

Zoo of flows in a 3D gauged supergravity with a periodic potential

Lev Astrakhantsev^{1,2,*}, Anastasia A. Golubtsova^{1,†} and Mikhail A. Podoinitsyn^{1,‡}

¹*Bogoliubov Laboratory of Theoretical Physics, JINR, Joliot-Curie Street 6, Dubna 141980, Russia*

²*Moscow Institute of Physics and Technology, Institutskii per, 9, Dolgoprudny 141700, Russia*



(Received 27 February 2026; accepted 13 May 2026; published 6 July 2026)

In this paper we construct solutions with AdS/dS asymptotics for $D = 3$ truncated gauged supergravity with a periodic scalar potential. In a holographic perspective, assuming Dirichlet boundary conditions, the solutions can be interpreted as deformations of two-dimensional dual conformal field theories triggered by nonzero vacuum expectation values (VEVs) of irrelevant operators. In addition to the domain-wall-type solutions, we incorporated in the analysis a black string solution, which can also be interpreted as a deformation by VEV of an irrelevant operator. Generalizing the flows to finite temperature, we find that the corresponding geometries are singular but have horizons. For certain flows, we provide an analytical description near the horizon region. For an exact renormalization group (RG) flow solution, we explicitly compute the Brown-York stress-energy tensor on a cutoff surface and show that the $T\bar{T}$ operator factorizes along the holographic RG flow. We also define an effective, scale-dependent deformation parameter $\mu(\phi)$, whose running is governed by the scalar field at the cutoff.

DOI: [10.1103/mzdz-33s5](https://doi.org/10.1103/mzdz-33s5)

I. INTRODUCTION

The renormalization group (RG) [1–6] provides a powerful framework for understanding how physical theories evolve with energy scale. In this context, RG flows represent deformations of conformal field theories (CFTs), typically induced either by adding relevant operators to the action or by turning on vacuum expectation values (VEVs) for such operators.

The holographic framework provides a geometric realization of the renormalization group. Within this duality, a boundary quantum field theory and its dynamics are fully encoded in the geometry and fields of a higher-dimensional gravitational bulk [7–9]. This correspondence establishes a precise dictionary: deformations of the conformal field theory are mapped to specific asymptotically anti-de Sitter (AdS) spacetime geometries that solve the bulk equations of motion [10–17]. In [18–22], a systematic and covariant approach was developed that employs an eigenfunction expansion of the dilatation operator acting on the phase space of the bulk gravitational theory.

Deformations by relevant operators ($\Delta < d$) correspond to introducing fields in the bulk gravitational theory, which modify the infrared (IR) geometry and trigger a flow to a new fixed point. In contrast, by irrelevant deformations ($\Delta > d$) growing in the ultraviolet (UV) dictate the short-distance behavior of the theory and break conformal symmetry in the UV. A generic irrelevant deformation will backreact strongly on the AdS geometry. The holographic dual is related to a diverging non-normalizable mode, which means a drastic deformation of the boundary CFT, potentially leading to a nonlocal boundary theory or a “non-AdS” asymptotic geometry. For certain cases, deformations caused by irrelevant operators can be classified as exotic RG flows [23].

An extension of the holographic renormalization to the case of the presence of sources of irrelevant operators was discussed in [24,25], where it was shown that, to complete the analysis, one has to introduce a cutoff surface $r = r_c$. The effect of coupling integer-dimensional scalars that source irrelevant operators on the boundary theory was explored in [26]. The thermodynamics of observables and irrelevant deformations were studied in [27]. The proposal that a finite radial cutoff $r = r_c$ in AdS_3 spacetime corresponds to a $T\bar{T}$ deformation of the dual CFT was established in [28].

Beyond AdS/CFT, holographic RG flows have also been explored in the context of the dS/CFT duality [29–31], where the role of the radial coordinate is replaced by time, and the flow connects Euclidean CFTs in the asymptotic past and future.

In this work, we focus on a three-dimensional truncated gauged supergravity model [32,33], which features a real

*Contact author: levastr@theor.jinr.ru, lev.astrakhantsev@phystech.su

†Contact author: golubtsova@theor.jinr.ru

‡Contact author: mpod@theor.jinr.ru

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

scalar field with a periodic potential inherited from the full gauged supergravity action. This potential possesses multiple extrema corresponding to AdS, de Sitter (dS), and Minkowski vacua. The holographic RG flows for this supergravity with a hyperbolic potential were studied in [34,35], and Janus/RG-flow interfaces were found in [36]. The periodic case offers a distinct and rich structure since, besides AdS extrema, the potential includes dS and Minkowski extrema. At the same time, below we will show that all solutions in this model are related to deformations by irrelevant operators. In the five-dimensional case, holographic flows for the $D = 5$ gauged supergravity model with a cosine superpotential corresponding to deformations of irrelevant operators were found in [37].

Here we construct and study solutions from AdS and dS vacua to the Minkowski one and generalize them to the case of finite temperature. From the holographic point of view, the solutions interpolating between AdS and Minkowski vacua with the Dirichlet boundary conditions can be associated with a deformation of a two-dimensional (2D) dual CFT by a nonzero expectation value of an irrelevant operator, which spontaneously breaks the conformal symmetry. For the general type of scalar potentials, such flows were discussed as exotic irrelevant holographic RG flows between minima of potentials in [23].

For the de Sitter case, we find two types of solutions flowing to Minkowski vacua that differ by asymptotics near the Minkowski fixed point. Both solutions between dS and Minkowski fixed points can be interpreted as irrelevant deformations by VEV of a scalar operator associated with different signs of the VEV. In this case, the dual CFTs turn out to be nonunitary. An example of the dS_3/CFT_2 correspondence with a nonunitary CFT was discussed in [38].

A key aspect of our analysis is the use of a dynamical systems approach to study these RG flows [23,39–41], where flows appear as trajectories in the phase space of the scalar field and the β function. In this work, we focus on flows, which correspond to deformations caused by the VEVs of irrelevant operators. We further generalize these flows to finite temperature, finding that the corresponding thermal geometries are mostly singular and the only regular ones are the Bañados-Teitelboim-Zanelli (BTZ) and Schwarzschild–de Sitter (SdS) black holes. Our findings on the singularity of generic thermal flows are consistent with an analysis from [42]. We also discuss a black string solution [32], showing that it corresponds to a VEV-driven deformation where the VEV is related to the mass of the black string. We define the $T\bar{T}$ operator [43] for the exact holographic RG flow through the Brown-York stress-energy tensor and show that the operator is factorized along the flow. We also define an effective deformation parameter μ , which depends on the scalar field.

The paper is organized as follows. In Sec. II, we briefly review a $D = 3$, $\mathcal{N} = (2, 0)$ gauged supergravity model and discuss the scalar field behavior near AdS/dS extrema of the scalar potential. In Sec. III, we present exact and

numerical solutions flowing from AdS/dS vacua and discuss their holographic aspects. We also come from equations of motion (EOM) of the model to a 2D autonomous dynamical system on the plane and in the disk. In Sec. IV, we use the Poincaré transformation to project the phase space of the system with nonzero temperature into the unit cylinder and the ball, which helps us understand the global phase portrait and allows us to construct the flows numerically. We also construct analytic near-horizon solutions. In Sec. V, we compute the Brown-York stress-energy tensor on a cutoff surface for the exact holographic RG flow and show that the $T\bar{T}$ operator factorizes. We also introduce an effective deformation parameter μ , which decreases along the flow. We conclude in Sec. VI, where we discuss our main results and sketch further directions to explore. Some technical details are left for the Appendixes.

II. SETUP

A. The supergravity model

We consider the $\mathcal{N} = (2, 0)$ AdS₃ supergravity model coupled to n copies of $\mathcal{N} = (2, 0)$ scalar multiplets constructed in [32]. The field content of this three-dimensional supergravity is given by a vielbein e_μ^a , a doublet of gravitini ψ_μ , and a gauge field A_μ . The scalar multiplet consists of a complex scalar field Φ and a doublet of spinorial fields λ , where R-symmetry indices are omitted. In [32], the σ model manifold was considered to be a coset space G/H with a compact and noncompact G and its maximal compact subgroup H . In papers [34,35,44,45], the noncompact case with the hyperbolic target space $\mathbb{H}^2$ was studied. In this paper, we focus on a truncation of the σ model to that one, which includes a single complex scalar field Φ (that is, $n = 1$) and G is compact, so the target space is a two-dimensional sphere, i.e.,

$$\frac{SU(2)}{U(1)} = S^2. \quad (2.1)$$

In this case the bosonic part of the Lagrangian reads [32]

$$e^{-1}\mathcal{L} = \frac{1}{4}R - \frac{e^{-1}}{16ma^4}\epsilon^{\mu\nu\rho}A_\mu\partial_\nu A_\rho - \frac{|D_\mu\Phi|^2}{a^2(1+|\Phi|^2)} - V(\Phi), \quad (2.2)$$

where $e = \det e_\mu^a$ and $D_\mu\Phi = (\partial_\mu - iA_\mu)\Phi$. Note that the parameter m in (2.2) is related to the cosmological constant as $\Lambda \equiv -4m^2$ and the parameter a has a nonzero value and corresponds to the curvature of the scalar manifold (sphere).¹ The potential $V(\Phi)$ in (2.2) is given by

¹The $a = 0$ case corresponds to the flat σ model. To take this limit, first fields ϕ and A_μ have to be rescaled with appropriate powers of a , and after the limit, the potential in the action reduces to a cosmological constant [33,46].

$$V(\Phi) = 2m^2 C^2 (2a^2 |S|^2 - C^2), \quad (2.3)$$

where

$$C = \frac{1 - |\Phi|^2}{1 + |\Phi|^2}, \quad S = \frac{2\Phi}{1 + |\Phi|^2}. \quad (2.4)$$

To bring the kinetic term of the scalar field to the standard form, one splits the complex scalar field Φ into its modulus $|\Phi| \equiv \phi$ and phase θ . For the former, we additionally define

$$C \equiv \cos \phi, \quad |S| \equiv \sin \phi. \quad (2.5)$$

After this, the Lagrangian of the supergravity theory takes the form

$$e^{-1} \mathcal{L} = \frac{1}{4} R - \frac{e^{-1}}{a^4} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho - \frac{1}{4a^2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{4a^2} |S|^2 (\partial_\mu \theta + A_\mu) (\partial^\mu \theta + A^\mu) - V(\phi). \quad (2.6)$$

Note that the potential depends only on the field ϕ . So from the field equations of θ and the vector field A_μ , it is easy to see that setting $A_\mu = \theta = 0$ is a consistent truncation of the theory [32], which finally brings us to the action of the form

$$S = \frac{1}{4} \int d^3 x \sqrt{|g|} \left(R - \frac{1}{a^2} (\partial \phi)^2 - 4V(\phi) \right) + \frac{1}{2} \int_{\partial M} d^2 x \sqrt{|\gamma|} K, \quad (2.7)$$

with the potential

$$V(\phi) = -2m^2 \cos^2 \phi \left((1 + 2a^2) \cos^2 \phi - 2a^2 \right). \quad (2.8)$$

In (2.7) K is the trace of the extrinsic curvature and γ is the determinant of the induced metric $\gamma_{\alpha\beta}$, with $\alpha, \beta = 1, 2$.

On the period $[-\pi, \pi]$ the potential (2.8) has extrema that correspond to AdS vacua at

$$\phi_{\text{AdS}} = c_1 \pi, \quad c_1 = 0, \pm 1; \quad (2.9)$$

de Sitter extrema are located at

$$\phi_{\text{dS}} = \pm \arccos \left(\sqrt{\frac{a^2}{1 + 2a^2}} \right) \mp c_2 \pi, \quad c_2 = 0, 1; \quad (2.10)$$

and Minkowski extrema on the period are

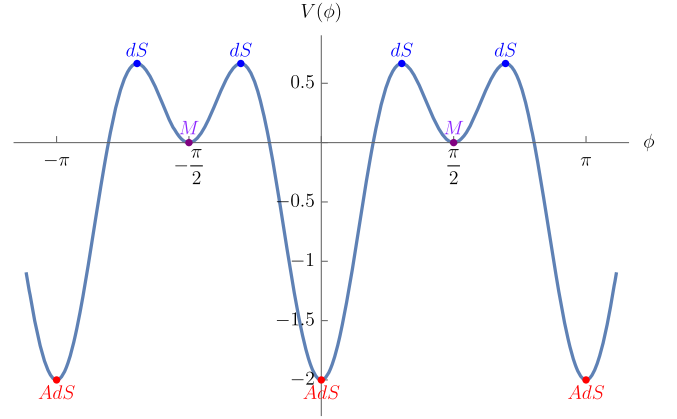


FIG. 1. The behavior of the potential (2.8) for $a^2 = 1$ and $m = 1$ on the interval $[-\pi, \pi]$.

$$\phi_M = \pm \frac{\pi}{2}. \quad (2.11)$$

We note that the number of the extrema of the potential (2.8) and their types do not depend on the parameter a at all, as we can see in Fig. 1. Further, for our convenience, we will set $a^2 = 1$ when we numerically draw phase portraits and trajectories.

The potential (2.8) is related to the superpotential W by the following relation:

$$V = \frac{a^2}{4} W^2 - \frac{1}{2} W^2, \quad (2.12)$$

where the exact supersymmetric superpotential [47] has the form

$$W_{\text{susy}} = -2m \cos^2 \phi. \quad (2.13)$$

The behavior of the superpotential W as a function of ϕ is presented in Fig. 2. Extrema of the superpotential include AdS and Minkowski vacua.

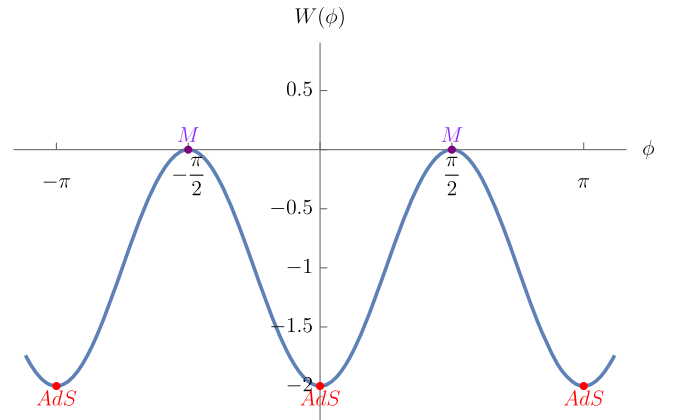


FIG. 2. Plot of the superpotential (2.13) with $m = 1$. There are three AdS and two Minkowski M extrema on the interval $[-\pi, \pi]$.

B. Equations of motion

In this paper, we will discuss solutions to (2.7) with the following ansatz of the metric:

$$ds^2 = e^{2A(w)} \left(-f(w) dt^2 + dx^2 \right) + \frac{dw^2}{f(w)}, \quad (2.14)$$

where $f(w)$ is the blackening function. It defines a location of the horizon, on either side of which f has a different sign. A solution, which is endowed by f in the ansatz, passes through a horizon and exchanges the coordinate w from space- to timelike and vice versa.

The asymptotically BTZ black hole is given by (2.14) with

$$A = \sqrt{-\frac{V(\phi_{\text{AdS}})}{2}} w + \dots \quad (2.15)$$

and

$$f = 1 - e^{-\sqrt{-2V(\phi_{\text{AdS}})}(w-w_h)} + \dots \quad (2.16)$$

For $f = 1$, the metric (2.14) describes an asymptotically AdS_3 spacetime without a black hole. Extending the possible set of values for the function f to the interval $[-1, 0)$, we can get the asymptotically Schwarzschild–de Sitter metric given by (2.14) with the scale factor

$$A = \sqrt{\frac{V(\phi_{\text{dS}})}{2}} w + \dots \quad (2.17)$$

and the blackening factor

$$f = -1 + e^{-\sqrt{2V(\phi_{\text{dS}})}(w-w_h)} + \dots \quad (2.18)$$

Putting $f = -1$, we come to an asymptotically dS space-time

$$ds^2 = e^{2A(w)} (dt^2 + dx^2) - dw^2. \quad (2.19)$$

Note, that using the redefinition for the coordinates $w \rightarrow \tau$ and $t \rightarrow r$ in (2.19), we obtain the dS_3 metric in the expanding patch [for another dS coordinate representation, see Sec. A 2 b and Appendix A 1].

In what follows, we will discuss solutions to EOM corresponding to the action (2.7) with the metric ansatz (2.14), which includes the blackening function. The Einstein equations to (2.7) with the metric (2.14) can be represented in the following form:

$$\ddot{A} + \frac{\dot{\phi}^2}{a^2} = 0, \quad (2.20)$$

$$\ddot{f} + 2\dot{f}\dot{A} = 0, \quad (2.21)$$

$$\dot{A}\dot{f} + 2\dot{A}^2 f - \frac{\dot{\phi}^2}{a^2} f + 4V = 0, \quad (2.22)$$

where the overdot indicates the derivative with respect to the radial coordinate w . The scalar field equation is

$$\ddot{\phi} + \dot{f}\dot{\phi}f^{-1} + 2\dot{A}\dot{\phi} - 2a^2 f^{-1} V_\phi = 0, \quad (2.23)$$

where V_ϕ is a derivative of the potential with respect to ϕ .

For certain flows, it will be convenient to consider the equations of motion with $f = 1$. Then the equations of motion (2.20)–(2.22) take the form

$$2\ddot{A} + 2\dot{A}^2 + 4V + \frac{\dot{\phi}^2}{a^2} = 0, \quad (2.24)$$

$$2\dot{A}^2 + 4V - \frac{\dot{\phi}^2}{a^2} = 0, \quad (2.25)$$

together with the scalar field equation

$$\ddot{\phi} + 2\dot{A}\dot{\phi} - 2a^2 V_\phi = 0. \quad (2.26)$$

Note that, for $f = 1$, from the variation of the gravitini and dilatino, we have the following first-order Bogomol'nyi-Prasad-Sommerfield (BPS) equations [47]:

$$\begin{aligned} \dot{A} &= -W, \\ \dot{\phi} &= a^2 W'. \end{aligned} \quad (2.27)$$

In the derivation of Eqs. (2.27) in [47], the constraint of the preservation of supersymmetry was used. The solutions to Eqs. (2.27) also solve the second-order equations of motion for (2.7) with (2.8). However, the superpotential does not have dS extrema; solutions to (2.27) interpolate between AdS and Minkowski spacetimes.

C. The scalar field near AdS and dS regions

Near AdS extrema, which correspond to $\phi_{\text{AdS}} = c_1 \pi$, with $c_1 = 0, \pm 1$, the scalar potential has the following expansion:

$$V = -2 + 4(1 + a^2)(\phi - \phi_{\text{AdS}})^2 + \dots \quad (2.28)$$

The AdS space curvature and the mass are given, correspondingly, as

$$\ell^2 = -\frac{1}{2V(\phi_{\text{AdS}})} = \frac{1}{4}, \quad M^2 = 16a^2(1 + a^2). \quad (2.29)$$

The central charge of the dual theory is related to ℓ via the Brown-Henneaux formula

$$c = \frac{3\ell}{2G_N}. \quad (2.30)$$

Because of the form of the potential (2.8), the AdS spacetimes for any ϕ such that $V'(\phi) = 0$ have the same radii, and this is also relevant for the case of the dS spacetime.

The generic solution to the scalar field near AdS regions has the following form:

$$\phi \sim \phi_- e^{-\Delta_- w} + \phi_+ e^{-\Delta_+ w}, \quad (2.31)$$

where the corresponding scaling dimensions $\Delta_{\pm}$ are given by

$$\Delta_{\pm} = 1 \pm (1 + 2a^2). \quad (2.32)$$

In the standard holographic dictionary, ϕ_+ , ϕ_- from (2.31) are related to the VEV of the dual operator and the source, correspondingly. For our model, Δ_- (2.32) is always negative for any $a^2 > 0$, while $\Delta_+ > d$, i.e., Δ_+ is irrelevant. In particular, for $a^2 = 1$ we have $\Delta_+ = 4$ and $\Delta_- = -2$.

The fact that Δ_- (2.32) is far below the unitarity bound precludes the use of Neumann and mixed boundary conditions. For the bulk scalar with the mass M given by (2.29), the Dirichlet boundary condition (BC) is the only admissible one, leading to a unitary CFT dual with a scalar operator of dimension $\Delta_+ = 2(1 + a^2)$. Since $\Delta_+ > d$ for any $a^2 > 0$, the corresponding deformation is irrelevant, we can assume the dual theory is defined on the cutoff surface $w = w_c$, so the Dirichlet BC reads

$$\phi \sim \phi_+ e^{-\Delta_+ w}. \quad (2.33)$$

Near dS extrema (2.10), the potential has the following form:

$$V = \frac{2a^4}{1 + 2a^2} - \frac{8a^2(1 + a^2)}{1 + 2a^2}(\phi - \phi_{\text{dS}})^2 + \dots \quad (2.34)$$

The parameter H and the mass M for the de Sitter case are defined by

$$H^2 = \frac{4V(\phi_*)}{2} = \frac{4a^4}{1 + 2a^2}, \quad M^2 = 2a^2 V''(\phi_*) = -\frac{32a^4(1 + a^2)}{1 + 2a^2}, \quad (2.35)$$

so for any $a^2 > 0$ we have $M^2 < 0$, i.e., the scalar field is tachyonic (for tachyons in the dS spacetime, see [48]). The scaling dimensions [29] are defined by

$$h^{\pm} = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} - \frac{M^2}{H^2}} = 1 \pm \sqrt{9 + 8a^2}. \quad (2.36)$$

Similar to the AdS case, here, h_- is negative for any $a^2 > 0$, while $h_+ > d$. Though the negative mass squared seems admissible in the dS spacetime satisfying the analog of the Breitenlohner-Freedman bound $M^2 < \frac{d^2}{4} H^2$ [49,50], nevertheless the scalar field is tachyonic in this case, so a

holographic dual theory is nonunitary. It is interesting to note, that for $a^2 = 2$, Δ_+ have the same value for both AdS and dS cases and are equal to $\Delta_+ = 6$.

The solution to the scalar field near the dS point with the metric in the form (A9) reads [51–54]

$$\phi \sim \phi_{\text{dS}} + \phi^+ e^{-2h_+ t} + \phi^- e^{-2h_- t}, \quad (2.37)$$

with $t \rightarrow +\infty$. Because h_- is negative for all real $a^2 > 0$, the second term in (2.37) is apparently divergent, which corresponds to unbounded energy for tachyons in nonunitary CFT. The central charge for the dS case can be related with H using (2.30) and $H = i\ell$ [29,55]. Note that in [17] it was shown that the results of the near-boundary analysis of asymptotically AdS spacetimes can be applied to asymptotically de Sitter spacetimes.

III. FLOWS BETWEEN ADS/DS AND MINKOWSKI VACUA

A. Exact flows driven by VEV of an irrelevant operator

In this subsection, we will discuss two exact solutions to the action (2.7), which solve the first-order equations (2.27).

1. Domain wall solution

There is an exact half-supersymmetric solution to (2.27), which starts at the AdS spacetime as $w \rightarrow -\infty$ and ends at the Minkowski one as $w \rightarrow \infty$, which both correspond to the minima of the scalar potential. The scale function A and scalar field ϕ have the following dependence on the radial coordinate w ,

$$A(w) = -\frac{1}{4a^2} \ln(e^{-8ma^2 w} + 1) + c_A, \quad \phi = \pm \arctan(e^{4ma^2 w}) \pm n\pi, \quad (3.1)$$

where $c_A \in \mathbb{R}$, $n \in \mathbb{Z}$, and $w \in (-\infty, \infty)$. In Fig. 3 we show possible supersymmetric flows of (2.27) with $a^2 = 1$ and $m = 1$.

The exact solution (3.1) solves the first-order equations including the superpotential W (2.13), with the Dirichlet boundary conditions (2.33) imposed near the AdS boundary

$$\phi \sim \phi_+ e^{-\Delta_+ w} + \dots, \quad (3.2)$$

i.e., $\phi_- = 0$. This asymptotic behavior can be seen from the expansion of the exact solution for the scalar field (3.1). Thus, the solution is dual to a deformation of CFT to a gapped phase, which is being driven by the VEV ϕ_+ of an irrelevant operator. Solutions of this type were discussed in the classification of exotic RG flows in [23]. Moreover, flows triggered by nonzero VEV of irrelevant operators were discussed in the context of deformations in the Coulomb branch in [56].

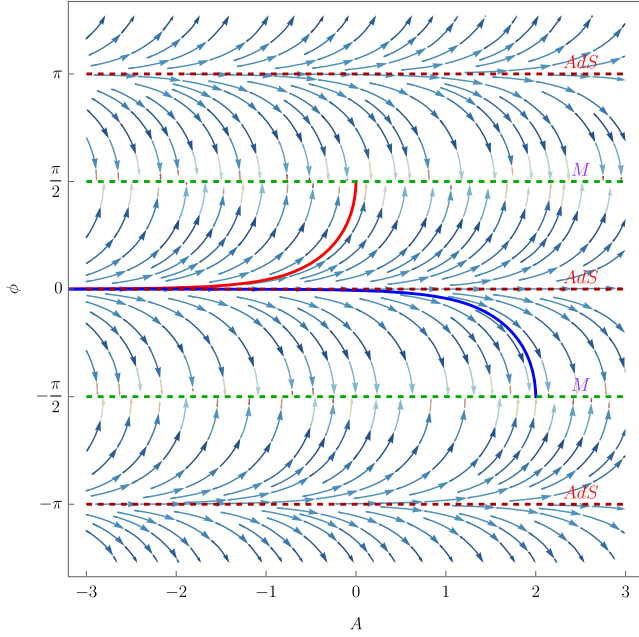


FIG. 3. Phase portrait of the supersymmetric system (2.27), with $m = 1$, $a^2 = 1$. Red and blue trajectories correspond to the same $n = 0$, but to opposite signs for the dilaton field, together with different constants c_A for the scale factor of the exact solution (3.1).

2. Black string

In [32], a black string solution was constructed for the model (2.2) with a single scalar field, which parametrizes the target space S^2 . The black string metric is given by

$$ds^2 = \left(1 - \frac{M}{r}\right)^{\frac{1}{2a^2}} (-dt^2 + dx^2) + \frac{1}{64m^2 a^4 r^2} \left(1 - \frac{M}{r}\right)^{-2} dr^2, \quad (3.3)$$

where M is a mass and the horizon is located at $r_h = M$. The scalar field of the black string solution is given by

$$\Phi_+ = \left(\frac{\sqrt{r} - \sqrt{M}}{\sqrt{r} + \sqrt{M}}\right)^{1/2}, \quad \Phi_- = \left(\frac{\sqrt{r} + \sqrt{M}}{\sqrt{r} - \sqrt{M}}\right)^{1/2}. \quad (3.4)$$

In (3.4), the fields Φ_+ , Φ_- are the stereographic coordinates of S^2 such that $0 \leq \Phi_+ \leq 1$ and $1 \leq \Phi_- \leq \infty$. The fields Φ_+ and Φ_- constitute a well-defined map on the upper and lower hemispheres, S_+^2 and S_-^2 , accordingly.

Doing the scalar field transformation, we come to

$$\phi = 2 \arctan \Phi = 2 \arctan \left(\frac{\sqrt{r} - \sqrt{M}}{\sqrt{r} + \sqrt{M}}\right)^{1/2}, \quad (3.5)$$

where we choose the scalar field on the upper hemisphere.

The Hawking temperature of the black string solution (3.3)–(3.5) can be calculated as follows:

$$T_H = \frac{2ma}{\pi} \sqrt{\frac{M}{r} \left(1 - \frac{M}{r}\right)^{\frac{1}{2a^2}}} \Big|_{r=M} = 0. \quad (3.6)$$

The vanishing temperature indicates that the horizon of the black string solution is degenerate, so the solution is extremal. Extremal charged black string solutions in three dimensions were studied in [57].

The metric of the black string can be rewritten in the form (2.14) with $f = 1$. This is achieved by a suitable reparametrization of the radial coordinate,

$$\frac{dr}{8a^2 r \left(1 - \frac{M}{r}\right)} = dw, \quad w = + \frac{1}{8a^2} \ln(r - M),$$

$$r = M + e^{8a^2 w}, \quad (3.7)$$

where we put $m^2 = 1$. Then, taking into account (3.7), the black string metric in terms of the w coordinate has the form

$$ds^2 = \left(\frac{e^{8a^2 w}}{M + e^{8a^2 w}}\right)^{\frac{1}{2a^2}} (-dt^2 + dx^2) + dw^2, \quad (3.8)$$

with the scalar field given by

$$\phi(w) = 2 \arctan \left(\frac{\sqrt{M + e^{8a^2 w}} - \sqrt{M}}{\sqrt{M + e^{8a^2 w}} + \sqrt{M}}\right)^{1/2}$$

$$= \pm \arctan \left(\frac{1}{\sqrt{M}} e^{4a^2 w}\right), \quad (3.9)$$

and the scale factor reads

$$A(w) = -\frac{1}{4a^2} \log \left(1 + M e^{-8a^2 w}\right). \quad (3.10)$$

One can check that (3.9) and (3.10) solve the first-order equations (2.27) with the supersymmetric potential (2.13).

Comparing (3.9) and (3.10) to the domain wall solution (3.1), the black string differs by the factor M . The scalar field near $w \rightarrow -\infty$ has the expansion

$$\phi(w \rightarrow -\infty) = \pm \frac{1}{\sqrt{M}} e^{4a^2 w} + \dots, \quad (3.11)$$

which corresponds to the contribution of the term with nonzero VEV ϕ_+ of the irrelevant operator with the scaling dimension $\Delta_+ = 4a^2$. Thus, the black string can be interpreted as a deformation triggered by a nonzero VEV of an irrelevant operator. The parameter M plays the role of a modulus controlling the scale of the condensate, i.e.,

$$\langle \mathcal{O} \rangle \propto \frac{1}{\sqrt{M}}. \quad (3.12)$$

Near Minkowski space with $w \rightarrow +\infty$, the scalar field has an expansion

$$\phi(w \rightarrow +\infty) = \frac{\pi}{2} \mp \sqrt{M} e^{-4a^2 w} + \dots, \quad (3.13)$$

which coincides with (A22) for $c_1 = 0, c_2 = \mp\sqrt{M}$ and $a^2 = 1$.

B. Flows with $f = 1$ through dynamical systems

In this section, following the works [34,44], we derive autonomous dynamical systems from the gravity equations of motion for the case $f = 1$. We consider dynamical systems with phase spaces $\mathbf{R}^2$ and $\mathbf{D}^2$. The latter will clarify the dynamics near extrema points corresponding to Minkowski spacetimes. First, we construct numerical trajectories for ϕ . Then, we place the system in the disk $\mathbf{D}^2$ to properly explore the asymptotic dynamics.

1. Dynamical system on the plane $\mathbf{R}^2$

To come to an autonomous dynamical system, it is convenient to use the scalar variable X [58],

$$X = \frac{\dot{\phi}}{A}, \quad (3.14)$$

which was also used for the case of the hyperbolic target space of the σ model [35,44,45]. Then we can represent the equations of motion as the autonomous system

$$\begin{aligned} \frac{d\phi}{dA} &= X, \\ \frac{dX}{dA} &= \left(\frac{X^2}{a^2} - 2 \right) \left(X + \frac{a^2 V'}{2V} \right), \end{aligned} \quad (3.15)$$

where the scalar field ϕ is defined on the interval $[-\pi, \pi]$.

The system (3.15) has equilibrium points with coordinates $(\phi, X) = (0, 0)$ and $(\phi, X) = \left(\pm \arccos\left(\sqrt{\frac{a^2}{1+2a^2}}\right), 0 \right)$, which are correspondingly related to AdS and dS vacua. It is important to note that the Minkowski vacua with $\phi = \pm\frac{\pi}{2}$, despite being the extrema of the potential, are not critical points of the system (3.15), since X is divergent in this case. Note that all equilibrium points of the system (3.15) are only saddles, so the phase portrait consists of stable/unstable manifolds. The manifolds act as separatrices.

The phase portrait of the system (3.15) is depicted in Fig. 4. Here and in the following, by using red and blue points, we explicitly label the AdS and dS points correspondingly. By purple points M we formally label the Minkowski space, but these points exist on the phase portrait implicitly because they coincide with the points where $V = 0$, where the vector field is singular. In Fig. 4,

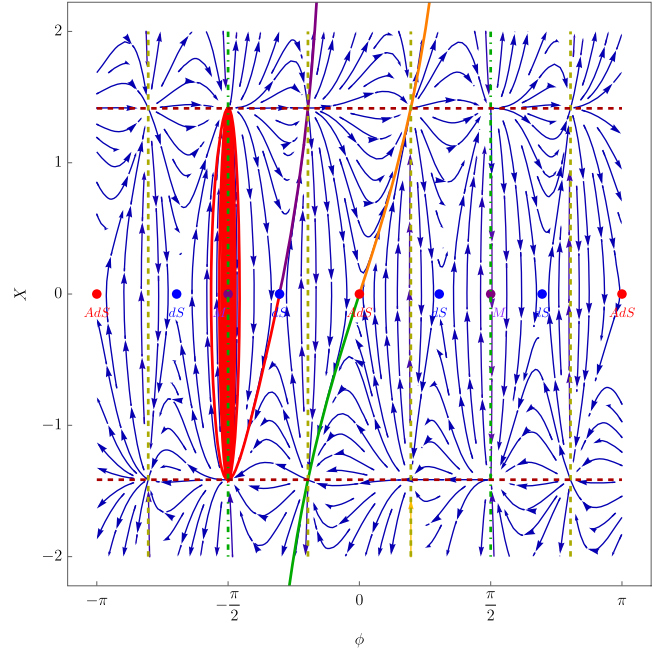


FIG. 4. Phase portrait and numerical trajectories on the period $\phi \in (-\pi, \pi)$. We set $a^2 = 1$. Orange and green curves from the AdS point are two branches of the supersymmetric explicit solution (3.1). Red and purple curves are the numerical solutions of the system (3.15) with initial conditions $(\phi, X) = (-0.955, \pm 0.025)$.

four yellow and two green dashed vertical lines correspond to the zeros of the potential, and horizontal lines are related to the equality $X^2 = 2a^2$, compensating for the divergence in the term V_ϕ/V . Hence, at the intersections of horizontal with vertical lines, the vector field is regular. As previously in [34], these intersection points are similar to the phase intersection points of an ordinary pendulum.

In Fig. 4, from the AdS equilibrium point, we observe flows, which correspond to branches of the exact supersymmetric flow (3.1). We show the branches of the exact flow between the AdS equilibrium and the Minkowski equilibrium by orange and green curves in Fig. 4. The flows are also presented in Fig. 3 corresponding to the first-order system (2.27). In Fig. 4, the branches of the flow, starting from the AdS saddle point, run to $X \rightarrow \pm\infty$, getting closer to singular lines, which are shown by red and represent Minkowski points. From the exact solution, we know that this flow should end in the Minkowski point, but from Fig. 4 corresponding to the system (3.15), we cannot see it precisely.

Linearizing the dynamical system (3.15), we can find the eigenvalues and eigenvectors and then write down an asymptotic solution. The general solution near the AdS equilibrium point at $\phi = 0$ can be presented in the following form:

$$\begin{bmatrix} \phi \\ X \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + k_1 e^{-\Delta_- A} u_1 + k_2 e^{-\Delta_+ A} u_2, \quad (3.16)$$

where the scaling dimensions $\Delta_{\pm}$ (2.32) coincide with the eigenvalues $\lambda_{1,2}$ taken with opposite signs, $\Delta_{\pm} = -\lambda_{1,2}$, the eigenvalues are $\lambda_{1,2} = -1 \pm (1 + 2a^2)$, and the eigenvectors read

$$u_1 = \begin{bmatrix} 1 \\ \lambda_1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 1 \\ \lambda_2 \end{bmatrix}. \quad (3.17)$$

Let us recall from [34] that setting k_1 or k_2 to zero in (3.16) can be associated with imposing the Dirichlet or Neumann boundary conditions on the dilaton, correspondingly. Here, as we see, $\Delta_- < 0$, which makes $e^{-\Delta_- A}$ divergent and (3.16) invalid for large A . It means that we cannot impose the Neumann BC in this case, i.e., we should always keep $k_1 = 0$, imposing the Dirichlet BC instead. Hence, on the quantum theory side, we can interpret the phase flows only as they are triggered by nonzero VEV of some irrelevant operator.

From dS equilibrium points, which are saddles, we see two numerical flows in Fig. 4 shown by the purple and the red solid curves. The first one tends to the singular line, crosses the red dashed line, and goes to $X \rightarrow \infty$. Again, in this region, we also cannot determine its asymptotic behavior due to an unsuitable coordinate system. We will find its asymptotic behavior considering the dynamical system in the disk $\mathbf{D}^2$ below. However, we can find an asymptotic behavior near the dS critical point using Lyapunov analysis. Thus, near the dS point we have the following general solution:

$$\begin{bmatrix} \phi \\ X \end{bmatrix} = \begin{bmatrix} \pm \arccos\left(\sqrt{\frac{a^2}{1+2a^2}}\right) \\ 0 \end{bmatrix} + k_1 e^{-\Delta_- A} u_1 + k_2 e^{-\Delta_+ A} u_2, \quad (3.18)$$

where $\Delta_{\pm} = -\lambda_{1,2}$ with the eigenvalues given by $\lambda_{1,2} = -1 \pm \sqrt{9 + 8a^2}$ and the eigenvectors are

$$u_1 = \begin{bmatrix} 1 \\ \lambda_1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 1 \\ \lambda_2 \end{bmatrix}. \quad (3.19)$$

The second flow from the dS point is shown by the red solid curve in Fig. 4. This curve starts from the dS point and tends to the singular line corresponding to the null of the potential coinciding with the Minkowski extremum. As we see in Fig. 4, the red curve is going in converging ellipses around the Minkowski point M . The size of the major axis is constant, while the minor axis gradually decreases to a very small size. We can write down this solution analytically. For that, we expand the function V_{ϕ}/V in the (3.15) system near the point $\pi/2$,

$$\begin{aligned} \frac{d\phi}{dA} &= X, \\ \frac{dX}{dA} &= \left(\frac{X^2}{a^2} - 2\right) \left(X + \frac{a^2}{\phi - \frac{\pi}{2}} - \frac{3 + 8a^2}{6} \left(\phi - \frac{\pi}{2}\right)\right). \end{aligned} \quad (3.20)$$

Leaving the leading term in the system (3.20) we are brought to

$$\begin{aligned} \frac{d\phi}{dA} &= X, \\ \frac{dX}{dA} &= \left(\frac{X^2}{a^2} - 2\right) \frac{a^2}{\phi - \frac{\pi}{2}}. \end{aligned} \quad (3.21)$$

The system (3.21) can be represented as a single equation for the scalar field

$$\left(\phi(A) - \frac{\pi}{2}\right) \phi''(A) - (\phi'(A))^2 + 2 = 0, \quad (3.22)$$

where we define $\phi'(A) := d\phi/dA$ and set $a^2 = 1$. Assuming the following initial conditions

$$\phi(0) = \frac{\pi}{2} + \psi, \quad \phi'(0) = v, \quad (3.23)$$

where $\psi^2 \simeq 0$ and $-\sqrt{2} < v < \sqrt{2}$, we get the solution near the Minkowski point in the following form:

$$\begin{aligned} \phi(A) &= \frac{\pi}{2} + \frac{\psi}{\sqrt{2-v^2}} \\ &\times \left(\sqrt{2-v^2} \cos \frac{A\sqrt{2-v^2}}{\psi} + v \sin \frac{A\sqrt{2-v^2}}{\psi} \right). \end{aligned} \quad (3.24)$$

The solution (3.24) is in agreement with the numerical solution of the system (3.15) under the initial conditions (3.23). This solution does not reach the Minkowski fixed point; however, it is worth noting that Eq. (3.22) is defined for the restricted region of ϕ . Solutions to (3.24) with different initial conditions form a family of nested ellipses with mutual vertices $(\phi, X) = (\frac{\pi}{2}, \pm\sqrt{2})$ and with Minkowski point M in their center. As we take into account the next nonlinear terms in (3.20), the solution to such a differential equation will be represented by the spiral curve covering these nested ellipses and eventually terminating at $\phi = \phi_M$. It is worth noting that an oscillatory approach to a Minkowski phase, originating from a de Sitter-like stage, was recently observed in [59]. In that work, the dilaton exhibits damped oscillations as it settles into the minimum of the effective potential after a regular bounce, leading to a final dust-dominated cosmology.

2. Dynamical system in the disk $\mathbf{D}^2$

As it was mentioned above, the phase space of the dynamical system (3.15) does not capture the behavior of the phase trajectories at infinity. However, we can improve our understanding of the dynamics near these regions if we map the system (3.15) into the disk $\mathbf{D}^2$.

To perform this, we introduce a new compact coordinate Z given by

$$Z = 2 \tan\left(\frac{\phi}{2}\right). \quad (3.25)$$

By owing (3.25), we translate all periodic critical points to the finite set.

In general, to obtain a polynomial function of Z for the term V_ϕ/V , a suitable change can also be $Z = n \tan(\frac{\phi}{k})$, where $n, k \in \mathbb{N}$, and here we have chosen the most convenient case, $n = k = 2$, which holds the structure of the critical points on the phase portrait. One can also choose $k = 1$, but in this case the points $\phi = \pm \frac{\pi}{2}$, which are related

to Minkowski space, correspond to $Z = \pm\infty$ and are not visible on the phase portrait.

Next, we perform the Poincaré transformation by introducing the following change of coordinates $(Z, X) \in \mathbf{R}^2 \rightarrow (z, x) \in \mathbf{D}^2$:

$$Z = \frac{z}{\sqrt{1-x^2-z^2}}, \quad X = \frac{x}{\sqrt{1-x^2-z^2}}, \quad (3.26)$$

with the constraint

$$z^2 + x^2 \leq 1. \quad (3.27)$$

In the coordinates (z, x) the autonomous dynamical (3.15) system takes the following form:

$$\begin{aligned} \dot{z} &= p(z, x), \\ \dot{x} &= q(z, x), \end{aligned} \quad (3.28)$$

where the rhs's of equations are

$$\begin{aligned} p(z, x) &= \frac{x}{4}(z^2 - 1)(4x^2 + 3z^2 - 4) - xz(3x^2 + 2z^2 - 2) \\ &\quad \cdot \left(x + \frac{16z(1-x^2-z^2)(48(x^2-1)z^2 + 16(x^2-1)^2 + 33z^4)}{588(x^2-1)z^4 + 368(x^2-1)^2z^2 + 64(x^2-1)^3 + 285z^6} \right), \\ q(z, x) &= \frac{x^2(x^2-1)(z-4x)}{4} + (1-x^2-z^2) \left(\frac{z}{380}(1408-1693x^2) + (x^2-1)x \right. \\ &\quad \left. - \frac{4z(7x^4-9x^2+2)}{5(4x^2+5z^2-4)} - \frac{4z(x^2-1)(524x^4+x^2(591z^2-820)-306z^2+296)}{19(72(x^2-1)z^2+16(x^2-1)^2+57z^4)} \right). \end{aligned} \quad (3.29)$$

The critical points and sets of the system in the disk $\mathbf{D}^2$ (3.28) with (3.29) include

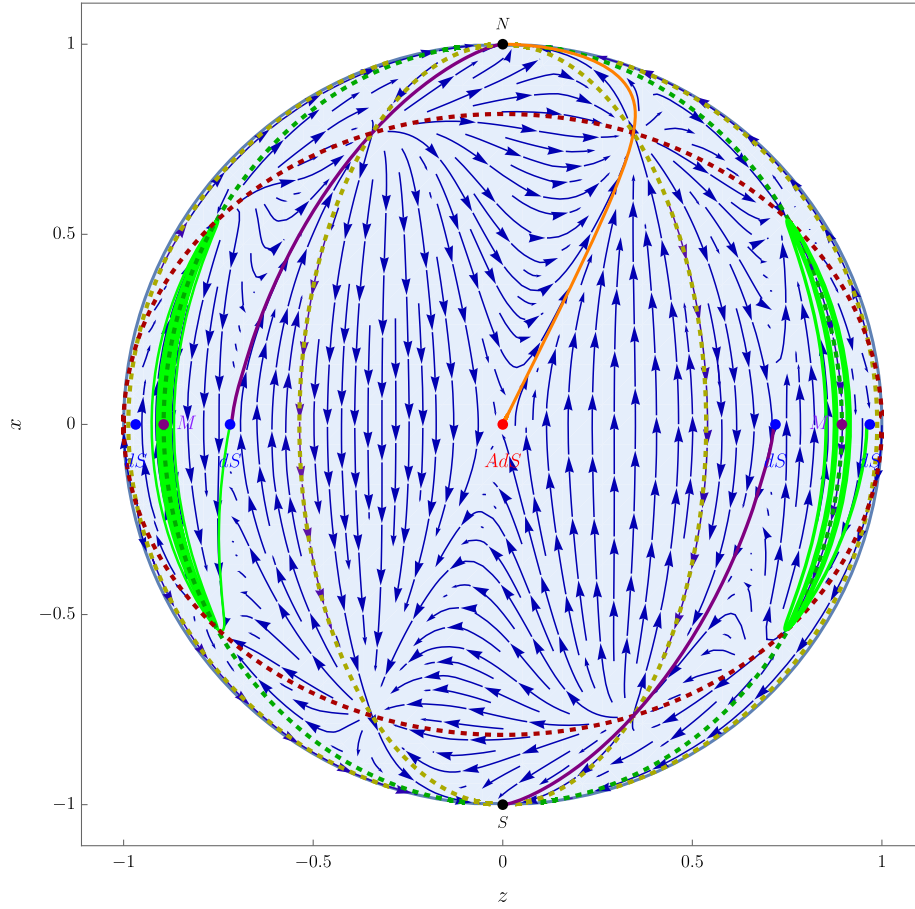
- (1) AdS, $(0, 0)$; dS, $(z_{\text{dS}}, 0)$; N , $(0, 1)$; S , $(0, -1)$;
- (2) two curves corresponding to zeros of the potential V , which are related to Minkowski extrema (shown by dashed green in Fig. 5); and
- (3) four curves corresponding to the four zeros of the potential except for zeros related to Minkowski extrema (shown by dashed dark yellow in Fig. 5).

Note that AdS and dS critical points are saddles, while N and S are stable nodes. The phase portrait of the system in the disk (3.28) with (3.29) is depicted in Fig. 5. Here we show the AdS point by red, four dS points by blue, two Minkowski M points by purple, and points N and S corresponding to the poles of the disk $\mathbf{D}^2$ by black. Note that all Minkowski points correspond to each point of the singular green dashed curves, which, as the yellow dashed curves, are related to $V = 0$. Here the latitudinal dashed red curves correspond to the lines $X^2 = 2a^2$ of the system (3.15) on the plane $\mathbf{R}^2$. The phase flows on the meridional curves are singular; however, on the intersections with the latitudinal dashed curves, they are

regular. From Fig. 5, we see that all flows pass these intersection points.

In Fig. 5, we observe the following trajectories:

- (i) The flow, which corresponds to the exact solution (3.1), starts in the AdS point and ends at the north pole N of the disk, which corresponds to Minkowski space. In Fig. 5, we show this flow by an orange curve. As mentioned above, this flow with the Dirichlet BC is triggered by a nonzero VEV of an irrelevant operator with the scaling dimension $\Delta_+ = 2 + 2a^2$. There is also a mirror flow from the AdS equilibrium point to the south pole S , corresponding to another branch of the solution (3.1). Note that these exotic RG flows differ by the sign of the source.
- (ii) Two numerical green flows begin in the dS points and then go by spirals down to the points M corresponding to Minkowski space. The scaling dimension of the irrelevant operator is $\Delta_+ = 1 + \sqrt{9 + 8a^2}$. This flow previously appeared in Fig. 4 (in red).
- (iii) The numerical purple flow, which also starts from the dS point, has already been observed in Fig. 4;


 FIG. 5. Global phase portrait for the system (3.28) with (3.29) in the disk $\mathbb{D}^2$.

now it can be seen that it approaches Minkowski space in the same manner as the solution from the AdS fixed point to the Minkowski one (the orange curve). The flow can be constructed for any dS point of the potential and can end either in the north pole N or in the south pole S . This flow is related to an irrelevant operator with the same scaling dimension Δ_+ as for the previous flow drawn in green.

IV. WARMING UP: FLOWS WITH HORIZONS

A. Flows with $f \neq 1$ through 3D dynamical system

In this section, we will consider the case when we also have solutions with the metric (2.14) such that $f \neq 1$. This case includes an asymptotically AdS black hole (BTZ), a dS black hole, and also a dS solution in the Poincaré coordinates. To describe such solutions, we introduce a new variable Y related to f ,

$$Y = \frac{\dot{f}}{f\dot{A}}, \quad (4.1)$$

such that $Y \rightarrow \infty$ as $f \rightarrow 0$. Then EOM (2.20)–(2.23) are brought to the form of the autonomous 3D dynamical

system defined on $\mathbf{R}^3$,

$$\begin{aligned} \frac{d\phi}{dA} &= X, \\ \frac{dX}{dA} &= \left(\frac{X^2}{a^2} - Y - 2 \right) \left(X + \frac{a^2 V_\phi}{2V} \right), \\ \frac{dY}{dA} &= Y \left(\frac{X^2}{a^2} - Y - 2 \right). \end{aligned} \quad (4.2)$$

In (4.2), for any fixed $\phi = \text{const}$, we have a critical curve given by

$$\frac{X^2}{a^2} - Y - 2 = 0, \quad (4.3)$$

similar to that which appears for the autonomous dynamical system with the hyperbolic potential [35].

Since $Y \rightarrow \infty$, it is convenient to come to a compact configuration space as it was considered for the case of the hyperbolic scalar potential in [35, 45]. In this section, first, we will project the dynamical system (4.2) into the cylinder and, then, into the ball. We will explore the space of solutions numerically and construct analytical solutions near the horizon.

1. Dynamical system in the cylinder

To study the flows, we apply the Poincaré transformation to map the dynamical system (4.2) into a unit cylinder as in [35]. We keep the coordinate ϕ to be periodic, and the other two coordinates are transformed as

$$X = \frac{x}{\sqrt{1-x^2-y^2}}, \quad Y = \frac{y}{\sqrt{1-x^2-y^2}}, \quad (4.4)$$

such that the new coordinates x and y obey the constraint

$$x^2 + y^2 \leq 1. \quad (4.5)$$

In these coordinates, the system (4.2) takes the following form:

$$\begin{aligned} \dot{\phi} &= \mathbf{m}(x, y), \\ \dot{x} &= \mathbf{p}(\phi, x, y), \\ \dot{y} &= \mathbf{q}(\phi, x, y), \end{aligned} \quad (4.6)$$

where the rhs's of Eqs. (4.6) are given by

$$\mathbf{m}(x, y) = x\sqrt{1-x^2-y^2}; \quad (4.7)$$

$$\begin{aligned} \mathbf{p}(\phi, x, y) &= 2a^2(x^2 - 1) \tan \phi \left(\frac{(x^2 + y^2 - 1)(2\sqrt{1-x^2-y^2} + y)}{(2a^2 + 1) \cos 2\phi - 2a^2 + 1} + \frac{x^2 \sqrt{1-x^2-y^2}((2a^2 + 1) \cos 2\phi + 1)}{(2a^2 + 1) \cos 2\phi - 2a^2 + 1} \right) \\ &\quad + \frac{x(x^2 + y^2 - 1)(a^2(y\sqrt{1-x^2-y^2} + 2(1-x^2-y^2)) - x^2)}{a^2}; \end{aligned} \quad (4.8)$$

$$\begin{aligned} \mathbf{q}(\phi, x, y) &= y \left(2x \tan \phi \left(\frac{a^2(x^2 + y^2 - 1)(2\sqrt{1-x^2-y^2} + y)}{(2a^2 + 1) \cos 2\phi - 2a^2 + 1} + \frac{x^2 \sqrt{1-x^2-y^2}((2a^2 + 1) \cos 2\phi + 1)}{(2a^2 + 1) \cos 2\phi - 2a^2 + 1} \right) \right. \\ &\quad \left. + \frac{(x^2 + y^2 - 1)(a^2(y\sqrt{1-x^2-y^2} + 2(1-x^2-y^2)) - x^2)}{a^2} \right); \end{aligned} \quad (4.9)$$

and we redefine the derivative as follows:

$$\chi' = (1 - x^2 - y^2)d\chi/dA. \quad (4.10)$$

This redefinition of the derivative is admissible (see [60]), since it does not change the behavior of flows and allows us to avoid divergencies that arise due to the constraint (4.5).

The phase portrait of the dynamical system (4.6) with (4.7)–(4.9) in period $\phi \in [-\pi, \pi]$ is presented in Fig. 6. The sets of the equilibrium points of (4.6) related to $f = 0$ and $f = 1$, correspondingly, are

- (1) AdS, $(\phi_{\text{AdS}}, 0, 0)$; dS, $(\phi_{\text{dS}}, 0, 0)$; and
- (2) AdS_h, $(\phi_{\text{AdS}}, 0, 1)$; dS_h, $(\phi_{\text{dS}}, 0, 1)$.

The classification of the equilibrium points of (4.6) is presented in Table I. The ϕx plane in Fig. 6 corresponds to the zero-temperature case, i.e., $f = 1$, and is an invariant manifold of the system (4.6). The curve (4.3) in terms of the coordinates of the cylinder (4.4) is shown by the white dashed closed curve.

Assuming certain initial conditions, we construct trajectories of (4.6)–(4.9) for $\phi \in (-\frac{\pi}{2}, 0]$, which we can see in Fig. 6, namely:

- (1) The flows that can be found exactly:

- (a) BTZ black hole with $f \neq 1$, which starts from the horizon h_1 to the equilibrium point, corresponding to the AdS point at $\phi = 0$ as in [35]. This solution is shown by the cyan straight line.
- (b) The Schwarzschild–de Sitter black hole with $f \neq 1$, which flows from the horizon dSh_1 to the dS₁ point. This flow is shown by the green straight line.
- (c) The half-supersymmetric exact flow between the AdS and Minkowski fixed points (3.1) with $f = 1$, shown by the orange solid curve. This trajectory also appears in Fig. 5 (shown by orange).
- (2) The flows that are constructed numerically:
 - (a) Solutions that begin near the BTZ horizon h_1 , pass close the AdS equilibrium point, and then go to infinity. These solutions are shown by cyan curves in Fig. 6. The initial conditions for such flows are $(\phi_0, x_0, y_0) = (0 + \delta, 0, 1 - \epsilon)$. Here $0 < \epsilon \ll 1$ and $\delta \in (-|\phi_{n1}|, \phi_{n2})$, where ϕ_{n1}/ϕ_{n2} corresponds to the null of the potential between AdS and dS₁/dS₂ points.
 - (b) Solutions that start near the SdS horizon dSh_i , pass near the dS point, and spiral down to

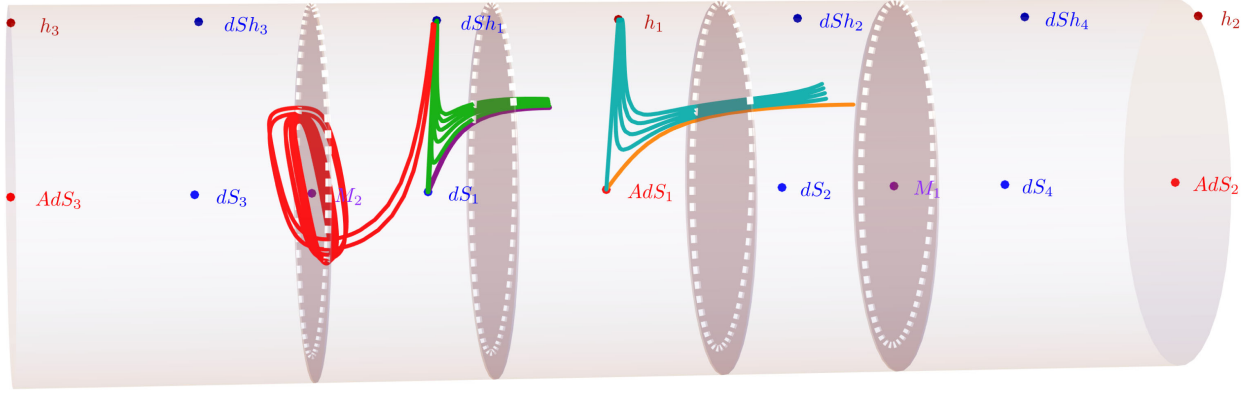


FIG. 6. The trajectories of the dynamical system (4.6), for which the phase space is a unit cylinder. By the orange curve we show the exact solution from the AdS_3 point to the Minkowski one (3.1) with $f = 1$, 1(a). By straight cyan and green lines we show BTZ and SdS black holes, correspondingly, 1(b) and 1(c). A family of solutions 2(a) is shown by cyan curves. The solutions 2(b) and 2(c) are shown by red and green curves, correspondingly. The solution 2(d) from the dS point to infinity is shown by purple. The singular surfaces are shown by gray, on which the white dashed curves are regular.

Minkowski space. These flows are shown by red curves in Fig. 6. The initial conditions for such flows are $(\phi_0, x_0, y_0) = (\phi_{dS_1} - \delta, 0, 1 - \epsilon)$. Here $0 < \epsilon \ll 1$ and $\delta \in (0, \frac{\pi}{2} - |\phi_{dS_1}|)$.

- (c) Solutions that start from SdS horizon dSh_i , pass near the dS point, and go to infinity. We show these solutions by green curves in Fig. 6. The initial conditions for such flows are $(\phi_0, x_0, y_0) = (\phi_{dS_1} + \delta, 0, 1 - \epsilon)$. Here $0 < \epsilon \ll 1$ and $\delta \in (0, |\phi_{n2}|)$, where ϕ_{n2} corresponds to the null of the potential between AdS and dS_1 points.
- (d) A flow that begins from the dS point and behaves similar to the exact solution (3.1) with $f = 1$. We show this flow by the purple curve in Fig. 6. This trajectory also appears in Fig. 5 (in purple).

We see that solutions 2(a)–2(d) cross the singular surface through the dashed white curves.

From Fig. 6, we observe that the BTZ and SdS black holes are the only black holes with AdS/dS asymptotics for the system (4.6). In that sense, we cannot construct a family of asymptotically AdS/dS black hole solutions that are characterized by some value of the scalar field on the horizon as it was done for the hyperbolic potential [35]. Instead, a small variation of the scalar field ϕ leads to solutions that begin near-horizon regions of BTZ/SdS black holes, then pass near AdS/dS critical points, and then go to infinity. For the AdS case, this can be interpreted as that for exotic RG flows driven by the VEV of irrelevant operators, and there is only a thermal state, which is described by the BTZ black hole.

Below, we will also discuss the system with $f \neq 1$ projected into $\mathbb{B}^3$.

2. Dynamical system in the ball

In this subsection, to clarify the behavior of the flows that we discussed above, we will map the 3D dynamical

system (4.2) into the ball. For this, we use a compact variable Z (3.25) instead of the periodic ϕ in (4.2), i.e., we get the following system:

$$\begin{aligned} \frac{dZ}{dA} &= X \left(1 + \frac{Z^2}{4} \right), \\ \frac{dX}{dA} &= \left(\frac{X^2}{a^2} - Y - 2 \right) \left(X + \frac{a^2 V_\phi}{2} \right), \\ \frac{dY}{dA} &= Y \left(\frac{X^2}{a^2} - Y - 2 \right). \end{aligned} \quad (4.11)$$

Then we apply the Poincaré projection to map the dynamical system (4.11) defined in $\mathbb{R}^3$ into the ball $\mathbb{B}^3$, namely,

$$\begin{aligned} Z &= \frac{z}{\sqrt{1 - z^2 - x^2 - y^2}}, & X &= \frac{x}{\sqrt{1 - z^2 - x^2 - y^2}}, \\ Y &= \frac{y}{\sqrt{1 - z^2 - x^2 - y^2}}, \end{aligned} \quad (4.12)$$

where $(z, x, y) \in \mathbb{B}^3$ and are also related by the constraint

$$x^2 + y^2 + z^2 \leq 1. \quad (4.13)$$

The coordinate transformation (4.12) allows us to map the infinite points of the system (4.2), which, as we have seen, are difficult to be caught studying the dynamical system in the unit cylinder, into the ball. Then the dynamical system in the ball takes the form

$$\begin{aligned} \dot{z} &= m(z, x, y), \\ \dot{x} &= p(z, x, y), \\ \dot{y} &= q(z, x, y), \end{aligned} \quad (4.14)$$

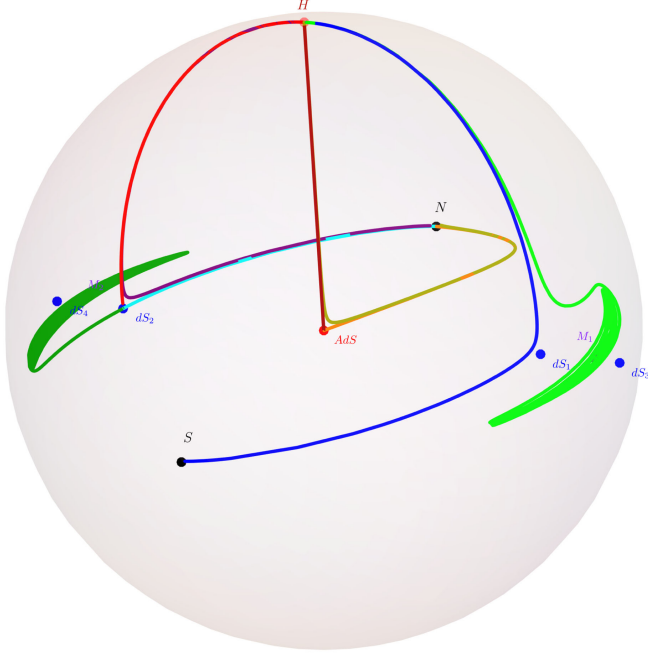


FIG. 7. The numerical flows of the system (4.2), for which the phase space is a 3D ball. The straight red line from H to the AdS fixed point is the BTZ solution. The solid red curve from H to the dS_2 point describes the Schwarzschild–de Sitter solution. The dark yellow curve from H to N corresponds to a singular deformation of the BTZ solution. The blue and purple curves from H to N and S , correspondingly, are related to deformations of the SdS solution. The green curve from H , which spirals down to Minkowski space, also corresponds to a deformation of the SdS solution. The orange curve from AdS to N is the exact zero-temperature flow. The solution from N (the Minkowski point) to the dS point is shown by cyan. The dark green curve corresponds to a zero-temperature flow between dS and Minkowski fixed points.

where the rhs of (4.14) can be found from (4.11) applying (4.12).

Under the coordinate transformation (4.12), the points of the system on $\mathbb{R}^3$ with the coordinate $Y \rightarrow +\infty$ are mapped to the point $(z, x, y) \rightarrow (0, 0, 1)$ in the ball $\mathbb{B}^3$, regardless of the values of Z and X if they are fixed. That is, all the points in the cylinder with coordinates $(\phi, 0, 1)$ for any ϕ in Fig. 6 are mapped into the point $(z, x, y) = (0, 0, 1)$ in $\mathbb{B}^3$. We denote this point by H in Fig. 7 as corresponding to horizons of both BTZ and Schwarzschild–de Sitter solutions. The types of critical points are considered in Table II.

TABLE II. Classification of critical points of the dynamical system (4.14) for $a^2 = 1$.

Point	Type	Eigenvalues $(-\Delta_z, -\Delta_x, -\Delta_y)$
H	Unstable node	$(1, 0, 2)$
N, S	Stable node	$(-2, -1, 0)$
AdS	Saddle	$(-4, 2, -2)$
dS	Saddle	$(-1 - \sqrt{17}, -1 + \sqrt{17}, -2)$

We show the phase portrait of the system in the unit ball (4.14) in Fig. 7. The center of the ball $(0, 0, 0)$ corresponds to the AdS solution at zero temperature. The dS points are inside the sphere and have coordinates $(z_{dS}, 0, 0)$. The Minkowski points with $\phi = \pm\pi/2$ are mapped to the critical curve, which includes the north and the south poles, N and S , with the coordinates $(0, \pm 1, 0)$ and points M . All trajectories corresponding to the finite-temperature solutions start from the horizon H with $(0, 0, 1)$.

We see the following flows to (4.14) in Fig. 7 are:

- (1) flows that can be found exactly:
 - (a) the exact nonrotating BTZ solution, shown by the straight dark red line going from point H to point AdS;
 - (b) the exact SdS solution shown by the red curve from H to the dS_2 point;
 - (c) the exact supersymmetric flow with $f = 1$ shown by the orange curve from point AdS to point N ;
- (2) flows, which can be constructed numerically:
 - (a) the solution, which flows from H , passes the AdS point and ends in the north pole N . We show this by the solid dark yellow curve. To construct the solution, we take the initial conditions as $(z_0, x_0, y_0) = (0, \delta, 1 - \epsilon)$, where $\delta, \epsilon > 0$ and $\delta, \epsilon \ll 1$. This solution appears as a deformation of the BTZ black hole, introducing a nonzero finite value of ϕ on the horizon;
 - (b) the flows from H , passing near dS and ending in N/S points, respectively. The flows are shown by purple and blue curves in Fig. 7. To find them we use the following initial conditions: $(z_0, x_0, y_0) = (\pm\epsilon, \pm\delta, 1 - \epsilon)$, where $\delta, \epsilon, \epsilon > 0$ and $\delta, \epsilon, \epsilon \ll 1$. These flows correspond to deformations of the SdS solution such that on the horizon we have $\phi(w_h) = \phi_h$;

TABLE I. Classification of the critical points of the dynamical system (4.6) (we set $a^2 = 1$). Here $\phi_0 \in [-\pi, \pi]$ and we suppose that $(\phi_0, 0, 1)$ represents the set of points on the cylinder axis with $y = 1$.

Point set	Type	Eigenvalues $(-\Delta_\phi, -\Delta_x, -\Delta_y)$
$(\phi_0, 0, 1)$	Unstable degenerate node	$(0, 0, 2)$
$(\phi_M, 1, 0)$	Stable degenerate node	$(-2, 0, 0)$
$(\phi_{AdS}, 0, 0)$	Saddle	$(-4, 2, -2)$
$(\phi_{dS}, 0, 0)$	Saddle	$(-1 - \sqrt{17}, -1 + \sqrt{17}, -2)$

TABLE III. Types of solutions depending on the constants in schematic Lyapunov solution (4.16).

Type	Solution	Zero constants	Nonzero constants
1(a)	BTZ	$c_{1z}, c_{2z}, c_{1x}, c_{2x}$	c_y
1(b)	SdS	c_{2z}, c_{1x}, c_{2x}	c_{1z}, c_y
2(a)	Deformed BTZ	c_{1z}, c_{2z}	c_{1x}, c_{2x}, c_y
2(b), 2(c)	Deformed SdS	$\dots$	$c_{1z}, c_{2z}, c_{1x}, c_{2x}, c_y$

(c) the trajectory shown by green corresponds to a deformation of the SdS solution and flows from H , passing near the dS_1 point, then spirals down to the Minkowski point M . The initial conditions for this flow read as $(z_0, x_0, y_0) = (\pm\epsilon, \mp\delta, 1 - \epsilon)$, where $\delta, \epsilon, \epsilon > 0$ and $\delta, \epsilon, \epsilon \ll 1$; and

(d) the flows with $f = 1$, previously discussed in Sec. III B, see Fig. 5, namely, the flow from point dS_2 to point N by cyan and the flow from dS_2 to Minkowski by dark green.

The trajectories of the dynamical system (4.14) in Fig. 7 can clarify the behavior of the flows from Fig. 6. Namely, we see that the flows 2(a) and 2(b), which correspond to deformations of the BTZ and SdS solutions, start from H , which is an unstable node, and end in the north pole N , which is a stable node, see Table II. We see that these deformations end in Minkowski space.

It is instructive to do the Lyapunov analysis of the dynamical system (4.14). The eigenvectors at the critical point H , which is an unstable node, are given by

$$u_z = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad u_x = \begin{bmatrix} 1 \\ \frac{1}{4} \\ 0 \end{bmatrix}, \quad u_y = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \quad (4.15)$$

Because the one eigenvalue is equal to zero, the solution near the point H is not linear. Though, keeping in mind the form of eigenvectors (4.15), we can schematically write down the solution near the point H ,

$$\begin{aligned} \begin{bmatrix} z \\ x \\ y - 1 \end{bmatrix} &= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \left(c_{1z} + c_{2z}N(A) \right) e^{-\Delta_z A} \\ &+ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \left(c_{1x} + c_{2x}N(A) \right) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} c_y e^{-\Delta_y A}, \end{aligned} \quad (4.16)$$

where $c_{1z}, c_{2z}, c_{1x}, c_{2x}$, and c_y are nonzero constants characterizing the solutions. Here, by $N(A)$ we denote the nonlinear and logarithmic terms, which contribute to the solution of the scalar field ϕ . We see the correspondence between constants and solutions in Table III.

In the next section, we will show that the scalar field near the horizon of the singular solutions 2(a)–2(c) contains the logarithmic divergences; such behavior of the scalar field allows us to bring the flows to Minkowski space.

B. Analytical solutions

1. Exact solutions with horizons

First, we recall how to derive dS_3 , AdS_3 , BTZ, and Schwarzschild–de Sitter solutions from the supergravity equations. All of them correspond to extrema of the potential. Assuming that a solution is defined for $V'(\phi) = 0$, the equation for the scale factor is easily integrated,

$$A = C_1 w + C_2, \quad C_1, C_2 = \text{const}, \quad (4.17)$$

where C_2 can be set to zero. A nonzero C_1 corresponds to AdS and dS solutions, while $C_1 = 0$ is valid for the Minkowski solution. Then, we plug (4.17) into (2.21), which yields

$$f = -\frac{2V}{C_1^2} + \bar{c} e^{-2C_1 w}, \quad \bar{c} = \text{const}. \quad (4.18)$$

From the latter, one can see that the dS_3 metric (2.19) corresponds to the scale factor (4.18) with $C_1 = \sqrt{2V}$, $V > 0$, and $\bar{c} = 0$. Note that, in terms of the coordinates of the dynamical system on $\mathbb{R}^3$ (4.2), the dS_3 equilibrium point is given by $X = 0, Y = 0, \phi = \phi_{dS}$. If $V < 0$, $C_1 = \sqrt{-2V}$, and $\bar{c} = 0$, we get the AdS_3 space metric. Taking $\bar{c}$ to be nonzero, $\bar{c} \neq 0$, we get AdS and dS black hole solutions, namely, the blackening function for the BTZ/SdS₃ solutions reads

$$f(w - w_h) = \pm 1 \mp e^{-2\sqrt{\mp 2V}(w - w_h)}, \quad (4.19)$$

where we fixed $\bar{c} = \mp 1$ for the BTZ and SdS solutions, correspondingly. The scale factor of BTZ/SdS black holes is given by (4.17).

Also note that for both BTZ and SdS₃ solutions the coordinate Y has the same sign of infinity in the limit $w \rightarrow w_h$,

$$Y = \frac{\dot{f}}{f\dot{A}} \xrightarrow{w \rightarrow w_h} +\infty, \quad \dot{A} = C_1 = \text{const}, \quad (4.20)$$

hence we can see both BTZ and SdS₃ solutions on the same phase portrait in Fig. 7.

2. Analytical solutions near horizon

To find an analytical description of the numerically constructed solutions from the previous subsection near the horizon regions, we follow the method from [35,45]. In the near-horizon region, $Y \rightarrow +\infty$, the system (4.2) simplifies,

so the equation for Y decouples from the system and can be easily solved as

$$Y(A - A_h) = \frac{1}{A - A_h}, \quad (4.21)$$

where the integration constant was chosen to properly obtain the boundary condition $Y(A_h) = +\infty$. The other two equations of the system (4.2) are reduced to a single second-order equation for the scalar field

$$(A - A_h) \frac{d^2 \phi}{dA^2} + \frac{d\phi}{dA} + \frac{a^2 V'(\phi)}{2 V(\phi)} = 0, \quad (4.22)$$

where we also used (4.21).

We are looking for near-horizon geometries when the value of the scalar field at the horizon ϕ_h is not necessarily the value at the extremum of $V(\phi)$. We can further simplify Eq. (4.22) by expanding the last term in a Taylor series up to the first order near ϕ_h . As a result, we obtain the following equation:

$$\mathcal{A} \frac{d^2 \Phi}{d\mathcal{A}^2} + \frac{d\Phi}{d\mathcal{A}} + \Delta_h \Phi + \Lambda_h = 0, \quad (4.23)$$

where we use $\Phi = \phi - \phi_h$ and $\mathcal{A} = A - A_h$. In (4.23), we also denote the first two terms in the Taylor expansion of V'/V as

$$\Delta_h = \frac{a^2}{2} \left(\frac{V''(\phi_h)}{V(\phi_h)} - \left(\frac{V'(\phi_h)}{V(\phi_h)} \right)^2 \right), \quad \Lambda_h = \frac{a^2}{2} \frac{V'(\phi_h)}{V(\phi_h)}. \quad (4.24)$$

One can see that, for any ϕ_h and a^2 , the quantity Δ_h in (4.24) is negative,

$$\Delta_h = -4a^2 \frac{1 - 4a^4 + (1 + 2a^2) \cos 2\phi_h + 4a^4 \sec^2 \phi_h}{(1 - 2a^2 + (1 + 2a^2) \cos 2\phi_h)^2} < 0. \quad (4.25)$$

Then, taking into account (4.25), the general solution to (4.23) can be represented as follows:

$$\phi(A) = \phi_h - \frac{\Lambda_h}{\Delta_h} + c_1 I_0 \left(2\sqrt{|\Delta_h|(A - A_h)} \right) + c_2 K_0 \left(2\sqrt{|\Delta_h|(A - A_h)} \right), \quad (4.26)$$

where I_0 and K_0 are modified Bessel functions of the first and second kinds, respectively.

Imposing the condition on the horizon

$$\phi(A_h) = \phi_h + \delta, \quad (4.27)$$

with $\delta^2 \sim 0$ and expanding in series the solution (4.26) near A_h , we get

$$\phi = \phi_h + \frac{\Lambda_h}{|\Delta_h|} + c_1 (1 + |\Delta_h| \bar{A}) + c_2 (-2\gamma - \log \bar{A} + (2 - 2\gamma + \log \bar{A}) \bar{A}), \quad (4.28)$$

where $\bar{A} = A - A_h$. Setting the integration constant $c_2 = 0$, we neglect the contribution of K_0 to the solution (4.26), which is divergent on the horizon. In this case, we obtain only solutions 1(a) and 1(b) in Fig. 7. For them, the constant c_1 is given by

$$c_1 = \delta - \frac{\Lambda_h}{|\Delta_h|}. \quad (4.29)$$

To obtain the solutions 2(a)–2(c) in Fig. 7, we should also include K_0 and for the initial conditions impose $\phi'(A_h) = v$ together with (4.27). The integration constants in this case become

$$c_1 = \frac{\left(\delta - \frac{\Lambda_h}{|\Delta_h|} \right) (1 + (-3 + 2\gamma + \log(|\Delta_h|\epsilon)) |\Delta_h|\epsilon) - (2\gamma + \log(|\Delta_h|\epsilon)) \epsilon v}{1 - 2|\Delta_h|\epsilon(1 + \log(|\Delta_h|\epsilon))},$$

$$c_2 = -\frac{(\Lambda_h - |\Delta_h|\delta + v)\epsilon}{1 - 2|\Delta_h|\epsilon(1 + \log(|\Delta_h|\epsilon))}, \quad (4.30)$$

where we fix $\epsilon = (A - A_h)$ to be small and neglect terms of order ϵ^2 . Near the horizon, $\epsilon \rightarrow 0$, the constant c_1 resembles (4.29), and the constant c_2 goes to zero. However, the term with c_2 grows as we move away from the horizon, causing the solution to deviate from the regular BTZ/SdS geometry and become singular.

For the case $c_2 = 0$ and c_1 as in (4.29) we can easily write down an analytical solution near the horizon.

Having (4.28), the equation on the scale factor $A(w)$ (2.20) leads to the solution

$$A(w) = \Theta_h \ln \left(1 + C_A \Theta_h^{-1} w \right), \quad \Lambda'_h = \delta |\Delta_h| - \Lambda_h,$$

$$\Theta_h = \frac{a^2}{\Lambda_h^2}, \quad (4.31)$$

where the constant $C_A = \sqrt{|2V(\phi_h)|}$ is fixed such that in the extremum of the potential the solution (4.31) reproduces the BTZ/SdS black hole solutions near the horizon.

As in [35,45], we have for the blackening function $f(A) \approx C_f(A - A_h)$, so to find the constant C_f we plug (4.28) and (4.31) into (2.22), which yields

$$\dot{A}^2 \left(\frac{1}{A - A_h} + 2 - \Theta_h^{-1} \right) + \frac{4V(\phi_h)}{C_f(A - A_h)} = 0. \quad (4.32)$$

Neglecting the second and the third terms in parentheses in the latter equation, we get C_f ,

$$C_f = 2 \operatorname{sgn}(V(\phi_h)) \left(1 + C_A \Theta_h^{-1} w_h \right)^2, \quad (4.33)$$

such that near extrema, where $\Lambda'_h \rightarrow 0$, we have

$$C_f = 2 \operatorname{sgn}(V(\phi_{\text{AdS/dS}})). \quad (4.34)$$

We see that the sign of C_f depends on the value of $V(\phi_h)$, i.e., for the trajectories on the interval $\phi_h \in (-\frac{\pi}{2}, 0]$ in Fig. 6, it is negative for $\phi_h \in (\frac{1}{2} \arccos(\frac{1-2a^2}{1+2a^2}), 0]$ and positive for $\phi_h \in (-\frac{\pi}{2}, \frac{1}{2} \arccos(\frac{1-2a^2}{1+2a^2}))$.

Then the metric of the solution in the near-horizon region looks like

$$ds^2 = \left(1 + C_A \Theta_h^{-1} w \right)^{2\Theta_h} (-f(w) dt^2 + dx^2) + \frac{dw^2}{f(w)}, \quad (4.35)$$

with blackening function

$$f(w) = C_f \Theta_h \ln \left(\frac{1 + C_A \Theta_h^{-1} w}{1 + C_A \Theta_h^{-1} w_h} \right), \quad (4.36)$$

and the scalar field near the horizon is given by

$$\phi = \phi_h + \delta + \frac{a^2}{\Lambda'_h} \ln \left(\frac{1 + C_A \Theta_h^{-1} w}{1 + C_A \Theta_h^{-1} w_h} \right), \quad (4.37)$$

with the constants C_f , C_A , Λ'_h , and Θ_h defined above.

If the horizon is located in the extrema of the potential, such that $\Lambda'_h \rightarrow 0$, the solution (4.35) comes to the near-horizon geometry of the BTZ/SdS black hole. The metric then turns into the form

$$ds^2 \approx e^{2C_A w_h} (-f(w) dt^2 + dx^2) + \frac{dw^2}{f(w)}, \quad (4.38)$$

with

$$f(w) \approx C_f C_A (w - w_h), \quad (4.39)$$

where C_f is given by (4.34) and $C_A = \sqrt{|2V(\phi_{\text{AdS/dS}})|}$ for the AdS/dS case, i.e.,

$$\begin{aligned} C_f &= 2, & C_A &= \sqrt{-2V(\phi_{\text{AdS}})} & \text{for } V(\phi_{\text{AdS}}) < 0; \\ C_f &= -2, & C_A &= \sqrt{2V(\phi_{\text{dS}})} & \text{for } V(\phi_{\text{dS}}) > 0. \end{aligned} \quad (4.40)$$

In [35,45], a special class of the flows with a slowly changing scalar field $X^2 \sim 0$ was found that almost matches the flows without the constraint. The geometry of the flows is described by a nondeformed BTZ metric, and the solution to the scalar field is given by a hypergeometric function, which can be extended from the near-horizon region to the boundary.

Under the constraint $X^2 \sim 0$, the solution to Y is $Y(A - A_h) = \frac{2}{e^{2(A-A_h)} - 1}$. Then, substituting $Y(A)$ back into the system (4.2), we derive the equation for the scalar field,

$$r(1-r) \frac{d^2 \Phi(r)}{dr^2} + (1-2r) \frac{d\Phi(r)}{dr} + \frac{|\Delta_h|}{2} \Phi(r) = 0, \quad (4.41)$$

where $r = e^{2(A-A_h)}$ and $\Phi = \phi - \phi_h + \frac{\Lambda_h}{\Delta_h}$. The fundamental solution to this equation near $r = 1$ can be represented in the form

$$\Phi(r) = {}_2F_1(a_h, 1 - a_h, 1, 1 - r), \quad (4.42)$$

where $a_h = \frac{1}{2}(1 + \sqrt{1 + 2|\Delta_h|}) \geq 1$, which means this solution diverges for $r \rightarrow \infty$. We see that, unlike in [35], such a solution is not able to be analytically defined from horizon to the conformal boundary.

V. EXACT RG FLOW AND “ $T\bar{T}$ ” OPERATOR

Let us calculate the Brown-York stress-energy tensor of the exact holographic RG flow given by (3.1) with $f = 1$, $a^2 = 1$, and $m = 1$ assuming a cutoff surface Σ_c at $w = w_c$. For the metric (3.1), the unit normal to the hypersurface Σ_c is ∂_w with the components

$$n^\mu = (0, 0, 1), \quad (5.1)$$

then the induced metric reads

$$\gamma_{ab} dx^a dx^b = e^{2A(w_c)} (-dt^2 + dx^2). \quad (5.2)$$

Taking into account (5.2), the components of the extrinsic curvature are defined as

$$K_{ab} = \frac{1}{2} \mathcal{L}_n \gamma_{ab} = \frac{1}{2} \partial_w \gamma_{ab} \quad (5.3)$$

and we have

$$K_{tt} = -K_{xx} = -A'(w)e^{2A(w)}, \quad K = 2A'(w). \quad (5.4)$$

The Brown-York tensor with the counterterm is given by

$$T_{ab}^{BY} = \frac{1}{8\pi G} \left(K_{ab} - K\gamma_{ab} + \frac{1}{\ell} \gamma_{ab} \right), \quad (5.5)$$

where $\ell = \frac{1}{2}$ from (2.29). The nonzero components of (5.5) at $w = w_c$ are

$$T_{tt}^{BY} = -T_{xx}^{BY} = \frac{1}{8\pi G} e^{2A} (A' - 2). \quad (5.6)$$

One can rewrite the Brown-York tensor at $w = w_c$ (5.6) in terms of the coupling constant ϕ ,

$$\begin{aligned} T_{ab}^{BY} &= \frac{e^{2A(w)}}{8\pi G} \begin{pmatrix} \frac{2}{1+e^{8w}} - 2 & 0 \\ 0 & -\frac{2}{1+e^{8w}} + 2 \end{pmatrix} \\ &= \frac{e^{2A(\phi)}}{4\pi G} \begin{pmatrix} -\sin^2 \phi & 0 \\ 0 & \sin^2 \phi \end{pmatrix}, \end{aligned} \quad (5.7)$$

where we used that

$$A'(w) = \frac{2}{1+e^{8w}} = 2 \cos^2 \phi. \quad (5.8)$$

Next, we can compute the trace of the Brown-York stress-energy tensor²

$$T_a^a = \frac{1}{2\pi G} \sin^2 \phi \quad (5.9)$$

and the square of the stress-energy tensor

$$T^{ab}T_{ab} = \gamma^{ac}\gamma^{bd}T_{ab}T_{cd} = \frac{1}{8\pi^2 G^2} \sin^4 \phi. \quad (5.10)$$

Taking into account (5.9) and (5.10), we can define the composite $T\bar{T}$ operator,

$$T\bar{T} = \frac{1}{8} \left(T^{ab}T_{ab} - (T_a^a)^2 \right) = -\frac{1}{64\pi^2 G^2} \sin^4 \phi. \quad (5.11)$$

For any stationary, translation-invariant state in any 2D quantum field theory (QFT), the expectation value of the $T\bar{T}$ factorizes as $\langle T\bar{T} \rangle = \langle T \rangle \langle \bar{T} \rangle - \langle \Theta \rangle^2$. From (5.7) we find that $\langle T \rangle = 0$, $\langle \bar{T} \rangle = 0$, and $\langle \Theta \rangle = \frac{1}{8\pi G} \sin^2 \phi$, confirming the identity holds.

In a $T\bar{T}$ -deformed CFT, the expectation values in a stationary state obey

$$\langle T_a^a \rangle = -4\pi\mu \langle T\bar{T} \rangle. \quad (5.12)$$

Although we are considering a holographic RG flow rather than a $T\bar{T}$ -deformed boundary theory, one may nevertheless use this relation to define an effective, scale-dependent parameter $\mu(\phi)$.

Using (5.12) together with (5.9) and (5.11), we obtain an effective, scale-dependent parameter

$$\mu(\phi(w_c)) = \frac{8G}{\sin^2 \phi(w_c)} = 8G \left(1 + e^{-8a^2 w_c} \right), \quad (5.13)$$

where w_c denotes the location of the cutoff surface. This function decreases monotonically along the RG flow: in the UV limit $w_c \rightarrow -\infty$ (equivalently $\phi \rightarrow 0$), we have $\mu \rightarrow +\infty$, while in the IR $w_c \rightarrow +\infty$ ($\phi \rightarrow \pi/2$) we obtain the finite value $\mu \rightarrow 8G$. Notably, μ does not depend on the parameter a^2 . A similar monotonic behavior is exhibited by the holographic c function

$$c \sim 1/\cos^2 \phi \propto 1/A', \quad (5.14)$$

in accordance with the c theorem.

VI. DISCUSSION

In this work, we have constructed solutions with AdS, dS, and Minkowski spaces asymptotics in a three-dimensional truncated supergravity coupled to a σ model with target space $S^2 = SU(2)/U(1)$. The action includes a scalar with a periodic scalar potential. The solutions interpolate between AdS or dS extrema of the potential to Minkowski vacuum, which also corresponds to an extremum of the potential. Depending on the value of a^2 , which is related to the curvature of the target space, the scalar potential of the holographic model gives rise to both AdS and dS solutions, which are dual to CFTs.

We provide a holographic interpretation for the exact half-supersymmetric AdS-to-Minkowski solutions flowing from a minimum to minimum of the potential. With Dirichlet boundary conditions, they are dual to deformations of the two-dimensional CFTs driven by nonzero VEV of irrelevant operators, spontaneously breaking conformal symmetry. In [23,42], similar solutions to holographic models with generic types of scalar potentials were discussed as holographic exotic RG flows. The solutions from de Sitter to Minkowski are also irrelevant deformations, but the dual theory is nonunitary. For the de Sitter case, the sign of the “VEV” factor affects the asymptotic behavior of the flows near Minkowski space. We have found that it can be either asymptotically periodic or linear.

Doing a generalization to the finite-temperature case, we have found that the only regular geometries in this case are BTZ and SdS black holes; the other thermal solutions are singular. This seems to be in agreement with the analysis of [42].

We have also discussed the black string solution [32] to our supergravity model, for which we have shown that it

²For readability, we now omit the superscript BY.

can be associated with a deformation triggered by a nonzero VEV of an irrelevant operator, such that the VEV is related to the mass of the black string. An interesting point is that the black string solution is extremal ($T = 0$) and, after a suitable change of radial coordinate, resembles the domain wall solution interpolating between the AdS to Minkowski spacetimes, while also satisfying the first-order BPS equations.

Reducing the equations of motion to an autonomous dynamical system defined on various phase spaces (on the plane, in the disk, in the cylinder, and in the ball), we have clarified the behavior of the flows for certain regions. We have also provided an analytical description of the solution near the horizon region. We have shown that the initial conditions for the scalar field on the horizon determine the behavior of the thermal solutions.

We have explicitly computed the Brown-York stress-energy tensor on a cutoff surface at $w = w_c$ for the exact holographic RG flow and demonstrated the factorization of the $T\bar{T}$ operator along the flow. Furthermore, we have introduced an effective, scale-dependent deformation parameter $\mu(\phi(w_c))$ whose running is governed by the scalar field on the cutoff surface.

An important future direction is to extend this analysis to higher-dimensional settings, particularly through the uplift of our three-dimensional gauged supergravity solutions to string and M-theory models. It is natural to ask whether our AdS₃ and dS₃ solutions can be realized as geometries of wrapped M2- or M5-branes, or as compactifications of AdS₄ or AdS₅ supergravity with suitable internal manifolds. Such an uplift would clarify the microscopic interpretation of the exotic RG flows and finite-temperature solutions we have observed. In particular, the black string solution we discussed may be related to a deformed M5-brane or NS5-brane system, where the VEV of the irrelevant operator finds a natural interpretation in terms of brane moduli or world volume fluxes [61].

For future research, it would also be interesting to apply this dynamical systems approach to five-dimensional gauged supergravity models. A promising candidate is the theory in [62], where the full $D = 5$, $\mathcal{N} = 8$ gauged supergravity is truncated to a two-scalar model. A comprehensive study of the RG-flow phase portrait for this theory is clearly desirable. Another relevant model, constructed for $D = 5$, $\mathcal{N} = 4$ gauged supergravity, can be found in [63]. We hope to address these models in future work.

An interesting direction of future research is to extend our analysis to the case of other exotic RG flows, such as limit cycles in the context of certain holographic models. Despite the substantial amount of work on these nonperturbative effects (for example, from the perspective of gravity, see [23,42,64]), the question of the fundamental nature and physical understanding of such nonperturbative effects remains open. To understand the nature of such effects, it is necessary to consider a large number of model examples in which exotic flows can exist.

It would also be intriguing to explore the scalar sector of $D = 3$, $\mathcal{N} = 2$ gauged supergravity extended with a Fayet-Iliopoulos term [65]. Beyond the supersymmetric solutions already known in this context [66], a particularly interesting direction is the search for gravitational instantons—vacuum solutions that satisfy the Einstein equations with vanishing energy-momentum tensor, as studied in [67,68]. Uncovering the connection between such gravitational configurations on the bulk side and RG flows on the dual QFT side remains an open problem for future investigation.

ACKNOWLEDGMENTS

The authors would like to thank E. T. Musaev and N. S. Deger for helpful discussions. L. A. thanks Boğaziçi University and especially I. Gahramanov for kind hospitality where a part of this work was done.

DATA AVAILABILITY

There are no publicly available research data or software supporting this manuscript. Requests for further information or data should be sent to the authors.

APPENDIX A: SOLUTIONS TO THE MODEL

1. dS, AdS, Minkowski, and SdS solutions

Here we consider the case $f = 1$. Combining Eqs. (2.20)–(2.22) we have

$$\ddot{A} + \frac{\dot{\phi}^2}{a^2} = 0. \quad (\text{A1})$$

Assuming that the scalar field is almost constant leads to $\frac{\dot{\phi}^2}{a^2} \sim 0$, so integrating (A1) we obtain

$$A = c_1 w + c_2. \quad (\text{A2})$$

At the same time, from (2.22) we find the following scale factor:

$$A = \sqrt{-2V(\phi_*)}w + c_2. \quad (\text{A3})$$

For de Sitter we have

$$V = \frac{2a^4}{1 + 2a^2}, \quad (\text{A4})$$

i.e.,

$$A = \frac{2a^2}{\sqrt{1 + 2a^2}} iw \quad (\text{A5})$$

with $c_2 = 0$.

Then the dS metric can be represented in the following form:

$$ds^2 = e^{\frac{4a^2}{\sqrt{1+2a^2}} iw} (-dt^2 + dx^2) + dw^2, \quad (\text{A6})$$

with the constant scalar field

$$\phi = \phi_{\text{ds}}. \quad (\text{A7})$$

One can reach the standard form of the de Sitter metric using double Wick rotation

$$t \rightarrow -ir, \quad -dt^2 = dr^2, \quad w \rightarrow -i\tau, \quad dw^2 \rightarrow -d\tau^2, \quad (\text{A8})$$

which yields the metric

$$ds^2 = -d\tau^2 + e^{a^2 \sqrt{\frac{1}{1+2a^2}} \tau} (dr^2 + dx^2). \quad (\text{A9})$$

It seems that these coordinates are called “flat slicing.”

Note that, at the same time, de Sitter spacetime can be represented in the following form (in the static coordinates):

$$ds^2 = -(1 - br^2)dt^2 + \frac{dr^2}{1 - br^2} + r^2 d\varphi^2, \quad (\text{A10})$$

where b is a positive constant. The cosmological horizon is located at $r = b^{-1/2}$.

The Schwarzschild–de Sitter solution is given by the following metric in the static coordinates [69]:

$$ds^2 = -(1 - 2m - r^2)dt^2 + \frac{dr^2}{1 - 2m - r^2} + r^2 d\varphi^2, \quad (\text{A11})$$

where $m = 4GE$. The horizon is located at $r_h = \sqrt{1 - 2m}$, and the radial coordinate changes as $0 < r < \sqrt{1 - 2m}$.

For the Minkowski solution, we have that the potential vanishes, $V = 0$. The scale factor in this case is just zero, $A = 0$. So the metric is

$$ds^2 = -dt^2 + dx^2 + dw^2. \quad (\text{A12})$$

2. Asymptotic behavior of flows from linearized EOM

In this subsection, we discuss how to find solutions to Eqs. (2.24) and (2.23) analytically, assuming that we are in the region of the extrema of the potential. In particular, we find an asymptotically periodic solution near Minkowski.

a. Flows near AdS and Minkowski extrema

We start from the AdS extremum with $\phi = 0$, so the equations of motion (2.24) and (2.23) take the following form:

$$\ddot{A} + 2\dot{A}^2 - 8 = 0, \quad (\text{A13})$$

$$\ddot{\phi} + 2\dot{A}\dot{\phi} - 2a^2 V_\phi = 0, \quad (\text{A14})$$

where we used that the linearized potential near the AdS extremum is given by

$$V = -2, \quad V_\phi = V_{\phi\phi}(\phi_\star) \cdot \phi, \quad V_{\phi\phi} = 16. \quad (\text{A15})$$

For Eqs. (A13) and (A14), the scale factor has a special solution given by

$$A = 2w. \quad (\text{A16})$$

The asymptotic behavior of the scale factor (A16) goes to infinity $A \rightarrow -\infty$, as $w \rightarrow -\infty$, which match with the scale for the exact half-supersymmetric solution (3.1) near the AdS fixed point.

The solution to the scalar field equation (A14) in this case reads

$$\phi = c_1 e^{-8w} + c_2 e^{4w}, \quad (\text{A17})$$

where c_1 and c_2 are constants of integration. This solution is consistent with the behavior of the exact solution near AdS (3.1) if one chooses $c_1 = 0$ in (A17).

We see that the reconstructed solution is in agreement with the Dirichlet boundary conditions

$$\phi_- = 0. \quad (\text{A18})$$

Now we turn to the Minkowski extremum $\phi = \frac{\pi}{2}$, for which the potential and its derivative are

$$V = 0, \quad V_\phi = 8\left(\phi - \frac{\pi}{2}\right). \quad (\text{A19})$$

The solution for the scale factor A is trivial,

$$A = 0. \quad (\text{A20})$$

The equation for the scalar field near $\phi = \frac{\pi}{2}$ reads

$$\ddot{\phi} - 16\left(\phi - \frac{\pi}{2}\right) = 0 \quad (\text{A21})$$

and has the following solution:

$$\phi = \frac{\pi}{2} + c_1 e^{4w} + c_2 e^{-4w}. \quad (\text{A22})$$

The exact solution has the Minkowski asymptotics for $w \rightarrow +\infty$. The solution (A22) is in agreement with this; if one sets $c_1 = 0$, then we reach $\phi = \frac{\pi}{2}$. Thus, we have reproduced the asymptotic behavior of the exact solution (3.1) correctly. Below, we will show that the solutions to the equations of motion near extrema of the potential match with those to dynamical system equations near equilibria points.

b. Flows near dS extrema

Similarly, we can find the asymptotic behavior near dS extrema. For this case, the equations of motion (2.24) and (2.23) take the following form:

$$\ddot{A} + 2\dot{A}^2 + 4\frac{2a^4}{1+2a^2} = 0, \quad (\text{A23})$$

$$\ddot{\phi} + 2\dot{A}\dot{\phi} - 2a^2V_{\phi} = 0, \quad (\text{A24})$$

where we used that the linearized potential near the dS extremum is given by

$$V = \frac{2a^4}{1+2a^2}, \quad V_{\phi} = V_{\phi\phi}(\phi_{\text{dS}}) \cdot (\phi - \phi_{\text{dS}}),$$

$$V_{\phi\phi} = -\frac{16a^2(1+a^2)}{1+2a^2}. \quad (\text{A25})$$

We have a special solution for the scale factor given by

$$A = \pm \frac{i2a^2}{\sqrt{1+2a^2}} w = C_A w. \quad (\text{A26})$$

The solution for the scalar field in this case is

$$\phi = c_1 e^{w \left(-\sqrt{C_A^2 + 2a^2 V_{\phi\phi} - C_A} \right)} + c_2 e^{w \left(\sqrt{C_A^2 + 2a^2 V_{\phi\phi} - C_A} \right)} + \phi_{\text{dS}}, \quad (\text{A27})$$

where

$$\phi_{\text{dS}} = \pm \arccos \left(\sqrt{\frac{a^2}{1+2a^2}} \right), \quad C_A^2 = -\frac{4a^4}{1+2a^2}. \quad (\text{A28})$$

One can see that the solution (A27) is complex in terms of the domain wall coordinates, since

$$-C_A \pm \sqrt{C_A^2 + 2a^2 V_{\phi\phi}} = \frac{2a^2 i}{\sqrt{1+2a^2}} \left(\pm \sqrt{9+8a^2} - 1 \right). \quad (\text{A29})$$

However, the solution for ϕ in terms of A is real and can be represented as follows:

$$\phi = c_1 e^{(-\sqrt{9+8a^2}-1)A} + c_2 e^{(\sqrt{9+8a^2}-1)A} \pm \arccos \left(\sqrt{\frac{a^2}{1+2a^2}} \right). \quad (\text{A30})$$

Below, we will also show that the solution near the dS point (A30) can be recovered from the analysis of the dynamical system (B11). The solution for the dS case with $f = 1$ has the complex scale factor (A26), but doing double Wick rotation, we come to the dS metric in an expanding patch [see (A8) and (A9)]. If we set $f = -1$ from the start, we obtain the real dS metric in the form (2.19) accurate to just the redefinition of coordinates corresponding also to the dS expanding patch.

APPENDIX B: LYAPUNOV ANALYSIS

Implement the Lyapunov analysis on the dynamical system. Define the functions

$$f = X, \quad g = \left(\frac{X^2}{a^2} - 2 \right) \left(X + \frac{a^2}{2} \frac{V'}{V} \right) \quad (\text{B1})$$

and its derivatives matrix

$$\mathcal{M} = \begin{pmatrix} \frac{\partial f}{\partial \phi} & \frac{\partial f}{\partial X} \\ \frac{\partial g}{\partial \phi} & \frac{\partial g}{\partial X} \end{pmatrix} \Big|_{X=X_c, \phi=\phi_c}. \quad (\text{B2})$$

Demonstrate the analysis explicitly for the dS point. This point has the following coordinates:

$$X_c = 0, \quad \phi_c = \phi_{\text{dS}} = \pm \arccos \left(\sqrt{\frac{a^2}{1+2a^2}} \right). \quad (\text{B3})$$

The matrix elements are as follows:

$$\mathcal{M}_{11} = 0; \quad \mathcal{M}_{12} = 1;$$

$$\mathcal{M}_{21} = -\frac{\sec^2 \phi (X^2 - 2)(13 + 3 \cos 4\phi)}{(1 - 3 \cos 2\phi)^2} \Big|_{X=X_c, \phi=\phi_c} = 8(1+a^2);$$

$$\mathcal{M}_{22} = \frac{3X^2}{a^2} - \frac{2X \tan \phi ((2a^2 + 1) \cos 2\phi + 1)}{(2a^2 + 1) \cos^2 \phi - 2a^2} - 2 \Big|_{X=X_c, \phi=\phi_c} = -2; \quad (\text{B4})$$

$$\det \mathcal{M} = -16. \quad (\text{B5})$$

The characteristic equation

$$\lambda^2 - \lambda(\mathcal{M}_{11} + \mathcal{M}_{22}) + \mathcal{M}_{11}\mathcal{M}_{22} - \mathcal{M}_{12}\mathcal{M}_{21} = 0 \quad (\text{B6})$$

has the following solutions:

$$\lambda_{1,2} = -1 \pm \sqrt{9 + 8a^2}. \quad (\text{B7})$$

Hence, we conclude that the de Sitter fixed point is a saddle.

The eigenvectors are defined by the following equation:

$$\mathcal{M}_{ij}^j u_{1,2}^j = \lambda_{1,2} u_{1,2}^i, \quad (\text{B8})$$

from where we can find them,

$$u_1 = \begin{bmatrix} 1 \\ \lambda_1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 1 \\ \lambda_2 \end{bmatrix}. \quad (\text{B9})$$

We know that eigenvalues of the Lyapunov exponent correspond to the operators scaling dimensions with the opposite sign,

$$\lambda_{1,2} = -\Delta_{1,2}. \quad (\text{B10})$$

Hence, we can obtain the solution near dS fixed points, which is given by

$$\begin{bmatrix} \phi \\ X \end{bmatrix} = \begin{bmatrix} \pm \arccos\left(\sqrt{\frac{a^2}{1+2a^2}}\right) \\ 0 \end{bmatrix} + k_1 e^{-\Delta_1 A} u_1 + k_2 e^{-\Delta_2 A} u_2. \quad (\text{B11})$$

In the same manner, the general solution for the AdS point at $X_c = 0$ and $\phi_c = \phi_{\text{AdS}} = 0$ is given by

$$\begin{bmatrix} \phi \\ X \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + k_1 e^{-\Delta_1 A} u_1 + k_2 e^{-\Delta_2 A} u_2 \quad (\text{B12})$$

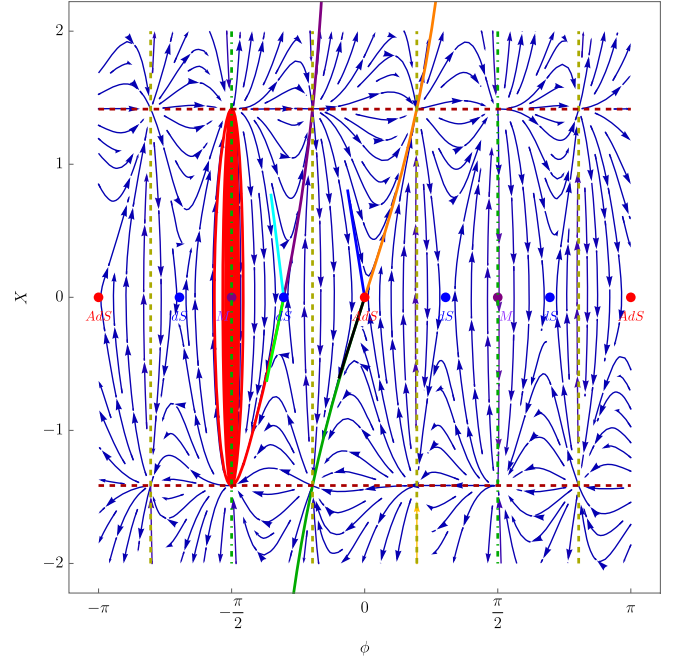


FIG. 8. Phase flows from AdS and dS equilibrium points to the Minkowski one with the depicted eigenvectors directions. Blue and black straight lines correspond to the AdS eigenvectors, and cyan and green straight lines correspond to the dS eigenvectors.

with the eigenvalues $\lambda_{1,2} = -\Delta_{1,2} = -1 \pm (1 + 2a^2)$ and the eigenvectors

$$u_1 = \begin{bmatrix} 1 \\ \lambda_1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 1 \\ \lambda_2 \end{bmatrix}. \quad (\text{B13})$$

We see that the AdS point is also a saddle. In Fig. 8, we draw the directions of the eigenvalues corresponding to the central AdS point and to its closest left dS point.

-
- [1] M. Gell-Mann and F. E. Low, Quantum electrodynamics at small distances, *Phys. Rev.* **95**, 1300 (1954).
 - [2] N. N. Bogolyubov and D. V. Shirkov, Charge renormalization group in quantum field theory, *Nuovo Cimento* **3**, 845 (1956).
 - [3] N. N. Bogolyubov and D. V. Shirkov, *Introduction to the Theory of Quantized Fields*, Interscience Monographs (Interscience Publishers, New York, 1959), Vol. 3.
 - [4] K. Symanzik, Small distance behavior in field theory and power counting, *Commun. Math. Phys.* **18**, 227 (1970).
 - [5] C. G. Callan, Jr., Broken scale invariance and asymptotic behavior, *Phys. Rev. D* **5**, 3202 (1972).
 - [6] K. G. Wilson, The renormalization group: Critical phenomena and the Kondo problem, *Rev. Mod. Phys.* **47**, 773 (1975).

- [7] J. M. Maldacena, The large N limit of superconformal field theories and supergravity, *Adv. Theor. Math. Phys.* **2**, 231 (1998).
- [8] E. Witten, Anti-de Sitter space and holography, *Adv. Theor. Math. Phys.* **2**, 253 (1998).
- [9] S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Gauge theory correlators from noncritical string theory, *Phys. Lett. B* **428**, 105 (1998).
- [10] E. T. Akhmedov, A remark on the AdS/CFT correspondence and the renormalization group flow, *Phys. Lett. B* **442**, 152 (1998).
- [11] J. de Boer, E. P. Verlinde, and H. L. Verlinde, On the holographic renormalization group, *J. High Energy Phys.* **08** (2000) 003.

- [12] D. Z. Freedman, S. S. Gubser, K. Pilch, and N. P. Warner, Renormalization group flows from holography supersymmetry and a c theorem, *Adv. Theor. Math. Phys.* **3**, 363 (1999).
- [13] S. de Haro, S. N. Solodukhin, and K. Skenderis, Holographic reconstruction of space-time and renormalization in the AdS/CFT correspondence, *Commun. Math. Phys.* **217**, 595 (2001).
- [14] J. de Boer, The holographic renormalization group, *Fortschr. Phys.* **49**, 339 (2001).
- [15] M. Bianchi, D. Z. Freedman, and K. Skenderis, Holographic renormalization, *Nucl. Phys.* **B631**, 159 (2002).
- [16] M. Bianchi, D. Z. Freedman, and K. Skenderis, How to go with an RG flow, *J. High Energy Phys.* **08** (2001) 041.
- [17] K. Skenderis, Lecture notes on holographic renormalization, *Classical Quantum Gravity* **19**, 5849 (2002).
- [18] I. Papadimitriou, Holographic renormalization made simple: An example, *Subnuclear series* **41**, 508 (2005).
- [19] I. Papadimitriou and K. Skenderis, Correlation functions in holographic RG flows, *J. High Energy Phys.* **10** (2004) 075.
- [20] I. Papadimitriou and K. Skenderis, AdS/CFT correspondence and geometry, *IRMA Lect. Math. Theor. Phys.* **8**, 73 (2005).
- [21] I. Papadimitriou, Multi-trace deformations in AdS/CFT: Exploring the vacuum structure of the deformed CFT, *J. High Energy Phys.* **05** (2007) 075.
- [22] I. Papadimitriou, Lectures on holographic renormalization, *Springer Proc. Phys.* **176**, 131 (2016).
- [23] E. Kiritsis, F. Nitti, and L. Silva Pimenta, Exotic RG flows from holography, *Fortschr. Phys.* **65**, 1600120 (2017).
- [24] B. C. van Rees, Holographic renormalization for irrelevant operators and multi-trace counterterms, *J. High Energy Phys.* **08** (2011) 093.
- [25] B. C. van Rees, Irrelevant deformations and the holographic Callan-Symanzik equation, *J. High Energy Phys.* **10** (2011) 067.
- [26] M. Broccoli, Irrelevant operators and their holographic anomalies, *J. High Energy Phys.* **05** (2022) 001.
- [27] R. A. Davison, S. A. Gentle, and B. Goutéraux, Impact of irrelevant deformations on thermodynamics and transport in holographic quantum critical states, *Phys. Rev. D* **100**, 086020 (2019).
- [28] L. McGough, M. Mezei, and H. Verlinde, Moving the CFT into the bulk with $T\bar{T}$, *J. High Energy Phys.* **04** (2018) 010.
- [29] A. Strominger, The dS/CFT correspondence, *J. High Energy Phys.* **10** (2001) 034.
- [30] F. Leblond, D. Marolf, and R. C. Myers, Tall tales from de Sitter space 1: Renormalization group flows, *J. High Energy Phys.* **06** (2002) 052.
- [31] F. Leblond, D. Marolf, and R. C. Myers, Tall tales from de Sitter space. 2. Field theory dualities, *J. High Energy Phys.* **01** (2003) 003.
- [32] N. S. Deger, A. Kaya, E. Sezgin, and P. Sundell, Matter coupled AdS(3) supergravities and their black strings, *Nucl. Phys.* **B573**, 275 (2000).
- [33] N. S. Deger, A. Kaya, E. Sezgin, P. Sundell, and Y. Tanii, (2,0) Chern-Simons supergravity plus matter near the boundary of AdS(3), *Nucl. Phys.* **B604**, 343 (2001).
- [34] K. Arkhipova, L. Astrakhantsev, N. S. Deger, A. A. Golubtsova, K. Gubarev, and E. T. Musaev, Holographic RG flows and boundary conditions in a 3D gauged supergravity, *Eur. Phys. J. C* **84**, 560 (2024).
- [35] A. Golubtsova, A. Nikolaev, and M. Podoinitsyn, Holographic RG flows in a 3D gauged supergravity at finite temperature, *Phys. Rev. D* **110**, 066011 (2024).
- [36] M. Gutperle and C. Hultgreen-Mena, Janus and RG-interfaces in minimal 3d gauged supergravity, *Int. J. Mod. Phys. A* **40**, 2550113 (2025).
- [37] K. Behrndt, A Nonsingular infrared flow from $D = 5$ gauged supergravity, *Phys. Lett. B* **487**, 30 (2000).
- [38] Y. Hikida, T. Nishioka, T. Takayanagi, and Y. Taki, CFT duals of three-dimensional de Sitter gravity, *J. High Energy Phys.* **05** (2022) 129.
- [39] S. Gukov, RG flows and bifurcations, *Nucl. Phys.* **B919**, 583 (2017).
- [40] I. Y. Aref'eva, Holographic renormalization group flows, *Theor. Math. Phys.* **200**, 1313 (2019).
- [41] E. Kiritsis, F. Nitti, and J.-L. Raymond, Improved holographic QCD on a curved background: An application of dynamical system theory in holography, *Fortschr. Phys.* **73**, e70046 (2025).
- [42] U. Gürsoy, E. Kiritsis, F. Nitti, and L. Silva Pimenta, Exotic holographic RG flows at finite temperature, *J. High Energy Phys.* **10** (2018) 173.
- [43] F. A. Smirnov and A. B. Zamolodchikov, On space of integrable quantum field theories, *Nucl. Phys.* **B915**, 363 (2017).
- [44] A. A. Golubtsova and M. K. Usova, Stability analysis of holographic RG flows in 3D supergravity, *Eur. Phys. J. Plus* **138**, 260 (2023).
- [45] A. A. Golubtsova, E. Gourgoulhon, and M. A. Podoinitsyn, More on thermal holographic RG flows in a 3D gauged supergravity, [arXiv:2412.06536](https://arxiv.org/abs/2412.06536).
- [46] J. M. Izquierdo and P. K. Townsend, Supersymmetric spacetimes in $(2 + 1)$ adS supergravity models, *Classical Quantum Gravity* **12**, 895 (1995).
- [47] N. S. Deger, Renormalization group flows from $D = 3$, $N = 2$ matter coupled gauged supergravities, *J. High Energy Phys.* **11** (2002) 025.
- [48] H. Epstein and U. Moschella, de Sitter tachyons and related topics, *Commun. Math. Phys.* **336**, 381 (2015).
- [49] E. Kiritsis and A. Tsouros, De Sitter versus anti de Sitter flows and the (super)gravity landscape, *J. High Energy Phys.* **08** (2023) 126.
- [50] E. Kiritsis, S. Morales-Tejera, and C. Rosen, De Sitter versus anti de Sitter flows and the (super)gravity landscape: Part II, *J. High Energy Phys.* **04** (2026) 111.
- [51] N. A. Chernikov and E. A. Tagirov, Quantum theory of scalar field in de Sitter space-time, *Ann. Inst. Henri Poincaré Phys. Theor. A* **9**, 109 (1968), <https://inspirehep.net/literature/53162>.
- [52] E. A. Tagirov, Consequences of field quantization in de Sitter type cosmological models, *Ann. Phys. (N.Y.)* **76**, 561 (1973).
- [53] R. Bousso, A. Maloney, and A. Strominger, Conformal vacua and entropy in de Sitter space, *Phys. Rev. D* **65**, 104039 (2002).
- [54] V. Balasubramanian, J. de Boer, and D. Minic, Notes on de Sitter space and holography, *Classical Quantum Gravity* **19**, 5655 (2002).

- [55] S. Collier, L. Eberhardt, and B. Mühlmann, A microscopic realization of dS_3 , *SciPost Phys.* **18**, 131 (2025).
- [56] N. Bobev, N. Halmagyi, K. Pilch, and N. P. Warner, Holographic, $N = 1$ supersymmetric RG flows on M2 branes, *J. High Energy Phys.* **09** (2009) 043.
- [57] N. Kaloper, Entropy count for extremal three-dimensional black strings, *Phys. Lett. B* **434**, 285 (1998).
- [58] U. Gursoy, E. Kiritsis, L. Mazzanti, and F. Nitti, Holography and thermodynamics of 5D dilaton-gravity, *J. High Energy Phys.* **05** (2009) 033.
- [59] P. Conzino, G. Fanizza, M. Gasperini, E. Pavone, L. Tedesco, and G. Veneziano, From the string vacuum to FLRW or de Sitter via α' corrections, *J. Cosmol. Astropart. Phys.* **12** (2023) 019.
- [60] L. Perko, *Differential Equations and Dynamical Systems* (Springer-Verlag, Berlin, Heidelberg, 1991).
- [61] K. Pilch and N. P. Warner, $N = 1$ supersymmetric renormalization group flows from IIB supergravity, *Adv. Theor. Math. Phys.* **4**, 627 (2002).
- [62] N. Bobev, E. Malek, B. Robinson, H. Samtleben, and J. van Muiden, Kaluza-Klein spectroscopy for the Leigh-Strassler SCFT, *J. High Energy Phys.* **04** (2021) 208.
- [63] P. Karndumri, AdS_5 vacua and holographic RG flows from 5D $N = 4$ gauged supergravity, *Eur. Phys. J. C* **83**, 164 (2023).
- [64] Y. Bea and D. Mateos, Heating up exotic RG flows with holography, *J. High Energy Phys.* **08** (2018) 034.
- [65] M. Abou-Zeid and H. Samtleben, Chern-Simons vortices in supergravity, *Phys. Rev. D* **65**, 085016 (2002).
- [66] N. S. Deger and C. A. Deral, Supersymmetric null and timelike warped AdS and strings in $D = 3$, $N = 2$ gauged supergravity, *Phys. Rev. D* **113**, 026024 (2026).
- [67] S. de Haro, I. Papadimitriou, and A. C. Petkou, Conformally coupled scalars, instantons and vacuum instability in $AdS(4)$, *Phys. Rev. Lett.* **98**, 231601 (2007).
- [68] S. de Haro and A. C. Petkou, Instantons and the Hartle-Hawking-Maldacena proposal for dS/CFT, *J. High Energy Phys.* **11** (2014) 126.
- [69] M. Spradlin, A. Strominger, and A. Volovich, Les Houches lectures on de Sitter space, in *Les Houches Summer School: Session 76: Euro Summer School on Unity of Fundamental Physics: Gravity, Gauge Theory and Strings* (2001), 10, pp. 423–453, [arXiv:hep-th/0110007](#).