

Electroweak η_w meson

Gia Dvali,¹ Archil Kobakhidze²,[✉] and Otari Sakhelashvili^{✉2}

¹*Arnold Sommerfeld Center, Ludwig-Maximilians-Universität,
Theresienstraße 37, 80333 München, Germany*
and *Max-Planck-Institut für Physik, Boltzmannstrasse 8, 85748 Garching, Germany*

²*Sydney Consortium for Particle Physics and Cosmology, School of Physics,
The University of Sydney, Sydney, New South Wales 2006, Australia*



(Received 19 August 2024; accepted 22 May 2025; published 9 June 2025)

We argue that the Standard Model is accompanied by a new pseudoscalar degree of freedom, η_w meson, which cancels the topological susceptibility of the electroweak vacuum and gets its mass from this effect. The prediction is based on the analyticity properties of the Chern-Simons correlator combined with the basic features of gravity. Depending on the quality level of the $U(1)_{B+L}$ symmetry, the η_w emerges as a $B + L$ pseudo-Goldstone boson or as a Stückelberg 2-form of the electroweak gauge redundancy. An intriguing scenario of the first category is the emergence of η_w in the form of the phase of a $U(1)_{B+L}$ -violating fermion condensate triggered by the instantons, somewhat similar to the η' meson in QCD. Regardless of its origin, the presence of the η_w meson in the theory appears to be a matter of consistency.

DOI: [10.1103/mxmdm-wb1p](https://doi.org/10.1103/mxmdm-wb1p)

I. INTRODUCTION

In this paper, we argue that the Standard Model is accompanied by a new degree of freedom, the η_w meson, which gets its mass from the topological susceptibility of the vacuum (TSV) of the electroweak theory.

The main evidence for η_w comes from noticing that elimination of the electroweak θ vacuum by the anomalous $B + L$ symmetry of the Standard Model necessarily implies the generation of the massive pole in electroweak TSV. Moreover, this pole has the quantum numbers that fully match the phase of the fermion condensate generated by the electroweak instantons.

The precise nature of the η_w meson depends on the quality of the $U(1)_{B+L}$ symmetry, which is defined as follows. We shall say that the quality of the $U(1)_{B+L}$ symmetry is good if it is explicitly broken exclusively by the electroweak instantons. In the opposite case, we shall say that the quality of the $U(1)_{B+L}$ symmetry is poor. This would be the case if, for example, the $U(1)_{B+L}$ symmetry were to be explicitly broken by some high-dimensional fermion operators generated by physics beyond the Standard Model.

We must stress that the fact of existence of the η_w meson in the spectrum is insensitive to the quality of $U(1)_{B+L}$ symmetry. However, its precise nature can be.

In arriving at the above conclusions, gravity plays an important role of a monitor. At the most conservative level, the role of gravity is in guaranteeing the impossibility of the decoupling of fields at finite Planck mass, M_P . Since this property is a direct consequence of general covariance, we take it as common knowledge rather than an extra assumption. The conclusion about the existence of the η_w particle is reached when the above feature of gravity is superimposed over the following correspondence [1]:

At finite coupling, the nullification of TSV is equivalent to a Higgs phase of the Chern-Simons 3-form.

The above correspondence is rather general and follows from the gauge invariance and the analytic properties of the TSV correlator.

Now, since TSV is nullified by the anomalous $B + L$ symmetry of the Standard Model, the existence of η_w is inevitable since the emergence of a massive pole is the only option for vanishing TSV.

We show that a possible explicit breaking of $B + L$ by high-dimensional operators coming from physics beyond the Standard Model (BSM) cannot change the above conclusion.

Next, we move beyond the basic framework. By imposing further restrictions, such as the incompatibility of TSV

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI. Funded by SCOAP³.

with the requirement of a well-defined S matrix for gravity [2–4], we gain more information about the nature of η_w . The latter requirement, although fully justified, goes beyond the common knowledge category. Using it as a guideline, in case of a poor-quality $U(1)_{B+L}$ symmetry, it motivates one to introduce η_w as a 2-form $B_{\mu\nu}$ that transforms under the electroweak $SU(2)$ gauge redundancy [1].

Concerning the most conservative case, an intriguing question, to which we devote a special discussion, is whether the η_w meson could emerge as a phase of a fermion condensate triggered by the electroweak instantons. For illustrative purposes, in Sec. III B, we explicitly compute the $(B + L)$ -violating condensate within a simplified toy model that carries some relevant features of the electroweak sector of the Standard Model. If the condensate indeed forms, η_w would play the role somewhat analogous to the η' meson of QCD in the limit of a massless up quark. The mass of the latter is generated from the TSV of QCD. Of course, an important difference is that, unlike QCD, $SU(2)_w$ is in the Higgs phase. This triggers a number of open questions about the validity domain and the role of the condensate that we shall discuss.

In summary, regardless of its origin, the η_w meson appears to be a crucial ingredient for a consistent coupling between the Standard Model and gravity.

II. EVIDENCE FOR η_w FROM TOPOLOGICAL SUSCEPTIBILITY

A. General argument

In this subsection, we shall deduce the inevitability of the η_w boson from TSV. Before discussing the $U(1)_{B+L}$ symmetry of the Standard Model, we shall give general arguments connecting the elimination of the θ vacuum with the existence of a pseudoscalar. We shall follow [1,3–8], relying on gauge redundancy and analyticity properties of the spectral representation of TSV.

We start by formulating the physics of the θ vacuum in the language of a topological susceptibility. Let us first consider an $SU(N)$ gauge theory with θ -term included in the action,

$$S_\theta = \int_{3+1} \theta F \tilde{F}, \quad F \tilde{F} \equiv \epsilon^{\mu\nu\alpha\beta} \text{tr} F_{\mu\nu} F_{\alpha\beta}, \quad (1)$$

where $F_{\mu\nu}$ is the standard field strength of the $N \times N$ gluon matrix $A_\mu \equiv A_\mu^c T^c$, with T^c the generators of $SU(N)$ and $c = 1, 2, \dots, N^2 - 1$ the color adjoint index. This term is a total derivative,

$$F \tilde{F} = \epsilon^{\mu\nu\alpha\beta} \partial_\mu C_{\nu\alpha\beta}^{(\text{CS})}, \quad (2)$$

where

$$C_{\mu\nu\alpha}^{(\text{CS})} \equiv \text{tr} \left(A_{[\mu} \partial_\nu A_{\alpha]} + \frac{2}{3} A_{[\mu} A_\nu A_{\alpha]} \right) \quad (3)$$

is the Chern-Simons 3-form. It thereby can be rewritten as a boundary term,

$$S_\theta = \theta \int_{2+1} dX^\mu \wedge dX^\nu \wedge dX^\alpha C_{\mu\nu\alpha}^{(\text{CS})}, \quad (4)$$

where X^μ are the embedding coordinates of the $2 + 1$ -dimensional boundary. Of course, the term is invariant since under the gauge transformation, $A_\mu \rightarrow U(x) A_\mu U^\dagger(x) + U \partial_\mu U^\dagger$ with $U(x) \equiv e^{-i\omega(x)^b T^b}$, the 3-form shifts by an exterior derivative

$$C_{\mu\nu\beta}^{(\text{CS})} \rightarrow C_{\mu\nu\beta}^{(\text{CS})} + \partial_{[\mu} \Omega_{\nu\beta]}, \quad (5)$$

where $\Omega_{\mu\nu} = \text{tr} A_{[\mu} \partial_{\nu]} \omega$.

First, let us assume that θ is a physically observable parameter. In particular, this implies that there exists no anomalous symmetry under which θ can be shifted to zero. For example, this is the case in QCD with no massless quarks as well as in the electroweak theory with explicitly broken $B + L$ symmetry beyond the electroweak anomaly. As it is well known, in both theories the corresponding θ -terms have observable physical effects. A well-known example of an observable quantity in QCD is the electric dipole moment of the neutron.

The physicality of the θ -term is directly linked with the correlator usually referred to as TSV,

$$FT \langle F \tilde{F}, F \tilde{F} \rangle_{p \rightarrow 0} \equiv \lim_{p \rightarrow 0} \int d^4x e^{ipx} \langle T[F \tilde{F}(x), F \tilde{F}(0)] \rangle = \text{const}, \quad (6)$$

where T stands for time ordering, FT stands for Fourier transformation, and p is a four-momentum. The θ -term is physical if the above correlator is nonzero and vice versa. Notice that the correspondence between a nonzero TSV and the existence of physical θ vacua is independent of the specifics of the physics that contributes in TSV. For example, in pure Yang-Mills theories, the well-established contributors are the instantons. However, for the time being, we shall keep the discussion maximally general.

We now come to the following important point. The nonzero value of the expression (6) implies that the Källén-Lehmann spectral representation of the Chern-Simons correlator includes a physical pole at $p^2 = 0$ [1,3,4],

$$FT \langle C^{(\text{CS})}, C^{(\text{CS})} \rangle = \frac{\rho(0)}{p^2} + \sum_{m \neq 0} \frac{\rho(m^2)}{p^2 - m^2}, \quad (7)$$

where $\rho(m^2)$ is a spectral function. The important thing is that $\rho(0) \neq 0$. That is, the physicality of θ is in one-to-one correspondence with the presence of the pole $p^2 = 0$ in (7).

We must stress that, while the correlator in (7) is gauge dependent, its pole structure is not. The massless pole $p^2 = 0$ cannot be gauged away, in full analogy with a physical pole in the correlator for a $U(1)$ photon field.

The gauge independence of the massless pole in (7) can be seen in several ways. First, this can be understood by computation of gauge-invariant force mediated by the 3-form field between the two conserved sources $J^{\alpha\beta\gamma}(x)$, where $\partial_\alpha J^{\alpha\beta\gamma}(x) = 0$. For example,

$$J^{\alpha\beta\gamma}(x) = q \int d^3\xi \delta(x - X(\xi)) \frac{\partial X^\alpha \partial X^\beta \partial X^\gamma}{\partial \xi^a \partial \xi^b \partial \xi^c} \epsilon^{abc} \quad (8)$$

describes a current of a membrane with $(2+1)$ -dimensional world-volume coordinates $\xi^a, a = 0, 1, 2$ and $(3+1)$ -dimensional target space coordinates $X^\alpha, \alpha = 0, 1, 2, 3$. Here q is a constant representing the 3-form charge of the membrane. In particular, such a membrane can also describe the boundary given in (4). For a planar membrane located in the plane $x_3 = L$, the current is $J^{\alpha\beta\gamma}(x) = \delta(x_3 - L) \delta_0^{[\alpha} \delta_1^\beta \delta_2^{\gamma]}$. It is easy to see that the correlator evaluated between two parallel static membranes located in the planes $x_3 = 0$ and $x_3 = L$ gives a long-range linear potential $V(L) \propto q^2 L$, which unambiguously indicated that the 3-form contains a massless field corresponding to the $p^2 = 0$ pole in the correlator (7).

The second way of demonstrating the gauge independence of the massless pole is by introducing the Stückelberg field 2-form field $B_{\mu\nu}$ for the gauge redundancy (5) as was done in [1]. Combined with Stückelberg, the 3-form becomes gauge invariant: $(\tilde{C}_{\mu\nu\beta}^{\text{CS}} \equiv C_{\mu\nu\beta}^{\text{CS}} + f \partial_{[\mu} B_{\nu\beta]})$, where f is a scale. Computation of the gauge-invariant version of the correlator, $FT\langle \tilde{C}^{(\text{CS})}, \tilde{C}^{(\text{CS})} \rangle$ shows that the pole gets shifted to a nonzero value $p^2 = m^2 \propto 1/f^2$, fully controlled by the scale f . Thus, taking $f \rightarrow \infty$, we gradually recover a massless pole in a fully gauge-invariant manner. We shall discuss this version of the theory in more detail later in connection with the gauge formulation of the axion [1].

Thus, in a theory with physical θ vacuum, the operator expansion of $C^{(\text{CS})}$ contains a massless field C . This is an exact statement. Notice that, since $C^{(\text{CS})}$ is a 3-form, its massless entry brings no propagating degrees of freedom. It is important to emphasize that the understanding of TSV in terms of a massless 3-form does not amount to any modification of the theory. It is just an alternative language for accounting the θ vacua. However, this language shall allow us to draw powerful conclusions using Eq. (7) as a guideline.

Let us now assume that some new physics added to the theory makes the previously nonzero TSV (6) zero. Two

different ways of achieving this will be discussed later. The important point is that, regardless of the particular dynamics, the vanishing TSV implies that the spectral representation (7) no longer contains a massless pole. This can only happen in two ways: (1) either the pole gets shifted to a nonzero value $p^2 = m_\eta^2 \neq 0$, or (2) the spectral weight of the massless entry vanishes $\rho(0) = 0$. That is, the massless 3-form field C either becomes massive [1] or decouples [9].

The decoupling option, $\rho(0) = 0$, however, is excluded by gravity. The point is that, at a finite value of the Planck scale M_P , gravity excludes the existence of any fully decoupled 3-form field. Indeed, even if we assume for a moment that, after being canonically normalized, this field is decoupled from all the particle excitations of the gauge sector, it must still couple to gravity. Because of this, the massless 3-form has a physical effect. For example, its field strength contributes to the vacuum energy that sources gravity. Thus, the physical effect of θ vacua would persist gravitationally. This would be in clear contradiction to the fact that θ is unphysical because of zero TSV. Thus, the decoupling of the $p^2 = 0$ entry is prohibited by gravity at finite M_P .

We are thereby left with the sole option that, with any deformation of the theory that makes TSV zero, the would-be massless pole shifts to a massive one, $p^2 = m_\eta^2$. A massive 3-form, however, propagates a single degree of freedom and is a pseudoscalar. This can be interpreted as a 3-form Higgs effect [1]: initially massless 3-form that contains no propagating degree of freedom acquires a single degree of freedom and becomes a propagating pseudoscalar. In conclusion:

Any physics that eliminates TSV, and thus renders θ unphysical, leads to the emergence of a massive pseudoscalar degree of freedom.

B. Two ways of removing TSV

We now discuss the two alternative physics that can ensure the vanishing TSV. As already established by the general argument, the emergence of a massive pseudoscalar is inevitable in both cases. However, the nature of this field is different.

1. Removing TSV via good-quality anomalous $U(1)$ symmetry

Let us assume that we endow the theory with an anomalous $U(1)$ symmetry with the corresponding $U(1)$ current J_μ exhibiting an anomalous divergence,

$$\partial^\mu J_\mu \propto F\tilde{F}. \quad (9)$$

In such a case, the θ -term (1) can be arbitrarily redefined and, in particular, can be set to zero by a proper $U(1)$ transformation,

$$\theta \rightarrow \theta + \text{const.} \quad (10)$$

This implies that the θ parameter must become unphysical.

Thus, the inclusion of an anomalous symmetry must make TSV (6) zero. As we have already established, this implies a 3-form Higgs effect and the corresponding emergence of a massive pseudoscalar [1]. The effect can be viewed as the topological mass generation [5] and represents a general consequence of Eqs. (6) and (9).

Now, matching the quantum numbers and the anomaly properties, it is clear that the above degree of freedom must realize the anomalous $U(1)$ symmetry nonlinearly. Hence, in the case of a good-quality anomalous $U(1)$ symmetry, the pseudoscalar must materialize as a pseudo-Goldstone boson emerging from the spontaneous breaking of the very same $U(1)$ symmetry.

A useful example illustrating this case is the removal of TSV of QCD by a good-quality anomalous symmetry. Such anomalous chiral symmetry in QCD can be obtained either by suppressing the Yukawa coupling constant of one of the quarks or by introducing an anomalous Peccei-Quinn symmetry [10] via the extension of the field content. In both cases, the θ -term can be rotated away by the corresponding anomalous $U(1)$ transformation. As a result, the θ -term becomes unphysical. Of course, simultaneously, the TSV vanishes.

Thereby, as discussed in [1], in both cases, we end up with a 3-form Higgs effect with the required pseudo-Goldstone degree of freedom automatically provided in both realizations of the anomalous $U(1)$ symmetry. In the case of the Peccei-Quinn scenario, this degree of freedom is an axion [11,12], which comes as a Goldstone boson of the enlarged scalar sector of the theory. Likewise, in the case of a massless quark, the role of the axion is assumed by the η' meson of QCD.

In the latter case, the degree of freedom emerges as the phase of the 't Hooft determinant [13],

$$\mathcal{L}_{\text{tHooft}} \propto |\det(\bar{q}_L q_R)| e^{i\left(\frac{\eta'}{f_\eta} - \theta\right)}, \quad (11)$$

where q_L and q_R are left- and right-handed components of N_f flavors of quark fields and f_η is the decay constant. This determinant has a nonzero vacuum expectation value (VEV). Correspondingly, η' emerges as a pseudo-Goldstone boson of the chiral symmetry,

$$q_L \rightarrow e^{i\alpha} q_L, \quad q_R \rightarrow e^{-i\alpha} q_R. \quad (12)$$

This symmetry is broken both spontaneously (via the quark condensate) as well as explicitly (via the chiral anomaly). Because of the anomaly, under the chiral transformation θ shifts as

$$\theta \rightarrow \theta + 2N_f \alpha, \quad (13)$$

which tells us that it is unphysical.

Of course, the above is fully matched by the dynamical picture. Dynamically, the vacuum is achieved by the minimization of the 't Hooft determinant (11). In the minimum, the θ -term is exactly canceled by the VEV of η' . Correspondingly, the generation of the η' mass in 't Hooft language is through the presence of the 't Hooft determinant and its nonzero VEV.

An alternative way of understanding the generation of the η' mass from TSV is via the Witten-Veneziano mechanism [14,15]. Both languages can be described as a 3-form Higgs effect, in which the η' is eaten up by the 3-form and forms a massive pseudoscalar [1].

2. Removing TSV via a gauge 2-form

Let us now consider the case in which either there exists no anomalous $U(1)$ symmetry or its quality is poor. In such a case, the TSV can be removed by the mechanism of a 2-form gauge axion introduced in [1].

The key ingredient is a 2-form field $B_{\mu\nu}$ that transforms under the $SU(N)$ gauge symmetry (5) as

$$B_{\mu\nu} \rightarrow B_{\mu\nu} + \frac{1}{f} \Omega_{\nu\beta}, \quad (14)$$

where f is a scale. Notice that in this formulation $B_{\mu\nu}$ is positioned as an intrinsic part of the gauge redundancy, without any reference to a global symmetry.

Because of this, it enters the Lagrangian through the following gauge-invariant combination, $\bar{C}_{\mu\nu\beta}^{\text{CS}} \equiv C_{\mu\nu\beta}^{\text{CS}} + f\partial_{[\mu} B_{\nu\beta]}$. The lowest-order term in the Lagrangian is

$$\frac{1}{f^2} (C_{\mu\nu\beta}^{\text{CS}} + f\partial_{[\mu} B_{\nu\beta]})^2. \quad (15)$$

From this form, it is clear that the scale f plays the role of the cutoff. Without loss of generality, we can tie it to the Planck mass $f = M_P$. This case has the advantage of being protected by the gauge symmetry against arbitrary deformation to all orders in the operator expansion [1,3,4,16]. Correspondingly, in this formulation, the axion has exact quality.

The vanishing of TSV to all orders can be understood as the result of the 3-form Higgs effect [1]. Indeed, without $B_{\mu\nu}$, the TSV of the $SU(N)$ is nonzero. Therefore, C^{CS} contains a massless 3-form, C ,

$$C^{\text{CS}} = \Lambda^2 C + \text{heavy modes}, \quad (16)$$

where Λ is the scale of TSV. All the $SU(N)$ fields can be integrated out, and the resulting EFT is

$$\mathcal{K}(E) + \frac{1}{f^2} (\Lambda^2 C_{\mu\nu\beta} + f \partial_{[\mu} B_{\nu\beta]})^2, \quad (17)$$

where $\mathcal{K}(E)$ is an algebraic function of the fields strength $E \equiv \partial_\alpha C_{\mu\nu\beta} \epsilon^{\alpha\mu\nu\beta}$ [1].¹ The higher-order derivative terms play no role in the vacuum structure and can be safely ignored. Correspondingly, the vacuum derived from (17) is exact.

It is clear that the 2-form $B_{\mu\nu}$ acts as a Stückelberg field for C , and the two combine into a massive 3-form $\bar{C}_{\mu\nu\beta}^{\text{CS}}$, which is equivalent to a massive pseudoscalar. Obviously, this form is gauge invariant. In the corresponding gauge-invariant counterpart of (7) the would-be massless pole is shifted to $p^2 = \Lambda^4/f^2$ and TSV vanishes.

As already discussed previously, this construction also shows that the existence of a massless pole in the correlator (7) of the original theory with nonzero TSV is a gauge-independent statement. This massless pole can be recovered in the above gauge-invariant correlator of $\bar{C}_{\mu\nu\beta}^{\text{CS}}$ by taking the limit $f \rightarrow \infty$ in which the Stückelberg $B_{\mu\nu}$ decouples.

C. Electroweak sector

We shall now apply the above understanding to the weak sector of the Standard Model. The main difference is that, unlike the color group of QCD, the $SU(2)$ weak symmetry is in the Higgs phase, due to a nonzero VEV of the Higgs doublet $\langle \Phi \rangle = v$. Despite this, in the absence of fermions, the TSV of $SU(2)$ is nonzero. Correspondingly, the weak θ is physical [17,18]. In other words, the mass gaps of the $SU(2)$ gauge bosons generated by the Higgs effect, do not eliminate the topological structure of the vacuum. Rather, they only constrain the range of the instantons that effectively contribute to TSV (6). The effect is purely quantitative.

Correspondingly, as in QCD, the θ -vacuum structure of the $SU(2)$ weak sector is controlled by TSV (6) and correspondingly by the pole at $p^2 = 0$ in the Chern-Simons correlator (7).

Let us now discuss the effect of fermions. Per generation, they consists of four $SU(2)$ doublets of left-handed Weyl fermions: three colors of the quark doublets and a single lepton doublet,

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad \ell_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}. \quad (18)$$

In addition, the theory contains their right-handed counterparts u_R, d_R, e_R, ν_R , which are singlets of $SU(2)$. The color and generation indices are not shown explicitly.

We assume that all these fermions (including neutrinos) form massive Dirac fermions due to their Yukawa couplings with the Higgs doublet $\Phi = (\phi^+, \phi^0)^T$,

¹Notice that, since E is an exterior derivative, it is the same for $\bar{C}_{\mu\nu\beta}^{\text{CS}}$ and $C_{\mu\nu\beta}^{\text{CS}}$.

$$\mathcal{L} = y_u \Phi \bar{q}_L u_R + y_\nu \Phi \bar{\ell}_L \nu_R + y_d \Phi^c \bar{q}_L d_R + y_e \Phi^c \bar{\ell}_L e_R, \quad (19)$$

where Φ^c is a conjugated doublet.

Despite generating a mass gap, these Yukawa couplings preserve the chiral $U(1)_{B+L}$ symmetry,

$$\begin{aligned} (q_L, u_R, d_R) &\rightarrow e^{i\alpha} (q_L, u_R, d_R), \\ (\ell_L, e_R, \nu_R) &\rightarrow e^{i3\alpha} (\ell_L, e_R, \nu_R). \end{aligned} \quad (20)$$

This symmetry is anomalous with respect to $SU(2)$, resulting into a corresponding shift of θ .

Although $U(1)_{B+L}$ symmetry has a good quality at the level of the Standard Model, we do not know whether this quality holds in general. In particular, it is not excluded that $U(1)_{B+L}$ symmetry is broken by high-dimensional fermion operators. They generically appear in various extensions of the Standard Model. A well-known example of this sort is grand unification. Since the nature of η_w depends on the quality of $U(1)_{B+L}$, we consider the two options separately.

III. GOOD-QUALITY $U(1)_{B+L}$

We start with the option that the quality of $U(1)_{B+L}$ is good, implying that the sole source of its explicit breaking is the electroweak anomaly. In such a case $U(1)_{B+L}$ renders the electroweak θ unphysical [17,18].² The effect is very similar to the one of a chiral symmetry of a massless quark on the θ -term of QCD. In both cases, as a result of the anomalous symmetry, the TSV vanishes.

Correspondingly, our general arguments apply to $U(1)_{B+L}$. The elimination of weak θ means that the massless pole in the correlator (7) is removed. As we already discussed, this implies the existence of a new pseudoscalar degree of freedom η_w , which realizes the $U(1)_{B+L}$ symmetry nonlinearly. The remaining question is the origin of this pseudo-Goldstone.

A. External η_w

One option is that the η_w boson is an external degree of freedom. This possibility looks especially reasonable in light of the fact that coupling to gravity was a crucial factor in justifying its existence. The realization of this scenario is straightforward. The η_w meson can be introduced as a Goldstone phase degree of freedom of a complex scalar field $\Phi = |\Phi| e^{i\frac{\eta_w}{f}}$ that breaks $U(1)_{B+L}$ symmetry spontaneously. This is a full analog of the Peccei-Quinn scenario in QCD. Such generalizations of the Peccei-Quinn axion have been considered previously in the literature [1,19].

The field Φ can couple to quarks and leptons via an arbitrary operator with nonzero $U(1)_{B+L}$ charge and can be

²Notice that an explicit breaking of either B or L symmetries (e.g., through the Majorana neutrino masses) does not change the situation qualitatively. The weak θ remains unobservable due to the remaining L or B rotations, respectively.

assigned an opposite charge. For example, the operator can be chosen as $\Phi q q l$. In this case, under the $B + L$ symmetry (20) Φ transforms as $\Phi \rightarrow e^{-i6\alpha}\Phi$. In other words, in this formulation good-quality $U(1)_{B+L}$ is not a symmetry only of the Standard Model species but is necessarily shared with an external field Φ .

B. The fermion condensate

The question that we now would like to ask is whether the η_w boson could emerge from the electroweak physics without any need for its extensions. Of course, in such a case it can only emerge as a collective degree of freedom. The composite operator that matches the desired quantum numbers and the transformation properties is the phase of the 't Hooft determinant, which is generated by the $SU(2)_w$ instantons. If this determinant would have a nonzero VEV, the corresponding phase would be an obvious candidate for η_w .

In order to gauge the plausibility of such a scenario, we shall explicitly calculate the $(B + L)$ -violating fermion condensate. However, we shall perform the calculation within a toy model that represents a simplified version of the electroweak sector of the Standard Model. Namely, we shall get rid of the color and the hypercharge, thereby reducing the gauge sector to a weak $SU(2)_w$ group. Correspondingly, we reduce the fermion content to two $SU(2)_w$ doublets of left-handed Weyl fermions (18) and their singlet right-handed partners, removing the color and generation quantum numbers. Basically, we shrink the fermion content of the Standard Model to a single generation of leptons and a single color of quarks. The Yukawa sector of the Lagrangian is still described by (19), modulo the above simplification.

Next, we switch to the Euclidean formulation. For convenience, we combine fermions into an eight-component spinor $\Psi = (\psi, \phi)^T$ [17,18], where

$$\psi = q_L + \ell_R^c, \quad \phi = \begin{pmatrix} u_R \\ d_R \end{pmatrix} + \begin{pmatrix} e_L^c \\ -\nu_L^c \end{pmatrix}. \quad (21)$$

Note, ψ comprises gauge $SU(2)$ -doublet spinors, while ϕ contains $SU(2)$ -singlet ones.

With the above notations, the fermionic Lagrangian in Euclidean space can be written in a compact form as³

$$\mathcal{L}_F = \Psi^\dagger \hat{\mathcal{D}} \Psi, \quad (22)$$

where $\hat{\mathcal{D}}$ is given by

³We recall that, when turning to the Euclidean space, a Minkowski spinor $\phi \rightarrow -i\phi^\dagger$ ($\phi_{L,R} \rightarrow -i\phi_{R,L}^\dagger$), whereas the Euclidean spinor $\phi^\dagger$ is an independent field rather than a complex conjugated field to ϕ . The Euclidean γ matrices are defined such that $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$. For further conventions, see, e.g., [20].

$$\begin{pmatrix} -i\cancel{D} & i\epsilon M_\ell^* \epsilon P_L - iM_q P_R \\ i\epsilon M_\ell^T \epsilon P_R - iM_q^\dagger P_L & -i\cancel{D} \end{pmatrix}. \quad (23)$$

Here, $\cancel{D} = \gamma_\mu(\partial_\mu - i\hat{W}_\mu)$; $P_{L,R} = (1 \pm \gamma_5)/2$ and M_q and M_ℓ embody the quark-Higgs and lepton-Higgs Yukawa interactions, respectively,

$$M_q = \begin{pmatrix} y_u \phi^{0*}, & y_d \phi^+ \\ -y_u \phi^{+*}, & y_d \phi^0 \end{pmatrix}; \quad (24)$$

M_ℓ is obtained from M_q by replacing the quark Yukawa couplings with the corresponding leptonic ones: $y_{u,d} \rightarrow y_{\nu,e}$.

The $U(1)_{B+L}$ symmetry (20) of the full Standard Model, with a proper normalization, translates as the following global transformation:

$$\Psi \rightarrow e^{i\alpha\Gamma_5/2}\Psi, \quad \Psi^\dagger \rightarrow \Psi^\dagger e^{i\alpha\Gamma_5/2}, \quad (25)$$

which leaves the Lagrangian (22) invariant. The generalized chirality operator has the form

$$\Gamma_5 = \begin{pmatrix} \gamma_5, & 0 \\ 0, & -\gamma_5 \end{pmatrix}. \quad (26)$$

Hence, both left- and right-handed quarks and leptons have positive Γ_5 chirality, while their antiparticles carry the negative Γ_5 chirality.

The nonperturbative sector in our model, like in the full electroweak theory, is dominated by $\nu = \pm 1$ one (anti) instanton contributions. These are exponentially suppressed relative to the topologically trivial sector, which is dominated by the perturbative physics. This is because, in the Higgs phase, the weak $SU(2)$ instantons are constrained and thereby are screened at distances larger than the electroweak length $\sim 1/v$ [13,21].

As long as the theory remains weakly coupled, the noninteracting ideal instanton gas is expected to be an excellent approximation. The explicit field configurations that describe the constrained (anti)instantons with the unit topological charge can be found in [20]. Also, notice that for nonzero Yukawa couplings, $y_{u,d}, y_{\nu,e}$, the theory is fully gapped, with no massless degrees of freedom. However, the massive fermions nevertheless exhibit normalizable zero modes in the background of the electroweak instantons [22].

Within this setup, the $B + L$ -violating fermionic condensate $\langle \Psi^\dagger(x)\Psi(x) \rangle$ can be straightforwardly computed. It is represented by the fermion propagator in the background of the instanton gas, $(\hat{\mathcal{D}} + i\mu)^{-1}$ averaged over the instanton configurations and positions as well as zero and massive modes of gauge and Higgs and fermion fields,⁴

⁴In dealing with integration over fermions, we introduce a regulator parameter μ , by adding the $(B + L)$ -violating term $i\mu\Psi^\dagger\Psi$ to the Lagrangian. It must be taken to 0 at the final stage of computation.

$$\langle \Psi^\dagger(x) \Psi(x) \rangle = \lim_{\mu \rightarrow 0} \int \frac{d^4 z d\rho}{\rho^5} D(\rho) \langle x | (\hat{D} + i\mu)^{-1} | x \rangle. \quad (27)$$

In the above equation, the quantity

$$D(\rho) = \left(\frac{2\pi}{\alpha(\rho)} \right)^4 e^{-\frac{2\pi}{\alpha(\rho)} - 2\pi^2 v^2 \rho^2} \rho \mu \quad (28)$$

is interpreted as the density of instantons of size ρ in the presence of fermion zero modes and $\alpha(\rho)$ is the effective $SU(2)$ gauge coupling constant evaluated at the scale ρ . As discussed, the weak θ is absorbed in the condensate phase and omitted here.

In order to show that the above condensate is indeed nonzero, we inspect the fermion propagator $(\hat{D} + i\mu)^{-1}$ and separate it into the propagator for zero modes P_0 and the propagator for massive modes Δ [23],

$$\frac{1}{\hat{D} + i\mu} = \frac{P_0}{i\mu} + \Delta - i\mu\Delta^2 + \mathcal{O}(\mu^2). \quad (29)$$

Plugging this propagator into Eq. (27), we observe that the only contribution that survives the $\mu \rightarrow 0$ limit comes from the fermion zero modes. After evaluating the integrals, we obtain

$$\langle \Psi^\dagger(x) \Psi(x) \rangle \simeq -iv^3 \left(\frac{2\pi}{\alpha} \right)^4 e^{-\frac{2\pi}{\alpha}}. \quad (30)$$

In the course of the above calculations, we have used $\langle x | P_0 | x \rangle = \Psi_0^\dagger(x-z) \Psi_0(x-z)$ and the normalization of the zero mode wave function: $\int d^4 x \Psi_0^\dagger(x) \Psi_0(x) = 1$. The condensate in Minkowski space is related to the one calculated in Euclidean space as $\langle \bar{\Psi} \Psi \rangle = -i \langle \Psi^\dagger \Psi \rangle$ and hence is real.

The fermion condensate provides an internal mechanism for spontaneous breaking of $U(1)_{B+L}$ symmetry. The corresponding pseudo-Goldstone excitation η_w is the right candidate for eliminating the massless pole in the correlator (7). In order to see this explicitly, let us consider the following anomalous Ward identity (see, e.g., [24] in the context of QCD) that follows from evaluating the vacuum expectation value of the variation $\delta(\Psi^\dagger \Gamma_5 \Psi) = 2i\Psi^\dagger \Psi$ under the generalized chiral transformations (25):

$$\begin{aligned} \int d^4 x \left\langle \left(i\mu \Psi^\dagger \Gamma_5 \Psi - \frac{\alpha}{4\pi} F \tilde{F} \right) (x), \Psi^\dagger \Gamma_5 \Psi(0) \right\rangle \\ = i \langle \Psi^\dagger \Psi \rangle \neq 0. \end{aligned} \quad (31)$$

The first term on the lhs of (31) comes from the variation of the classical Lagrangian, while the second term originates from the anomaly. In the absence of this anomalous term, using the relation $\partial_\mu J_{B+L}^\mu = 2i\mu \Psi^\dagger \Gamma_5 \Psi$, one immediately infers the existence of the massless pole corresponding to

the Goldstone boson. The anomaly contribution ensures that no massless particles exist in the spectrum. Taking $\mu \rightarrow 0$ in (31), we obtain

$$\int d^4 x \langle F \tilde{F}(x), \Psi^\dagger \Gamma_5 \Psi \rangle_{p=0} \propto \langle \Psi^\dagger \Psi \rangle. \quad (32)$$

It is therefore clear that, for satisfying the above identity, there must exist a state $|\eta\rangle$ such that $\langle 0 | F \tilde{F} | \eta \rangle = B(p) \neq 0$ and $\langle \eta | \Psi^\dagger \Gamma_5 \Psi | 0 \rangle = C(p) \neq 0$. In turn, this implies that the topological susceptibility contains a massive pole in its Källén-Lehmann spectral decomposition. Equivalently, the Chern-Simons 3-form becomes a massive propagating field [1],

$$FT \langle C^{(\text{CS})}, C^{(\text{CS})} \rangle = \frac{\rho(0)}{p^2 - m_\eta^2} + \dots, \quad (33)$$

where $\rho(0) = |B(0)|^2 \neq 0$.

A comment on the validity of the dilute instanton gas approximation employed in the above calculations is in order. It is certainly valid at energy scales below the sphaleron threshold $\sim 2\pi M_W/\alpha$. Above this threshold, multiple vector boson exchanges between instantons must be taken into account, and the series must be resummed. In this regime, the process likely becomes dominated by the instanton-anti-instanton bound states rather than the individual noninteracting instantons. Since such bound states carry the trivial topological charge, we expect fermion zero modes to delocalize and hence the fermion condensate to “evaporate.” We thus may regard the sphaleron threshold as an absolute upper bound on the ultraviolet scale beyond which the emergent η_w does not exist and the relevant effects are described by multiparticle states.

C. Physical meaning and validity of the fermion condensate

Let us reflect on the meaning of the above computation for our proposal. First, we can take it as an indication that a gauge theory that is in the Higgs phase, in principle, could accommodate the required η_w -type particle. Namely, if the condensate exists, η_w emerges as its phase. As already noted, this is somewhat analogous to the emergence of the η' meson as of the phase of the quark condensate in ordinary QCD.

However, this analogy must be weighted very carefully. For one, in the present case, the theory is in the Higgs phase, and fermions are not confined. This creates a set of questions. In particular, if η_w is a collective mode, the domain of validity of its EFT must impose further restrictions on the parameters, such as the Yukawa couplings. What these conditions are and whether they can be satisfied within a more realistic setup have to be studied separately.

Another obvious question is, what is the role of gravity? On one hand, in the case of an good-quality $U(1)_{B+L}$

symmetry, gravity demands the existence η_w in form of a pseudo-Goldstone boson of $U(1)_{B+L}$. However, if the condensate can be provided entirely by the nonperturbative electroweak $SU(2)$ dynamics, the emergence of η_w appears to be guaranteed without gravity. Such a scenario exhibits no *a priori* inconsistency. Indeed, if an interacting η_w exists already in the limit of $M_P = \infty$, it easily accommodates the demands of gravity also for a finite M_P . However, at least at the level of EFT, such a scenario appears to be a lucky coincidence in which the consistency demands of gravity are met by the EFT already for $M_P = \infty$.

Notably, similar precedents do exist. An example is the cancellation of the gravitational anomalies within the low-energy EFT. Such cancellations must take place for arbitrary values of M_P , including $M_P = \infty$.

A more specific example, which is directly relevant to the present case, is the demand for the cancellation of the chiral gravitational anomaly among the spin-1/2 fermions [28,29].⁵ As argued in the latter work, this condition is imposed by the Eguchi-Hanson instantons [26,27] since such fermions give no zero modes in their background [28,29]. Because of this, the chiral gravitational anomaly must be taken up by a spin-3/2 fermion. This leads to the formation of their condensate [30,31], which breaks the anomalous R symmetry spontaneously. The R axion emerges as the phase of the gravitino condensate.

Despite some striking similarities, the above precedent cannot be directly transported to the case of the Higgsed gauge symmetries and especially to the Standard Model. Even if in a toy model, in which the fermion masses are free parameters, the fermion condensation takes place, it is far from being clear whether this is possible within the Standard Model.⁶

Once again, we would like to stress that the question of the possible emergence of η_w from the fermion condensate changes nothing about the necessity of its existence and its finite-strength coupling to the electroweak TSV at finite M_P .

IV. POOR-QUALITY $B+L$ AND 2-FORM η_w

So far, we have been considering the situation in which the explicit breaking of the $U(1)_{B+L}$ symmetry was coming exclusively from the $SU(2)$ instantons. In such a case, irrespective of its precise origin, the particle η_w represents a pseudo-Goldstone boson of $B+L$ symmetry.

We shall now discuss how the situation changes if the $U(1)_{B+L}$ symmetry is of poor quality; that is, if we assume that this symmetry is explicitly broken beyond the

electroweak anomaly. We shall split the discussion in the following two stages.

First, without any extra assumptions, we show that the existence of η_w is not jeopardized by the quality of $U(1)_{B+L}$ symmetry. Next, we show that, by employing certain well-justified assumptions about gravity, one can go further and motivate a gauge formulation of η_w .

A. Insensitivity of η_w toward $B+L$ breaking

In order to understand the effect of this breaking on η_w , we notice that the explicit breaking of $U(1)_{B+L}$ requires physics from BSM, since, as it is well known, among the Standard Model fields there exist no gauge-invariant renormalizable operators that can break $U(1)_{B+L}$. At the same time, we know from the existing experimental constraints that $B+L$ -violating high-dimensional operators must be extremely suppressed. Equipped with this knowledge, it is easy to see that the existence of the η_w meson, derived in the limit of exact quality $B+L$ symmetry, is insensitive toward any phenomenologically acceptable explicit breaking of this symmetry.

For example, the $U(1)_{B+L}$ symmetry can be explicitly broken by dimension-six operators of the type,

$$\frac{1}{M^2} qqql, \quad (34)$$

where M is the scale of some BSM physics. The well-known phenomenological constraints on baryon and lepton number-violating processes imply that the scale M has to be extremely high, $M > 10^{16}$ GeV. This immediately tells us that the effect of such operators on the η_w meson is totally negligible. For example, the relative correction to the fermion condensate calculated for $M = \infty$ (i.e., for exact quality $B+L$) is suppressed by powers of the scale M . It is obvious that, for all practical purposes, this correction is irrelevant.

We thus conclude that the explicit breaking of $U(1)_{B+L}$ symmetry cannot jeopardize the very existence of the η_w meson deduced in the limit of exact quality $B+L$ symmetry.

B. The case from gravity for gauge realization of η_w

There exists an additional (and fully independent) reasoning which in the case of a poor-quality $B+L$ symmetry not only suggests the necessity of the η_w meson but also makes the case for its gauge realization. This reasoning is based on more fundamental knowledge about gravity. Although it relies on different assumptions, it does not affect the previous conclusion about the existence of η_w . Rather, it provides additional supporting evidence for it and at the same time makes the case for its fully gauge-invariant realization. This reasoning goes as follows.

In the case of a poor-quality $B+L$ symmetry, the TSV cannot be fully nullified by the anomalous symmetry that is

⁵On a separate note, this requirement can have interesting phenomenological implications for Standard Model neutrinos, such as the presence of their right-handed partners.

⁶Further analysis performed recently indicates the validity of the fermion condensate in the full Standard Model.

explicitly broken beyond the anomaly by the operators of the type (34). Correspondingly, the θ vacua, although highly suppressed in energy, remain physical.

This, however, is incompatible with the S -matrix formulation of gravity [2–4]. This is due to the fact [32] that the S -matrix formulation, currently the only existing formulation of quantum gravity, is incompatible with the vacua with nonasymptotically flat cosmologies. These include the de Sitter vacua [33–35] as well as any anti-de Sitter-type vacua leading to big crunch cosmologies [3,4]. Correspondingly, the θ vacua must be eliminated. However, this is not possible via a $U(1)_{B+L}$ Goldstone, since this symmetry is of poor quality. Instead, the goal has to be achieved via the introduction of η_w in the form of a gauge axion [1].

As we already discussed, in this scenario, we introduce η_w as a 2-form field $B_{\mu\nu}$ that transforms under the $SU(2)$ gauge symmetry (5) as (14). The construction follows the steps that were already outlined for the generic $SU(N)$ symmetry. The 2-form $B_{\mu\nu}$ plays the role of the Stückelberg field that compensates the $SU(2)$ gauge shift (5) of C . This puts C in the Higgs phase, shifting the pole in the correlator (7) to $p^2 \neq 0$. Because of the protection by the gauge symmetry [(5) and (14)], the $B_{\mu\nu}$ realization of η_w guarantees that the electroweak θ vacuum is nullified to all orders in operator expansion [1,3,4,16].

Notice that, although at the level of the low-energy EFT, the $B_{\mu\nu}$ formulation can be dualized into a pseudoscalar axion with an arbitrary potential [1], the UV completion of the latter in the form of a Goldstone phase of a complex field breaks duality, explaining the stability of $B_{\mu\nu}$ quality relative to the Goldstone case [3].

V. THE DOUBLE ROLE OF GRAVITY

We are learning that gravity provides a dual motivation for the existence of η_w . The concrete arguments depend on the status of $U(1)_{B+L}$ symmetry and on how far one is willing to take the gravitational constraints.

At most conservative level of reasoning, it suffices to employ the very basic feature of gravity: its universal nature. This ensures that at a finite value of M_P no fully decoupled fields can exist in the theory. As already said, since this feature is a direct consequence of general covariance, we regard it as a common sense rather than an extra assumption.

However, for the full clarity, let us elaborate on this topic. It is well known that by the power of general covariance, the Einstein gravity couples to all canonically normalized fields with a universal strength set by $1/M_P$. In particular, this applies to a massless 3-form field. Gravitational effects of this field are well understood (see, e.g., [36–39]). Applied to the present case, this feature guarantees the impossibility of decoupling of a massless $p^2 = 0$ pole from the TSV of a gauge theory.

Notice that this nondecoupling argument is very different from the so-called weak gravity conjecture [40], which independently leads to the same conclusion. Indeed, this conjecture implies that gravity must be the weakest force. Although, initially formulated for Abelian $U(1)$ gauge forces, if applied to the force mediated by a massless 3-form, it would also exclude the possibility of decoupling at finite M_P .

As already explained, the impossibility of decoupling of the 3-form leaves the only option for vanishing of the TSV: the shift of the pole toward a nonzero mass, $p^2 = m_\eta^2$. This leads to the existence of η_w as of $U(1)_{B+L}$ pseudo-Goldstone.

The existence of η_w is insensitive to the quality of $B + L$ symmetry and persists even if the symmetry is of poor quality, i.e., it is explicitly broken by the sources beyond the $SU(2)$ anomaly. For example, the explicit breaking can originate from higher-dimensional operators of the type (34).

As already discussed, due to its extremely suppressed strength, such breaking cannot jeopardize the existence of η_w , since the corrections to the effects generated by the electroweak instantons are totally negligible. Correspondingly, the pseudo-Goldstone nature of η_w derived for $M = \infty$ is unaffected.

What changes, however, is that the pseudo-Goldstone η_w cannot fully cancel the electroweak θ -term. In particular, this is clear from the fact that the fermionic zero modes in the instanton background become gapped. Correspondingly, the θ vacuum, although extremely suppressed, is physical.

As explained, by employing the additional requirement of a well-defined S matrix, we can obtain further restrictions on the nature of η_w . Since in this case the quality requirement is exact, it is most natural that η_w is introduced as the 2-form $B_{\mu\nu}$, which transforms as a Stückelberg under the $SU(2)$ gauge symmetry [1]. In this case, no anomalous global symmetry is required. The θ vacua of $SU(2)$ theory are absent in all orders in the operator expansion.

A few comments are in order. First, depending on the level of $U(1)_{B+L}$ quality, external η_w can coexist with the phase of the condensate and mix with it. This is similar to the mixing between the hidden axion and the η' meson in QCD. Second, gravity demands that the mass of the proper η_w must be generated exclusively from the electroweak TSV. This follows from the S -matrix requirement of exact vanishing of TSV [3], as well as from the requirement that spin-1/2 fermions must not contribute to the gravitational anomaly due to the absence of their zero modes in the Eguchi-Hanson instantons [25].

VI. DISCUSSIONS

In conclusion, in the present paper, we have provided different evidences that lead us, as the only rational

explanation, to the prediction of a new particle, the η_w boson, in the electroweak sector of the Standard Model. The key ingredients are the presence of the massive pole in the TSV correlator on one hand and the presence of the $B + L$ -violating condensate with matching quantum numbers on the other. Both ingredients were achieved by using the well-accepted tools such as the Ward identities, gauge invariance, and instanton calculus, without going beyond the validity of approximations used (e.g., such as the dilute instanton gas). At the same time, gravity played the role of an additional useful “spectator” tool that enabled the monitoring of the nondecoupling of the pole in TSV.

In light of our analysis, under the above conservative assumptions that are organic for the currently accepted framework, the existence of the η_w boson comes up as a matter of consistency for the topological structure of the Standard Model vacuum and its embedding in gravity.

A highly intriguing possibility is the emergence of η_w in the form of the phase of the $U(1)_{B+L}$ -violating fermionic condensate of quarks and leptons. As a step in this direction, we performed an illustrative computation in a toy version of the Standard Model, which appears to support the generation of the fermion condensate by the instantons. However, several open questions remain. Namely, the role of gravity and the range of validity of EFT of η_w , as well as the extrapolation of the results to a fully realistic version of the Standard Model, must be further scrutinized.

The phenomenological and cosmological implications of η_w depend on its decay constant f . Since the mass of η_w is generated from the electroweak TSV, for all the reasonable values of the scale f , η_w is expected to be an extraordinarily light and weakly interacting particle. In the case of emergent η_w , the phenomenological relevant parameters can, in principle, be extracted from two-instanton correlators, employing the formalism of [41].

Irrespective of the actual origin, the (pseudo)Goldstone nature of η_w and the anomaly relations dictate its interactions with electroweak gauge bosons, which would be suppressed by $\propto 1/f$. Note, however, that coupling of η_w to two photons is absent due to the absence of the mixed $B + L$ -electromagnetic anomaly. The interactions with fermions that are generated nonperturbatively are further suppressed by a large exponential factor stemming from instantons. Therefore, the most significant coupling of η_w to

quarks and leptons emerges at (effectively) a two-loop level through the loop of massive electroweak gauge bosons. This may induce rare decays of mesons with the characteristic signature of missing energy carried away by η_w . These processes can be distinguished from the Standard Model background processes where the missing energy is carried away by neutrinos if f , as we expect, is not far from the electroweak scale.

Finally, going beyond the minimal knowledge and making more advanced (although fully justified) assumptions, such as the necessity of a well-defined S matrix in gravity, allows us to further restrict the nature of η_w motivating its gauge formulation *à la* gauge axion [1]. However, the level of minimalism in the above two steps must be distinguished very clearly.

ACKNOWLEDGMENTS

We would like to thank Misha Shaposhnikov, for discussions on fermionic zero modes and condensate, and Lasha Berezhiani, for the discussion on the validity of the instanton gas approximation. O. S. would like to thank the Arnold Sommerfeld Center at LMU, the Max-Planck-Institute for Physics, Munich, and MITP Mainz (Cluster of Excellence PRISMA+ Project ID 390831469) for the hospitality extended during the completion of this work. The work of G. D. was supported in part by the Humboldt Foundation under the Humboldt Professorship Award, by the European Research Council Gravities Horizon Grant AO No. 850 173-6, by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany’s Excellence Strategy—EXC-2111–390814868, Germany’s Excellence Strategy under Excellence Cluster Origins EXC 2094–390783311. The work of A. K. and O. S. was partially supported by the Australian Research Council under the Discovery Projects Grants No. DP210101636 and No. DP220101721. Funded by the European Union.

The views and opinions expressed herein are those of the authors only and do not necessarily reflect those of the European Union or European Research Council. Neither the European Union nor the granting authority can be held responsible for them.

-
- [1] G. Dvali, Three-form gauging of axion symmetries and gravity, [arXiv:hep-th/0507215](#).
 - [2] G. Dvali, C. Gomez, and S. Zell, A proof of the axion?, [arXiv:1811.03079](#).
 - [3] G. Dvali, Strong-CP with and without gravity, [arXiv:2209.14219](#).

- [4] G. Dvali, The role of gravity in naturalness versus consistency: Strong-CP and dark energy, *Phil. Trans. R. Soc. A* **382**, 20230084 (2023).
- [5] G. Dvali, R. Jackiw, and S. Y. Pi, Topological mass generation in four dimensions, *Phys. Rev. Lett.* **96**, 081602 (2006).

- [6] G. Dvali, S. Folkerts, and A. Franca, How neutrino protects the axion, *Phys. Rev. D* **89**, 105025 (2014).
- [7] G. Dvali and L. Funcke, Small neutrino masses from gravitational θ -term, *Phys. Rev. D* **93**, 113002 (2016).
- [8] G. Dvali, Topological origin of chiral symmetry breaking in QCD and in gravity, *arXiv:1705.06317*.
- [9] A. Hebecker and P. Henkenjohann, Gauge and gravitational instantons: From 3-forms and fermions to weak gravity and flat axion potentials, *J. High Energy Phys.* **09** (2019) 038; G. Dvali (private communications).
- [10] R. D. Peccei and H. R. Quinn, CP conservation in the presence of instantons, *Phys. Rev. Lett.* **38**, 1440 (1977).
- [11] S. Weinberg, A new light boson?, *Phys. Rev. Lett.* **40**, 223 (1978).
- [12] F. Wilczek, Problem of strong P and T invariance in the presence of instantons, *Phys. Rev. Lett.* **40**, 279 (1978).
- [13] G. 't Hooft, Computation of the quantum effects due to a four-dimensional pseudoparticle, *Phys. Rev. D* **14**, 3432 (1976); **18**, 2199(E) (1978).
- [14] E. Witten, Current algebra theorems for the $U(1)$ Goldstone boson, *Nucl. Phys.* **B156**, 269 (1979).
- [15] G. Veneziano, $U(1)$ without instantons, *Nucl. Phys.* **B159**, 213 (1979).
- [16] O. Sakhelashvili, Consistency of the dual formulation of axion solutions to the strong CP problem, *Phys. Rev. D* **105**, 085020 (2022).
- [17] A. A. Anselm and A. A. Johansen, Baryon nonconservation in standard model and Yukawa interaction, *Nucl. Phys.* **B407**, 313 (1993).
- [18] A. A. Anselm and A. A. Johansen, Can electroweak theta term be observable?, *Nucl. Phys.* **B412**, 553 (1994).
- [19] A. A. Anselm, Periodic universe and condensate of pseudoGoldstone field, *Phys. Lett. B* **260**, 39 (1991).
- [20] M. Shifman, *Advanced Topics in Quantum Field Theory* (Cambridge University Press, Cambridge, England, 2022). ISBN 978-1-108-88591-1, 978-1-108-84042-2.
- [21] I. Affleck, On constrained instantons, *Nucl. Phys.* **B191**, 429 (1981).
- [22] N. V. Krasnikov, V. A. Rubakov, and V. F. Tokarev, Zero fermion modes in models with spontaneous symmetry breaking, *J. Phys. A* **12**, L343 (1979).
- [23] L. S. Brown, R. D. Carlitz, D. B. Creamer, and C. k. Lee, Propagation functions in pseudoparticle fields, *Phys. Rev. D* **17**, 1583 (1978).
- [24] G. M. Shore, The $U(1)_A$ anomaly and QCD phenomenology, *Lect. Notes Phys.* **737**, 235 (2008).
- [25] G. Dvali, A. Kobakhidze, and O. Sakhelashvili, Hint to supersymmetry from GR vacuum, *Phys. Rev. D* **110**, 086008 (2024).
- [26] T. Eguchi and A. J. Hanson, Asymptotically flat selfdual solutions to Euclidean gravity, *Phys. Lett.* **74B**, 249 (1978).
- [27] T. Eguchi and A. J. Hanson, Selfdual solutions to Euclidean gravity, *Ann. Phys. (N.Y.)* **120**, 82 (1979).
- [28] M. F. Atiyah and I. M. Singer, The index of elliptic operators on compact manifolds, *Bull. Am. Math. Soc.* **69**, 422 (1963).
- [29] M. F. Atiyah, V. K. Patodi, and I. M. Singer, Spectral asymmetry and Riemannian geometry. I, *Math. Proc. Cambridge Philos. Soc.* **77**, 43 (1975).
- [30] S. W. Hawking and C. N. Pope, Symmetry breaking by instantons in supergravity, *Nucl. Phys.* **B146**, 381 (1978).
- [31] K. Konishi, N. Magnoli, and H. Panagopoulos, Spontaneous breaking of local supersymmetry by gravitational instantons, *Nucl. Phys.* **B309**, 201 (1988).
- [32] G. Dvali, S -matrix and anomaly of de Sitter, *Symmetry* **13**, 3 (2020).
- [33] G. Dvali and C. Gomez, Quantum compositeness of gravity: Black holes, AdS and inflation, *J. Cosmol. Astropart. Phys.* **01** (2014) 023.
- [34] G. Dvali and C. Gomez, Quantum exclusion of positive cosmological constant?, *Ann. Phys. (Berlin)* **528**, 68 (2016).
- [35] G. Dvali, C. Gomez, and S. Zell, Quantum break-time of de Sitter, *J. Cosmol. Astropart. Phys.* **06** (2017) 028.
- [36] J. D. Brown and C. Teitelboim, Neutralization of the cosmological constant by membrane creation, *Nucl. Phys.* **B297**, 787 (1988).
- [37] G. Dvali and A. Vilenkin, Cosmic attractors and gauge hierarchy, *Phys. Rev. D* **70**, 063501 (2004).
- [38] G. Dvali, Large hierarchies from attractor vacua, *Phys. Rev. D* **74**, 025018 (2006).
- [39] G. Dvali, A vacuum accumulation solution to the strong CP problem, *Phys. Rev. D* **74**, 025019 (2006).
- [40] N. Arkani-Hamed, L. Motl, A. Nicolis, and C. Vafa, The String landscape, black holes and gravity as the weakest force, *J. High Energy Phys.* **06** (2007) 060.
- [41] R. D. Pisarski and F. Rennecke, Multi-instanton contributions to anomalous quark interactions, *Phys. Rev. D* **101**, 114019 (2020).