$\begin{aligned}
h_1 &= 2M \left(F_1(q^2) + \frac{E_c^2}{M^2} F_2(q^2) \right) = 2MG_E, \\
h_2 &= 2\sqrt{2}E_c(F_1(q^2) + F_2(q^2)) = 2\sqrt{2}E_c G_M, \\
h_3 &= i \frac{2E_c}{M} \sqrt{E_c^2 - M^2} F_4(q^2), \\
h_4 &= 2\sqrt{2} \sqrt{E_c^2 - M^2} F_3(q^2),
\end{aligned} \tag{36}$$

where $E_c = \sqrt{q^2}/2$ represents the beam energy in the center-of-mass frame. We parametrize these helicity amplitudes in the following scheme:

$$\begin{aligned}
h_1 &= \frac{\sqrt{2}}{2} \sqrt{1 - \alpha_\psi}, \\
h_2 &= \sqrt{1 + \alpha_\psi} e^{-i\Phi}, \\
h_3 &= \sqrt{\alpha_1} e^{i\phi_1}, \\
h_4 &= \sqrt{2\alpha_2} e^{i\phi_2}.
\end{aligned} \tag{37}$$

The estimated sizes of the parity violation parameters for various baryon production processes are summarized in Table I.

B. Decay amplitudes for the subprocess $J/\psi \rightarrow B_1\bar{B}_2$

For subprocesses $J/\psi \rightarrow B_1\bar{B}_2$, where $\bar{B}_2$ is not the antiparticle of B_1 (e.g., $J/\psi \rightarrow \Lambda\bar{\Sigma}^0$), the helicity amplitudes can be expressed as

$$A_{\lambda_1, \lambda_2} = \begin{pmatrix} h_1 + h_3 & h_2 + h_4 \\ h_2 - h_4 & h_1 - h_3 \end{pmatrix}, \tag{38}$$

where λ_1 and λ_2 denote the helicities of B_1 and $\bar{B}_2$, respectively. Under parity transformations, the helicity amplitudes follow the transformation relations:

$$A_{\lambda_1, \lambda_2}^{B_1\bar{B}_2} \xrightarrow{\text{Parity Transform}} P_{B_1} P_{\bar{B}_2} P_{J/\psi} (-1)^{J-s_1-s_2} A_{-\lambda_1, -\lambda_2}^{B_1\bar{B}_2}, \tag{39}$$

where P_{B_1} , $P_{\bar{B}_2}$, and $P_{J/\psi}$ are the parities of B_1 , $\bar{B}_2$, and J/ψ , and J , s_1 , and s_2 are their spins. For the process $J/\psi \rightarrow \Lambda\bar{\Sigma}^0$, we obtain that h_1 and h_2 correspond to the amplitudes where parity is conserved, and h_3 and h_4 correspond to amplitudes with parity violation.

For subprocesses $J/\psi \rightarrow B_1\bar{B}_2$, the helicity amplitudes are related to the transition form factors of $B_1\bar{B}_2$, which are introduced by [34,66]:

$$\begin{aligned}
\mathcal{M}_{\lambda_1 \lambda_2} &= \bar{u}(p_1, \lambda_1) \left[F_1(q^2) \gamma^\mu + F_2(q^2) \frac{i\sigma^{\mu\nu} q_\nu}{M_1 + M_2} \right. \\
&\quad \left. + F_3(q^2) \gamma^\mu \gamma_5 + F_4(q^2) \frac{\sigma^{\mu\nu} q_\nu}{M_1 + M_2} \gamma_5 \right] \\
&\quad \times v(p_2, \lambda_2) \varepsilon_\mu(q, \kappa),
\end{aligned} \tag{40}$$

where $\bar{u}$ and v are the spinors of the baryons B_1 and $\bar{B}_2$, and p_1 , p_2 , M_1 , M_2 are their momenta and masses. ε_μ is the polarization vector of J/ψ , and $q = p_1 + p_2$, $\kappa = \lambda_1 - \lambda_2$ are its momentum and helicity. F_1 , F_2 , F_3 , and F_4 are the transition form factors for the production process of $J/\psi \rightarrow B_1\bar{B}_2$. Using the CP transformation properties of fermion bilinears, we obtain the following relations between the form factors for the conjugate processes $J/\psi \rightarrow B_1\bar{B}_2$ and $J/\psi \rightarrow B_2\bar{B}_1$,

$$\begin{aligned}
F_1^{B_1\bar{B}_2} &= -P_{B_1} P_{\bar{B}_2} P_{J/\psi} \eta_{J/\psi} F_1^{B_2\bar{B}_1}, \\
F_2^{B_1\bar{B}_2} &= -P_{B_1} P_{\bar{B}_2} P_{J/\psi} \eta_{J/\psi} F_2^{B_2\bar{B}_1}, \\
F_3^{B_1\bar{B}_2} &= -P_{B_1} P_{\bar{B}_2} P_{J/\psi} \eta_{J/\psi} F_3^{B_2\bar{B}_1}, \\
F_4^{B_1\bar{B}_2} &= P_{B_1} P_{\bar{B}_2} P_{J/\psi} \eta_{J/\psi} F_4^{B_2\bar{B}_1}.
\end{aligned} \tag{41}$$

Since Eqs. (38) and (40) are equivalent, i.e., $A_{\lambda_1 \lambda_2} = \mathcal{M}_{\lambda_1 \lambda_2}$, the correspondence between helicity amplitudes and form factors is given by

$$\begin{aligned}
h_1 &= \frac{2\sqrt{1 - \xi_2^2}}{\xi_1} E_c (\xi_1^2 F_1 + F_2), \\
h_2 &= 2\sqrt{2} \sqrt{1 - \xi_2^2} E_c (F_1 + F_2), \\
h_3 &= \frac{2\sqrt{1 - \xi_1^2}}{\xi_1} E_c (iF_4 \mp \xi_1 \xi_2 F_3), \\
h_4 &= \frac{2\sqrt{2} \sqrt{1 - \xi_1^2}}{\xi_1} E_c (\xi_1 F_3 \mp i\xi_2 F_4),
\end{aligned} \tag{42}$$

where $\xi_1 = (M_1 + M_2)/\sqrt{q^2}$ and $\xi_2 = (M_1 - M_2)/\sqrt{q^2}$, with M_1 and M_2 representing the masses of B_1 and $\bar{B}_2$, respectively. The sign $-$ before ξ_2 corresponds to the process $J/\psi \rightarrow B_1\bar{B}_2$, and the sign $+$ corresponds to the conjugate process $J/\psi \rightarrow B_2\bar{B}_1$. For specific reaction channels such as $J/\psi \rightarrow \Lambda\bar{\Sigma}^0$ and its conjugate process

$J/\psi \rightarrow \Sigma^0 \bar{\Lambda}$, the helicity amplitudes satisfy the following relations:

$$\begin{aligned} h_1^{\Lambda \bar{\Sigma}^0} &= h_1^{\Sigma^0 \bar{\Lambda}} = h_1^{\bar{\Lambda} \Sigma^0}, \\ h_2^{\Lambda \bar{\Sigma}^0} &= h_2^{\Sigma^0 \bar{\Lambda}} = h_2^{\bar{\Lambda} \Sigma^0}, \\ h_3^{\Lambda \bar{\Sigma}^0} &= -h_3^{\Sigma^0 \bar{\Lambda}} = -h_3^{\bar{\Lambda} \Sigma^0}, \\ h_4^{\Lambda \bar{\Sigma}^0} &= h_4^{\Sigma^0 \bar{\Lambda}} = -h_4^{\bar{\Lambda} \Sigma^0}. \end{aligned} \quad (43)$$

The helicity amplitudes are parametrized as follows:

$$\begin{aligned} \{\alpha_{\psi}^{\Lambda \bar{\Sigma}^0}, \alpha_1^{\Lambda \bar{\Sigma}^0}, \alpha_2^{\Lambda \bar{\Sigma}^0}\} &= \{\alpha_{\psi}^{\Sigma^0 \bar{\Lambda}}, \alpha_1^{\Sigma^0 \bar{\Lambda}}, \alpha_2^{\Sigma^0 \bar{\Lambda}}\} = \{\alpha_{\psi}^{\bar{\Lambda} \Sigma^0}, \alpha_1^{\bar{\Lambda} \Sigma^0}, \alpha_2^{\bar{\Lambda} \Sigma^0}\}, \\ \{\Phi^{\Lambda \bar{\Sigma}^0}, \phi_1^{\Lambda \bar{\Sigma}^0}, \phi_2^{\Lambda \bar{\Sigma}^0}\} &= \{\Phi^{\Sigma^0 \bar{\Lambda}}, \pi + \phi_1^{\Sigma^0 \bar{\Lambda}}, \phi_2^{\Sigma^0 \bar{\Lambda}}\} = \{\Phi^{\bar{\Lambda} \Sigma^0}, \pi + \phi_1^{\bar{\Lambda} \Sigma^0}, \pi + \phi_2^{\bar{\Lambda} \Sigma^0}\}. \end{aligned} \quad (45)$$

Notably, the amplitudes h_3 and h_4 involve mixtures of the CP -even and CP -odd form factors. To separate these, we introduce an alternative parametrization scheme. We define new helicity amplitudes

$$\begin{aligned} h_3 &= h'_3 \mp \xi_1 \xi_2 h'_4, \\ h_4 &= \sqrt{2}(\xi_1 h'_4 \mp \xi_2 h'_3), \end{aligned} \quad (46)$$

Then we have

$$\begin{aligned} h'_3 &= i \frac{2\sqrt{1-\xi_1^2}}{\xi_1} E_c F_4, \\ h'_4 &= \frac{2\sqrt{1-\xi_1^2}}{\xi_1} E_c F_3. \end{aligned} \quad (47)$$

For the specific channels $J/\psi \rightarrow \Lambda \bar{\Sigma}^0$ and its conjugate $J/\psi \rightarrow \Sigma^0 \bar{\Lambda}$, the CP conservation requires

$$\begin{aligned} h_1^{\Lambda \bar{\Sigma}^0} &= h_1^{\Sigma^0 \bar{\Lambda}}, \\ h_2^{\Lambda \bar{\Sigma}^0} &= h_2^{\Sigma^0 \bar{\Lambda}}, \\ h_3^{\Lambda \bar{\Sigma}^0} &= -h_3^{\Sigma^0 \bar{\Lambda}}, \\ h_4^{\Lambda \bar{\Sigma}^0} &= h_4^{\Sigma^0 \bar{\Lambda}}. \end{aligned} \quad (48)$$

We then adopt the following parametrization scheme for the helicity amplitudes:

$$\begin{aligned} h_1 &= \frac{\sqrt{2}}{2} \sqrt{1-\alpha_{\psi}}, \\ h_2 &= \sqrt{1+\alpha_{\psi}} e^{-i\Phi}, \\ h'_3 &= \sqrt{\alpha'_1} e^{i\phi'_1}, \\ h'_4 &= \sqrt{\alpha'_2} e^{i\phi'_2}. \end{aligned} \quad (49)$$

We have

$$\begin{aligned} h_1 &= \frac{\sqrt{2}}{2} \sqrt{1-\alpha_{\psi}}, \\ h_2 &= \sqrt{1+\alpha_{\psi}} e^{-i\Phi}, \\ h_3 &= \sqrt{\alpha'_1} e^{i\phi'_1}, \\ h_4 &= \sqrt{2\alpha'_2} e^{i\phi'_2}. \end{aligned} \quad (44)$$

Under CP conservation, the parameters satisfy the following relations:

$$\begin{aligned} \alpha_1 &= \alpha'_1 + \xi_1^2 \xi_2^2 \alpha'_2 \mp 2\xi_1 \xi_2 \sqrt{\alpha'_1 \alpha'_2} \cos(\phi'_1 - \phi'_2), \\ \alpha_2 &= \xi_2^2 \alpha'_1 + \xi_1^2 \alpha'_2 \mp 2\xi_1 \xi_2 \sqrt{\alpha'_1 \alpha'_2} \cos(\phi'_1 - \phi'_2), \\ \sin \phi_1 &= \frac{\sqrt{\alpha'_1} \sin \phi'_1 \mp \xi_1 \xi_2 \sqrt{\alpha'_2} \sin \phi'_2}{\sqrt{\alpha_1}}, \\ \cos \phi_1 &= \frac{\sqrt{\alpha'_1} \cos \phi'_1 \mp \xi_1 \xi_2 \sqrt{\alpha'_2} \cos \phi'_2}{\sqrt{\alpha_1}}, \\ \sin \phi_2 &= \frac{\xi_1 \sqrt{\alpha'_2} \sin \phi'_2 \mp \xi_2 \sqrt{\alpha'_1} \sin \phi'_1}{\sqrt{\alpha_2}}, \\ \cos \phi_2 &= \frac{\xi_1 \sqrt{\alpha'_2} \cos \phi'_2 \mp \xi_2 \sqrt{\alpha'_1} \cos \phi'_1}{\sqrt{\alpha_2}}. \end{aligned} \quad (50)$$

By comparing these conjugate production processes, one can search for signals of CP violation. For example, for the processes $J/\psi \rightarrow \Lambda \bar{\Sigma}^0$ and $J/\psi \rightarrow \Sigma^0 \bar{\Lambda}$, based on Eq. (48), CP conservation requires

$$\begin{aligned} \{\alpha_{\psi}^{\Lambda \bar{\Sigma}^0}, \alpha_1^{\Lambda \bar{\Sigma}^0}, \alpha_2^{\Lambda \bar{\Sigma}^0}\} &= \{\alpha_{\psi}^{\Sigma^0 \bar{\Lambda}}, \alpha_1^{\Sigma^0 \bar{\Lambda}}, \alpha_2^{\Sigma^0 \bar{\Lambda}}\}, \\ \{\Phi^{\Lambda \bar{\Sigma}^0}, \phi_1^{\Lambda \bar{\Sigma}^0}, \phi_2^{\Lambda \bar{\Sigma}^0}\} &= \{\Phi^{\Sigma^0 \bar{\Lambda}}, \pi + \phi_1^{\Sigma^0 \bar{\Lambda}}, \pi + \phi_2^{\Sigma^0 \bar{\Lambda}}\}. \end{aligned} \quad (51)$$

For the baryon-antibaryon pair production processes $J/\psi \rightarrow B \bar{B}$, the relationships between the form factors and the partial decay widths of J/ψ have been provided in Ref. [54]. Here, we extend a similar analysis to the $J/\psi \rightarrow B_1 \bar{B}_2$ case. The partial decay width for $J/\psi \rightarrow B_1 \bar{B}_2$ is given by

$$\Gamma_{J/\psi \rightarrow B_1 \bar{B}_2} = \frac{|\vec{p}|}{8\pi M_{J/\psi}^2} \frac{1}{3} \sum_{\lambda_1, \lambda_2} |\mathcal{M}_{\lambda_1 \lambda_2}|^2, \quad (52)$$

where $|\vec{p}|$ is the momentum of $B_1 (\bar{B}_2)$ in the rest frame of J/ψ . By substituting Eq. (40) into above equation, we obtain

$$\Gamma_{J/\psi \rightarrow B_1 \bar{B}_2} = \frac{M_{J/\psi}}{24\pi} (1 - \xi_1^2)^{1/2} (1 - \xi_2^2)^{3/2} \times (2|G_1|^2 + \xi_1^2|G_2|^2 + 2|G_3|^2 + |G_4|^2), \quad (53)$$

where the recombined form factors G_i are defined as

$$\begin{aligned} G_1 &= F_1 + F_2, \\ G_2 &= F_1 + \frac{F_2}{\xi_1^2}, \\ G_3 &= \frac{\sqrt{1 - \xi_1^2}(\xi_1 F_3 - i\xi_2 F_4)}{\xi_1 \sqrt{1 - \xi_2^2}}, \\ G_4 &= \frac{\sqrt{1 - \xi_1^2}(iF_4 - \xi_1 \xi_2 F_3)}{\xi_1 \sqrt{1 - \xi_2^2}}. \end{aligned} \quad (54)$$

For the process $J/\psi \rightarrow \Lambda \bar{\Sigma}^0$, the form factors G_1 and G_2 account for parity conservation contributions, with the relative phase defined as $\Phi^{\Lambda \bar{\Sigma}^0} = \arg(G_2/G_1)$. Since the parity-violating form factors G_3 and G_4 are expected to be small, and their contributions to the decay width are quadratically suppressed, they can be safely neglected. Based on measurements from Ref. [46], we obtain

$$\begin{aligned} G_1 &= 1.41 * 10^{-4}, \\ G_2 &= 1.21 * 10^{-4}, \\ \Phi &= 1.011 \text{ rad}. \end{aligned} \quad (55)$$

C. The polarization transfer matrix for subprocess $J/\psi \rightarrow B_1 \bar{B}_2$

The polarization correlations of $B_1 \bar{B}_2$ for the subprocess $J/\psi \rightarrow B_1 \bar{B}_2$ can be expressed as

$$\begin{aligned} \rho_{\lambda_1, \lambda_2; \lambda'_1, \lambda'_2}^{B_1 \bar{B}_2} &= \sum_{\kappa, \kappa'} \mathcal{D}_{\kappa, \lambda_1 - \lambda_2}^{1*}(0, \theta, \phi) \mathcal{D}_{\kappa', \lambda'_1 - \lambda'_2}^1(0, \theta, \phi) \\ &\times \rho_{\kappa, \kappa'}^{J/\psi} A_{\lambda_1, \lambda_2} A_{\lambda'_1, \lambda'_2}^*, \end{aligned} \quad (56)$$

where θ and ϕ are the helicity angles of B_1 in the rest frame of J/ψ . $\rho_{\kappa, \kappa'}^{J/\psi}$ is the spin density matrix of J/ψ , and $\rho_{\lambda_1, \lambda_2; \lambda'_1, \lambda'_2}^{B_1 \bar{B}_2}$ represents the polarization correlation density matrix (production density matrix) for $B_1 \bar{B}_2$. In the helicity formalism, this density matrix can be written as

$$\rho_{\lambda_1, \lambda_2; \lambda'_1, \lambda'_2}^{B_1 \bar{B}_2} = \sum_{\mu, \nu=0} S_{\mu\nu} \Sigma_{\mu}^{B_1} \otimes \Sigma_{\nu}^{\bar{B}_2}, \quad (57)$$

where $S_{\mu\nu}$ is the polarization correlation matrix for $B_1 \bar{B}_2$, and $\Sigma_{\mu}^{B_1}$, $\Sigma_{\nu}^{\bar{B}_2}$ are their polarization expansion matrices, given by Eq. (2). We introduce a novel form to describe the decay process,

$$S_{\mu\nu} = S_{\rho}^{\psi} a_{\rho, \mu\nu}, \quad (58)$$

where S_{ρ}^{ψ} represents the polarization expansion coefficients for J/ψ , as given in Eq. (12). $a_{\rho\mu\nu}$ represents the polarization transfer matrix from J/ψ to $B_1 \bar{B}_2$, which is defined by

$$\begin{aligned} a_{\rho, \mu\nu} &= 4 \sum_{\lambda_1, \lambda'_1} \sum_{\lambda_2, \lambda'_2} (\Sigma_{\mu}^{B_1})_{\lambda'_1, \lambda_1} (\Sigma_{\nu}^{\bar{B}_2})_{\lambda'_2, \lambda_2} A_{\lambda_1, \lambda_2} A_{\lambda'_1, \lambda'_2}^* \\ &\times \sum_{\kappa, \kappa'} (\Sigma_{\rho}^{J/\psi})_{\kappa, \kappa'} \mathcal{D}_{\kappa, \lambda_1 - \lambda_2}^{1*}(0, \theta, \phi) \mathcal{D}_{\kappa', \lambda'_1 - \lambda'_2}^1(0, \theta, \phi), \end{aligned} \quad (59)$$

where $\Sigma_{\rho}^{J/\psi}$ is the polarization projection matrix for J/ψ , as outlined in Appendix B. By substituting Eqs. (31) and (37) into the above equation, we obtain the specific form of $a_{\rho, \mu\nu}$ for $J/\psi \rightarrow B \bar{B}$, as detailed in Appendix C. Then the full polarization of $B \bar{B}$ can be expressed as

$$\begin{aligned} S_{00} &= \frac{1}{3} (3 + \alpha_{\psi} + 2\alpha_1 + 4\alpha_2) + 2\sqrt{2} D_3^c [S_L^{\psi} \cos \theta + (S_T^{x, \psi} \cos \phi + S_T^{y, \psi} \sin \phi) \sin \theta] \\ &\quad + (\alpha_{\psi} - \alpha_1 + \alpha_2) \left[\frac{1}{3} S_{LL} (1 + 3 \cos 2\theta) + (S_{LT}^{x, \psi} \cos \phi + S_{LT}^{y, \psi} \sin \phi) \sin 2\theta \right. \\ &\quad \left. + (S_{TT}^{xx, \psi} \cos 2\phi + S_{TT}^{xy, \psi} \sin 2\phi) \sin^2 \theta \right], \\ S_{10} &= -(D_0^c - 2D_5^c) [S_L^{\psi} \sin \theta - (S_T^{x, \psi} \cos \phi + S_T^{y, \psi} \sin \phi) \cos \theta] \\ &\quad - \sqrt{2} (D_1^s + D_4^s) (S_T^{x, \psi} \sin \phi - S_T^{y, \psi} \cos \phi) + \sqrt{2} (D_1^c - D_4^c) \left[S_{LL} \sin 2\theta \right. \end{aligned} \quad (60)$$

$$\begin{aligned}
& - (S_{LT}^{x,\psi} \cos \phi + S_{LT}^{y,\psi} \sin \phi) \cos 2\theta - \frac{1}{2} (S_{TT}^{xx,\psi} \cos 2\phi + S_{TT}^{xy,\psi} \sin 2\phi) \sin 2\theta \Big] \\
& + (D_0^s - 2D_5^s) [(S_{LT}^{x,\psi} \sin \phi - S_{LT}^{y,\psi} \cos \phi) \cos \theta + (S_{TT}^{xx,\psi} \sin 2\phi - S_{TT}^{xy,\psi} \cos 2\phi) \sin \theta], \tag{61}
\end{aligned}$$

$$\begin{aligned}
S_{20} = & \sqrt{2}(D_1^s + D_4^s) [S_L^\psi \sin \theta - (S_T^{x,\psi} \cos \phi + S_T^{y,\psi} \sin \phi) \cos \theta] \\
& - (D_0^c - 2D_5^c) (S_T^{x,\psi} \sin \phi - S_T^{y,\psi} \cos \phi) - (D_0^s - 2D_5^s) \Big[S_{LL} \sin 2\theta \\
& - (S_{LT}^{x,\psi} \cos \phi + S_{LT}^{y,\psi} \sin \phi) \cos 2\theta - \frac{1}{2} (S_{TT}^{xx,\psi} \cos 2\phi + S_{TT}^{xy,\psi} \sin 2\phi) \sin 2\theta \Big] \\
& + \sqrt{2}(D_1^c - D_4^c) [(S_{LT}^{x,\psi} \sin \phi - S_{LT}^{y,\psi} \cos \phi) \cos \theta + (S_{TT}^{xx,\psi} \sin 2\phi - S_{TT}^{xy,\psi} \cos 2\phi) \sin \theta], \tag{62}
\end{aligned}$$

$$\begin{aligned}
S_{30} = & \frac{2\sqrt{2}}{3} (D_2^c + 2D_3^c) + (1 + \alpha_\psi + 2\alpha_2) [S_L^\psi \cos \theta + (S_T^{x,\psi} \cos \phi + S_T^{y,\psi} \sin \phi) \sin \theta] \\
& - \sqrt{2}(D_2^c - D_3^c) \Big[\frac{1}{3} S_{LL}^\psi (1 + 3 \cos 2\theta) + (S_{LT}^{x,\psi} \cos \phi + S_{LT}^{y,\psi} \sin \phi) \sin 2\theta \\
& + (S_{TT}^{xx,\psi} \cos 2\phi + S_{TT}^{xy,\psi} \sin 2\phi) \sin^2 \theta \Big], \tag{63}
\end{aligned}$$

$$\begin{aligned}
S_{01} = & -(D_0^c + 2D_5^c) [S_L^\psi \sin \theta - (S_T^{x,\psi} \cos \phi + S_T^{y,\psi} \sin \phi) \cos \theta] \\
& + \sqrt{2}(D_1^s - D_4^s) (S_T^{x,\psi} \sin \phi - S_T^{y,\psi} \cos \phi) - \sqrt{2}(D_1^c + D_4^c) \Big[S_{LL}^\psi \sin 2\theta \\
& - (S_{LT}^{x,\psi} \cos \phi + S_{LT}^{y,\psi} \sin \phi) \cos 2\theta - \frac{1}{2} (S_{TT}^{xx,\psi} \cos 2\phi + S_{TT}^{xy,\psi} \sin 2\phi) \sin 2\theta \Big] \\
& + (D_0^s + 2D_5^s) [(S_{LT}^{x,\psi} \sin \phi - S_{LT}^{y,\psi} \cos \phi) \cos \theta + (S_{TT}^{xx,\psi} \sin 2\phi - S_{TT}^{xy,\psi} \cos 2\phi) \sin \theta], \tag{64}
\end{aligned}$$

$$\begin{aligned}
S_{11} = & \frac{1}{3} (1 - \alpha_\psi - 2\alpha_1) - S_{LL}^\psi \Big[\frac{2}{3} (1 - \alpha_\psi - 2\alpha_1) - 2(1 - \alpha_1 - \alpha_2) \sin^2 \theta \Big] \\
& - (1 - \alpha_1 - \alpha_2) (S_{LT}^{x,\psi} \cos \phi + S_{LT}^{y,\psi} \sin \phi) \sin 2\theta \\
& + 2\sqrt{2}D_3^s [(S_{LT}^{x,\psi} \sin \phi - S_{LT}^{y,\psi} \cos \phi) \sin \theta - (S_{TT}^{xx,\psi} \sin 2\phi - S_{TT}^{xy,\psi} \cos 2\phi) \cos \theta] \\
& + [(1 + \alpha_\psi - 2\alpha_2) - (1 - \alpha_1 - \alpha_2) \sin^2 \theta] (S_{TT}^{xx,\psi} \cos 2\phi + S_{TT}^{xy,\psi} \sin 2\phi), \tag{65}
\end{aligned}$$

$$\begin{aligned}
S_{21} = & -\frac{2\sqrt{2}}{3} D_2^s + \sqrt{2} S_{LL}^\psi \Big[\frac{1}{3} D_2^s (1 + 3 \cos 2\theta) - 2D_3^s \sin^2 \theta \Big] \\
& + \sqrt{2}(D_2^s + D_3^s) (S_{LT}^{x,\psi} \cos \phi + S_{LT}^{y,\psi} \sin \phi) \sin 2\theta \\
& + (1 + \alpha_\psi - 2\alpha_2) [(S_{LT}^{x,\psi} \sin \phi - S_{LT}^{y,\psi} \cos \phi) \sin \theta - (S_{TT}^{xx,\psi} \sin 2\phi - S_{TT}^{xy,\psi} \cos 2\phi) \cos \theta] \\
& + \frac{\sqrt{2}}{2} [2D_2^s \sin^2 \theta - D_3^s (3 + \cos 2\theta)] (S_{TT}^{xx,\psi} \cos 2\phi + S_{TT}^{xy,\psi} \sin 2\phi), \tag{66}
\end{aligned}$$

$$\begin{aligned}
S_{31} = & -\sqrt{2}(D_1^c + D_4^c) [S_L^\psi \sin \theta - (S_T^{x,\psi} \cos \phi + S_T^{y,\psi} \sin \phi) \cos \theta] \\
& + (D_0^s + 2D_5^s) (S_T^{x,\psi} \sin \phi - S_T^{y,\psi} \cos \phi) - (D_0^c + 2D_5^c) [S_{LL}^\psi \sin 2\theta \\
& - (S_{LT}^{x,\psi} \cos \phi + S_{LT}^{y,\psi} \sin \phi) \cos 2\theta - \frac{1}{2} (S_{TT}^{xx,\psi} \cos 2\phi + S_{TT}^{xy,\psi} \sin 2\phi) \sin 2\theta] \\
& + \sqrt{2}(D_1^s - D_4^s) [(S_{LT}^{x,\psi} \sin \phi - S_{LT}^{y,\psi} \cos \phi) \cos \theta + (S_{TT}^{xx,\psi} \sin 2\phi - S_{TT}^{xy,\psi} \cos 2\phi) \sin \theta], \tag{67}
\end{aligned}$$

$$\begin{aligned}
S_{02} = & \sqrt{2}(D_1^s - D_4^s)[S_L^\psi \sin \theta - (S_T^{x,\psi} \cos \phi + S_T^{y,\psi} \sin \phi) \cos \theta] \\
& + (D_0^c + 2D_5^c)(S_T^{x,\psi} \sin \phi - S_T^{y,\psi} \cos \phi) + (D_0^s + 2D_5^s) \left[S_{LL}^\psi \sin 2\theta \right. \\
& \left. - (S_{LT}^{x,\psi} \cos \phi + S_{LT}^{y,\psi} \sin \phi) \cos 2\theta - \frac{1}{2}(S_{TT}^{xx,\psi} \cos 2\phi + S_{TT}^{xy,\psi} \sin 2\phi) \sin 2\theta \right] \\
& + \sqrt{2}(D_1^c + D_4^c)[(S_{LT}^{x,\psi} \sin \phi - S_{LT}^{y,\psi} \cos \phi) \cos \theta + (S_{TT}^{xx,\psi} \sin 2\phi - S_{TT}^{xy,\psi} \cos 2\phi) \sin \theta], \tag{68}
\end{aligned}$$

$$\begin{aligned}
S_{12} = & -\frac{2\sqrt{2}}{3}D_2^s + \sqrt{2}S_{LL}^\psi \left[\frac{1}{3}D_2^s(1 + 3\cos 2\theta) + 2D_3^s \sin^2 \theta \right] \\
& + \sqrt{2}(D_2^s - D_3^s)(S_{LT}^{x,\psi} \cos \phi + S_{LT}^{y,\psi} \sin \phi) \sin 2\theta \\
& - (1 + \alpha_\psi - 2\alpha_2)[(S_{LT}^{x,\psi} \sin \phi - S_{LT}^{y,\psi} \cos \phi) \sin \theta - (S_{TT}^{xx,\psi} \sin 2\phi - S_{TT}^{xy,\psi} \cos 2\phi) \cos \theta] \\
& + \frac{\sqrt{2}}{2}(2D_2^s \sin^2 \theta + D_3^s(3 + \cos 2\theta))(S_{TT}^{xx,\psi} \cos 2\phi + S_{TT}^{xy,\psi} \sin 2\phi), \tag{69}
\end{aligned}$$

$$\begin{aligned}
S_{22} = & -\frac{1}{3}(1 - \alpha_\psi - 2\alpha_1) + S_{LL}^\psi \left[\frac{2}{3}(1 - \alpha_\psi - 2\alpha_1) + 2(\alpha_\psi + \alpha_1 - \alpha_2) \sin^2 \theta \right] \\
& - (\alpha_\psi + \alpha_1 - \alpha_2)(S_{LT}^{x,\psi} \cos \phi + S_{LT}^{y,\psi} \sin \phi) \sin 2\theta \\
& + 2\sqrt{2}D_3^s[(S_{LT}^{x,\psi} \sin \phi - S_{LT}^{y,\psi} \cos \phi) \sin \theta - (S_{TT}^{xx,\psi} \sin 2\phi - S_{TT}^{xy,\psi} \cos 2\phi) \cos \theta] \\
& + [(1 + \alpha_\psi - 2\alpha_2) - (\alpha_\psi + \alpha_1 - \alpha_2) \sin^2 \theta](S_{TT}^{xx,\psi} \cos 2\phi + S_{TT}^{xy,\psi} \sin 2\phi), \tag{70}
\end{aligned}$$

$$\begin{aligned}
S_{32} = & (D_0^s + 2D_5^s)[S_L^\psi \sin \theta - (S_T^{x,\psi} \cos \phi + S_T^{y,\psi} \sin \phi) \cos \theta] \\
& + \sqrt{2}(D_1^c + D_4^c)(S_T^{x,\psi} \sin \phi - S_T^{y,\psi} \cos \phi) + \sqrt{2}(D_1^s - D_4^s) \left[S_{LL}^\psi \sin 2\theta \right. \\
& \left. - (S_{LT}^{x,\psi} \cos \phi + S_{LT}^{y,\psi} \sin \phi) \cos 2\theta - \frac{1}{2}(S_{TT}^{xx,\psi} \cos 2\phi + S_{TT}^{xy,\psi} \sin 2\phi) \sin 2\theta \right] \\
& + (D_0^c + 2D_5^c)[(S_{LT}^{x,\psi} \sin \phi - S_{LT}^{y,\psi} \cos \phi) \cos \theta + (S_{TT}^{xx,\psi} \sin 2\phi - S_{TT}^{xy,\psi} \cos 2\phi) \sin \theta], \tag{71}
\end{aligned}$$

$$\begin{aligned}
S_{03} = & \frac{2\sqrt{2}}{3}(D_2^c - 2D_3^c) - (1 + \alpha_\psi + 2\alpha_2)[S_L^\psi \cos \theta + (S_T^{x,\psi} \cos \phi + S_T^{y,\psi} \sin \phi) \sin \theta] \\
& - \sqrt{2}(D_2^c + D_3^c) \left[\frac{1}{3}S_{LL}^\psi(1 + 3\cos 2\theta) + (S_{LT}^{x,\psi} \cos \phi + S_{LT}^{y,\psi} \sin \phi) \sin 2\theta \right. \\
& \left. + (S_{TT}^{xx,\psi} \cos 2\phi + S_{TT}^{xy,\psi} \sin 2\phi) \sin^2 \theta \right], \tag{72}
\end{aligned}$$

$$\begin{aligned}
S_{13} = & -\sqrt{2}(D_1^c - D_4^c)[S_L^\psi \sin \theta - (S_T^{x,\psi} \cos \phi + S_T^{y,\psi} \sin \phi) \cos \theta] \\
& - (D_0^s - 2D_5^s)(S_T^{x,\psi} \sin \phi - S_T^{y,\psi} \cos \phi) + (D_0^c - 2D_5^c) \left[S_{LL}^\psi \sin 2\theta \right. \\
& \left. - (S_{LT}^{x,\psi} \cos \phi + S_{LT}^{y,\psi} \sin \phi) \cos 2\theta - \frac{1}{2}(S_{TT}^{xx,\psi} \cos 2\phi + S_{TT}^{xy,\psi} \sin 2\phi) \sin 2\theta \right] \\
& + \sqrt{2}(D_1^s + D_4^s)[(S_{LT}^{x,\psi} \sin \phi - S_{LT}^{y,\psi} \cos \phi) \cos \theta + (S_{TT}^{xx,\psi} \sin 2\phi - S_{TT}^{xy,\psi} \cos 2\phi) \sin \theta], \tag{73}
\end{aligned}$$

$$\begin{aligned}
S_{23} = & (D_0^s - 2D_5^s)[S_L^\psi \sin \theta - (S_T^{x,\psi} \cos \phi + S_T^{y,\psi} \sin \phi) \cos \theta] \\
& - \sqrt{2}(D_1^c - D_4^c)(S_T^{x,\psi} \sin \phi - S_T^{y,\psi} \cos \phi) - \sqrt{2}(D_1^s + D_4^s) \left[S_{LL}^\psi \sin 2\theta \right.
\end{aligned}$$

$$\begin{aligned}
& - (S_{LT}^{x,\psi} \cos \phi + S_{LT}^{y,\psi} \sin \phi) \cos 2\theta - \frac{1}{2} (S_{TT}^{xx,\psi} \cos 2\phi + S_{TT}^{xy,\psi} \sin 2\phi) \sin 2\theta \Big] \\
& + (D_0^c - 2D_5^c) [(S_{LT}^{x,\psi} \sin \phi - S_{LT}^{y,\psi} \cos \phi) \cos \theta + (S_{TT}^{xx,\psi} \sin 2\phi - S_{TT}^{xy,\psi} \cos 2\phi) \sin \theta], \tag{74}
\end{aligned}$$

$$\begin{aligned}
S_{33} = & -\frac{1}{3} (1 + 3\alpha_\psi - 2\alpha_1 + 4\alpha_2) - 2\sqrt{2}D_3^c [S_L^\psi \cos \theta + (S_T^{x,\psi} \cos \phi + S_T^{y,\psi} \sin \phi) \sin \theta] \\
& - (1 + \alpha_1 + \alpha_2) \left[\frac{1}{3} S_{LL}^\psi (1 + 3 \cos 2\theta) + (S_{LT}^{x,\psi} \cos \phi + S_{LT}^{y,\psi} \sin \phi) \sin 2\theta \right. \\
& \left. + (S_{TT}^{xx,\psi} \cos 2\phi + S_{TT}^{xy,\psi} \sin 2\phi) \sin^2 \theta \right]. \tag{75}
\end{aligned}$$

For convenience, we introduce the following D -type functions:

$$\begin{aligned}
D_0^s &= \sqrt{1 - \alpha_\psi^2} \sin \Phi, \\
D_0^c &= \sqrt{1 - \alpha_\psi^2} \cos \Phi, \\
D_1^s &= \sqrt{\alpha_1(1 + \alpha_\psi)} \sin(\Phi + \phi_1), \\
D_1^c &= \sqrt{\alpha_1(1 + \alpha_\psi)} \cos(\Phi + \phi_1), \\
D_2^s &= \sqrt{\alpha_1(1 - \alpha_\psi)} \sin \phi_1, \\
D_2^c &= \sqrt{\alpha_1(1 - \alpha_\psi)} \cos \phi_1, \\
D_3^s &= \sqrt{\alpha_2(1 + \alpha_\psi)} \sin(\Phi + \phi_2), \\
D_3^c &= \sqrt{\alpha_2(1 + \alpha_\psi)} \cos(\Phi + \phi_2), \\
D_4^s &= \sqrt{\alpha_2(1 - \alpha_\psi)} \sin \phi_2, \\
D_4^c &= \sqrt{\alpha_2(1 - \alpha_\psi)} \cos \phi_2, \\
D_5^s &= \sqrt{\alpha_1\alpha_2} \sin(\phi_1 - \phi_2), \\
D_5^c &= \sqrt{\alpha_1\alpha_2} \cos(\phi_1 - \phi_2). \tag{76}
\end{aligned}$$

The terms D_1^s, D_1^c, D_2^s , and D_2^c arise from CP -violating effects in baryon production, specifically due to contributions from the baryon electric dipole moment. The terms D_3^s, D_3^c, D_4^s , and D_4^c originate from parity-violating effects, which are associated with the contribution of the parity-violating form factor F_3 . Finally, the terms D_5^s and D_5^c are affected by both CP -violating and parity-violating effects in the baryon production process. When neglecting the electron form factors and assuming unpolarized beams, $S_{LL}^\psi = 1/2$ and all other polarization components of J/ψ can be set to zero. Furthermore, if we also neglect parity-violating helicity amplitudes for baryon-antibaryon pair production processes, the expression simplifies to the form given in Ref. [30].

For the subprocess $J/\psi \rightarrow \Lambda \bar{\Sigma}^0$ and its conjugate channel $J/\psi \rightarrow \Sigma^0 \bar{\Lambda}$, the polarization correlation expressions

derived above remain applicable. The angles θ and ϕ represent the production angles of the first baryon appearing in the expression. For example, for the process $J/\psi \rightarrow \Lambda \bar{\Sigma}^0$, θ and ϕ denote the production angles of the Λ in the J/ψ helicity frame. Correspondingly, for the process $J/\psi \rightarrow \Sigma^0 \bar{\Lambda}$, these angles describe the production of the Σ^0 in the same reference frame. The constraints imposed by CP conservation on the parameters are given in Eq. (45). Alternatively, an equivalent parametrization can be adopted, in which the parameters $\alpha_1, \alpha_2, \phi_1$, and ϕ_2 are replaced by $\alpha'_1, \alpha'_2, \phi'_1$, and ϕ'_2 , as defined in Eq. (50). The corresponding CP conservation relations for these parameters in the conjugate processes are presented in Eq. (51).

Moreover, certain polarization components of J/ψ produced directly in electron-positron annihilation are

small. However, for J/ψ production via alternative mechanisms, significant polarization may be present across all independent components. For example, the decay chain $e^+e^- \rightarrow \psi(2S) \rightarrow J/\psi X$ can lead to a more diverse polarization structure. Our formalism provides a complete framework for experimentally accessing all independent polarization components of J/ψ .

D. Symmetry analysis of baryon-antibaryon polarization correlations

In the absence of parity and CP -violating effects, the polarization correlations of baryon-antibaryon pairs satisfy the symmetry relation

$$S_{\mu\nu}(\theta, \phi) = S_{\nu\mu}(\pi - \theta, \pi + \phi). \quad (77)$$

This symmetry no longer holds when parity and CP -violating effects are included. To analyze the impact of parity and CP violation in hyperon productions, we define the following observables:

$$\begin{aligned} \Delta S_{ij} &= S_{ij} + S_{ji}, \\ \delta S_{ij} &= S_{ij} - S_{ji}. \end{aligned} \quad (78)$$

When the azimuthal angle ϕ in hyperon production processes is not considered, i.e., by integrating over ϕ , the baryon-antibaryon polarization correlations reduce to those that depend only on the purely longitudinal polarization components S_L^ψ and S_{LL}^ψ . In this case, a parity-violating observable sensitive to baryon-antibaryon production is given by

$$\delta S_{12} = 4\sqrt{2}S_{LL}^\psi D_3^s \sin^2 \theta. \quad (79)$$

According to Table I, the parity violation effects in different production channels are illustrated in Fig. 4, assuming

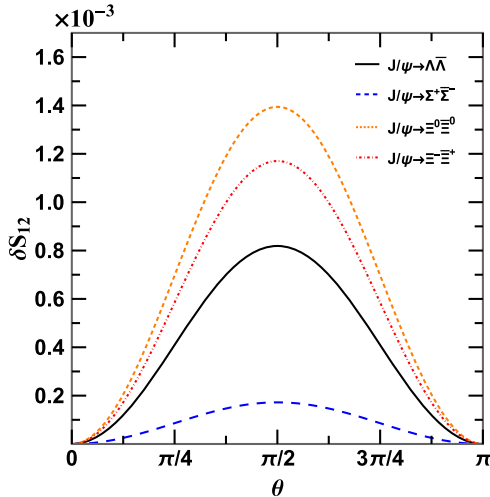


FIG. 4. Parity-violating effects in different hyperon production processes under unpolarized beam conditions.

$S_{LL}^\psi = 1/2$. For processes $J/\psi \rightarrow \Lambda \bar{\Lambda}$ and $J/\psi \rightarrow \Sigma^+ \bar{\Sigma}^-$, the effects are of order $\mathcal{O}(10^{-4})$, and for $J/\psi \rightarrow \Xi^0 \bar{\Xi}^0$ and $J/\psi \rightarrow \Xi^- \bar{\Xi}^+$, they reach $\mathcal{O}(10^{-3})$.

Similarly, we define the following CP -violating observables:

$$\begin{aligned} \Delta S_{02} &= 2\sqrt{2}D_1^s S_L^\psi \sin \theta + 4D_5^s S_{LL} \sin 2\theta, \\ \Delta S_{03} &= \frac{4\sqrt{2}}{3}D_2^c - \frac{2\sqrt{2}}{3}D_2^s S_{LL}^\psi (1 + 3 \cos 2\theta), \\ \Delta S_{12} &= -\frac{4\sqrt{2}}{3}D_2^s + \frac{2\sqrt{2}}{3}D_2^s S_{LL}^\psi (1 + 3 \cos 2\theta), \\ \Delta S_{13} &= -2\sqrt{2}D_1^c S_L^\psi \sin \theta - 4D_5^c S_{LL}^\psi \sin 2\theta, \\ \delta S_{01} &= -4D_5^c S_L^\psi \sin \theta - 2\sqrt{2}D_1^c S_{LL}^\psi \sin 2\theta, \\ \delta S_{23} &= -4D_5^s S_L^\psi \sin \theta - 2\sqrt{2}D_1^s S_{LL}^\psi \sin 2\theta. \end{aligned} \quad (80)$$

These observables serve as sensitive probes for detecting the violation of CP in baryon production, which is contributed by the baryon's electric dipole moment.

IV. THE JOINT ANGULAR DISTRIBUTION OF PRODUCTS

The characteristics of hyperon weak decays make them as self-analyzers for spin analysis. The expressions for baryon decays have been extensively discussed in Refs. [19,28–31,33,34,59,60,67–76], with various forms of representation. To maintain the completeness of the article, we follow the decay expression scheme used in [30,33,34] and provide a brief review of hadronic decays and radiative decays of baryons. By combining the polarization analysis of the processes $e^+e^- \rightarrow J/\psi$, $J/\psi \rightarrow B_1 \bar{B}_2$, and baryon decays, we present the full expression for the joint angular distribution of the final-state particles. Focusing on the process $J/\psi \rightarrow \Lambda \bar{\Sigma}^0$, we provide predictions for the statistical uncertainties of the helicity amplitude parameters and their corresponding transition form factors using the joint angular distribution.

A. Hadronic decay of baryon

We focus on the most common hadronic decay process for baryons, $B_1(J^{P_{B_1}}) \rightarrow B_3(1/2^+) + \pi(0^-)$, in which the polarization of the daughter baryon B_3 can be expressed as

$$S_\nu^{B_3} = \sum_\mu S_\mu^{B_1} a_{\mu\nu}, \quad (81)$$

where $S_\mu^{B_1}$ and $S_\nu^{B_3}$ denote the polarization expansion coefficients for B_1 and B_3 , respectively. The polarization transfer matrix $a_{\mu\nu}$ is defined as

$$\begin{aligned} a_{\mu\nu}^h &= \frac{2J+1}{2\pi} \sum_{\kappa, \kappa'} \sum_{\lambda, \lambda'} A_\lambda A_\lambda^* (\Sigma_\mu^{B_1})_{\kappa, \kappa'} (\Sigma_\nu^{B_3})_{\lambda' \lambda}, \\ &\times \mathcal{D}_{\kappa, \lambda}^{\prime*}(\Omega) \mathcal{D}_{\kappa', \lambda'}^J(\Omega), \end{aligned} \quad (82)$$

where $\mathcal{D}_{\kappa,\lambda}^{J*}(\Omega) = \mathcal{D}_{\kappa,\lambda}^{J*}(0, \theta, \phi)$ is the Wigner- $\mathcal{D}$ matrix, with J being the spin of the parent particle and A_λ representing the helicity amplitude for the decay process. The indices κ and λ correspond to the helicities of B_1 and B_3 , respectively. For parity-conserving decay processes, the helicity amplitude satisfies the following relation:

$$A_{1/2} = (-1)^{J+1/2} P_{B_1} A_{-1/2}. \quad (83)$$

For the case $J^{B_1} = 1/2^+$, the relationship between the helicity amplitude and the canonical amplitude is [30,61]

$$\begin{aligned} A_{1/2} &= \frac{\sqrt{2}}{2} (A_S + A_P), \\ A_{-1/2} &= \frac{\sqrt{2}}{2} (A_S - A_P). \end{aligned} \quad (84)$$

Under the normalization condition $|A_S|^2 + |A_P|^2 = |A_{1/2}|^2 + |A_{-1/2}|^2 = 1$, the amplitudes can be parametrized as [19]

$$\begin{aligned} \alpha_D &= 2\text{Re}[A_S^* A_P] = |A_{1/2}|^2 - |A_{-1/2}|^2, \\ \beta_D &= 2\text{Im}[A_S^* A_P] = 2\text{Im}[A_{1/2} A_{-1/2}^*], \\ \gamma_D &= |A_S|^2 - |A_P|^2 = 2\text{Re}[A_{1/2} A_{-1/2}^*], \end{aligned} \quad (85)$$

where $\beta_D = \sqrt{1 - \alpha_D^2} \sin \phi_D$ and $\gamma_D = \sqrt{1 - \alpha_D^2} \cos \phi_D$. With this parametrization scheme, the expression for the polarization transfer matrix $a_{\mu\nu}$ is given by [30]

$$\begin{aligned} a_{00}^h &= 1, \\ a_{03}^h &= \alpha_D, \\ a_{10}^h &= \alpha_D \cos \phi \sin \theta, \\ a_{11}^h &= \gamma_D \cos \theta \cos \phi - \beta_D \sin \phi, \\ a_{12}^h &= -\beta_D \cos \theta \cos \phi - \gamma_D \sin \phi, \\ a_{13}^h &= \sin \theta \cos \phi, \\ a_{20}^h &= \alpha_D \sin \theta \sin \phi, \\ a_{21}^h &= \beta_D \cos \phi + \gamma_D \cos \theta \sin \phi, \\ a_{22}^h &= \gamma_D \cos \phi - \beta_D \cos \theta \sin \phi, \\ a_{23}^h &= \sin \theta \sin \phi, \\ a_{30}^h &= \alpha_D \cos \theta, \\ a_{31}^h &= -\gamma_D \sin \theta, \\ a_{32}^h &= \beta_D \sin \theta, \\ a_{33}^h &= \cos \theta. \end{aligned} \quad (86)$$

For parity-conserving decay processes, we have $\alpha_D = \beta_D = 0$ and $\gamma_D \rightarrow (-1)^{J+1/2} P_{B_1}$.

Under the CP -conservation constraint, the decay parameters for the particle and antiparticle should satisfy $\alpha_D = -\alpha_{\bar{D}}$ and $\phi_D = -\phi_{\bar{D}}$. Therefore, one can define these observables to characterize CP violation,

$$\begin{aligned} A_{CP} &= \frac{\alpha_D + \alpha_{\bar{D}}}{\alpha_D - \alpha_{\bar{D}}}, \\ B_{CP} &= \frac{\beta_D + \beta_{\bar{D}}}{\alpha_D - \alpha_{\bar{D}}}. \end{aligned} \quad (87)$$

These observables are directly related to the weak CP -violating phases and the strong interaction phases from the final-state interactions [23,24,60].

In experimental measurements, CP violation is sometimes directly represented by the decay parameter ϕ_D , which serves as an observable for CP violation,

$$\Phi_{CP} = \frac{\phi_D + \phi_{\bar{D}}}{2}. \quad (88)$$

At the leading-order approximation for α_D and ϕ_D , we have [56]

$$B_{CP} = \Phi_{CP} \frac{\sqrt{1 - \langle \alpha_D \rangle^2}}{\langle \alpha_D \rangle} \cos \langle \phi_D \rangle - A_{CP} \frac{\langle \alpha_D \rangle}{\sqrt{1 - \langle \alpha_D \rangle^2}} \sin \langle \phi_D \rangle, \quad (89)$$

where

$$\begin{aligned} \langle \alpha_D \rangle &= \frac{\alpha_D - \alpha_{\bar{D}}}{2}, \\ \langle \phi_D \rangle &= \frac{\phi_D - \phi_{\bar{D}}}{2}. \end{aligned} \quad (90)$$

B. Radiative decay of baryon

We focus on the most common radiative decay process for baryons, $B_1(J^{P_{B_1}}) \rightarrow B_3(1/2^+) + \gamma(1^-)$, in which the polarization of the daughter baryon B_3 can be expressed as

$$S_\nu^{B_3} = \sum_\mu S_\mu^{B_1} a_{\mu\nu}, \quad (91)$$

The polarization transfer matrix $a_{\mu\nu}$ is defined as

$$\begin{aligned} a_{\mu\nu}^\gamma &= \frac{2J+1}{2\pi} \sum_{\kappa,\kappa'} \sum_{\lambda,\lambda'} A_{\lambda_1,\lambda_2} A_{\lambda'_1,\lambda'_2}^* (\Sigma_\mu^{B_1})_{\kappa,\kappa'} (\Sigma_\nu^{B_3})_{\lambda'_1,\lambda_1} \\ &\quad \times \delta_{\lambda'_2,\lambda_2} D_{\kappa,\lambda_1-\lambda_2}^{J*}(\Omega) D_{\kappa',\lambda'_1-\lambda'_2}^J(\Omega), \end{aligned} \quad (92)$$

where A_{λ_1,λ_2} denotes the helicity amplitude of the decay process, with λ_1 and λ_2 the helicities of the B_3 and photon, respectively. Based on helicity conservation and the fact that the helicity of a real photon cannot be zero, we obtain

$$A_{\lambda_1, \lambda_2} = \begin{pmatrix} A_{1/2,1} & 0 & 0 \\ 0 & 0 & A_{-1/2,-1} \end{pmatrix}. \quad (93)$$

For parity-conserving decay processes, the helicity amplitudes satisfy the following relation:

$$A_{1/2,1} = (-1)^{J-1/2} P_{B_1} A_{-1/2,-1}. \quad (94)$$

Under the normalization condition $|A_{1/2,1}|^2 + |A_{-1/2,-1}|^2 = 1$, the amplitudes can be parametrized as

$$\alpha_D = |A_{-1/2,-1}|^2 - |A_{1/2,1}|^2. \quad (95)$$

With this parametrization scheme, the polarization transfer matrix $a_{\mu\nu}$ is given by

$$\begin{aligned} a_{00}^\gamma &= 1, \\ a_{03}^\gamma &= -\alpha_D, \\ a_{10}^\gamma &= \alpha_D \sin \theta \cos \phi, \\ a_{13}^\gamma &= -\sin \theta \cos \phi, \\ a_{20}^\gamma &= \alpha_D \sin \theta \sin \phi, \\ a_{23}^\gamma &= -\sin \theta \sin \phi, \\ a_{30}^\gamma &= \alpha_D \cos \theta, \\ a_{33}^\gamma &= -\cos \theta. \end{aligned} \quad (96)$$

Under the CP -conservation constraint, the decay parameters for the particle and antiparticle should satisfy $\alpha_D = -\alpha_{\bar{D}}$, and thus, we can define an observable sensitive to CP violation:

$$A_{CP} = \frac{\alpha_D + \alpha_{\bar{D}}}{\alpha_D - \alpha_{\bar{D}}}. \quad (97)$$

C. The joint angular distribution of final products

Combining the polarization analysis of the production and decay subprocesses of J/ψ discussed in Secs. II and III, along with the discussion of baryon decays in this section, we can express the joint angular distribution of the final-state particles as

$$W \propto S_{\mu\nu} a_{\mu 0}^{(h/\gamma)B_1} a_{\nu 0}^{(h/\gamma)\bar{B}_2}, \quad (98)$$

where $S_{\mu\nu}$ is the polarization correlation matrix for the $B_1 \bar{B}_2$ system, as detailed in Eqs. (60)–(75). The matrices $a_{\mu 0}^{(h/\gamma)B_1}$ and $a_{\nu 0}^{(h/\gamma)\bar{B}_2}$ represent the polarization transfer matrices for the decays of B_1 and $\bar{B}_2$, which are given by Eqs. (86) and (96) for hadronic and radiative decays, respectively. For cascade decay particles, the transfer chain can be extended, e.g., $a_{\mu 0}^{(h/\gamma)B_1} \rightarrow \sum_{\mu_2 \dots \mu_n} a_{\mu \mu_2}^{(h/\gamma)B_1} \dots a_{\mu_n 0}^{(h/\gamma)B_n}$.

As shown in Eq. (26), the polarization of J/ψ is determined by both beam polarization and electron form factors. In this paper, we estimate the electron form factors for the $e^+e^- \rightarrow J/\psi$ subprocess at leading order, as shown in Eq. (22). We recommend that the experimental analyzes fit the polarization expansion coefficients $S_L^\psi, \dots, S_{TT}^{xy,\psi}$ directly as free parameters. Given the beam polarization, the electron form factors can be extracted using Eqs. (22) and (26). Comparison between experimental measurements and our theoretical predictions allows for a precise test of the electron- J/ψ interaction.

D. Statistical significance

While CP violation has been widely studied in subprocesses such as $J/\psi \rightarrow \Lambda \bar{\Lambda}$ and $J/\psi \rightarrow \Xi \bar{\Xi}$ [56,58,60], fewer investigations have focused on associated baryon production, such as $J/\psi \rightarrow \Lambda \bar{\Sigma}^0$. These isospin-violating processes, including $J/\psi \rightarrow \Lambda \bar{\Sigma}^0$ and its conjugate channel $J/\psi \rightarrow \Sigma^0 \bar{\Lambda}$, proceed primarily via electroweak interactions and thus offer unique opportunities for probing CP violation. In this work, we analyze the sensitivity to the parameters α'_1, α'_2 , and the relevant transition form factors in this channel.

For the subprocess $J/\psi \rightarrow \Lambda \bar{\Sigma}^0$, the joint angular distribution of the final-state particles is given by

$$\frac{d\sigma}{d\Omega} \propto \mathcal{W}(\vec{\omega}; \vec{\eta}) = \sum_{\mu, \nu, \nu'} S_{\mu\nu}^{\Lambda \bar{\Sigma}} a_{\mu 0}^{h\Lambda} a_{\nu\nu'}^{\gamma \bar{\Sigma}} a_{\nu' 0}^{h\bar{\Lambda}}, \quad (99)$$

where $\vec{\omega} = \{\theta, \phi, \theta_p, \phi_p, \theta_{\bar{\Lambda}}, \phi_{\bar{\Lambda}}, \theta_{\bar{p}}, \phi_{\bar{p}}\}$ is the set of angular variables, and $\vec{\eta} = \{\alpha_\psi, \Phi, \alpha_1, \dots, S_L^\psi, \dots, S_{TT}^{xy,\psi}\}$ includes parameters associated with baryon production, decays, and polarization of J/ψ .

For J/ψ produced directly in electron-positron annihilation, the most beam-sensitive polarization components are S_L^ψ (primarily affected by longitudinal polarization of the beam) and $S_{TT}^{xx,\psi}, S_{TT}^{xy,\psi}$ (primarily affected by transverse polarization of the beam). The effects of beam polarization on other polarization components of J/ψ are generally suppressed. At BESIII, electron beams are typically unpolarized, although some spontaneous transverse polarization may arise during beam propagation due to the Sokolov–Ternov effect [59,62]. Future experiments such as STCF aim to prepare polarized beams [51]. According to the analysis in Ref. [60], longitudinal polarization of the beams significantly improves the sensitivity of the parameters compared to the transverse polarization. Therefore, our sensitivity analysis focuses primarily on the longitudinal case, setting $S_{LL}^\psi \rightarrow 1/2$, $S_L^\psi \rightarrow -P_e(S_L)$, and all other components to zero.

However, it should be noted that although most of the polarization components of J/ψ are only slightly influenced by beam polarization, they can still induce asymmetry effects in final-state particles. Such asymmetries can

TABLE II. The statistical uncertainties of the parity-violating helicity amplitude parameters α'_1 , α'_2 , and the corresponding form factors F_3 , F_4 in the $J/\psi \rightarrow \Lambda \bar{\Sigma}^0$ process. The predictions on the statistics at BESIII are conducted with unpolarized beams, and those at STCF are conducted with 80% beam longitudinal polarization.

$J/\psi \rightarrow \Lambda \bar{\Sigma}^0$	BESIII	STCF
$n_{\text{tag}}^{\text{evt}}$	1.3×10^4	4.4×10^6
$\sigma(\sqrt{\alpha'_1})$	2.3×10^{-2}	8.3×10^{-4}
$\sigma(\sqrt{\alpha'_2})$	2.1×10^{-2}	7.9×10^{-4}
$\sigma(F_3)$	4.01×10^{-6}	1.48×10^{-7}
$\sigma(F_4)$	4.22×10^{-6}	1.55×10^{-7}

overlap with the genuine CP -violating effects. Therefore, these components must be retained in experimental fits to ensure accurate isolation of CP -violating observables.

To evaluate statistical uncertainties, we apply the maximum likelihood method to the joint angular distribution. The covariance matrix of the parameters can be obtained from the Fisher information matrix [77]:

$$I_{ij}(\eta_i, \eta_j) = n \int \frac{1}{\tilde{\mathcal{W}}} \left(\frac{\partial \tilde{\mathcal{W}}}{\partial \eta_i} \right) \left(\frac{\partial \tilde{\mathcal{W}}}{\partial \eta_j} \right) d\vec{\omega}, \quad (100)$$

where n is the number of selected events, and $\tilde{\mathcal{W}} = \mathcal{W} / \int \mathcal{W} d\vec{\omega}$ is the normalized joint angular distribution. The covariance matrix is the inverse of the Fisher information matrix, $V = I^{-1}$. The statistical uncertainty for a specific parameter η_i is then given by

$$\sigma(\eta_i) = \sqrt{V_{ii}}. \quad (101)$$

In Table II, we present the sensitivities for the parameters α'_1 and α'_2 for the process $J/\psi \rightarrow \Lambda \bar{\Sigma}^0$, based on projected

statistics from BESIII [46] and STCF [51]. The corresponding parameters in the conjugate channel have similar sensitivities. Using Eq. (42), we also estimate the sensitivity to the transition form factors, whose values are constrained by Eq. (53). Although the statistical yield for $J/\psi \rightarrow \Lambda \bar{\Sigma}^0$ is relatively low, the sensitivities of the form factors reach levels comparable to those for processes such as $J/\psi \rightarrow \Lambda \bar{\Lambda}$ [56,60]. Moreover, since the preparation of polarized beams is a key design objective of the planned STCF accelerator [51], we illustrate in Fig. 5 how parameter sensitivities vary with polarization degree of the electron beam. It is clear that beam polarization significantly enhances measurement precision.

V. SUMMARY

In this paper, we present a comprehensive polarization analysis of the process $e^+e^- \rightarrow J/\psi \rightarrow B_1 \bar{B}_2$. The analysis is divided into three parts: the subprocess $e^+e^- \rightarrow J/\psi$, the subprocess $J/\psi \rightarrow B_1 \bar{B}_2$, and the subsequent decays of baryons.

As a spin-1 particle, the polarization state of J/ψ is characterized by eight independent polarization components. For the subprocess $e^+e^- \rightarrow J/\psi$, we analyze how the polarization state of J/ψ is influenced by both the beam polarization and the electron form factors. A detailed investigation of the initial beam polarization is provided, emphasizing the transformation effects from the laboratory frame to the center-of-mass frame. We present all possible helicity amplitudes for the annihilation process and explicitly establish their relationship to the electron form factors. By including both beam polarization and all relevant helicity amplitudes, we systematically analyze the resulting polarization of J/ψ produced in e^+e^- annihilation.

When parity-violating and CP -violating effects are included in the subprocess $J/\psi \rightarrow B_1 \bar{B}_2$, the polarization

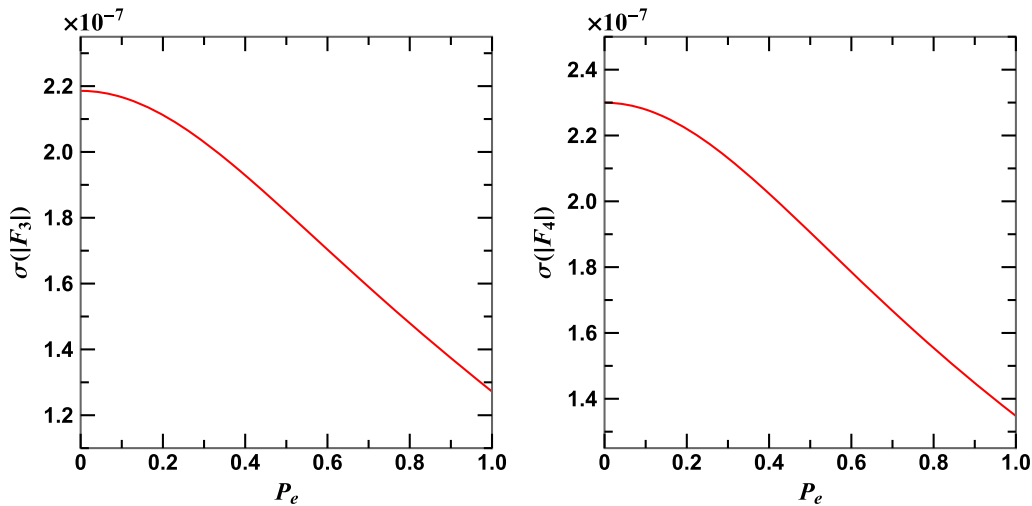


FIG. 5. Predicted statistical uncertainties for the transition form factors F_3 and F_4 for the process $J/\psi \rightarrow \Lambda \bar{\Sigma}^0$ as a function of the degree of electron beam longitudinal polarization P_e .

correlations of the baryon-antibaryon system are modified. This opens the possibility of searching for CP violations in baryon production processes. We classify these subprocesses into two types: baryon-antibaryon pair production and associated production of different baryons. For the pair production process $J/\psi \rightarrow B\bar{B}$, we introduce two helicity amplitudes that conserve both parity and CP , one amplitude that violates parity but conserves CP , and one that violates both parity and CP . The explicit correspondence between these helicity amplitudes and the form factors of the baryon B is provided. For the associated production process $J/\psi \rightarrow B_1\bar{B}_2$, we define two parity-conserving and two parity-violating helicity amplitudes and establish their relationship to the transition form factors of the $B_1\bar{B}_2$ system. Based on the bilinear covariance of these transition form factors, we derive their transformation properties under CP conservation. Subsequently, we determine the CP constraints on the helicity amplitudes for the conjugate processes $J/\psi \rightarrow B_1\bar{B}_2$ and $J/\psi \rightarrow B_2\bar{B}_1$. To describe polarization transfer in these subprocesses, we introduce a polarization transfer matrix $a_{\rho,\mu\nu}$ and provide the complete expression for the polarization correlations of the $B_1\bar{B}_2$ system. Dedicated observables sensitive to parity and CP violations in baryon production processes are proposed for future experimental investigation.

Due to the weak decay properties of hyperons, their polarization can be extracted experimentally from the angular distributions of their decay products. We express the polarization transfer matrices for both hadronic and radiative hyperon decays in terms of the corresponding decay parameters. Differences in the decay parameters between hyperons and their antiparticles serve as important observables for detecting violations of CP in hyperon decays. By combining the polarization analyzes of the subprocesses $e^+e^- \rightarrow J/\psi$, $J/\psi \rightarrow B_1\bar{B}_2$, and subsequent baryon decays, we provide the complete joint angular distribution of all final-state particles. Focusing specifically on the process $J/\psi \rightarrow \Lambda\bar{\Sigma}^0$, we present sensitivity projections for the helicity amplitude parameters α'_1 and α'_2 , along with their related transition form factors. These results offer essential theoretical guidance for future experimental measurements of CP violations in associated baryon production channels.

At the mass threshold of J/ψ , the electron-positron annihilation cross section experiences significant enhancement, enabling high-statistics data samples in experiments. This provides a powerful opportunity to search for CP violations in the production and decay of ordinary baryons through the $e^+e^- \rightarrow J/\psi \rightarrow B_1\bar{B}_2$ process. This paper presents a refined polarization analysis of these processes and provides the complete joint angular distribution of the final-state products, offering valuable theoretical support for future CP violation studies involving ordinary baryons.

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DATA AVAILABILITY

The data that support the findings of this article are openly available [40,42,44,45,46,51].

APPENDIX A: THE DEFINITION AND PROBABILITY OF THE SPIN COMPONENTS FOR SPIN ONE PARTICLES

In the Cartesian coordinate system, the spin density matrix for a spin-1 particle can be expressed as [78]

$$\rho_1 = \frac{1}{3} \left(\mathbf{1} + \frac{3}{2} S^i \Sigma^i + 3 T^{ij} \Sigma^{ij} \right), \quad (\text{A1})$$

where Σ^i and Σ^{ij} represent independent, orthogonal, and Hermitian basis matrices. The matrix Σ^i , in the S_z representation, can be written as

$$\begin{aligned} \Sigma^x &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, & \Sigma^y &= \frac{i}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \\ \Sigma^z &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \end{aligned} \quad (\text{A2})$$

The remaining basis matrices can be expressed in terms of Σ^i ,

$$\Sigma^{ij} = \frac{1}{2} (\Sigma^i \Sigma^j + \Sigma^j \Sigma^i) - \frac{2}{3} \delta^{ij} \mathbf{1}. \quad (\text{A3})$$

For the polarization vector S^i , there are three independent polarization components, which can be represented as

$$S^i = (S_T^x, S_T^y, S_L). \quad (\text{A4})$$

The definitions of these three polarization components are

$$S_T^x = \langle \Sigma^x \rangle, \quad S_T^y = \langle \Sigma^y \rangle, \quad S_L = \langle \Sigma^z \rangle, \quad (\text{A5})$$

where $\langle \Sigma^a \rangle = \text{Tr}[\Sigma^a \rho_1]$. For the rank-2 polarization tensor T^{ij} , it consists of five independent polarization tensors, expressed as

$$T^{ij} = \frac{1}{2} \begin{pmatrix} -\frac{2}{3}S_{LL} + S_{TT}^{xx} & S_{TT}^{xy} & S_{LT}^x \\ S_{TT}^{xy} & -\frac{2}{3}S_{LL} - S_{TT}^{xx} & S_{LT}^y \\ S_{LT}^x & S_{LT}^y & \frac{4}{3}S_{LL} \end{pmatrix}. \quad (\text{A6})$$

The definitions of these five polarization components are

$$\begin{aligned} S_{LL} &= \frac{3}{2} \langle \Sigma^{zz} \rangle, & S_{LT}^x &= 2 \langle \Sigma^{xz} \rangle, & S_{LT}^y &= 2 \langle \Sigma^{yz} \rangle, \\ S_{TT}^{xx} &= \langle \Sigma^{xx} - \Sigma^{yy} \rangle, & S_{TT}^{xy} &= 2 \langle \Sigma^{xy} \rangle. \end{aligned} \quad (\text{A7})$$

To analyze the probabilistic interpretation of these polarization components, we introduce the eigenstate vector $|m_{\theta,\phi}\rangle$ along a specific direction, where m represents the eigenvalue. Therefore, the probability of the system being in a specific polarization state is

$$P(m_{(\theta,\phi)}) = \text{Tr}[\rho_1 |m_{(\theta,\phi)}\rangle \langle m_{(\theta,\phi)}|]. \quad (\text{A8})$$

For convenience, we introduce the following notation:

$$\begin{aligned} |m_x\rangle &= |m_{(\frac{\pi}{2},0)}\rangle, |m_y\rangle = |m_{(\frac{\pi}{2},\frac{\pi}{2})}\rangle, |m_z\rangle = |m_{(0,0)}\rangle, \\ |m_{x+y}\rangle &= |m_{(\frac{\pi}{2},\frac{\pi}{4})}\rangle, |m_{x-y}\rangle = |m_{(\frac{\pi}{2},-\frac{\pi}{4})}\rangle, |m_{x+z}\rangle = |m_{(\frac{\pi}{4},0)}\rangle, \\ |m_{x-z}\rangle &= |m_{(-\frac{\pi}{4},0)}\rangle, |m_{y+z}\rangle = |m_{(\frac{\pi}{4},\frac{\pi}{2})}\rangle, |m_{y-z}\rangle = |m_{(-\frac{\pi}{4},\frac{\pi}{2})}\rangle. \end{aligned} \quad (\text{A9})$$

Based on Eqs. (A5) and (A7), one can interpret the probability of the polarization components as

$$\begin{aligned} S_T^x &= P_x(1) - P_x(-1), \\ S_T^y &= P_y(1) - P_y(-1), \\ S_L &= P_z(1) - P_z(-1), \\ S_{LL} &= \frac{1}{2}P_z(1) + \frac{1}{2}P_z(-1) - P_z(0), \\ S_{LT}^x &= P_{x-z}(0) - P_{x+z}(0), \\ S_{LT}^y &= P_{y-z}(0) - P_{y+z}(0), \\ S_{LT}^{xy} &= P_{x-y}(0) - P_{x+y}(0), \\ S_{TT}^{xx} &= P_y(0) - P_x(0). \end{aligned} \quad (\text{A10})$$

Thus, we can derive the threshold values for these polarization components as

$$\begin{aligned} S_T^x, S_T^y, S_L &\in [-1, 1], \\ S_{LL} &\in \left[-1, \frac{1}{2}\right], \\ S_{LT}^x, S_{LT}^y, S_{TT}^{xx}, S_{TT}^{xy} &\in [-1, 1]. \end{aligned} \quad (\text{A11})$$

The total degree of polarization is

$$\begin{aligned} d &= \frac{1}{\sqrt{2s}} \sqrt{(2s+1)\text{Tr}[\rho^2] - 1} \\ &= \left[\frac{3}{4} S^i S^i + \frac{3}{2} T^{ij} T^{ij} \right]^{1/2} \\ &= \left\{ \frac{3}{4} [(S_T^x)^2 + (S_T^y)^2 + S_L^2] + \frac{3}{2} \left[\frac{2}{3} S_{LL}^2 \right. \right. \\ &\quad \left. \left. + \frac{1}{2} ((S_{LT}^x)^2 + (S_{LT}^y)^2 + (S_{TT}^{xx})^2 + (S_{TT}^{xy})^2) \right] \right\}^{1/2}. \end{aligned} \quad (\text{A12})$$

APPENDIX B: SPIN-ONE POLARIZATION PROJECTION MATRICES

Following the approach outlined in [34], we map the polarization expansion coefficients $S_0, S_1, \dots, S_8$ in the helicity framework for a spin-1 particle to the Cartesian spin components $1, S_L, \dots, S_{TT}^{xy}$, as detailed in Eq. (12). In this scheme, the polarization expansion matrices in the helicity framework are chosen as follows:

$$\begin{aligned} \Sigma_0 &= \frac{1}{3} \mathbf{I}, \Sigma_1 = \frac{1}{2} \Sigma^z, \Sigma_2 = \frac{1}{2} \Sigma^x, \\ \Sigma_3 &= \frac{1}{2} \Sigma^y, \Sigma_4 = \Sigma^{zz}, \Sigma_5 = \Sigma^{xz}, \\ \Sigma_6 &= \Sigma^{yz}, \Sigma_7 = \frac{1}{2} (\Sigma^{xx} - \Sigma^{yy}), \Sigma_8 = \Sigma^{xy}. \end{aligned} \quad (\text{B1})$$

The explicit expressions for these matrices are

$$\Sigma_0 = \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \Sigma_1 = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad \Sigma_2 = \frac{\sqrt{2}}{4} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\begin{aligned}
\Sigma_3 &= \frac{i\sqrt{2}}{4} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, & \Sigma_4 &= \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, & \Sigma_5 &= \frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}, \\
\Sigma_6 &= \frac{i}{2\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, & \Sigma_7 &= \frac{1}{2} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, & \Sigma_8 &= \frac{i}{2} \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}.
\end{aligned} \tag{B2}$$

APPENDIX C: SPIN TRANSFER MATRIX FOR $J/\psi \rightarrow B\bar{B}$

Based on Eq. (59), we present the complete expression for the polarization transfer matrix in the $J/\psi \rightarrow B\bar{B}$ subprocess in this section.

The components of the polarization transfer matrix associated with the production cross section S_0 of the J/ψ are given by

$$\begin{aligned}
a_{0,00} &= \frac{1}{3}(3 + \alpha_\psi + 2\alpha_1 + 4\alpha_2), \\
a_{0,30} &= \frac{2\sqrt{2}}{3}(D_2^c + 2D_3^c), \\
a_{0,11} &= \frac{1}{3}(1 - 2\alpha_1 - \alpha_\psi), \\
a_{0,21} &= -\frac{2\sqrt{2}}{3}D_2^s, \\
a_{0,12} &= -\frac{2\sqrt{2}}{3}D_2^s, \\
a_{0,22} &= -\frac{1}{3}(1 - 2\alpha_1 - \alpha_\psi), \\
a_{0,03} &= \frac{2\sqrt{2}}{3}(D_2^c - 2D_3^c), \\
a_{0,33} &= -\frac{1}{3}(1 - 2\alpha_1 + 4\alpha_2 + 3\alpha_\psi).
\end{aligned} \tag{C1}$$

The components of the polarization transfer matrix associated with the vector polarization component $S_1(S_L)$ are

$$\begin{aligned}
a_{1,00} &= 2\sqrt{2}D_3^c \cos \theta, \\
a_{1,10} &= -(D_0^c - 2D_5^c) \sin \theta, \\
a_{1,20} &= \sqrt{2}(D_1^s + D_4^s) \sin \theta, \\
a_{1,30} &= (1 + 2\alpha_2 + \alpha_\psi) \cos \theta, \\
a_{1,01} &= -(D_0^c + 2D_5^c) \sin \theta, \\
a_{1,31} &= -\sqrt{2}(D_1^c + D_4^c) \sin \theta, \\
a_{1,02} &= \sqrt{2}(D_1^s - D_4^s) \sin \theta, \\
a_{1,32} &= (D_0^s + 2D_5^s) \sin \theta, \\
a_{1,03} &= -(1 + \alpha_\psi + 2\alpha_2) \cos \theta, \\
a_{1,13} &= -\sqrt{2}(D_1^c - D_4^c) \sin \theta, \\
a_{1,23} &= (D_0^s - 2D_5^s) \sin \theta, \\
a_{1,33} &= -2\sqrt{2}D_3^c \cos \theta.
\end{aligned} \tag{C2}$$

The components of the polarization transfer matrix associated with the vector polarization component $S_2(S_T^x)$ are

$$\begin{aligned}
a_{2,00} &= 2\sqrt{2}D_3^c \sin \theta \cos \phi, \\
a_{2,10} &= (D_0^c - 2D_5^c) \cos \theta \cos \phi - \sqrt{2}(D_1^s + D_4^s) \sin \phi, \\
a_{2,20} &= -(D_0^c - 2D_5^c) \sin \phi - \sqrt{2}(D_1^s + D_4^s) \cos \theta \cos \phi, \\
a_{2,30} &= (1 + \alpha_\psi + 2\alpha_2) \sin \theta \cos \phi, \\
a_{2,01} &= (D_0^c + 2D_5^c) \cos \theta \cos \phi + \sqrt{2}(D_1^s - D_4^s) \sin \phi, \\
a_{2,31} &= (D_0^s + 2D_5^s) \sin \phi + \sqrt{2}(D_1^c + D_4^c) \cos \theta \cos \phi, \\
a_{2,02} &= (D_0^c + 2D_5^c) \sin \phi - \sqrt{2}(D_1^s - D_4^s) \cos \theta \cos \phi, \\
a_{2,32} &= -(D_0^s + 2D_5^s) \cos \theta \cos \phi + \sqrt{2}(D_1^c + D_4^c) \sin \phi, \\
a_{2,03} &= -(1 + \alpha_\psi + 2\alpha_2) \sin \theta \cos \phi, \\
a_{2,13} &= -(D_0^s - 2D_5^s) \sin \phi + \sqrt{2}(D_1^c - D_4^c) \cos \theta \cos \phi, \\
a_{2,23} &= -(D_0^s - 2D_5^s) \cos \theta \cos \phi - \sqrt{2}(D_1^c - D_4^c) \sin \phi, \\
a_{2,33} &= -2\sqrt{2}D_3^c \sin \theta \cos \phi.
\end{aligned} \tag{C3}$$

The components of the polarization transfer matrix associated with the vector polarization component $S_3(S_T^y)$ are

$$\begin{aligned}
a_{3,00} &= 2\sqrt{2}D_3^c \sin \theta \sin \phi, \\
a_{3,10} &= (D_0^c - 2D_5^c) \cos \theta \sin \phi + \sqrt{2}(D_1^s + D_4^s) \cos \phi, \\
a_{3,20} &= (D_0^c - 2D_5^c) \cos \phi - \sqrt{2}(D_1^s + D_4^s) \cos \theta \sin \phi, \\
a_{3,30} &= (1 + 2\alpha_2 + \alpha_\psi) \sin \theta \sin \phi, \\
a_{3,01} &= (D_0^c + 2D_5^c) \cos \theta \sin \phi - \sqrt{2}(D_1^s - D_4^s) \cos \phi, \\
a_{3,31} &= -(D_0^s + 2D_5^s) \cos \phi + \sqrt{2}(D_1^c + D_4^c) \cos \theta \sin \phi, \\
a_{3,02} &= -(D_0^c + 2D_5^c) \cos \phi - \sqrt{2}(D_1^s - D_4^s) \cos \theta \sin \phi, \\
a_{3,32} &= -(D_0^s + 2D_5^s) \cos \theta \sin \phi - \sqrt{2}(D_1^c + D_4^c) \cos \phi, \\
a_{3,03} &= -(1 + 2\alpha_2 + \alpha_\psi) \sin \theta \sin \phi, \\
a_{3,13} &= (D_0^s - 2D_5^s) \cos \phi + \sqrt{2}(D_1^c - D_4^c) \cos \theta \sin \phi, \\
a_{3,23} &= -(D_0^s - 2D_5^s) \cos \theta \sin \phi + \sqrt{2}(D_1^c - D_4^c) \cos \phi, \\
a_{3,33} &= -2\sqrt{2}D_3^c \sin \theta \sin \phi.
\end{aligned} \tag{C4}$$

The components of the polarization transfer matrix associated with the tensor polarization component $S_4(S_{LL})$ are

$$\begin{aligned}
a_{4,00} &= \frac{1}{3}(\alpha_\psi - \alpha_1 + \alpha_2)(1 + 3 \cos 2\theta), \\
a_{4,10} &= \sqrt{2}(D_1^c - D_4^c) \sin 2\theta, \\
a_{4,20} &= -(D_0^s - 2D_5^s) \sin 2\theta, \\
a_{4,30} &= -\frac{\sqrt{2}}{3}(D_2^c - D_3^c)(1 + 3 \cos 2\theta), \\
a_{4,01} &= -\sqrt{2}(D_1^c + D_4^c) \sin 2\theta, \\
a_{4,11} &= -\frac{2}{3}(1 - \alpha_\psi - 2\alpha_1) + 2 \sin^2 \theta (1 - \alpha_1 - \alpha_2), \\
a_{4,21} &= \frac{\sqrt{2}}{3}D_2^s(1 + 3 \cos 2\theta) - 2\sqrt{2}D_3^s \sin^2 \theta, \\
a_{4,31} &= -(D_0^c + 2D_5^c) \sin 2\theta,
\end{aligned} \tag{C5}$$

$$\begin{aligned}
a_{4,02} &= (D_0^s + 2D_5^s) \sin 2\theta, \\
a_{4,12} &= 2\sqrt{2}D_3^s \sin^2 \theta + \frac{\sqrt{2}}{3}D_2^s(1 + 3\cos 2\theta), \\
a_{4,22} &= \frac{2}{3}(1 - \alpha_\psi - 2\alpha_1) + 2(\alpha_\psi + \alpha_1 - \alpha_2) \sin^2 \theta, \\
a_{4,32} &= \sqrt{2}(D_1^s - D_4^s) \sin 2\theta, \\
a_{4,03} &= -\frac{\sqrt{2}}{3}(D_2^c + D_3^c)(1 + 3\cos 2\theta), \\
a_{4,13} &= (D_0^c - 2D_5^c) \sin 2\theta, \\
a_{4,23} &= -\sqrt{2}(D_1^s + D_4^s) \sin 2\theta, \\
a_{4,33} &= -\frac{1}{3}(1 + \alpha_1 + \alpha_2)(1 + 3\cos 2\theta).
\end{aligned} \tag{C6}$$

The components of the polarization transfer matrix associated with the tensor polarization component $S_5(S_{LT}^x)$ are

$$\begin{aligned}
a_{5,00} &= (\alpha_\psi - \alpha_1 + \alpha_2) \sin 2\theta \cos \phi, \\
a_{5,10} &= (D_0^s - 2D_5^s) \cos \theta \sin \phi - \sqrt{2}(D_1^c - D_4^c) \cos 2\theta \cos \phi, \\
a_{5,20} &= (D_0^s - 2D_5^s) \cos 2\theta \cos \phi + \sqrt{2}(D_1^c - D_4^c) \cos \theta \sin \phi, \\
a_{5,30} &= -\sqrt{2}(D_2^c - D_3^c) \sin 2\theta \cos \phi, \\
a_{5,01} &= (D_0^s + 2D_5^s) \cos \theta \sin \phi + \sqrt{2}(D_1^c + D_4^c) \cos 2\theta \cos \phi, \\
a_{5,11} &= -(1 - \alpha_1 - \alpha_2) \sin 2\theta \cos \phi + 2\sqrt{2}D_3^s \sin \theta \sin \phi, \\
a_{5,21} &= (1 + \alpha_\psi - 2\alpha_2) \sin \theta \sin \phi + \sqrt{2}(D_2^s + D_3^s) \sin 2\theta \cos \phi, \\
a_{5,31} &= (D_0^c + 2D_5^c) \cos 2\theta \cos \phi + \sqrt{2}(D_1^s - D_4^s) \cos \theta \sin \phi, \\
a_{5,02} &= -(D_0^s + 2D_5^s) \cos 2\theta \cos \phi + \sqrt{2}(D_1^c + D_4^c) \cos \theta \sin \phi, \\
a_{5,12} &= -(1 + \alpha_\psi - 2\alpha_2) \sin \theta \sin \phi + \sqrt{2}(D_2^s - D_3^s) \sin 2\theta \cos \phi, \\
a_{5,22} &= -(\alpha_\psi + \alpha_1 - \alpha_2) \sin 2\theta \cos \phi + 2\sqrt{2}D_3^s \sin \theta \sin \phi, \\
a_{5,32} &= (D_0^c + 2D_5^c) \cos \theta \sin \phi - \sqrt{2}(D_1^s - D_4^s) \cos 2\theta \cos \phi, \\
a_{5,03} &= -\sqrt{2}(D_2^c + D_3^c) \sin 2\theta \cos \phi, \\
a_{5,13} &= -(D_0^c - 2D_5^c) \cos 2\theta \cos \phi + \sqrt{2}(D_1^s + D_4^s) \cos \theta \sin \phi, \\
a_{5,23} &= (D_0^c - 2D_5^c) \cos \theta \sin \phi + \sqrt{2}(D_1^s + D_4^s) \cos 2\theta \cos \phi, \\
a_{5,33} &= -(1 + \alpha_1 + \alpha_2) \sin 2\theta \cos \phi.
\end{aligned} \tag{C7}$$

The components of the polarization transfer matrix associated with the tensor polarization component $S_6(S_{LT}^y)$ are

$$\begin{aligned}
a_{6,00} &= (\alpha_\psi - \alpha_1 + \alpha_2) \sin 2\theta \sin \phi, \\
a_{6,10} &= -(D_0^s - 2D_5^s) \cos \theta \cos \phi - \sqrt{2}(D_1^c - D_4^c) \cos 2\theta \sin \phi, \\
a_{6,20} &= (D_0^s - 2D_5^s) \cos 2\theta \sin \phi - \sqrt{2}(D_1^c - D_4^c) \cos \theta \cos \phi, \\
a_{6,30} &= -\sqrt{2}(D_2^c - D_3^c) \sin 2\theta \sin \phi, \\
a_{6,01} &= -(D_0^s + 2D_5^s) \cos \theta \cos \phi + \sqrt{2}(D_1^c + D_4^c) \cos 2\theta \sin \phi, \\
a_{6,11} &= -(1 - \alpha_1 - \alpha_2) \sin 2\theta \sin \phi - 2\sqrt{2}D_3^s \sin \theta \cos \phi,
\end{aligned}$$

$$\begin{aligned}
a_{6,21} &= -(1 + \alpha_\psi - 2\alpha_2) \sin \theta \cos \phi + \sqrt{2}(D_2^s + D_3^s) \sin 2\theta \sin \phi, \\
a_{6,31} &= (D_0^c + 2D_5^c) \cos 2\theta \sin \phi - \sqrt{2}(D_1^s - D_4^s) \cos \theta \cos \phi, \\
a_{6,02} &= -(D_0^s + 2D_5^s) \cos 2\theta \sin \phi - \sqrt{2}(D_1^c + D_4^c) \cos \theta \cos \phi, \\
a_{6,12} &= (1 + \alpha_\psi - 2\alpha_2) \sin \theta \cos \phi + \sqrt{2}(D_2^s - D_3^s) \sin 2\theta \sin \phi, \\
a_{6,22} &= -(\alpha_\psi + \alpha_1 - \alpha_2) \sin 2\theta \sin \phi - 2\sqrt{2}D_3^s \sin \theta \cos \phi, \\
a_{6,32} &= -(D_0^c + 2D_5^c) \cos \theta \cos \phi - \sqrt{2}(D_1^s - D_4^s) \cos 2\theta \sin \phi, \\
a_{6,03} &= -\sqrt{2}(D_2^c + D_3^c) \sin 2\theta \sin \phi, \\
a_{6,13} &= -(D_0^c - 2D_5^c) \cos 2\theta \sin \phi - \sqrt{2}(D_1^s + D_4^s) \cos \theta \cos \phi, \\
a_{6,23} &= -(D_0^c - 2D_5^c) \cos \theta \cos \phi + \sqrt{2}(D_1^s + D_4^s) \cos 2\theta \sin \phi, \\
a_{6,33} &= -(1 + \alpha_1 + \alpha_2) \sin 2\theta \sin \phi.
\end{aligned} \tag{C8}$$

The components of the polarization transfer matrix associated with the tensor polarization component $S_7(S_{TT}^{xx})$ are

$$\begin{aligned}
a_{7,00} &= (\alpha_\psi - \alpha_1 + \alpha_2) \sin^2 \theta \cos 2\phi, \\
a_{7,10} &= (D_0^s - 2D_5^s) \sin \theta \sin 2\phi - \frac{\sqrt{2}}{2}(D_1^c - D_4^c) \sin 2\theta \cos 2\phi, \\
a_{7,20} &= \frac{1}{2}(D_0^s - 2D_5^s) \sin 2\theta \cos 2\phi + \sqrt{2}(D_1^c - D_4^c) \sin \theta \sin 2\phi, \\
a_{7,30} &= -\sqrt{2}(D_2^c - D_3^c) \sin^2 \theta \cos 2\phi, \\
a_{7,01} &= (D_0^s + 2D_5^s) \sin \theta \sin 2\phi + \frac{\sqrt{2}}{2}(D_1^c + D_4^c) \sin 2\theta \cos 2\phi, \\
a_{7,11} &= [(1 + \alpha_\psi - 2\alpha_2) - (1 - \alpha_1 - \alpha_2) \sin^2 \theta] \cos 2\phi - 2\sqrt{2}D_3^s \cos \theta \sin 2\phi,
\end{aligned} \tag{C9}$$

$$\begin{aligned}
a_{7,21} &= -(1 + \alpha_\psi - 2\alpha_2) \cos \theta \sin 2\phi + \frac{\sqrt{2}}{2}[2D_2^s \sin^2 \theta - D_3^s(3 + \cos 2\theta)] \cos 2\phi, \\
a_{7,31} &= \frac{1}{2}(D_0^c + 2D_5^c) \sin 2\theta \cos 2\phi + \sqrt{2}(D_1^s - D_4^s) \sin \theta \sin 2\phi,
\end{aligned} \tag{C10}$$

$$\begin{aligned}
a_{7,02} &= -\frac{1}{2}(D_0^s + 2D_5^s) \sin 2\theta \cos 2\phi + \sqrt{2}(D_1^c + D_4^c) \sin \theta \sin 2\phi, \\
a_{7,12} &= (1 + \alpha_\psi - 2\alpha_2) \cos \theta \sin 2\phi + \frac{\sqrt{2}}{2}[2D_2^s \sin^2 \theta + D_3^s(3 + \cos 2\theta)] \cos 2\phi, \\
a_{7,22} &= [(1 + \alpha_\psi - 2\alpha_2) - (\alpha_\psi + \alpha_1 - \alpha_2) \sin^2 \theta] \cos 2\phi - 2\sqrt{2}D_3^s \cos \theta \sin 2\phi,
\end{aligned} \tag{C11}$$

$$\begin{aligned}
a_{7,32} &= (D_0^c + 2D_5^c) \sin \theta \sin 2\phi - \frac{\sqrt{2}}{2}(D_1^s - D_4^s) \sin 2\theta \cos 2\phi, \\
a_{7,03} &= -\sqrt{2}(D_2^c + D_3^c) \sin^2 \theta \cos 2\phi, \\
a_{7,13} &= -\frac{1}{2}(D_0^c - 2D_5^c) \sin 2\theta \cos 2\phi + \sqrt{2}(D_1^s + D_4^s) \sin \theta \sin 2\phi, \\
a_{7,23} &= \frac{\sqrt{2}}{2}(D_1^s + D_4^s) \sin 2\theta \cos 2\phi + (D_0^c - 2D_5^c) \sin \theta \sin 2\phi, \\
a_{7,33} &= -(1 + \alpha_1 + \alpha_2) \sin^2 \theta \cos 2\phi.
\end{aligned} \tag{C12}$$

The components of the polarization transfer matrix associated with the tensor polarization component $S_8(S_{TT}^{xy})$ are

$$\begin{aligned}
a_{8,00} &= (\alpha_\psi - \alpha_1 + \alpha_2) \sin^2 \theta \sin 2\phi, \\
a_{8,10} &= -(D_0^s - 2D_5^s) \sin \theta \cos 2\phi - \frac{\sqrt{2}}{2} (D_1^c - D_4^c) \sin 2\theta \sin 2\phi, \\
a_{8,20} &= \frac{1}{2} (D_0^s - 2D_5^s) \sin 2\theta \sin 2\phi - \sqrt{2} (D_1^c - D_4^c) \sin \theta \cos 2\phi, \\
a_{8,30} &= -\sqrt{2} (D_2^c - D_3^c) \sin^2 \theta \sin 2\phi, \\
a_{8,01} &= \frac{\sqrt{2}}{2} (D_1^c + D_4^c) \sin 2\theta \sin 2\phi - (D_0^s + 2D_5^s) \sin \theta \cos 2\phi, \\
a_{8,11} &= [(1 + \alpha_\psi - 2\alpha_2) - (1 - \alpha_1 - \alpha_2) \sin^2 \theta] \sin 2\phi + 2\sqrt{2} D_3^s \cos \theta \cos 2\phi, \\
a_{8,21} &= (1 + \alpha_\psi - 2\alpha_2) \cos \theta \cos 2\phi + \frac{\sqrt{2}}{2} [2D_2^s \sin^2 \theta - D_3^s (3 + \cos 2\theta)] \sin 2\phi, \\
a_{8,31} &= \frac{1}{2} (D_0^c + 2D_5^c) \sin 2\theta \sin 2\phi - \sqrt{2} (D_1^s - D_4^s) \sin \theta \cos 2\phi, \tag{C13}
\end{aligned}$$

$$\begin{aligned}
a_{8,02} &= -\frac{1}{2} (D_0^s + 2D_5^s) \sin 2\theta \sin 2\phi - \sqrt{2} (D_1^c + D_4^c) \sin \theta \cos 2\phi, \\
a_{8,12} &= -(1 + \alpha_\psi - 2\alpha_2) \cos \theta \cos 2\phi + \frac{\sqrt{2}}{2} [2D_2^s \sin^2 \theta + D_3^s (3 + \cos 2\theta)] \sin 2\phi, \\
a_{8,22} &= [(1 + \alpha_\psi - 2\alpha_2) - (\alpha_\psi + \alpha_1 - \alpha_2) \sin^2 \theta] \sin 2\phi + 2\sqrt{2} D_3^s \cos \theta \cos 2\phi, \tag{C14}
\end{aligned}$$

$$\begin{aligned}
a_{8,32} &= -(D_0^c + 2D_5^c) \sin \theta \cos 2\phi - \frac{\sqrt{2}}{2} (D_1^s - D_4^s) \sin 2\theta \sin 2\phi, \\
a_{8,03} &= -\sqrt{2} (D_2^c + D_3^c) \sin^2 \theta \sin 2\phi, \\
a_{8,13} &= -\frac{1}{2} (D_0^c - 2D_5^c) \sin 2\theta \sin 2\phi - \sqrt{2} (D_1^s + D_4^s) \sin \theta \cos 2\phi, \\
a_{8,23} &= -(D_0^c - 2D_5^c) \sin \theta \cos 2\phi + \frac{\sqrt{2}}{2} (D_1^s + D_4^s) \sin 2\theta \sin 2\phi, \\
a_{8,33} &= -(1 + \alpha_1 + \alpha_2) \sin^2 \theta \sin 2\phi. \tag{C15}
\end{aligned}$$

These polarization transfer matrices provide a complete description of polarization transfer in the $J/\psi \rightarrow B\bar{B}$ process. Experimentally, the polarization information of J/ψ can be obtained by analyzing the polarization and angular distributions of $B\bar{B}$.

APPENDIX D: CONVENTION FOR THE SPIN-1/2 AND SPIN-1 SPINORS

In this section, we provide the conventions for the spin-1/2 and spin-1 field spinors in the Dirac-Pauli representation. For a particle with mass m , the four-momentum in spherical coordinates can be expressed as

$$p^\mu = (E, |\vec{p}| \sin \theta \cos \phi, |\vec{p}| \sin \theta \sin \phi, |\vec{p}| \cos \theta). \tag{D1}$$

For a spin-1/2 field, the Dirac equation must be satisfied [79]. The spinor field with $\lambda = \pm 1$ can be written as

$$\begin{aligned}
u(p, \pm) &= \begin{pmatrix} \sqrt{E + m} \chi_\pm \\ \pm \sqrt{E - m} \chi_\pm \end{pmatrix}, \\
v(p, \pm) &= \begin{pmatrix} \sqrt{E - m} \chi_\mp \\ \mp \sqrt{E + m} \chi_\mp \end{pmatrix}, \tag{D2}
\end{aligned}$$

where $\chi_\pm$ represents the two-component spinor,

$$\begin{aligned}
\chi_+ &= \begin{pmatrix} \cos \frac{\theta}{2} \\ \sin \frac{\theta}{2} e^{i\phi} \end{pmatrix}, \\
\chi_- &= \begin{pmatrix} -\sin \frac{\theta}{2} \\ \cos \frac{\theta}{2} e^{i\phi} \end{pmatrix}. \tag{D3}
\end{aligned}$$

For a spin-1 particle, the field must satisfy the Proca equation [80]. The polarization vector with $\lambda = \pm 1, 0$ can be expressed as

$$\begin{aligned}\epsilon^\mu(p, \pm) &= \begin{pmatrix} 0 \\ \mp \cos \theta \cos \phi + i \sin \phi \\ \mp \cos \theta \sin \phi - i \cos \phi \\ \pm \sin \theta \end{pmatrix}, \\ \epsilon^\mu(p, 0) &= \frac{1}{m} \begin{pmatrix} |\vec{p}| \\ E \sin \theta \cos \phi \\ E \sin \theta \sin \phi \\ E \cos \theta \end{pmatrix}.\end{aligned}\tag{D4}$$

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- [1] T. D. Lee and C. N. Yang, Question of parity conservation in weak interactions, *Phys. Rev.* **104**, 254 (1956).
- [2] C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, Experimental test of parity conservation in β decay, *Phys. Rev.* **105**, 1413 (1957).
- [3] R. L. Garwin, L. M. Lederman, and M. Weinrich, Observations of the failure of conservation of parity and charge conjugation in meson decays: The magnetic moment of the free muon, *Phys. Rev.* **105**, 1415 (1957).
- [4] J. I. Friedman and V. L. Telegdi, Nuclear emulsion evidence for parity nonconservation in the decay chain $\pi^+ \rightarrow \mu^+ \rightarrow e^+$, *Phys. Rev.* **106**, 1290 (1957).
- [5] A. D. Sakharov, Violation of CP invariance, C asymmetry, and baryon asymmetry of the universe, *Pis'ma Zh. Eksp. Teor. Fiz.* **5**, 32 (1967).
- [6] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Evidence for the 2π decay of the K_2^0 meson, *Phys. Rev. Lett.* **13**, 138 (1964).
- [7] B. Aubert *et al.* (BABAR Collaboration), Observation of CP violation in the B^0 meson system, *Phys. Rev. Lett.* **87**, 091801 (2001).
- [8] K. Abe *et al.* (Belle Collaboration), Observation of large CP violation in the neutral B meson system, *Phys. Rev. Lett.* **87**, 091802 (2001).
- [9] R. Aaij *et al.* (LHCb Collaboration), Observation of CP violation in charm decays, *Phys. Rev. Lett.* **122**, 211803 (2019).
- [10] N. Cabibbo, Unitary symmetry and leptonic decays, *Phys. Rev. Lett.* **10**, 531 (1963).
- [11] M. Kobayashi and T. Maskawa, CP violation in the renormalizable theory of weak interaction, *Prog. Theor. Phys.* **49**, 652 (1973).
- [12] D. E. Morrissey and M. J. Ramsey-Musolf, Electroweak baryogenesis, *New J. Phys.* **14**, 125003 (2012).
- [13] G. 't Hooft, Symmetry breaking through Bell-Jackiw anomalies, *Phys. Rev. Lett.* **37**, 8 (1976).
- [14] G. 't Hooft, Computation of the quantum effects due to a four-dimensional pseudoparticle, *Phys. Rev. D* **14**, 3432 (1976); **18**, 2199(E) (1978).
- [15] R. Jackiw and C. Rebbi, Vacuum periodicity in a Yang-Mills quantum theory, *Phys. Rev. Lett.* **37**, 172 (1976).
- [16] C. G. Callan, Jr., R. F. Dashen, and D. J. Gross, The structure of the gauge theory vacuum, *Phys. Lett.* **63B**, 334 (1976).
- [17] S. Navas *et al.* (Particle Data Group), Review of particle physics, *Phys. Rev. D* **110**, 030001 (2024).
- [18] S. Okubo, Decay of the Σ^+ hyperon and its antiparticle, *Phys. Rev.* **109**, 984 (1958).
- [19] T. D. Lee and C. N. Yang, General partial wave analysis of the decay of a hyperon of spin 1/2, *Phys. Rev.* **108**, 1645 (1957).
- [20] A. Pais, Notes on antibaryon interactions, *Phys. Rev. Lett.* **3**, 242 (1959).
- [21] T. Brown, S. F. Tuan, and S. Pakvasa, CP nonconservation in hyperon decays, *Phys. Rev. Lett.* **51**, 1823 (1983).
- [22] L. L. Chau and H. Y. Cheng, Partial rate differences from CP violation in hyperon nonleptonic decays, *Phys. Lett.* **131B**, 202 (1983).
- [23] J. F. Donoghue and S. Pakvasa, Signals of CP nonconservation in hyperon decay, *Phys. Rev. Lett.* **55**, 162 (1985).
- [24] J. F. Donoghue, X. G. He, and S. Pakvasa, Hyperon decays and CP nonconservation, *Phys. Rev. D* **34**, 833 (1986).
- [25] R. Aaij *et al.* (LHCb Collaboration), Study of Λ_b^0 and Ξ_b^0 decays to $\Lambda h^+ h'^-$ and evidence for CP violation in $\Lambda_b^0 \rightarrow \Lambda K^+ K^-$ decays, *Phys. Rev. Lett.* **134**, 101802 (2025).
- [26] R. Aaij *et al.* (LHCb Collaboration), Observation of charge-parity symmetry breaking in baryon decays, *Nature (London)* **643**, 1223 (2025).
- [27] L. Kopke and N. Wermes, J/ψ decays, *Phys. Rep.* **174**, 67 (1989).
- [28] J. T. Donoghue, Spin and parity analysis for bosons decaying into spin 1/2 pairs, *Phys. Rev.* **178**, 2288 (1969).
- [29] H. Chen and R. G. Ping, Helicity amplitude analysis of hyperon nonleptonic decays in J/ψ or $\psi(2S)$ decays, *Phys. Rev. D* **76**, 036005 (2007).
- [30] E. Perotti, G. Fäldt, A. Kupsc, S. Leupold, and J. J. Song, Polarization observables in e^+e^- annihilation to a baryon-antibaryon pair, *Phys. Rev. D* **99**, 056008 (2019).

- [31] P. C. Hong, R. G. Ping, T. Luo, X. R. Zhou, and H. Li, Phenomenological study of $J/\psi \rightarrow \Xi^0(\Lambda\pi^0)\bar{\Xi}^0(\bar{\Lambda}\gamma)$ decays*, *Chin. Phys. C* **47**, 093103 (2023).
- [32] Z. Zhang and J. J. Song, Spin density matrix for Ω^- and its polarization alignment in $\psi(3686) \rightarrow \Omega^-\bar{\Omega}^+$, *Chin. Phys. C* **47**, 093101 (2023).
- [33] Z. Zhang, J. J. Song, and Y. j. Zhou, Refined analysis of $\Omega^-\bar{\Omega}^+$ polarization in electron-positron annihilation process, *Phys. Rev. D* **109**, 036005 (2024).
- [34] Z. Zhang, R. G. Ping, T. Liu, J. J. Song, W. Yang, and Y. j. Zhou, Polarization analysis of two baryons with various spin combinations produced in electron-positron annihilation, *Phys. Rev. D* **110**, 034034 (2024).
- [35] A. Z. Dubnickova, S. Dubnicka, and M. P. Rekalo, Investigation of the nucleon electromagnetic structure by polarization effects in $e^+e^- \rightarrow B\bar{B}$ processes, *Nuovo Cimento Soc. Ital. Fis.* **109A**, 241 (1996).
- [36] E. Tomasi-Gustafsson, F. Lacroix, C. Duterte, and G. I. Gakh, Nucleon electromagnetic form-factors and polarization observables in space-like and timelike regions, *Eur. Phys. J. A* **24**, 419 (2005).
- [37] H. Czyz, A. Grzelinska, and J. H. Kuhn, Spin asymmetries and correlations in Λ -pair production through the radiative return method, *Phys. Rev. D* **75**, 074026 (2007).
- [38] G. Fäldt and A. Kupsc, Hadronic structure functions in the $e^+e^- \rightarrow \bar{\Lambda}\Lambda$ reaction, *Phys. Lett. B* **772**, 16 (2017).
- [39] M. Ablikim *et al.* (BESIII Collaboration), Polarization and entanglement in baryon-antibaryon pair production in electron-positron annihilation, *Nat. Phys.* **15**, 631 (2019).
- [40] M. Ablikim *et al.* (BESIII Collaboration), Precise measurements of decay parameters and CP asymmetry with entangled $\Lambda - \bar{\Lambda}$ pairs, *Phys. Rev. Lett.* **129**, 131801 (2022).
- [41] M. Ablikim *et al.* (BESIII Collaboration), Σ^+ and $\bar{\Sigma}^-$ polarization in the J/ψ and $\psi(3686)$ decays, *Phys. Rev. Lett.* **125**, 052004 (2020).
- [42] M. Ablikim *et al.* (BESIII Collaboration), Stringent test of CP symmetry in Σ^+ hyperon decays, [arXiv:2503.17165](#).
- [43] M. Ablikim *et al.* (BESIII Collaboration), Probing CP symmetry and weak phases with entangled double-strange baryons, *Nature (London)* **606**, 64 (2022).
- [44] M. Ablikim *et al.* (BESIII Collaboration), Tests of CP symmetry in entangled $\Xi^0 - \bar{\Xi}^0$ pairs, *Phys. Rev. D* **108**, L031106 (2023).
- [45] M. Ablikim *et al.* (BESIII Collaboration), Investigating the $\Delta I = 1/2$ rule and CP violation through the measurement of decay asymmetry parameters in Ξ^- decays, *Phys. Rev. Lett.* **132**, 101801 (2024).
- [46] M. Ablikim *et al.* (BESIII Collaboration), Extracting the femtometer structure of strange baryons using the vacuum polarization effect, *Nat. Commun.* **15**, 8812 (2024).
- [47] M. Ablikim *et al.* (BESIII Collaboration), Measurement of the absolute branching fraction and decay asymmetry of $\Lambda \rightarrow n\gamma$, *Phys. Rev. Lett.* **129**, 212002 (2022).
- [48] M. Ablikim *et al.* (BESIII Collaboration), Precision measurement of the decay $\Sigma^+ \rightarrow p\gamma$ in the process $J/\psi \rightarrow \Sigma^+\Sigma^-$, *Phys. Rev. Lett.* **130**, 211901 (2023).
- [49] M. Ablikim *et al.* (BESIII Collaboration), Strong and weak CP tests in sequential decays of polarized Σ^0 hyperons, *Phys. Rev. Lett.* **133**, 101902 (2024).
- [50] M. Ablikim *et al.* (BESIII Collaboration), Measurement of the decay $\Xi^0 \rightarrow \Lambda\gamma$ with entangled $\Xi^0\bar{\Xi}^0$ pairs, *Sci. Bull.* **70**, 454 (2025).
- [51] M. Achasov *et al.*, STCF conceptual design report (Volume 1): Physics & detector, *Front. Phys. (Beijing)* **19**, 14701 (2024).
- [52] X. G. He, J. P. Ma, and B. McKellar, CP violation in $J/\psi \rightarrow \Lambda\bar{\Lambda}$, *Phys. Rev. D* **47**, R1744 (1993).
- [53] X. G. He and J. P. Ma, Testing of P and CP symmetries with $e^+e^- \rightarrow J/\psi \rightarrow \Lambda\bar{\Lambda}$, *Phys. Lett. B* **839**, 137834 (2023).
- [54] Y. Du, X. G. He, J. P. Ma, and X. Y. Du, Fundamental tests of P and CP symmetries using octet baryons at the J/ψ threshold, *Phys. Rev. D* **110**, 076019 (2024).
- [55] A. Bondar, A. Grabovsky, A. Reznichenko, A. Rudenko, and V. Vorobyev, Measurement of the weak mixing angle at a Super Charm-Tau factory with data-driven monitoring of the average electron beam polarization, *J. High Energy Phys.* **03** (2020) 076.
- [56] N. Salone, P. Adlarson, V. Batzskaya, A. Kupsc, S. Leupold, and J. Tandean, Study of CP violation in hyperon decays at super-charm-tau factories with a polarized electron beam, *Phys. Rev. D* **105**, 116022 (2022).
- [57] S. Zeng, Y. Xu, X. R. Zhou, J. J. Qin, and B. Zheng, Prospects of CP violation in Λ decay with a polarized electron beam at the STCF, *Chin. Phys. C* **47**, 113001 (2023).
- [58] J. Fu, H. B. Li, J. P. Wang, F. S. Yu, and J. Zhang, Probing hyperon electric dipole moments with a full angular analysis, *Phys. Rev. D* **108**, 9 (2023).
- [59] X. Cao, Y. T. Liang, and R. G. Ping, Production and decay of hyperons in a transversely polarized electron-positron collider, *Phys. Rev. D* **110**, 014035 (2024).
- [60] M. Guo, Z. Zhang, R. Ping, and J. Jiao, Search for CP violations in the production and decay of the hyperon-antihyperon pairs, [arXiv:2502.20274](#).
- [61] M. Jacob and G. C. Wick, On the general theory of collisions for particles with spin, *Ann. Phys. (N.Y.)* **7**, 404 (1959).
- [62] A. A. Sokolov and I. M. Ternov, On polarization and spin effects in the theory of synchrotron radiation, *Sov. Phys. Dokl.* **8**, 1203 (1964).
- [63] M. Ablikim *et al.* (BESIII Collaboration), Design and construction of the BESIII detector, *Nucl. Instrum. Methods Phys. Res., Sect. A* **614**, 345 (2010).
- [64] J. S. Schwinger, On quantum electrodynamics and the magnetic moment of the electron, *Phys. Rev.* **73**, 416 (1948).
- [65] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Addison-Wesley, Reading, USA, 1995).
- [66] J. G. Korner and M. Kuroda, e^+e^- annihilation into baryon-antibaryon pairs, *Phys. Rev. D* **16**, 2165 (1977).
- [67] S. U. Chung, Spin formalism, CERN Report No. CERN-1971-008.
- [68] J. Kim, J. Lee, J. S. Shim, and H. S. Song, Polarization effects in spin $3/2$ hyperon decay, *Phys. Rev. D* **46**, 1060 (1992).
- [69] N. Byers and S. Fenster, Determination of spin and decay parameters of fermion states, *Phys. Rev. Lett.* **11**, 52 (1963).
- [70] J. Button-Shafer, Fermion decay into spin- $\frac{3}{2}$ fermion plus spin-0 boson, *Phys. Rev.* **139**, B607 (1965).

- [71] R. Lednicky, About spin parity of a $p\omega$ -enhancement near 1800-MeV, *Phys. Lett.* **58B**, 89 (1975).
- [72] R. Lednicky, On evaluation of polarization of the charmed baryon Λ_c^+ , *Sov. J. Nucl. Phys.* **43**, 817 (1986).
- [73] M. Ademollo and R. Gatto, Complete spin tests for fermions, *Phys. Rev.* **133**, B531 (1964).
- [74] S. M. Berman and M. Jacob, Spin and parity analysis in two-step decay processes, SLAC Report No. 43, 1965.
- [75] X. L. Xia, H. Li, X. G. Huang, and H. Z. Huang, Feed-down effect on Λ spin polarization, *Phys. Rev. C* **100**, 014913 (2019).
- [76] R. X. Shi, Z. Jia, L. S. Geng, H. Peng, Q. Zhao, and X. Zhou, Status and prospect of weak radiative hyperon decays, *Chin. Phys. Lett.* **42**, 032401 (2025).
- [77] R. A. Fisher, On the mathematical foundations of theoretical statistics, *Phil. Trans. R. Soc. A* **222**, 309 (1922).
- [78] A. Bacchetta and P. J. Mulders, Deep inelastic leptonproduction of spin-one hadrons, *Phys. Rev. D* **62**, 114004 (2000).
- [79] P. A. M. Dirac, The quantum theory of the electron, *Proc. R. Soc. A* **117**, 610 (1928).
- [80] A. Proca, Sur la theorie ondulatoire des electrons positifs et negatifs, *J. Phys. Radium* **7**, 347 (1936).