

# Vacuum pair production in spatially asymmetric time-oscillating electric fields

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We investigate electron-positron pair production from the quantum vacuum in spatially asymmetric, oscillating electric fields using the Dirac-Heisenberg-Wigner formalism in  $1 + 1$  dimensions. The electric field is strongly space-time dependent  $[E(x, t)\mathbf{e}_x]$  and characterized by both spatial asymmetry and temporal oscillations. The temporal dynamics incorporate multiple modulation mechanisms, including variations in central frequency, frequency chirping, and phase modulation. Our results show that spatial asymmetry significantly enhances pair production compared to symmetric field configurations. Furthermore, optimal tuning of temporal parameters, such as the frequency  $\omega$  and chirp parameter  $b$ , can further amplify the yield. For  $\omega \gtrsim 0.4m$ , the momentum spectra exhibit oscillatory structures dominated by multiphoton processes, whereas low-frequency fields ( $\omega \lesssim 0.3m$ ) result in tunneling-dominated Gaussian distributions. Chirped fields introduce spectral asymmetry and interference patterns, with peak yields increasing by up to a factor of 9 for  $\omega = 0.7m$  and  $b = 0.5\omega/\tau$ . These findings offer a promising approach for optimizing vacuum pair production in experimentally realizable spatiotemporal field configurations.

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## I. INTRODUCTION

Vacuum electron-positron pair production is one of the most fascinating predictions of quantum electrodynamics (QED), the theory describing the fundamental interaction between light and matter. In the framework of QED, the vacuum is not a static void but a dynamic medium characterized by the presence of virtual particle-antiparticle pairs, which continuously undergo generation and annihilation due to quantum fluctuations. When an extremely strong electromagnetic field is applied, it can supply enough energy to convert these virtual particles into real, observable ones—namely, electrons and positrons.

The possibility of pair creation was first proposed by Sauter in 1931 [1] as a resolution to the Klein paradox in relativistic quantum mechanics [2]. Sauter's idea was extended to QED by Schwinger in 1951 [3], who explicitly computed the vacuum persistence probability using the proper-time method in a constant electric field, expressed through the one-loop Heisenberg-Euler effective action. The number of created pairs was first calculated by Nikishov [4]

by solving the Dirac equation in a constant electric field and applying the Green's function technique [5]. However, this effect has not yet been observed experimentally due to the extremely high electric field strength required for electron-positron pair production, known as the Schwinger critical field strength,  $E_{\text{cr}} \approx 1.3 \times 10^{18}$  V/m (corresponding to a laser intensity of  $I_{\text{cr}} \sim 4.3 \times 10^{29}$  W/cm<sup>2</sup>). With rapid advancements in laser technology, the intensity of high-power lasers is expected to reach  $I \sim 10^{26}$  W/cm<sup>2</sup> in the near future. This development may enable the observation of the nonperturbative Schwinger effect in laboratory settings [6,7].

There are two primary mechanisms for vacuum electron-positron pair production: tunneling [8,9] and the multiphoton process [10,11]. The tunneling mechanism dominates in the presence of a strong, low-frequency electric field. It is analogous to a particle surmounting a potential barrier that it cannot classically overcome. In contrast, the multiphoton effect becomes more significant under high-frequency, weak-field conditions. In this regime, multiple photons coherently combine their energy to produce an electron-positron pair. Consequently, high-frequency fields—even if their amplitude is lower than that of the Schwinger critical strength—can still induce pair production through the cumulative effect of photon interactions. These two mechanisms are distinguished by the Keldysh parameter,  $\gamma = m_e \omega / e E_0$  [12], where  $m_e$ ,  $e$ ,  $\omega$ , and  $E_0$  denote the electron mass, charge, frequency, and

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external electric field strength, respectively. For  $\gamma \ll 1$ , the process is dominated by tunneling, whereas for  $\gamma \gg 1$ , the multiphoton effect prevails. The multiphoton effect was observed in SLAC experiments involving the collision of a high-energy electron beam with an intense laser pulse [13,14]. However, the tunneling process has yet to be experimentally demonstrated due to the challenge of reaching the required Schwinger critical field strength.

Another promising approach to enhance electron-positron pair production under subcritical field strengths is the dynamically assisted Schwinger mechanism [15–17]. This mechanism involves the combination of two electric fields: one with high intensity and low frequency, and the other with high frequency and low intensity. The high-intensity field facilitates tunneling, whereas the high-frequency field supports the multiphoton process. In this scenario, the Keldysh parameter satisfies  $\gamma \sim 1$ . Although tunneling and multiphoton effects are individually suppressed, their combined presence significantly enhances pair production through dynamical assistance, thereby reducing the required field intensity [18,19]. As a result, recent studies have concentrated on increasing the pair production rate by exploiting various combinations of electric fields within the dynamically assisted Schwinger framework [16–24]. Meanwhile, many investigations have demonstrated that pair production rates and particle dynamics are highly sensitive to external field parameters such as pulse duration, carrier-envelope phase, pulse number, and spatial scale [25–39]. Therefore, optimizing the configuration of external fields is crucial for advancing the study of vacuum pair production.

Although many previous studies have confirmed that dynamically assisted mechanisms can reduce the required field intensity, most of them have primarily focused on spatially homogeneous, time-dependent external fields rather than space-dependent ones. This is because the Schwinger effect is generally suppressed in spatially inhomogeneous electric fields [40–44]. Specifically, spatial inhomogeneities impart momentum to the Dirac sea, effectively increasing the mass gap and thereby reducing pair production efficiency. In contrast, temporal inhomogeneities typically enhance pair production by supplying energy to the Dirac sea [45]. Another consequence of spatial inhomogeneities is the immediate acceleration of pairs by the electric field upon creation. Depending on the spatial extent of the electric field, the momentum imparted to the pairs at creation varies. Consequently, the spatial profile of the field influences postcreation dynamics, affecting momentum spectra [39,46,47] and leading to phenomena such as self-bunching [25,27] and ponderomotive effects [48]. Additionally, these spatial inhomogeneities can also modify temporal effects, including the dynamically assisted Schwinger effect and quantum interference.

Notably, electric fields realized in realistic physical or experimental settings are inherently inhomogeneous in both

spatial and temporal dimensions. These inhomogeneities can significantly influence the vacuum pair production process, underscoring the need for further investigation into the Schwinger effect under realistic spacetime-dependent field configurations in  $3 + 1$  dimensions. However, problems in  $3 + 1$  dimensions are notoriously difficult for both theoretical analysis and numerical simulation. Fortunately, because pair production predominantly occurs along the direction of the electric field, the system can be effectively reduced to a  $1 + 1$ -dimensional framework by neglecting particle momenta orthogonal to this dominant direction under the spatially one-dimensional time-dependent electric fields  $[E(x, t)\mathbf{e}_x]$  [25,36,49]. Accordingly, some recent studies have investigated space-dependent electric fields in  $1 + 1$  dimensions and have sought to mitigate the suppression of pair production due to spatial inhomogeneity by combining spatially inhomogeneous, time-varying electric fields [18,27,39,50–52]. However, most of these studies have focused on symmetric spatially inhomogeneous fields due to the convenience they offer in simplifying theoretical and numerical analysis. Additionally, the significant impact of the temporal asymmetry of the external field on vacuum pair production has been well established [53–56]. Nevertheless, the effects of spatial asymmetry in the electric field remain poorly understood and merit further investigation.

In this study, we conduct a systematic investigation of vacuum pair production in spatially and temporally inhomogeneous electric fields  $(E(x, t)\mathbf{e}_x)$  in  $1 + 1$  dimensions. Specifically, we examine a spatially asymmetric electric field whose temporal component is characterized by various fundamental frequencies, frequency chirps, and initial phases. When a spatially asymmetric field configuration is employed, the interplay among the fields spatial parameters becomes more complex compared to the symmetric case, resulting in distinct outcomes as the frequency and spatial scale of the field vary. Furthermore, the results are sensitive to the initial phase of the field. Notably, we identify an optimal combination of field parameters that may be applied in future strong-field QED experiments.

The rest of this paper is structured as follows. In Sec. II, we briefly review the Dirac-Heisenberg-Wigner (DHW) formalism and present its key equations (Sec. II A). We also introduce a model of a spatially asymmetric external field with temporal oscillations (Sec. II B). In Sec. III, we first compare the DHW results for spatially symmetric and asymmetric fields. We then examine how the results vary with changes in spatial scale, frequency, chirp parameters, and initial phase, and discuss the associated physical implications. Finally, Sec. IV summarizes the main conclusions.

Throughout this paper, we adopt natural units, where  $\hbar = c = 1$ , and all physical quantities are expressed in terms of the electron mass  $m$ . For instance, the units of field frequency and momentum are expressed in  $m$ , and the characteristic timescale of the electric field is  $1/m$ .

## II. METHOD AND EXTERNAL FIELD DESCRIPTION

### A. DHW formalism

The DHW formalism, a relativistic phase-space approach, is widely used to study vacuum pair production. It effectively investigates pair creation under time-dependent and spatially varying electromagnetic fields. In this framework, the external electromagnetic field is treated using a mean-field approximation of the Hartree type [35,57], *i.e.*,

$$F^{\mu\nu}(x) \approx \langle \hat{F}^{\mu\nu}(x) \rangle. \quad (1)$$

Under this approximation, the background gauge field  $A(\mathbf{x}, t)$  is treated as a fixed  $C$ -number field rather than a  $Q$ -number operator, thus eliminating the need for path ordering. The particles themselves are treated as quantum fields. For details on the underlying calculations and numerical strategies, see Refs. [25,36]. A brief introduction to the key concepts of the DHW method is provided below.

In the presence of a background field, the QED Lagrangian is written as

$$\mathcal{L}(\Psi, \bar{\Psi}, A) = \frac{1}{2}(\bar{\Psi}\gamma^\mu D_\mu \Psi - \bar{\Psi}D_\mu^\dagger \gamma^\mu \Psi) - m\bar{\Psi}\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad (2)$$

where  $\mathcal{D}_\mu = (\partial_\mu + ieA_\mu)$  and  $\mathcal{D}_\mu^\dagger = (\overleftarrow{\partial}_\mu - ieA_\mu)$  denote the covariant derivatives with the vector potential  $A_\mu$ , which vanishes at asymptotic times, and  $\gamma^\mu$  are the gamma matrices. To describe particle dynamics, the Dirac equation is essential. Applying the Euler-Lagrange equation to the Lagrangian in Eq. (2) yields the Dirac equation and its adjoint,

$$\begin{aligned} (i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu - m)\Psi &= 0, \\ \bar{\Psi}(i\overleftarrow{\partial}_\mu \gamma^\mu + e\gamma^\mu A_\mu + m) &= 0. \end{aligned} \quad (3)$$

Using the Dirac spinors  $\Psi$  and  $\bar{\Psi}$ , a gauge-covariant density operator is constructed in the Heisenberg picture,

$$\hat{C}_{\alpha\beta}(r, s) = \mathcal{U}(A, r, s)[\bar{\Psi}_\beta(r - s/2), \Psi_\alpha(r + s/2)], \quad (4)$$

where  $r$  and  $s$  represent the center-of-mass and relative coordinates, respectively. To maintain gauge invariance in the density operator defined in Eq. (4), a Wilson line factor is introduced, formally defined as

$$\mathcal{U}(A, r, s) = \exp\left(ies \int_{-1/2}^{1/2} d\xi A(r + \xi s)\right). \quad (5)$$

The covariant Wigner operator is then defined as the Fourier transform of the density operator in Eq. (4),

$$\hat{\mathcal{W}}_{\alpha\beta}(r, p) = \frac{1}{2} \int d^4s e^{ips} \hat{C}_{\alpha\beta}(r, s). \quad (6)$$

Taking the vacuum expectation value of Eq. (6), we obtain the covariant Wigner function  $\mathbb{W}(r, p)$ ,

$$\mathbb{W}(r, p) = \langle \Phi | \hat{\mathcal{W}}(r, p) | \Phi \rangle, \quad (7)$$

where  $|\Phi\rangle$  (or  $\langle\Phi|$ ) denotes the initial vacuum state.

The mean-field approximation simplifies the evaluation of the vacuum expectation value of the Wigner operator, as the electromagnetic field tensor can be factored out in the correlation function,

$$\langle \Phi | F_{\mu\nu} \hat{C} | \Phi \rangle = F_{\mu\nu} \langle \Phi | \hat{C} | \Phi \rangle. \quad (8)$$

This factorization truncates what would otherwise be an infinite hierarchy of correlation functions, thereby simplifying subsequent computations.

Following the mean-field approximation and using the Dirac algebra, the Wigner function is decomposed into 16 covariant Wigner coefficients,

$$\mathbb{W} = \frac{1}{4}(\mathbb{S} + i\gamma_5 \mathbb{P} + \gamma^\mu \mathbb{V}_\mu + \gamma^\mu \gamma_5 \mathbb{A}_\mu + \sigma^{\mu\nu} \mathbb{T}_{\mu\nu}). \quad (9)$$

Here,  $\mathbb{S}$ ,  $\mathbb{P}$ ,  $\mathbb{V}_\mu$ ,  $\mathbb{A}_\mu$ , and  $\mathbb{T}_{\mu\nu}$  denote the scalar, pseudoscalar, vector, axial-vector, and tensor components, respectively. Specifically,  $\mathbb{S}$  is related to mass density,  $\mathbb{P}$  to condensate density,  $\mathbb{V}_\mu$  to fermion current density,  $\mathbb{A}_\mu$  to polarization density, and  $\mathbb{T}_{\mu\nu}$  to electric dipole-moment density [58].

As shown in Refs. [25,58], the dynamical equation for the Wigner function is given by

$$D_t \mathbb{W} = -\frac{1}{2} \mathbf{D}_x [\gamma^0 \boldsymbol{\gamma}, \mathbb{W}] + im[\gamma^0, \mathbb{W}] - i\mathbf{P}\{\gamma^0 \boldsymbol{\gamma}, \mathbb{W}\}, \quad (10)$$

where  $D_t$ ,  $\mathbf{D}_x$ , and  $\mathbf{P}$  are pseudo-differential operators. To describe pair creation in vacuum, it is useful to formulate the problem as an initial value problem and then transition to an equal-time formalism using the equal-time approach [59–61]. The equal-time Wigner function is obtained by energy averaging the Wigner function,

$$\mathbb{w}(\mathbf{x}, \mathbf{p}, t) = \int \frac{dp_0}{2\pi} \mathbb{W}(r, p), \quad (11)$$

where  $\mathbf{x}$  and  $\mathbf{p}$  represent the particle's position and kinetic momentum, respectively. For vacuum pair production, only the equal-time Wigner function can describe the system's evolution. Therefore, all quantities discussed here refer to its components.

We now apply the DHW formalism to a 1+1-dimensional system with one spatial dimension and a time-varying electric field  $E(x, t)$ . In this case, only four Wigner components remain:  $\mathbb{S}$ ,  $\mathbb{V}_0$ ,  $\mathbb{V}_x$ , and  $\mathbb{P}$ . The complete

set of equations of motion is thus reduced to four equations [25,27,58],

$$D_t \mathbb{S} - 2p_x \mathbb{P} = 0, \quad (12)$$

$$D_t \mathbb{V}_0 + \partial_x \mathbb{V} = 0, \quad (13)$$

$$D_t \mathbb{V} + \partial_x \mathbb{V}_0 = -2m \mathbb{P}, \quad (14)$$

$$D_t \mathbb{P} + 2p_x \mathbb{S} = 2m \mathbb{V}, \quad (15)$$

The pseudodifferential operator  $D_t$  is defined as

$$D_t = \partial_t + e \int_{-1/2}^{1/2} d\xi E_x(x + i\xi \partial_{p_x}, t) \partial_{p_x}. \quad (16)$$

Note that in our 1 + 1-dimensional field configuration, transverse momentum has negligible influence. Therefore, for simplicity in both theoretical and numerical calculations, the transverse momentum of particles is ignored [25,39,46,47,52].

Among the four Wigner components described in Eqs. (12)–(15), only two have nonvanishing vacuum initial conditions [36],

$$\mathbb{S}_{\text{vac}} = -\frac{2m}{\Omega}, \quad \mathbb{V}_{\text{vac}} = -\frac{2p_x}{\Omega}, \quad (17)$$

where  $\Omega = \sqrt{p_x^2 + m^2}$  denotes the particle energy. By subtracting the vacuum contributions, the modified Wigner components are written as

$$\tilde{\mathbb{W}} = \mathbb{W} - \mathbb{W}_{\text{vac}}. \quad (18)$$

The particle number density in phase space is formally defined as [25]

$$n(x, p_x, t) = \frac{m \mathbb{S}(x, p_x, t) + p_x \mathbb{V}(x, p_x, t)}{\Omega(p_x)}, \quad (19)$$

where the total energy of the created particles is divided by the energy of a single particle  $\Omega(p_x)$ .

The position and momentum distributions of the created particles can be derived from  $n(x, p_x, t)$  as follows:

$$n(x, t) = \int dp_x n(x, p_x, t), \quad (20)$$

$$n(p_x, t) = \int \frac{dx}{2\pi} n(x, p_x, t). \quad (21)$$

The total particle yield is obtained by integrating over the entire phase space,

$$N(t) = \int \frac{dx}{2\pi} \int dp_x n(x, p_x, t). \quad (22)$$

To account for spatial dependence on the spatial scale  $\lambda$  and enable comparisons across different conditions, we define the following reduced quantities:

$$\bar{n}(p_x, t) = \frac{n(p_x, t)}{\lambda} \quad \text{and} \quad \bar{N}(t) = \frac{N(t)}{\lambda}. \quad (23)$$

## B. External field model

Previous studies have demonstrated that the efficiency of pair creation depends on both the temporal and spatial characteristics of the external field, such as its temporal shape [54,55] and frequency chirp [37,52]. One of our prior investigations [47] explored the influence of various spatial configurations of two spatially separated, space-dependent, temporally Sauter-type electric fields on the efficiency of pair production. The findings revealed that the spatial configuration of the electric field plays a critical role in enhancing the pair production rate and shaping the momentum spectrum.

In this study, we analyze electron-positron pair production in 1 + 1 dimensions by considering a spatially asymmetric electric field with distinct temporal oscillations, as illustrated in Figs. 1 and 5. The field configuration is described as follows:

$$\begin{aligned} E(x, t) &= e E_{\text{cr}} [E_1(x) + E_2(x)] g(t) \\ &= e E_{\text{cr}} \left[ \epsilon_1 \text{sech}^2 \left( \frac{x}{\lambda} - d \right) + \epsilon_2 \text{sech}^2 \left( \frac{x}{\lambda} + d \right) \right] \\ &\quad \times \text{sech}^2 \left( \frac{t}{\tau} \right) \cos(\omega t), \end{aligned} \quad (24)$$

where  $g(t)$  is the temporal component of the electric field;  $\lambda$  and  $\tau$  represent the spatial and temporal scales of the electric field, respectively; and  $\omega$  denotes the fields fundamental frequency. It is important to note that the equations of motion for the Wigner components, given by Eqs. (12)–(15), are numerically solvable only for a limited class of space-dependent electric fields. This limitation arises because applying the pseudodifferential operator  $D_t$  [Eq. (16)] introduces an error function that yields valid results only for specific field configurations, such as spatially homogeneous Gaussian or Sauter-type envelopes. To address this challenge, we construct a spatially asymmetric effective field by combining two spatially superposed electric fields with different intensities, as given in Eq. (24). In this expression,  $E_{\text{cr}}$  represents the Schwinger critical field strength, while  $\epsilon_1$  and  $\epsilon_2$  denote the amplitudes of the two spatially superposed electric fields. Based on optimal values identified in our previous work [47], we fix the field strengths at  $\epsilon_1 = 1.8$  and  $\epsilon_2 = 0.2$ , with a spatial

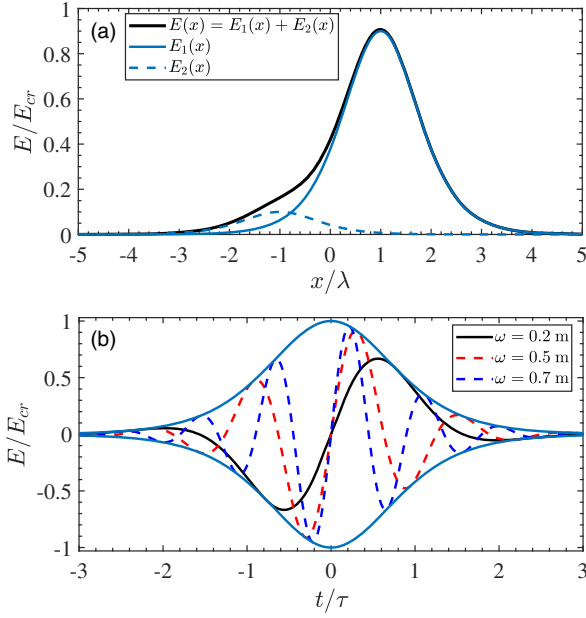


FIG. 1. Schematic diagrams of the (a) spatial and (b) temporal profiles of the combined electric field, normalized to the critical field. Two symmetric, spatially superposed fields separated by a fixed distance  $d = 1 \text{ m}^{-1}$  are shown as solid and dashed blue curves. The resulting effective field is represented by the black curve in (a). In (b), the color-coded curves correspond to different original frequencies  $\omega$ . For these field configurations, the parameters are set to  $\epsilon_1 = 1.8$ ,  $\epsilon_2 = 0.2$ , and  $\epsilon = 0.5$ . The spatial scale  $\lambda$  and frequency  $\omega$  are varied, while the temporal scale is fixed at  $\tau = 10 \text{ m}^{-1}$  throughout the numerical computations.

separation of  $d = 1 \text{ m}^{-1}$ . For all numerical calculations, we set  $E_0 = 0.5E_{\text{cr}}$  ( $\epsilon = 0.5$ ) and use a temporal profile of  $\text{sech}^2(t/\tau)$  (a Sauter-type temporal envelope) with a fixed temporal scale  $\tau = 10 \text{ m}^{-1}$ . The electric field is oriented along the  $x$ -axis and varies as a function of both  $x$  and  $t$  ( $E(x, t)\mathbf{e}_x$ ).

Figure 1(a) shows that the spatial superposition of two fields results in a new, asymmetric effective field that substantially alters the dynamics of pair production and the resulting particle phase-space distribution. In Fig. 1(b), the temporal component of the field is depicted for various frequencies. By employing this idealized field model, we investigate the effects of a spatially asymmetric electric field with a specific time-dependent oscillation on the number density, momentum distribution, and total reduced particle number of the created pairs in phase space.

### III. RESULTS AND ANALYSIS

In this section, we present results calculated using the DHW formalism described in Sec. II A, based on the external field configurations outlined in Sec. II B. We also analyze these results in detail for various field parameters. We first compare the reduced total particle number  $\bar{N}$  for different field configurations, as shown in Fig. 2(a). The

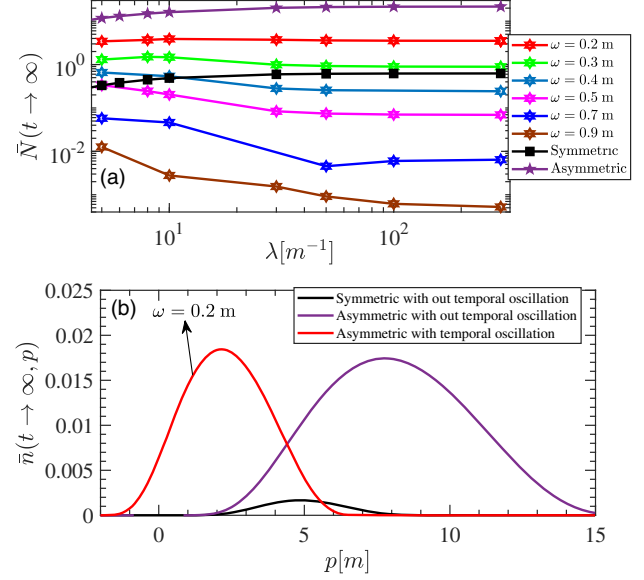


FIG. 2. (a) Reduced total particle number  $\bar{N}$  for various spatial scales  $\lambda$  under different central frequencies in Eq. (24) and various spatial configurations. (b) Momentum spectra of particles for different fields with fixed spatial scale  $\lambda = 50 \text{ m}^{-1}$ . Other parameters are the same as those in Fig. 1.

results indicate that  $\bar{N}$  is higher for the spatially asymmetric field without temporal oscillation (purple line) than for both the spatially symmetric field without temporal oscillation (black line) and the spatially asymmetric field with temporal oscillation (red line). This suggests that optimal pair production can be achieved using spatially asymmetric electric fields without temporal oscillation. However, in realistic scenarios, electric fields are inherently spatiotemporally inhomogeneous and oscillate with a specific frequency. These inhomogeneities can significantly affect the vacuum pair production process. Therefore, it is of critical importance to examine the effects of spatially asymmetric fields with varying temporal oscillation frequencies on pair production. It is also important to note that magnetic fields may influence the pair production process in realistic  $3 + 1$ -dimensional conditions—an effect that purely electric  $1 + 1$ -dimensional models do not account for. Although magnetic fields cannot independently generate particles from the vacuum, as they do not supply energy, they do affect the subsequent dynamics of the produced particles via Lorentz forces and spin interactions. This can result in spin-dependent momentum spectra [16,30,62,63]. However, magnetic field effects are not included in our  $1 + 1$ -dimensional model.

Figure 2(a) shows that the reduced total particle number  $\bar{N}$  decreases as the field frequency increases. This observation is somewhat counterintuitive, as higher field frequencies imply higher photon energies, which might be expected to produce more pairs. However, when fast (high-frequency) and spatially inhomogeneous electric

fields are superimposed, the momentum transferred by the fast fields increases the vacuum energy gap, thereby diminishing the effectiveness of dynamical assistance [18,64]. As a result, dynamical assistance is most effective when the perturbation is purely time- or space-dependent. Figure 2(b) presents the particle momentum spectrum for various field configurations with a fixed spatial scale  $\lambda = 50 \text{ m}^{-1}$ . We observe that the momentum spectrum is highly sensitive to changes in the field configuration. The number density peaks more sharply under an asymmetric temporally oscillating field, while it broadens and shifts toward the positive momentum direction in the case of purely spatial dependence.

It is well known that increasing the frequency of the electric field can facilitate perturbative pair creation through multiphoton absorption. The dominant pair creation mechanism—nonperturbative or perturbative—depends on the frequency and spatial scale of the field. We examine the momentum spectra of the created pairs under a spatially asymmetric electric field with varying spatial scales and temporal frequencies, as illustrated in Fig. 3. At a low frequency of  $\omega = 0.2m$ , the pair creation process is primarily nonperturbative due to the small Keldysh parameter. As a result, the momentum spectrum exhibits a soft Gaussian distribution with approximate bilateral symmetry, relatively insensitive to spatial scale, as shown in Fig. 3(a). As the field frequency increases to  $\omega = 0.3m$ , the momentum spectra remain similar for small spatial scales ( $\lambda = 5 \text{ m}^{-1}$  and  $\lambda = 10 \text{ m}^{-1}$ ), as shown in Fig. 3(b). In contrast, for larger spatial scales ( $\lambda = 50 \text{ m}^{-1}$  and  $\lambda = 300 \text{ m}^{-1}$ ), the spectra develop multipeak structures, a characteristic feature of the perturbative multiphoton pair production mechanism. With further increases

in frequency, the peak value of the momentum spectrum increases, the distribution narrows, and the spectrum shifts toward negative  $p$  values.

At higher frequencies, as illustrated in Figs. 3(c)–3(f), the perturbative pair creation process becomes more prominent, resulting in harder oscillatory components in the momentum spectrum. High-frequency electric field components, which correspond more closely to the perturbative regime, enhance pair production by enabling the absorption of multiple high-energy photons. The number of absorbed photons  $n$  can be calculated using the energy conservation relation  $n\omega = 2\sqrt{m_*^2 + p^2}$  with, where the effective mass is given by  $m_* = m[1 + (mE_0/\omega E_{cr})^2/2]^{1/2}$  [65]. For instance, the three peaks observed in Fig. 3(c) correspond to pair production through absorption of seven, eight, and ten photons, respectively.

The pronounced oscillations observed in the momentum spectra at higher frequencies, particularly for  $\omega = 0.4m$  and  $\omega = 0.5m$ , can be attributed to quantum interference effects. Specifically, pair creation does not occur at a single moment but at multiple time instances. Electrons created at different times accumulate a nonvanishing relative phase. As a result, pair creation events occurring at different times interfere with each other, leading to constructive or destructive interference in the pair production process. This interference modifies the momentum spectra and gives rise to characteristic oscillatory patterns.

As the frequency increases further to  $\omega = 0.7m$  and  $\omega = 0.9m$ , the interference effect is likely suppressed, and peak splitting becomes evident, as shown in Figs. 3(e) and 3(h). When high-frequency electric fields that are spatially inhomogeneous are superimposed, the pair production rate decreases. This is because the momentum provided

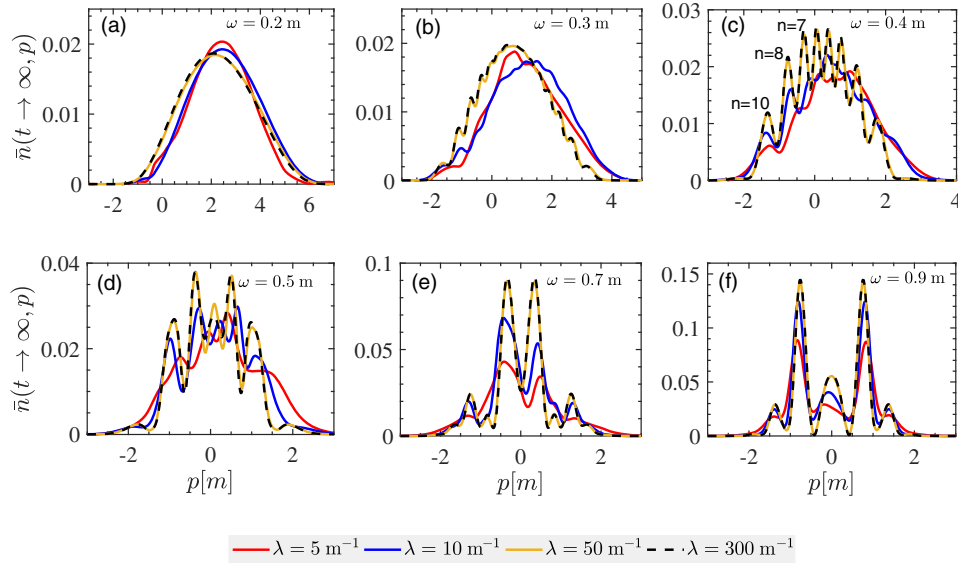


FIG. 3. Momentum distributions for four representative spatial scales  $\lambda$  under a spatially asymmetric field with various temporal frequencies, (a) for  $\omega = 0.2m$ , (b) for  $\omega = 0.3m$ , (c) for  $\omega = 0.4m$ , (d) for  $\omega = 0.5m$ , (e) for  $\omega = 0.7m$ , (f) for  $\omega = 0.9m$ . Other parameters are the same as those in Fig. 1.

by the fast fields increases the vacuum energy gap, as previously discussed in Fig. 2(a). Moreover, quantum interference effects tend to be suppressed by spatial inhomogeneities [51,52]. Specifically, it becomes difficult for pairs created at different spacetime points to converge in phase space after undergoing inhomogeneous acceleration. Consequently, different pair creation events rarely interfere, leading to the suppression of oscillatory structures in the momentum spectra [27]. Meanwhile, once created, pairs are accelerated by the electric field. If the spatial extent of the field is sufficiently large, the pairs remain within the field after creation and consequently gain substantial momentum. Thus, spatial inhomogeneities, by altering postcreation dynamics, influence the momentum spectra and result in self-bunching effects [25,27]. Accordingly, we observe pronounced symmetric splitting of the momentum spectrum for large spatial scales,  $\lambda = 50 \text{ m}^{-1}$  and  $\lambda = 300 \text{ m}^{-1}$ , as shown in Figs. 3(e) and 3(h), which is indicative of particle self-bunching.

In Fig. 4(a), the frequency dependence of the momentum spectra is illustrated for a fixed, relatively large spatial scale  $\lambda = 50 \text{ m}^{-1}$ , indicating a high degree of spatial asymmetry in the field. At low frequencies ( $\omega = 0.2m, 0.3m$ ), the momentum spectrum is broad and exhibits nonperturbative characteristics. At higher frequencies ( $\omega = 0.4m, 0.5m$ ), the spectrum becomes narrower and displays clear perturbative oscillations. Furthermore, it is centered around  $p = 0$  and exhibits a self-bunching profile with a higher peak for frequencies  $\omega = 0.7m$  and  $\omega = 0.9m$ . As the frequency  $\omega$  increases, the number of cycles in the envelope

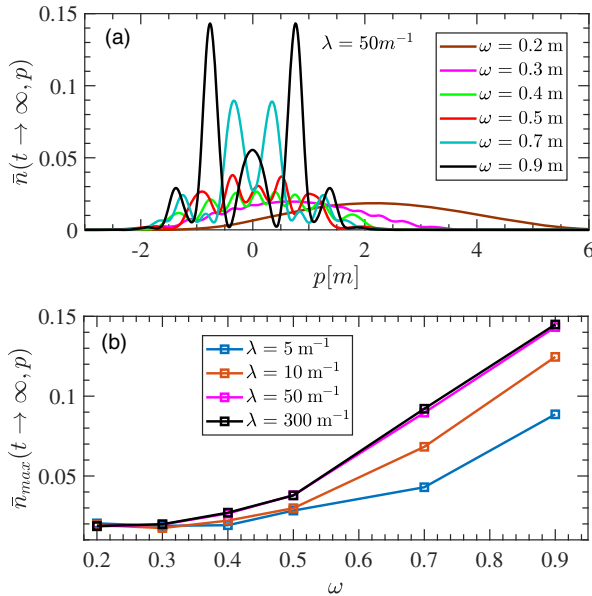


FIG. 4. (a) Momentum distributions for various temporal frequencies  $\omega$  in a spatially asymmetric field with fixed spatial scale  $\lambda = 50 \text{ m}^{-1}$ . (b) Maximum reduced particle density  $\tilde{n}_{\text{max}}$  as a function of temporal frequency  $\omega$  for four representative spatial scales  $\lambda$ . Other parameters are the same as those in Fig. 1.

of the field configuration also increases, as illustrated in Fig. 1(b). This variation alters the timing of pair creation events, thereby influencing the interference pattern and giving rise to complex momentum spectra. These oscillatory spectra are markedly distinct from the monotonic temporal oscillations of the applied electric field  $E(x, t)$ . The behavior shown in Fig. 4 arises from the interference of multiple pair creation events, rather than directly from the temporal structure of the applied field. The interference pattern is shaped by both spatial and temporal inhomogeneities. These findings suggest that careful design of the field characteristics—such as the spatial and temporal profiles and the cycle structure of laser pulses—could enable control over both the timing of pair production and the resulting momentum spectrum. This control is crucial for optimizing pulse shaping strategies aimed at maximizing the Schwinger effect in future strong-field QED experiments.

From Figs. 2(a), 3, and 4(b), we systematically analyze the influence of the spatial extent  $\lambda$  of the electric field on vacuum pair production. It is observed that for large values of  $\lambda$  ( $\lambda \geq 50 \text{ m}^{-1}$ ), the reduced total particle number  $\bar{N}$  remains nearly constant across different field configurations and frequencies, as shown in Fig. 2(a). Furthermore, the maximum particle density increases consistently with both  $\omega$  and  $\lambda$ , with similar trends for  $\lambda = 50 \text{ m}^{-1}$  and  $\lambda = 300 \text{ m}^{-1}$ , as illustrated in Fig. 4(b). These phenomena can be attributed to the spatially quasihomogeneous nature of the electric field configuration at large  $\lambda$ . The quasihomogeneous regime has been analyzed using the local density approximation, as discussed in Refs. [25,27,37]. For large spatial scales, the spatially homogeneous field approximation can be applied. In this approximation, the temporal oscillation  $\cos(\omega t)$  in the time-dependent field  $E(t)$  is preserved at each point  $x$  while the field is treated as spatially uniform. This preserves time dependence and captures oscillatory effects, following a mean-field approach where the field dynamics are decoupled from the particle backreaction. Accordingly, the momentum distribution (or reduced particle density  $\tilde{n}$ ) can be computed by summing results for homogeneous fields with varying field strengths [27]:  $\tilde{n}(p, t \rightarrow \infty) = \sum_x \frac{n(\epsilon(x)|p, t \rightarrow \infty)}{\lambda}$ , where  $n(\epsilon(x)|p, t \rightarrow \infty)$  is the momentum distribution corresponding to a homogeneous field with an effective field strength at specific point  $x$ . It is important to note that the homogeneous field approximation is valid only when the pair-formation length  $l = 2m/|eeE_{\text{cr}}|$  is much smaller than the spatial width  $\lambda$  of the external field [25]. Fields with sufficiently large  $\lambda$  satisfy this condition. Figure 3 confirms the quasihomogeneous character of the field for large spatial extents. For  $\lambda = 50 \text{ m}^{-1}$  and  $\lambda = 300 \text{ m}^{-1}$ , the momentum spectra are nearly identical. This indicates that, under the parameters used in this study, the homogeneous field approximation holds for  $\lambda \geq 50 \text{ m}^{-1}$ , allowing the field to be treated as spatially quasi-homogeneous. As a

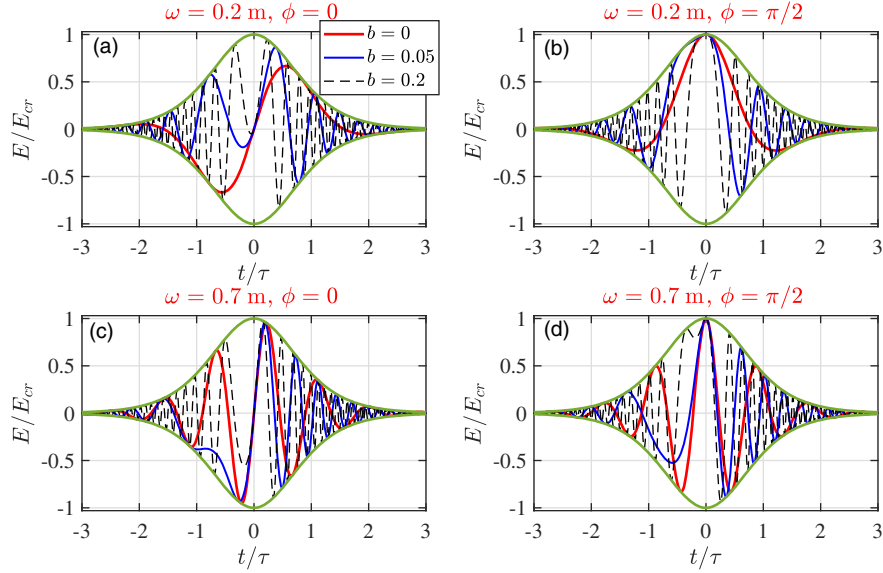


FIG. 5. Schematic diagrams of the temporal component of the electric field for different frequency chirp parameters  $b$ , featuring an asymmetric spatial profile as shown in Fig. 1(a). The top row [(a) and (b)] corresponds to the low-frequency case ( $\omega = 0.2m$ ), while the bottom row [(c) and (d)] corresponds to the high-frequency case ( $\omega = 0.7m$ ). The first column [(a) and (c)] shows results for initial phase  $\phi = 0$ , and the second column [(b) and (d)] for  $\phi = \pi/2$ . Other parameters are the same as those in Fig. 1.

result, the momentum spectrum remains unchanged for larger spatial scales, and thus, even larger values of  $\lambda$  were not considered in this work.

Previous studies have demonstrated that frequency chirp has a significant impact on pair production. Specifically, a chirp introduces a time-dependent variation in the original field frequency. Figure 5 illustrates the temporal evolution of the electric field under various chirp conditions. It is evident that different chirp parameters alter the oscillation patterns and cycle counts within the same temporal envelope. These changes affect the pair production process and modify postcreation dynamics, ultimately altering the momentum spectrum. However, most prior studies have focused on spatially homogeneous or symmetric inhomogeneous fields. The effect of temporal frequency chirp in spatially asymmetric fields remains largely unexplored. In this work, we introduce a linear frequency chirp into a spatially asymmetric field. An effective frequency for the chirped electric field is defined as  $\omega_{\text{eff}} = \omega + bt$ , where  $b$  is the chirp parameter. The temporal component of the external field in Eq. (24) is formulated as  $g(t) = \text{sech}^2(\frac{t}{\tau}) \cos(\omega t + bt^2)$ , and the external electric field can therefore be written as

$$\begin{aligned}
 E(x, t) &= eE_{\text{cr}}[E_1(x) + E_2(x)]g(t) \\
 &= eE_{\text{cr}} \left[ \epsilon_1 \text{sech}^2\left(\frac{x}{\lambda} - d\right) + \epsilon_2 \text{sech}^2\left(\frac{x}{\lambda} + d\right) \right] \\
 &\quad \times \text{sech}^2\left(\frac{t}{\tau}\right) \cos(\omega t + bt^2). \quad (25)
 \end{aligned}$$

The momentum spectra of created particles under low-frequency ( $\omega = 0.2m$ ) and high-frequency ( $\omega = 0.7m$ ) chirped fields, with various chirp parameters  $b$  and spatial scales, are presented in Fig. 6. The patterns and peak values of the spectra differ noticeably with different chirp parameters. For the low-frequency case and small chirp values ( $b = 0.05\omega/\tau$ ,  $0.2\omega/\tau$ ), the momentum spectra show little deviation from the unchirped case ( $b = 0$ ) shown in Fig. 3(a), as illustrated in Figs. 6(a) and 6(b). As the chirp parameter increases to  $b = 0.5\omega/\tau$ , the momentum spectrum begins to exhibit nonperturbative multiphoton oscillations, accompanied by a slight increase in peak value, as shown in Fig. 6(c). Although the original frequency is relatively small ( $\omega = 0.2m$ ), the introduction of a chirp increases the effective frequency  $\omega_{\text{eff}}$ , leading to visible oscillations in the momentum spectrum at larger spatial scales. Similar oscillatory structures typically appear in unchirped fields only at higher original frequencies, such as  $\omega = 0.3m$  [see Fig. 3(b)].

The momentum spectrum for a higher-frequency ( $\omega = 0.7m$ ) electric field with varying chirp parameter  $b$  is shown in Figs. 6(d)–6(f). It is evident that both the peak value and the oscillatory behavior of the momentum spectrum are enhanced as the chirp parameter  $b$  increases. The oscillatory pattern can be attributed to interference between two temporally separated pair-creation events, further modulated by the complex oscillations induced by a large chirp parameter. Electrons created at different times possess a nonvanishing relative phase, which leads to interference in the pair creation process. This relative phase is momentum-dependent, as the momentum is modulated

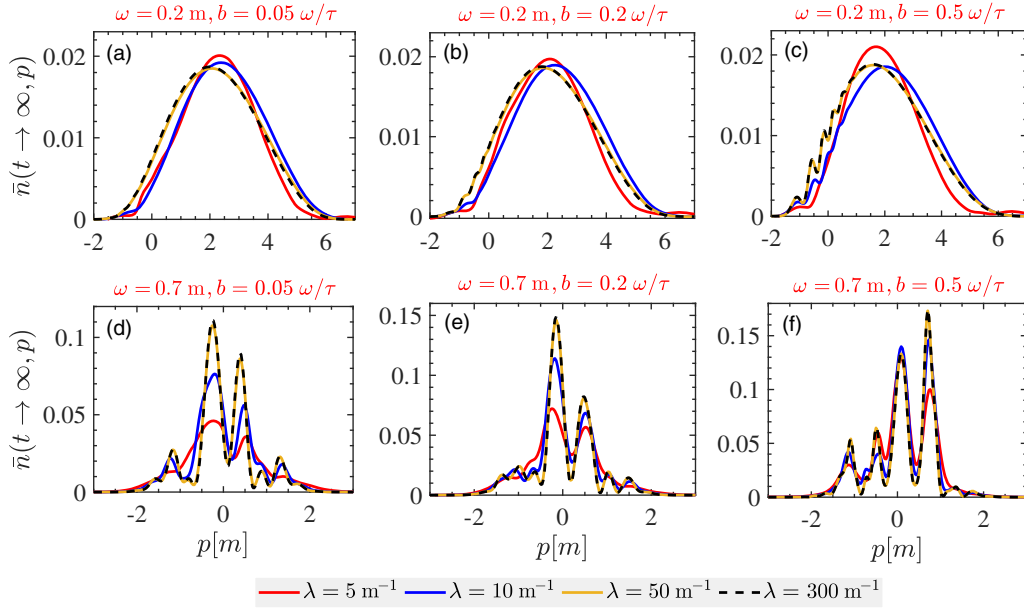


FIG. 6. Momentum distributions at four representative spatial extents  $\lambda$  for a spatially asymmetric field with low [top row, (a)–(c)] and high [bottom row, (d)–(f)] temporal frequencies, for various chirp parameters  $b$ . The other parameters are the same as in Fig. 1.

by the electric field. Consequently, constructive and destructive interference alternates, resulting in characteristic variations in the momentum spectra that are highly sensitive to the form of the applied electric field. As also shown in Figs. 6(d)–6(f), introducing a chirp enhances the peak value of the momentum spectrum while breaking its symmetry. In the absence of chirping, for high original frequencies and large spatial scales, the momentum spectrum is symmetric about  $p = 0$ , as illustrated in Figs. 3(d)–3(f). However, under chirped fields, this symmetry is broken, and the highest peak shifts toward the positive momentum side, as shown in Figs. 6(d)–6(f). This asymmetry may be attributed to the postcreation dynamics of the pairs, which are influenced by the chirped electric field, leading to asymmetric temporal oscillations within the pulse envelope (see Fig. 5). The broken symmetry results in distinct particle dynamics after creation. These findings indicate that the peak of the momentum spectrum shifts progressively toward the positive  $p$  region with increasing chirp parameter  $b$ .

As shown in Fig. 3, for large spatial scales ( $\lambda \geq 50 \text{ m}^{-1}$ ), the spatially inhomogeneous approximation is applicable. Therefore, in Fig. 7, we present the momentum spectra under a fixed spatial scale of  $\lambda = 50 \text{ m}^{-1}$ , considering both low- and high-frequency electric fields with two distinct chirp parameters:  $b = 0.05\omega/\tau$  (small chirp) and  $b = 0.5\omega/\tau$  (large chirp). It is observed that the peak value of the number density increases with both the original frequency and the chirp parameter. Specifically, in the high-frequency case with a large chirp parameter ( $\omega = 0.7 \text{ m}$ ,  $b = 0.5\omega/\tau$ ), the peak number density is approximately 9 times greater than in the low-frequency case with the same

chirp parameter ( $\omega = 0.2 \text{ m}$ ,  $b = 0.5\omega/\tau$ ). In the unchirped scenario, the peak number density of the high-frequency field ( $\omega = 0.7 \text{ m}$ ) is approximately 5 times that of the low-frequency field ( $\omega = 0.2 \text{ m}$ ). The enhancement becomes more significant in the presence of chirping, as the frequency chirp increases the effective frequency of the field, thereby enhancing the pair production rate. Additionally, in the low-frequency case, the momentum spectrum is broader and exhibits weak oscillations only when the chirp parameter is large ( $\omega = 0.2 \text{ m}$ ,  $b = 0.5\omega/\tau$ ). In contrast, for the high-frequency case, the momentum

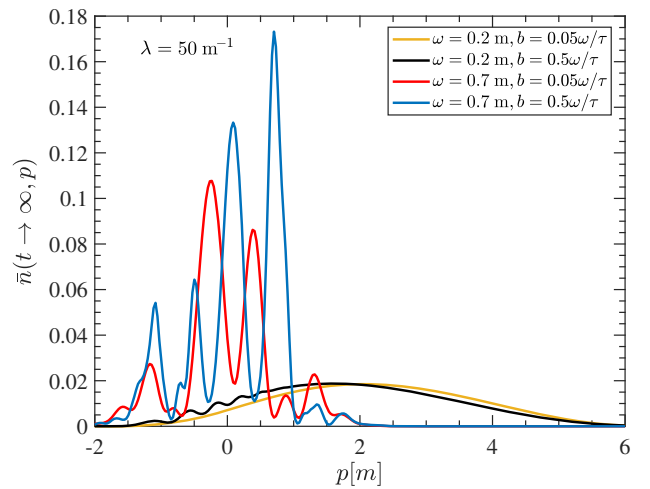


FIG. 7. Momentum distribution for two different temporal frequencies  $\omega$ , with various chirp parameters  $b$  and a fixed spatial scale of  $\lambda = 50 \text{ m}^{-1}$ . The other parameters are the same as in Fig. 1.

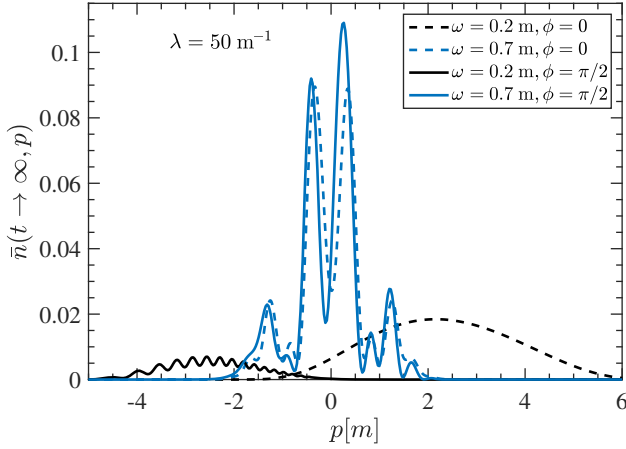


FIG. 8. Momentum distribution for two different initial phases,  $\phi = 0$  and  $\phi = \pi/2$ , with a fixed spatial scale of  $\lambda = 50 \text{ m}^{-1}$ . The other parameters are the same as in Fig. 1.

spectrum demonstrates strong oscillations regardless of the chirp parameter value. These oscillatory features are caused by interference effects, as previously discussed.

Finally, we investigate the influence of the initial phase of spatially asymmetric electric fields on the efficiency of pair production. As shown in Fig. 5, when the initial phase changes from  $\phi = 0$  to  $\phi = \pi/2$ , the maximum of the electric field shifts to  $t = 0$ . This shift modifies both the effective field strength and the number of cycles within a fixed envelope, thereby affecting the pair production efficiency and the resulting momentum spectrum. The results for unchirped electric fields at both low and high frequencies with the two selected initial phases are presented in Fig. 8. We systematically compare and analyze the momentum distributions for the initial phases  $\phi = 0$  and  $\phi = \pi/2$ . For the low-frequency case ( $\omega = 0.2m$ ), the peak of the momentum distribution initially lies in the negative momentum region and shifts toward the positive region as the initial phase changes from  $\phi = 0$  to  $\phi = \pi/2$ . Additionally, the momentum spectrum corresponding to  $\phi = 0$  exhibits an oscillatory pattern.

In contrast, for the high-frequency case ( $\omega = 0.7m$ ), both momentum spectra show a similar shape, but with a significantly higher peak when the initial phase is  $\phi = \pi/2$ . This suggests that the pair production efficiency increases with increasing initial phase, which is attributed to the higher field strength for  $\phi = \pi/2$ . However, the trend reverses in the low-frequency scenario: the peak value for  $\phi = 0$  exceeds that for  $\phi = \pi/2$ . This may be due to the dominance of different pair production mechanisms at different initial phases, resulting from changes in the number of cycles. For instance, in the low-frequency case with  $\phi = \pi/2$ , the strong oscillations observed in the momentum spectrum (see Fig. 8) are characteristic of multiphoton perturbative processes. Conversely, for  $\phi = 0$ , the momentum spectrum of the produced pairs

exhibits a soft Gaussian distribution with approximate bilateral symmetry, which originates from the typical nonperturbative tunneling process.

#### IV. CONCLUSION

In this work, we have systematically investigated vacuum pair production in spatially asymmetric electric fields with time-dependent oscillations using the Dirac-Heisenberg-Wigner (DHW) formalism. Our analysis demonstrates that spatial asymmetry and temporal modulation of electric fields significantly affect the efficiency and characteristics of vacuum pair production. The results indicate that appropriately designed field configurations can enhance pair production rates and allow control over the resulting momentum spectra. These findings offer valuable insights for future high-intensity laser experiments and theoretical studies on the Schwinger effect. The main conclusions are summarized as follows:

- (i) *Spatial asymmetry* enhances pair production rates compared to symmetric field configurations. The highest yields are achieved at large spatial scales ( $\lambda \geq 50 \text{ m}^{-1}$ ), where the electric field becomes effectively quasihomogeneous.
- (ii) *Temporal characteristics* play a critical role in determining the pair production mechanism: low-frequency fields ( $\omega \lesssim 0.3m$ ) favor nonperturbative tunneling, while high-frequency fields ( $\omega \gtrsim 0.4m$ ) lead to multiphoton processes, as evidenced by the emergence of oscillatory structures in the momentum spectra.
- (iii) *Frequency chirp* ( $b$ ) not only breaks the symmetry of the momentum spectra but also enhances the pair production rate. A notable enhancement—up to a ninefold increase in peak number density—was observed for the case  $\omega = 0.7m$  with  $b = 0.5\omega/\tau$ .
- (iv) *Initial phase*  $\phi$  affects interference patterns in the momentum spectrum. In high-frequency regimes, the phase  $\phi = \pi/2$  maximizes the particle yield, due to stronger effective field strength and favorable cycle timing.

These results underscore the importance of spatiotemporal field engineering in the realization and optimization of the Schwinger effect. Future work should extend this analysis to multidimensional field configurations and investigate quantum interference dynamics to further improve the efficiency and controllability of vacuum pair production.

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## DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

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